Design of a new type of particle separator

Kalle Sträng (kast0075@student.umu.se)

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Abstract

An electrofuge is a new concept for a particle separation technique that combines the two classical particle separation systems: The centrifugal and the electrostatic. The major problem in going from concept to a practical particle separator is how the voltage that is required to separate the particles using electrical fields in the rotational system is going to be transferred.

The electrical field required, and thereby the voltage that needs to be transferred to the rotational system, is derived in the thesis. This is done based on the particle velocities and by looking at the particle charging in an electric field created by a controlled corona discharge from a thin wire in a circular pipe.

Wireless power transfer using two planar coils that are based on resonant inductive coupling are investigated and the efficiency in the power transfer is derived. Small scale tests to verify the theory is performed.

The voltage that is required in the rotational system is in the order of 400 Volts depending on the disk stack design. The theoretical wireless power transfer system had a maximum efficiency of 93% with an optimal load. The experimental unoptimized power transfer system using a signal generator as power source had a maximum efficiency of 24% and the effects of resonant circuits are demonstrated.
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1 Introduction

Oil mist is a by product created when processing metal in industries. This mist contains small particles of oil, metals, chemicals used to make the oil cleaner and bacteria. This mist is toxic, inhaling this mist can in worst case cause cancer. The oil mist also causes problems in the factory, the mist will form a coating in the factory that needs to be cleaned constantly, otherwise the machines, electrical instruments and the ventilation-system can take damage. The work presented in this thesis is performed at the company 3nine that are developing oil-mist separators to remove the dangerous particles out of the mist.

During the history two different system have been used to separate the dangerous particles with success: The Centrifugal particle separators and the Electrostatic precipitators. The Centrifugal particle separator was invented in 1864 [1] and was used to separate the fat from milk but it took a couple of years before the separators got commercial whereas the electrostatic precipitator was invented 1908 [2]. The centrifugal system is more effective at removing larger particles and often have the possibility to be cleaned during operation and the electrostatic precipitator is more effective at removing smaller particles but can have the problem of the particles stacking up at the separation electrodes and thus decreasing the efficiency. If one was able to combine the advantages between the two systems into a electrofuge that could separate both large and small particles with the possibility of cleaning the system during operation with both an external cleaning system in place (CIP) and the centrifugal force a state of the art particle separator could be invented.

3nine is developing disk-stack centrifuges where the cleaning process can be separated into three parts: First by using the cyclone-effect for particles larger than 10 µm, then the rotating lamellas will remove the particles in the range of 1-10 µm and the particles that are smaller than 1 µm will be separated in a high efficiency particulate arresting (HEPA)-filter that is a filter that is specified for removing small particles from an air stream.

The electrostatic precipitator separates the particles with the help of electric fields created between two discharge electrodes. The particles gets charged by an electric field by powering a wire with a high voltage (∼ kV) and a low current (∼ mA) to create a corona discharge either between the discharge electrodes (a one-stage separator) or before the discharge electrodes (a two-stage separator) where the electric field used to charge the particles has to be larger than the break down voltage to successfully create a corona. [3].
The biggest problem to develop a combination of the two systems is to create the electric field between the rotational disks. One possible solution to this problem is to use two axially aligned coils that creates a magnetic field when a voltage is applied to transfer the power wireless, as demonstrated by Tesla in 1927 [4]. Today the same technique is used to charge phones without wires and systems that can transfer effects with good efficiency in the order of kW exist. [5]

In this thesis, the electric field and thus the voltage that need to be transferred into the rotational system will be calculated by combining the theory behind centrifugal separators and electrostatic precipitators. Then the theory behind wireless power transfer using resonant coupled planar coils will be presented and investigated if it is a possible solution to the problem of transfer the voltage into the rotational system.

1.1 Objective

The objective of this thesis is to find out the voltage needed to separate 0.3 \( \mu m \) particles in an electrofuge and find a concept to transfer the calculated voltage between two, each other facing, conducting surfaces of two adjacent rotating conical lamellas in a rotating system.
2 Theory

This section explains the theory used to find the electric field required to separate the particles in the electrofuge (a combined electrostatic precipitator and centrifugal particle separator). First the particle charging process by a corona field will be explained, then the particles sediment velocity (i.e. the velocity the removed particle has when it enters the disk stack) will be derived. With the derived sediment velocity the boundary particle can be found and used to calculate the required electric field. Finally how the electric field will be created will be explained. In this thesis spherical, conductive particles will be assumed.

2.1 Particle charging by corona discharge

To understand how to charge particles in particle separation applications the easiest solution is to look how it is done in electrostatic precipitators. In electrostatic precipitators the particles get charged when they pass an electric field created by the corona discharge effect [3]. To create the corona, a high voltage is applied to a wire inside a grounded pipe, see Figure 2.1. The wire will discharge against the pipe and create an electric field. The electric field produced can be calculated as

$$V_{corona} = r_0 E_{Corona} \ln \left( \frac{R}{r_0} \right)$$

where $R$ is the radius of the pipe, $r_0$ is the radius of the wire and $E_{Corona}$ is the corona onset field that is [3], [6].

$$E_{Corona} = \delta \left( 32.2 + 8.64 \cdot \frac{10^4}{(r_0 \delta)^{1/2}} \right)$$

where $\delta$ is the relative gas density relative to air and the numbers are found experimentally.

To create a corona plasma the corona voltage in equation (2.1) has to be larger than the break down voltage that is found using Paschen’s law:

$$V_B = \frac{Bpd}{\ln (Apd) - \ln \left[ \ln \left( 1 + \frac{1}{\gamma_{Se}} \right) \right]}$$

where $V_B$ is the break down voltage, $p$ is the pressure, $d$ is the gap distance, $\gamma_{Se}$ is the secondary-electron emission coefficient i.e. the number of secondary electrons
produced per incident positive ion, \( A \) is the saturation ionization constant and \( B \) is the ionization energies constant. Using this equation for air, the breakdown voltage is 43MV/m \([7]\). \( E_{\text{Corona}} \) has to be larger than that value for a corona to occur.

