Corporate Social Responsibility and Strategic
Managerial Delegation

Georgios Miaris

Umeå School of Business, Economics and Statistics
Department of Economics
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Supervisor: Thomas Aronsson
Abstract

This thesis examines the strategic delegation model in a duopoly market. A strategic analysis is used to integrate Corporate Social Responsibility (CSR) into managerial incentive design. We develop and examine two different scenarios of the delegation game. The first scenario is the model of simultaneous managerial delegation in which firms compete sequentially in quantities (Stackelberg fashion), while the second scenario is the model of sequential managerial delegation in which firms compete in quantities simultaneously (Cournot fashion). In light of this, the purposes of this thesis is to measure the business performance of the quantity competing firms and clarify under which circumstances the first mover has the advantage of commitment or not in these two scenarios.

Keywords: Strategic delegation, Corporate Social Responsibility, Cournot competition, Stackelberg competition, Duopoly.
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1 Introduction

Economic theory typically examines firms as economic units, whose main goal is to maximize their profits. Any deviation from strategies defined in the context of strict maximization of profits is indicative of a malfunctioning firm. However, current empirical evidence as we refer to later on (see section 1.1) suggests that business strategies often deviate from profit maximizing behavior, with the ultimate goal of gaining a competitive advantage over the competitors. Strategic delegation and Corporate Social Responsibility (CSR) are included in this category of strategies. The purpose of this thesis is to examine the impact of the aforementioned strategies on business performance considering the order in which the decisions are made. For this reason, we analyze two scenarios of managerial delegation. Each scenario contains three stages of quantity and incentive competition.

Current research presents the following main motivations for assigning the responsibility of certain decisions by firm owners to their directors. A firm owner can gain a strategic advantage over its competitors by hiring a manager whose objective function deviates from rigorous profit maximization. This kind of representation is defined in the literature as Strategic Delegation.

The present thematic section of this thesis is specialized in CSR. Although there is no generally accepted definition of CSR, the European Commission refers to it as "a concept by which companies voluntarily incorporate social and environmental concerns into their business activities and their contacts with other stakeholders" (European Commission, Green Paper, 2001). However, Milton Friedman (1970) is critical to the concept of CSR as he argues that manager’s social responsibility behavior wastes resources and has different opinion on social responsibility into business. Therefore, we can conclude that CSR is a debatable topic drawing increasing attention both in academia and among policymakers.

1.1 General Purpose, Motivation and Significance of CSR

Despite the aforementioned complexity, due to the difficulty of defining CSR, it draws an increasing attention among firms. According to a survey conducted by PricewaterhouseCoopers (PwC) in 2010, 81% of the global companies which were examined had information regarding CSR activities on their websites by the end of July of 2010. The survey reports that one year earlier, 75% of the firms had CSR information on their websites.
The survey also shows that more than 94% of all European firms post on their websites information related to CSR activities.

In 2013, PwC published a survey which concludes that awareness of corporate responsibility is rising and shows an increased sense of responsibility by companies towards people, the environment, and the society. Another survey conducted by KPMG (2017), 250 of the largest firms (according to revenue as classification criterion) globally, report facts regarding their CSR activities. In this light, Becchetti et al. (2006) refer that more than half of the firms that belong to the top 100 firms, located in the 16 most industrialized countries, reported their CSR activities during 2005. Thus, the impact analysis of CSR is very important, since CSR is a major concern for modern firms. To this context, firms considering CSR try to recognize the consequences of their actions for society. On the other hand, consumers have paid even more attention to CSR lately as well. For instance, before accomplishing a purchase consumers check for other qualitative features of the product such as non-animal testing procedures, abating pollution, recycling and so on apart from the price (McWilliams & Siegel 2001).

Furthermore, researchers strive towards the relationship between CSR and firm’s performance. In the light of previous studies, Margolis and Walsh (2003) created a survey which includes 127 journal publications from 1970’s to 2000’s studying the relationship between CSR and firm’s performance. The findings of the survey suggest positive relationship between CSR and finance performance. More specifically, Margolis and Walsh (2003) examined 127 empirical studies, 17 studies in 1970’s, 30 studies in 1980’s, 68 studies in 1990’s and the rest in 2000’s. In the majority of the studies (in 109 out of 127), CSR was treated as an independent variable, trying to predict firm’s financial performance. The results show positive relationship between firm’s financial performance in more than half of these studies (54 out of 109). In 22 out of 127 CSR was treated as the dependent variable predicted by firm’s financial performance. The results show the positive relationship in 16 out of 22 studies between CSR and financial performance. We have four more results (131 results) than studies (127) because in four of the studies, the authors examined the relationship of CSR and firm’s performance in both directions.

Another topic that should be clarified thoroughly is the use of imperfect competition model. The present thesis follows earlier studies which are based on duopoly models (Kim & Kwon 2017; Sklivas 1987; Basu 1996; Stamatopoulos 2016; Lee & Matsumura 2017).
Moreover, we use imperfect competition model such as oligopoly and henceforth duopoly, which is a special type of oligopoly, because global markets are often structured in oligopolistic fashion (Das & Donnenfeld, 1989). As an illustration of this, the two leading firms on electronic commerce are eBay and Amazon. Further, there is a dominant player in the airlines market in each country and the rest are followers. This dominant player is most likely the leader, while other smaller airline firms that are followers such as the case of Air France and Aigle Azur in France, British Airways and Thomas Cook Airlines in UK, Lufthansa and Germania in Germany. Furman & Orszag (2015) presented significant results regarding market concentration over the last years from 1997 to 2007. Furman & Orszag (2015) show that in 75% of sectors such as transportation and warehousing, finance and insurance, retail trade, have had significant increases in market concentration. Furthermore, Furman (2016) demonstrates that a large number of industries have seen increases in the revenue share among the 50 of the largest firms from 1997 to 2012. Accordingly, Economist (2016) reports that in more than 40% of approximately 900 different industries which were examined, the top 4 firms (in terms of average share of total industry revenue), controlled more than 30% of the US market in 2012. The above information strengthens our assumption to use a duopolistic context, it is close to reality in many cases.

A duopoly model deviates from perfect competition which is the ideal market structure in many aspects. The most important differentiation is that in duopoly there are only two sellers and many buyers. The existence of two sellers in a market inevitably leads to strategic planning by the firms. Strategic planning means that a firm takes into consideration its rival’s actions before it takes an action (Varian, 1992). Therefore, firms constantly watch carefully their competitor and strive to exploit every vulnerability of them. More than that, we use a delegation model because nowadays there is separation of ownership and management in companies. More specifically, we use an oligopolistic model in which firm owners choose the type of incentive contract they will communicate to their manager.

1.2 Agency in duopoly

This section discusses the strategic decisions of the owners of a firm to incentivize the managers. Even though the term was introduced in the economic literature by Schelling (1960), the managerial incentive literature developed by Vickers (1985), Fershtman & Judd
(1987) and Sklivas (1987) henceforth (VFJS). They created the theoretical background on the use of strategic representation to establish conditions of competitive advantage in oligopolistic markets. In particular, in a theoretical framework of competition in quantities, the owner of each firm has the ability to recruit a manager and offer him/her an incentive contract that will lead the manager to a more “aggressive behavior”\(^1\) than strict maximization of profits.

