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Greenhouse Gas Footprint Minimization of Credit Default Swap Baskets

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Abstract

Global bond market capitalization amounts to approximately $100 trillion, compared to $60 trillion in the equity markets. Despite debt financing being a large part of the global financial market, the measurements and greenhouse gas reduction investment strategies to date are not nearly as thorough as for equity financing. More recently, the problem has been brought into light by the World Bank, expressing concerns about the crucial role of debt financing activities in the current and upcoming threats caused by climate change.

A commonly used credit derivative in debt financing is credit default swaps (CDS), which is an agreement between two parties to exchange the credit risk of a reference entity. The buyer of the contract makes fixed periodic payments to the seller of the contract, who collects the premiums in exchange for making the protection buyer whole in the case of a defaulting reference entity.

This thesis aims to minimize the greenhouse gas emission exposure for two CDS indices, iTraxx Main and CDX.IG, each consisting of 125 equally weighted constituents, or companies. The CDS indices are widely used high liquid fixed income instruments. In 2017, iTraxx Main had a monthly trading volume of $330-440 billion notional, and CDX.IG a corresponding volume of $200-275 billion. In order to rate the greenhouse gas emissions of the constituents, the ECOBAR model was used. The model utilizes a discrete ranking score system, where the aim is to obtain as low score as possible. To minimize the ECOBAR score for the baskets, Markowitz Modern Portfolio Theory was used, implemented by using a quadratic programming algorithm. By optimizing the portfolios while retaining a low tracking error and high correlation toward the CDS indices, underlying investment properties were retained.

We show that one can construct replicated portfolios of the CDS indices that have significantly lower ECOBAR scores than the indices themselves, whilst still maintaining a low tracking error and high correlation with the actual indices. When constructing baskets of fewer constituents, one can replicate the indices with merely 10-30 constituents, without worsening the tracking error or correlation substantially, and obtain an even lower ECOBAR score for the respective portfolios.

Sammanfattning

Det globala marknadsvärdet för obligationer uppgår till cirka 100 biljoner USD, i jämförelse med 60 biljoner USD för den globala kapitalmarknaden. Trots att lånebaserad finansiering är en stor del av den globala finansiella marknaden, är modellerna för att mäta och minska utsläppsexponeringen inte alls lika välutformade som för investeringar som görs med eget kapital. Detta uppmärksammades nyligen av Världsbanken som uttryckt oro kring den avgörande roll som lånebaserad finansiering har för såväl pågående som kommande hot orsakade av klimatförändringar.

Ett vanligt förekommande kreditinstrument som används inom lånebaserad finansiering är credit default swaps (CDS). Kontraktet är en överenskommelse mellan två parter för att överföra kreditrisk gällande ett specifikt institut från den ena parten till den andra. Köparen av kontraktet gör periodiska utbetalningar till säljaren av kontraktet, som tar emot premierna i utbyte mot att betala ut en förutbestämd summa till köparen om institutet går i konkurs.

Uppsatsen syftar till att minimera utsläppsexponeringen av växthusgaser för två kända CDS-index, iTraxx Main och CDX.IG, vardera bestående av 125 likaviktade konstituenter, eller företag. Under 2017 hade iTraxx Main en månatlig handelsvolym på 330-440 miljarder USD nominellt och CDX.IG motsvarande 200-275 miljarder USD. För att kunna ranka utsläppen hos konstituenterna användes ECOBAR-modellen, som baseras på ett diskret bedömningssystem, där målet är att uppnå ett så lågt värde som möjligt. Vid minimering av ECOBAR-värdet hos CDS-indexen användes Markowitz Moderna Portföljteori, implementerad genom en kvadratisk programmeringsalgoritm. Genom att optimera portföljerna och samtidigt vidhålla ett lågt tracking error och hög korrelation gentemot de replikerade indexen, bibehölls de underliggande investeringsegenskaperna.

Vi visar att man kan konstruera replikerade portföljer av CDS-indexen som har ett markant lägre ECOBAR-värde än själva indexen, medan man vidhåller ett lågt tracking error och en hög korrelation med de faktiska indexen. Vid konstruktionen av CDS-korgar med färre konstituenter kan man replikera indexen med endast 10-30 tillgångar, utan att försämra tracking error eller korrelation väsentligt, och dessutom nå ett ännu lägre ECOBAR-värde.

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Glossary

Basket	A basket is a group of assets or securities created for the purpose of simultaneous buying or selling. May also be referred to as a portfolio.
Constituent	A constituent is a company or corporate group included in a credit default swap index. The constituent plays the reference entity role in the CDS contract. May also be referred to as a portfolio asset.
Long Position/Risk	A long position means that a speculator has purchased an asset, believing that it will increase in value. Long risk equals buy risk, which is the same as selling protection, often expressed "sell CDS". This position makes profit when the spread decreases and becomes "tigther".
Short Position/Risk	A short position means that a speculator has lent and sold an asset, expecting a decrease in asset value. Short risk equals sell risk, which is the same as buying protection, often expressed "buy CDS". This position makes profit when the spread increases and becomes "wider".
Spread	The spread, or bid-ask spread is the difference between the quoted prices by a market maker for an immediate sale (ask) and an immediate purchase (bid) of a financial security or contract. The size of the bid-offer spread is a measure of the market liquidity, and also the size of the transaction cost.
Volatility	Volatility, another term for standard deviation, typically describes the degree of variation for price observations over time. Volatility is one of the most commonly used risk measures for financial securities.

Abbreviations

bp	Basis Point
cdf	Cumulative Distribution Function
CDS	Credit Default Swap
DV01	Dollar Value of a Basis Point
ESG	Environmental, Social and Governance
GHG	Greenhouse Gas
KKT	Karush–Kuhn–Tucker Conditions
LGD	Loss Given Default
MMV	Markowitz Mean Variance
NPV	Net Present Value
OTC	Over-The-Counter
P&L	Profit & Loss
pdf	Probability Density Function
PV	Present Value
ZCB	Zero Coupon Bond

1 Introduction

This chapter serves as an introduction to the thesis and describes in what context it exists as well as the specific problem that is addressed. The financial derivatives to be included in the analysis will be introduced, along with environmental measurements within finance. Finally, the outline of the thesis structure is presented.

1.1 Background

At the One Planet Summit in Paris in December 2017, the World Bank announced that they "will no longer finance upstream oil and gas". The reasons for this announcement are the current and upcoming threats caused by climate change. The World Bank's goal is to have climate action lending adding up to 28% of their total lending by 2020. Gyorgy Dallos at Greenpeace International commented "The world's financial institutions now need to take note and decide whether their financing is going to be part of the problem or the solution" [1]. Some companies have already showed that they want to be part of the solution by issuing green bonds, i.e. bonds that finance green projects, which has increased significantly over the last years. From merely $3 billion green bonds issued in 2012 [2], the total summation of issued green bonds in 2017 adds up to $156.7 billion, and the estimate for 2018 is close to $250 billion [3].

Britain's six leading banks, amongst others, are all supporting the Task Force on Climate-Related Financial Disclosures, which aims to make companies disclose their direct and indirect exposures to global warming. Also, banks have to disclose how high lending exposure they have to companies with climate-related risks. Carney, chairman of the Financial Stability board, stated that 20 of the 30 systemically important banks globally as well as a significant amount of the largest insurance companies, asset managers, transport, consumer goods and energy companies are now committed to inform investors about their exposures to global warming [1].

The announcement from the World Bank is a major breakthrough in the debt financing world, as expressed by Stephen Kretzmann, executive director at Oil Change International: "It is hard to overstate the significance of this historic announcement by the World Bank" [1]. Global bond market capitalization amounts to approximately $100 trillion compared to $60 trillion in the equity markets [4]. Despite debt financing being a large part of the global financial market, the measurements and greenhouse gas reduction strategies are not nearly as thorough as for eq-

uity financing.

With this statement from the World Bank, it is clear that debt financing has a large impact on the environment. There are new, important findings to be made within this area of finance, thus leading to the topic of this thesis.

1.2 Objectives and Scope

The thesis will look to develop and evaluate ways to reduce the implicit carbon footprint for baskets of credit default swaps, which will be constructed from two commonly traded CDS indices.

More specifically, the study will include a long only format, i.e. the equivalent derivative position of a traditional bond buyer. The study will also utilize a long-short perspective, to go long risk in greenhouse gas (GHG) effective companies (implicitly providing financing), versus short risk in GHG ineffective companies (implicitly withdrawing financing).

Moreover, the study will investigate how reducing the size of the portfolios significantly, allowing to create more effective carbon reduction strategies, affects the results.

The ambition is to reduce the implicit carbon footprint of the portfolios, while retaining a low tracking error and high correlation with the actual CDS indices, to retain the underlying investment properties of the CDS indices.

1.3 Limitations

Due to the fact that GHG emission data is historically inadequate and is not yet covered on a large scale, the study is limited to two major CDS indices, iTraxx Main and CDX.IG, to be analyzed during recent years.

The included GHG data in the study measures scope 1 and scope 2 emissions, see section 1.6.1. Scope 3 emissions are significantly harder to measure, which could potentially give a misrepresented view of the reality and were therefore left out of this study. Moreover, the study do not assess the methods used to measure the GHG data, which is a limitation of this thesis.

1.4 Strukturinvest Fondkommission

The thesis project was conducted at Strukturinvest Fondkommission AB in Stockholm. The company was founded in 2009 and develops structured investment products for private investors, companies and institutions. Strukturinvest is a standalone Swedish security paper company under supervision by the Swedish Financial Supervisory Authority. The company has no placements themselves, thus only trading for their customers exclusively [5].

1.4.1 Glacier Impact Climate Fund

The thesis project was set up by Ulf Erlandsson, Chief Investment Officer at Strukturinvest. The findings of this thesis will be used to create sufficient trading strategies, possibly to be used in the financial management of the Glacier Impact Climate Fund. The hedge fund is a climate total return strategy managed by Erlandsson, to be launched in 2018 with a target of $100 million in assets under management [6].

1.5 Credit Default Derivatives

This section introduces the financial derivatives which will be included in the conducted analysis of this thesis. The major risks of the contracts, the structure of the contracts and how they are used will be explained.

1.5.1 Counterparty Credit Risk

Financial risk is normally divided into subcategories which corresponds to different types of risks. An important part is the counterparty credit risk, also known as counterparty risk. Counterparty risk can be further divided into two subcategories of risk exposures [7]:

- **Credit Risk** is the risk that a debtor is unable or unwilling to conduct a payment in order to fulfill its contractual obligations. Generally this is known as a default.

- **Market Risk** is the risk of losses in financial positions or contracts arising from movements in market prices. It can arise from movements in underlying variables such as credit spreads or stock prices. Market risk can be reduced or eliminated by entering into an offsetting contract, which is also known as hedging.

Counterparty risk has a major importance for over-the-counter (OTC) traded derivatives. The counterparty risk is mitigated in otherwise exchange traded derivatives [8].

1.5.2 Over-The-Counter

The OTC market is the off-exchange market. The major difference from the exchange market is that the trading participants are directly exposed to the risk of a default of the counterparty. For trades on the exchange market, a clearing house acts as a middle part and eliminates the credit risk for the trading participants. The clearing house is compensated by collateral from the trading participants, a fee depending on the risk aspects of the trade.

1.5.3 Credit Default Swap

A Credit Default Swap (CDS) contract is an agreement between two parties to exchange the credit risk of a reference entity. The buyer of the contract makes fixed periodic payments to the seller of the contract, who collects the premiums in exchange for making the protection buyer whole in the case of a defaulting reference entity. When the contract expires, the seller has received regular payments and does not pay anything back. CDS are OTC transactions, thus resulting in full credit risk exposure for the involved participants [9]. CDS contracts are similar to buying or selling insurance contracts on a corporation or sovereign entity's debt. However unlike insurance, it is not necessary to own the underlying debt to buy protection using CDS [10]. For a graphic illustration of the contract, see Figure 1.

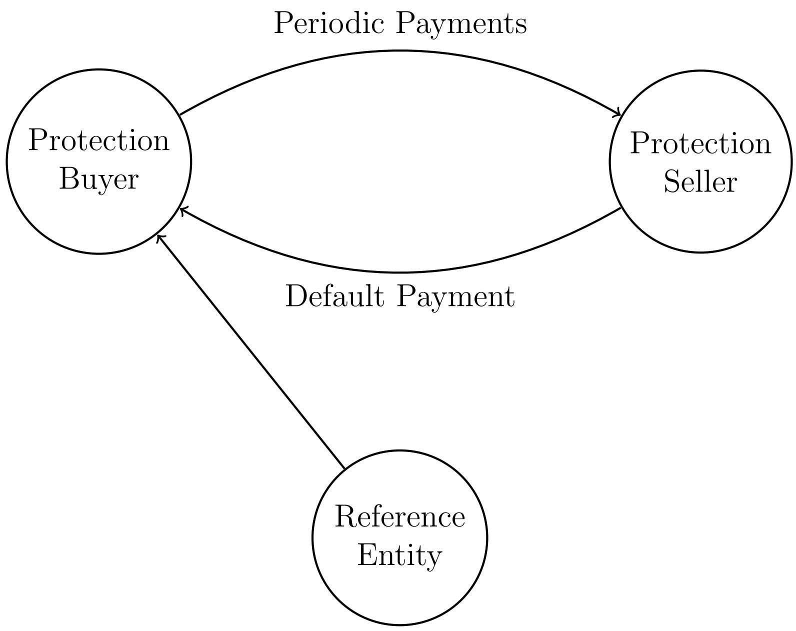

Figure 1: The CDS contract consists of three participants: the protection buyer, the protection seller and a reference entity.

Credit Event

The trigger of a CDS contract is called a credit event. These are the most common credit events [9]:

- **Bankruptcy** includes insolvency, appointment of administrators and creditor arrangements.
- **Failure to pay** is when a payment on one or more obligations fails after any grace period.
- **Restructuring** is a change in the agreement between the reference entity and the obligation holder due to the downturn in creditworthiness or financial condition to the reference entity. In respect to reduction of principal or interest, postponement of payment of principal or interest, change of currency or contractual subordination.

Settlement

Assume that an investor is exposed to an entity through a bond and needs to hedge the default risk of the entity. The investor can then buy a CDS contract with this entity as the reference. If the reference entity would default before the maturity of the CDS, the investor of the CDS will receive a singular default payment from the seller of the contract, which is called settlement. Usually the settlement is physical or in cash.

If the settlement is physical, the investor deliver the bond to the seller of the CDS, in exchange for a payment corresponding to the face value of the bond. If the settlement is in cash, the investor receives a payment equal to the difference of the face value and the market value of the bond.

1.5.4 CDS Indices

A Credit Default Swap index, also known as a CDS basket, is a credit derivative used to hedge credit risk or to take a position in a basket of credit entities. Unlike a CDS, which is an over-the-counter credit derivative, a CDS index is a completely standardized credit security and may therefore be more liquid and trade at a smaller bid-offer spread. Therefore, it can be cheaper to hedge a portfolio of CDS or bonds with a CDS index, than it would be to buy several single name CDS to achieve a similar effect. CDS indices can contain both long and short positions. A long position in a CDS index corresponds to selling protection against default of a reference entity, thus a short position corresponds to buying protection against default of a reference entity. Over the past few

years, CDS indices have been traded through clearing houses in addition to OTC.

There are currently two main families of corporate CDS indices: CDX and iTraxx. CDX indices contain North American and Emerging Market companies, and are administered by the CDS Index Company. iTraxx indices contain companies from the rest of the world and are managed by the International Index Company [11].

In 2017, iTraxx Main had a monthly trading volume of $330-440 billion notional, and represented 41% of all global cleared credit derivatives. The corresponding volume for CDX.IG was $200-275 billion, which represented 28% [12]. These indices are two very relevant high liquid fixed income instruments.

iTraxx Main

iTraxx Main, also known as Markit iTraxx Europe Index, is composed of 125 liquid European entities with investment grade credit ratings that trade in the CDS market. The index is renewed and enrolled every six months, roll dates are March 20 and September 20. Each index that has a roll date of September 20 shall be issued with the maturity date of December 20, occuring 3, 5, 7 and 10 years following the roll date. In a similar way, each index that has a roll date of March 20 shall be issued with the maturity date of June 20, for corresponding future years.

The index can be be further divided into sub-indices to represent specific portions of the credit market, sector sub-indices are Autos & Industrials, Consumers, Energy, Technology, Media & Telecommunication and Financials. The index is constructed by selecting the highest ranking entities in each sector from the Liquidity List. The Liquidity List is created using the average weekly trading activity over the last six months prior to the Roll Date. Entities are required to demonstrate trading activity greater than zero during the last eight weeks preceding the last Friday of the month, prior to the month in which the Roll Date occurs, as well as have Investment Grade Relevant Rating [10].

CDX.IG

CDX.IG, also known as Markit CDX Investment Grade, is composed of 125 entities, which are the most liquid North American entities with investment grade credit ratings that trade in the CDS market. The index is renewed and enrolled every six months, roll dates are March 20 and

September 20. Each index that has a roll date of September 20 shall be issued with the maturity date of December 20, occuring 1, 2, 3, 5, 7 and 10 years following the roll date. In a similar way, each index that has a roll date of March 20 shall be issued with the maturity date of June 20, for corresponding future years.

The index can be be further divided into sub-indices to represent specific portions of the credit market, sector sub-indices are Consumer Cyclical, Energy, Financials, Industrial, Telecommunication, Media & Technology and HiVolatility. The index is constructed by selecting the highest ranking entities in each sector from the Liquidity List. The Liquidity List is created by using the average weekly trading activity, by determining all entities which single-name CDS are traded under the Standard North American Corporate Transaction Type. Entities must have been assigned a relevant credit rating of BBB-, Baa3 or above. The entities are then ranked after liquidity level based on the notional market risk activity [13].

1.6 ESG Investments

Environmental, social and governance (ESG) investments has increased significantly over the last couple of years, and has developed from merely excluding companies with a bad impact on society to now stand on more delicate CO_2-reduction strategies in the capital markets. The situation today however, is such that focus regarding these strategies almost exclusively concern equity investment portfolios. Although several projects such as the Carbon Disclosure Project, which alludes to make companies report their greenhouse gas emissions, has a large impact on the market and society, the debt part of the financial spectrum is somewhat overlooked [14].

