Optimal Mixed Taxation and Multiple Externalities

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Abstract

This paper explores the optimal policy rules for mixed taxation in a static economy with multiple externalities, where there is asymmetric information between the government and individuals. Our main contribution is to characterize the optimal commodity and marginal income tax structure under different assumptions about externalities and the available tax instruments. The exploration starts with our benchmark model with atmospheric positional and environmental externalities, accessible linear commodity and non-linear income taxes as well as two ability-type consumers. The outcome shows that the marginal value of externalities enters additively into the commodity tax formula without altering the policy rule for marginal income taxation. The analysis surveys two scenarios (the failure of implementing a tax on the positional commodity and upward comparisons) of how other taxes become indirect instruments to correct for the positional externality when the tax on the positional commodity does not fully internalize the positional externality. The results also imply that cross-price effect between two externality-generating commodities matters in the whole tax system.

Key words: Positional and environmental externalities, additivity property, mixed taxation, linear commodity taxes, non-linear income taxes
1. Introduction

1.1 General purpose and motivation

This thesis focuses on optimal mixed taxation in economies with multiple externalities and distributional objectives when relative consumption concerns matter. Mixed taxation denotes a combination of different tax instruments such as commodity taxes, labor income taxes, capital income taxes, production taxes, emission taxes, etc. In this paper, we focus on a mix of linear commodity taxes and a non-linear income tax. We will analyze the implications of taxation policy on simultaneous environmental and positional externalities. The literature that addresses the policy implications of simultaneous positional and environmental externalities simultaneously is sparse (Howarth 1996, 2005; Brekke and Howarth, 2002; Wendner, 2005). Yet, the literature that considers the individual externalities’ effects with respect to different modes of taxation is large\(^1\). Since these two externalities are operative simultaneously, their policy implications are, however, worth to be addressed simultaneously.

To this end, a static model with asymmetric information is established as a benchmark for optimal mixed taxation when social-status seeking behavior through conspicuous consumption also gives rise to environmental damage. Two types of consumers are characterized by different earning abilities given an identical time endowment. Their utility function is defined over a numeraire and an externality-generating commodity, leisure, relative consumption, and environmental quality. In the model, by assumption, consumers choose their utility maximizing consumption bundles, subject to their budget constraint which is affected by the given tax instruments in this paper. The information structure allows the government to implement a mix of a non-linear income tax and linear commodity taxes. On one hand, individual earnings-ability measured by the before-tax hourly wage rate is private information, while the government can only observe an individual’s pre-tax income. Due to this, the government can use a non-linear income tax. Consumption, on the other hand, is only observed at the aggregate level. Consequently, the government is required to resort to linear commodity taxes. The self-selection

constraint is designed to make mimicking unattractive. In absence of such a constraint, a high-ability individual could mimic the low-ability type, which would potentially undermine the redistribution system. The assumption of atmospheric consumption externalities, which implies identical marginal contribution to externality across each individual allows Sandmo’s additivity property to carry over. The additive property means that the marginal value of an externality enters the tax on the externality-generating good as an additive term, whereas the policy rules for the other commodity taxes remain unaffected. In this setting, the tax policy outcomes result to be an application of the principle of targeting: if a consumption externality has been fully corrected by the available tax on this externality-generating commodity, this perfect correction implies that the government has no motivation for further corrective measures to alter the present consumption pattern and labor supply. The policy incentives in other tax instruments keep intact, indicating these tax instruments will not used for this externality correction.

Two additional extensions are made for a further analysis of tax implication. The first extension is when the government is unable to tax the good that generates the positional externality, which is exemplified by low housing property tax rates in housing market. Consequently, the government will use other commodity taxes and the marginal income taxes as indirect instruments to correct the positional externality. In doing so, we consider a three-commodity economy, where a dirty good, whose consumption only leads to environmental disamenity, is separated from a conspicuous commodity, to derive the tax on the dirty good and the marginal income taxes. The outcome indicates that the lack of a direct tax instrument to internalize the positional externality yields that other tax instruments are needed to correct for this externality, and that the compensated cross-price effects between the two commodities are essential.

The second extension is upward comparison as one example of a non-atmospheric externality, such that marginal contribution to this externality differs by heterogeneous individuals. Under upward comparisons, the consumption by the low-ability type is non-positional. An optimal linear tax on the positional good is also set along with the other policy instruments in the above-defined three-good economy. However, the inclusion of this linear tax does not solve the positional externality due to its inflexibility. The other
commodity tax and the marginal income taxes simultaneously are used to correct for the positional externality.

1.2 Previous studies

Previous literature that is relevant for this thesis covers mixed taxation under externalities, restrictions on policy feasibility, and non-atmospheric externalities. The respective literature is shortly reviewed in the following:

(i) Mixed taxation under externalities

Ramsay (1927) and Mirrlees (1927) are classic examples of the sole use of a commodity tax by the former and the exclusive use of income tax by the latter. In contrast, mixed-tax models that use both commodity taxes and non-linear income taxes, are expected to be promising for practical tax design. This is partially due to the application of mixed taxation in the current tax systems (typically) used in developed countries. For the previously elaborated informational assumptions that income is observed at the individual level, whereas consumption is only observed at the aggregate level, the tax system is possibly the most flexible. The literature on optimal mixed taxation has been developed by Atkinson (1977), Atkinson and Stiglitz (1976) as well as Mirrlees (1976). Edwards et al. (1994) characterize the government’s limited information on consumer’s earning-ability in a self-selection approach to present policy implications in mixed taxes. None of the literature, however, takes externality correcting properties of taxation into account in the design of a tax system.

Further research includes the study on mixed taxation in the presence of one type of externality. Eckerstorfer (2014) derives mixed taxation rules in an economy with asymmetric information about consumers’ earning-ability and with a non-atmospheric positional externality. The preferences are defined over leisure, a non-positional good, two positional commodities, and relative consumption. Most earlier studies in this area assumed that all consumption was positional and derived optimal income tax policies based on this assumption (for example, Oswald, 1983; Tuomala, 1990; Aronsson and Johansson-Stenman, 2008, 2010). The study by Eckerstorfer (2014) differs from these (earlier) studies in that he made a distinction between positional and non-positional consumption goods and analyzed the corrective role of commodity taxes in this
framework. Among other results, relaxing the atmospheric assumption causes a deviation to Sandmo’s additivity property. The positionality would thus not be perfectly rectified if a tax on the positional commodity were imposed. The reason why this no longer applies under a non-atmospheric externality is that a linear taxation of the externality-generating good shows to be insufficiently flexible to stimulate the internalization of the occurring externality. In addition, whether private consumption goods and leisure are complements or substitutes plays a vital role in the marginal income tax structure.

Pirttilä and Tuomala (1997) analyze the efficient use of commodity taxes, non-linear income tax and public good provision in the presence of environmental externalities. One of their main contributions is the combination of an analysis in environmental externality and a self-selection approach. This paper is based on the rule of Pareto optimal allocation in an economy with two ability-type consumers whose earning ability is private information and the government who needs to solve the self-selection constraint of high-ability groups. Two main outcomes in the second-best setting are yielded. Firstly, the self-selection constraint could be relaxed if leisure is complementary with environmental quality. Mimicking behaviors thus prove to be less attractive, which leads to lower marginal income tax rates. Secondly, Sandmo’s additivity property carries over in the optimal commodity tax structure.

However, neither Eckerstorfer nor Pirttilä and Tuomala consider the interacting effects among different externalities, although Eckerstorfer (2014) does consider two positionalities in a three-good economy, where one non-positional commodity is separated from the two remaining positional ones. More specifically, the consumption externality in Pirttilä and Tuomala (1997) does not contain positional externalities, and the analysis in Eckerstorfer (2014), on the contrary, ignores the environmental externality caused by conspicuous consumption. Exemptions are present in Howarth (1996; 2006), Brekke and Howarth (2002) and Wendner (2005). The taxation implications in an economy with multiple externalities are discussed there, but no tax implications in the second-best case are considered by the authors.

(ii) A restriction on one of the policy instruments
Fewer policy instruments than targeted variables to control are most likely relevant in many cases. If a specific type of tax instrument is politically controversial, its implementation will face strong opposition from the media, affected social and economic associations or opposing political parties for the purpose of achieving the withdrawal of such kind of policies even though the policy may benefit the whole country. This can be exemplified by public protests against environmental taxes in developing countries and very low housing property tax in western countries compared to its degree of positionality. Johansson-Stenman et al.(2002) and Aronsson and Johansson-Stenman (2014) hereby refer to the magnitude to which the marginal utility of consumption is derived from relative consumption. The degree of positionality for housing is around 50% in the calculation of Alpizar et al. (2005), but the tax on housing wealth ranges from 0.2 to 0.75 percent in Scandinavian countries for instance. Intuitively, positional externality caused by housing consumption has thus not been fully corrected by the housing tax. Aronsson and Mannberg (2015) summarize two possible reasons for this low tax rate: the homeowners’ association constitutes a strong lobby group in Sweden that lobbies against higher taxes on housing wealth. Also, regionally optimal property taxes are confined to its jurisdictional boundaries. This implies that the positional externality caused by cross-boundary consumption behaviors might not be fully internalized. However, to our knowledge, previous research has not devised a theoretical model to discuss the failure of one tax instrument and to describe this economic phenomenon.