More specifically, a two-stage game is considered. In the first stage of the VFJS model, the firm owners, who are profit maximizers, decide the incentive parameters in an exogenously given incentive contract, which is a linear combination of profits and sales of their firm. In the second stage, as the terms of the contracts are common knowledge, the directors compete in the market by setting quantities. The owner who delegates the production of quantities to a manager earns higher profits than their competitor who does not delegate. In balance, the result is that both owners choose strategic representation by their manager, so we are led to a prisoner's dilemma due to increased competition among rival companies. The prisoner’s dilemma means that if the owners do not delegate, they would earn higher profits but eventually both delegate because it is their dominant strategy. If the owner does not delegate to a manager, then the firm would earn less profits than its rival. Therefore, in this context, the best response of the owner is to delegate. (For an extensive analysis of prisoner’s dilemma see: Varian, 1992)

1.3 Literature Review on Cournot and Stackelberg competition

The models of Cournot and Stackelberg have been used in Industrial Organization (IO) extensively for more than five decades. These two models differ in their assumptions and the most important difference is the following; The Cournot model assumes that competing firms act simultaneously in the market, while the Stackelberg model assumes that firms act sequentially in the market (Varian, 1992). The incorporation of a timing sequence into market analysis leads to totally different results than simultaneous games. The Stackelberg game gives rise to questions such as: which firm will be the leader/follower in the market, which firm will obtain higher profits at the end of the game, is it always the leading firm that obtains higher profits and under what conditions will the leader/follower have the advantage in the market? In the following sections we endeavor to answer these questions thoroughly.

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\(^1\) With the term “aggressive behavior” we mean the choice of the manager to produce more quantity compared to the quantity produced at profit maximizing case (see Sklivas, 1987).
Kim & Kwon (2017), seek the duopoly context using strategic delegation with sales-contract managerial incentives, where manager’s compensation scheme is based on linear combination between profits and revenues. In this paper the authors use three different games and each game contains two stages. In the first stage, the owners of the firms select sequential/simultaneous decision regarding managerial delegation and in the second stage the managers choose sequential/simultaneous decisions regarding quantity/price competition. The results in two out of three games of their study show that in quantity competition, the leading firm commit itself to the profit maximizing behavior, therefore the leader does not delegate to the manager. In the first game (model of sequential firm decision), the commitment to profit maximizer behavior for the leading firm gives the first mover advantage and obtains higher payoff than its rival (Kim & Kwon, 2017). However, in the second game (model of simultaneous managerial delegation), the follower exploits the commitment of the leader to the profit maximizer behavior and obtains higher profits (Kim & Kwon, 2017). The second game shows that there is no status quo in the market and that the follower can have advantage of being follower in the market compared to the leader firm.

In the same spirit, Basu (1996) examines the issue of strategic delegation in duopoly on quantity competition using the VFJS model. He shows that even though the use of strategic delegation with incentives to sales which leads to prisoner’s dilemma (another way to state it: firms will delegate the production to a manager knowing that eventually, they will end up with lower profits) the subgame perfect Nash equilibrium\(^2\) (SPNE) can coincide with the standard Stackelberg outcome by incorporating in the model, the cost of hiring manager. On top of that, he concludes that the results hold when the cost of hiring a manager is positive and equal for both firms. Additionally, Sharma (2017) uses CSR in a duopoly context and shows that the SPNE can coincide with standard Stackelberg result as well. Even though this result is similar to Basu (1996), Sharma (2017) uses different assumptions. Sharma (2017) assumes that the cost of production for both firms is symmetric and the cost of hiring a manager differs for the rival firms. However, in both papers if the cost of hiring the manager equals zero the Stackelberg outcome cannot occur.

Similarly, Kopel and Löffler (2008), consider a duopoly-Stackelberg market in which managers are compensated by the linear combination between profits and sales (more precisely quantities sold). The profits of the firms are calculated by the revenue minus the

\(^2\) SPNE typically means an equilibrium where all players satisfy their first order conditions simultaneously. For extensive analysis regarding SPNE see Varian (1992).
option of the reduction of cost production by investing in RnD. Before the game starts the authors select which firm leads and which one follows in quantity competition. The leading firm loses the advantage of moving first and follower gains higher profits (Kopel and Löffler, 2008). They examine an innovation- delegation game where the owners of the leader firm and the follower firm have the following options: to delegate the production to a manager or not, to invest in RnD or not and the combination of these cases. Kopel and Löffler (2008) conclude that the advantage of the leader vanishes because the follower hires a manager and incentivizes him to behave aggressively. Therefore, the follower obtains more profits than the leader. So, the delegation game can help the follower to reverse the disadvantage of being follower.

Another slightly different paper on this topic written by Kopel and Brand (2012), investigate the SPNE in a mixed duopoly- quantity framework. They use two different types of firms; one is a firm with corporate social responsibility, while the other is a profit-maximizing firm. In their results, the SPNE of the game implies that both firms will hire a manager (dominant strategy of the game). Furthermore, an important contribution of the paper is that it shows in mixed oligopoly a firm that uses CSR can achieve higher profits than a profit-maximizing firm with having as motivation non profit-maximizing goals. Moreover, Bian et al. (2016) incorporate CSR into managerial incentive scheme in duopoly context and conclude that under Cournot competition if the products are substitutes the owners will always choose to delegate in order to achieve higher output. So the CSR scheme increases the competition. However, the profits for the firms will not necessarily be higher compared to the non-delegation model.

Moreover, Stamatopoulos (2016) addresses strategic delegation incentives to show that Cournot and Stackelberg fashion quantity competition games are likely to give equivalent results when the rival firms delegate the managerial incentives sequentially in Cournot quantity competition and delegate simultaneously the managerial incentives under Stackelberg quantity competition. In other words, Stamatopoulos (2016) shows that the first mover at the contract delegation stage under Cournot competition has equivalent payoff with the second mover at the quantity decision stage under Stackelberg competition and vice versa.

Another study that adopts a similar research question as in this paper is written by Lee and Matsumura (2017). They show the first mover’s advantage under price competition, which is
in contrast with the existing literature in price competition without delegation\(^3\). They use environmental corporate social responsibility (ECSR) as delegation device from the owner to the manager. ECSR presupposes that manager’s compensation depends on profits minus a monetary amount for environmental improvement. They investigate sequential price competition while firms can freely choose if they would delegate or not. In the first stage while firms delegate simultaneously, the leader in price competition does not delegate, however, the follower delegates. Eventually, even though the leader in price competition does not delegate he/she ends up with higher profits than the follower by the end of the game. Therefore, they conclude that first mover’s advantage holds under price competition.

Finally, the milestones regarding the topic of first and second mover’s advantage, Gal-Or (1985) and Dowrick (1986) analyze under which circumstances the first or second mover has the advantage. The authors show that when the products are strategic substitutes the pay-off of the leader will be higher than the pay-off of the follower. On the other hand, when the products are strategic complements follower obtain higher profits than the leader. Therefore, Dowrick (1986) rise a serious question of whether the roles of being leader or being follower have been assigned to the firms by the researchers or whether it is a choice by the firms. Our thesis differs from Gal-Or (1985) and Dowrick (1986) analysis, since we focus on strategic delegation scheme to obtain our results while they do not use strategic delegation to obtain their results.

1.4 Corporate Social Responsibility Literature Review

As we have already said, the idea of firms’ owners employing managers with different objectives than strict profit maximization, in order to achieve competitive advantage against their rivals, has been formalized in the theory of strategic managerial delegation.

However, CSR has not only been examined under the scheme of strategic delegation. Baron (2001, 2003) examines CSR under the prism of private politics\(^4\). Baron’s main finding is that private politics and CSR can affect the strategic position of a firm in an industry under the existence of activist consumers, who can boycott firms with non-socially friendly behavior. More recently, Baron (2008), in a principal-agent context, argues that firms may include social incentives to managers besides profit maximization if the following three assumptions hold, first consumers reward the firm for its social initiatives (consumers are willing to pay

\(^3\) Gal-Or (1985) and Dowrick (1986) show second mover’s advantage under price competition.