1.6.1 Greenhouse Gas Footprint

Greenhouse gas emissions are generally divided into three separate subcategories or scopes, to avoid the risk of double counting when trying to manage and report stated emissions. Another reason for this separation is that it simplifies the recognition of which emissions the organizations can control directly, and those they may only be able to influence [15].

Scope 1

Scope 1, or Direct GHG, consists of company controlled or owned emissions, i.e. emissions directly from the organization's process. It could

be from fossil fuels used in boilers and furnaces or for transportation of materials. Other types are emissions from the manufacturing process or unintentional releases from equipment leaks or from refrigeration systems [16].

Scope 2

Scope 2, also known as Electricity Indirect GHG, are emissions from purchased electricity, steam, cooling or other energy types which are a consequence of the activities of the organization, but occur at another organization's sources, e.g. an electricity distributor [17].

Scope 3

Scope 3, or Other Indirect GHG, are emissions that are not directly controlled by the organization, but rather created as a consequence of their operations. These are emissions including, but not limited to, business travel, employee commuting and the emissions that are released when using the products the company has produced [15].

1.7 Outline

The thesis is structured as follows: In Chapter 2, a theoretical background will be provided to give the reader some knowledge within the area. The chapter includes the ECOBAR model, used to rank and compare the GHG exposure of the constituents in the CDS indices, CDS pricing theory based on the ISDA CDS Standard Model and Markowitz Modern Portfolio Theory, which is the foundation of today's portfolio construction and management. Moreover, the chapter includes theory from linear algebra to prove strict convexity for a quadratic optimization problem, as well as the theory of interpolation which is a commonly used numerical method to construct new data points. In Chapter 3, the implementation of the above models will be explained. Additionally, the data preparation process and methods to adjust the data will be presented. The chapter will also list the validations used to verify the applied methodology. In Chapter 4, a presentation and visualization of the results of the GHG minimizations are shown. First, the general characteristics of the analysis is demonstrated, followed by the base case of the CDS indices to be replicated. Then the results of the three investment strategies long only, long-short and green-brown baskets is presented. In Chapter 5, a discussion of the results will be provided as well as a real world GHG reduction example. In Chapter 6, recommendations for further studies of the subject will be presented.

2 Theory of Methodology

This chapter presents information that was condensed from a literature study and intends to create a fundamental understanding of the theoretical background, and describe a general approach to conduct the requested analysis. The chapter presents the ECOBAR model, used to rank the constituents based on GHG exposure, the theory behind CDS pricing using the ISDA CDS Standard Model. Markowitz Modern Portfolio Theory is presented, which is the fundament of today's portfolio construction and management. Moreover, the chapter introduces theory from linear algebra to prove strict convexity for a quadratic optimization problem, as well as the theory of a commonly used numerical method to construct new data points, known as interpolation.

2.1 ECOBAR Model

In order to be able to rank the constituents of the CDS indices in terms of their GHG exposure, a model called ECOBAR was used. The model utilizes a discrete score system and plays an important role in the conducted analysis, since the ECOBAR scores are minimized in the constructed portfolios. The implementation of the model can be seen in section 3.6, with some additional adjustments mentioned in section 3.3.5.

The theory in the following section is based on Erlandsson [14].

The model gives a score on three levels: the first level is a sector score which is a value between 1 and 3 in how the sectors perform compared to each other in terms of GHG emissions, denoted by C_t^r, for sector r at time t. The second level is a score between 1 and 3 in how the constituents perform compared to other constituents in the same sector, denoted by K_t^n, for constituent n in time t. The third level is a score of 0 if it is considered a green bond, otherwise it is given a score of 1, denoted by G^m, for green bond m. This level is constructed to reward green bonds by giving them 0 since they are not emitting any GHG. The scores are combined to give a total ECOBAR score S_t^m:

$$S_t^m = G^m \cdot (C_t^r \cdot K_t^n) \tag{2.1}$$

where $S_t^m \in [0, 1, 2, 3, 4, 6, 9]$, with 9 being the score of the largest GHG emitters and 0 the score of a green bond.

When constructing a portfolio which allows to go both long and short

risk, the ECOBAR model is constructed to assign alternative ECOBAR scores for the short positions. Instead of just assigning an ECOBAR score of -9 when shorting a company bond with an original score of 9, an estimated inverse function should be used. When the resulting score of being long risk in a bond with a score of 9 and being short risk in a CDS in the exact same amount and risk with a score of -9, hence a total ECOBAR score of 0, this relation could be exploited. Here, a portfolio manager could buy a substantial amount of the bond and at the same time having the same ECOBAR score of being long risk in a green bond, which is not what this model is trying to fulfill.

Instead, one assigns a long score of 9 with a corresponding short score of 1, and a long score of 1 with a short score of 9. Thus, one uses the inverse function to compute the ECOBAR score when shorting. A fitted inversion function of the ECOBAR score used for short risk positions in CDS can be derived from:

$$\begin{aligned} &y = 0.0025x^4 - 0.0849x^3 + 1.024x^2 - 5.5653x + 13.633 \\ &\text{where } 1 \leq y \leq 9. \end{aligned} \tag{2.2}$$

To be able to use the inverse function in the chosen implementation, one needs to assign which constituents to go long in and which to go short in beforehand. This is further explained in section 3.6.5.

When going long risk in a CDS, one implicitly provides financing for said company, i.e. sell protection, and the opposite when going short risk. From an environmental point of view it is therefore most effective to go long risk in companies with low GHG exposure and short risk in companies with high GHG exposure. This is the reason why the ECOBAR scores in this thesis are aimed to be minimized. For further discussion regarding the impact credit derivatives have on the underlying reference companies, see section 5.3.

2.2 Credit Default Swap Pricing

The valuation and pricing process of a CDS contract involves aspects such as the default probability, loss amount, recovery rate and timing of default. The fundament of CDS pricing is that the present value of all CDS premium payments should equal the present value of the expected payoff from the CDS, for the NPV to be 0 for both parties of the contract. This results in each party being equally well off. The implementation of the CDS pricing can be found in section 3.4.

The theory in the following section is based on Brigo and Mercurio [18].

A credit event of the reference entity will be denoted as a default and the three parts of the contract are denoted as:

- 0 = Investor
- 1 = Reference entity
- 2 = Counterparty

The time of default is denoted by τ_i where $i = 0, 1, 2$ represents the different parts of the contract. The protection buyer makes regular payments at the rate S, the spread, at fixed times $T_{a+1}, T_{a+2}, ..., T_b$, until expiration of the contract or a default of the reference entity occurs. In exchange the protection buyer receives a payment of the loss given default (LGD) on the contract notional amount, in a default event of the reference entity. The maximum value of the LGD is 1 when the full notional amount is paid, and the minimum value is 0 when nothing is paid.

2.2.1 Premium Leg

The value of the premium leg is the present value of the payments made by the protection buyer [19]. Given the assumption that the stochastic discount factor $D(s,t)$ is independent from the default time τ_1, for all $0 < s < t$, the value of the premium leg of the CDS at time 0 can be defined as:

$$\begin{aligned}
\text{PremiumLeg}_{a,b}(S) = \mathbf{E}\Big[& D(0,\tau_1)(\tau_1 - T_{\gamma(\tau_1)-1}) S \mathbf{1}_{\{T_a<\tau_1<T_b\}}\Big]^+ \\
& + \sum_{i=a+1}^{b} \mathbf{E}\Big[D(0,T_i)\alpha_i S \mathbf{1}_{\{\tau_1 \geq T_i\}}\Big] \\
= S & \int_{t=T_a}^{T_b} P(0,t)(\tau_1 - T_{\gamma(\tau_1)-1}) Q(\tau_1 \in [t, t+dt)) \\
& + S \sum_{i=a+1}^{b} P(0,T_i)\alpha_i Q(\tau_1 \geq T_i) \\
= - S & \int_{t=T_a}^{T_b} P(0,t)(\tau_1 - T_{\gamma(\tau_1)-1}) d_t Q(\tau_1 \geq t) \\
& + S \sum_{i=a+1}^{b} P(0,T_i)\alpha_i Q(\tau_1 \geq T_i)
\end{aligned} \tag{2.3}$$

where α_i is the time (year fraction) between T_{i-1} and T_i. $T_{\gamma(\tau_1)-1}$ is the final payment date before τ_1. $P(0,t)$ is the zero coupon bond (ZCB) marked to market which discounts the cash flows from time t to 0. $Q(\tau_1 \geq T)$ is the survival probability, which is described further in section 2.2.5. τ_1 denotes the default time of the reference entity.

The summation term represents the discounted payments, and the integral term represents the accrued premium, which is a fraction of the premium accrued from the preceding payment date up until the time of default [20].

2.2.2 Protection Leg

The value of the protection leg is the present value of the amount which the protection buyer receives in the event of a default of the reference entity [19].

Given the assumptions that the interest rates and the default time τ_1 are independent, the value of the protection leg of the CDS at time 0 can be defined as:

$$\begin{aligned} \mathrm{ProtectionLeg}_{a,b}(\mathrm{LGD}) &= \mathbf{E}\left[\mathbf{1}_{\{T_a<\tau_1\leq T_b\}} D(0,\tau_1)\mathrm{LGD}\right] \\ &= \mathrm{LGD}\int_{t=T_a}^{T_b} P(0,t)Q(\tau_1 \in [t, t+dt)) \\ &= -\ \mathrm{LGD}\int_{t=T_a}^{T_b} P(0,t)d_tQ(\tau_1 \geq t)) \end{aligned} \tag{2.4}$$

2.2.3 Payoff

Given the assumptions that the interest rates and default time τ_1 are independent, and further assume that the recovery rate is deterministic, the value of a CDS contract for the seller at time t is then given by [21]:

$$\mathrm{CDS}_{a,b}(t;S) = \mathrm{PremiumLeg}_{a,b}(t;S) - \mathrm{ProtectionLeg}_{a,b}(t;S) \tag{2.5}$$

In order to obtain the value of the CDS contract to the protection buyer, the operators prior to the legs are switched.

For a CDS contract at time 0, given a default of the reference entity

between time T_a and T_b, a periodic premium rate of S_1, and a loss given default LGD_1, the value of the CDS contract to the protection seller is given by:

$$
\begin{aligned}
\mathrm{CDS}_{a,b}(0, S_1, \mathrm{LGD}_1) = S_1 \Bigg[- \int_{t=T_a}^{T_b} & P(0,t)(t - T_{\gamma(t)-1}) d_t Q(\tau_1 \geq t) \\
& + \sum_{i=a+1}^{b} P(0,T_i)\alpha_i Q(\tau_1 \geq T_i) \Bigg] \\
& + \mathrm{LGD}_1 \Bigg[\int_{t=T_a}^{T_b} P(0,t) d_t Q(\tau_1 \geq t) \Bigg]
\end{aligned}
\tag{2.6}
$$

where $\gamma(t)$ is the first payment in period T_j following time t.

One can now denote the residual net present value (NPV) of a receiver CDS contract between T_a and T_b evaluated at T_j, where $T_a < T_j < T_b$:

$$
\mathrm{NPV}(T_j, T_b) := \mathrm{CDS}_{a,b}(T_j, S, \mathrm{LGD}_1) \tag{2.7}
$$

Equation (2.7) can be written on the same form as Equation (2.6) but for evaluation at time T_j:

$$
\begin{aligned}
& \mathrm{NPV}(T_j, T_b) = \mathrm{CDS}_{a,b}(T_j, S_1, \mathrm{LGD}_1) \\
= \mathbf{1}_{\tau_1 > T_j} \Bigg\{ S_1 \Bigg[- \int_{max\{T_a,T_j\}}^{T_b} & P(T_j,t)(t - T_{\gamma(t)-1}) d_t Q(\tau_1 \geq t \mid \mathcal{F}_{T_j}) \\
& + \sum_{i=max\{a,j\}+1}^{b} P(T_j,T_i)\alpha_i Q(\tau_1 \geq T_i \mid \mathcal{F}_{T_j}) \Bigg] \\
& + \mathrm{LGD}_1 \Bigg[\int_{max\{T_a,T_j\}}^{T_b} P(T_j,t) d_t Q(\tau_1 \geq t \mid \mathcal{F}_{T_j}) \Bigg] \Bigg\}
\end{aligned}
\tag{2.8}
$$

where evaluation is based on the information which is available on the market at time $T_j, \mathcal{F}_{T_j}$ [22].

2.2.4 Return

CDS returns in the form of profit and losses, also known as P&L, are calculated from the P&L on the underlying CDS legs.

The theory in the following section is based on Rennison et al. [23] and Bomfim [24].

Consider a CDS initiated at time t and closed after T years, S_t^M denotes the M-year spread at time t. N is the notional on the M-year leg. Notional is the initial investment amount invested in the CDS at time t, which typically amounts to \$10 million. $\mathrm{DV01}_t^M$ denotes the DV01 for M years starting at time t. DV01 is the expected present value of 1 basis point paid on the premium leg until default or maturity, whichever comes first. The P&L on the underlying M-year CDS leg, which by definition is equal to the M-year CDS return, is then given by:

$$\begin{aligned} R_t^M &= \Big[\text{Carry} + \text{Roll-Down}\Big] \cdot \text{Notional} \\ &= \Big[T \cdot S_t^M + \mathrm{DV01}_T^{M-T}\big(S_t^M - S_T^{M-T}\big)\Big] \cdot N \end{aligned} \tag{2.9}$$

Carry

The most obvious component of why time matters for the P&L of a CDS trade is the premium accrued or received/paid while the trade is in place. Consider that an investor is selling protection, premium accrual will then benefit the position, in which case the position is said to have positive carry. Naturally, the opposite is true for a protection-buying trade, where the investor expects the par CDS spreads of the reference entity to widen. Prior to entering into such a trade, the investor needs to consider the time that it might take for the expected spread widening to take place, because the investor will have to pay for any premium accrued while the position is held. Thus, the position has negative carry.

Roll-Down

The roll-down component measures the P&L that would be realized entirely due to the decaying time-to-maturity of the different CDS legs, assuming that the spread curve remains constant. Roll-down is merely one potential P&L path. Unlike the carry, roll-down is not linear with the horizon of the trade. The magnitude and sign of the P&L from roll-down can change with different trade horizons.

2.2.5 Survival and Hazard Function

As previously mentioned, the pricing process of a CDS contract involves the default probability aspect. The probability of default at different

times is given by the survival function, and the hazard function gives the instantaneous default rate.

The theory in the following section is based on Rodriguez [25].

Assume that T is a continuous random variable, $f(t)$ is the probability density function (pdf), $F(t) = Pr\{T < t\}$ is the cumulative distribution function (cdf), that gives the probability of an event which has occurred up until duration t. One can now define the survival probability function as the complement of the cdf:

$$Q(t) = Pr\{T \geq t\} = 1 - F(t) = \int_t^\infty f(x)dx \tag{2.10}$$

The survival function states the probability that a default has not yet occurred until time t. The hazard rate is the instantaneous default rate which one can define as:

$$\lambda(t) = \lim_{dt \to 0} \frac{Pr\{t \leq T < t + dt \mid T \geq t\}}{dt} \tag{2.11}$$

The numerator represents the conditional probability of default in interval $[t, t+dt)$, given that a default has not yet occurred. The denominator is the width of the time interval. By computing the limit of the expression and letting dt go to zero, one obtains the instantaneous rate of default, also known as hazard rate. The hazard rate can be rewritten according to [26] as:

$$\lambda(t) = \frac{f(t)}{Q(t)} \tag{2.12}$$

One can see that the default rate at time t is given by the pdf at t divided by the survival probability until time t. One can combine Equation (2.10) and Equation (2.12) and express the hazard rate as:

$$\lambda(t) = -\frac{d}{dt} \log Q(t) \tag{2.13}$$

If one integrates the expression from 0 to t, the survival probability can be expressed as a function of the hazard rates up to time t:

$$Q(t) = exp\left\{ -\int_0^t \lambda(x)dx \right\} \tag{2.14}$$

The integral is the cumulative hazard function and can be viewed as the summation of the risks from time 0 to t:

$$\Lambda(t) = \int_0^t \lambda(x)dx \tag{2.15}$$

Given the hazard rates, one can calculate the survival function, and vice versa. The survival function gives the probability of default at different times, while the hazard rate gives the short time probability of default.

2.3 Markowitz Modern Portfolio Theory

Markowitz introduced modern portfolio theory in terms of mean variance portfolio optimization. Markowitz Mean Variance (MMV) aims to construct a portfolio for which the risk is minimized, given a certain level of the expected return, μ_p. The variance, σ_p^2, of the portfolio is used to quantify the level of risk. The implementation of this theory can be found in section 3.6 where it is used to minimize the tracking error as well as the ECOBAR score.

The theory in the following section is based on Markowitz [27].

Let r_i be the random variable associated with the rate of return for asset i, for $i = 1, 2, ..., n$, and define the random vector:

$$z = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

Set $\mu_i = \mathbf{E}(r_i)$, $\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_n)^T$ and $\mathrm{cov}(z) = \Sigma$. If $\boldsymbol{w} = (w_1, w_2, ..., w_n)^T$ is a set of weights related to an investment portfolio, then the rate of return of this portfolio, $r = \sum_{i=1}^n r_i w_i$, is also a random variable with mean $\boldsymbol{\mu}^T \boldsymbol{w}$ and variance $\boldsymbol{w}^T \sum \boldsymbol{w}$. If μ_p is the acceptable baseline expected rate of return for the portfolio, then according to the Markowitz portfolio theory, an optimal portfolio is any portfolio solving the following quadratic program:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \frac{1}{2}\boldsymbol{w}^T \sum \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{\mu}^T \boldsymbol{w} = \mu_p \\ & \mathbf{1}^T \boldsymbol{w} = 1 \end{aligned} \tag{2.16}$$

where **1** denotes the vector of ones. The first constraint reflects that the expected portfolio return is fixed to μ_p. The second constraint states that the weights are regarded as the proportion of the total portfolio contained in each asset. The Karush-Kuhn-Tucker (KKT) conditions for this quadratic program are:

$$\begin{aligned} &(1) \quad \Sigma\boldsymbol{w} - \lambda_1\boldsymbol{\mu} - \lambda_2\mathbf{1} = 0 \\ &(2) \quad \mu_p \leq \boldsymbol{\mu}^T\boldsymbol{w},\ \mathbf{1}^T\boldsymbol{w} = 1,\ 0 \leq \lambda_1 \\ &(3) \quad \boldsymbol{\mu}^T\boldsymbol{w} - \mu_p = 0 \end{aligned} \tag{2.17}$$

for some $\lambda_1, \lambda_2 \in \mathbb{R}$. Because the covariance matrix Σ is symmetric and positive definite, it is known that if $(\boldsymbol{w}, \lambda_1, \lambda_2)$ is any triple satisfying the KKT conditions, then $\boldsymbol{w}$ is inevitably a solution to Equation (2.16). One can show that if (2.16) is feasible, then a solution to the minimization problem must always exist. Thus, a KKT triple can always be found for (2.16).

