<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *International journal of mathematical education in science and technology*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Bengmark, S., Thunberg, H., Winberg, M. (2017)

Success-factors in Transition to University Mathematics.

International journal of mathematical education in science and technology, 48(7): 988-1001

<https://doi.org/10.1080/0020739X.2017.1310311>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:umu:diva-132707>

To appear in the *International Journal of Mathematical Education in Science and Technology*, Vol. 00, No. 00, Month 20XX, 1–13

Success-factors in Transition to University Mathematics

S. Bengmark^a, H. Thunberg^b and T.M. Winberg^c

^a*Mathematical Sciences, Chalmers University of Technology, Gothenburg, Sweden*

^a*Mathematical Sciences, University of Gothenburg, Gothenburg, Sweden*

^b*Department of Mathematics, KTH Royal Institute of Technology, Stockholm, Sweden*

^c*Department of Science and Mathematics Education, Umeå University, Umeå, Sweden*

(This is an Accepted Manuscript of an article published by Taylor & Francis in *International Journal of Mathematical Education in Science and Technology* on 18 April 2017, available online:
<http://dx.doi.org/10.1080/0020739X.2017.1310311>)

This study examines the relative importance of different factors influencing students' performance in first year university mathematics. Characteristics (motivation, actions and beliefs) of students at three universities in Sweden (n=1007) were measured twice; when entering the university and at the end of the first year. Principal Component Analysis (PCA) and Orthogonal Projection to Least Squares (OPLS) analysis were used for the identification of constructs and analysis of the predictive power of the constructs, respectively. Four important constructs were found which we label *Self-efficacy*, *Motivation type*, *Study habits* and *Views of mathematics*.

When entering university, these four constructs predicted 14% of the variation in students' level of success at university math, as measured by their grades in university mathematics, while 37% could be predicted at the end of their first year at the university. When comparing the four constructs, self-efficacy is the best predictor, yet, by it self, only explaining 5% of the variation when measured in the beginning of the university studies, but 21% at the end of the first year. Upper secondary grades alone predicted 17% of the variation in first year university grades. Together, students' upper secondary grades and the four constructs predicted 21% and 43% of students' university mathematics grades, when constructs pertained to upper secondary school and university, respectively.

Study habits were found to be more important for predicting university achievement for the third of the students' with the lowest upper secondary grades. Relying on the textbook and frequent interaction with peers during the university studies, at the expense of using internet-based resources for learning, contributed positively to university achievement for this same group. The association between students' view of mathematics and their university performance was low for students with intermediate and high upper secondary grades, and non-existing for students with lower grades.

Keywords: mathematics; transition; tertiary; self-efficacy, motivation; study habits; views of mathematics; pre-knowledge; grades

1. Background

The problems connected to the transition from secondary to tertiary mathematics have been extensively studied. Entering university studies in mathematics have been shown to require a fair amount of adaptability of the novice students, both due to a partly new mathematical content and to a new learning environment. [1] This may call for changes of students' approaches to learning and views on knowledge, [2] which seem to be a crucial step for students to successfully undertake the transition into university mathematics.

Stadler [3] identified three categories for describing students' approaches to learning

mathematics during the transition from the secondary to tertiary mathematics. These are *Learning objects of mathematics*, i.e. the students' ideas about what mathematics and learning mathematics is all about, *Mathematical resources*, referring to objects, humans, or ideas that students use for learning mathematics and *Actions as learners*, i.e. how students act and how they use various mathematical resources to achieve the goals expressed in the mathematical learning objects category.

Motivation has been identified as a perceived key factor for succeeding in first year mathematics at university level, by the students themselves and by their teachers. [4] Expectancy-value-theory, [5] claims that a high degree of self-efficacy (expectations of success) and positive value beliefs will lead to improved learning and performance. This theory also postulates that students' beliefs about, and attitudes towards, university mathematics learning (i.e., activity stereotypes), and locus of control over the outcomes from learning, affects their expectations of success and value beliefs. [6] This is partially supported by the results of Liston and Donoghue [7] who found that factors like *Enjoyment of mathematics* and *Mathematics self-concept* (ability beliefs) correlated strongly with performance in first year university math courses for first year students for whom mathematics was a service subject.

On the other hand, Liston and Donoghue [7] also showed that approaches to learning (surface or deep) and conceptions of mathematics had no significant influence on the students' exam results. The latter result was interpreted as a consequence of the way the courses at university are taught and examined, i.e., that 'surface- type approaches to learning are an adequate means to success'. [7, p. 85]

Inglis, Palipana, Trenholm and Ward [8] examined the correlation between students' attainment and their choice of learning resources in a blended teaching environment; students having access to traditional lectures, online lectures and a mathematics learning support center. It was shown that students who preferred traditional lectures tended to achieve higher scores on the final exam, while the groups characterized by frequent use of online lectures had below average examination scores.