\(^4\) Private politics is the activist actions made by groups of people to affect the economic activity (Baron, 2001).
higher price), second managers have personal preferences towards corporate responsibility because they gain personal satisfaction of thinking about corporate social responsibility, and third investors prefer social expenses by firm even though the financial return is lower. However, the present analysis departs from Baron (2008), since the research of this thesis concentrates exclusively on the strategic delegation use of CSR and owners have complete knowledge about their managers and incentivize them respectively to their goal. However, Baron (2008) assumes that profits of the owners depend on how consumers valuate the social expenditures, how productive are the employees and the ability of the manager.

McWilliams and Siegel (2001) model firms’ incentives to engage in CSR activities in oligopolistic markets with homogeneous goods. In the context of the Resource Based View of the firm, managers should treat CSR decisions in the same way they treat decisions of investments, in order to determine the level of firms’ resources that should be allocated to CSR activities. More specifically the authors refer: “The ideal level of CSR can be determined by cost benefit analysis. To maximize profit, the firm should offer precisely that level of CSR for which the increased revenue (from increased demand) equals the higher cost (of using resources to provide CSR). By doing so, the firm meets the demands of relevant stakeholders—both those that demand CSR (consumers, employees, community) and those that "own" the firm (shareholders).”

Bagnoli and Watts (2003) examine the case in which an oligopolistic firm links the provision of a public good (such as CSR activities or environmental friendly goods) to the sale of their private product. The researchers find that the provision of CSR by firms is negatively related to the number of the firms in the market and positively related to the consumers’ willingness to pay for the supply of the public good.

1.5 Research Question and Contribution

Derived from the above discussion, we contribute to the aforementioned literature in the following manner:

1) We include CSR incentives for contract construction to delegate output production while Kim & Kwon (2017), Basu (1996), Kopel and Löffler (2008) use the linear combination of profits and sales as incentive scheme (VFJS model).
2) We use sequential and simultaneous action in each of the stages of the two games. Therefore, our analysis differs from Bian et al. (2016) who focused only on simultaneous output and managerial decision.

3) This paper strives to show under what conditions two competing firms (or the owners of them) in sequential and simultaneous fashion under quantity and CSR managerial delegation scheme incentivize their managers.

*Can the leader using CSR incentive scheme always achieve higher profits than the follower? Under which conditions would the leader achieve higher profits? On the other hand, can the follower obtain higher profits than the leader? Under which circumstances would the follower achieve it?*

In order to succeed and answer these questions our goal is to examine the payoffs (business performance) of been leader or follower correspondingly. Additionally, our research questions differ from Gal-Or (1986), Stamatopoulos (2016), Kim & Kwon (2017) for the following reasons: 1) Gal-Or (1986) did not use the context of strategic delegation, 2) Stamatopoulos (2016) does not use CSR scheme and uses different managerial objective function as well. Additionally, he strives to show equivalence of Stackelberg and Cournot competition which is beyond the scope of this thesis. However, we show equivalence between Stackelberg and Cournot models as Stamatopoulos (2016), 3) Kim & Kwon (2017) use the VFJS model, while we use CSR as incentive model. Our analysis uses the methodological analysis Stamatopoulos (2016) and Kim & Kwon (2017). Furthermore, our basic model to derive inverse demand functions accrues from Singh and Vives (1984) and is based on Sharma (2017) as well.

The present work is important to be read for the following reasons: This paper takes a theoretical approach of reality and tries to simplify the complexity of markets and decode the strategical approach of competition among the owners. It develops two different duopoly competition models, which may have characteristics of online service firms or the airline market as mentioned in subsection 1.1. We explicitly assume observability of the delegation of each player’s choice in both models. Observability is an essential assumption in our modeling because in some countries firms are forced by the law to announce their manager’s contracts or owners announce the contracts publicly to attract managers, or repeated competition among firms will eventually help firms to find out the managerial incentives of their rival (Scalera & Zazzaro, 2008). The rest of the thesis is organized as follows: In section 2 we present the general model of strategic delegation with CSR related incentives, in section
3 and 4 we present Stackelberg fashion quantity competition and Cournot fashion quantity competition. Afterward, in section 5 we provide welfare implications under both models and finally, section 6 contains concluding remarks.
2 The model

We use a framework which is closely related to the well-known framework of VFJS (strategic delegation model) and suppose that a duopoly market exists, where we examine Cournot and Stackelberg competition (quantity-setting duopoly) in a homogeneous good market. Considering two firms, firm i and j which are characterized by the distinction between management and ownership. Each firm belongs to a private owner (owner i, j) and each owner hires a manager (manager i, j) for their firm. Each owner delegates the market decisions to the manager. In turn, the manager takes market decisions based on the incentive contract that they signed with the owner. The owner of each firm offers the manager a contract that is a weighted average of profit and the consumer surplus (CS). The owners’ of the firms engage in CSR activities while using CS to show that they care about their customers. Therefore, CS works as a proxy of CSR in our model (Bian et al., 2016).

2.1 Inverse demand functions

Following Singh and Vives (1984), we assume that the representative consumer maximizes

\[ U(q_i, q_j) - \sum_{k=i,j} q_k p_k \] , where \( q_i \) is the quantity of good i and \( p_i \) is its price. The utility function takes the following form:

\[ U(q_i, q_j) = a_i q_i + a_j q_j - (\beta_i q_i^2 + 2\gamma q_i q_j + \beta_j q_j^2)/2 \]

where \( a_i, \beta_i \) are positive constant parameters and \( \gamma \geq 0 \) (\( \gamma \) cannot be signed without additional assumptions because it depends on, whether the goods are strategic complements, substitutes or independent). Additionally, \( U(q_i, q_j) \) can be thought of as the gross utility, \( \sum_{k=i,j} q_k p_k \) is the expenditure, \( CS = U(q_i, q_j) - \sum_{k=i,j} q_k p_k \), the consumer surplus and \( a_i \) is the reservation price.

By using the (consumer’s) first order conditions (F.O.C) for \( q_i, q_j \) we obtain the inverse demand functions:

\[ p_i = a_i - \beta_i q_i - \gamma q_j \] \hspace{1cm} (1)

\[ p_j = a_j - \beta_j q_j - \gamma q_i \] \hspace{1cm} (2)
Following Sharma (2018) we suppose that \( p_i = p_j = p \), \( a_i = a_j = a \), \( \beta_i = \beta_j = 1 \) and \( \gamma = 1 \). By setting \( \gamma = 1 \) we suppose the goods are perfect substitutes and consumers are indifferent which good will consume. Consequently, the inverse demand function is:

\[
P = a - q_i - q_j
\] (3)

where \( a > 0 \) (reservation price), \( p \) is the price market, while the utility function simplifies to read:

\[
U(q_i, q_j) = a q_i + a q_j - (q_i^2 + 2q_i q_j + q_j^2)/2.
\] (4)

The two firms are assumed to have common, constant marginal production costs \( c_i = c_j = c \) (> 0). The cost of producing \( q_i \) units of product \( i \) is given by \( c q_i \), and to ensure that \( q_i > 0 \) we need to assume that \( c \in (0, a) \), which implies \( a > c > 0 \).