2.3.1 Efficient Frontier

The points (σ_p, μ_p) is referred to as the efficient frontier. The set of all efficient portfolios represents the choice the asset manager must take between risk σ_p, and return, μ_p. According to Markowitz, an efficient portfolio maximizes returns given a level of risk, which is represented by the standard deviation. All portfolios (σ_p, μ_p) to the right of the efficient frontier are possible, however not optimal.

2.4 Mathematical Proofs

The linear algebraic proofs and properties listed below can be used to investigate strict convexity of the optimization function expressed in Equation (2.16). The implementation was made as a part of the validation process, which can be found in section 3.7.6.

The theory in the following section is based on Sasane and Svanberg [28] and Oliveira [29].

2.4.1 Convex Quadratic Functions

Lemma 2.1 *Let f:* $\mathbb{R}^n \to \mathbb{R}$ *be a quadratic function given by:*

$$f(x) = \frac{1}{2}x^T Ax + c^T x + c_0$$
$$\text{where } x \in \mathbb{R}^n,$$

where $A \in \mathbb{R}^{n \times n}$ *is a symmetric matrix,* $c \in \mathbb{R}^n,\ c_0 \in \mathbb{R}$. *Then*

(1) *f is convex iff A is positive semi-definite.*
(2) *f is strictly convex iff A is positive definite.*

2.4.2 Positive Definite Matrices

Definition 2.2 *The symmetric matrix A is positive definite iff all its eigenvalues are positive.*

Theorem 2.3 *A is positive definite iff* $x^T Ax > 0$, $\forall x \neq 0$.

Proof. *Assume there is* $x \neq 0$ *such that* $x^T Ax \leq 0$ *and A is positive definite. Then there exists* $Q^T Q = I$ *such that* $A = Q^T \Lambda Q$ *with* $\Lambda_{ii} = \lambda_i > 0$. *Then for* $y \neq 0$ *such that* $x = Q^T y$

$$0 \geq x^T Ax = y^T QAQy = y^T QQ^T \Lambda QQ^T y = y^T \Lambda y = \sum_{i=1}^{n} \lambda_i y_i^2 > 0$$

which is a contradiction.

2.5 Polynomial Interpolation

Interpolation is a method in numerical analysis to construct new data points within the range of a discrete set of known data points. By fitting a curve function to the data set using polynomials, one can estimate the value of that function for the intermediate values of the data. The implementation was made to approximate missing CDS spreads, for further details see section 3.3.2.

The theory in the following section is based on Brandimarte [30].

Consider a set of support points (x_i, y_i), $i = 0, 1, ..., n$, where $y_i = f(x_i)$ and $x_i \neq x_j$ for $i \neq j$. It is simple to find a polynomial of degree n (at most) for which $P_n(x_i) = y_i$ for any i. One may rely on the Lagrange polynomials defined as:

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \tag{2.18}$$

Note that these polynomials are of degree n and that:

$$L_i(x_k) = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} \tag{2.19}$$

Now an interpolating polynomial can be written as:

$$P_n(x) = \sum_{i=0}^{n} y_i L_i(x) \tag{2.20}$$

One may see that the polynomial passes through the data set, unfortunately, one may also note that the interpolating polynomial has some undesirable oscillation behavior near the end points of the interval. The oscillation of high-degree interpolating polynomials is a typical difficulty, and there are a few ways to overcome it.

2.5.1 Piecewise Cubic Interpolation

One way to avoid oscillating polynomials in function interpolation is resorting to low-degree polynomials, interpolating the data points piecewise.

Given the $N + 1$ knots (x_i, y_i), one can use N first-degree polynomials $S_i(x)$, each one valid on the interval (x_i, x_{i+1}). The resulting function is required to be continuous, i.e. $S_i(x_{i+1}) = S_{i+1}(x_{i+1})$. From the Lagrange polynomials defined in Equation (2.18) it follows that:

$$\begin{aligned} &S_i(x) = y_i \frac{x - x_{i+1}}{x_i - x_{i+1}} + y_{i+1} \frac{x - x_i}{x_{i+1} - x_i} \\ &\text{where } x \in [x_i, x_{i+1}] \end{aligned} \tag{2.21}$$

This interpolation type is called linear spline. While the interpolating function is continuous, its derivative is not. If the data which is interpolated are prices of an asset as a function of an underlying factor, non-differentiability prevents the ability to estimate sensitivities. If one would like to approximate a function which shall then be optimized, non-differentiability is a complication.

One can enforce the continuity of the derivatives of the spline by increasing the degree of the polynomials. The most commonly used spline is obtained by joining N third-degree polynomials $S_i(x)$ with coefficients $s_{i0}, s_{i1}, s_{i2}, s_{i3}$, which must satisfy the following requirements:

$$
\begin{aligned}
& S(x) = S_i(x) = s_{i0} + s_{i1}(x - x_i) + s_{i2}(x - x_i)^2 + s_{i3}(x - x_i)^3 \\
& \qquad\qquad\qquad x \in [x_i, x_{i+1}], \quad i = 0, 1, ..., N-1 \\
& S(x_i) = y_i, \qquad\qquad\qquad\quad i = 0, 1, ..., N \\
& S_i(x_{i+1}) = S_{i+1}(x_{i+1}), \qquad i = 0, 1, ..., N-2 \\
& S_i^{'}(x_{i+1}) = S_{i+1}^{'}(x_{i+1}), \qquad i = 0, 1, ..., N-2 \\
& S_i^{''}(x_{i+1}) = S_{i+1}^{''}(x_{i+1}), \qquad i = 0, 1, ..., N-2
\end{aligned}
\tag{2.22}
$$

The resulting spline $S(x)$ is a cubic spline. The condition stated above require the spline itself, and its first and second derivatives to be continuous. In order to specify a spline, one must give $4N$ coefficients. Passage through the support points gives $N+1$ conditions. The continuity of the spline and the two derivatives invoke $3(N-1)$ conditions, resulting in a total of $4N-2$ conditions. Thus, one have two degrees of freedom which may be eliminated by enforcing further requirements. Most often, they involve some conditions at, or near, the end points x_0 and x_N. Among the most common conditions, recall the following ones:

- $S^{''}(x_0) = S^{''}(x_N) = 0$, which leads to natural splines.
- $S^{'}(x_0) = f^{'}(x_0)$ and $S^{'}(x_N) = f^{'}(x_N)$, which may be used if one have an exact idea of the behavior of $f(x)$ near the end points.
- The not-a-knot condition, obtained by requiring that the third order derivative $S^{'''}(x)$ is continuous in x_1 and x_{N-1}, which implies that $S(x)$ would be a spline for the knots $x_0, x_2, x_3, ..., x_{N-2}, x_N$, but it interpolates through x_1 and x_{N-1} as well.

3 Applied Methodology

This chapter describes the exact methodology applied to the conducted analysis, including a more in-depth description of the process as a whole, as well as the specific methods used. The chapter shows how the the raw data was collected and lists the methods used to reduce and process the data. Moreover, it is described how the ECOBAR model, the CDS pricing process and portfolio optimization was implemented and validated.

3.1 Data Collection

The data needed to conduct the analysis was extracted first hand from the Bloomberg trading system, unless stated otherwise. The Bloomberg system is the standard software for trading among financial institutions. For an overview of the general characteristics of the collected GHG and CDS data, see Figures 7-11 in Appendix A and Figures 12-16 in Appendix B.

The data set for greenhouse gas emissions was extracted from Trucost. Trucost is a company which estimates the hidden costs of unsustainable use of natural resources by companies. The data covers carbon footprint measured in scope 1 and scope 2, for a large quantity of bond issuers, on a yearly basis of 2017.

The constituents data set included data regarding the constituents which are included in the CDS indices that was analyzed. The set covered 125 constituents for iTraxx Main and CDX.IG, respectively. Parameters included which sector the constituents operate within and tickers used in the Bloomberg trading system, to name a few.

The credit default swap data consisted of issued single name 5Y CDS mid rate closing spreads, for a large quantity of issuers, which covered issued contracts in the period 1^{st} January 2015 to 31^{st} December 2017. Furthermore, data for the 5Y spreads for CDS indices iTraxx Main S25 and CDX.IG S26 was extracted from the same period. S25 and S26 denotes Series 25 and 26 respectively, since the indices are rolled, or renewed, every six months. Note that Series 25 for iTraxx Main rolls at the same time as Series 26 of CDX.IG, since there is a shift between the series. Onward, the CDS indices will be referred to as iTraxx Main and CDX.IG, rather than iTraxx Main S25 and CDX.IG S26.

The forward swap data consisted of daily money market spot Libor rates with maturities of 1M, 2M, 3M, 6M, 9M and 12M, and forward swap

rates with maturities of 2Y-10Y, 12Y, 15Y, 20Y and 30Y. Both which covers the period 1^{st} January 2015 to 31^{st} December 2017.

3.2 Data Reduction

First, the data set was reduced in terms of the GHG data coverage, which resulted in a remaining 109 out of 125 constituents for iTraxx Main, and 114 out of 125 constituents for CDX.IG.

Secondly, the data set was reduced in terms of the single name CDS spreads data coverage, both regarding specific issuers but also in terms of specific trading days during the period. The CDS data was filtered with thresholds of missing data points set to a maximum of 15% on an issuer basis, and 20% on trading day basis for both iTraxx Main and CDX.IG.

This resulted in the final CDS data sets, with a remainder of 107 out of 125 constituents on 737 out of the total 783 trading days for iTraxx Main. Respectively, 102 out of 125 constituents for 734 out of the total 785 trading days for CDX.IG.

After the data reduction process, the amount of missing data points in the remainder data sets were 1.09% for iTraxx Main and 0.52% for CDX.IG.

3.3 Data Processing

In order to fill the remainder of the missing spread data points, two methods were used which are described in more detail in sections 3.3.1-3.3.2.

To accomplish a fair comparison between the parameters of the constituents, a few methods were used, which are further described in sections 3.3.3-3.3.5.

3.3.1 Correlation Trajectory

The idea behind the method was to use spread trajectories from other constituents to determine what the missing spread was for a certain constituent. Moreover, the trajectories were taken from the constituents with the highest spread correlation with the missing spread value constituent. The correlations were calculated between the actual spreads, during the period 1^{st} January 2015 to 31^{st} December 2017.

The method is iterative, where the first iteration takes the highest correlating spread trajectory into consideration. The second iteration checks the second highest correlating trajectory, and so forth. The correlation threshold was set to 0.90, meaning that all used correlation trajectories had a correlation of 0.90 or higher with the missing spread. Below follows an example of how a missing spread was calculated with this method.

Example

Consider a data sheet of spreads $S_{i,t}$, for a time series $t = 1, 2, ..., T$ for n constituents, $i = 1, 2, ..., n$. A correlation matrix is constructed from the spread data, $\sigma^{n\times n}$. Denote the missing spread data point as $S_{i,t}$ for any $i,\ t$.

First, track down the spread with the highest correlation to S_i, $max\{\sigma^{n\times n}\}$ $= \sigma_{i,j}$. It follows that the spread trajectory of constituent j is used, to calculate the missing spread of constituent i. Now, the missing spread data point is calculated as:

$$S_{i,t} = S_{i,t-1} \cdot \frac{S_{j,t}}{S_{j,t-1}} \tag{3.1}$$

Note that naturally, the method does not work in the first time point $t = 1$, since no data is available for earlier time points. Thus, the method is skipping such missing data points. If $S_{j,t-1} = 0$, $S_{i,t}$ will remain 0. Furthermore, if $S_{j,t} = 0$, the trajectory cannot be calculated and the method moves on to the next iteration.

3.3.2 Interpolation

The interpolation implementation was done in MATLAB using the interpolation algorithm *pchip*, which is a piecewise cubic interpolation method. The theoretical background behind the method can be found in section 2.5.

3.3.3 Greenhouse Gas Normalization

To accomplish a fair comparison between the carbon footprint for the constituents, the GHG emissions were normalized. The general theory presented in section 2.1 have been applied to the CDS indices, with some modifications. Before assigning the constituents their within sector ECO-BAR scores, the GHG emissions were normalized as following:

$$\mathrm{GHG}_i^{\mathrm{Norm}} = \frac{\mathrm{GHG}_i}{\mathrm{MV}_i} \tag{3.2}$$

where GHG_i denotes the GHG emissions in metric tons on a yearly basis for constituent i. MV_i is the total bond market value for constituent i.

Due to the equally weighted characteristics of the CDS indices, this was the most straightforward way of taking the constituent size aspect into consideration when comparing their corresponding GHG emissions.

3.3.4 Green Bond Adjustment

To take green bonds into consideration, the market values for each constituent were adjusted in regard to green bond market values. A ratio of the green bond value for each constituent, with regard to the total market value of the bonds of the constituent, was calculated as:

$$\text{GBA}_i = \frac{\text{MV}_i - \text{GBV}_i}{\text{MV}_i} \tag{3.3}$$

where MV_i denotes the total bond market value for constituent i. GBV_i is the total green bond market value for constituent i.

This is the chosen interpretation of including green bonds as explained in section 2.1, taking $G^m \in \{0, 1\}$ into consideration by using GBA_i instead.

3.3.5 ECOBAR Score Normalization

In order to calculate a correct ECOBAR score for a portfolio with both long and short positions, a normalization of the investment weights was made, computing the normalized ECOBAR score as:

$$\text{ECOBAR}^{\text{Norm}} = \sum_{i=1}^{n} \frac{|0 - w_i|}{|0 - w_1| + |0 - w_2| + ... + |0 - w_n|} \cdot \text{ECOBAR}_i \tag{3.4}$$

where ECOBAR_i denotes the ECOBAR score for constituent i, w_i is the investment weight for constituent i and $w_1, w_2, ..., w_n$ are the investment weights for each constituent $1, 2, ..., n$.

With this normalization technique, all normalized weights sum up to 1 and are thus expressed as a fraction of the total weights they represent. Each normalized weight w_i is then multiplied by the corresponding ECOBAR_i, to obtain a normalized ECOBAR score in the same way as for a long only portfolio.

The above ECOBAR normalization is under the assumption that a long position in a constituent with an ECOBAR score of 1, equals a short position in a constituent with a long ECOBAR score of 9, i.e. a short ECOBAR score of 1. The main reason why the long and short spectrum of the ECOBAR score is equal, is to keep the ECOBAR model stable and to not be able to exploit short positions, as explained in section 2.1. Another aspect is that promoting companies that are low emitters, i.e. good for the environment, by taking long positions, is just as important as punishing the heavy emitters, i.e. those that are bad for the environment, by taking short positions.

3.4 CDS Pricing Implementation

The ISDA CDS Standard Model maintained by Markit, has been used for CDS pricing. The model intends to standardize the way in which the running spread can be converted to an upfront fee, as well as how the cash settlement amount is calculated for a CDS. The pricing has been implemented in MATLAB using functions *cdsbootstrap* for the hazard rate and survival function, and *cdsprice* for the contract pricing. The day-count basis used for the contract was actual/360, following the ISDA CDS Standard Model [31]. The CDS pricing implementation is based on the theory introduced in section 2.2.

3.4.1 Roll and Maturity Dates

The roll and maturity dates of the contracts were specified in the following way:

- CDS contracts with a roll date between [20 September, 20 March], are issued with a maturity date of 20 December, occuring at any specified year in the future.
- CDS contracts with a roll date between [21 March, 19 September], are issued with a maturity date of 20 June, occuring at any specified year in the future.

CDS contracts with five years to maturity, $5Y$, were used in the implementation. Thus, resulting in a time to maturity T of the CDS contracts, where $[5.25 \geq T \geq 4.75]$, depending on the roll and maturity date grouping described above.

3.4.2 Zero Curve

The zero curve was constructed using MATLAB function *pyld2zero*, which creates the zero curve given the par yield curve. The yield curve was constructed using forward swap data from Bloomberg.

3.5 Portfolio Construction

The analysis was conducted on two categories of portfolios. The first portfolio category was a replication of the CDS indices iTraxx Main and CDX.IG. The idea was to create as similar portfolios as possible as the actual indices themselves, with matching underlying investment properties. The other portfolio category was constructed from only the greenest and brownest constituents of each CDS index. The idea was to create a very efficient green-brown investment strategy by constructing a portfolio of significantly less constituents, but still keeping the underlying characteristics of the actual indices.

3.5.1 iTraxx Main Replication

The actual index was replicated by combining the constituents data for iTraxx Main with issued single name CDS spreads for each and every constituent of the index during the period 1^{st} January 2015 to 31^{st} December 2017.

3.5.2 CDX.IG Replication

The actual index was replicated by combining the constituents data for CDX.IG with issued single name CDS spreads for each and every constituent of the index during the period 1^{st} January 2015 to 31^{st} December 2017.

3.5.3 Green-Brown Portfolios

This portfolio category was constructed from the greenest and brownest constituents of the CDS indices, i.e. the constituents with the lowest and highest ECOBAR scores. The idea was to create long-short strategies by taking advantage of the most interesting constituents from a GHG perspective, while retaining the properties of the CDS indices.

The number of constituents to compose each of the green-brown portfolio was chosen arbitrary by varying the portfolio size. The tested portfolios were created consisting of the $5, 10, ..., 25$ greenest and brownest constituents, adding up to a total of $10, 20, ..., 50$ constituents per portfolio.

The portfolio consisting of the 20 greenest and 20 brownest constituents for the indices were further analyzed.

3.6 ECOBAR Minimization Implementation

The objective in terms of portfolio optimization is to minimize the GHG footprint of the CDS basket, i.e. minimizing the ECOBAR score, whilst trying to keep as low tracking error against the CDS index as possible. The tracking error is measured in terms of standard deviation, i.e. volatility, hence by minimizing the tracking error, the volatility will remain stable. To solve this problem, Markowitz Modern Portfolio Theory introduced in section 2.3 was used.