The importance of the *students' pre-knowledge*, the amount and level of mathematics studied in upper secondary school, has in several studies been shown to be positively correlated with first year tertiary achievements in mathematics. [9, 10] Upper secondary grades have in some studies been shown to have large influence on students' performance in mathematics during the first year at the university. [7, 10, 11] In a recent Irish study [12] it is shown that discriminant analysis of measures of mathematical pre-knowledge gives a very good prediction of students performance in first year university mathematics, grades from upper secondary school being the predictor variable carrying most information. But James, Montelle and Williams [11, p. 1047] also found that

the occasional student who has performed brilliantly at school will, contrary to expectations, function poorly at university. But, more importantly, when it comes to formulating eligibility requirements, there are also a significant number of students with poor or average NCEA results who shine at university.

The studies above all suggest factors that influence or predict the success in the transition to tertiary level mathematics. However, studies involving several possible predictive factors are scarce. One exception is the study by Pampaka, Williams and Hutcheson [13] which is a mixed-method study, including a quantitative measurement of students' transitional experience, similar in spirit to the study reported on in the present paper. They reported that students' perceptions of the transition to university mathematics, their degree of positivity towards the transition and their self-efficacy are good predictors for the development of positive dispositions to university studies in mathematics. However, it is not clear according to this study if this affects students' learning, "We have thus far found no strong predictors of university end of first year performance". [13, p. 1066]

2. Aim

The aim of this study is to describe the transition as a truly multidimensional phenomenon, investigating the relative importance of different factors affecting the performance in mathematics during the first year at the university. By comparing different factors, already considered important by other researchers, we want to investigate what is more urgent to attend to for those university or upper secondary school teacher who want to facilitate a successful transition to tertiary mathematics learning for their students. Our goal is to investigate the relative importance of different factors for the students' success in tertiary mathematics, both for the student group as a whole and for different groups defined by the students' grades from upper secondary school.

3. Conceptual framework

In order to capture the multi-dimensionality of the transition we put together a conceptual framework comprising factors previously considered important in research. The framework includes the following four aspects:

The first aspect concerns *student's motivation*. Self-determination theory (SDT) [14] distinguishes between *intrinsic motivation*, where the individual engages in the activity/behavior for its own sake, i.e., because it is enjoyable, and *extrinsic motivation*, where the behavior is initiated and sustained because of expectancies on the tangible outcomes of the activities (e.g. a good grade). Several forms of extrinsic motivation have been identified, differing in to which extent the motive for a behavior has been internalized by the individual, e.g., how enjoyable, valuable or important an activity is perceived by the individual. Generally speaking, low degree of internalization is associated with negative emotions such as frustration, irritation and shame while pursuit of external goals that have been internalized would lead to greater satisfaction with the situation and more positive activating [15] emotions, and higher academic performance. [16] In this study, the degree of internalization has been measured in terms of the perceived value of mathematics for learning other subjects, interest, and perceived importance for the individuals' intellectual development. Internalization was also measured in terms of to what extent the student feels driven by a need to appear intelligent in the eyes of others or follows teachers advise on appropriate study strategies rather than creating own.

The second aspect concerns *students' actions*. In this framework this refers to a broad spectrum of actions, including the students' use of learning resources such as teachers, peers, books, internet, etc. It also includes help-seeking behavior and activities both during lectures and outside of class. In this study we look both for the students' self-reported activities and their valuation of these.

Thirdly we have the *students' subject specific beliefs* about learning and knowledge that are known to be important for their reasoning ability [17] and performance [18] within the subject. In this framework we include both the *students' views on mathematics* and *learning of mathematics*, i.e., their perception of mathematics as a field and how it is learnt.

The fourth and final aspect is *students' pre-knowledge*, which in this study is measured by self-reported upper secondary grades in mathematics. Upper secondary grades reflect a broad spectrum of mathematical knowledge. Some would say that they also measure other aspects of the student, such as level of ambition, general study preparedness etcetera.

Table 1. Query themes in the questionnaires.

Questionnaire 1	Questionnaire 2
(1) Entry requirements, and level of students' mathematics pre-knowledge.	(1) Entry requirements.
(2) Lesson activities in upper secondary school.	(2) Lesson activities at university.
(3) Valuation of lesson activities in upper secondary school.	(3) Valuation of lesson activities at university.
(4) Help-seeking behaviour during mathematics lessons in upper secondary school.	(4) Help-seeking behaviour during mathematics lessons at university.
(5) Valuation of homework activities outside school.	(5) Valuation of homework activities outside university.
(6) Help-seeking behaviour during homework in upper secondary school.	(6) Help-seeking behaviour during homework at university.
(7) Valuation of resources for the learning of mathematics.	(7) Valuation of resources for the learning of mathematics.
(8) Valuation of the mathematics teacher's actions	(8) Valuation of the mathematics teacher's actions.
(9) Valuation of working with peers.	(9) Valuation of working with peers.
(10) Valuation of the textbook.	(10) Valuation of the textbook.
(11) Views on mathematics and the learning of mathematics.	(11) View on mathematics and the learning of mathematics.
(12) Expectations concerning forthcoming studies of mathematics at university.	(12) Experiences of mathematics studies at university.
(13) Expected requirements to succeed with mathematics studies at university.	(13) Experienced requirements to succeed with mathematics studies at university.