### 2.2 Corporate Social Responsibility Framework

Our analysis differs from the Profit- Sales delegation model in the following fashion. Firm \( i \)'s profit function is given by:

\[
\Pi_i = \Pi_i(q_i, q_j) = (p - c)q_i = (a - q_i - q_j - c)q_i.
\] (5)

Similarly, firm \( j \)'s profit function is given by:

\[
\Pi_j = \Pi_j(q_i, q_j) = (p - c)q_j = (a - q_i - q_j - c)q_j.
\] (6)

Firm \( i \)'s manager and firm \( j \)'s manager have the following objective functions which are CSR-related:

Manager \( i \)'s objective function:

\[
O_i = O_i(u_i, q_i, q_j) = \Pi_i(q_i, q_j) + u_i CS(q_i, q_j).
\] (7)

Manager \( j \)'s objective function:

\[
O_j = O_j(u_i, q_i, q_j) = \Pi_j(q_i, q_j) + u_j CS(q_i, q_j).
\] (8)
By using eq. (3) the consumer surplus can be written as: (see eq. A1- A6, Appendix)

$$CS = \frac{1}{2}(q_i + q_j)^2.$$ 

$u_i$, $u_j$ can take values $[0, +\infty)$. However from the second order conditions (S.O.C) the analysis is restricted to the domain $u_i, u_j \in [0, 2)$, (see eq. (A9) Appendix). The incentive parameter $u_i, u_j$ is of concern to owner of each firm.

Following Bian et al. (2016) the CRS incentives are designed endogenously by the owners of the firms in the following fashion:

$$\max_{D_i,g_i,u_i} \Pi_{oi} = \Pi_i - (D_i + g_i(\Pi_i + u_i CS)) \tag{9}$$

Such that:

$$U_{Mi} = (D_i + g_i(\Pi_i + u_i CS)) \geq U^r \tag{10}$$

Where, $D_i$ is a certain amount of money that the manager earns, $g_i$ is a positive scale parameter, $u_i$ is the incentive parameter for the manager, $\Pi_{oi}$ is owner’s profit, $U_{Mi}$ is manager’s compensation, $U^r$ is the minimum utility for the manager, or in other words, it is all the alternative offers that manager has. Obviously, the owner compensates the manager exactly at the point where the manager would agree with the compensation. Therefore, form weak inequality (10) becomes an equality. We substitute equation (10) into (9), consequently, the owner’s maximization scheme alternates and becomes $\Pi_{oi} = \Pi_i - U^r$. However, $U^r$ is constant term, so owner’s problem is identical with the profit maximization problem. Additionally, the owner strives to optimize his problem using only the managerial incentive parameter $u_i$. Lastly, the manager endeavors to maximize his compensation scheme by inflating the CSR incentive parameter as much as possible while the owner’s motives is to find out the optimal incentive parameter to maximize his profits.

We consider two games, game 1 and game 2 respectively, and the games run as follows:

Game 1:

- In stage 1 the owner of firm i and the owner of firm j choose $u_i, u_j$ respectively
- In stage 2 the manager of firm i chooses output $q_i$
- In stage 3 the manager of firm j chooses output $q_j$

---

5 The behavior of firm j is analogous with firm i.
Game 2:

- In stage 1 the owner of firm $i$ chooses $u_i$
- In stage 2 the owner of firm $j$ chooses $u_j$
- In stage 3 the manager of firm $i$ and the manager of firm $j$ choose $q_i, q_j$

To state it more precisely, in the first game, the two firms compete in quantities in Stackelberg fashion (sequentially) at the third and second stage of the game, while in the first stage compete to choose incentive parameter for the managers. However, in the first stage of the game, they compete simultaneously. We assume that firm $i$ is leading at the quantity competition and firm $j$ is follower.

In the second game, the two firms compete in quantities as well, but in Cournot fashion (simultaneously) at the third stage of the game. However, in stage 2, firm $j$ chooses incentive parameter and in stage 1, firm $i$ chooses incentive parameter. We assume that in each stage each choice is publicly observable in both games which means that the owners observe each other choices.

For simplicity, and without any loss of generality, we set $a=1$ and $c=0$ in both games and we use the method of backward induction to reach the results.
3 The Model of simultaneous managerial decision and sequential output decision (game 1)

Stage 3:
Manager j’s objective function takes the form:
\[ O_j = O_j(q_i, q_j) = \Pi_j(q_i, q_j) + u_j CS(q_i, q_j). \]  
(8)
Manager j treats \( q_i \) as exogenous. Taking the F.O.C. with respect to \( q_j \), we can learn manager j’s best response:
\[ q_j(q_i, u_j) = \frac{q_i - u_j q_j + u_i}{u_j - 2}. \]  
(11)
As we notice in equation (11) manager j’s best response depends directly on firm i’s output, indirectly on firm i’s incentive parameter and directly on the firm’s own incentive parameter.

Stage 2:
The objective function facing manager i can now be written as:
\[ O_i = O_i(q_i, q_j(q_i)) = \Pi_i(q_i, q_j(q_i)) + u_i CS(q_i, q_j(q_i)). \]  
(7)
We notice that manager i’s objective function depends on manager j’s reaction function. Therefore, manager i take this information into consideration when he/she tries to find out his/her best response.

Taking the F.O.C. with respect to \( q_i \) we have:
\[ q_i^i(u_i, u_j) = \frac{-u_i^3 + 3u_i - u_i - 2}{2u_j + u_i - 4}. \]  
(12)
Equation (12) explains that the optimal output for firm i depends exclusively on the incentive parameters of both firms.

Substituting equation (12) into equation (11) we obtain the output of firm j:
\[ q_j^j(u_i, u_j) = \frac{u_i^3 + u_i u_j - 4u_i^2 - 2u_j + 3u_j + 2}{u_i u_j + 2u_i^2 - 2u_j - 8u_j + 8}. \]  
(13)
Equations (12), (13) are quantities defined conditional on the incentive parameters.
From equation (13) we can conclude that firm j’s optimal output depends exclusively on the incentive parameters of firm i and firm j, as well.

To this point, it is important to mention that if we assume that both inventive parameters equal zero then, we obtain the optimal quantities of the regular Stackelberg game with firm i as leader and firm j as follower. The results of this regular Stackelberg game and the advantage of the first mover can be easily solved out or even more easily drawn out from a microeconomic textbook (for further information regarding Stackelberg Duopoly see: Varian 1992).

**Stage 1:**

The profit function for firm j can be written as:

\[ \Pi_j(q_i^j(u_i, u_j), q_j^j(u_i, u_j)) = (1 - q_i^j(u_i, u_j) - q_j^j(u_i, u_j))q_j^j(u_i, u_j). \] (14)

Pay-off for owner j depends directly on incentive parameters of both firms via the optimal quantities as well. Taking the F.O.C. of (14) with respect to \( u_j \) we have the following result:

\[ \frac{2u_j^3 + (3u_i - 12)u_j^2 + (2u_i^2 - 12u_i + 22)u_j - 5u_i^2 + 13u_i - 8}{(2u_j + u_i - 4)^3} = 0. \] (15)

The profit function for firm i similarly can be written as:

\[ \Pi_i(q_i^j(u_i, u_j), q_j^j(u_i, u_j)) = (1 - q_i^j(u_i, u_j) - q_j^j(u_i, u_j))q_i^j(u_i, u_j). \] (16)

Pay-off for owner i depends directly from the optimal quantities and from incentive parameters of both firms as well. Taking the F.O.C. with respect to \( u_i \) we have the following result:

\[ \frac{(u_i^2 - 6u_j + 9)u_i}{(u_i + 2u_j - 4)^3} = 0. \] (17)

From (17) we conclude that the owner of firm i behaves as profit maximizer and that the manager’s objectives are identical with the owner’s objectives, since \( u_i = 0 \).