(iii) Non-atmospheric externality

A non-atmospheric externality is often more consistent with the economic reality since the marginal contribution to a positional externality might differ across different consumers. Earlier research also deals with the tax policy implications of such externalities. Diamond (1973) discusses public policies dealing with congestion in a second-best setting when a uniform price is not enough to internalize a non-atmospheric externality. In his results, the optimal pricing relies on whether a demand for congested facilities is sensitive, in addition to price, to congestion. Green and Sheshinski (1976) show that given the differed marginal contribution to the consumption externality across individuals, a hybrid mixed policy including direct and indirect taxes outweighs direct

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2 The concept here is different from “asymmetric externality” by Eckerstorfer and Wendner (2013). It means that the individual’s choices of reference group affect their well-beings if they care about relative consumption. An asymmetric externality may either be atmospheric or non-atmospheric
taxes alone. In the discussion of optimal labor and capital income taxation, Aronsson and Johansson-Stenman (2010) employ upward comparisons in an intertemporal optimization problem, where consumers are featured by two-period lifespan, in the presence of a hypothetical welfarist government. An upward comparison leads to non-atmospheric externality, where all individuals face the same measure of reference consumption, i.e., the average consumption among high-ability individuals. For the motivation of upward comparisons, Major et al. (1991) suggest that such comparison could be expected to improve an individual’s performance. In the model of Aronsson and Johansson-Stenman (2010), the reference level is exclusively contributed by the current young productive individuals’ consumption. Similar to consumption comparisons for old consumers, the ongoing consumption of the old high-ability type is compared by all old consumers. Aronsson and Johansson-Stenman (2018) derive non-linear income tax implications when comparing the outcome in welfarist and paternalist governments. Their research is also extended to alternative reference points. One specific alternative is upward social comparison. Eckerstorfer and Wendner (2013) set a framework with non-atmospheric externalities caused by relative consumption to derive optimally proportionate commodity taxes in a similar defined economy as in Eckerstorfer (2014). They show that a first-best outcome requires the government to observe individual-specific consumption pattern for implementing the optimal commodity tax.

1.3 Contribution

This paper primarily makes three contributions based on the previous literature. Firstly, combining Eckerstorfer (2014) and Pirttilä and Tuomala (1997), this paper extends the research framework with single externality and social-status seeking to that pertaining to the inclusion of both environmental and positional externalities. Furthermore, it indicates a higher extent of economic distortion and therefore more potential welfare losses. In the presence of such effects, taxes no longer are pure distortionary taxes but have a corrective tax element that addresses externalities from consumption. If the externalities are atmospheric, the commodity tax is used to achieve corrective purposes, while the income taxes are used for redistribution objectives. Furthermore, the income tax is partly used to compensate the consumers for distortions created by the commodity taxes. These distortions associated with the commodity tax are represented by the non-corrective part of this tax, i.e., the discrepancy between the tax and the marginal value of the externalities.
Secondly, compared to Aronsson and Mannberg (2015), we modify the benchmark model to discuss tax implications in the presence of multiple externalities when the government is unable to implement an optimal tax on the commodity that causes the positional externality. This case means that the principle of targeting does not hold. Other commodity taxes and the income tax will serve for the positional externality corrective purpose. The compensated cross-price effect between the positional good and the dirty good acts as an essential factor in the taxation of the dirty good. The tax rate could be higher (lower) if the compensated cross-price effect is negative (positive), i.e. with them being complements (substitutes).

Thirdly, in this defined framework, we use the non-atmospheric consumption externality from Eckerstorfer (2013) to explore commodity and income taxes. For simplicity, we take one extreme case as Aronsson and Johansson-Stenman (2018) do: upward comparisons where all consumers compare their consumption with the consumption among high-ability individuals, meaning that the marginal contribution made by low ability-type consumers to the reference level is zero. The results presented below imply that the environmental externality is atmospheric and is internalized by the tax on the dirty good. The linear tax on the positional good, however, does not fully internalize the positional externality. Marginal income taxation has to correct the positionality in addition to its redistribution objectives among different types of consumers.

1.4 Outline

The demarcation of this paper will thus be the following: In section 2, we set up the benchmark model and address Pareto-efficient marginal income taxes and commodity taxes in a second best. Two extensions are made thereafter by modifying our benchmark model to a three-good economy, where we separate a dirty good from a positional good. Section 3 describes mixed tax policies when the government is unable to tax the positional good. Section 4 discusses the policy rules under upward comparisons, as one case of a non-atmospheric externality. Section 5 summarizes the results and concludes along with a discussion of possible extensions for future research.

2. Benchmark model
2.1 Preferences and individual behavior

The model consists of one country with a fixed population. There are two types of consumers, a low type consumers \( j = 1 \) and a high type consumer \( j = 2 \). \( n^j \) denotes the number of individuals of ability-type \( j \). They distinguish themselves from one another by their different innate ability as reflected in the pre-tax wages \( w^j \), where \( w^2 > w^1 \).

The individual has preferences over two private goods, a good \( C^j \) that does not generate any externalities and an externality-generating good \( X^j \). For good \( X \) an individual consumer both cares about his or her absolute and relative consumption compared to others, and two externalities are generated from its consumption. One externality comes from relative consumption. Following Aronsson and Johansson-Stenman (2008), we use the difference between the individual’s own consumption and the average consumption \( R^j = X^j - \bar{X} \) to describe the individual’s relative consumption\(^3\), where the reference consumption level is therefore expressed by \( \bar{X} = \frac{n^1 X^1 + n^2 X^2}{n^1 + n^2} \), which is treated exogenously to the individual. Increased consumption of the positional good by any individual raises consumption reference levels in terms of average consumption, which in turn harms society as a whole (Frank, 1997). The other externality is environment pollution \( E = \sum_j n^j X^j \), which is interpretable to imply that pollution depends on the aggregate consumption. Therefore, both the positional and environmental externality is atmospheric meaning that the marginal contribution by each individual is the same. The utility also depends on his or her leisure time \( Z^j \). The time endowment for each consumer is normalized to be one and labor time used to earn income is given by the share of time spent on working \( l^j \). \( Z^j \) can therefore be expressed by \( Z^j = 1 - l^j \). The utility function faced by an individual of ability-type \( j \) can therefore be written as

\[
U^j = U(C^j, X^j, Z^j, E, R^j).
\] (1)

By assumption \( U^j \) is twice continuously differentiable in each argument and strictly concave. We also assume \( \frac{\partial U^j}{\partial C^j} > 0, \frac{\partial U^j}{\partial X^j} > 0, \frac{\partial U^j}{\partial R^j} > 0, \frac{\partial U^j}{\partial Z^j} > 0, \frac{\partial U^j}{\partial E} < 0 \). The economics

\(^3\) There are of course other ways to describe relative consumption concern, for example \( R^j = \frac{X^j}{\bar{X}} \) in Clark & Oswald (1998)
intuition is easy to understand: more utility can be obtained directly from more private consumption, a higher relative consumption level as well as more leisure time. Conversely, labor time - which counterbalances leisure - and environmental pollution weaken consumer’s straightforward satisfactions.

We set good $C$ as a numeraire. The price of it is set to unity. Suppose that the set of tax instruments consists of a linear commodity tax $t_x$ and a non-linear income tax $T\left(\frac{w}{l}\right)$. The government poses a proportionate tax $t_x$ on the externality-generating commodity, so the relative price of good $X$ is $q_x = 1 + t_x$, where we assume the producer price of this good to be unity for simplicity. The government can observe the earnings $Y = w/l$ but does not observe the wage (which reflects the innate ability) of any particular consumer. The consumption pattern is only observed at an aggregate level. The asymmetric information prevents the government from redistribution through lump-sum taxes based on ability. Therefore the design of taxation has to deal with the revelation of a real preference for each consumer.

The consumer’s optimization problem then becomes to choose $C', X'$ and $l'$ in order to maximize his/her utility function as presented in (1) subject to the budget constraint (2). Compatible with a static model, the individual here has to consume all his/her earning and no savings exist,

$$w'l' - T\left(\frac{w}{l}\right) = C' + q_x X' = C' + (1 + t_x) X'.$$

The before-tax wage rate $w'$, the consumer prices $q_x$, the average consumption level of the positional commodity $\bar{X}$, and the environmental damage $E$ are treated as exogenously given by the individual(s).

We follow Christiansen (1984) when solving the individual’s optimization problem. The optimal taxation will be derived with the use of commodity demand function and indirect utility function conditioned on labor hours. We, therefore, divide the decisions into two
stages. In the first stage, we choose $C^j$ and $X^j$ to maximize the utility subject to the budget constraint

$$B^j = C^j + q_x X^j = C^j + (1 + t_x) X^j,$$

where $B^j$ is a fixed post-tax income. The following conditional demand functions can, therefore, be derived from the solutions of this first-stage problem

$$C^j = C(B^j, q_x, Z^j, E, \bar{X})$$

$$X^j = X(B^j, q_x, Z^j, E, \bar{X})$$

where $\frac{\partial X^j}{\partial B^j} > 0$ if $X$ is a normal good, in which case $\frac{\partial X^j}{\partial q_x} < 0$. Furthermore, we interpret $X^j$ and $Z^j$ as complements (substitutes) if $\frac{\partial X^j}{\partial Z^j} > 0$ ($< 0$). The substitution of the conditional demand functions (4) and (5) into the direct utility function (1) yields the conditional indirect utility function

$$V^j = V(B^j, q_x, Z^j, E, \bar{X})$$

where $\frac{\partial V^j}{\partial B^j} > 0$, $\frac{\partial V^j}{\partial q_x} < 0$, $\frac{\partial V^j}{\partial Z^j} > 0$, $\frac{\partial V^j}{\partial E} < 0$, $\frac{\partial V^j}{\partial X} < 0$.