Table 2. Example question

11. Here are some questions about your views on mathematics and learning of mathematics.

	Strongly disagree			Strongly agree	
a) Its easy for me to learn mathematics.	1	2	3	4	5
b) I can solve most exercises by myself.	1	2	3	4	5
⋮			⋮		
w) I learn new concepts by solving exercises.	1	2	3	4	5

4. The instrument

An instrument was developed, consisting of two questionnaires with 13 query themes (Table 1), aiming at characterizing the students with respect to the four aspects above. Motivation was investigated using adapted items from the Learning Self-Regulation Questionnaire [19] which is specifically designed for older students, and restricted to only measure the extent to which the individual is under autonomous or controlled regulation, rather than all the different forms of extrinsic or internal motivation. To capture students' actions and their views of mathematics, we developed a number items inspired by Biggs et al [20] and Stadler [3]. Altogether, each questionnaire consists of approximately 130 items and takes about 15-25 minutes for the student to complete. The questionnaires were validated in a pilot study. [21]

The first questionnaire focus on the students' previous experiences of, and beliefs about, their mathematics studies at upper secondary level, and also their expectations on mathematics studies at tertiary level. As a measure of students' pre-knowledge, their upper secondary grades in mathematics is also asked for in this questionnaire. In the second questionnaire, almost the same questions were asked, but with a focus on their experiences of university mathematics studies. In addition, questions about gender, age, and university program are included.

Questions 3 to 13 are formulated as five-step Likert scales. Examples of questions about views on mathematics and the learning of mathematics can be seen in Table 2.

Information about students' earned credits and grades on their mathematics courses at university was gathered in order to construct measures of students' achievements in first year university mathematics.

5. The sample

In this study we have followed first-year engineering students at three universities in Sweden. The universities were chosen according to availability while the engineering programs were purposely selected to capture the typical but also the specificity of different student groups. However, in this report, we have chosen not to discriminate between students from different programs or universities.

Questionnaire 1 (Qe1) was administered during the first days at the university. At the end of the first year, the same cohort of students were approached at a regular lecture and asked to answer Questionnaire 2 (Qe2). By then, all students had studied the equivalent of at least one quarter of a year of fulltime mathematics. At two programs at one of the universities, the lectures selected for this purpose had extremely low rates of attendance. Therefore, for these programs, the students were given a second chance to answer Questionnaire 2 after their summer holidays right in the beginning of their second year at the university.

The first questionnaire was answered by 843 students and the second by 525 students. The intersection, i.e. the students that answered both questionnaires, consisted of 361 students. In total, 1007 students answered at least one of the questionnaires. Orthogonal Partial Least Squares Discriminant Analysis (OPLS-DA) was performed to investigate if students who only answered the first questionnaire differed from the students who answered both questionnaires. The analysis was made on students' scores on the constructs in Qe1, as well as which university and program they attended, age, sex, educational background, and first year university mathematics achievement. The students who answered both questionnaires showed significantly higher grades on the upper secondary school D-level mathematics course ($M = 2.42, SD = 0.71$ versus $M = 2.27, SD = 0.72$, $t(801) = -3.07, p = .002$), higher normalized average university mathematics grades ($M = 0.29, SD = 0.94$ versus $M = -0.17, SD = 0.97$, $t(684) = -6.35, p < 0.001$), and normalized proportion of accomplished math ECTS points ($M = 0.30, SD = 0.79$ versus $M = -0.18, SD = 1.04$, $t(684) = -6.79, p < 0.001$). Together these variables accounted for 6% of the dropout. Hence, there was some bias towards more high-performing students completing both questionnaires. This would be expected since low performing students tend to drop out of university to higher extent than high performing students.

For some analyses, the students were divided into three percentiles, according to their average grades on the three most advanced upper secondary mathematics courses (Mathematics C,D and E) that were required for entering the university programs. These groups were labeled *low grades*, *intermediate grades*, and *high grades*, respectively.

6. Method of analysis

6.1. Overview

First, the data from the two questionnaires were subjected to principal component analysis (PCA) using the statistical software Simca P+ [22] to identify the main constructs in the factorial structure of the questionnaire. Second, all constructs were then subjected to Orthogonal Partial Least Squares analysis (OPLS), a regression extension of PCA, to investigate the ability of each construct to individually predict students' achievement. Third, hierarchical OPLS was performed to investigate the constructs' ability to jointly predict achievement. Finally, the models in step two and three were computed for three subgroups of students, with low, intermediary, and high upper secondary school grades, respectively.

6.2. *Pre-treatment of data*

As a measure of student achievement in university mathematics, we used their *normalized university grades*. Grades were normalized within each course. For each individual, a weighted average grade was calculated, where the weights were proportional to the number of credits for the respective course.