3.6.1 Algorithm

To optimize the portfolio, the quadratic programming algorithm *quadprog* available in MATLAB was used. The algorithm solves convex optimization problems expressed on the following standard form:

$$
\begin{aligned}
\min_{\boldsymbol{w}} \quad & f(w) := \frac{1}{2} w^T \sum w + c^T w \\
\text{s. t.} \quad & w \in \mathbb{R}^n \\
& A \cdot w \leq b \\
& Aeq \cdot w = beq \\
& lb \leq w \leq ub
\end{aligned}
\tag{3.5}
$$

where $\Sigma \in \mathbb{R}^{n \times n},\ c \in \mathbb{R}^n,\ A \in \mathbb{R}^{m \times n},\ b \in \mathbb{R}^m,\ Aeq \in \mathbb{R}^{q \times n},\ beq \in \mathbb{R}^q$ and $\{lb, ub\} \in \mathbb{R}^n$ [32]. Here n denotes the number of constituents in the CDS index and $\{m, q\}$ the number of constraints to minimize over.

3.6.2 Tracking Error

The general definition for calculating the tracking error is:

$$
\text{TE} = \sqrt{\frac{\sum_{t=1}^{n} (R_t - R_t^I)^2}{n-1}} \tag{3.6}
$$

where R_t is the return for the portfolio manager at time t, R_t^I is the return for the benchmark index at time t and n the number of return periods to calculate over [33].

In this thesis, Equation (3.6) is used to calculate the tracking error for the validation period, 2017, when the optimized investment weights

are tested against the index. The summation in the numerator is used to compute optimized weights during the calibration period, 2015-2016, which is the loss function below.

3.6.3 Loss Function

To be able to replicate the indices sufficiently, one needs to capture the difference between the returns of the CDS index and the returns of the replicated portfolios and then minimize that difference, i.e. minimize the tracking error. To do so, the following quadratic loss function was used:

$$L = \sum_{t=1}^{T} \left[\sum_{i=1}^{n} \left(w_i \cdot R_{t,i} \right) - R_t^I \right]^2 \tag{3.7}$$

where $R_{t,i}$ is the CDS return for constituent i in time period $[t-1, t]$. R_t^I is the CDS index return in time period $[t-1, t]$. w_i denotes the investment weight for constituent i. $t = 1, 2, ..., T$ is the time period and n denotes the number of constituents.

Example

Here follows an example of how the loss function in Equation (3.7) can be expressed on the form of the MMV problem in Equation (2.16), for two constituents $i = 1, 2$ in time period $t = 1, 2, ..., T$:

$$
\begin{aligned}
L &= \sum_{t=1}^{T} \left[\sum_{i=1}^{2} \left(w_i \cdot R_{t,i} \right) - R_t^I \right]^2 \\
&= \sum_{t=1}^{T} \left[\sum_{i=1}^{2} w_i \cdot \left(R_{t,i} - R_t^I \right) \right]^2 \\
&= \sum_{t=1}^{T} \left[w_1 \cdot \left(R_{t,1} - R_t^I \right) + w_2 \cdot \left(R_{t,2} - R_t^I \right) \right]^2 \\
&= \sum_{t=1}^{T} \Bigg[w_1^2 \cdot \left(R_{t,1} - R_t^I \right)^2 + 2w_1 w_2 \cdot \left(R_{t,1} - R_t^I \right)\left(R_{t,2} - R_t^I \right) \\
&\qquad + w_2^2 \cdot \left(R_{t,2} - R_t^I \right)^2 \Bigg] \\
&= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \cdot \sum_{t=1}^{T} \begin{bmatrix} \left(R_{t,1} - R_t^I \right)^2 & \left(R_{t,1} - R_t^I \right)\left(R_{t,2} - R_t^I \right) \\ \left(R_{t,2} - R_t^I \right)\left(R_{t,1} - R_t^I \right) & \left(R_{t,2} - R_t^I \right)^2 \end{bmatrix} \\
&\qquad \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\
&= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \cdot \Bigg\{ \begin{bmatrix} \left(R_{1,1} - R_1^I \right)^2 & \left(R_{1,1} - R_1^I \right)\left(R_{1,2} - R_1^I \right) \\ \left(R_{1,2} - R_1^I \right)\left(R_{1,1} - R_1^I \right) & \left(R_{1,2} - R_1^I \right)^2 \end{bmatrix} \\
&\qquad + \begin{bmatrix} \left(R_{2,1} - R_2^I \right)^2 & \left(R_{2,1} - R_2^I \right)\left(R_{2,2} - R_2^I \right) \\ \left(R_{2,2} - R_2^I \right)\left(R_{2,1} - R_2^I \right) & \left(R_{2,2} - R_2^I \right)^2 \end{bmatrix} \\
&\qquad + \ldots + \begin{bmatrix} \left(R_{T,1} - R_T^I \right)^2 & \left(R_{T,1} - R_T^I \right)\left(R_{T,2} - R_T^I \right) \\ \left(R_{T,2} - R_T^I \right)\left(R_{T,1} - R_T^I \right) & \left(R_{T,2} - R_T^I \right)^2 \end{bmatrix} \Bigg\} \\
&\qquad \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}
\end{aligned}
\tag{3.8}
$$

It becomes clear that the exact form of $\boldsymbol{w}^T \sum \boldsymbol{w}$ is obtained. Furthermore, this can be extended for any number of constituents n, where $i = 1, 2, ..., n$.

3.6.4 Investment Weights

Constraints for the investment weight limitations for each individual constituent were set as lower and upper bounds, to prevent the individual weights from taking too extreme values.

Constraints for the investment weight limitations for each sector of the CDS indices were applied in order to keep the sector allocation of the portfolio stable. The goal was to retain a certain diversification among the investments in the portfolio. The base line of the limitations were set by calculating each sector's original investment weight for the CDS indices. A threshold was then applied to the sector base line, allowing the portfolio manager to vary the sector allocations, after preferences.

A total investment allocation constraint was applied, specifying that the summation of all investment weights must be equal to 1. Thus, a greater portion invested in short positions must result in an equally greater portion invested in long positions, since the summation must always be the same. This is a fundamental property of how the value of the total investment portfolio is calculated.

3.6.5 Long and Short Positions

Due to the nature of using the inverse function to assign ECOBAR scores for short positions as presented in section 2.1, constraints for long and short positions for each individual constituent had to be set before initiating the optimization. Thus, it is only possible to take long positions in a particular constituent during the entire optimization period, or a short position, respectively. As an example, one could choose the 15 constituents with highest ECOBAR scores to be short positions only, and the remaining constituents set to be long positions only.

3.7 Validation

This section lists the different validation techniques used to verify the applied methodology and the conducted analysis.

3.7.1 CDS Pricing

The calculated CDS prices from the CDS pricing implementation in MATLAB, were numerically validated with high accuracy toward CDS pricing in the Bloomberg trading system. The pricing in Bloomberg is based on the same ISDA CDS Standard Model as the chosen implementation, hence providing strong credibility for the implemented pricing

model. The conclusion was that the calculated CDS prices did not differ from the Bloomberg prices.

3.7.2 Portfolio Replication

To conclude whether the constructed portfolios have similar characteristics as the actual indices, the correlations were studied. High correlations suggest that the constructed portfolios follow the actual indices.

The ultimate washed CDS data to replicate iTraxx Main had a correlation of 0.967 with the actual index during the period 2015-2017. Moreover, the washed CDS data to replicate CDX.IG had a correlation of 0.944 with the actual index during the period 2015-2017, for further details see section 4.2.

It was concluded that the replicated portfolios have high correlations with the actual CDS indices and thus can be used as benchmarks for the optimized portfolios.

3.7.3 Long and Short Portfolio Combinations

A test was conducted to conclude whether the optimization algorithm used to minimize the ECOBAR score chooses the "correct" constituents for long and short investment positions, i.e. the "correct" portfolio combinations. To do so, the portfolio problem of 107 constituents for iTraxx Main and 102 for CDX.IG, was reduced to merely four constituents with different ECOBAR scores. Reducing a technical problem in this way is an effective practice, where the aim is to give the observer a better understanding of the underlying mechanics of a complicated problem.

It was showed in the conducted test illustrated in Table 20 in Appendix D, that the method of assigning the weights for long and short positions before initiating the optimization process, seemingly provided the optimal conditions to compute as low ECOBAR score as possible for the portfolios. All 15 combinations for the portfolio illustrated in Table 20 were empirically tested for 1000 different portfolios consisting of four assets from each of the two indices. Note that the portfolio combination of short only positions is excluded, since the investment weight constraints do not allow short only portfolios, as described in section 3.6.4. Thus resulting in 15 possible portfolio combinations for four assets.

The claim that the best portfolio combination for minimizing the ECOBAR score is where you short the assets that receive a lower inverse

ECOBAR score when shorted, than their respective regular ECOBAR score, holds for all of the tested portfolios. The data used in the test was from the calibration period of 2015-2016. The comparison between the combinations are the lowest possible ECOBAR scores that can be computed from the quadratic programming sequence for a specific combination.

3.7.4 Model Robustness

To test the robustness of the MMV model when minimizing the ECOBAR scores while retaining a low tracking error, the model was applied on two separate CDS indices. By acquiring results from different data sets, one gets a better idea whether the model holds sufficiently for a broader set of applications.

3.7.5 Model Backtesting

The Σ matrix in the MMV problem in Equation (2.16) was constructed from data during the calibration period 1^{st} January 2015 to 31^{st} December 2016 (training data set). The Σ matrix was then used to calibrate the optimal investment weights $\boldsymbol{w} = (w_1, w_2, ..., w_n)$. These weights were then tested during the validation period 1^{st} January 2017 to 31^{st} December 2017 (validation data set). This way one can measure the performance of the model when employed on historical data, providing information that is otherwise not available when models are tested on synthetic data.

3.7.6 Optimization Convexity

In order to verify that the optimization function expressed in Equation (2.16) is strictly convex and thus can be solved with a quadratic programming function, *Lemma 2.1* under section 2.4.1, *Definition 2.2* and *Theorem 2.3* under section 2.4.2 were used. MATLAB function *eig* was used to verify that all eigenvalues, λ_i, of the Σ matrix satisfied $\lambda_i \geq 0,\ i = 1, 2, ..., n$.

Verification plots for the eigenvalues of the Σ matrices for iTraxx Main and CDX.IG can be found in Figures 17 and 18 in Appendix C. This has also been verified for the constructed green-brown baskets.

It was concluded that the MMV problem is strictly convex for all portfolios and thus only has one global minimum solution, which can be solved with the implemented quadratic programming algorithm *quadprog*.

3.7.7 Replication Error Distribution

Since tracking errors generally do not show whether the portfolio returns underperform or overperform compared to the replicated indices, fitted distribution tests were conducted to check that particular behavior.

Here, the difference between the portfolio return and index return, $R_t - R_t^I$, in time $t = 1, ..., 252$, during the validation period 2017, was fitted to a normal distribution. The test was conducted on IWV1 and IWV10 portfolios for both iTraxx Main and CDX.IG, from section 4.4.1. Moreover, the green-brown portfolios 15G15B for both iTraxx Main and CDX.IG from section 4.5 were tested as well. The distribution plots can be seen in Figures 19-24 in Appendix E.

The distributions in Figures 19-24 were tested with a one sample t-test at the 5% significance level for the null hypothesis, that the mean is equal to zero and that the data is normally distributed. The null hypothesis was not rejected in any of the cases. The mean values for the different fitted portfolios alter between being slightly negative and positive, meaning that there are both portfolios that to some degree underperform, as well as overperform. Generally, the plotted histograms are taller and narrower than the fitted normal distribution curves, hence there are more values which are close to zero. Since a tracking error of zero is the most preferred, this shape is not a concern. By analyzing the distribution fits, there does not seem to be any evidence that the optimized portfolios would systematically underperform.

It should be clarified that what is denoted as tracking error in this section, merely is how the portfolio return differ from the index return. Not the tracking error for the entire validation period, which is always positive.

4 Results

This chapter presents the results obtained from the applied methodology described in previous chapters. First, the general characteristics of the analysis is demonstrated, followed by the base case of the CDS indices to be replicated. Then the results of the three investment strategies long only, long-short and green-brown baskets is presented.

The following notations are used to describe the optimization parameters in the sections below:

- **ECOBAR** is the normalized minimum ECOBAR score of the optimized portfolio.
- **Tracking Error** of the optimized portfolio in terms of returns between the portfolio and the index during the validation period. It is expressed annualized in percentage units.
- **Correlation** between the optimized portfolio and the index during the validation period.
- **TotWeightLong** is the total investment weight in long positions for the optimized portfolio, expressed in percentages.
- **TotWeightShort** is the total investment weight in short positions for the optimized portfolio, expressed in percentages.
- **WeightLb** is the lower bound (limit), for individual investment weights in the optimized portfolio.
- **WeightUb** is the upper bound (limit), for individual investment weights in the optimized portfolio.
- **Assets (L, S, Tot)** is the number of assets the optimized portfolio is invested in long, short and total (long + short) positions.
- **InvWeight** is the investment weight, expressed as a percentage of the investment portfolio.
- **ECOBAR*** is the ECOBAR score when applying the inverse function for short positions.
- **Risk Share** is the risk contribution of a single investment to the investment portfolio, based on the size of the single investment weight, in relation to the size of the total investment portfolio.

4.1 General Characteristics

The following section intends to give a brief understanding of the overall behavior of the portfolio optimization, before studying the obtained results in more detail.

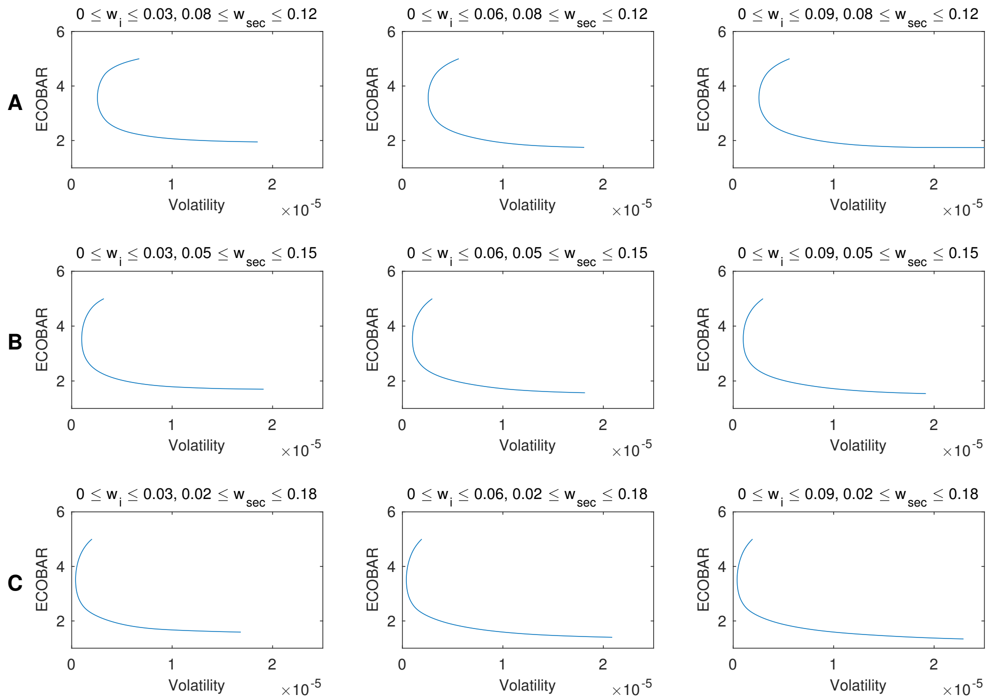

Figure 2: An illustration of how the efficient frontiers, described in section 2.3.1 and represented by the blue lines, behave when alternating sector and individual investment weight bounds for long only portfolios. The data is from the calibration period and the x-axis is the volatility of the loss function.

Above in Figure 2, several subplots can be seen that show how the minimization was affected when the sector bounds, w_{sec}, as well as the individual weights, w_i, were loosened. Here, no shorting was allowed as it was done to get an idea how the bounds affect the minimization process in general. Note that the sector constraints has the greatest impact of whether an optimal portfolio can be found. The data in Figures 2 and 3 is from the calibration period for iTraxx Main. The data from CDX.IG behaves in the same way.

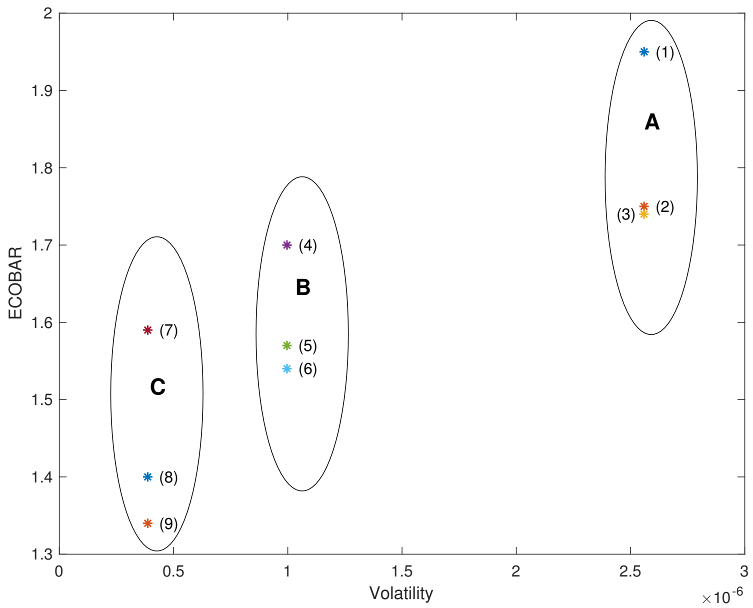

Figure 3: Minimum ECOBAR scores and minimum volatility for each subplot in Figure 2. Letters A, B and C indicate the row of corresponding subplots in Figure 2.

In Figure 3, the minimum volatility of the loss function as well as the minimum ECOBAR value in each of the subplots in Figure 2 are shown. Labels 1-3 represents the first row of subplots in Figure 2 from left to right, Labels 4-6 represents the middle row of subplots in Figure 2, and so forth. Here it becomes even clearer that loosening the individual weights leads to a smaller ECOBAR value of the portfolio, but not to a change in the minimum volatility. Moreover, loosening the sector bounds leads to both a smaller ECOBAR value as well as a smaller volatility, i.e. a smaller tracking error.