For particles with radius larger than 1\( \mu m \) the ion-charging process is dominant and the charge obtained by the particle in the corona field can be calculated as \([8]\)

\[
q_i(t) = \left(1 + \frac{2\epsilon_r - 1}{\epsilon_r + 2}\right) E_{\text{Corona}} r_p^2 \frac{t}{t + \tau},
\]

where \( r_p \) is the particle radius, \( t \) is the time of exposure to the ions, \( e \) is the electron charge, \( N_0 \) is the number of ions from the corona, \( \mu_i \) is the ion mobility and \( \epsilon_r \) is the electrical permittivity of the particle and

\[
q_s(t) = \left(1 + \frac{2\epsilon_r - 1}{\epsilon_r + 2}\right) E_{\text{Corona}} r_p^2,
\]

is the saturation charge (i.e. the maximum charge a particle can achieve) and the particle charge constant, \( \tau \) is defined as

\[
\tau = \left(\pi N_0 e \mu_i\right)^{-1}.
\]

For particles with a radius smaller than 1 \( \mu m \) the diffusion charging process is dominant for particle charging. The charge obtained by a particle by diffusion charging is
where $k_b$ is Boltzmann’s constant, $T$ is the temperature and $V_{rms}$ is the root-mean-square of the thermal velocity. For numerical solutions one can approximate that both charging processes occurs at the same time, that is, adding equation (2.2) and (2.3)

$$q(t) = q_i(t) + q_d(t).$$

### 2.2 Particle velocity

In an electrofuge there are three different forces acting on the particles; a centrifugal–, an electrical– and a viscous–force. All the quantities will be projections of the normal direction and thus the quantities in this section will be scalars. The centrifugal force acting on a particle perpendicular to an insert plate is using Newtons second law with the acceleration \( \ddot{a} = -\omega^2 r \hat{r} = -\omega^2 r \cos \alpha \hat{n} \)

$$F_c = \frac{4\pi r^3 \Delta \rho}{3} \omega^2 r \cos \alpha,$$

where $r_p$ is the radius of the particle, $\Delta \rho$ is the difference in the density of the air and the particle, $\omega$ is the rotational velocity of the disk-stack, $r$ is the distance to the rotational axis and $\alpha$ is the angle of incidence of the particle defined in Figure 2.2 and $\hat{n}$ is the normal unit vector.

![Figure 2.2: Cross-section view of the disk configuration.](image)

The electrical force acting on a particle in a electric field is

$$F_E = qE,$$
where $q$ is the charge of the particle obtained in equation (2.4) and $\vec{E}$ is the electric field between the disks.

A viscous breaking force will act on the sedimenting particle, using Stoke’s law

$$F_v = -6\pi \mu r_p v K_{Cu},$$

(2.7)

where $\mu$ is the viscosity of the medium the particle moves in and

$$K_{Cu} = \left[1 + 1.246 + \frac{2\lambda}{d_p} + 0.42 \frac{2\lambda}{d_p} \exp\left(-0.87 \frac{d_p}{2\lambda}\right)\right]^{-1},$$

(2.8)

is the Cunningham correction factor where $d_p$ is the particle diameter and $\lambda$ is the mean-free-path of the particle. [9] The particle is accelerated initially until the viscous breaking force is in equilibrium with the accelerating centrifugal and electrical forces, that is

$$-F_v = F_E + F_c.$$ 

Using equation (2.5), (2.6) and (2.7) the sedimenting particle velocity is

$$v = \frac{2r^2 \Delta \rho}{9\mu K_{Cu}} \omega^2 r \cos \alpha + \frac{qE}{6\pi \mu r_p K_{Cu}}.$$ 

(2.9)

### 2.3 Boundary particle

To get an estimate of the potential needed for the parallel plate capacitor stack the boundary particle need to be found. The boundary particle is the particle size which the separator can remove all the particles of that size, i.e. if a separator has a boundary particle of $r_p = 1 \mu m$ all the particles larger than 1 $\mu m$ will be separated.

To find the boundary particle the sedimenting distance can be calculated as

$$\ell = \int_0^T v \, dt,$$

(2.10)

where $v$ is the sedimenting velocity in equation (2.9). Assuming that the particle follow the airflow $Q$ between the parallel capacitor plates a relation between the particles movement in the radial direction and time can be achieved as

$$\frac{dr}{dt} = \frac{Q \sin \alpha}{2\pi r h N} \implies dt = \frac{2\pi r h N}{Q \sin \alpha} \, dr,$$

(2.11)
where $h$ is the distance between the plates and $N$ is the number of disks in the stack. Using equation (2.9) and (2.11) in equation (2.10):

$$
\ell = \int_{r_0}^{R} \left[ \frac{qE}{6\pi \mu \rho_p K_{Cu}} + \frac{2r^2 \Delta \rho \omega^2}{9\mu K_{Cu}} \cos \alpha \right] \frac{2\pi hN}{Q \sin \alpha} dr
$$

$$
= \frac{2\pi hN}{Q \sin \alpha} \left[ \frac{qE}{6\pi \mu \rho_p K_{Cu}} r + \frac{2r^2 \Delta \rho \omega^2}{9\mu K_{Cu}} r^2 \cos \alpha \right] dr
$$

$$
= \frac{2\pi hN}{Q \sin \alpha} \left[ \frac{qE}{6\pi \mu \rho_p K_{Cu}} \frac{r^2}{2} + \frac{2r^2 \Delta \rho \omega^2}{9\mu K_{Cu}} \frac{r^3}{3} \cos \alpha \right]_{r=r_0}^{r=R}
$$

$$
= \frac{2\pi hN}{Q \sin \alpha} \left[ \frac{qE}{6\pi \mu \rho_p K_{Cu}} \frac{(R^2 - r_0^2)}{2} + \frac{2r^2 \Delta \rho \omega^2}{9\mu K_{Cu}} \frac{(R^3 - r_0^3)}{3} \cos \alpha \right]. \quad (2.12)
$$