In all multivariate analyses, data were scaled to unit variance and centered to facilitate fitting of the principal components and compensate for differences in variance between the variables/constructs. This procedure corresponds to the assumption that all variables are equally important, and is regarded as the most objective approach when there is no information about the data prior to analysis. [23] In all analyses, students with more than 50% missing data in the independent or dependent variables were excluded.

6.3. *Validation and reliability testing of models*

Cross validation [24, 25] was used for deciding on the number of principal components to retain in the analyses of construct dimensionality and in the OPLS regression models. In the cross validation procedure, one seventh of the data is systematically left out and predicted by the remaining data until all data has been left out once, and the precision of each prediction calculated. The proportion of the variation that can be predicted is denoted Q^2 , which can be considered as a measure of how well the model can predict new data that are not part of the model, while the R^2 is a measure of the proportion of the variation that can be described by the model (i.e., how well the model fits existing data). For PCA models that did not have enough predictive ability to reach default Q^2 limits in SIMCA P+ 12, decisions on whether the construct should be used in subsequent OPLS analyses were based on: 1) whether the dimensionality (i.e., correlation structure between items) was intelligible in relation to the theoretical underpinnings of the construct or not, and 2) if the items in the construct were able to predict a significant proportion of students' university mathematics achievement. In the PCA models, default limits for Q^2 means a significant reduction of prediction residuals while in OPLS models, they reflect an ability of the model to predict a significant amount of the variation in outcomes, in both cases significance is on the 5% level.

Response permutation testing was performed on the final OPLS models to assess the risk of the model being spurious, i.e., fitting the data well but predicting new observations well purely by chance. This means that the original model was compared, with respect to R^2 and Q^2 , to 100 parallel models where the outcome data had been randomly permuted while keeping the independent variables, i.e., students' responses to the questionnaires, intact. The permutation testing showed that the prediction models using results from all responding students were valid, i.e., not predicting any of the achievement measures merely by chance (Table 4).

Cross validation ANOVA, [26] testing the null hypothesis that there is no difference between the residuals of the prediction model and the variation around the average outcome, supported the reliability of all prediction models, p values being lower than .05 for all models. Furthermore, Hotellings, Distance to model (DModX) analysis and Observation risk analysis was performed to assess any undue leverage of single observations (i.e., students) on the direction of the components in the model. Deletion of the few outliers that were found did not change the correlation patterns (i.e., loadings of variables) or R^2 and Q^2 of the models. Thus, it was concluded that the models reflected general patterns, i.e., were not influenced by a few observations with large deviations from the average student' responses. Due to multicollinearity among the predictor variables, variable importance for projection values (VIP), rather than individual MLR correlation coefficients, were used for determining the relative importance of items within constructs

Table 3. Validation statistics for the constructs in questionnaire 1 (Qe1) and questionnaire 2 (Qe2), i.e., for beginner and experienced students respectively. Comp is the number of components in the model, n is the number of students included in the model, R^2X and Q^2 is the proportion (in percent) of the variation in students' responses that could be described and predicted in the cross validation procedure, respectively. Sign. Indicates if the Q^2 reached the cross-validation limits, i.e., if the model is statistically significant.

Construct	Comp	n	R^2X	Q^2	Sign.
Self-efficacy, Qe1	1	855	38.9	14.9	Yes
Self-efficacy, Qe2	1	529	39.6	14.9	Yes
Motivation, Qe1	1	847	25.4	4.3	No
Motivation, Qe2	1	529	27.6	10.0	Yes
Study habits, Qe1	5	856	33.2	18.5	Yes
Study habits, Qe2	7	454	41.5	20.9	Yes
View on mathematics, Qe1	1	849	19.0	-3.4	No
View on mathematics, Qe2	1	529	18.8	-2.3	No

for predicting students' university achievement. A variable's VIP value is the sum of the variable's squared loading weights on the predictive components of a model, weighted by the amount of variation in the predicted variable that is explained by each component, and by the number of terms in the model. The VIP value has been shown to be a good measure of a variables' relative importance when collinearity is present among the predictor variables. [27]

7. Results and discussion

7.1. Validation models

In the student questionnaire responses, four general constructs were found which we have labeled: *Self-efficacy*, *Motivation type*, *Study habits*, and *View on mathematics*. Most constructs, except *View of mathematics*, in both Qe1 and Qe2 and *Motivation* in Qe1, were significant according to cross validation in the PCA, i.e., had high internal consistency (Table 3). However, all constructs were significant in the OPLS, when individually predicting students' achievement during the first year studying mathematics (Table 4). Therefore the constructs *Views of mathematics* and *Motivation* were also retained for the hierarchical OPLS analyses of the constructs' ability to jointly predict students' study success. The construct *Study habits* showed 5 and 7 statistically significant dimensions in Qe1 and Qe2 respectively. However, only three of the dimensions were considered theoretically intelligible, showed similar loading patterns in Qe1 and Qe2, and describe meaningful amounts of the variation (i.e., $R^2 > 5\%$). The three dimension of study habits were: *Peers as resource for learning*, *Textbooks/individual work as resources for learning*, and *Use of internet for learning*.