In the second stage, we choose labor hours $l^j$ to maximize the conditional indirect utility function subject to

$$Z^j = 1 - l^j$$

$$B^j = w^j l^j - T(w^j l^j)$$

The individual’s F.O.C is accordingly described by

$$w^j - MRS_{Z,B}^j = w^j T'(w^j l^j).$$

where $MRS_{Z,B}^j = \frac{\partial V^j}{\partial Z^j} / \frac{\partial V^j}{\partial B^j}$ is the marginal rate of substitution between leisure and disposable income.
2.2 Government behavior

The government is assumed to internalize positional and environmental externalities, and aims at reaching a Pareto efficient resource allocation. The government would therefore face the two externality constraints

$$\bar{X} = \frac{n^1 X^1 + n^2 X^2}{n^1 + n^2} \quad (10)$$

$$E = \sum_j n^j X^j. \quad (11)$$

With equation (8) the government’s budget constraint given by

$$\sum_j n^j \left[T \left(w^j l^j \right) + t_x X^j \right] = 0$$

can be recast as

$$\sum_j n^j \left[w^j l^j - B^j + t_x X^j \right] = 0. \quad (12)$$

We consider the realistic case where the government wants to redistribute from the high-ability to the low-ability type\(^4\). We must, therefore, impose a self-selection constraint such that each high-ability individual prefers the allocation intended for his/her type over the allocation intended for the low-ability type. Otherwise, high-type individuals may choose to mimic the low-ability type, and the redistribution system would therefore be undermined. The following inequation (13) embodies the incentive of self-selection constraint

$$V^2 = V \left(B^2, q_x, Z^2, E, \bar{X} \right) \geq V \left(B^1, q_x, Z^2, E, \bar{X} \right) = \hat{V}^2. \quad (13)$$

\(\hat{V}^2\) denotes the utility of the mimicker who as a high ability-type individual pretends to be a low ability-type one. The mimicker would reduce labor time to derive identical disposable wage, \(B^1\), as a low ability-type individual. \(\hat{Z}^2 = 1 - \phi l^1 = 1 - \frac{w^1}{w^2} l^1\) is the mimicker’s leisure time, where \(\phi l^1 < l^1\) is the corresponding time spent on work and \(\phi = w^1 / w^2 < 1\) as the wage ratio implies more leisure time for the mimicker.

\(^4\) Stiglitz (1982) addressed both directions of redistribution.
We formulate the public decision problem as if the government maximizes the utility of the low ability-type subject to a minimum utility restriction and self-selection binding for high ability-type consumers, as well as externality generation and government’s budget feasibility. We also write the externalities as separate constraints, which is convenient for the derivation of the associated shadow prices. The Lagrangean for the social planner reads

\[ L = V^1 + \eta (V^2 - \bar{V}^2) + \lambda (V^2 - \bar{V}^2) \]

\[ + \mu \left( E - \sum_j n^j X^j \right) + \delta \left( \sum_j \frac{n^j X^j}{n^j + n^2} \right) + \gamma \sum_j n^j \left( w^j l^j - B^j + t_x X^j \right) \]  \tag{14}

where \( \eta, \lambda, \mu, \delta \) and \( \gamma \) are Lagrange multipliers associated with the minimum utility restriction, self-selection constraint, the environmental constraint, the positional constraint, and the government’s budget constraint, respectively. The social F.O.Cs for \( B^j, l^j, q_x, E, X \) can now be written as follows:

\[ \frac{\partial V^1}{\partial B^i} - \lambda \frac{\partial V^2}{\partial B^i} + \left( \gamma t_x - \frac{1}{n^j + n^2} \delta - \mu \right) n^i \frac{\partial X^1}{\partial B^i} - \gamma n^i = 0 \]  \tag{15a}

\[ - \frac{\partial V^1}{\partial Z^j} + \lambda \phi \frac{\partial V^2}{\partial Z^j} - \left( \gamma t_x - \frac{1}{n^j + n^2} \delta - \mu \right) n^j \frac{\partial X^1}{\partial Z^j} + \gamma n^j w^j = 0 \]  \tag{15b}

\[ (\eta + \lambda) \frac{\partial V^2}{\partial B^2} = \left( \gamma t_x - \frac{1}{n^j + n^2} \delta - \mu \right) n^2 \frac{\partial X^2}{\partial B^2} - \gamma n^2 = 0 \]  \tag{15c}

\[ - (\eta + \lambda) \frac{\partial V^2}{\partial Z^2} = \left( \gamma t_x - \frac{1}{n^j + n^2} \delta - \mu \right) n^2 \frac{\partial X^2}{\partial Z^2} + \gamma n^2 w^2 = 0 \]  \tag{15d}

\[ \left( \begin{array}{c} \gamma n^i - \frac{\partial V^1}{\partial B^i} \\ \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \end{array} \right) X^i + \left( \begin{array}{c} \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \\ \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial Z^2} \end{array} \right) X^2 \]

\[ + \lambda \frac{\partial V^2}{\partial B^l} \tilde{X}^l + \left( \gamma t_x - \frac{\delta}{n^j + n^2} - \mu \right) \sum_j n^j \frac{\partial X^j}{\partial q_x} = 0 \]  \tag{15e}

\[ ^5 \text{ We use Roy’s identity } \frac{\partial V^j}{\partial q_x} = - \frac{\partial V^j}{\partial B^j} X^j \text{ and } q_x = 1 + t_x \text{ in equation (15e).} \]
\[
\frac{\partial V^1}{\partial E} + (\eta + \lambda) \frac{\partial V^2}{\partial E} - \lambda \frac{\partial V^2}{\partial X} - \left(\gamma t_x - \frac{\delta}{n^1 + n^2} - \mu \right) \sum_j n^j \frac{\partial X^j}{\partial E} + \mu = 0
\]

\[
\frac{\partial V^1}{\partial X} + (\eta + \lambda) \frac{\partial V^2}{\partial X} - \lambda \frac{\partial V^2}{\partial X} - \left(\gamma t_x - \frac{\delta}{n^1 + n^2} - \mu \right) \sum_j n^j \frac{\partial X^j}{\partial X} + \delta = 0.
\]

The efficient tax structure hinges on the social shadow prices of the environmental damage and positional externality measured in terms of public funds, \(\mu/\gamma\) and \(\delta/\gamma\) respectively. These ratios are interpretable as the government’s marginal willingness to lessen environmental damage and dispassionate status-seeking behaviors measured in terms of tax revenues. We define \(MWP^i_{E,B} = -\frac{\partial V^j/\partial E}{\partial V^j/\partial B^i} > 0\) as individual \(j\)’s (for ability-type \(j=1,2\)) marginal willingness to pay for an improvement in environmental quality. Similarly \(\overline{MWP}^2_{E,B} = -\frac{\partial V^2/\partial E}{\partial V^2/\partial B^i} > 0\) denotes the marginal willingness for any mimickers. We combine (15f), \(MWP^j_{E,B} = -\frac{\partial V^j/\partial E}{\partial V^j/\partial B^i}\) and \(\overline{MWP}^2_{E,B} = -\frac{\partial V^2/\partial E}{\partial V^2/\partial B^i}\) to obtain the shadow price of environmental externality (See Appendix)

\[
\frac{\mu}{\gamma} = \frac{1}{\Omega^E} \left\{ \sum_j n^j MWP^j_{E,B} + \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial B^i} \left( MWP^i_{E,B} - \overline{MWP}^2_{E,B} \right)^2 \right\} - \left( t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} \right) \sum_j n^j \frac{\partial X^j,\text{com}}{\partial E}
\]

where \(\frac{\partial X^j,\text{com}}{\partial E} = \frac{\partial X^j}{\partial E} + \frac{\partial X^j}{\partial B^i} MWP^j_{E,B}\) and \(\Omega^E = \left[ 1 - \sum_j n^j \frac{\partial X^j,\text{com}}{\partial E} \right].\)

The variable \(\Omega^E\) denotes an environmental feedback parameter. It describes how the demand equations respond to a change in the environmental externality. Following Sandmo (1980), stability requires this term to be positive. The first term in curly brackets refers to the sum of all individuals’ marginal willingness to pay for a cleaner environment. The third term measures how the low-ability type and the mimicker differ in the marginal value of less pollution. The forth term represents tax revenue effects as the total volume

\[\text{n We use Slutsky condition to substitute the conditional compensated demand function.}\]
of pollution affects the commodity tax bases, exemplified by the sum of compensated conditional demand for the externality-generating good responding to an increase in pollution. In this component, \( \frac{1}{n' + n''} \delta \) denotes the marginal value of positional externality. The perfect correction for this positional externality by the commodity tax can be used to derive the unconditional shadow price, where \( \left( t_x - \frac{1}{n' + n''} \delta \right) \) will vanish in (16).