The electric field needed to sediment a particle, i.e. $\ell = h$ (that is, the particle has to sediment between the plates separated with a distance $h$) can be found using equation (2.12):

$$
E = \frac{6}{q} \left( \frac{Q\mu \rho_p K_{Cu} \sin \alpha}{N(R - r_0)^2} - \frac{4}{27} \pi r_0^3 \Delta \rho \omega^2 (R - r_0) \cos \alpha \right). \quad (2.13)
$$

### 2.4 Electric potential

To create the electric field in section 2.2 the disks in Figure 2.2 has to be electrical conductive. For the static parallel plate capacitor ignoring fringe effects and for small distances between the plates the electric field is

$$
E = \frac{\Delta V}{h}, \quad (2.14)
$$

where $\Delta V$ is the voltage difference between the upper and lower adjacent plates and $h$ is the distance between the plates.

#### 2.4.1 Induced magnetic field due to rotation

In this case, the parallel plates will rotate and the surface current will induce a magnetic field if the plates rotate fast enough. To investigate if the rotation is fast enough to induce a magnetic field one can do a Lorentz transformation of
the electric and magnetic fields of the initial system at rest to get the electric and magnetic field of the rotational system [11]

\[
\vec{E}' = \gamma \left( \vec{E} + \vec{v} \times \vec{B} \right) + (1 - \gamma) \frac{\vec{v} (\vec{v} \cdot \vec{E})}{\vec{v}^2},
\]

\[
\vec{B}' = \gamma \left( \vec{B} - \frac{\vec{v}}{c^2} \times \vec{E} \right) + (1 - \gamma) \frac{\vec{v} (\vec{v} \cdot \vec{B})}{\vec{v}^2},
\]

where \( \gamma \) is the Lorentz factor and \( \vec{v} = \vec{\omega} \times \vec{r} \). Since the velocity is much lower than the speed of light, \( c \), and thus \( \gamma \approx 1 \) the equations above reduce to

\[
\vec{E}' = \vec{E} + \vec{v} \times \vec{B},
\]

\[
\vec{B}' = \vec{B} - \frac{\vec{v}}{c^2} \times \vec{E} \approx \vec{B}.
\]

Using these fields it can be shown that Amperes and Faradays law in a rotational frame transforms to [12]

\[
\nabla \times \vec{B}' = \frac{4\pi}{c} \vec{J}' + \frac{1}{c} \frac{\partial \vec{E}'}{\partial t'} - \frac{\vec{v}}{c^2} \times \frac{\partial \vec{B}'}{\partial t'} + \nabla \times \left[ \frac{\vec{v}}{c} \times \left( \vec{E}' - \frac{\vec{v}}{c} \times \vec{B}' \right) \right]
\]

\[
\nabla \times \vec{E}' = -\frac{1}{c} \frac{\partial \vec{B}'}{\partial t'},
\]

that is known as Schiffs equations. Using the equations above one can calculate that the magnetic field in both the frames will be negligible as demonstrated by [13].
3 Wireless Power Transfer system

In this section a concept to transfer high voltage to the rotating system is proposed. The system is based on two magnetically coupled planar coils that transfer the energy using induced magnetic fields. A simple circuit will be demonstrated then the resonant frequency condition to increase the efficiency will be explained followed by the Q-factor and the circuit components. A derivation for the efficiency and the power transferred will be shown using circuit analysis and a method for designing the coils will be presented. The section will end with a diagram of a power generator circuit and an explanation of the experiments performed in this thesis. The transmitter coil will be referred to as either Tx or the index 1 and the receiver coil will be referred to as either Rx or the index 2.

3.1 Inductive power transfer

A inductive coupled wireless power transfer system can be described with the simplified circuit diagram in Figure 3.1.

![Circuit Diagram](image)

Figure 3.1: A circuit diagram for the wireless power transfer system where $u_g$ is the input voltage, $R_1$ is the resistance of the circuit including the resistance impedance of the power generator, $C_1$ and $C_2$ is the capacitors of the circuits, $L_1$ and $L_2$ is the transmitter and receiver coil and $R_L$ is the load resistance of the object that should get the power.

The voltage generated in the first circuit, $u_g$ is supplied from a power generator and the resistance $R_1$ is the resistance of the generator. The inductors with inductance $L_1$ and $L_2$ are inductively coupled to transfer power over an air gap with the mutual inductance $M$. The coupling between the coils can be described with the coupling
factor

\[ k = \frac{M}{\sqrt{L_1 L_2}}. \]  

(3.1)

The resistance \( R_L \) in the second circuit represent the load that the power is transferred to. The circuit can be explained using Kirchoff’s voltage and current law

\[
\begin{pmatrix}
Z_1 & j\omega M \\
-j\omega M & Z_2
\end{pmatrix}
\begin{pmatrix}
i_1 \\
i_2
\end{pmatrix}
=
\begin{pmatrix}
u_g \\
0
\end{pmatrix}
\]

where the impedance \( Z_1 \) and \( Z_2 \) are

\[
Z_1 = R_1 + \frac{1}{j\omega C_1} + j\omega L_1, \\
Z_2 = R_2 + \frac{1}{j\omega C_2} + j\omega L_2,
\]

and \( \omega \) is the frequency of the power generators output voltage. Here we have used the convention of circuit theory to use \( j \) for the imaginary unit. The circuit will be solved in section 3.4 with circuit analysis.