7.2. Joint prediction models

Table 4 shows that for beginner students (Qe1), the four main constructs together with upper secondary grades predicted 21% of students' university grades. For experienced students (Qe2), the four constructs together with upper secondary grades explained 43% of students' variation in university grades. Without upper secondary grades, the constructs together explained 14% and 37% of the results for the beginner and experienced students, respectively.

7.3. *Individual constructs as predictors*

Looking at the individual constructs, we found that the students' self-efficacy after one year of university studies was a strong predictor for their achievement ($Q^2=21\%$), while self-efficacy among beginners was not ($Q^2=5\%$) (Table 4). The finding that beginners' self-efficacy is not a good predictor is at edge with what you would expect from previous research. [7] This contradicts the results of Wigfield and Eccles, [5] who found that self-efficacy was predictive of future achievement for students. We suggest that one explanation for this ambiguity is that the characteristics and demands of the past and present environment in their study were probably not so different from each other. On the other hand Liston and Donoghue, [7] just like our study, considered the transition from secondary to tertiary level, so differences in findings are harder to explain. One can speculate that the nature of national differences between studies at the secondary level, the self-efficacy generating environment, and the tertiary level, where students' achievements are measured, affects the result.

The motivation type of experienced students predicted 12% of their university achievement, compared to only 4% for beginner students (Table 4). For students with intermediate and high upper secondary grades, the motivation type developed in the end of the first university year was particularly important for predicting university achievement, predicting 19% and 12% respectively, while it played a small role for students with low upper secondary grades ($Q^2=4\%$) (Table 5). According to VIP values, the individual motivation item that was most predictive of university achievement for all three groups was 'I engage in mathematics learning because it is interesting'. Furthermore, the item 'I engage in mathematics learning because I want to appear intelligent' was also among the most important items for predicting achievement in all three groups. However, studying mathematics was only considered important for the intellectual development by successful students in the high and intermediate groups, while unimportant as predictor of achievement in the group with low upper secondary grades. Hence, successful university mathematics students, irrespective of their previous upper secondary school performance, seem to be driven by a genuine interest for mathematics, which is in line with the assumption that autonomous regulation is beneficial for learning. However, the desire to appear as intelligent, which in the original SRQ-L is considered as an indicator of controlled regulation, was also conducive to learning for all three groups. In a similar fashion, striving for good study results in order to feel pride was conducive to learning for the high' and low' groups, although being considered as indicating a controlled regulation which is theoretically associated with low performance.

In general, students' study habits explained little of the students achievement ($Q^2=2\%$ and 8% for beginners and experienced respectively) (Table 4). However, for students with low grades from upper secondary school, frequent use of literature/textbook and intense interaction with peers at the university ($Qe2$) correlated positively with higher achievement in university mathematics ($Q^2=16\%$) (Table 5). For the sub-construct Use of internet for learning, there was a weak negative correlation with achievement. In contrast, there was a weak positive correlation between achievement and the more traditional approach (using the textbook and lecture notes for reviewing theory and solution examples, and a focus on old exams and problem solving exercises in rehearsal). This is in line with the results of Inglis, Palipana, Trenholm and Ward. [8] This trend was similar in all three groups of students, but emphasized for the group of students with the lowest upper secondary grades, indicating that this group benefits most from a traditional approach to learning. Solving textbook problem was beneficial for all three groups. For the group with the low upper secondary grades, working with previous exams was among the most important variables for predicting success at the university, while this variable was decreasing in importance for the groups with intermediate and high grades - having

Table 4. Statistics for OPLS models, predicting students normalized average university mathematics grades by individual constructs and hierarchical top-models'. The table shows the number of predictive components in the model (Comp), number of students (n), and proportion of the variation in the independent variables (R^2X) that was used for predicting a certain proportion of variation in students' university mathematics grades (Q^2). R^2Y is the proportion of students' university mathematics grades that was described by each model. CV, Perm., and CV-A indicates if the models were significant according to Cross-validation, Response permutation testing, and Cross-validation ANOVA, respectively.

Models	Comp	n	R^2X	R^2Y	Q^2	CV	Perm.	CV-A
Individual prediction constructs								
Upper secondary math grades	1	663	77.6	17	17	Yes	Yes	**
Self-efficacy, Qe1 and Qe2	1	444	41.8	25.2	22.3	Yes	Yes	**
Motivation, Qe1 and Qe2	1	436	30.3	19.7	14.6	Yes	Yes	**
Study habits, Qe1 and Qe2	1	450	7.4	32.9	8.4	Yes	Yes	**
View on mathematics, Qe1 and Qe2	1	507	13.8	7.5	4.2	Yes	Yes	**
Self-efficacy, Qe1	1	685	37.5	6.0	5.1	Yes	Yes	**
Self-efficacy, Qe2	1	454	52.0	22.7	20.8	Yes	Yes	**
Motivation, Qe1	1	680	17.1	6.5	3.7	Yes	Yes	**
Motivation, Qe2	1	454	37.4	17.0	12.4	Yes	Yes	**
Study habits, Qe1	1	686	5.3	10.2	2.0	Yes	Yes	*
Study habits, Qe2	1	454	10.6	27.4	8.1	Yes	Yes	**
View on mathematics, Qe1	1	682	14.8	3.8	1.5	Yes	Yes	*
View on mathematics, Qe2	1	454	16.1	7.8	5.3	Yes	Yes	**
Hierarchical, all prediction constructs								
Qe1, incl. upper sec. grades	1	679	53.0	22.3	20.8	Yes	Yes	**
Qe2, incl. upper sec. grades	1	355	58.4	44.5	42.6	Yes	Yes	**
Qe1, excl. upper sec. grades	1	677	47.8	14.2	13.6	Yes	Yes	**
Qe2, excl. upper sec. grades	1	454	70.5	38.1	36.9	Yes	Yes	**