4.2 Base Case

The following section contains the results obtained from the base case of the portfolio, i.e. the replication of the two actual CDS indices, iTraxx Main and CDX.IG.

Table 1: Illustration of a portfolio in the base case.

Constituent	ECOBAR	InvWeight
Constituent 1	1	10%
Constituent 2	1	10%
Constituent 3	2	10%
Constituent 4	3	10%
Constituent 5	3	10%
Constituent 6	4	10%
Constituent 7	4	10%
Constituent 8	6	10%
Constituent 9	9	10%
Constituent 10	9	10%
Portfolio ECOBAR	**4.2**	

Table 1 illustrates a portfolio in the base case. The investments are equally weighted for ten constituents. GHG effective constituents are marked in green and GHG ineffective constituents are marked in brown. The resulting ECOBAR portfolio score is 4.2.

Table 2: Parameters for the base case of the replicated CDS indices, iTraxx Main and CDX.IG.

Strategy / Parameter	iTraxx Main Replication	CDX.IG Replication
ECOBAR	3.41	3.89
Tracking Error	0	0
Correlation	1	1
TotWeightLong (%)	100	100
TotWeightShort (%)	0	0
WeightLb	1/107	1/102
WeightUb	1/107	1/102
Assets (L, S, Tot)	(107, 0, 107)	(102, 0, 102)

Table 2 shows the general characteristics of the replicated iTraxx Main and CDX.IG indices. These are the reduced indices after the data has been washed, that contain 107 constituents for iTraxx Main and 102 constituents for CDX.IG. When all assets have equal weights, i.e. 1/107 for iTraxx Main and 1/102 for CDX.IG, the total ECOBAR amounts to 3.41 for iTraxx Main and 3.89 for CDX.IG. These are the values that the optimization aims to reduce, whilst keeping the tracking error as close to 0 as well as a correlation as close to 1 as possible. Note that the ECOBAR scores for both indices differs from the expected ECOBAR portfolio value of 4.0. This is due to the fact that the sectors contained different

number of constituents, which lead to uneven ECOBAR sector scores for the constituents in some cases.

The reason why iTraxx Main had a lower score than CDX.IG was also due to the green bond reduction ratio, which reduced the ECOBAR value for iTraxx Main more since the constituents in that index have issued more green bonds than the CDX.IG constituents. Furthermore, Erlandsson has observed slightly lower mean ECOBAR scores than 4.0 for iTraxx Main, CDX.IG and a portfolio of Fortune 500 companies in previous research [14]. These ECOBAR portfolio deviations are therefore not uncommon.

4.3 Long Strategy

The following section contains the results obtained from the portfolio optimization when investments are made in long positions only. The results are from the validation period.

Table 3: Illustration of a portfolio for a long only investment strategy.

Constituent	**ECOBAR**	**InvWeight**
Constituent 1	1	20%
Constituent 2	1	20%
Constituent 3	2	10%
Constituent 4	3	10%
Constituent 5	3	10%
Constituent 6	4	10%
Constituent 7	4	10%
Constituent 8	6	10%
Constituent 9	9	0%
Constituent 10	9	0%
Portfolio ECOBAR	**2.6**	

Table 3 illustrates a portfolio for a long only investment strategy. The investment positions are increased in GHG effective constituents, while no positions are taken in GHG ineffective ones. GHG effective constituents are marked in green and GHG ineffective constituents are marked in brown. The resulting ECOBAR portfolio score is thereby reduced to 2.6.

Table 4: Long strategy comparison when number of assets invested in, i.e. assets that have a weight significantly larger than 0, are forced to increase, for iTraxx Main.

Strategy / Parameter	L0	L1	L2	L3	L4
ECOBAR	1.77	1.81	1.85	1.88	1.92
Tracking Error (%)	0.1124	0.1070	0.1006	0.1019	0.0957
Correlation	0.987	0.988	0.989	0.989	0.990
TotWeightLong (%)	100	100	100	100	100
TotWeightShort (%)	0	0	0	0	0
WeightLb	0	0	0	0	0
WeightUb	3/107	3/107	3/107	3/107	3/107
Assets (L, S, Tot)	(41, 0, 41)	(56, 0, 56)	(70, 0, 70)	(81, 0, 81)	(97, 0, 97)

Table 5: Long strategy comparison when number of assets invested in, i.e. assets that have a weight significantly larger than 0, are forced to increase, for CDX.IG.

Strategy / Parameter	L0	L1	L2	L3	L4
ECOBAR	2.00	2.04	2.08	2.11	2.14
Tracking Error (%)	0.1462	0.1459	0.1456	0.1532	0.1460
Correlation	0.969	0.969	0.968	0.965	0.968
TotWeightLong (%)	100	100	100	100	100
TotWeightShort (%)	0	0	0	0	0
WeightLb	0	0	0	0	0
WeightUb	3/102	3/102	3/102	3/102	3/102
Assets (L, S, Tot)	(36, 0, 36)	(52, 0, 52)	(66, 0, 66)	(79, 0, 79)	(91, 0, 91)

As can be seen in Tables 4 and 5, the ECOBAR value increases when the number of assets used in the optimization are forced to increase. This is quite logical since a weight of zero contributes by an ECOBAR score of zero for the portfolio. The optimization algorithm aims to invest as little as possible in assets with high ECOBAR scores, leading to a decreased total ECOBAR score when it is allowed to go zero weights in more assets. Moreover, even when one is taking short positions in assets with high ECOBAR scores, obtaining the inverse ECOBAR score, it is still more favourable to have zero investment weights in those assets. This property of the ECOBAR model becomes very obvious when one minimizes the ECOBAR score, and no other constraints are taken into consideration.

The ECOBAR scores for CDX.IG are larger than for iTraxx, which can be explained by the fact that CDX.IG has a higher base case ECOBAR score, as can be seen in Table 2.

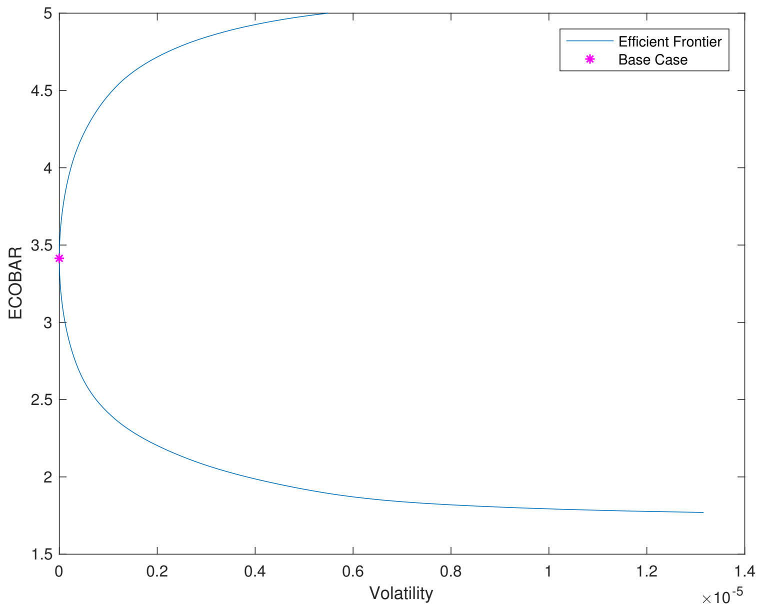

Figure 4: The long only portfolio efficient frontier and the replicated index base case for iTraxx Main. The efficient frontier for CDX.IG is similar.

Figure 4 shows the efficient frontier from the optimization conducted in the calibration period when trying to minimize the ECOBAR score as well as the volatility of the loss function. The base case has the values from Table 2. It is observed that the ECOBAR score can be lowered significantly, with the trade-off of higher volatility for the loss function, i.e. higher tracking error.

4.4 Long-Short Strategy

The following section contains the results obtained from the portfolio optimization when investments are made in both long and short positions. The results are from the validation period.

Table 6: Illustration of a portfolio for a long-short investment strategy.

Constituent	ECOBAR	InvWeight	Sector	ECOBAR*	Risk Share
Constituent 1	1	25%	A	1	11.4%
Constituent 2	1	25%	B	1	11.4%
Constituent 3	2	22%	C	2	10%
Constituent 4	3	22%	C	3	10%
Constituent 5	3	22%	B	3	10%
Constituent 6	4	22%	B	4	10%
Constituent 7	4	22%	A	4	10%
Constituent 8	6	−20%	A	2	9.1%
Constituent 9	9	−20%	B	1	9.1%
Constituent 10	9	−20%	C	1	9.1%
Portfolio ECOBAR	**2.2**				

Table 6 illustrates a portfolio for a long-short investment strategy. Long positions are taken in GHG effective constituents and short positions are taken in GHG ineffective ones. GHG effective constituents are marked in green and GHG ineffective constituents are marked in brown. The resulting ECOBAR portfolio score is 2.2.

4.4.1 Individual Weight Variation

Varying the individual investment weights changes how large and small values the investment weights illustrated in Table 6 can take, when optimizing the portfolio. For instance, if the lower bound is set to $-1/10$, the investment weights for Constituent 8-10 would only be able to be between -10% and 0%. Correspondingly, an upper bound set to $3/10$ would allow the investment weights for Constituent 1-7 to be between 0% and 30%.

Table 7: How relaxing individual weights bounds, IWV, affect the optimization outputs when shorting 15 assets for iTraxx Main.

Strategy / Parameter	IWV1	IWV3	IWV10	IWV100
ECOBAR	1.85	1.80	1.78	1.78
Tracking Error (%)	0.1321	0.1376	0.2016	0.2032
Correlation	0.980	0.980	0.963	0.962
TotWeightLong (%)	114	114	114	114
TotWeightShort (%)	−14	−14	−14	−14
WeightLb	−1/107	−3/107	−10/107	−100/107
WeightUb	5/107	15/107	50/107	500/107
Assets (L, S, Tot)	(37, 15, 52)	(38, 6, 44)	(32, 4, 36)	(32, 4, 36)

Table 8: How relaxing individual weights bounds, IWV, affect the optimization outputs when shorting 15 assets for CDX.IG.

Strategy / Parameter	IWV1	IWV3	IWV10	IWV100
ECOBAR	2.01	1.99	1.99	1.99
Tracking Error (%)	0.1953	0.1794	0.1746	0.1756
Correlation	0.942	0.950	0.953	0.953
TotWeightLong (%)	114.7	114.7	114.7	114.7
TotWeightShort (%)	-14.7	-14.7	-14.7	-14.7
WeightLb	$-1/102$	$-3/102$	$-10/102$	$-100/102$
WeightUb	5/102	15/102	50/102	500/102
Assets (L, S, Tot)	(35, 15, 50)	(36, 8, 44)	(36, 4, 40)	(36, 4, 40)

By relaxing the individual weight bounds, one obtains a lower ECOBAR score, as seen in Tables 7 and 8. The tracking errors and correlations behave differently between the two indices, with values generally worsening for iTraxx Main and improving for CDX.IG as IWV increases. Note that the parameters stabilizes at IWV10 for iTraxx Main, and IWV3 for CDX.IG.

4.4.2 Sector Weight Variation

Varying the sector weights adjusts how close to the sector allocation of the actual CDS indices the sector weights are required to be. In Table 6 the sector weight allocation for sector A is $25\% + 22\% - 20\% = 27\%$. To variate this with $\pm 10\,\%$, would be to relax the sector weight to be between values of $(1 - 0.1) \cdot 27\%$ and $(1 + 0.1) \cdot 27\%$, i.e. between 24.3% and 29.7%. Thus, the optimization is less restricted in this case.

In Tables 9 and 10, the starting sector allocations are those of the base cases of the replicated indices, which are all varied in the same way as in the example above. In all other performed portfolio optimizations presented in sections 4.3 and 4.4, the sector allocations are fixed, hence only allowed to be the exact allocations of the base cases of the replicated indices.

Table 9: How relaxing the sector weights, SWV, by ±10 %, ±20 %,...,±50 %, affects the optimization outputs when shorting 15 assets for iTraxx Main.

Strategy / Parameter	SWV0	SWV10	SWV20	SWV30	SWV40	SWV50
ECOBAR	1.84	1.75	1.73	1.70	1.68	1.65
Tracking Error (%)	0.1151	0.1176	0.1064	0.1033	0.1057	0.1091
Correlation	0.987	0.987	0.988	0.988	0.987	0.986
TotWeightLong (%)	107	107	107	107	107	107
TotWeightShort (%)	−7	−7	−7	−7	−7	−7
WeightLb	−1/107	−1/107	−1/107	−1/107	−1/107	−1/107
WeightUb	3/107	3/107	3/107	3/107	3/107	3/107
Assets (L, S, Tot)	(43, 8, 51)	(45, 9, 54)	(45, 10, 55)	(51, 8, 59)	(53, 8, 61)	(50, 9, 59)

Table 10: How relaxing the sector weights, SWV, by ±10 %, ±20 %,...,±50 %, affects the optimization outputs when shorting 15 assets for CDX.IG.

Strategy / Parameter	SWV0	SWV10	SWV20	SWV30	SWV40	SWV50
ECOBAR	2.12	2.03	1.98	1.93	1.91	1.89
Tracking Error (%)	0.1683	0.1603	0.1572	0.1778	0.1794	0.1778
Correlation	0.963	0.966	0.967	0.952	0.953	0.952
TotWeightLong (%)	107.4	107.4	107.4	107.4	107.4	107.4
TotWeightShort (%)	−7.4	−7.4	−7.4	−7.4	−7.4	−7.4
WeightLb	−1/102	−1/102	−1/102	−1/102	−1/102	−1/102
WeightUb	3/102	3/102	3/102	3/102	3/102	3/102
Assets (L, S, Tot)	(41, 9, 50)	(42, 9, 51)	(40, 8, 48)	(41, 9, 50)	(41, 9, 50)	(42, 9, 51)

The base case sector weights of the replicated portfolios of iTraxx Main and CDX.IG, SWV0, in Tables 9 and 10, can be found in Figure 11 in Appendix A and Figure 16 in Appendix B. These weights have then been relaxed in portfolios SWV10,...,SWV50 to show how they affect the optimization. As can be seen in Tables 9 and 10, the sector constraints impact the ECOBAR score more than it influence the correlation or tracking error. A fully disclosed optimized portfolio SWV0 for iTraxx Main, where WeightUb is set to 5/107, can be seen in Table 21 in Appendix F.

If a portfolio manager wants to keep the sector allocations totally stable, by looking at SWV0, one sees that the ECOBAR score can be reduced significantly compared to the base case scores of 3.41 and 3.89, found in Table 2. It is also noted however, that an even lower ECOBAR score can be obtained, if the sector constraints are relaxed.

4.4.3 Leverage Variation

Varying the leverage adjusts what the summations of all long weights, and all short weights respectively, adds up to. As illustrated in Table 6, the summation of the long position weights are 160%, and the summation of short position weights are −60%. As long as the total sum of all weights

adds up to 100% (160% + (−60%)), the leverage can be increased or decreased as preferred. Given the example of 300% in long weights and −200% in short weights, this portfolio would be denoted LV200 in Tables 11-12.

Table 11: Variation of leverage, from 0% to 420%, when shorting 15 assets for iTraxx Main.

Strategy / Parameter	LV0	LV7	LV14	LV112	LV189	LV420
ECOBAR	1.75	1.76	1.78	1.85	1.88	1.88
Tracking Error (%)	0.1308	0.1746	0.2032	0.8461	1.1223	2.8574
Correlation	0.982	0.971	0.962	0.734	0.644	0.456
TotWeightLong (%)	100	107	114	212	289	520
TotWeightShort (%)	0	−7	−14	−112	−189	−420
WeightLb	0	−300/107	−300/107	−300/107	−300/107	−300/107
WeightUb	900/107	900/107	900/107	900/107	900/107	900/107
Assets (L, S, Tot)	(36, 0, 36)	(32, 1, 33)	(32, 4, 36)	(20, 6, 26)	(19, 9, 28)	(13, 4, 17)

Table 12: Variation of leverage, from 0% to 441%, when shorting 15 assets for CDX.IG.

Strategy / Parameter	LV0	LV7	LV15	LV118	LV199	LV441
ECOBAR	1.99	1.99	1.99	2.00	2.01	2.01
Tracking Error (%)	0.1491	0.1603	0.1746	0.4000	0.5985	1.2493
Correlation	0.966	0.961	0.953	0.805	0.674	0.393
TotWeightLong (%)	100	107.4	114.7	218	299	541
TotWeightShort (%)	0	−7.4	−14.7	−118	−199	−441
WeightLb	0	−300/102	−300/102	−300/102	−300/102	−300/102
WeightUb	900/102	900/102	900/102	900/102	900/102	900/102
Assets (L, S, Tot)	(36, 0, 36)	(36, 2, 38)	(36, 4, 40)	(31, 12, 43)	(29, 13, 42)	(25, 14, 39)

If a portfolio manager wants to increase the leverage of their portfolio, it could be done without increasing the ECOBAR score substantially, as seen in Tables 11 and 12. It does however impact the tracking error and correlation of the replicated index negatively, as can be seen when comparing, for example, portfolios LV420 and LV441 with corresponding LV0 portfolios.

4.4.4 Utilize All Assets

As illustrated in Table 6, all assets are utilized since no investment weights are 0%. However as illustrated in Table 3, Constituents 9-10 have investment weights 0% and are thus not invested in the portfolio.

If one would like to perform a portfolio optimization where no investment weights are 0%, i.e. to utilize all assets, each investment weight of the portfolio could be forced to be at least 5%, for example. In the

section below, all investment weights are forced to be at least $\pm 0.1\,\%$, depending on if the weights are to be long or short positions.

Table 13: Variation of the number of assets to take short positions in, while forcing to invest at least $\pm 0.1\,\%$ in each of the 107 constituents of iTraxx Main.

Strategy / Parameter	UAS0	UAS5	UAS10	UAS15	UAS20	UAS25
ECOBAR	1.94	1.92	1.90	1.89	1.88	1.87
Tracking Error (%)	0.1041	0.1019	0.1059	0.1075	0.1162	0.1157
Correlation	0.988	0.988	0.987	0.987	0.985	0.985
TotWeightLong (%)	100	102.3	104.7	107	109.4	111.7
TotWeightShort (%)	0	−2.3	−4.7	−7	−9.4	−11.7
WeightLb	1/1000	−1/107	−1/107	−1/107	−1/107	−1/107
WeightUb	5/107	5/107	5/107	5/107	5/107	5/107
Assets (L, S, Tot)	(107, 0, 107)	(102, 5, 107)	(97, 10, 107)	(92, 15, 107)	(87, 20, 107)	(82, 25, 107)

Table 14: Variation of the number of assets to take short positions in, while forcing to invest at least $\pm 0.1\,\%$ in each of the 102 constituents of CDX.IG.