This result is interesting because the owner i commit his manager to profit maximizing behavior.
Substituting (17) into (13) we have $u_j^3 - 6u_j^2 + 11u_j - 4 = 0$. \hspace{1cm} (18)

Using equations (17), (18) we reach the optimal values of incentive parameters$^6$:

\[ u_i^* = 0. \] \hspace{1cm} (19)

\[ u_j^* = 0.47862. \] \hspace{1cm} (20)

So, the follower has higher incentive parameter than the leader which implies the following:

\[ q_i^*(u_i^*, u_j^*) = 0.26096. \] \hspace{1cm} (21)

\[ q_j^*(u_j^*, u_j^*) = 0.567959. \] \hspace{1cm} (22)

Obtaining the above result, we observe that follower produces more than the leader and achieves higher level output.

The optimal price in the market accrues by substituting equation (21), (22) into inverse demand function:

\[ p^* \left( q_i^*(u_i^*, u_j^*), q_j^*(u_j^*, u_j^*) \right) = 0.171081. \] \hspace{1cm} (23)

Consequently,

\[ \Pi_i^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) = 0.04466. \] \hspace{1cm} (24)

\[ \Pi_j^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) = 0.0567959. \] \hspace{1cm} (25)

Comparing the profits of each firm we notice that the leader obtains less profits than the follower and loses the first mover advantage while (24), (25) is the equilibrium of the game. This result contradicts with the regular Stackelberg model which gives the first mover advantage. Intuitively, this result stems from the fact that firm j predicts firm i’s behavior that owner i will choose $u_i^* = 0$ because as a Stackelberg leader this is the dominant strategy. Owner j knowing this fact behaves more aggressive and increases his/her output. The outputs are strategic substitutes which implies that the rise of one´s output leads to decrease in the other´s output. Consequently, owner j is better off by selecting this strategy and owner i is worse off.

$^6$Equations (17), (18) satisfy S.O.C (see Appendix eq. (A9), (A12)). Additionally, equation (A18) has one unique solution in the domain \([0,2)\), (see Appendix eq. (A18), (A19)).
**Proposition 1:** Under Stackelberg quantity competition and simultaneous contract delegation, the follower achieves higher pay-off than the leader under the scheme of CSR. (i.e. second mover’s advantage shows up).

It is worth underlining that, if we set both incentive parameters equal to zero then the leading firm $i$ achieves higher profits than the follower under Stackelberg quantity competition.
4 The Model of sequential managerial decision and simultaneous output decision (game 2)

**Stage 3:**

Manager $j$’s objective function can be written as:

$$O_j = O_j(u_i, q_i, q_j) = \Pi_j(q_i, q_j) + u_j CS(q_i, q_j).$$  \hfill (8)

Taking the F.O.C with respect to $q_j$ we can derive manager $j$’s reaction function can be written as follows:

$$q_j (q_i, u_j) = \frac{q_j - q_i u_j^{-1}}{u_j^{-2}}.$$  \hfill (11)

Manager $i$’s objective function can be written as:

$$O_i = O_i(u_i, q_i, q_j) = \Pi_i(q_i, q_j) + u_i CS(q_i, q_j).$$  \hfill (7)

Comparing equation (3) with (14) we observe that in (3) owner of firm $i$ takes into consideration the rival’s best response while in (14) he/she does not. Therefore, the difference in mathematical terms is expressed by using $q_j$ instead of $q_j(q_i)$ in (14) compared to (3). Here each firm treats the other firm’s output as exogenous.

Manager $i$’s reaction function is given by:

$$q_i (q_j) = \frac{q_j - q_i u_i^{-1}}{u_i^{-2}}.$$  \hfill (26)

Looking into equations (11), (26) we notice that the game at this stage is symmetric. Plugging equation (11) into (26) and vice versa we have:

$$q_i^*(u_i, u_j) = \frac{-u_i + u_j + 1}{-u_i + u_j + 3}.$$  \hfill (27)

$$q_j^*(u_i, u_j) = \frac{-u_i + u_j + 1}{-u_i + u_j + 3}.$$  \hfill (28)

Equations (27), (28) are symmetric as well. Additionally, we notice that the firm with the higher incentive parameter will produce more than his/her rival.

To this point we should mention that if the incentive parameters of both firms are equal to zero then we obtain the results of the regular Cournot game. The results for the regular
Cournot game that both quantities are $\frac{1}{3}$ and can be easily drawn by a microeconomic textbook (for further information regarding Cournot Duopoly see: Varian 1992).

**Stage 2:**

Firm $j$'s profit function is written as:

$$\Pi_j(q_i^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j))q_j^*(u_i, u_j).$$  \hspace{1cm} (29)

Firm $j$ treats $u_i$ as exogenous. Taking the F.O.C. with respect to $u_j$ we have:

$$u_j(u_i) = \frac{u_i^2 - 2u_i + 1}{-u_i + 3}. \hspace{1cm} (30)$$

We observe that equation (30) depends exclusively on firm $i$'s choice regarding managerial incentives $u_i$.

**Stage 1:**

Firm $i$'s profit can then be written as (by using firm $j$'s reaction function):

$$\Pi_i(q_i^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j)) - q_j^*(u_i, u_j)q_i^*(u_i, u_j).$$  \hspace{1cm} (31)

Equation (31) reflects firm $i$'s profit function incorporating owner $j$'s best managerial response. Taking the F.O.C. with respect to $u_i$ we have the following result:

$$\frac{u_i^3 - 6u_i^2 + 11u_i - 4}{4(u_i - 2)^3} = 0. \hspace{1cm} (32)$$

Solving equation (32) we obtain following result:

$$u_i^* = 0.47862. \hspace{1cm} (33)$$

By substituting equation (33) into (30) we obtain:

$$u_j^* = 0.107813. \hspace{1cm} (34)$$

From equation (33) and (34) we can conclude that:

$$q_i^*(u_i^*, u_j^*) = 0.5711. \hspace{1cm} (35)$$

$$q_j^*(u_i^*, u_j^*) = 0.25755. \hspace{1cm} (36)$$

Plugging in equation (35), (36) into inverse demand function we obtain the equilibrium price:
\[ p^* \left( q^*_i(u^*_i, u^*_j), q^*_j(u^*_j, u^*_j) \right) = 0.17135. \]  

(37)

Optimal pay-off for owner i and owner j:

\[ \Pi^*_i(q^*_i(u^*_i, u^*_j), q^*_j(u^*_j, u^*_j)) = 0.09785798. \]  

(38)

\[ \Pi^*_j(q^*_i(u^*_i, u^*_j), q^*_j(u^*_j, u^*_j)) = 0.04413119. \]  

(39)

Intuitively the explanation is the following: Owner i knows that he/she has the advantage of moving first and signing a contract with the manager. Therefore, the owner incentivizes the manager to be as aggressive as possible in order to produce as much as possible, even though the rival manager competes simultaneously to produce output in stage 3. The aggressive contracting behavior on behalf of owner i leads firm j to obtain less profits.

**Proposition 2:** Under Cournot quantity competition and sequential contract delegation, the first mover has the advantage of setting a more aggressive contract and obtaining more profits than his/her rival.

**Proposition 3:** Under Cournot quantity competition, the first mover obtains the same profits as the second mover under Stackelberg quantity competition and vice versa.