In a similar pattern, let \( MWP_{X,B}^j = \frac{\partial V^j}{\partial X} \frac{\partial X}{\partial V^j} \) denote the marginal willingness to pay to avoid the positional externality for an individual with ability \( j \) and \( MWP_{X,B}^{-2} = \frac{\partial V^{-2}}{\partial X} \frac{\partial X}{\partial V^{-2}} \) denote the corresponding willingness for a mimicker. The combination with equation (16), \( MWP_{X,B} = \frac{\partial V^j}{\partial X} \frac{\partial X}{\partial V^j} \) and \( MWP_{X,B}^{-2} = \frac{\partial V^{-2}}{\partial X} \frac{\partial X}{\partial V^{-2}} \) allows us to derive the shadow price of the positional externality

\[
\frac{\delta}{\gamma} = \frac{1}{\Omega^X} \left\{ \sum_j n' \frac{\partial X^j}{\partial X} \frac{\partial X^j}{\partial V^j} \left( MWP_{X,B}^j - MWP_{X,B}^{-2} \right) - \left( t_x - \frac{\mu}{\gamma} \right) \sum_j n' \frac{\partial X^{\text{com}}_j}{\partial X} \right\}, \quad (18)
\]

where \( \frac{\partial X^{\text{com}}_j}{\partial X} = \frac{\partial X^j}{\partial X} \frac{\partial X^j}{\partial V^j} MWP_{X,B}^j \) and \( \Omega^X = \left[ 1 - \frac{1}{n' + n''} \sum_j n' \frac{\partial X^{\text{com}}_j}{\partial X} \right]. \quad (19) \]

The variable \( \Omega^X \) in (18) denotes positional feedback parameter and keeps positive for stability as \( \Omega^E \) in (17). The first term in curly brackets refers to the sum of consumers’ marginal willingness to pay for a decrease in the level of reference consumption. The third term measures the difference in this marginal willingness to pay for a lower reference level between the low-ability type and the mimicker. The forth term represents tax revenue effects as the reference level influences the commodity tax bases, since the sum of compensated conditional demand for the positional good responds to increased reference consumption. There, \( \mu/\gamma \) is the conditional shadow price of environmental externality and has been perfectly rectified by the commodity tax, which allows us to
derive the unconditional shadow price of positional externality by substituting the following efficient commodity tax implication into (18). The optimal commodity taxation is described by (See Appendix)

\[ t_x = \frac{1}{M} \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial B} \left( X^1 - \hat{X}^2 \right) + \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} \mu, \]  

(20a)

where \( M = \sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial q_x} \) and \( \frac{\partial X^{j,\text{com}}}{\partial q_x} = \frac{\partial X^j}{\partial q_x} + \frac{\partial X^j}{\partial B} X^j. \)  

(21)

Equation (20a) reflects three different motives for commodity taxation: (i) to relax the self-selection constraint (ii) to adjust the level of the environmental damage (iii) and to balance the social status-seeking behaviors. The first term in (20a) is proportional to \( \left( X^1 - \hat{X}^2 \right) \) and independent of shadow prices of positional and environmental externalities, implying the necessity of the tax instrument regardless of the presence of any externalities. The social shadow price of the environmental damage and that of positional externality enter additively in the tax formula on the externality-generating good \( X \). In spite of identical disposable income across the low-ability type and the mimicker, and therefore the same consumption possibility, their choices possibly end up with differentiated consumption pattern, because the possible correlation between consumption bundles and leisure plays a vital role.

If leisure is complementary with the externality-generating good such that the mimicker consumes more of this good than the low-ability type, i.e. \( X^1 < \hat{X}^2 \), a higher commodity tax would, for this reason, impair the mimicker’s utility more. As such, the government would tend to execute a higher tax on the externality-generating good so as to relax the self-selection constraint. The argument for a lower tax rate goes analogously if there is a substitute relationship between good \( X \) and leisure.

\[ \text{The Slutsky condition with respect to price effects on conditional compensated demand functions.} \]
The policy incentive to internalize the environmental externality is captured by $\mu/\gamma$, where $\mu/\gamma$ reflects a mixture of corrective and redistributive motives for environmental quality. A cleaner environment would not only without doubt benefit consumers but possibly also alter the flexibility of self-selection constraint. The later effect relies on whether leisure is complementary with, or substitutable for, marginal willingness to pay for a cleaner environment $MWP_{E,B}^{j}$.

If leisure is complementary with environmental quality such that the marginal willingness to pay for a cleaner environment $MWP_{E,B}^{j}$ increases with leisure, $MWP_{E,B}^{j}$ will be less than $MWP_{E,B}^{2}$. In this context, A decrease in environmental quality gives rise to relatively more disutility for the mimicker. The self-selection constraint is therefore relaxed. We could expect a socially favorable outcome because the government will face fewer impediments (slackness of the self-selection constraint) when reallocationing economic resources. A lower marginal value of environmental quality will be judged by the government. This “downplay” thus implies a lower commodity tax rate. Parallel to this case, if leisure and environmental quality are substitutes, then $MWP_{E,B}^{j} > MWP_{E,B}^{2}$ holds. An increase in environmental quality would contribute to relaxing the self-selection binding. A higher marginal value of environment would therefore be attached by the government, which in turn raises the commodity tax.

Analogously, the policy motivation to internalize the positional externality is described by $\delta/\gamma$ with both corrective and redistributive motives for a change in $\overline{X}$. First, a lower average level of positional consumption would contribute to consumer’s utility. Second, the total effect depends on how self-selection constraints respond to a change in the reference level, i.e. possible correlation between leisure and the marginal willingness to pay for a lower reference level $MWP_{X,B}^{j}$. We could derive similar patterns as in the analysis of policy incentives in the environmental externality, which means if $MWP_{X,B}^{j}$ increases (decreases) with the use of leisure, $MWP_{X,B}^{j} < (>) MWP_{X,B}^{2}$ holds. A(n) decrease
(increase) in positional consumption would hurt (benefit) the mimicker more. This contributes to relaxing (tighten) the self-selection constraint.

By using the marginal rate of substitution between leisure and private disposable income

\[
MRS_{j,B}^i = \frac{\partial V^i / \partial Z^i}{\partial V^i / \partial B^i}
\]

and corresponding conditional compensated commodity demand

\[
\frac{\partial X^{j,\text{com}}}{\partial Z^i} = \frac{\partial X^{j}}{\partial Z^i} - \frac{\partial X^{j}}{\partial B^i} MRS_{j,B}^i
\]

the non-linear marginal income taxes can be formulated as follows (See Appendix)

\[
T'(w^i l^i) = \frac{1}{w^i} \left\{ \lambda \frac{\partial \tilde{V}^2}{\partial B^i} \left[ MRS_{j,B}^i - \tilde{\phi MRS}_{j,B}^2 \right] + \left( t_x - \frac{1}{n^i + n^2} \frac{\delta}{\gamma} - \frac{\mu}{\gamma} \right) \frac{\partial X^{1,\text{com}}}{\partial Z^i} \right\},
\]

\[
T'(w^2 l^2) = \frac{1}{w^2} \left( t_x - \frac{1}{n^i + n^2} \frac{\delta}{\gamma} - \frac{\mu}{\gamma} \right) \frac{\partial X^{2,\text{com}}}{\partial Z^i},
\]

where

\[
t_x = \frac{1}{n^i + n^2} \frac{\delta}{\gamma} - \frac{\mu}{\gamma} = \frac{1}{M \gamma} \frac{\lambda}{\gamma} \frac{\partial \tilde{V}^2}{\partial B^i} \left( X^i - \tilde{X}^2 \right).
\]

The marginal income tax formulas show that the two social shadow prices of the externalities do not affect the policy rules for marginal income taxation. Since the commodity tax on the externality-generating good, \( t_x \), is available to target the overall consumption externalities, distortion in the labor supply in response to the externalities would not be necessary. Particularly, the substitution of equation (20b) into (22a) and (22b) implies that the shadow prices of both positional and environmental externalities vanish from the marginal income tax rules. This is why Sandmo’s (1975) additivity property, which is a consequence of the principle of targeting, is applicable in the tax structure. The optimal income taxes are therefore not used for externality correction in this model. The marginal income tax reflects two basic motives for taxation: (i) to relax the self-selection binding, which is accomplished by exploiting the difference in marginal valuation on leisure between a low ability-type and a mimicker and (ii) for compensation for the distortions caused by the commodity taxation.
The interactive term \( t_x - \frac{1}{n_1 + n_2} \frac{\delta}{\gamma} \mu \frac{\partial X_{\text{com}}}{\partial Z^j} \) in (22a) and (22b) describes a compensation effect of how marginal income \( T'(w/l^j) \) responds to commodity taxation \( t_x \). This compensation effect will not appear if there are no externalities and commodity taxes. The marginal income tax implications in this case are consistent with those in Stiglitz (1982) that a positive marginal income tax rate of the low-ability type and a zero marginal tax rate of the high-ability type. Such an interaction effect arises since the income tax, which is more flexible compared to the commodity tax, is used to compensate the consumers for commodity tax-generating distortions. For example, if the commodity tax exceeds the tax that would be motivated by pure externality correction

\[
t_x > \frac{1}{n_1 + n_2} \frac{\delta}{\gamma} + \frac{\mu}{\gamma},
\]

the commodity tax distorts the consumption of \( X \) downwards. Therefore, more consumption of the externality-generating good, as a compensation for the distortion caused by the inflexible commodity tax, is recommended for social welfare improvement. The policy maker will for this reason set a higher marginal income tax rate if the externality-generating good is complementary with leisure and a lower marginal income tax rate if they are substitutable. Policy incentives in a similar manner switch to diminish the consumption of good \( X \) if

\[
t_x < \frac{1}{n_1 + n_2} \frac{\delta}{\gamma} + \frac{\mu}{\gamma}.
\]

This compensation effect also differs from that derived by Pirttilä and Tuomala (1997) and Eckerstorfer (2014). Their optimal marginal income tax formulas contain a single externality (environmental externality in Pirttilä and Tuomala (1997) and positional externality in Eckerstorfer (2014)) affecting the interaction between the income and commodity taxation. Because we consider positional and environmental externalities simultaneously when deriving optimal policy rules for taxation.

3. Extension to a case where taxation of the positional good is not feasible

The economic intuition of fewer instruments than variables to control stems from economic realities. The phenomenon could be ascribed to social association efficient boycott, administration efficiency and anti-policy propaganda made by political
competitors, etc. For example, the draft on carbon dioxide emission tax has been strongly resisted by energy-intensive industries in developing countries, such as China, which impairs firms’ competition by adding additional financial costs. In case of inefficient emission monitoring firms may resort to higher emission levels to circumvent the emission tax, though the tax rule has been ratified by national juristic system. Another example is low housing property tax rates in (not confined to) Europe analyzed by Aronsson and Mannberg (2015), where two possible reasons are mentioned: homeowners’ association has become an influential lobby group on housing policy in Sweden; local housing tax will not fully correct this positional externality, if the consumption of housing is made by residents in other regions. In their research, the tax rate on housing wealth does not exceed 0.75 percent in Scandinavian countries, but more than half of the utility is derived from positional effects in housing consumption (Alpizar et al., 2005).