Strategy / Parameter	UAS0	UAS5	UAS10	UAS15	UAS20	UAS25
ECOBAR	2.18	2.14	2.12	2.10	2.08	2.06
Tracking Error (%)	0.1392	0.1403	0.1478	0.1514	0.1622	0.1708
Correlation	0.971	0.970	0.967	0.965	0.960	0.955
TotWeightLong (%)	100	102.5	104.9	107.4	109.8	112.3
TotWeightShort (%)	0	−2.5	−4.9	−7.4	−9.8	−12.3
WeightLb	1/1000	−1/102	−1/102	−1/102	−1/102	−1/102
WeightUb	5/102	5/102	5/102	5/102	5/102	5/102
Assets (L, S, Tot)	(102, 0, 102)	(97, 5, 102)	(92, 10, 102)	(87, 15, 102)	(82, 20, 102)	(77, 25, 102)

When forcing the portfolio optimization to not set any investment weights to zero, but rather to invest in all assets composing the index, the results differ slightly from earlier optimizations. By observing Tables 13 and 14, it becomes clear that when utilizing all assets in the optimization, one will decrease the ECOBAR score when taking short positions in more assets. Compare portfolios UAS0 where no short positions are taken, with portfolios UAS25 where short positions are taken in 25 assets, for instance. The result is logical, since the assets which are shorted are the ones with the highest ECOBAR scores, thus obtaining lower scores from the ECOBAR inverse function when shorted. The values of the tracking errors and correlations are worsened when more assets are shorted, which also appears logical since the portfolio composition then moves further away from the base cases of the indices.

4.5 Green-Brown Baskets

The following section contains the results obtained from the replication of the CDS indices, by only using the greenest and brownest constituents to compose the baskets, and apply long-short strategies. Thus obtaining

an efficient green-brown basket composition, where the aim is to decrease the ECOBAR score even further than before, while still retaining as low tracking error and high correlation as possible. The results are from the validation period.

Table 15: Illustration of a portfolio for a green-brown basket investment strategy.

Constituent	**ECOBAR**	**InvWeight**	**ECOBAR***	**Risk Share**
Constituent 1	1	50%	1	16.7%
Constituent 2	1	50%	1	16.7%
Constituent 3	1	50%	1	16.7%
Constituent 4	2	25%	2	8.3%
Constituent 5	2	25%	2	8.3%
Constituent 6	6	−12.5%	2	4.2%
Constituent 7	6	−12.5%	2	4.2%
Constituent 8	9	−25%	1	8.3%
Constituent 9	9	−25%	1	8.3%
Constituent 10	9	−25%	1	8.3%
Portfolio ECOBAR	**1.25**			

Table 15 illustrates a portfolio for a green-brown basket investment strategy. Long positions are taken in GHG effective constituents and short positions are taken in GHG ineffective ones. GHG effective constituents are marked in green and GHG ineffective constituents are marked in brown. The resulting ECOBAR portfolio score is 1.25.

4.5.1 Portfolio Size Variation

Table 16: Variation of basket size of $n/2$ green and $n/2$ brown constituents, 5G5B, 10G10B,...,25G25B for green-brown portfolios of iTraxx Main, with fixed total shorting investment weights of −10%. Individual investment weight bounds are $-1/n$ and $3/n$.

Strategy / Parameter	**5G5B**	**10G10B**	**15G15B**	**20G20B**	**25G25B**
ECOBAR	0.99	1.00	1.00	1.00	1.00
Tracking Error (%)	0.3737	0.2864	0.2212	0.2051	0.2067
Correlation	0.911	0.908	0.945	0.952	0.953
TotWeightLong (%)	110	110	110	110	110
TotWeightShort (%)	−10	−10	−10	−10	−10
WeightLb	−1/10	−1/20	−1/30	−1/40	−1/50
WeightUb	3/10	3/20	3/30	3/40	3/50
Assets (L, S, Tot)	(5, 3, 8)	(10, 4, 14)	(15, 3, 18)	(19, 5, 24)	(19, 5, 25)

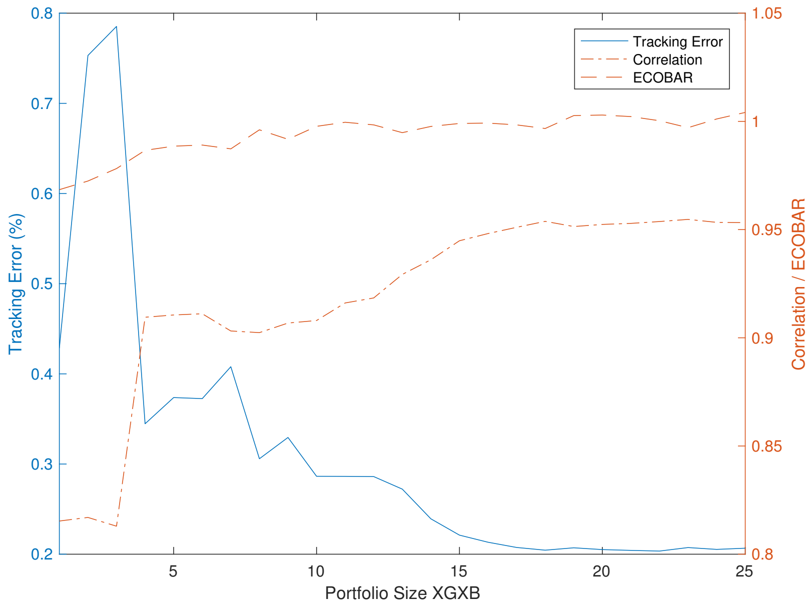

Figure 5: Variation of ECOBAR score, tracking error and correlation for green-brown portfolios of iTraxx Main, for portfolio size variations of 1G1B, 2G2B,...,25G25B.

Table 17: Variation of basket size of $n/2$ green and $n/2$ brown constituents, 5G5B, 10G10B,...,25G25B for green-brown portfolios of CDX.IG, with fixed total shorting investment weights of -10%. Individual investment weight bounds are $-1/n$ and $3/n$.

Strategy / Parameter	5G5B	10G10B	15G15B	20G20B	25G25B
ECOBAR	1.00	1.00	1.25	1.42	1.52
Tracking Error (%)	0.3363	0.3078	0.2579	0.2337	0.2312
Correlation	0.814	0.852	0.897	0.915	0.917
TotWeightLong (%)	110	110	110	110	110
TotWeightShort (%)	-10	-10	-10	-10	-10
WeightLb	$-1/10$	$-1/20$	$-1/30$	$-1/40$	$-1/50$
WeightUb	$3/10$	$3/20$	$3/30$	$3/40$	$3/50$
Assets (L, S, Tot)	(5, 2, 7)	(9, 2, 11)	(14, 3, 17)	(17, 5, 22)	(22, 6, 28)

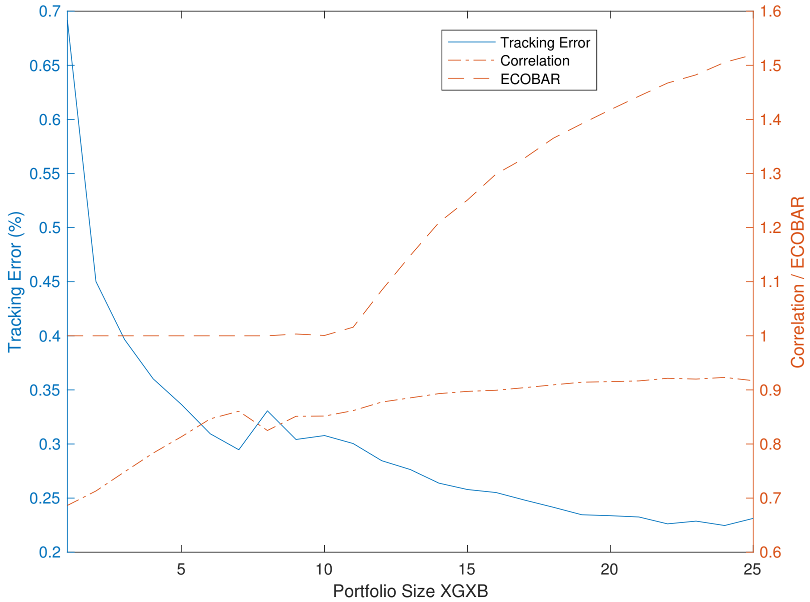

Figure 6: Variation of ECOBAR score, tracking error and correlation for green-brown portfolios of CDX.IG, for portfolio size variations of 1G1B, 2G2B,...,25G25B.

In Tables 16 and 17, different sized green-brown baskets have been optimized, with an ECOBAR score of around 1.0 for iTraxx Main, and ranging between 1.0-1.52 for CDX.IG. Note that there are no sector weight allocation constraints applied in this section. It becomes clear that when the baskets contain a broader set of assets from the index, generally, a lower tracking error and lower correlation is obtained. Since the indices contain over 100 assets, that seems logical. It is, however, notable that one can mimic the CDS indices with barely ten assets, as in the 5G5B portfolio case, and still obtain a small tracking error and a correlation of 0.910 for iTraxx Main, and 0.814 for CDX.IG respectively.

Figures 5 and 6 displays how the ECOBAR scores, tracking errors and correlations change when the size of the baskets are varied, for the corresponding portfolios seen in Tables 16 and 17. When observing the two figures, it becomes clear that the portfolios in general obtain lower tracking errors and higher correlations when the size of the baskets increases, at the cost of a higher ECOBAR score. There are some irregularities in this trend however, thus a portfolio manager should study this behaviour carefully before constructing a specific basket.

The reason why the ECOBAR score increases more for the green-brown CDX.IG baskets, than for the green-brown iTraxx Main baskets, is simply

the fact that there are fewer constituents in CDX.IG that have ECOBAR scores of 1 (for long positions) and 9 (for short positions).

The reason for this is mainly because the sectors which have received an ECOBAR sector score of 1 in CDX.IG contain remarkably fewer constituents than the corresponding sectors in iTraxx Main. This leads to fewer constituents in CDX.IG being able to obtain a total ECOBAR score of 1. In a perfect world where all sectors in both indices contain the same amount of constituents, the shown trend for the ECOBAR scores would be more similar for the two.

4.5.2 Leverage Variation

Table 18: Variation of leverage, LV, ranging between 0% and 150% for a 20G20B basket of iTraxx Main.

Strategy / Parameter	LV0	LV30	LV60	LV90	LV120	LV150
ECOBAR	1.05	0.97	0.98	0.99	0.99	0.99
Tracking Error (%)	0.1873	0.4969	0.5397	0.6080	0.6969	0.7953
Correlation	0.961	0.780	0.740	0.690	0.630	0.560
TotWeightLong (%)	100	130	160	190	220	250
TotWeightShort (%)	0	−30	−60	−90	−120	−150
WeightLb	0	−100/40	−100/40	−100/40	−100/40	−100/40
WeightUb	300/40	300/40	300/40	300/40	300/40	300/40
Assets (L, S, Tot)	(21, 0, 21)	(6, 3, 9)	(8, 3, 11)	(8, 3, 11)	(11, 3, 14)	(13, 3, 16)

Table 19: Variation of leverage, LV, ranging between 0% and 150% for a 20G20B basket of CDX.IG.

Strategy / Parameter	LV0	LV30	LV60	LV90	LV120	LV150
ECOBAR	1.00	1.00	1.00	1.00	1.00	1.00
Tracking Error (%)	0.2969	0.3492	0.4445	0.5556	0.6731	0.7953
Correlation	0.858	0.811	0.740	0.663	0.595	0.537
TotWeightLong (%)	100	130	160	190	220	250
TotWeightShort (%)	0	−30	−60	−90	−120	−150
WeightLb	0	−100/40	−100/40	−100/40	−100/40	−100/40
WeightUb	300/40	300/40	300/40	300/40	300/40	300/40
Assets (L, S, Tot)	(9, 0, 9)	(9, 3, 12)	(10, 4, 14)	(9, 4, 13)	(9, 4, 13)	(9, 4, 13)

When increasing the leverage for a basket of the 20 greenest and 20 brownest constituents, as observed in Tables 18 and 19, the ECOBAR scores are kept steady around 1, even when setting the shorting leverage as high as 150%. It does however affect the tracking error and correlation negatively, similar to the behaviour observed in previous sections.

5 Discussion and Conclusions

This chapter holds an overall discussion and presents the conclusions from the analysis of the results described in previous chapters. Furthermore, the shortcomings and assumptions made are reviewed. A real world example illustrating the potential impact of the techniques presented in this thesis is also revealed.

5.1 Portfolio Optimization

When observing the results in the previous section, some general conclusions can be drawn. First and foremost, it is clearly possible to lower the GHG emission exposure when optimizing the portfolio. Moreover, it is possible to optimize the portfolio while keeping the sector allocations totally intact for each respective index. By studying the tracking errors and correlations, it is noticed that they deviate from the base case of the CDS indices. In addition, they vary between the different optimized portfolios tested. Nonetheless, it cannot be expected to have a correlation of 1 and a tracking error of 0 when optimizing portfolios consisting of over 100 assets, and the parameters are quite close in general. It should also be stated that portfolio managers construct and manage investment portfolios for different applications and purposes. Thus, it is difficult to determine whether a specific level of tracking error or correlation is sufficient, since it is highly individual.

The different optimized portfolio variations presented in the previous section affects the tracking errors and correlations differently. The leverage variation seems to have the largest impact on both correlations and tracking errors, worsening both parameters considerably, as can be seen in Tables 11 and 12. It is also obvious that when the constraints are less restricted, i.e. when loosening the individual weights and sector weights as in Tables 7-8 and 9-10, the ECOBAR score can obtain lower values.

When comparing the results for the two different indices, one can draw the conclusion that the portfolio optimization have similar behaviour for the majority of the portfolios tested. The model appears robust and should work sufficiently for other data sets and applications. Generally, the optimized portfolios of iTraxx Main seem to outperform the optimized portfolios of CDX.IG in terms of minimum ECOBAR score, tracking error and correlation. One explanation could be the data coverage aspect, where the iTraxx Main portfolio optimizations included 107 constituents, compared to 102 constituents for CDX.IG. This combined with the fact that the number of constituents in each sector differs

between the two indices, leading to iTraxx Main having notably more constituents with ECOBAR scores of 1 for long positions, respectively 9 for short positions to optimize over. Furthermore, iTraxx Main has additional constituents with outstanding green bonds, as well a lower ECOBAR score in the base case. Thus, the portfolio optimization seems to have a similar effect for both indices, with iTraxx Main having a superior starting point of the two.

An interesting observation is the fact that the long only optimized portfolios of the CDS indices obtain lower ECOBAR scores than the long-short optimized portfolios. Note that this is the case when not all assets are utilized, i.e. not in Tables 13 and 14. The explanation is simply because the long only portfolio assign zero investment weights to assets with high ECOBAR scores. This results in a lower ECOBAR score than when taking a short position in the respective asset, resulting in the inverse ECOBAR score. However, when forcing the optimization model to short a specific amount, it optimizes admirably over that constraint. If a portfolio manager requires all assets to be invested in, the long-short portfolio will result in a lower ECOBAR score than the long only portfolio. This can be seen in Tables 13 and 14, when comparing long only portfolios UAS0, with long-short portfolios UAS5-UAS25.

When observing the green-brown baskets of the CDS indices in section 4.5, it is evident that one can create portfolios with ECOBAR scores as low as 1. This is the lowest score a constituent can obtain, without having issued any green bonds. For these baskets the tracking errors are higher and the correlations are lower, compared to the portfolios which are constructed from the entire indices. However, the parameters are still decent and it is therefore suggested that these baskets could be used to mimic the indices, with slightly less accuracy.

It has been shown that by implementing long only as well as long-short portfolio strategies of the CDS indices, one can obtain significantly lower ECOBAR scores than the base case of the indices. Moreover, it is evident that it can be done whilst still maintaining a low tracking error and high correlation with the indices. If the individual investment weights and sector constraints are to be loosened, the ECOBAR score becomes even lower. When constructing baskets of fewer constituents, one can replicate the indices with merely 10-30 constituents, without worsening the tracking error and correlation substantially. By doing so, one can create highly effective CDS baskets with considerably lower exposures of GHG emissions.

5.2 Shortcomings and Assumptions

The methods used in the conducted analysis have some drawbacks and are based on some assumptions, which are presented in this section.

The collected data sets were not complete which led to the fallout of some constituents of the CDS indices, either because of GHG data shortage, or insufficient CDS spreads data. Despite this however, the indices could be replicated sufficiently well as is presented in section 3.7.2, even though there was a fallout of constituents.

One reason why the spread data sets were not complete is because constituents are added and removed from the CDS indices every six months when they are re-rolled. If one wants to have consistency within the scraped data, it is difficult to use a larger time span than the three years used. Thus, two years of data was used for the calibration period and one year for the validation period. The data used for the calibration period could otherwise have been extended, which could potentially have lead to even more stable portfolio optimizations.

Potential errors could arise from using correlation trajectories to fill missing spread data. In the case when several spreads are missing in a row for a particular CDS, the calculated synthetic spread trajectory could be influenced by a number of different CDS, e.g. period 1-2 by CDS X, period 2-3 by CDS Y and so forth. While all of them have a high individual correlation with the approximated CDS spread, the resulting CDS path might have deceptive behaviour. Nonetheless, the amount of approximated data in each data set using this method was minor, 0.78% for iTraxx Main and 0.39% for CDX.IG.

One part of the optimization process that might affect the results, was that when implementing long-short strategies, the long-short positions for each constituent had to be assigned before initiating the optimization process. This was a requirement to be able to solve the MMV problem with the implemented quadratic programming algorithm *quadprog*. Once the weights are set, *quadprog* could optimize freely over all assets for both long and short positions. However, it was not possible to let *quadprog* choose freely which assets to short and which assets to take long positions in. This problem arose from the nature of using the inverse function to assign ECOBAR scores for short positions. Thus, the weights had to be assigned with the regular score and the inverse score at the same time, for which no optimal solutions could be found.

