UMEÅ UNIVERSITY

Optimal Thinning
A Theoretical Investigation on Individual-Tree Level

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“Da steh ich nun, ich armer Tor!

Und bin so klug als wie zuvor;

Heiße Magister, heiße Doktor gar

Und ziehe schon an die Zehen Jahr

Herauf, herab und quer und krumm

Meine Schüler an der Nase herum-

Und sehe, daß wir nichts wissen können!”

— Johann Wolfgang von Goethe, Faust: Eine Tragödie

Abstract

Paper I: In paper I, we asked how a tree should optimally allocate its resources to maximize its fitness. We let a subject tree grow in an environment shaded by nearby competing trees. The competitors were assumed to have reached maturity and had stopped growing, thus creating a static light environment for the subject tree to grow in. The light environment was modeled as a logistic function. For the growth model we used the pipe model as a foundation, linking tree width and leaf mass. This allowed us to construct a dynamic tree-growth model where the tree can allocate biomass from photosynthesis (net productivity) to either stem-height growth, crown-size growth, or reproduction (seed production). Using Pontryagin's maximum principle we derived necessary conditions for optimal biomass allocation, and on that built a heuristic allocation model. The heuristic model states that the tree should first invest into crown-size and then switch to tree height-growth, and lastly invest into crown-size before the growth investments stop and all investments are allocated to reproduction. To test our heuristic method, we used it to determine the growth in several different light environments. The results were then compared to the optimal growth trajectories. The optimal growth was determined by applying dynamic programming. Our less computationally demanding heuristic performed very well in comparison. We also found there exist a critical crown-size: if the subject tree possessed a larger crown-size, the tree would be unable to reach up to the canopy height.

Paper II: One of the most important aspects of modelling forest growth, and modelling growth of individual trees in general, is the competition between trees. A high level of competition pressure has a negative impact on the growth of individual trees. There are many ways of modelling competition, the most common one is by using a competition index. In this paper we tested 16 competition indices, in conjunction with a log-linear growth model, in terms of the mean squared error and the coefficient of determination. 5 competition indices are distance-independent (i.e. distance between the competitors are not taken into consideration) and 11 are distance-dependent. The data we used to fit our growth model, with accompanying competition index, was taken from an experimental site, in northern Sweden, of Norway spruce. The growth data for the Norway spruce comes from stands which were treated with one of two treatments, solid fertilization, liquid fertilization, or no treatment (control stand). We found that the distance-dependent indices perform better than the distance-independent. However, both the best distance-dependent and independent index performed overall well. We also found that the ranking of the indices was unaffected by the stand treatment, i.e. indices that work well for one treatment will work well for the others.

Paper III: In this paper we studied how spatial distribution and size selection affect the residual trees, after a thinning operation, in terms of merchantable wood production and stand economy. We constructed a spatially explicit individual-based forest-growth model and fitted and validated the model against empirical data for Norway spruce stands in northern Sweden. To determine the cost for the forest operation we employed empirical cost functions for harvesting and forwarding. The income from the harvested timber is calculated from volume-price lists. The thinnings were determined by three parameters: the spatial evenness of residual trees, the size selection of removed trees, and the basal area reduction. In order to find tree selections fulfilling these constraints we used the metropolis algorithm. We varied these three constrains and applied them for thinning of different initial configurations of Norway spruce stands. The initial configurations for the stands where collected from empirical data. We found that changing the spatial evenness and size selection improved the net wood production and net present value of the stand up to 8%. However, the magnitude of improvement was dependent on the initial configuration (the magnitude of improvement varied between 1.7%—8%).

Paper IV: With new technology and methods from remote sensing, such as LIDAR, becoming more prevalent in forestry, the ability to assess information on a detailed scale has become more available. Measurements for each individual tree can be more easily gathered on a larger scale. This type of data opens up for using individual-based model for practical precision forestry planning. In paper IV we used the individual-based model constructed in paper III to find the optimal harvesting time for each individual tree, such that the land expectation value is maximized. We employed a genetic algorithm to find a near optimal solution to our optimization. We optimized a number of initial Norway spruce stands (data obtained from field measurements). The optimal management strategy was to apply thinning from above. We also found that increasing the discount rate will decrease the time for final felling and increase basal area reduction for the optimal strategy. Decreasing relocation costs (the cost to bring machines to the stand) led to an increase in the number of optimal thinnings and postponed the first thinning. Our strategy was superior to both the unthinned strategy and a conventional thinning strategy, both in terms of land expectation value (>20% higher) and merchantable wood production.