Note that the maximum voltage in the system is related to the operational frequency and the mutual inductance from Amperes law as

\[ u_g = j\omega M i. \]

3.1.1 Resonant frequency

In this theses the frequency of operation will be tuned to the frequency of the LC-tank in Figure 3.1. When resonance occurs the reactance of the inductor and the capacitor is equal, that is

\[
X_C = X_L \\
1 \omega C = \omega L \\
\omega \equiv \omega_0 = \frac{1}{\sqrt{LC}},
\]

(3.2)

(3.3)

that is, the energy in the circuit oscillates between the magnetic energy stored in the inductor and the electric energy stored in the capacitor. To maximize the power transferred between the coils, the frequency of both the circuits has to be equal:

\[
\omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}.
\]

(3.4)
3.1.2 Quality factor

The quality factor (Q) is used to describe a systems dampening effects, for a high Q-factor the system will have less dampening, that is, there is low power loss in the system. The Q-factor at resonant frequency $\omega_0$ is defined as

$$Q = \frac{\omega_0 W}{P} = \omega_0 \frac{\text{Stored energy in the system}}{\text{Power loss in the system}}.$$ 

If the circuit is a series resonant circuit, the Q-factor can be calculated as

$$Q = 1 + \frac{1}{R \sqrt{\frac{L}{C}}}.$$ (3.5)

The Q-factor is related to the band width of the frequency as

$$\text{BW} = \frac{\omega_0}{Q},$$

that is, the circuit will keep its resonant behavior in the frequency range $\omega_0 \pm \text{BW}$.

3.2 Coil Resistance

The main power loss in the WPT-system is due to conduction losses and radiation. For high frequency of the AC voltage the conduction losses dominate and the radiation losses can be neglected. The main effects giving rise to the conduction losses are the skin– and the proximity– effect [14], [15]. The AC will induce a magnetic field in the conductor, this induced magnetic field will give rise to eddy currents that cancels the current in the center of the conductor forcing the currents to the edge of the conductor (i.e. the current density is not uniformed). Taking this effect into account the AC-resistance, $R_{AC}$, of a circular conductor is

$$R_{AC} = R_{DC} \frac{w}{2\delta},$$ (3.6)

where the $R_{DC}$ is the DC resistance, $\delta$ is the skin–depth and $w$ is the width of the wire defined in Figure 3.2. The DC resistance is

$$R_{DC} = \frac{\ell}{\sigma \pi (w/2)^2},$$ (3.7)
where \( \sigma \) is the conductivity and \( \ell \) is the length of the wire that can be calculated using the variables in Figure 3.2 as \[15\]

\[
\ell = \frac{1}{2} N \pi (D_o + D_i)
\]

For low frequencies the skin depth can be computed as

\[
\delta = \sqrt{\frac{1}{\pi \mu_0 \sigma f}}
\]

(3.8)

where \( \mu_0 \) is the permeability of free space and \( f \) is the frequency of the AC sinusoidal voltage.

Figure 3.2: Cross-section view of a flat spiral coil where the parameters outer diameter \( D_o \), the inner diameter \( D_i \), the pitch \( p \) and the wire width \( w \) are defined. \[15\]

### 3.3 Inductance

If the wavelength of the current used in the system is much larger than the physical size of the system, the magnetic field produced by the current in the coil can be found using Ampère’s law

\[
\nabla \times \vec{B} = \mu_0 \vec{J},
\]

(3.9)

where \( \vec{J} \) is the current density. With the \( \vec{J} \) calculated the induction, \( L \), of the coil can be found as

\[
L_i = \frac{\mu_0}{4\pi i} \int \int \frac{\vec{J}(x_i) \cdot \vec{J}(x'_i)}{|x_i - x'_i|} \, d^3x_i \, d^3x'_i,
\]

where \( i \) is the current in the circuit \( i \) and \( x_i \) and \( x'_i \) are the positions we calculate the inductance at.
For a circular single loop of wire, the inductance can be estimated to [16], [17]

\[ L_i = \mu_0 R_i \left( \ln \left( \frac{8R_i}{r_i} \right) - 2 \right) \] (3.10)

where \( R_i\) is the radius of the circular conductor and \( r_i\) is the radius of the wire.

The mutual inductance between two thin coaxial wire loops can be calculated as [18]

\[ M = \pi \mu_0 \sqrt{R_1 R_2} \int_0^\infty J_1 \left( x \sqrt{\frac{R_1}{R_2}} \right) J_1 \left( x \sqrt{\frac{R_2}{R_1}} \right) J_0 \left( x \frac{\rho}{\sqrt{R_1 R_2}} \right) \exp \left( -x \frac{d}{\sqrt{R_1 R_2}} \right) \, dx, \]

where \( J_0 \) and \( J_1 \) are the zeroth and first order Bessel functions, \( R_1 \) and \( R_2 \) are the radius of the loop. \( \rho \) is the distance between the Cross-sections of the loops, i.e. when the coils cross-sections are perfectly align \( \rho = 0 \), see Figure 3.3, and \( d\) is the distance between the loops.

![Figure 3.3](image_url)

Figure 3.3: A figure of two coaxial loop of wires placed a distance \( d\) away from each other and where one coil is placed a distance \( \rho\) away from the common axis.