Modern portfolio theory and the efficient frontier have several assumptions that may not properly represent reality. Markowitz Mean Variance model only describes the complete distribution when the returns follow a normal distribution. In other cases the model could miss out on information about the distribution, which could have importance. The spread returns used to calibrate the Σ matrix to solve the MMV problem seems to rather follow a t-distribution.

MMV theory also assumes that investors are rational and avoid risk when possible, that there are not large enough investors to influence market prices and that investors have unlimited access to borrowing and lending money at the risk-free interest rate. However, in reality the market rather includes irrational and risk-seeking investors, large market participants who could influence market prices, and investors do not have unlimited access to borrowing and lending money.

5.3 Greenhouse Gas Reduction

The main goal of this thesis is to provide evidence showing that it is possible to construct sufficient GHG effective portfolio strategies for commonly used fixed income instruments such as CDS indices. Hopefully, this will contribute toward a more long term goal and possibly shift the investment trends within debt financing in order to fight climate change.

Moreover, the intention of this thesis is not to guarantee explicit reductions of the GHG emitted into the atmosphere. However, to give an idea in distinct numbers and hopefully an eye opener, a real world example is here provided, illustrating the reduction of GHG exposure in the optimized portfolios presented.

Take a hedge fund similar to the Glacier Impact Climate Fund with \$100 million in assets and implement the techniques presented in this thesis to minimize the ECOBAR score for the portfolio. If one were to lower the ECOBAR score of the long only portfolio from the iTraxx Main index benchmark of 3.41 to 1.77, as has been shown is possible in section 4.3, it would reduce the GHG exposure by roughly 100,000 tons of GHG emissions per year, calculated according to Equation (6.1) and Equation (6.2) in Appendix G. This corresponds to a GHG footprint of approximately 2600 flying hours for a fully loaded Boeing 747, during which it could manage a distance equal to 59 laps around Earth. For CDX.IG it has been showed that for a similar long only portfolio, one can lower the ECOBAR score from 3.89 to 2.00, reducing the carbon footprint by

approximately 35,000 tons of GHG emissions per year, which in the very same way corresponds to 21 laps for a 747 around Earth. Expressing the same numbers in terms of percentages, these portfolio optimizations reduce the GHG exposures by 86% for iTraxx Main and 84% for CDX.IG, respectively. Hence, it is noted that the effectiveness of the implemented portfolio optimizations is similar for the two indices. There is merely a difference in how much GHG exposure the indices have in each of their base case, which influence the corresponding amount of laps for a 747 around Earth, for the two indices.

Be aware that this does not necessarily lead to such amounts of GHG are avoided from being emitted into the atmosphere, since the constituents are still operative. Imagine however if numerous portfolio managers used the techniques presented in this thesis simultaneously. As mentioned earlier, iTraxx Main and CDX.IG had a combined trading volume of roughly half a trillion USD notional per month in 2017. Consequently, these highly liquid CDS indices do have a substantial impact in the financial markets. It is widely argued among market practitioners that CDS have an influence on bond prices.

A recent example on this subject was during the European sovereign crisis in 2010-2012, where CDS affected the bond prices. So called naked CDS were banned due to the large implication they had on the bond market, escalating the crisis [34]. With the trading volumes of CDS in mind, it becomes evident what impact the ECOBAR minimization of CDS baskets could have on underlying bond prices, leading to increased corporate bond prices for the worst emitters. By so, a real effect is observed regarding the financing for so-called brown companies, which could impact and hopefully reduce the GHG emitted into the atmosphere.

6 Delimitations and Further Studies

This chapter describes how the thesis objectives and scope can be delimited and how the applied methodology can be extended for further analysis in future work.

A natural extension of the thesis work would be to study other types of applications for CDS, such as first-to-default swaps, tranching techniques or other CDS indices. Similarly, other types of financial instruments could be studied by applying equivalent methods as the ones presented in this thesis. There is still a lot to be explored regarding climate impact within fixed income markets.

Alternative implementations of the portfolio optimization algorithm could be conducted, such as improving the optimization algorithm to enable it to choose which assets to take long and short positions in, which has to be done manually in the current implementation. As mentioned, the properties of the ECOBAR model using the inverse function for short positions, is the cause of this obstacle when implementing a quadratic programming algorithm such as *quadprog*.

The dependence modelling of the assets in the calibration process of the portfolio optimization could be further improved. As mentioned, Markowitz Mean Variance model assumes that asset returns are normally distributed, which is required to be able to fully describe their dependence behavior. Copula based dependence modelling capturing the actual t-distribution of the CDS returns, could potentially calibrate the model more accurately and thus improve the results.

Further work could be made to confirm the stability of the optimization model. By conducting scenario based testing for instance, one could observe how the model behaves when tested under different conditions.

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Appendices

A iTraxx Main Data Overview

This appendix provides visualizations of the replicated data of iTraxx Main, to show an overview of the underlying data.

A.1 Constituents

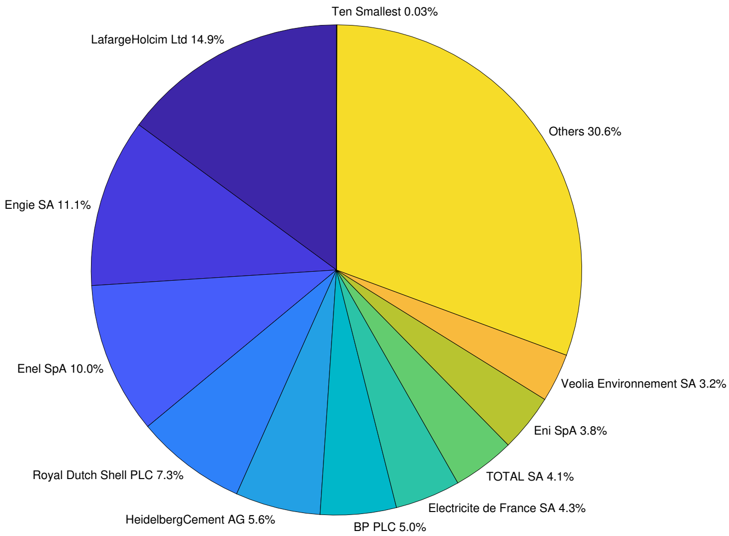

Figure 7: GHG footprint at constituent level of iTraxx Main.

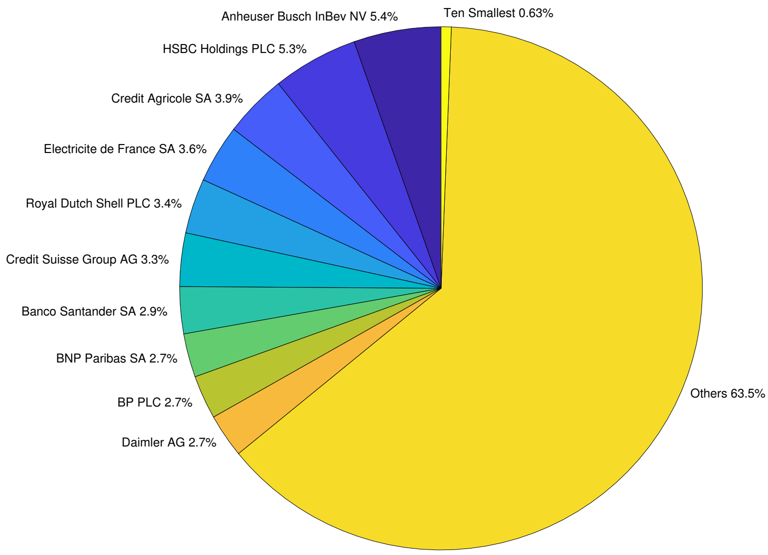

Figure 8: Market value of outstanding bonds at constituent level of iTraxx Main.

A.2 Sectors

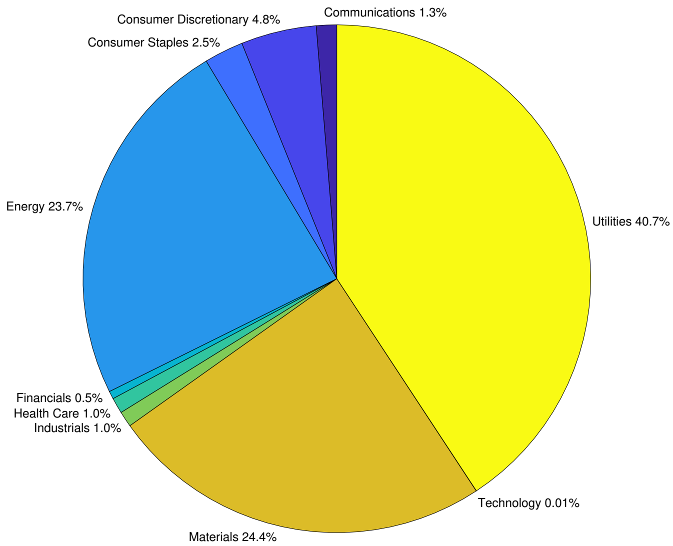

Figure 9: GHG footprint at sector level of iTraxx Main.

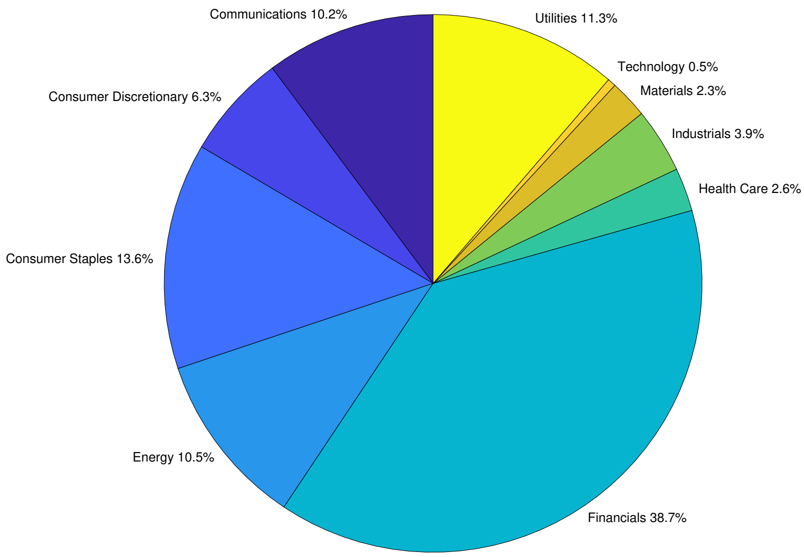

Figure 10: Market value of outstanding bonds at sector level of iTraxx Main.

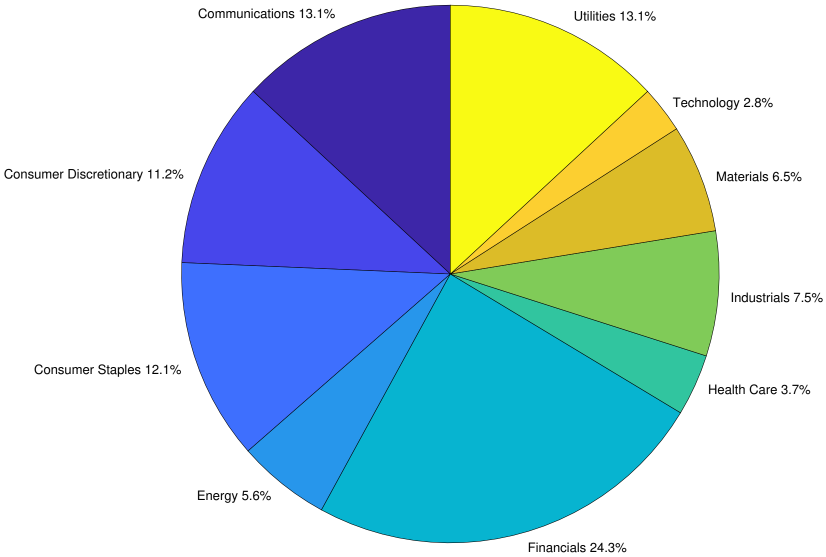

Figure 11: Sector composition of iTraxx Main.

B CDX.IG Data Overview

This appendix provides visualizations of the replicated data of CDX.IG, to show an overview of the underlying data.

B.1 Constituents

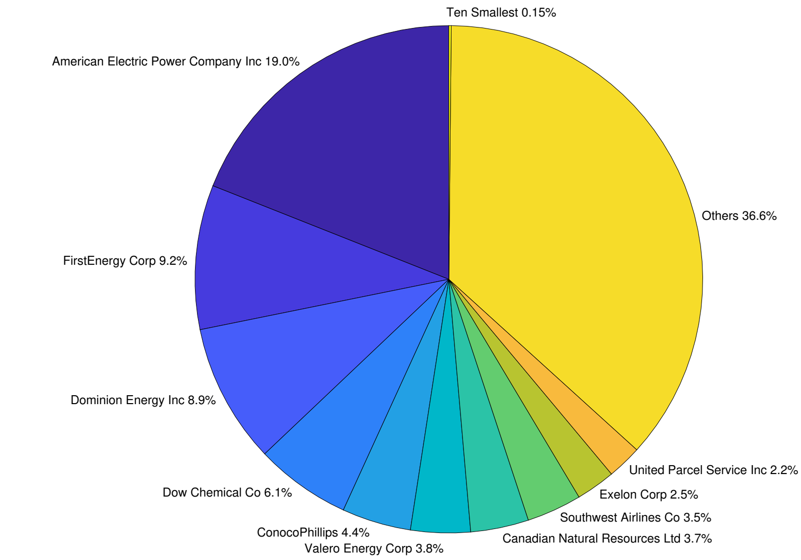

Figure 12: GHG footprint at constituent level of CDX.IG.

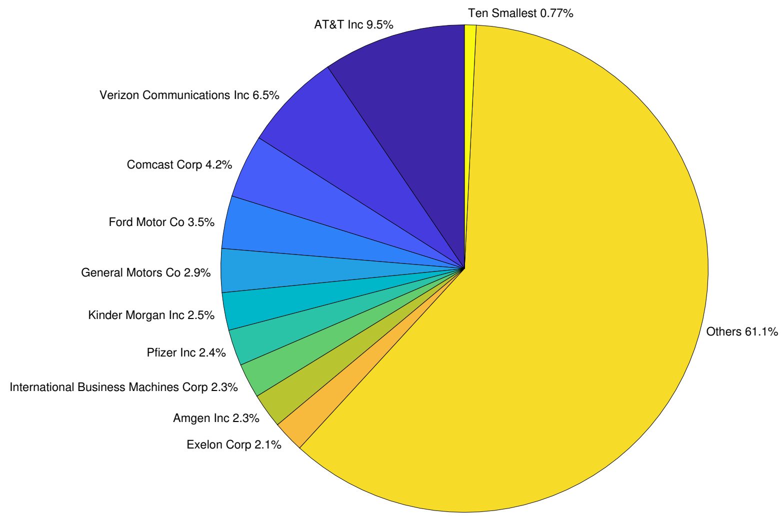

Figure 13: Market value of outstanding bonds at constituent level of CDX.IG.

B.2 Sectors

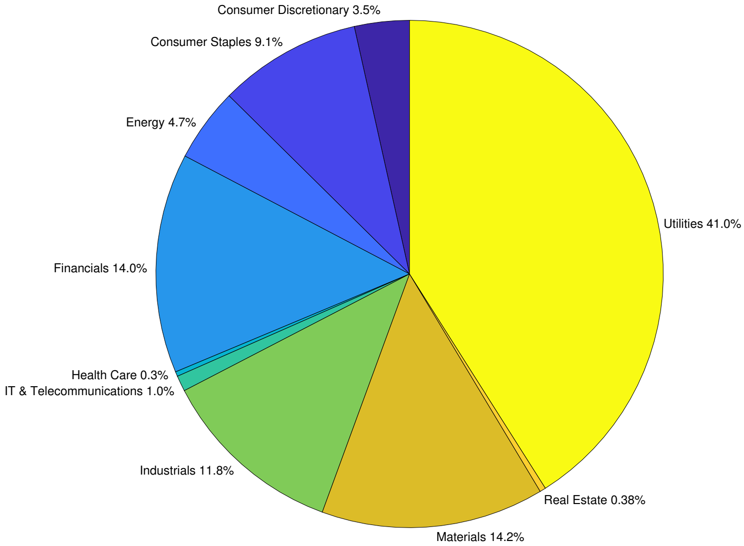

Figure 14: GHG footprint at sector level of CDX.IG.

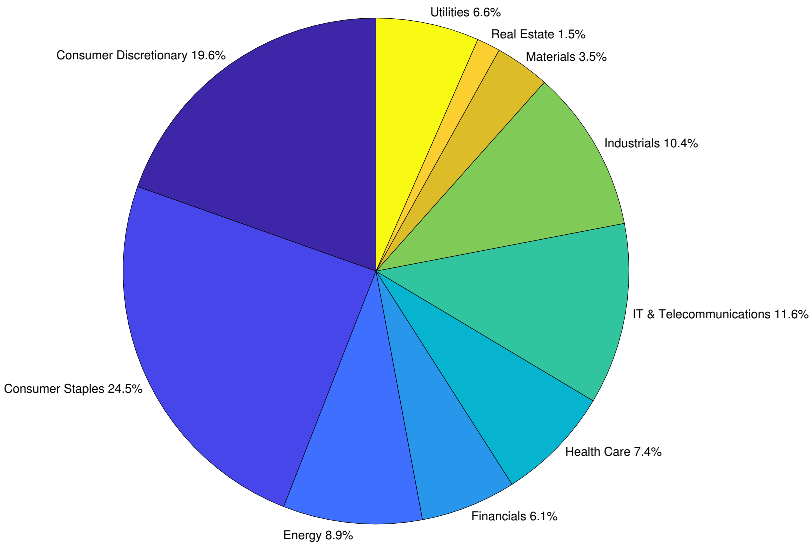

Figure 15: Market value of outstanding bonds at sector level of CDX.IG.