Enkel sammanfattning på svenska

Artikel I: För att kunna göra rimliga uppskattningar om hur träd kommer att växa i framtiden behövs bra modeller för att bestämma hur ett träd ändrar sin tillväxt som respons till sin omgivning. I denna artikel undersöker vi hur ett träd optimalt ska fördela sina resurser för att maximera sin reproduktionsframgång (fitness). Vi föreställer oss att ett träd växer i en statisk skuggad miljö, det vill säga att ljusmiljön inte förändras med tiden. Trädet kan välja mellan att investera i trädhöjd, kronstorlek eller reproduktion (fröproduktion). Med hjälp av matematisk analys har vi erhållit nödvändiga förutsättningar för hur trädet optimalt ska fördela sina resurser och på dessa byggt en heuristisk modell, vilken approximerar den sanna optimala tillväxten. Den heuristiska modellen säger att trädet först i ett tidigt stadium av sitt liv bör investera i kronstorlek och sedan byta till trädhöjdstillväxt och slutligen investera i kronstorlek innan tillväxtinvesteringarna upphör och alla investeringar tilldelas till reproduktion. För att testa vår heuristiska modell använder vi den för att bestämma tillväxten i olika ljusmiljöer. Resultaten jämfördes sedan med optimal tillväxt, vilken har bestämts med den beräkningskrävande metoden dynamisk programmering. Vår heuristiska metod fungerade mycket bra i jämförelse. Med denna metod kan vi mer realistiskt förutsäga hur ett träd växer i olika ljusmiljöer, jämfört med vanliga allometriska metoder.

Artikel II: En av de viktigaste aspekterna för att kunna förutsäga en skogs tillväxt, är att kunna beskriva konkurrensen mellan träden. En av de vanligaste sätten att beskriva konkurrensen är att använda så kallade konkurrensindex. Konkurrensindex är enkla matematiska modeller vilka anväder storheter som höjden på trädet, diameter, och avstånd mellan träd, för att kvantifiera konkurrensen mellan träden. Det finns många olika konkurrensensindex och de kan delas in i två kategorier: distansberonde (index som använder avstånd mellan de konkurrerande träden) och distansoberoende (index som inte beror på avstånd). Resultat från tidigare undersökningar har visat olika svar på frågan vilken typ av index (även vilka index) som fungerar bäst för att kunna uppskatta tillväxten. I denna artikel så studerade vi sexton olika index (fem avståndsoberoende och elva avståndsberoende). Vi testade hur väl dessa kan användas för att modellera mätningar på granskog i norra Sverige. Tillväxtdatat kommer från granbestånd som behandlades med, fast gödning, flytande gödning eller ingen gödning. Vi fann att de distansberoende indexen fungerar bättre än de distansoberoende, dock så fungerade både det bästa distansberoende indexet och det oberoende indexet bra. Vi fann också att om vi ordnade indexen efter hur bra de fungerade så var denna rangordningen mer eller mindre oförändrad för de tre behandlingarna. Detta indikerar att index som fungerar bra för en typ av behandling kommer att fungera bra för andra.

Artikel III: Gallring (avverkning av träd i beståndet) utförs bland annat för att minska kunkurrensen mellan kvarvarande träd i skogen och öka diametertillväxten, detta leder till ett högre värde på beståndet. Valet av träd som ska avverkas kan göras på många olika sätt. Två olika tänkbara sätt är att: 1) avverka alla träd i ett givet område eller 2) att avverka jämnt i beståndet. Om vi antar att vi avverkar samma mängd träd, så blir skillnaden mellan de två fallen att i fall ett så är träden mer rumsligt utspridda (större avstånd mellan dem) än i det andra fallet. I denna studie undersökte vi hur detta och storleksvalet (om vi väljer att avverka större eller mindre träd) påverkar gagnvirkesvolymen samt värdet på beståndet. Vi konstruerade en skogstillväxtmodell, vilken simulerar tillväxten för varje enskilt träd. Vi anpassade och validerade modellen för granbestånd i norra Sverige. För att bestämma kostnaden för skogsverksamheten användes empiriska kostnadsfunktioner för skördare och skotare. Vi simulerade olika gallringar där vi varierade den rumsliga utspridningen samt storleksvalet för fyra olika initiala granbestånd. Vi fann att valet av rumslig jämnhet och storleksval kan öka gagnvirkesvolymen samt värdet för beståndet med 8%. Storleken på ökningen är dock beroende av det ursprungliga utseendet på beståndet.

Artikel IV: Med ny teknik och metoder från fjärranalys, som t.ex. LIDAR, blir det allt lättare att få detaljerade mätningar. Dessa mätningar inkluderar till exempel position, diameter och höjd för varje enskilt träd. Detta öppnar upp möjligheten för att planera gallringar på enskild trädnivå, så kallat precisionsskogsbruk. För denna artikel använde vi skogstillväxtmodellen som vi konstruerade för artikel III för att hitta den optimala avverkningstiden för varje enskilt träd, så att vi maximerar nettointäkten för beståndet. Detta optimeringsproblem är stort, som ett exempel kan nämnas att antalet olika sätt att planera avverkningstid för 7 träd, där vi har 5 val på avverkningstid, kan göras på 78 125 olika sätt. Vi använde en numerisk metod för att hitta lösning på vårt optimeringsproblem. Vi optimerade gallringen för fyra olika granbestånd. För alla fyra bestånd var vår strategi överlägsen i jämförelse med ogallrade bestånd och bestånd som gallrats med en konventionell gallringsplan, både vad gäller nettoinkomst (> 20% högre) samt gagnvirkesvolymen.