In this section, we separate a positional good $X$ from a ‘dirty’ good, $D$, to discuss how the inability to directly tax the positional good, due to the lack of compelling reasons to tax “envy”, affects marginal income taxes $T'(w/l')$ and the tax on the dirty good $t_D$. A dirty good here can be described as a commodity that generates environment degradation. Note first that if both of the commodity taxes on a positional good $t_X$ and a dirty good $t_D$ were available, the shadow prices of positional and environmental externalities would additively enter into the associated commodity tax instruments respectively. The principle of targeting would carry over, because the hypothetical commodity taxes are enough to internalize the externalities. Income taxes would not be designed for corrective purposes. Two interactive terms between marginal income and commodity taxation \[ \left( t_X - \frac{1}{n' + n''} \frac{\delta}{\gamma} \right) \text{ and } \left( t_D - \frac{\mu}{\gamma} \right) \] are expected to appear and to affect the governmental willingness in compensation for different private consumption. Let us now add the restriction that $t_X = 0$ in the optimization problem.

3.1 Preferences and individual behavior

The separation in externality-generating commodities modify the utility function faced by the representative individual and it now becomes
As before, we solve the consumer’s problem in two stages. The first stage means choosing \( C^j, X^j \) and \( D^j \) to maximize the utility subject to the budget constraint

\[
B^j = C^j + X^j + q_D D^j = C^j + X^j + (1 + t_D) D^j ,
\]

where \( B^j \) is a fixed post-tax income. The following conditional demand functions and indirect utility function can, therefore, be derived with the solutions of this optimal problem

\[
C^j = C \left( B^j, q_D, Z^j, E, \overline{X} \right), \quad (26)
\]

\[
X^j = X \left( B^j, q_D, Z^j, E, \overline{X} \right), \quad (27)
\]

\[
D^j = D \left( B^j, q_D, Z^j, E, \overline{X} \right), \quad (28)
\]

\[
V^j = V \left( B^j, q_D, Z^j, E, \overline{X} \right). \quad (29)
\]

The second stage means choosing labor hours \( l^j \) to maximize the conditional indirect utility function. The individual’s F.O.C is the same as the expression (9) in section 2.

### 3.2 Government behavior

For the government, the generation mechanism of environmental and positional externalities changes to

\[
E = \sum_j n^j D^j , \quad (30)
\]

\[
\overline{X} = \frac{\sum_j n^j X^j}{n^j + n^2} . \quad (31)
\]

Since \( t_X = 0 \), the public budget now reads as

\[
\sum_j n^j \left( w^j l^j - B^j + t_D D^j \right) = 0 . \quad (32)
\]

We can accordingly recast the government’s optimization problem as the following Lagrangean function
\[
L = V^i + \eta \left( V^2 - V^1 \right) + \lambda \left( V^2 - \hat{V}^2 \right) \\
+ \mu \left( E - \sum_j n^j X^j \right) + \delta \left( X - \frac{\sum_j n^j X^j}{n^1 + n^2} \right) + \gamma \sum_j n^j \left( w^j t^j - B^j + t_D D^j \right)
\]

The outcome of the optimization process allows us to derive the efficient commodity tax on the dirty good (See Appendix)

\[
t_D = \frac{1}{N} \left\{ \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( D^j - \hat{D}^2 \right) + \frac{1}{n^1 + n^2} \delta \sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial q_D} \right\} + \frac{\mu}{\gamma},
\]

where \( N = \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial q_D} < 0 \) and

\[
\frac{\mu}{\gamma} = \frac{1}{\Omega^{E'}} \left\{ \sum_j n^j MWP^{j}_{E,B} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( MWP^{j}_{E,B} - \hat{MWP}^2_{E,B} \right) + \frac{1}{n^1 + n^2} \delta \sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial E} - t_D \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial E} \right\}
\]

with an environmental externality feedback parameter \( \Omega^{E'} = 1 - \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial E} \). Similarly,

\[
\frac{\delta}{\gamma} = \frac{1}{\Omega^{X'}} \left\{ \sum_j n^j MWP^{j}_{X,B} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( MWP^{j}_{X,B} - \hat{MWP}^2_{X,B} \right) - \left( t_D - \frac{\mu}{\gamma} \right) \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial X} \right\}
\]

with a positional externality feedback parameter \( \Omega^{X'} = 1 - \frac{1}{n^1 + n^2} \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial X} \).

In addition to relaxing the self-selection constraint and correcting for the environmental externality, the commodity tax on the dirty good will also be used to indirectly correct for the positional externality. The shadow price of positionality measured in terms of fiscal income enters the tax formula for the dirty good. For purpose of interpretation, we keep the sign of \( \delta/\gamma \) positive, since \( \delta/\gamma = \sum_j n^j MWP^{j}_{E,B} > 0 \) in the first-best case. We could accordingly predict a lower tax rate on the dirty good if good \( X \) and good \( D \) are substitutes.

\[8\] The way to derive the shadow prices of both positional and environmental externalities is the same as what we have done in section 2.
\[
\frac{\partial X^{j,\text{com}}}{\partial q_D} > 0 \text{ since } N < 0 \text{ in (34). The intuition is that an increase in the price of the dirty good will lead to relatively more demand for good } X \text{ and less for good } D. \text{ The environmental degradation accordingly lessens. The shadow price of environment externality enters (34) additively, which is ascribed to the assumption that the environmental externality is atmospheric, indicating that this externality is internalized by the tax on the dirty good.}
\]

In a similar manner, we derive the efficient marginal income taxes for the two types of consumer

\[
T'(w^1l^1) = \frac{1}{w^1} \left\{ \frac{1}{n^1} \frac{\partial \hat{V}}{\partial B^1} \left[ MRS^1_{Z,\beta} - \phi MRS^2_{Z,\beta} \right] - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} \frac{\partial X^{1,\text{com}}}{\partial Z^1} + \left( t_D - \frac{\mu}{\gamma} \right) \frac{\partial D^{1,\text{com}}}{\partial Z^1} \right\},
\]

(37a)

\[
T'(w^2l^2) = \frac{1}{w^2} \left\{ \left( t_D - \frac{\mu}{\gamma} \right) \frac{\partial D^{2,\text{com}}}{\partial Z^2} - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} \frac{\partial X^{2,\text{com}}}{\partial Z^2} \right\}.
\]

(37b)

As Micheletto (2008) shows, the income tax schedule will have Pigouvian elements if the additive property does not hold. The presence of \( \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} \frac{\partial X^{j,\text{com}}}{\partial Z^j} \) in both income tax formulas shows that income taxes are used for correcting positionality which a commodity tax on the positional good should otherwise have done. Consequently, since there is no tax on the positional good (which would otherwise internalize the positional externality), the government uses the tax on the dirty good and the marginal income taxes as indirect instruments to correct for the positional externality. These indirect instruments are, of course, imperfect from the point of view of correcting for the positional externality.

4. Extension to upward comparisons

In the analyses of Sections 2 and 3, the individual’s marginal contribution to the externalities is identical. We described each externality as an atmospheric externality. Non-atmospheric externality, on the contrary, refers to that individuals contribute to the externalities in different degrees at the margin. Positional externality compared to environmental externality, however, is more likely to be non-atmospheric, since it may
depend on the reference group consumers are concerned with and individual’s marginal contribution to reference standards. For simplicity, we consider an extreme case: upward comparisons. All consumers, therefore, compare their own consumption with that of the high-productivity type with a common reference measure

\[ \overline{X} = X^2, \]  

which means that the high-productivity type exclusively contributes to positional externalities, despite relative consumption concerns across all consumers.

In the economy with non-atmospheric externality, we also consider a linear tax, \( t_X \), on this positional good, which is to be optimally chosen together with the other tax instruments. The individual behavior remains unaltered since \( q_x \) and \( q_d \) are exogenous to the individual consumers. The social optimization problem, however, changes, because the government has to optimally choose the tax together with other variables and its budget constraint now reads

\[
\sum_j n^j \left( w^j l^j - B^j + t_d D^j + t_x X^j \right) = 0. \]  

We show in Appendix that the efficient commodity taxes on the positional good and the dirty good are formed respectively as

\[
t_X = \frac{1}{|A|} \left( \frac{\partial X^{2,\text{com}}}{\partial q_x} \sum_j n^j \frac{\partial D^j_{\text{com}}}{\partial q_d} - \frac{\partial X^{2,\text{com}}}{\partial q_d} \sum_j n^j \frac{\partial D^j_{\text{com}}}{\partial q_x} \right) \delta \gamma \]

\[+ \frac{1}{|A|} \frac{1}{\gamma} \frac{1}{\partial B^i} \left[ \left( X^{1} - \overline{X}^{2} \right) \sum_j n^j \frac{\partial D^j_{\text{com}}}{\partial q_d} - \left( D^1 - \overline{D}^{2} \right) \sum_j n^j \frac{\partial D^j_{\text{com}}}{\partial q_x} \right] \]  

\[ t_d = \frac{\mu}{|A|} + \frac{1}{|A|} \left( \frac{\partial X^{2,\text{com}}}{\partial q_d} \sum_j n^j \frac{\partial X^j_{\text{com}}}{\partial q_x} - \frac{\partial X^{2,\text{com}}}{\partial q_x} \sum_j n^j \frac{\partial X^j_{\text{com}}}{\partial q_d} \right) \delta \gamma \]

\[+ \frac{1}{|A|} \frac{1}{\gamma} \frac{1}{\partial B^i} \left[ \left( D^1 - \overline{D}^{2} \right) \sum_j n^j \frac{\partial X^j_{\text{com}}}{\partial q_x} - \left( X^{1} - \overline{X}^{2} \right) \sum_j n^j \frac{\partial X^j_{\text{com}}}{\partial q_d} \right] \]  

where
The commodity tax formulas (40a) and (40b) depend in a complicated way on cross-price effects, the correlation between leisure and externality-generating commodities, and the shadow prices of the two externalities as in section 3. The last term, two types of self-selection constraint, on the right hand side in (40a) and (40b), is analogous to its counterparts in (20a) and (34). The two shadow prices and the externality feedback parameters are also similar to those in previous sections, we therefore focus on the discrepancies to explain. The sign of $|A|$ depends on whether the negative own price effects dominate the cross-price effects. Normally under the assumption of normal goods for the two commodities, where the first product (of two negative terms) on the right hand side must be positive, we would therefore assume that $|A|$ is positive.