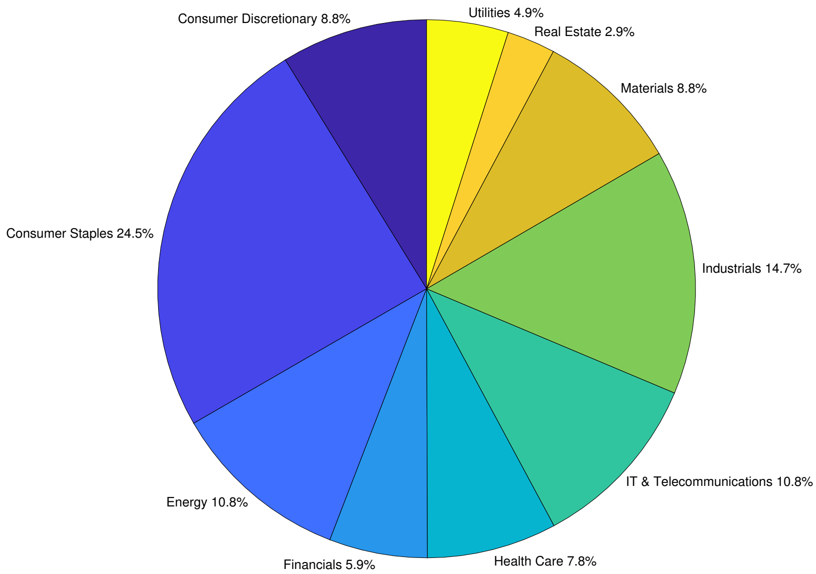

Figure 16: Sector composition of CDX.IG.

C Optimization Convexity

This appendix provides plots of the eigenvalues for the Σ matrices of iTraxx Main and CDX.IG.

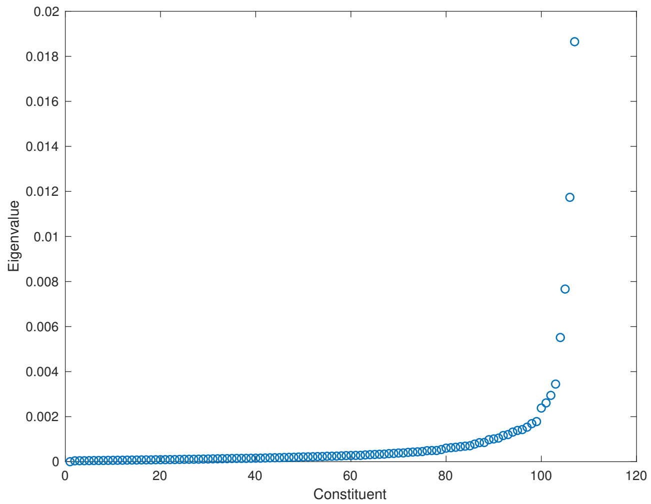

Figure 17: Eigenvalues for the Σ matrix of iTraxx Main, where all $\lambda_i \geq 0$.

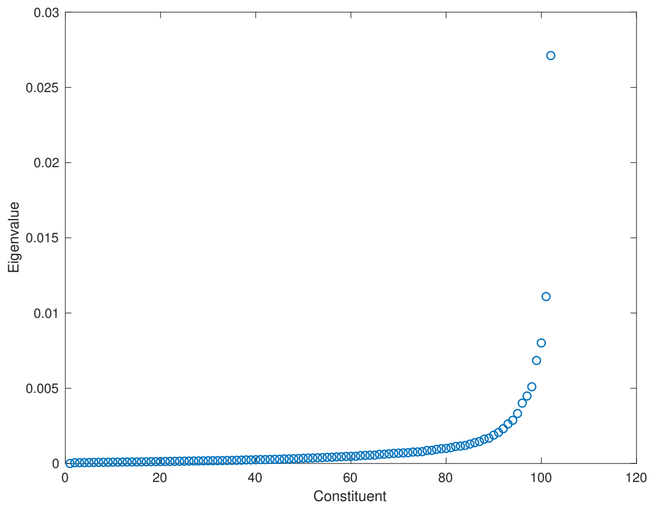

Figure 18: Eigenvalues for the Σ matrix of CDX.IG, where all $\lambda_i \geq 0$.

D Portfolio Combinations

This appendix provides a table of all possible portfolio combinations which were empirically tested for one portfolio.

Table 20: Minimum ECOBAR scores for all possible portfolio combinations of 4 assets with different ECOBAR scores, when optimizing over minimum variance with CDS indices. All constraints and limits are stable and total shorting investment weights is set to 5%. The column headers shows the (long, short) ECOBAR scores for the individual constituents tested. Note that the portfolio combination of short only positions is excluded, since the investment weight constraints do not allow short only portfolios.

(Long, Short) / Combination	**(1, 9)**	**(2, 6)**	**(4, 3)**	**(6, 2)**	**ECOBAR**
Combination 1	S	S	L	L	4.1
Combination 2	S	L	S	L	2.7
Combination 3	S	L	L	S	2.3
Combination 4	L	S	S	L	2.2
Combination 5	L	S	L	S	1.8
Combination 6	L	L	S	S	1.1
Combination 7	S	S	S	L	6.1
Combination 8	S	S	L	S	4.1
Combination 9	S	L	S	S	2.2
Combination 10	L	S	S	S	1.5
Combination 11	L	L	L	S	2.0
Combination 12	L	L	S	L	2.1
Combination 13	L	S	L	L	4.1
Combination 14	S	L	L	L	4.2
Combination 15	L	L	L	L	2.0

E Replication Error Distribution

This appendix provides histogram plots of the tracking errors for the replications of iTraxx Main, CDX.IG and green-brown portfolios for the validation period.

E.1 iTraxx Main

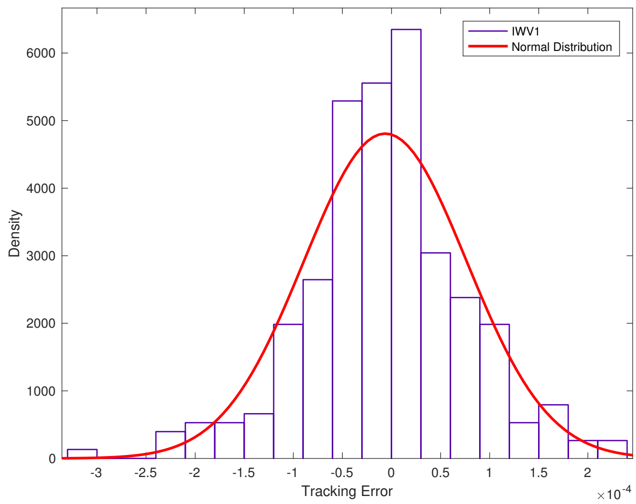

Figure 19: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for IWV1 of iTraxx Main, with $\mu = -6.4 \cdot 10^{-6}$, $\sigma = 8.3 \cdot 10^{-5}$.

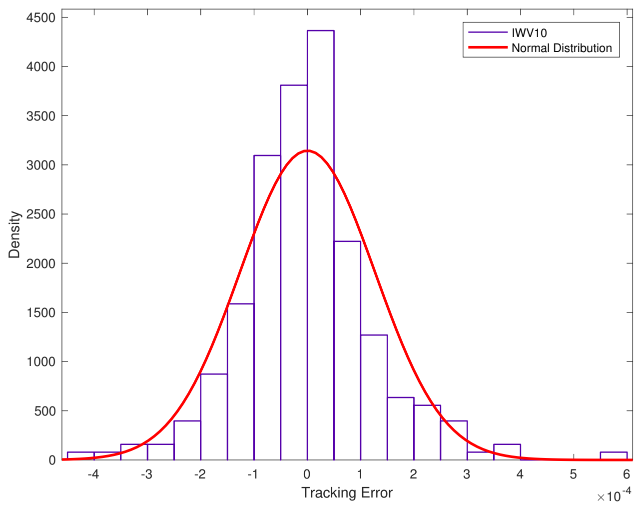

Figure 20: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for IWV10 of iTraxx Main, with $\mu = 2.1 \cdot 10^{-7}$, $\sigma = 1.3 \cdot 10^{-4}$.

E.2 CDX.IG

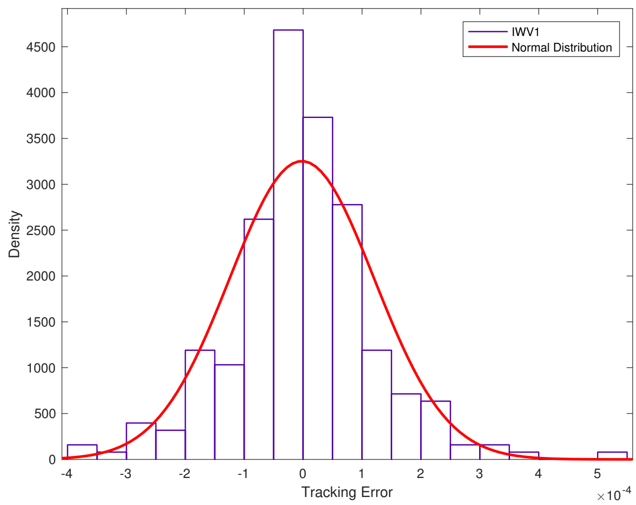

Figure 21: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for IWV1 of CDX.IG, with $\mu = -2.2 \cdot 10^{-6}$, $\sigma = 1.2 \cdot 10^{-4}$.

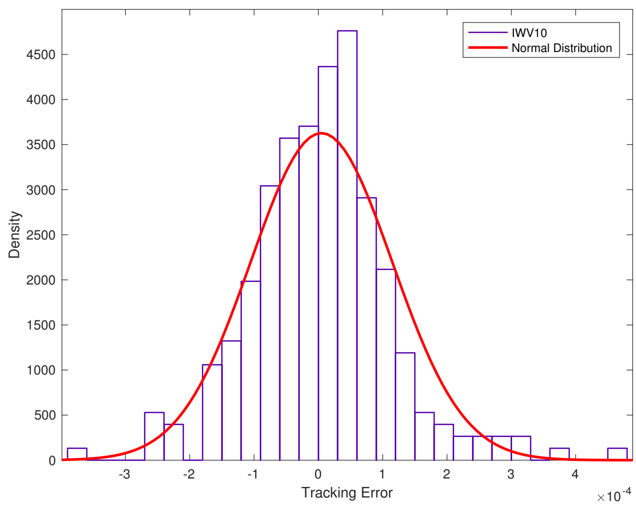

Figure 22: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for IWV10 of CDX.IG, with $\mu = 4.8 \cdot 10^{-6}$, $\sigma = 1.1 \cdot 10^{-4}$.

E.3 Green-Brown Baskets

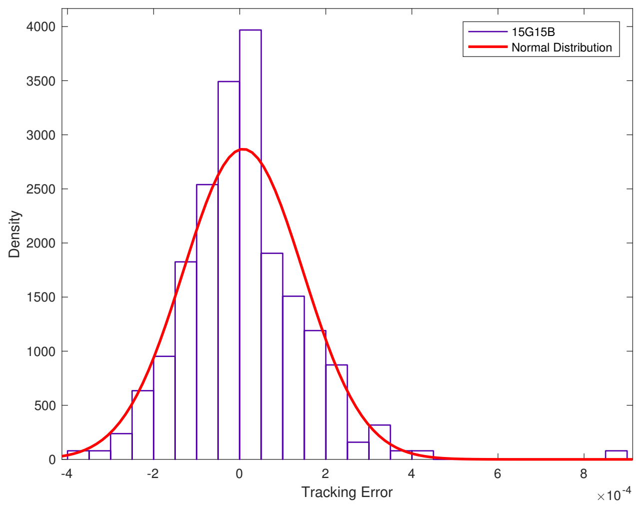

Figure 23: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for a 15G15B basket of iTraxx Main, with $\mu = 8.0 \cdot 10^{-6}$, $\sigma = 1.4 \cdot 10^{-4}$.

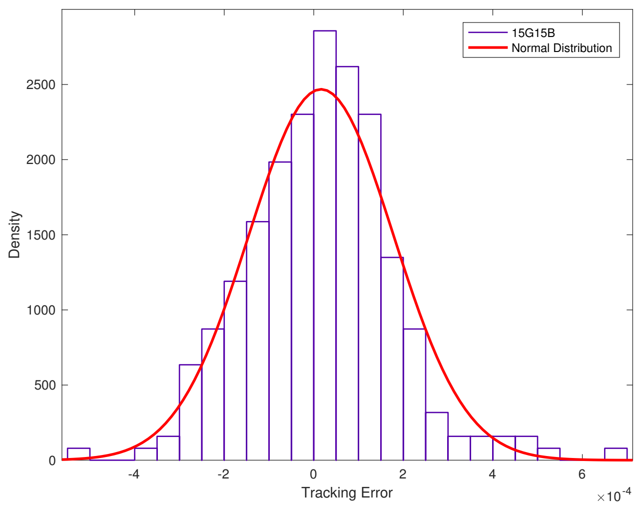

Figure 24: Normal distribution fit of the underlying tracking errors, $R_t - R_t^I$, for a 15G15B basket of CDX.IG, with $\mu = 1.7 \cdot 10^{-5}$, $\sigma = 1.6 \cdot 10^{-4}$.

F Disclosed Optimized Portfolio

This appendix provides a table of a fully disclosed optimized portfolio.

Table 21: An optimized portfolio, SWV0 for iTraxx Main, where WeightUb is set to 5/107. Note that the constituent names have been masked.

Constituent	Sector	ECOBAR	InvWeight
Constituent 1	Financials	2	0
Constituent 2	Consumer Staples	4	0
Constituent 3	Utilities	6	0
Constituent 4	Consumer Discretionary	6	−0.009345
Constituent 5	Financials	0.99	0.046489
Constituent 6	Financials	0.97	0.002991
Constituent 7	Communications	1	0.030140
Constituent 8	Financials	1	0.042908
Constituent 9	Financials	3	0.001189
Constituent 10	Financials	2	0
Constituent 11	Communications	1	0.010728
Constituent 12	Financials	1	0.012176
Constituent 13	Health Care	2	0.039822
Constituent 14	Consumer Discretionary	2	0.018017
Constituent 15	Energy	3	0.029588
Constituent 16	Communications	3	0
Constituent 17	Financials	1	0
Constituent 18	Financials	2	0
Constituent 19	Communications	3	0
Constituent 20	Consumer Staples	4	0
Constituent 21	Utilities	3	0.024841
Constituent 22	Utilities	9	−0.009345
Constituent 23	Communications	2	0
Constituent 24	Utilities	4.20	0
Constituent 25	Utilities	3	0.007879
Constituent 26	Utilities	9	−0.009345
Constituent 27	Energy	6	0
Constituent 28	Industrials	1	0.000414
Constituent 29	Energy	3	0.035835
Constituent 30	Communications	2	0
Constituent 31	Energy	6	0
Constituent 32	Consumer Staples	4	0
Constituent 33	Utilities	9	−0.009345
Constituent 34	Communications	3	0
Constituent 35	Consumer Discretionary	6	−0.004265
Constituent 36	Industrials	3	0

Constituent 37	Consumer Discretionary	4	0
Constituent 38	Financials	3	0
Constituent 39	Materials	3	0.046707
Constituent 40	Financials	2	0
Constituent 41	Financials	2	0
Constituent 42	Financials	2	0
Constituent 43	Financials	3	0
Constituent 44	Financials	3	0
Constituent 45	Financials	1	0.040964
Constituent 46	Materials	3	0.018696
Constituent 47	Health Care	6	−0.009345
Constituent 48	Financials	1.98	0
Constituent 49	Consumer Staples	2	0.010985
Constituent 50	Consumer Staples	6	0
Constituent 51	Utilities	3	0.046720
Constituent 52	Financials	3	0
Constituent 53	Financials	1	0.035413
Constituent 54	Financials	1	0.024536
Constituent 55	Consumer Discretionary	2	0.044959
Constituent 56	Consumer Discretionary	6	0
Constituent 57	Utilities	2.79	0.046727
Constituent 58	Utilities	6	0
Constituent 59	Utilities	8.54	−0.009344
Constituent 60	Industrials	1	0.039277
Constituent 61	Technology	2	0
Constituent 62	Utilities	6	0
Constituent 63	Consumer Staples	2	0.010637
Constituent 64	Consumer Staples	6	0
Constituent 65	Materials	9	0
Constituent 66	Financials	0.98	0.036299
Constituent 67	Consumer Staples	6	0
Constituent 68	Materials	3	0
Constituent 69	Communications	2	0
Constituent 70	Health Care	4	0
Constituent 71	Materials	6	0
Constituent 72	Consumer Discretionary	2	0.018611
Constituent 73	Consumer Discretionary	4	0
Constituent 74	Financials	3	0
Constituent 75	Consumer Staples	6	0
Constituent 76	Consumer Discretionary	4	0
Constituent 77	Communications	3	0
Constituent 78	Communications	2	0
Constituent 79	Technology	1	0.023830

Constituent 80	Energy	9	0
Constituent 81	Industrials	2	0
Constituent 82	Health Care	2	0.006905
Constituent 83	Financials	1.92	0
Constituent 84	Materials	6	0
Constituent 85	Financials	2	0
Constituent 86	Communications	3	0
Constituent 87	Financials	3	0
Constituent 88	Utilities	3	0.042458
Constituent 89	Industrials	2	0
Constituent 90	Communications	1	0.039574
Constituent 91	Technology	1	0.004206
Constituent 92	Communications	1	0.016500
Constituent 93	Financials	3	0
Constituent 94	Energy	9	-0.009345
Constituent 95	Consumer Discretionary	2	0.044170
Constituent 96	Industrials	3	0
Constituent 97	Industrials	3	0
Constituent 98	Materials	9	0
Constituent 99	Consumer Staples	2	0.040789
Constituent 100	Industrials	1	0.035072
Constituent 101	Communications	1	0.033893
Constituent 102	Consumer Staples	2	0.020307
Constituent 103	Consumer Discretionary	4	0
Constituent 104	Utilities	6.91	-0.000406
Constituent 105	Consumer Staples	4	0
Constituent 106	Consumer Staples	2	0.038774
Constituent 107	Consumer Discretionary	6	0

G Greenhouse Gas Exposure Calculation

This appendix provides the calculation formulas for the real world GHG exposure reduction example in section 5.3.

The GHG exposure of a CDS index with n constituents, per €1 invested, was calculated by:

$$\mathrm{GHG}^{\mathrm{EUR}} = \sum_{i=1}^{n} \frac{\mathrm{GHG}_i}{\mathrm{MV}_i} \cdot w_i \tag{6.1}$$

For a portfolio manager investing in the CDS index with a long only investment strategy, the total GHG exposure was calculated by:

$$\text{Total GHG Investment Exposure} = \mathrm{GHG}^{\mathrm{EUR}} \cdot \text{Investment Amount} \tag{6.2}$$