If \( \rho = 0 \) the mutual inductance can be calculated as

\[ M = \mu_0 \sqrt{R_1 R_2} \left[ \left( \frac{2}{m} - m \right) K(m) - \frac{2}{m} E(m) \right] \] (3.11)

where \( K(m) \) and \( E(m) \) is the complete elliptic integrals of the first and second kind and \( m \) is defined as

\[ m \equiv \left( \frac{4R_1 R_2}{(R_1 + R_2)^2 + d^2} \right)^{1/2}. \]
If the coils are made out of \( N \) turns of wire, the inductance and the mutual inductance between the coils using equation (3.10) and (3.11) for \( N \) turns is

\[
L_1 = \mu_0 R_1 N_1^2 \left( \ln \left( \frac{8R_1}{r_1} \right) - 2 \right),
\]

\[
L_2 = \mu_0 R_2 N_2^2 \left( \ln \left( \frac{8R_2}{r_2} \right) - 2 \right),
\]

\[
M_{12} = \mu_0 \sqrt{R_1 R_2} N_1 N_2 \left[ \left( \frac{2}{m} - m \right) K(m) - \frac{2}{m} E(m) \right],
\]

where \( N_1 \) and \( N_2 \) are the number of turns for coil 1 and coil 2 respectively.

### 3.4 Circuit analysis

The circuit in Figure 3.1 can be solved by circuit analysis by assuming that the coupled coils can be seen as a transformer with leakage inductance \( L_L \) and magnetic inductance \( L_M \). The new circuit can be seen in Figure 3.4.

![Figure 3.4: Equivalent circuit of the inductive power transfer system.](image)

The circuit can be simplified by assuming that the inductance of the coils are equal, i.e. the coils are equal. Then the mutual inductance and the leakage inductance can be written as

\[
M = K \sqrt{L_1 L_2} = KL
\]

\[
L_L = L - L_M = (1 - K)L
\]

where \( K \) is the coupling factor.
The circuit can be more simplified by looking at impedance of the circuit, the new equivalent circuit can be seen in Figure 3.5.

![Figure 3.5: Equivalent circuit of the wireless power transfer system shown in Figure 3.4.](image)

The impedance of the parameters is

\[
Z_1 = \frac{1}{j\omega C_1} + R_1 + j\omega L_1 \tag{3.15}
\]

\[
Z_2 = \frac{1}{j\omega C_2} + R_2 + j\omega L_2 \tag{3.16}
\]

\[
Z_M = j\omega M. \tag{3.17}
\]

### 3.4.1 Power transfered

This section is a reproduction of the work done in [19], however, this is a common method to solve circuits. [20]

Starting from Figure 3.5, the voltage over the impedance \(Z_M\) is

\[
u_{ZM} = \frac{u_g[Z_M \parallel (Z_2 + R_L)]}{Z_M \parallel (Z_2 + R_L) + Z_1} \]

\[
= \frac{u_g(Z_M Z_2 + Z_2 R_L)}{Z_M Z_2 + Z_M R_L + Z_M Z_1 + Z_2 Z_1 + Z_2 R_L}
\]

From this expression, the voltage over the load resistance can be found as

\[
u_{RL} = \frac{u_{ZM} R_L}{Z_2 + R_L} = \frac{u_{ZM} R_L}{Z_M Z_2 + Z_M R_L + Z_M Z_1 + Z_2 Z_1 + Z_2 R_L}.
\]
From Ohms law, the current in the secondary side is

\[ I_2 = \frac{u_{RL}}{R_L}. \]

The primary side current depends on the primary side impedance seen by the voltage source, defined as \( Z_T \) is calculated as

\[
Z_T = Z_1 + \frac{Z_M Z_2 + Z_M R_L}{Z_M + Z_2 + R_L} \\
= \frac{Z_M Z_2 + Z_M R_L + Z_1(Z_M + Z_2 + R_L)}{Z_M + Z_2 + R_L}.
\]

From the voltage source impedance the primary side current can be calculated

\[
i_1 = \frac{u_g}{Z_T} = \frac{u_g(Z_M + Z_2 + R_L)}{Z_M Z_2 + Z_M R_L + Z_1(Z_M + Z_2 + R_L)}.
\]

### 3.4.2 Efficiency

Losses in the circuit comes from the resistance of the two sides and the currents. The power loss can be found using the relationship \[19\]

\[
P_{\text{Loss}} = |i_1|^2 R_1 + |i_2|^2 R_2, \quad (3.18)
\]

that is, power loss to e.g. heat.

The output power can be found in a similar manner as

\[
P_O = |i_2|^2 R_L = i_2 u_{RL}. \quad (3.19)
\]

Using equation \[3.18\] and \[3.19\] and assuming that circuit operate at the resonant frequency, i.e. \( Z_M + Z_2 = R_2 \) the efficiency of the power transfer can be found as

\[
\eta = \frac{P_O}{P_{\text{Loss}} + P_O} = \frac{1}{1 + \frac{R_2}{R_L} + \frac{R_1|R_2 + R_L|^2}{R_L Z_M}}. \quad (3.20)
\]

Looking at equation \[3.20\] one can note that \( \eta \) maximizes when the denominator is minimized, thus one can find a optimal load resistance by differentiating w.r.t. \( R_L \).
\[
\frac{dF}{dR_L} = \frac{d}{dR_L} \left( \frac{R_2}{R_L} + \frac{R_1|R_2 + R_L|^2}{R_L|Z_M|^2} \right).
\]

When calculating the derivative the optimal load is found to be

\[
R_{L_{opt}} = \sqrt{\frac{R_2(R_1R_2 + |Z_M|^2)}{R_1}}.
\] (3.21)

### 3.5 Designing the coils

From Figure 3.2 a relation between the coils inner diameter, \(D_i\), the outer diameter, \(D_o\), the number of turns, \(N\), the pitch (i.e. the distance between the turns \(N\)), \(p\), and the wire width, \(w\) can be found as

\[
D_i = D_o - 2N(w + p).
\] (3.22)

Using equation (3.22) and assuming that the \(p > 2.5w\) (that is, the coils will be planar and the turns would not touch each other) a number of possible coils can be generated for a fixed outer diameter. The objective is to maximize the efficiency \(\eta\) in equation (3.20).