Comparing the above results expressed at Propositions 1, 2 and 3 we reach similar results with Stamatopoulos (2016). It is worthy to mention that we reach equivalent results even though our models have three major differences. First, in Stamatopoulos (2016), the owners’ use strategic delegation with sales related objective function for the manager, while we consider CSR related. Second, we have distinction in the structure of managerial objective function. This distinction is major, since in Stamatopoulos (2016) the objective function of the manager is a linear combination of profits and output, while in our model is weighted linear sum of firm´s profit and consumer surplus. Third, our model’s setting is built in the way that firms care about their customers while in Stamatopoulos (2016) firms are interested in their sales.

Additionally our results similar with Stamatopoulos (2016) show equivalence between the models of simultaneous and sequential quantity competition in oligopoly market.
Stamatopoulos (2016) finds that the produced quantities of each firm in Cournot quantity competition model are equivalent with the produced quantities in Stackelberg quantity competition. Furthermore, Stamatopoulos (2016) argues that the first mover under sequential delegation obtains the same payoff as the second mover in the game under Stackelberg competition in quantities. The equivalence between our results and Stamatopoulos (2016) results shows that firms, on the one hand use CSR to show that they care about their customers. On the other hand, use CSR for strategic reasons to obtain advantage over their rival into the market competition.
5 Welfare implications

In this section we discuss the implications of the above two games on the consumer and total social surplus.

To calculate consumer surplus, we use the following equation following Singh and Vives (1984):

\[ CS = aq_i + aq_j - (q_i^2 + 2q_iq_j + q_j^2)/2 - \sum_{k=l,j} q_k p_k. \]  

(40)

Additionally, we define total social surplus as the sum of firms´ surplus and consumer surplus and we follow Lee and Matsumura (2017):

\[ TSS(q_i, q_j) = \sum_{i=1}^n \Pi_i + CS(q_i, q_j). \]  

(41)

Using the equilibrium price and quantities from equations (21), (22), (23) for game 1 the results are as follows:\n
\[ U_{g_1}(q_i, q_j) \approx 0,343283. \]  

(42)

\[ TSS_{g_1}(q_i, q_j) \approx 0,48511. \]  

(43)

On the other hand, using the equilibrium price and quantities (35), (36), (37) for game 2 we obtain:

\[ U_{g_2}(q_i, q_j) = \frac{10}{81} \approx 0,34333. \]  

(44)

\[ TSS_{g_2}(q_i, q_j) = \frac{28}{81} \approx 0,48532. \]  

(45)

Proposition 4: The consumer surplus is equal under Stackelberg quantity competition with under Cournot quantity competition. This result carries over to the total social surplus as well.

---

7 The index \( g_1 \) and \( g_2 \) are used to mention the corresponding games.
Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Stackelberg quantity competition</th>
<th>Cournot quantity competition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Firm i</td>
<td>Firm j</td>
</tr>
<tr>
<td>Incentive weight</td>
<td>0</td>
<td>0,47862</td>
</tr>
<tr>
<td>Price</td>
<td>0,171081</td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>0,26096</td>
<td>0,0567959</td>
</tr>
<tr>
<td>Profit</td>
<td>0,044669</td>
<td>0,097167</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>0,343282</td>
<td></td>
</tr>
<tr>
<td>Total social surplus</td>
<td>0,48511</td>
<td></td>
</tr>
</tbody>
</table>

Table 1\(^8\) presents the summary of all results of the CSR related incentive models under quantity competition. Looking into the above table, we find:

1) Even though the incentive parameter of firm \(i\) under Stackelberg quantity competition is not equal with firm \(j\)’s incentive parameter under Cournot quantity competition; the firms obtain equivalent profits.

2) Under Stackelberg quantity of competition, the follower uses CSR delegation scheme for strategic purpose to overpass leader’s profits.

3) Under Cournot quantity of competition, the leader uses CSR delegation scheme for strategic purpose as well, to overpass the follower’s profits.

4) Comparing these two different market structures we observe that the first model’s results are equal the second model’s results. Therefore the consumers are indifferent between these two models.

---

\(^8\) The results differ slightly between the games due to rounding
6 Conclusion

This paper examines the strategic delegation under CSR-related managerial incentives in duopolistic market and compares models of Cournot and Stackelberg competition. We showed that under Cournot quantity competition and sequential managerial incentives the first mover’s dominant strategy is to incentivize the manager aggressively enough in order to achieve maximum profits. On the other hand, under Stackelberg quantity competition and simultaneous delegation of managerial incentives, the second mover has the advantage. The follower’s dominant strategy is to incentivize the manager aggressively while the leader does not delegate at all. On top of that, surprisingly we find equivalent results between the two models. Finally, we showed that total social surplus is equal under Stackelberg quantity competition with under Cournot quantity competition.

Our results have been derived under the assumption of the same production cost among the rival firms. Therefore, a possible extension of the present thesis is to incorporate asymmetric costs in the model. Moreover, our thesis is conducted into duopolistic context. Therefore a possible extension would be to examine if our results hold in Cournot and Stackelberg fashion of competition in more than two firms model. Finally, it is realistic to use the case of mixed duopoly under sequential quantity competition, where one firm would be profit maximizer and the rival firm would be owned by the public which behaves as non-profit maximizer, as analyzed by Kopel & Brand (2012).
7 References


Appendix

Derivation of consumer surplus:

\[ U(q_i, q_j) = a_i q_i + a_j q_j - (\beta_i q_i^2 + 2\gamma q_i q_j + \beta_j q_j^2)/2 \]  
(A1)

\[ \sum_{k=i,j} q_k p_k \]  
(A2)

\[ CS = U(q_i, q_j) - \sum_{k=i,j} q_k p_k \]  
(A3)

For \( p_i = p_j = p \), \( a_i = a_j = \alpha \), \( \beta_i = \beta_j = 1 \) and \( \gamma = 1 \)

(A1) \implies U(q_i, q_j) = a q_i + a q_j - (q_i^2 + 2q_i q_j + q_j^2)/2

(A2) \implies pq_i + pq_j

(A3) \implies CS(q_i, q_j) = a q_i + a q_j - \frac{q_i^2 + 2q_i q_j + q_j^2}{2} + pq_i + pq_j

F.O.C w.r.t. \( q_i \), \( q_j \)

\[ \frac{\partial CS(q_i, q_j)}{\partial q_i} = a - q_i - q_j - p = 0 \implies p = a - q_i - q_j \]  
(A4)

\[ \frac{\partial CS(q_i, q_j)}{\partial q_j} = a - q_i - q_j - p = 0 \implies p = a - q_i - q_j \]  
(A5)

At this point; we substitute (A4), (A5) into (A3) we get:

\[ CS = \frac{1}{2} (q_i + q_j)^2 \]  
(A6)

Derivation of proposition 1:

Stage 3:

\[ O_j = O_j(u_i, q_i, q_j) = \Pi_j(q_i, q_j) + u_j CS(q_i, q_j) = (1 - q_i - q_j) q_j + u_j \frac{1}{2}(q_i + q_j)^2 \]  
(A7)

F.O.C. w.r.t. \( q_j \)

\[ \frac{\partial O_j(u_i, q_i, q_j)}{\partial q_j} = \frac{\partial \Pi_j(q_i, q_j)}{\partial q_j} + u_j \frac{\partial CS(q_i, q_j)}{\partial q_j} = 0 \implies (u_j - 2) q_j + q_i u_j - q_i + 1 = 0 \implies

q_j(q_i, u_j) = \frac{q_i - q_i u_j^{-1}}{u_j - 2} \]  
(A8)
S.O.C. w.r.t. \( q_j \)

\[
\frac{\partial^2 O}{\partial q_j^2}(u_i, q_i, q_j) = u_j - 2 < 0 \Rightarrow u_j < 2
\]

(A9)

Therefore, \( u_j \in (0, 2) \). We assume that the minimum value for \( u_j \) can be zero because negative values do not have economic interpretation.