$$|A| = \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} - \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_D} > 0$$

$$\delta = \frac{1}{\Omega^X} \left( \sum_j n_j \cdot \frac{\partial X^{j,\text{com}}}{\partial q_D} \sum_j n_j \cdot \frac{\partial D^{j,\text{com}}}{\partial q_D} - \left( \frac{\partial X^{j,\text{com}}}{\partial q_X} - t_d \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_X} - t_d \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_X} \right) \right)$$

$$\mu = \frac{1}{\Omega^D} \left( \sum_j n_j \cdot \frac{\partial X^{j,\text{com}}}{\partial q_D} \sum_j n_j \cdot \frac{\partial D^{j,\text{com}}}{\partial q_D} + \frac{\partial X^{j,\text{com}}}{\partial q_D} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} - t_d \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_X} - t_d \sum_j n_j \frac{\partial X^{j,\text{com}}}{\partial q_X} \right)$$

$$\Omega^X = 1 - \frac{\partial X^{2,\text{com}}}{\partial X} \quad \text{and} \quad \Omega^D = 1 - \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D}$$

In (40a) term \(1 \left[ \frac{\partial X^{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} - \frac{\partial X^{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} \right] \frac{\delta}{\gamma} \) depicts how own price effects and cross-price effects collectively affect the marginal value of the positional externality. \(1 \left[ \frac{\partial X^{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} - \frac{\partial X^{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D^{j,\text{com}}}{\partial q_D} \right] \) is the weight attached to the positional externality in the tax formula for the positional good. Note that this term would be equal to one, if the positional externality were atmospheric. We can also show that this tax on the positional commodity is not a flexible enough instrument to correct for this externality. It is thus necessary to resort to the other commodity tax and the marginal income taxes to yield a correction of the occurring externality. More specifically, based on the assumption that the marginal contribution made by a low-ability
type consumer falls short of that by a high-ability type, which is described by $0 < \frac{1}{n^2} \frac{\delta}{\gamma}$, the high-ability individual consumes more positional commodity than socially preferred. The government will use $t_d$ indirectly to correct for this overconsumption by decreasing his/her consumption of the dirty good.

In (40b), we interpret $\frac{1}{A} \left( \frac{\partial X_{2,\text{com}}}{\partial q_D} - \sum_j n_j \frac{\partial X_{j,\text{com}}}{\partial q_X} - \frac{\partial X_{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial X_{j,\text{com}}}{\partial q_D} \right)$ as the weight attached to the positional externality in the tax formula for the dirty good. Note that this term would be equal to zero, if the positional externality were atmospheric. The presence of this term thus denotes how $t_d$ corrects for the positional externality.

The inability of the linear tax on the positional good $t_x$ to fully internalize the non-atmospheric positional externality is the key issue. This makes the government use the other tax instruments (referring to the tax on the dirty good and marginal income taxes) as supplementary instruments to correct for the positional externality for social welfare maximization. With the same method in Section 2, we use the following formulas for income taxation

$$T'(w^l) = \frac{1}{w^l} \left\{ \frac{1}{n^l} \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial B^1} \left( MRS_{Z,B}^2 - \phi MRS_{Z,B}^2 \right) + \left( t_d - \frac{\mu}{\gamma} \right) \frac{\partial D_{1,\text{com}}}{\partial Z^1} + t_x \frac{\partial X_{1,\text{com}}}{\partial Z^1} \right\} \quad (41a)$$

$$T'(w^2) = \frac{1}{w^2} \left\{ \left( t_d - \frac{\mu}{\gamma} \right) \frac{\partial D_{2,\text{com}}}{\partial Z^2} + t_x \left( 1 - \frac{\delta}{n^2} \right) \frac{\partial X_{2,\text{com}}}{\partial Z^2} \right\} \quad (41b)$$

where

$$t_x = \frac{1}{A} \left( \frac{\partial X_{2,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D_{j,\text{com}}}{\partial q_D} - \sum_j n_j \frac{\partial X_{j,\text{com}}}{\partial q_X} \sum_j n_j \frac{\partial D_{j,\text{com}}}{\partial q_D} \right) \frac{\delta}{\gamma}$$

$$+ \frac{1}{A} \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial B^1} \left( X^1 - \tilde{X}^1 \right) \sum_j n_j \frac{\partial D_{j,\text{com}}}{\partial q_D} - \left( D^1 - \tilde{D}^1 \right) \sum_j n_j \frac{\partial D_{j,\text{com}}}{\partial q_X} \right) \quad (40a)$$
The appearance of \( t_D \) and \( t_X \) explains how income taxation becomes an ancillary choice for the positional externality rectification, where the three terms are the function of \( \delta / \gamma \) indicating the principle of targeting is not fully applicable by \( t_X \).

For corrective objectives, the non-linear income taxes compared to the proportionate commodity taxes allow the government flexibly to set different tax rates for different consumers. Since low-ability type’s consumption of the good \( X \) is not positional under upward comparisons, we know \( t_X \) exceeds the marginal socially positional externality generated by low-ability consumers, meaning \( t_X > 0 \). The last term in (40a) is accordingly positive (negative) as long as leisure and the positional commodity are complements (substitutes). A higher (lower) marginal income tax on low-ability consumers is naturally extrapolated. This tax allows the government to motivate (prevent) low-ability consumption of this good by restraining (increasing) work hours. In this case, \( t_X \) is less than the marginal socially positional externality generated by high-ability consumers, with \( t_X < \frac{1}{n^2} \frac{\delta}{\gamma} \). A lower (higher) marginal income tax is therefore implemented for high-ability individuals if the condition that leisure is complementary with (substitutable to) the positional commodity holds. This income tax policy will discourage positional consumption for high income groups by motivating them to use more (less) leisure time on working.
5. Conclusions

This paper broadens our primarily theoretical findings on optimal taxation in the presence of both positional and environmental externalities. Under the asymmetric information, where the government can only screen individual income and aggregate consumption level, rather than individual-specific wage and consumption pattern, a mixture of linear commodity taxes and non-linear income taxes is optimal. The limited information also requires the government to account for possible mimicking behaviors (by high skilled individuals) in its tax design.

We depart from a benchmark model that combines the modelling frameworks of Pirttilä and Tuomala (1997) and Eckerstorfer (2014). In this standard model, the assumption of atmospheric externalities on both positional and environmental externalities means that the principle of targeting applies. Thus, the tax on the externality-generating good leads to full internalization of the externalities. The policy rules for income taxation hereby take the same form regardless of the appearance of these externalities. When the inflexible commodity is distortionary to rectify externalities, the non-linear income taxes are flexible enough to compensate the consumers for the distortions created by the commodity tax. The possible correlations between leisure and (i) the externality-generating commodity, (ii) environmental quality and (iii) consumption reference levels are vital in the whole tax system.

Two extensions of the model are made where Sandmo’s additive property does not carry over. The first is the unavailability of a direct tax on the positional good; the other is the presence of upward comparisons. This means that the positional externality is not atmospheric any longer, where low-ability types’ consumption is not positional and all exclusively care about the comparisons with the high-ability consumers. In the former case, the failure of implementing a tax on the positional commodity has implications for the tax on the dirty and income taxes. The accessible taxes work as indirect instruments to correct for the positional externality. In addition, cross-price effects between the positional good and the dirty good affect the whole commodity tax system. A lower (higher) tax rate on the dirty good will be implemented, if the demand for the positional
good increases (decreases) with that for the dirty good, which we describe as complementarity (substitutability).

In the latter case, upward comparisons, as one type of a non-atmospheric positional externality indicate that the principle of targeting is not applicable based on the tax instruments the government uses. The tax on the positional commodity, which is optimally set along with the tax on the dirty good, is not flexible enough to internalize the positional externality due to its uniformity among individuals. This optimal commodity tax thus wrongly incentivizes low-ability individuals to consume less of this positional commodity than socially desirable amounts, and gives rise to overconsumption of the positional commodity for high-ability individuals. Thus, a comprehensive positional externality correction has to rely on a mixture of both linear taxes on the positional and dirty goods as well as non-linear marginal taxes on income. Among other factors, the way income taxes correct for this positional externality relies on how leisure changes along with the demand for the positional commodity. A lower marginal income tax rate will be set for high-ability individuals, while low-ability consumers will face a higher marginal income tax rate, if leisure covaries with the positional commodity in the same direction.