To find the highest efficiency for a fixed outer diameter all possible combinations of \(N, D_i\) and \(p\) are tested for the constraint in equation (3.22) using Matlab. The inductance, AC resistance, the capacitance and the quality factor is calculated using equation (3.12), (3.6), (3.3) and equation (3.5) respectively. This procedure are repeated for both the transmitter (Tx) coil and receiver (Rx) coil.

When all the coils are created, the mutual inductance for each combination of Tx and Rx coils are calculated using equation (3.14), the coupling coefficient is calculated using equation (3.1) and the efficiency is calculated using equation (3.20).

### 3.6 Full wireless power transfer system

To get a functional wireless power transfer system the 230V 50Hz voltage need to be changed to the desired voltage output and frequency. A simple diagram over a power generator circuit can be seen in Figure 3.6.
Figure 3.6: A circuit diagram of a power generator circuit that changes the 230V 50Hz AC to DC in the rectifier (diode H-bridge) where the capacitor smooths out the wave. The mosfets then changes the DC into high frequency AC to operate the wireless power transfer system.

The 230V power source will get converted to a DC in the rectifier (the diode H-bridge). The capacitor after the rectifier is used to smoothen out the wave from the rectifier. The DC-voltage will then be converted to the desired frequency in the power inverter that consists of 4 field electric transistors (e.g. MOSFETS). The high frequency voltage is then increased or decreased to the desired voltage value in the transformer. This entire circuit represent the voltage source $u_g$ in Figure 3.1.

### 3.7 Experiment

To verify the theory explained above the coils derived in section 3.5 were created. The inductance of the coils was measured using a capacitor connected in series with the inductor then the maximum output voltage was found using a probe connected to an oscilloscope.

The coils were connected to the circuit shown in Figure 3.1 with $C_1 = C_2$ and $R_L = 2\Omega$. The resistances $R_1$ and $R_2$ was measured over the coils.

The coils were mounted on a plate that can be moved in both the $d$ and $\rho$ directions, see Figure 3.7 with their cross section facing each other, i.e. they had a common axis through origo ($\rho = 0$) at a distance $d$ as close to each other as
possible. The voltage in the two circuits was measured using two probes connected to an oscilloscope and the current was measured using a digital multimeter. The distance $d$ was then changed to 1 cm using the wheel on the movable plate to get the correct distance and the same measurements was repeated. This procedure was repeated for the distances $d = 2$ cm and 3 cm.

The coils were moved back to $d = 1$ cm and this time the coil were moved in horizontal direction, i.e. changed $\rho$ to 1 cm with $d = 1$ cm. The same measurement procedure as explained above was preformed. Then the rx coil was moved to $\rho = 2$ cm and the same measurements was performed.

All these tests was performed for two different capacitors, in the first test the capacitances was $C_1 = C_2 = 0.1 \mu F$ to make the circuit resonant at the operational frequency and for the other test the capacitances was $C_1 = C_2 = 64 nF$. 

Figure 3.7: A picture of the experimental setup where the Tx coil to the right in figure (a) is connected to the left side of the circuit diagram in Figure 3.1 and the Rx coil (to the left in the picture) is connected to the left side of the circuit diagram. Figure (b) demonstrates a different distance $d$ between the coils.
4 Results

In this section the results obtained during the thesis will be presented. The theoretical results will be presented first followed by the experimental results for the wireless power transfer.

4.1 Theoretical results

Assuming that the particles are spherical and conductive, the number of elementary charges an oil-mist particle obtain in a Corona-field due to ion charging for $N_0 = 5 \cdot 10^{12} \text{ m}^{-3}$, $\mu_i = 1.6 \cdot 10^{-4} \text{ m}^2/\text{Vs}$ and a corona field larger than the break down voltage [21], [22] in equation (2.2) can be seen in Figure 4.1.

![Figure 4.1: The number of elementary charges a oil mist particle obtained in a corona-field vs the particle radius.](image)

The number of elementary charges a oil-mist particle obtain due to diffusion charging at room temperature using equation (2.3) can be seen in Figure 4.2.
Figure 4.2: The number of elementary charges a oil mist particle obtain due to diffusion charging vs the particle radius.

The total charge number of elementary charges the particle obtain in the corona field when taking both charging effects into account can be seen in Figure 4.3.

Figure 4.3: The total number of elementary charges on a oil mist particle vs the particle radius.

Using the charge obtained above the electric field required can be found using equation (2.13) and can be seen in Figure 4.4 using $Q = 1000 \text{ m}^3/\text{h}$, $\Delta \rho = 1000 \text{ kg/m}^3$, $\omega = 45 \text{ m/s}$, $\mu = 18 \cdot 10^{-6} \text{ kg/m} \cdot \text{s}$ and $R = 0.15 \text{ m}$. 
The electric field required to separate a particle with radius 0.3µm for three different incline angles can be seen in Table 1.

Table 1: The electric field required to separate a particle with a radius of 0.3µm for three different incline angles.

<table>
<thead>
<tr>
<th>Incline angle α [°]</th>
<th>30</th>
<th>45</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(r_p = 0.3µm) [kV/m]</td>
<td>175</td>
<td>249</td>
<td>307</td>
</tr>
</tbody>
</table>

The optimal tx and rx coil using the methodology described in section 3.5 for a outer diameter of D_0 = 5cm can be seen in Table 2.

Table 2: The coil pair that results in the highest efficiency using the algorithm described in section 3.5 for f = 276kHz and D_0 = 5 cm.