**Stage 2:**

We substitute (A8) into (A10):

\[
O_i = O_i(u_i, q_i, q_j(q_i)) = \Pi_i(q_i, q_j(q_i)) + u_i CS(q_i, q_j(q_i)) = (1 - q_i \frac{q_i - q_i u_j - 1}{u_j - 2}) q_i + \\
u_i \frac{1}{2} q_i + \left(\frac{q_i - q_i u_j - 1}{u_j - 2}\right)^2
\]

(A10)

F.O.C. w.r.t. \( q_i \)

\[
\frac{\partial O_i(u_i, q_i, q_j(q_i))}{\partial q_i} = \frac{\partial \Pi_i(q_i, q_j(q_i))}{\partial q_i} + u_i \frac{\partial CS(q_i, q_j(q_i))}{\partial q_i} = 0 \Rightarrow (2u_j + u_i - 4)q_i + u_j^2 - 3u_j + u_i + 2 = 0 \Rightarrow \\
q_i^*(u_i, u_j) = \frac{u_j^2 - 3u_j + u_i + 2}{-2u_j - u_i + 4}
\]

(A11)

S.O.C. w.r.t. \( q_i \)

\[
\frac{\partial^2 O_i(u_i, q_i)}{\partial q_i^2} = 2u_j + u_i - 4 < 0 \Rightarrow 2u_j + u_i < 4 \Rightarrow u_i < 4 - 2u_j
\]

(A12)

Therefore, \( u_i \) belongs to the domain \([0, 4 - 2u_j]\). We assume that the minimum value for \( u_i \) can be zero because negative values do not have economic interpretation.

\[
q_j^*(q_i^*) = \frac{q_i - q_i u_j - 1}{u_j - 2} \Rightarrow q_j^*(u_i, u_j) = \frac{(u_j^2 - 3u_j + u_i + 2)}{-2u_j - u_i + 4} \left(\frac{u_j^2 - 3u_j + u_i + 2}{u_j - 2}\right) u_j - 1
\]
\[ q_j^*(u_i, u_j) = \frac{u_j^3 + u_i u_j - 4u_i^2 - 2u_i + 3u_j + 2}{u_i u_j + 2u_j^2 - 2u_i - 8u_j + 8} \]  \quad \text{(A13)}

**Stage 1:**

We substitute (A11), (A13) into objective function of firm j:

\[ \Pi_j(q_j^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j) - q_j^*(u_i, u_j))q_j^*(u_i, u_j) = \frac{(1 - u_j^3 + 3u_j - u_i - 2)}{2u_j + u_i - 4} - \]

\[ \frac{u_j^3 + u_i u_j - 4u_i^2 - 2u_i + 3u_j + 2}{u_i u_j + 2u_j^2 - 2u_i - 8u_j + 8} \left(\frac{u_j^3 + u_j u_i - 4u_j^2 - 2u_i + 3u_j + 2}{u_i u_j + 2u_j^2 - 2u_i - 8u_j + 8}\right) \]  \quad \text{(A14)}

F.O.C. w.r.t. \( u_j \)

\[ \frac{\partial \Pi_j(q_j^*(u_i, u_j), q_j^*(u_i, u_j))}{\partial u_j} = \frac{\partial \Pi_j}{\partial q_j^*} \frac{\partial q_j^*}{\partial u_j} + \frac{\partial \Pi_j}{\partial q_j^*} \frac{\partial q_j^*}{\partial u_j} = 0 \Rightarrow \]

\[ \frac{2u_j^3 + (3u_i - 12)u_j^2 + (2u_j^2 - 12u_i + 22)u_j - 5u_j^2 + 13u_i - 8}{(2u_j + u_i - 4)^3} = 0 \Rightarrow 2u_j^3 + (3u_i - 12)u_j^2 + (2u_j^2 - 12u_i + 22)u_j - 5u_j^2 + 13u_i - 8 = 0 \]  \quad \text{(A15)}

\[ \Pi_i(q_i^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j) - q_j^*(u_i, u_j))q_i^*(u_i, u_j) = (1 - \frac{u_j^3 + 3u_j - u_i - 2}{2u_j + u_i - 4}) - \]

\[ \frac{u_j^3 + u_i u_j - 4u_i^2 - 2u_i + 3u_j + 2}{u_i u_j + 2u_j^2 - 2u_i - 8u_j + 8} \left(\frac{-u_j^3 + 3u_j - u_i - 2}{2u_j + u_i - 4}\right) \]  \quad \text{(A16)}

F.O.C. w.r.t. \( u_i \)

\[ \frac{\partial \Pi_i(q_i^*(u_i, u_j), q_j^*(u_i, u_j))}{\partial u_i} = \frac{\partial \Pi_i}{\partial q_i^*} \frac{\partial q_i^*}{\partial u_i} + \frac{\partial \Pi_i}{\partial q_i^*} \frac{\partial q_i^*}{\partial u_i} = 0 \Rightarrow \frac{(u_j^3 - 6u_j^2 + 9)u_i}{(u_i + 2u_j - 4)^3} = 0 \Rightarrow \]

\[ u_i^* = 0 \]  \quad \text{(A17)}

Substituting (A17) into (A15):

\[ 2u_j^3 + (3u_i^* - 12)u_j^2 + (2u_i^2 - 12u_i^* + 22)u_j - 5u_j^2 + 13u_i^* - 8 = 0 \Rightarrow \]

\[ u_j^3 - 6u_j^2 + 11uj - 4 = 0 \]  \quad \text{(A18)}
Even though (A18) is cubic there is one unique solution at 0.47862 for \( f(u_j) = 0 \).

Solving equation (A18) with respect to \( u_j \) we obtain:

\[
u_j^* = 0.47862 \quad \text{(A19)}
\]

We substitute (A17), (A19) into (A11), (A13):

\[
q_i^* (u_i^*, u_j^*) = \frac{-u_i^{*2} + 3u_j^* - u_i^* + 2}{2u_j^* + u_i^* - 4} \Rightarrow q_i^* (u_i^*, u_j^*) = 0.26096 \quad \text{(A20)}
\]

\[
q_j^* (u_i^*, u_j^*) = \frac{u_j^{*2} - 4u_j^* + 3u_j^* + 2}{u_i^* u_j^* + 2u_j^* - 2u_i^* - 8u_j^* + 8} \Rightarrow q_j^* (u_i^*, u_j^*) = 0.567959 \quad \text{(A21)}
\]

We substitute (A20), (A21) into (A5):

\[
p^* (q_i^* (u_i^*, u_j^*), q_j^* (u_i^*, u_j^*)) = q_i^* (u_i^*, u_j^*) - q_j^* (u_i^*, u_j^*) = 1 - 0.26096 - 0.567959 = 0.171081 \quad \text{(A22)}
\]

We substitute (A21), (A22) into (A14):

\[
\Pi_i (q_i^* (u_i, u_j), q_j^* (u_i, u_j)) = (1 - q_i^* (u_i^*, u_j^*) - q_j^* (u_i^*, u_j^*)) q_j^* (u_i^*, u_j^*) = p^* (q_i^* (u_i^*, u_j^*), q_j^* (u_i^*, u_j^*)) = 0.171081 * 0.567959 = 0.097167 \quad \text{(A23)}
\]