The results of the present thesis is based on a static model and the assumption of a small or closed economy. Possibilities for future research are the extension to the use of a dynamic model and relaxing the assumption to an open economy. The analysis of mixed taxation in an OLG framework may yield interesting results. Thus, a dynamic analysis could take into consideration the effect of pollutant accumulation and intertemporal consumption decisions. Following Aronsson and Johansson-Stenman (2010), capital income taxation may also be added as an additional policy instrument into the analysis. Comprehensive interactive effects among commodity, labor income and capital income taxes affecting policy rules for taxation are expected. Elaborating on the assumptions of an open economy, at least two possible ideas come to our mind. The first is inspired by Aronsson and Blomquist (2003) and could claim to analyze the effects of international labor mobility on the optimal global policy in an open economy. A second possible extension could be the inclusion of international capital mobility as operationalized in Aronsson et al. (2016). The inflow and outflow of capital would affect intertemporal
decisions by influencing monetary discounting rates and most likely capital income taxation as well.

Appendix

Derivation of the shadow price of environmental externality in equation (16).

\[
\frac{\partial V^1}{\partial E} + (\eta + \lambda) \frac{\partial V^2}{\partial E} - \lambda \frac{\partial \tilde{V}^2}{\partial E} + \left( \gamma t_x - \frac{1}{n^1 + n^2} \delta - \mu \right) \sum_j n^j \frac{\partial X^j}{\partial E} + \mu = 0 \tag{15f}
\]

Combined with \( MWP'_{E,B} = -\frac{\partial V^1}{\partial E}/\partial B^1 \), \( MWP_{E,B} = -\frac{\partial \tilde{V}^2}{\partial B^1} \) and equation (15a) 

(15c), (15f) is reformulated as

\[
\left( \gamma t_x - \frac{\delta}{n^1 + n^2} - \mu \right) \sum_j n^j \frac{\partial X^j,\text{com}}{\partial X} 
- \gamma \sum_j n^j MWP'_{X,B} - \lambda \frac{\partial \tilde{V}^2}{\partial B^1} \left( MWP'_{X,B} - \frac{MWP_{X,B}}{2} \right) + \delta = 0
\]

(A1)

Dividing \( \gamma \) in both sides and rearranging the equation, we can derive the result.

The proof of equation (20a): the efficient commodity tax implication.

\[
F.O.C \left( t_x \right) \left( \gamma n^1 - \frac{\partial V^1}{\partial B^1} \right) X^1 + \left( \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \right) X^2 
+ \lambda \frac{\partial \tilde{V}^2}{\partial B^1} \tilde{X}^2 + \left( \gamma t_x - \frac{1}{n^1 + n^2} \delta - \mu \right) \sum_j n^j \frac{\partial X^j}{\partial q_x} = 0 \tag{15e}
\]

\[
F.O.C \left( B^1 \right) * X^1
\]

\[
\left( \frac{\partial V^1}{\partial B^1} - \gamma n^1 \right) X^1 - \lambda \frac{\partial \tilde{V}^2}{\partial B^1} X^1 + \left( \gamma t_x - \frac{1}{n^1 + n^2} \delta - \mu \right) n^1 \frac{\partial X^1}{\partial B^1} X^1 = 0 \tag{A2}
\]

\[
F.O.C \left( B^2 \right) * X^2
\]

\[
\left[ (\eta + \lambda) \frac{\partial \tilde{V}^2}{\partial B^2} - \gamma n^2 \right] X^2 + \left( \gamma t_x - \frac{1}{n^1 + n^2} \delta - \mu \right) n^2 \frac{\partial X^2}{\partial B^2} X^2 = 0 \tag{A3}
\]
The sum of (15e), (A2) and (A3) gives the following equation (A4)

\[
\frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^i} \left( \hat{X}^2 - X^i \right) + \left( t_x - \frac{1}{n^i + n^2} \frac{\delta}{\gamma} - \frac{\mu}{\gamma} \right) \sum_j n^j \frac{\partial X_{j,\text{com}}}{\partial q_x} = 0
\]

(A4)

\[
\Rightarrow t_x = \frac{1}{M} \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^i} \left( X^i - \hat{X}^2 \right) + \frac{1}{n^i + n^2} \frac{\delta}{\gamma} + \frac{\mu}{\gamma}
\]

(20a)

**Derivation of the Marginal Income Taxation for low-ability and high-ability types.**

We first reformulate (15a) and (15b)

\[
F.O.C (B^i) \frac{\partial V^1}{\partial B^i} = \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^i} \left( \gamma t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} - \mu \right) n^1 \frac{\partial X^1}{\partial B^i} + \gamma n^1
\]

(A5)

\[
F.O.C (l^i) \frac{\partial V^1}{\partial Z^i} = \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial Z^i} \left( \gamma t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} - \mu \right) n^1 \frac{\partial X^1}{\partial Z^i} + \gamma n^1 l^i
\]

(A6)

\[
\frac{\partial V^1}{\partial Z^i} = MRS^1_{Z, B} = \frac{\frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial Z^i} \left( \gamma t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} - \mu \right) n^1 \frac{\partial X^1}{\partial Z^i} + \gamma n^1 l^i}{\frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^i} \left( \gamma t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} - \mu \right) n^1 \frac{\partial X^1}{\partial B^i} + \gamma n^1}
\]

(A7)

\[
\frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^i} \frac{1}{n^1} \left( MRS^1_{Z, B} - \phi MRS^2_{Z, B} \right) + \left( t_x - \frac{1}{n^1 + n^2} \frac{\delta}{\gamma} - \frac{\mu}{\gamma} \right) \frac{\partial X_{1,\text{com}}}{\partial Z^i} = l^i - MRS^1_{Z, B}
\]

(A8)

Substituting equation (9) \( w^j \left[ 1 - T^j \left( w^j l^j \right) \right] = MRS^j_{Z, B} \) for \( j=1 \), into (A8), we get the optimal income tax implications for the low-ability type (22a). (22b) can be derived in the same way.

**The proof of the commodity tax implication in section 3.**
\[ L = V^i + \eta \left( V^2 - \tilde{V}^2 \right) + \lambda \left( V^2 - \tilde{V}^2 \right) \]
\[ + \mu \left( E - \sum_j n_j X^j \right) + \delta \left( \tilde{X} - \frac{\sum_j n_j X^j}{n^1 + n^2} \right) + \gamma \sum_j n_j \left( w^i l^j - B^j + t_d D^j \right) \]  

\[ \text{(33)} \]

\[ \text{F.O.C.} \left( B^j \right) \frac{\partial V^i}{\partial B^j} - \lambda \frac{\partial \tilde{V}^2}{\partial B^i} - \frac{n^1}{n^1 + n^2} \delta \frac{\partial X^1}{\partial B^i} + \left( \gamma t_d - \mu \right) n^1 \frac{\partial D^j}{\partial B^i} - \gamma n^1 = 0 \]  

\[ \text{(A9a)} \]

\[ \text{F.O.C.} \left( I^i \right) - \frac{\partial V^i}{\partial Z^i} + \lambda \phi \frac{\partial \tilde{V}^2}{\partial Z^i} + \frac{n^1}{n^1 + n^2} \delta \frac{\partial X^1}{\partial Z^i} - \left( \gamma t_d - \mu \right) n^1 \frac{\partial D^j}{\partial Z^i} + \gamma n^1 r^i = 0 \]  

\[ \text{(A9b)} \]

\[ \text{F.O.C.} \left( B^i \right) \left( \eta + \lambda \right) \frac{\partial \tilde{V}^2}{\partial B^i} - \frac{n^2}{n^1 + n^2} \delta \frac{\partial X^2}{\partial B^i} + \left( \gamma t_d - \mu \right) n^2 \frac{\partial D^2}{\partial B^i} - \gamma n^2 = 0 \]  

\[ \text{(A9c)} \]

\[ \text{F.O.C.} \left( I^i \right) - \left( \eta + \lambda \right) \frac{\partial \tilde{V}^2}{\partial Z^i} + \frac{n^2}{n^1 + n^2} \delta \frac{\partial X^2}{\partial Z^i} - \left( \gamma t_d - \mu \right) n^2 \frac{\partial D^2}{\partial Z^i} + \gamma n^2 r^2 = 0 \]  

\[ \text{(A9d)} \]

\[ \text{F.O.C.} \left( t_d \right) \]

\[ \left( \gamma n^1 - \frac{\partial V^1}{\partial B^1} \right) D^1 + \left[ \gamma n^2 - \left( \eta + \lambda \right) \frac{\partial V^2}{\partial B^2} \right] D^2 + \lambda \frac{\partial \tilde{V}^2}{\partial B^i} D^2 \]
\[ - \frac{1}{n^1 + n^2} \delta \sum_j n_j \frac{\partial X^j}{\partial q_d} + \left( \gamma t_d - \mu \right) \sum_j n_j \frac{\partial D^j}{\partial q_d} = 0 \]  

\[ \text{(A9e)} \]

\[ \text{F.O.C.} \left( E \right) \]

\[ \frac{\partial V^1}{\partial E} + \left( \eta + \lambda \right) \frac{\partial V^2}{\partial E} - \lambda \frac{\partial \tilde{V}^2}{\partial E} - \frac{1}{n^1 + n^2} \delta \sum_j n_j \frac{\partial X^j}{\partial E} + \left( \gamma t_d - \mu \right) \sum_j n_j \frac{\partial D^j}{\partial E} + \mu = 0 \]  

\[ \text{(A9f)} \]

\[ \text{F.O.C.} \left( X \right) \]

\[ \frac{\partial V^1}{\partial X} + \left( \eta + \lambda \right) \frac{\partial V^2}{\partial X} - \lambda \frac{\partial \tilde{V}^2}{\partial X} - \frac{1}{n^1 + n^2} \delta \sum_j n_j \frac{\partial X^j}{\partial X} + \left( \gamma t_d - \mu \right) \sum_j n_j \frac{\partial D^j}{\partial X} + \delta = 0 \]  

\[ \text{(A9g)} \]