* $p \leq 0.05$; ** $p \leq 0.01$

almost zero importance for the latter. However, it is not clear what the casual structure is. Is for example a traditional approach beneficial for learning, or is it that already more successful students prefer a traditional approach?

Beginner students' view of mathematics showed very low association ($Q^2=1\%$) with their university achievement. For experienced students, their views of mathematics predicted 5% of their university achievement. This predictive ability was entirely accounted for by the students with intermediate and high upper secondary school grades, while there was no association between beliefs and achievement for the students with low upper secondary grades. Successful students in the intermediate' and high' group tended to believe that mathematics needs to be theoretical, that it is important to be able to derive formulas by mathematical reasoning, and that it is important to understand how different aspects of the mathematical content are interrelated. The low correlation between students' beliefs and their performance could be interpreted as there was little reward for students striving for a deeper understanding of mathematics in their learning of university mathematics. This is probably in conflict with most university teachers' goals but in sync with the result by Liston and Donoghue. [7]

Upper secondary grades in mathematics turned out to be a strong predictor of university achievement (alone predicting 17% of student university grades). This supports previous results [7, 10–12]. More precisely, there is a strong tendency that upper secondary grades set an upper limit, but not a lower limit, for student achievements at the university (Figure 1). In fact there is no single example in our study of a student with poor upper secondary grades in mathematics and top average grades in first year university math courses, so our study does not unequivocally confirm the result of James, Montell and Williams, [11] even though we also find a rather large group of students with top upper secondary grades with medium or low university grades.

Table 5. OPLS models predicting students normalized university mathematics grades, using constructs individually or together for prediction, by students' upper secondary grades (Low, Intermediate, and High, respectively). For individual constructs, only models based on questionnaire 2 are shown. For joint models (containing all constructs), results for questionnaire 1 and 2 are displayed. For each model, the table show the number of predictive components in the model (Comp), number of students (n), and proportion of the variation in the independent variables (R^2X) that was used for predicting a certain proportion of variation in students' university mathematics grades (Q^2). R^2Y is the proportion of the variation in students' university mathematics grades that was described by each model. CV., Perm., and CV-A indicates if the models were significant according to Cross-validation, Response permutation testing, and Cross-validation ANOVA, respectively.

Models	Comp	n	R^2X	R^2Y	Q^2	CV	Perm.	CV-A
Self-efficacy Qe2, low grades	1	108	30,6	23,4	16,9	Yes	Yes	**
Self-efficacy Qe2, intermediate grades	1	118	39,9	12,7	10,2	Yes	Yes	**
Self-efficacy Qe2, high grades	1	127	53,6	29,3	25,7	Yes	Yes	**
Motivation Qe2, low grades	1	108	26,2	9,7	4,1	Yes	Yes	**
Motivation Qe2, intermediate grades	1	118	45,2	28,3	18,7	Yes	Yes	**
Motivation Qe2, high grades	1	127	27,3	18,6	12,4	Yes	Yes	**
Study habits Qe2, low grades	1	108	7,1	37,5	15,7	Yes	Yes	**
Study habits Qe2, intermediate grades	1	118	16,7	42,9	-4,6	No	No	No
Study habits Qe2, high grades	1	127	8,9	36,6	7,5	Yes	Yes	**
View on mathematics Qe2, low grades	1	108	14,5	11,1	-1,8	No	No	No
View on mathematics Qe2, intermediate grades	1	118	27,6	19,6	8,4	Yes	Yes	*
View on mathematics Qe2, high grades	1	127	18,3	12,3	5	Yes	Yes	*
Hierarchical, all prediction constructs (incl. upper secondary grades)								
All constructs Qe1, Low grades	1	253	67,1	8,9	7,0	Yes	Yes	**
All constructs Qe1, Intermediate grades	1	260	44,1	9,6	6,7	Yes	Yes	**
All constructs Qe1, High grades	1	260	43,9	8,0	4,9	Yes	Yes	*
All constructs Qe2, Low grades	1	137	50,2	27,6	24,8	Yes	Yes	**
All constructs Qe2, Intermediate grades	1	215	53	13,9	12,6	Yes	Yes	**
All constructs Qe2, High grades	1	215	50,1	20,7	19,9	Yes	Yes	**

* $p \leq 0.05$; ** $p \leq 0.01$

8. Implications for education and research

We have investigated the relative importance of a spectrum of variables for predicting student's achievement in the transition from secondary to tertiary mathematics. Not surprisingly, at the time of entrance to university, i.e., using only data from Qe1, the students' average grade from mathematics courses at upper secondary school was the strongest predictor of first-year university mathematics achievement. Hence the foundation laid by the secondary mathematics education is of utter importance. Strong mathematical knowledge, as measured by grades, are necessary for success in our study, however it is not sufficient.