We substitute (A20), (A21) into (A16):

\[
\Pi_i^* (q_i^* (u_i^*, u_j^*), q_j^* (u_i^*, u_j^*)) = (1 - q_i^* (u_i^*, u_j^*) - q_j^* (u_i^*, u_j^*)) q_i^* (u_i^*, u_j^*) = p^* (q_i^* (u_i^*, u_j^*), q_j^* (u_i^*, u_j^*)) = 0.171081 * 0.26096 = 0.04466 \quad \text{(A24)}
\]
Derivation of proposition 2:

Stage 3:

\[ O_j = O_j(u_i, q_i, q_j) = \Pi_j(q_i, q_j) + u_jCS(q_i, q_j) = (1 - q_i - q_j) q_j + u_j \frac{1}{2}(q_i + q_j)^2 \quad (A7) \]

F.O.C. w.r.t. \( q_j \)

\[ \frac{\partial O_j}{\partial q_j} = \frac{\partial \Pi_j(q_i, q_j)}{\partial q_j} + u_j \frac{\partial CS(q_i, q_j)}{\partial q_j} = 0 \implies (u_j - 2) q_j + q_i u_j - q_j + 1 = 0 \implies \]

\[ q_j(q_i, u_j) = \frac{q_i - q_i^{u_j^{-1}}}{u_j - 2} \quad (A8) \]

S.O.C. w.r.t. \( q_j \)

\[ \frac{\partial^2 O_j(u_i, q_i, q_j)}{\partial q_j^2} = u_j - 2 < 0 \implies u_j < 2 \quad (A9) \]

\[ O_i = O_i(u_i, q_i, q_j) = \Pi_i(q_i, q_j) + u_iCS(q_i, q_j) = (1 - q_i - q_j) q_i + u_i \frac{1}{2}(q_i + q_j)^2 \quad (A25) \]

F.O.C. w.r.t. \( q_i \)

\[ \frac{\partial O_i}{\partial q_i} = \frac{\partial \Pi_i(q_i, q_j)}{\partial q_i} + u_j \frac{\partial CS(q_i, q_j)}{\partial q_i} = 0 \implies (u_j - 2) q_i + q_j u_i - q_j + 1 = 0 \implies \]

\[ q_i(q_j, u_i) = \frac{q_j - q_j^{u_i^{-1}}}{u_i - 2} \quad (A26) \]

S.O.C. w.r.t. \( q_i \)

\[ \frac{\partial^2 O_i(u_i, q_i, q_j)}{\partial q_i^2} = u_i - 2 < 0 \implies u_i < 2 \quad (A27) \]

We substitute (A26) into (A8):

\[ q_j = \frac{q_i(q_j) - q_i(q_j)^{u_j^{-1}}}{u_j - 2} = \frac{q_j - q_j^{u_i^{-1}}}{u_i - 2} \frac{q_i - q_i^{u_i^{-1}}}{u_i - 2} u_j - 1 \implies \]

\[ q_j^*(u_i, u_j) = \frac{-u_i + u_j + 1}{-u_i - u_j + 3} \quad (A28) \]
We substitute (A33), (A34) into (A29):

\[ q_i = q_j (q_i) - q_j (q_i) u_{i-1} = \frac{q_i - q_j}{u_{i-2}} - \frac{q_i - q_j u_{i-1}}{u_{i-2}} = \Rightarrow q_i^*(u_i, u_j) = \frac{+u_i - u_j + 1}{-u_i - u_j + 3} \quad (A29) \]

**Stage 2:**

\[ \Pi_j(q_i^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j) - q_j^*(u_i, u_j)) q_j^*(u_i, u_j) = \\
(1 - \frac{u_i - u_j + 1}{-u_i - u_j + 3} - \frac{-u_i + u_j + 1}{-u_i - u_j + 3}) (\frac{-u_i + u_j + 1}{-u_i - u_j + 3}) \quad (A30) \]

F.O.C. w.r.t. \( u_j \)

\[ \frac{\partial \Pi_j(q_i^*(u_i, u_j), q_j^*(u_i, u_j))}{\partial u_j} = \frac{\partial \Pi_j}{\partial q_i^*} \frac{\partial q_i^*}{\partial u_j} + \frac{\partial \Pi_j}{\partial q_j^*} \frac{\partial q_j^*}{\partial u_j} = 0 = \Rightarrow -2((u_i - 3)u_j + u_i^2 - 2u_i + 1) = 0 = \Rightarrow \\
-2((u_i - 3)u_j + u_i^2 - 2u_i + 1) = 0 = \Rightarrow u_j = \frac{u_i^2 - 2u_i + 1}{-u_i + 3} \quad (A31) \]

**Stage 1:**

\[ \Pi_i(q_i^*(u_i, u_j(u_i)), q_j^*(u_i, u_j(u_i))) = (1 - q_i^*(u_i, u_j(u_i)) - q_j^*(u_i, u_j(u_i))) q_i^*(u_i, u_j(u_i)) = \\
(1 - \frac{-u_i^2 - 2u_i + 1}{-u_i + 3} - \frac{-u_i^2 + 2u_i + 1}{-u_i + 3}) (\frac{-u_i^2 + 2u_i + 1}{-u_i + 3}) \quad (A32) \]

F.O.C. w.r.t. \( u_i \)

\[ \frac{\partial \Pi_i(q_i^*(u_i, u_j(u_i)), q_j^*(u_i, u_j(u_i)))}{\partial u_i} = \frac{\partial \Pi_i}{\partial q_i^*} \frac{\partial q_i^*}{\partial u_i} + \frac{\partial \Pi_i}{\partial q_j^*} \frac{\partial q_j^*}{\partial u_i} = 0 = \Rightarrow \frac{u_i^3 - 6u_i^2 + 11u_i - 4}{4(u_i - 2)^3} = 0 = \Rightarrow \\
u_i^3 - 6u_i^2 + 11u_i - 4 = 0 = \Rightarrow u_i = 0.47862 \quad (A33) \]

We plug into (A31) eq. (A33):

\[ u_j = 0.107813 \quad (A34) \]

We substitute (A33), (A34) into (A29):

\[ q_i^*(u_i, u_j) = \frac{+u_i - u_j + 1}{-u_i - u_j + 3} = 0.5711 \quad (A35) \]
We substitute (A33), (A34) into (A28):

\[ q_j^*(u_i^*, u_j^*) = \frac{-u_i^* + u_j^* + 1}{-u_i^* - u_j^* + 3} = 0.25755 \]  
\[ \text{(A36)} \]

We substitute (A35), (A36) into (A5):

\[ p^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) = 1 - q_i^*(u_i^*, u_j^*) - q_j^*(u_i^*, u_j^*) = 1 - 0.5711 - 0.25755 = 0.17135 \]  
\[ \text{(A37)} \]

We substitute (A35), (A36) into (A30):

\[ \Pi_j^*(q_i^*(u_i, u_j), q_j^*(u_i, u_j)) = (1 - q_i^*(u_i, u_j) - q_j^*(u_i, u_j))q_j^*(u_i, u_j) = 
\[ p^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) \times q_j^*(u_i^*, u_j^*) = 0.26069 \times 0.17135 = 0.04413119 \]  
\[ \text{(A38)} \]

We substitute (A35), (A36) into (A32):

\[ \Pi_i^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) = (1 - q_i^*(u_i^*, u_j^*) - q_j^*(u_i^*, u_j^*))q_i^*(u_i^*, u_j^*) = 
\[ p^*(q_i^*(u_i^*, u_j^*), q_j^*(u_i^*, u_j^*)) \times q_i^*(u_i^*, u_j^*) = 0.09785798 \]  
\[ \text{(A39)} \]