List of papers

The thesis is based on the following four papers:

I. Peter Fransson, Åke Brännström, and Oskar Franklin. A tree's quest for light – optimal height and diameter growth under a shading canopy (submitted manuscript).

II. Peter Fransson, Oskar Franklin, Urban Nilsson, and Åke Brännström. Comparing distance-independent and distance-dependent competition indices for *Picea abies* (manuscript).

III. Peter Fransson, Urban Nilsson, Ola Lindroos, Oskar Franklin, and Åke Brännström (2019). Model-based investigation on the effects of spatial evenness, and size selection in thinning of *Picea abies* stands. Scandinavian Journal of Forest Research **34**(3): 189-199.

IV. Peter Fransson, Oskar Franklin, Ola Lindroos, Urban Nilsson, and Åke Brännström. A simulation-based approach to a near optimal thinning strategy – when allowing for individual harvesting times for individual trees (submitted manuscript).

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Chapter 1. Introduction

Swedish forestry

Sweden's land area is estimated to 40 million hectares, out of which 28 million hectares are covered by forest (70%)[1]. With this fact in mind, it is no wonder that forestry plays a major role in Swedish industry. Around 9—12 % of all employees in Swedish industry are working in the forest product industry and Sweden is the third biggest exporter in the world of forest products (pulp, paper, and sawn wood)[2]. Because of their importance, a great effort has been put into silviculture. The first forestry act was legislated a little over 100 years ago in 1903. The main focus of the first forestry act was that of the renewal of foraged forest areas (Lindahl et al. 2017). Around the early 50s even-aged (trees in the forest stand are of the same age) stand management became the norm as it was thought to increase yield (Lindahl et al. 2017). The most common spices in Swedish production forest are Norway spruce (*Picea abies*) and Scots pine (*Pinus sylvetris*).

A typical rotation-cycle

The rotation-cycle (life-cycle) of a production forest is dependent on the location of the stand, i.e. climate, soil quality, and species, but one example of a Norway spruce stand (which has been focal point of this thesis) in Sweden could be the following: the rotation-cycle starts with a regeneration phase in which soil-preparation (scarification) and planting, either with seeds or seedlings (most common), are undertaken to repopulate the area with at least circa 2500 trees per hectare, after planting the number of trees per hectare will increase due to natural regeneration. When the trees in the stand reach a height of around 2 m, a pre-commercial thinning (artificial removal of trees, and leaving the removed trees in the stand) is performed in which the number of trees is drastically reduced (e.g. from 10,000 trees per hectare to around 2,500). The next event is a first commercial thinning (thinning where the harvest is sold for commercial purpose, hence forth referred to as thinning) when the mean height is around 12 m and the density is lowered from 2,500 trees per hectare to around 1,200. The second thinning takes place when the mean height is around 18 m and the tree density is lowered from 1,200 to around 700. The rotation-cycle is ended by performing a final felling in which the remaining trees are harvested (a clear-cut), and the cycle

[1] Source: www.slu.se
[2] Source: www.skogsstyrelsen.se

starts anew. As explained the specifics of the rotation (i.e. densities and heights) vary based on the biotic and abiotic environment and the species, but even-aged forest stands with the described cycle elements is the norm in Swedish forestry today (Wallentin 2007). Scarification is mechanized, whereas planting and pre-commercial thinning is done manually. Thinning and final felling operations are mechanized and executed by use of a harvester, that fell, debranches and cross-cut trees into logs, and a forwarder, that transports the logs from the forest to road-side landings.