Neither *Self-efficacy*, nor *Motivation type*, *Study habits* or *View on mathematics*, measured at the time of entering the university, could alone predict the level of success in university mathematics to any substantial degree (Q^2 ranging from 1.5% to 5%). However, together these four constructs, pertaining to upper secondary school, had almost the same predictive power as upper secondary grades. This corroborates our proposition that the transition is a multivariate phenomenon. Further research is necessary if we want to truly understand this issue. We argue that interventions in upper secondary school, aiming at facilitating the transition to tertiary level, must take a broad approach. Just focusing on one aspect is not likely to be a successful strategy, especially considering the needs of different student groups.

We found that it was important that the students developed both a strong self-efficacy and a productive motivation type. None of these characteristics could, at the time of entering university, predict the level of success for the student. However they were strongly correlated with the level of success after one year. The relationships between university achievement on one hand, and self-efficacy and motivation on the other, probably are bi-directional. [6] Still, our results suggest that attention given to these aspects can

Figure 1. A scatter plot of the students at one of the universities in the study ($n=266$). Each dot represents a student, where the horizontal coordinate is determined by the students average grade on courses C, D and E in upper secondary school, and the vertical coordinate is determined by the students average normalized grade on his/her first year university mathematics courses. The picture is similar for the other two universities.

influence students' level of success. We argue there is need for more research on how university teachers can promote the development of productive motivation (i.e., autonomous regulation).

In general, there was a low degree of association between students' university achievement and their study habits. Study habits employed in upper secondary school predicted only 2% of students' university achievement, while study habits at the university predicted 8%. However, the association between university study habits and achievement was substantial for students with low grades from upper secondary school. For these students, the course literature and peers were important resources for learning, as measured by their grades on first year courses in mathematics. This is in line with previous research that have found that when working on mathematics problems uncertain students can get necessary help from peers which helps them stay on task. [28]

Furthermore, the benefit of using previous exams as a resource was pronounced for the same group. Being successful at the university for this group of students means passing courses with low grades (Figure 1). This suggests that the examination at the university is designed so that you can pass with a low grade by studying for the test, an option that is attractive for students with a weak background in mathematics and who are striving not to fail. For students with a stronger background in mathematics and aspiring to pass with higher grades, studying previous exams is not enough.

It would be interesting to know more about the reason for the clear association between study habits and achievement for the group with low upper secondary grades. Is it so that these students have been more dependent on their teacher during secondary schooling? Entering tertiary level means less time with the teacher, and hence the level of success are influenced by the ability to adopt and make use of other resources, such as the book and

peers. This has implications for how secondary school teachers and university teachers interact with this group of students. Research investigating the mechanisms behind this finding is needed to clarify how students with low upper secondary grades best can be supported, regarding their use of different resources for learning.

The students' views of mathematics had little effect on their level of success at university. Views of mathematics showed low correlation with achievement for students with intermediate and high upper secondary grades, while no correlation was found for students with low upper secondary grades. Liston and Donoghue [7] also found low correlations between students' views of mathematics and achievement, and concluded that a surface approach might be sufficient for studies at university level. As mentioned above, it seems that weaker students manage by studying for the test, a result that points in the same direction. This should be a matter of concern when designing courses at the university.

The importance of study habits and views of mathematics played different roles for the different groups of students. Weaker students seemed to employ less epistemologically sophisticated approaches to learning, relied on old test, and showed low association between their views of mathematics and their achievement. As a contrast, students with intermediate and high upper secondary grades to a greater extent acknowledged the necessity of theoretical understanding and independent reasoning in learning. Furthermore, the relation between motivation and achievement was strongest for students with intermediate or high upper secondary grades, while for students with low upper secondary grades, their study habits were more important for predicting success. To conclude, diversified approaches for enhancing achievement for different groups of students seem warranted. A main concern should be how to help students with a weaker background in mathematics to develop a more deep-learning oriented approach.