<table>
<thead>
<tr>
<th>Coil</th>
<th>N</th>
<th>p</th>
<th>w</th>
<th>L</th>
<th>C</th>
<th>R</th>
<th>Q</th>
<th>D_i</th>
<th>D_o</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx</td>
<td>7</td>
<td>0.0035 m</td>
<td>0.001 m</td>
<td>5.077 µH</td>
<td>65.4 nF</td>
<td>0.024 Ω</td>
<td>366.79</td>
<td>0.001m</td>
<td>0.05m</td>
</tr>
<tr>
<td>Rx</td>
<td>7</td>
<td>0.0035 m</td>
<td>0.001 m</td>
<td>5.077 µH</td>
<td>65.4 nF</td>
<td>0.024 Ω</td>
<td>366.79</td>
<td>0.001m</td>
<td>0.05m</td>
</tr>
</tbody>
</table>

The efficiency calculated using (3.20) for different load resistances can be seen in Figure 4.5 for M = 2.26µH and k = 0.446.
Figure 4.5: The theoretical efficiency vs the optimal load.

As seen in the figure, the optimal load agrees with the optimal load calculated in equation 3.21 and is found to be $R_{\text{opt}} = 0.624 \, \Omega$ and results in a maximum efficiency of $\eta = 93.24\%$ with a distance of 1 cm between the coils.

The efficiency, the normalized load voltage $V_{RL}/V_{in}$ and the normalized output power $P_o/P_{o,max}$ versus the optimal load can be seen in Figure 4.6.

Figure 4.6: The efficiency, normalized load voltage and the normalized output power vs the optimal load.
4.2 Experimental results

To measure the inductance the resistor was replaced with a capacitor. The circuit was connected to an oscilloscope and the resonant frequency was found using a signal generator and the inductance was found using equation 3.3 and the result can be seen in Table 3.

Table 3: The measured inductance of the Rx and Tx coils using a capacitor and a signal generator, the frequency listed is the resonance frequency.

<table>
<thead>
<tr>
<th>C [μF]</th>
<th>0.15</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{Tx} ) [kHz]</td>
<td>192</td>
<td>276</td>
</tr>
<tr>
<td>( f_{Rx} ) [kHz]</td>
<td>208</td>
<td>276</td>
</tr>
<tr>
<td>( L_{Tx} ) [μH]</td>
<td>154</td>
<td>128</td>
</tr>
<tr>
<td>( L_{Rx} ) [μH]</td>
<td>180</td>
<td>128</td>
</tr>
</tbody>
</table>

The input voltage and the output voltage for the Tx and Rx coils in Table 2 for different distances \( d \) defined in Figure 3.3 when \( \rho \) is fixed at 0 cm can be seen in Figure 4.7.
Figure 4.7: The Tx coil voltage vs the Rx voltage for three different distances between the coaxial planar coils. In the upper Figure $C = 64 \text{nF}$ and in the lower Figure $C = 0.10 \mu\text{F}$.

The input voltage and the output voltage for the Tx and Rx coils in Table 2 for different distances $\rho$ for the different capacitors can be seen in Figure 4.8 and Figure 4.9.
Figure 4.8: The Tx coil voltage vs the Rx voltage for different $\rho$ for the 0.1\u03b5F capacitor

Figure 4.9: The Tx coil voltage vs the Rx voltage for different $\rho$ for the 64\u03b5F capacitor

All the measured voltages and the currents for the 64\u03b5F capacitors have been summarized in Table 4 combined with the efficiency $\eta$. 
Table 4: The input power $P_1$ versus the output power over a resistance load $R_L = 2\Omega$ in series with a capacitor $C = 64\text{nF}$, $R_1 = 8\Omega$, $R_2 = 0.4\Omega$ and the tx and rx coils at 276 kHz.

<table>
<thead>
<tr>
<th>$d - \rho$ [cm]</th>
<th>$i_1$ [mA]</th>
<th>$v_1$ [mV]</th>
<th>$P_1$ [mW]</th>
<th>$i_2$ [mA]</th>
<th>$v_L$ [mV]</th>
<th>$P_L$ [mW]</th>
<th>$P_L/P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 — 0</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>21.1</td>
<td>212.6</td>
<td>4.48</td>
<td>0.10</td>
</tr>
<tr>
<td>1 — 0</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>14.5</td>
<td>157.5</td>
<td>2.28</td>
<td>0.0558</td>
</tr>
<tr>
<td>2 — 0</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>10.37</td>
<td>126</td>
<td>1.3</td>
<td>0.0319</td>
</tr>
<tr>
<td>3 — 0</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>9.13</td>
<td>70.87</td>
<td>0.29</td>
<td>0.0071</td>
</tr>
<tr>
<td>1 — 1</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>17.28</td>
<td>173</td>
<td>3</td>
<td>0.067</td>
</tr>
<tr>
<td>1 — 2</td>
<td>64.8</td>
<td>692</td>
<td>45</td>
<td>11.19</td>
<td>126</td>
<td>1.41</td>
<td>0.0313</td>
</tr>
</tbody>
</table>

All the measured voltages and currents for the $0.1\mu F$ capacitor have been summarized in Table 5.

Table 5: The input power $P_1$ versus the output power over a resistance load $R_L = 2\Omega$ in series with a capacitor $C = 0.1\mu F$, $R_1 = 8\Omega$, $R_2 = 0.4\Omega$ and the tx and rx coil at 276 kHz.