The proof of equation (33) in section 3: the dirty good taxation.

\[ \text{F.O.C.} \left( t_d \right) \]
\[
\left( n^1 \frac{\partial V^1}{\partial B^1} - \gamma n^1 \right)D^1 + \left[ \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \right]D^2 + \lambda \frac{\partial V^2}{\partial B^2} \hat{D}^2
\]
\[
- \frac{1}{n + n^2} \delta \sum_j n^j \frac{\partial X_j}{\partial q_D} + (\gamma t_D - \mu) \sum_j n^j \frac{\partial D^j}{\partial q_D} = 0
\]  
\[\text{F.O.C}(B^i) \ast D^i\]
\[
\left( \frac{\partial V^1}{\partial B^1} - \gamma n^1 \right)D^1 - \lambda \frac{\partial V^2}{\partial B^2} \hat{D}^1 - \frac{n^1}{n + n^2} \delta \frac{\partial X^1}{\partial B^1} D^3 + (\gamma t_D - \mu) n^1 \frac{\partial D^1}{\partial B^1} D^1 = 0
\]  
\[\text{F.O.C}(B^2) \ast D^2\]
\[
\left[ (\eta + \lambda) \frac{\partial V^2}{\partial B^2} - \gamma n^2 \right]D^2 - \frac{n^2}{n + n^2} \delta \frac{\partial X^2}{\partial B^2} D^2 + (\gamma t_D - \mu) n^2 \frac{\partial D^2}{\partial B^2} D^2 = 0
\]  
The aggregation of equation (A9e), (A10) and (A11) gives the policy rule for the tax on the dirty good.

The proof of commodity taxes (40a) and (40b) in section 4.

\[
L = V^1 + \eta \left( V^2 - \hat{V}^2 \right) + \lambda \left( V^2 - \hat{V}^2 \right) + \mu \left( E - \sum_j n^j D^j \right)
\]
\[
+ \delta \left( X - X^2 \right) + \gamma \sum_j n^j \left( w^j l^j - B^j + t_x X^j + t_D D^j \right)
\]  
\[\text{F.O.C}(B^i) \frac{\partial V^1}{\partial B^i} - \lambda \frac{\partial V^2}{\partial B^i} \left( \gamma t_D - \mu \right) \frac{\partial D^1}{\partial B^i} n^1 + \gamma n^1 t_x \frac{\partial X^1}{\partial B^i} - \gamma n^1 = 0
\]  
\[\text{F.O.C}(l^i) - \frac{\partial V^1}{\partial Z^i} + \lambda \frac{\partial V^2}{\partial Z^i} + n^1 \frac{\partial D^1}{\partial Z^i} \left( \mu - \gamma t_D \right) - \gamma n^1 t_x \frac{\partial X^1}{\partial Z^i} + n^1 w^i = 0
\]  
\[\text{F.O.C}(B^2) \left( \eta + \lambda \right) \frac{\partial V^2}{\partial B^2} + (\gamma t_x n^2 - \delta) \frac{\partial X^2}{\partial B^2} + n^2 \frac{\partial D^2}{\partial B^2} \left( \gamma t_D - \mu \right) - \gamma n^2 = 0
\]  
\[\text{F.O.C}(l^2) - \left( \eta + \lambda \right) \frac{\partial V^2}{\partial Z^2} + \delta \frac{\partial X^2}{\partial Z^2} - n^2 \left( \gamma t_D - \mu \right) \frac{\partial D^2}{\partial Z^2} - \gamma n^2 t_x \frac{\partial X^2}{\partial Z^2} + n^2 w^2 = 0
\]  
\[\text{F.O.C}(t_x)\]
\[
\left( \gamma n^1 - \frac{\partial V^1}{\partial B^1} \right) X^1 + \left( \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \right) X^2 + \lambda \frac{\partial V^2}{\partial B^1} \hat{X}^2 \\
+ \gamma t_x \sum_j n^i \frac{\partial X^j}{\partial q_x} - \delta \frac{\partial X^2}{\partial q_x} + (\gamma t_D - \mu) \sum_j n^i \frac{\partial D_j^i}{\partial q_x} = 0
\]

(A13e)

**F.O.C.** (\(t_D\))

\[
\left( \gamma n^1 - \frac{\partial V^1}{\partial B^1} \right) D^1 + \left[ \gamma n^2 - (\eta + \lambda) \frac{\partial V^2}{\partial B^2} \right] D^2 + \lambda \frac{\partial V^2}{\partial B^1} D^2 \\
+ \gamma t_x \sum_j n^i \frac{\partial X^j}{\partial q_D} - \delta \frac{\partial X^2}{\partial q_D} + (\gamma t_D - \mu) \sum_j n^i \frac{\partial D_j^i}{\partial q_D} = 0
\]

(A13f)

**F.O.C.** (\(E\))

\[
\frac{\partial V^1}{\partial E} (\eta + \lambda) \frac{\partial V^2}{\partial E} - \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial E} - \delta \frac{\partial X^2}{\partial E} + \gamma t_x \sum_j n^i \frac{\partial X^j}{\partial E} (\gamma t_D - \mu) \sum_j n^i \frac{\partial D_j^i}{\partial E} + \mu
\]

(A13g)

\[
\mu = \frac{1}{\Omega^D} \left\{ \sum_j n^i MWP_E^{j,i} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial E} \left( MWP_E^{j,i} - \vec{MWP}_E^{j,i} \right) \right\} \\
+ \frac{\delta}{\gamma} \frac{\partial X^2,\text{com}}{\partial E} - t_x \sum_j n^i \frac{\partial X^j,\text{com}}{\partial E} - t_D \sum_j n^i \frac{\partial D_j^i,\text{com}}{\partial E}
\]

(A14)

**F.O.C.** (\(\bar{X}\))

\[
\frac{\partial V^1}{\partial X} + (\eta + \lambda) \frac{\partial V^2}{\partial X} - \frac{\lambda}{\gamma} \frac{\partial V^2}{\partial X} - \delta \frac{\partial X^2}{\partial X} \\
+ \gamma t_x \sum_j n^i \frac{\partial X^j}{\partial X} + (\gamma t_D - \mu) \sum_j n^i \frac{\partial D_j^i}{\partial X} + \delta = 0
\]

(A13h)

\[
\delta = \frac{1}{\Omega^\bar{X}} \left\{ \sum_j n^i MWP_{\bar{X},b}^{j,i} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial \bar{X}} \left( MWP_{\bar{X},b}^{j,i} - \vec{MWP}_{\bar{X},b}^{j,i} \right) \right\} \\
- \left( t_D - \frac{\mu}{\gamma} \right) \sum_j n^i \frac{\partial D_j^i,\text{com}}{\partial X} - t_D \sum_j n^i \frac{\partial X^j,\text{com}}{\partial X}
\]

(A15)

The aggregation of (13e) and (A13a) multiplied by \(X^1\) and (A13c) multiplied by \(X^2\) gives

\[
\left( t_D - \frac{\mu}{\gamma} \right) \left( \sum_j n^i \frac{\partial D_j^i,\text{com}}{\partial q_x} \right) + t_x \sum_j n^i \frac{\partial X^j,\text{com}}{\partial q_x} = \delta \frac{\partial X^2,\text{com}}{\partial q_x} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^1} \left( X^1 - \hat{X}^2 \right)
\]

(A16)
where we use Slutsky substitution equation for compensated demand function regarding price effects of the positional good \( \frac{\partial X^{j,\text{com}}}{\partial q_X} = \frac{\partial X^j}{\partial q_X} + \frac{\partial X^j}{\partial B^j} X^j \) and \( \frac{\partial D^{j,\text{com}}}{\partial q_X} = \frac{\partial D^j}{\partial q_X} + \frac{\partial D^j}{\partial B^j} X^j \).

Analogously, the sum of (A13f), (A13a) multiplied by \( D^j \) and (A13c) multiplied by \( D^2 \) gives

\[
\left( t_D - \frac{\mu}{\gamma} \right) \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial q_D} + t_X \sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial q_D} = \frac{\delta}{\gamma} \frac{\partial X^{2,\text{com}}}{\partial q_D} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( D^j - \hat{D}^2 \right),
\]

(A17)

where we use Slutsky substitution equation for compensated demand function regarding price effect of the dirty good \( \frac{\partial X^{j,\text{com}}}{\partial q_D} = \frac{\partial X^j}{\partial q_D} + \frac{\partial X^j}{\partial B^j} D^j \) and \( \frac{\partial D^{j,\text{com}}}{\partial q_D} = \frac{\partial D^j}{\partial q_D} + \frac{\partial D^j}{\partial B^j} D^j \). We, therefore, get the following equation system

\[
\begin{bmatrix}
\sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial q_X} & \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial q_X} \\
\sum_j n^j \frac{\partial X^{j,\text{com}}}{\partial q_D} & \sum_j n^j \frac{\partial D^{j,\text{com}}}{\partial q_D}
\end{bmatrix}
\begin{bmatrix}
t_X \\
t_D - \frac{\mu}{\gamma}
\end{bmatrix}
= \begin{bmatrix}
\frac{\delta}{\gamma} \frac{\partial X^{2,\text{com}}}{\partial q_D} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( X^j - \hat{X}^2 \right) \\
\frac{\delta}{\gamma} \frac{\partial X^{2,\text{com}}}{\partial q_D} + \frac{\lambda}{\gamma} \frac{\partial \hat{V}^2}{\partial B^j} \left( D^j - \hat{D}^2 \right)
\end{bmatrix}
\]

(A18)

Using Cramer’s rule produces (40a) and (40b).
References