References

- [1] Wood L. The secondary-tertiary interface. In: D. Holton (ed.), *The teaching and learning of mathematics at university level: An ICMI study*, Dordrecht: Kluwer Academic Publishers. 2001;87–98.
- [2] Perry WG. *Forms of intellectual and ethical development in the college years: a scheme*. New York: Holt, Rinehart and Winston. 1970;
- [3] Stadler E. *Stadieövergången mellan gymnasiet och universitetet: matematik och lärande ur ett studerandeperspektiv*. (The transition secondary school to university: Mathematics education as seen from a student perspective). Acta Wexionensia nr 195/2009. Växjö: Växjö University Press. 2009;
- [4] Anthony G. Factors influencing first-year students' success in mathematics. *International Journal of Mathematical Education in Science and Technology*. 2000;31(1),3–14.
- [5] Wigfield A, Eccles JS. Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology*. 2000;25(1),68–81.
- [6] Schunk DH, Pintrich PR, Meece JL. *Motivation in education: theory, research and applications* (3 ed.). Upper Saddle River, New Jersey: Pearson Education, Inc. 2008;
- [7] Liston M, O'Donoghue J. Factors influencing the transition to university service mathematics: part I a quantitative study. *Teaching Mathematics and Its Applications*. 2009;29,53-68.
- [8] Inglis M, Palipana A, Trenholm S, Ward J. Individual differences in students' use of optional learning resources. *Journal of Computer-Aided Learning*. 2011;27(6),490–502.
- [9] Kajander A, Lovric M. Transition from secondary to tertiary mathematics: McMaster University experience. *International Journal of Mathematical Education in Science and Technology*. 2005;36(2-3),149–160.
- [10] Rylands LJ, Coady C. Performance of students with weak mathematics in first-year mathematics and science. *International Journal of Mathematical Education in Science and Technology*. 2009;40(6),741–753.

- [11] James A, Montelle C, Williams P. From lessons to lectures: NCEA mathematics results and first-year mathematics performance. *International Journal of Mathematical Education in Science and Technology*. 2008;39(8),1037–1050.
- [12] Faulkner F, Hannigan A, Fitzmaurice O. The role of prior mathematical experience in predicting mathematics performance in higher education. *International Journal of Mathematical Education in Science and Technology*. 2014;45(5):648–667.
- [13] Pampaka M, Williams J, Hutcheson G. Measuring student’s transition into the university and its association with learning outcomes. *British Educational Research Journal*. 2012;38(6),1041-1071.
- [14] Ryan RM, Deci EL. Intrinsic and extrinsic motivations: classic definitions and new directions. *Contemporary Educational Psychology*. 2000;25(1),54–67.
- [15] Pekrun R, Elliot AJ, Maier MA. Achievement goals and discrete achievement emotions: A theoretical model and prospective test. *Journal of Educational Psychology*. 2006;98(3),583–597.
- [16] Vansteenkiste M, Zhou M, Lens W, Soenens B. Experiences of autonomy and control among chinese learners: vitalizing or immobilizing? [Reports - Research]. *Journal of Educational Psychology*. 2005;97(3),468–483.
- [17] Winberg TM, Berg CAR. Students’ cognitive focus during a chemistry laboratory exercise: Effects of a computer-simulated prelab. *Journal of Research in Science Teaching*. 2007;44(8),1108–1133.
- [18] Bodin M, Winberg M. Role of beliefs and emotions in numerical problem solving in university physics education. *Physical Review Physics Education Research*. 2012;8.
- [19] Black AE, Deci EL. The effects of instructors’ autonomy support and students’ autonomous motivation on learning organic chemistry: A self-determination theory perspective. *Science Education*. 2000;84(6),740–756.
- [20] Biggs JB, Kember D, Leung DYP. The revised two factor study process questionnaire: R-SPQ-2F. *British Journal of Educational Psychology*. 2001;71,133–149.
- [21] Stadler E, Bengmark S, Thunberg H, Winberg TM. Novice mathematics students at the university: Experiences, orientations and expectations. Bergsten C, Jablonka E, Raman M. (ed.) *Evaluation and comparison of mathematical achievement: Dimensions and perspectives: Proceedings of MADIF8*. Linköping: Svensk förening för MatematikDidaktisk Forskning - SMDF (2012);
- [22] Umetrics. SIMCA-P+. [cited 2016 May 1]. Available from: <http://umetrics.com/>. 2010;
- [23] Eriksson L, Johansson E, Kettaneh-Wold N, Trygg J, Wikström C, Wold S. *Multi and megavariate data analysis. Part I: Basic principles and applications* (Second ed.). Umeå: Umetrics Inc. 2006;
- [24] Eastment H, Krzanowski W. Crossvalidatory choice of the number of components from a principal component analysis. *Technometrics*. 1982;24,73–77.
- [25] Van der Voet H. Comparing the predictive accuracy of models using a simple randomization test. *Chemometrics and intelligent laboratory systems*. 1994;25,313–323.
- [26] Eriksson L, Trygg J, Wold S. CV-ANOVA for significance testing of PLS and OPLS models. *Journal of Chemometrics*, 2008; 22(11-12), 594–600.
- [27] Palermo G, Piraino P, Zucht HD. Performance of PLS regression coefficients in selecting variables for each response of a multivariate PLS for omics-type data. *Advances and applications in bioinformatics and chemistry: AABC*. 2009;2,57–70.
- [28] Baker S, Gersten R, Lee D. A Synthesis of Empirical Research on Teaching Mathematics to Low-Achieving Students. *The Elementary School Journal*, 2002;103(1):51–73.