<table>
<thead>
<tr>
<th>$d - \rho$ [cm]</th>
<th>$i_1$ [mA]</th>
<th>$v_1$ [mV]</th>
<th>$P_1$ [mW]</th>
<th>$i_2$ [mA]</th>
<th>$v_L$ [mV]</th>
<th>$P_L$ [mW]</th>
<th>$P_L/P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 — 0</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>20.45</td>
<td>244</td>
<td>4.99</td>
<td>0.24</td>
</tr>
<tr>
<td>1 — 0</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>13.1</td>
<td>165</td>
<td>2.1667</td>
<td>0.10</td>
</tr>
<tr>
<td>2 — 0</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>5.4</td>
<td>78</td>
<td>0.43</td>
<td>0.021</td>
</tr>
<tr>
<td>3 — 0</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>1.2</td>
<td>39.37</td>
<td>0.0472</td>
<td>0.0023</td>
</tr>
<tr>
<td>1 — 1</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>12.6</td>
<td>165.4</td>
<td>2.08</td>
<td>0.104</td>
</tr>
<tr>
<td>2 — 1</td>
<td>65.9</td>
<td>307</td>
<td>20.23</td>
<td>7.97</td>
<td>110</td>
<td>0.878</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The normalized efficiency from Table 4 and 5 versus the normalized theoretical efficiency for different distances $d$ can be seen in Figure 4.10.
Figure 4.10: The Theoretical efficiency scaled with its maximum value $\eta_{\text{Theory}}/\eta_{\text{Theory,max}}$ versus the scaled measured efficiencies at different distances using their maximum values where the values are defined in Table 4 and 5.

In Figure 4.10 the resistances $R_L = 2 \ \Omega$, $R_1 = 9 \ \Omega$ and $R_2 = 0.4 \ \Omega$ have been used.
5 Discussion

The main objective of this thesis have been to find a concept to transfer high voltage to supply power to a rotational parallel plate capacitor system. To get an estimate for what voltages that is required the electric field that is needed have been derived, then a method for transfer of the voltage has been proposed and experiments for low voltage have been deducted to verify the theory behind the proposed voltage transfer system.

5.1 The electric field required

The assumption that the oil mist particles are conductive can be validated by referring to the experiment performed by Millikan that demonstrated that the oil particles obtain a charge that is an integer value of the elementary charge \[23\]. However tests could be performed to verify the number of charges on a particle by a similar experiment. If the particle separator are going to be used on a different application an investigation is needed about if the ozone created by the corona discharge will chemically interact and create problems.

The theoretical electrical field required to separate particles with a radius of \(0.3\mu m\) has been presented and according to the theory, the disks should have an incline angle of \(\alpha = 30^\circ\) to have the best effect in separating particles, with this \(\alpha\) the electric field required is \(E_{\alpha=30^\circ} = 175\) kV/m that is much lower than the electrical break down voltage. This value is lower than the electric fields used in electrostatic precipitators \[3\] as expected. We see that \(E_{\text{required}}\) goes to zero as the particle radius increase, this is also expected as this means that the centrifugal separation is dominant for larger particles.

5.2 Wireless power transfer

Looking at the experimental result the values efficiency was far away from the theoretical for the different distances. However if one normalized the values the experimental values followed the same pattern as the theoretical. The experimental values might have been improved if a power generator would have been used instead of a signal generator due to the lower impedance of the power generator. Then the coil parameters can be optimized using Roberts equation for impedance

The inductance measured for the coils did not agree with the theory either. The fact that the inductance was not the theoretical value combined with the fact that the impedance of the signal generator was ignored in the calculations can be the reason for the low efficiency in the practical power transfer system. One of the major problems with the coils was that the script had a lower limit of the inner diameter equal to the wire width which is difficult to manufacture without the proper machines.

The proposed wireless power transfer system have been able to transfer voltages of about 600 V [5] for larger coils, thus if the distance between the parallel plates is of the order \(\text{mm}\) the proposed system would suffice if the coil size is increased. An investigation about the I-V characteristics could be performed on the disk stack to be sure about how much voltage that is required.

Looking at the efficiency for the two different circuits tested we can see that for the 0.1\(\mu\)F the efficiency is higher but the voltage in the primary system is lower and vice versa for the 64\(n\)F capacitor. The higher efficiency for the 0.1\(\mu\)F system has to do with that the system operates close to or at its resonant frequency. If both coil had been identical the efficiency would most likely be higher. One can also note that the load voltage is higher for the 0.1\(\mu\)F system but the current in the rx sides are the same.

Looking at the output voltages in Figure 4.7 one can note that the input and output voltage is out of phase with a phase of \(\approx \pi/2\) with the 64\(n\)F capacitor as expected since the system did not operate at its resonant frequency. However looking at the 10\(\mu\)F capacitor one can see that the input and output voltage are out of phase as well. this means that the impedance of the tx circuit is not equal to the impedance of the rx circuit and thus the system is not operating at its resonant frequency as expected.
6 Conclusions

The theoretical electrical field required to separate particles with a radius of 0.3 µm has been presented and according to the theory, the disks should have an incline angle of $\alpha = 30^\circ$ to have the best effect in separating particles, with this $\alpha$ the electric field required is $E_{\alpha=30^\circ} = 175 \text{ kV/m}$ for a separator with similar dimensions as 3nines centrifugal separators and the voltage required is approximately 175-400 Volt, depending on the distance between the stacks. The electric field required decrease as the particle radius $r_p$ increase as expected.

The tests for the wireless power transfer system showed that the resonant frequency gives higher efficiency as expected. For further testings for the wireless power transfer system a power generator circuit as described in section 3.6 to decrease the impedance in the primary circuit and to increase the input voltage and tests with rotations has to be performed. Then the maximum output voltage for the small coils used in this thesis can be determined and it can be seen if larger coils are needed.

The theoretical model using optimized parameters and a optimal load resistance gives a maximum efficiency of $\eta = 93\%$ when the circuit is operating at its resonant frequency.
References