The thinning operation

As described in the preceding section thinning is an operation in which select trees are removed from a forest stand, and biomass is harvested. There are a couple of reasons for this undertaking. One reason is to generate income before the clear-cut. The second is to reallocate growth to the residual trees in order to promote diameter increase, which has a positive effect on future revenue (Wallentin 2007). The selection of trees for harvesting, can be made in a vast number of ways and can be characterised in different ways. The obvious one is the number of trees harvested from the stand. In addition to the amount of harvested trees, we can imagine a contingency of different strategies of selecting which trees to harvest with two extremes on both sides. On the one side of the contingency we don't make any choice of which individual trees we want to harvest; we simply remove the trees in a selected area of the stand, and this is known as systematic thinning. On the other side of the spectrum we have predetermined which trees we want to harvest, based on some characteristics, and this is known as selective thinning. One of the most common characteristics is the preference of the size of the harvested trees in relation to the residual trees in the forest, e.g. we remove the larger trees in the forest (thinning from above) or the smaller ones (thinning from below) (Wallentin 2007). In Swedish forestry the first thinning is a semi-systematic thinning, because corridors (extraction trails) in the forest stand has to be created (systematic thinning) to allow for the harvester and forwarder to access into the forest. Given the normal width of 4 m and a distance of 20 m between extraction trails, about 20% of the area is systematically thinned. The remaining amount that is desired to remove is done as a selective thinning in between the extraction trails, where most commonly smaller trees a selected for harvesting. The amount of trees to be removed and the timing of the thinnings are often determined by predefined thinning guidelines, which typically assumes how a stand of a certain species in a certain region is expected to grow under a typical management regime. However, these guidelines do not account for local variation in forest structure, inside of the stand, thus may not suggest the most efficient way to manage the forest.

From empirical studies we know that thinning operations produce fewer trees, but with larger diameters, and promote the production of merchantable timber, thereby increasing the value of the stand (Wallentin 2007; Nilsson et al. 2010) than in unthinned stands. Moreover, since the harvested trees are recovered, thus preventing self-thinning (loss of trees due to competition), the net (merchantable) wood production is higher in the thinned stands (Nilsson et al. 2010). Beside the positive effect on the production, several theoretical investigation have shown a positive effect on the net value (Valsta 1992; Zeide 2001; Hyytiäinen and Tahvonen 2003; Cao et al. 2006). Empirical investigation between selective and systematic thinning has shown that selective thinning has potential for increasing net production (Baldwin et al. 1989; Mäkinen et al. 2006).

Remote sensing – a road to precision forestry

Forest inventory and measuring are usually conducted to gather information on a stand level, such as mean diameter, stem density, basal area (The aggregated cross-sectional area of the trees' trunks at a height of 1.3 m divided by the forest area), etc. This type of data can give an estimation on the average state of a forest stand but neglects the local variation and spatial-pattern of the individual trees. The reason for neglecting more detailed data is the time consumption and cost of measuring every individual tree by hand. However, methods from remote sensing are now being used more frequently in forest management (Dubayah and Drake 2000; Carson et al. 2004). LIDAR (light detection and ranging) is a measuring method which uses lasers to measure distance between reflective objects and the laser source. This method makes it easier to assess highly detailed data on an individual-tree level (Persson et al. 2002; Holmgren and Persson 2004; Holmgren et al. 2008), such as tree height, diameter, position, etc. This opens for a lot of possibilities, such as accounting for local variation in the harvesting decision. One can even imagine going further and making a plan for each individual tree when the specific tree should be harvested, based on the expectation of growth. Commonly growth models are used to make a good guess on the future growth. The kind of models suited for this type of high detailed level of harvesting planning, are so called individual-based models (IBM) (Grimm and Railsback 2005). Individual-based models simulate the growth of each tree in the simulated forest stand. But simulating the forest growth in this way introduces new problems, not only from the point of view of the initial data needed to run the simulation (initial size for each individual tree), but also from a computational standpoint. The interaction between the individual tree and their environment has to be explicitly taken into account.

Model-based forest management

Empirical investigations take a lot of time to conduct due the comparatively long growth period of trees. In Sweden, the rotation-cycle is around typically at least 60 years under productive conditions and more than 100 years under less productive conditions. Thus, empirical studies on how different management regimes affect the stand on the whole rotation-cycle are rare. One way of inferring the effects of a particular regime in a reasonable timeframe is to conduct a theoretical investigation, with the aid of mathematical models.

Models have been used extensively in forest management, particularly for economic investigation and optimization. Most of these theoretical investigations were conducted with so-called stand-growth models, e.g. Hyytiäinen and Tahvonen (2002, 2003). Unlike individual based models, stand models do not consider the growth of individual trees but the stand as a whole, only using a few numbers of variables to describe the state of the stand. The drawback is the loss of detailed information, but on the other had these models are more computationally efficient. Typically, the decision variables that have been studied and optimized are the frequency, timing, and number of thinnings, as well as thinning intensity, and thinning form (thinning from above/below).

By contrast individual-based models have not been used as extensively, but a few investigations in management simulations exist. These include studies of optimizing the management for scenarios where the forest stand is even-aged (the age of the trees is uniform) (Valsta 1992; Pukkala and Miina 1998; Cao et al. 2006), uneven-age stands (Pukkala et al. 2009; Tahvonen et al. 2010), and any-age forest (Pukkala et al. 2014). In these studies, the IBM in conjunction with numerical optimization methods were used to optimize the management. In contrast to the stand-growth models, the number of decision variables are higher. Beyond the usual variables such as the number of thinnings, and timing, a number of variables have been introduced to decide the proportion of removal from either a predefined number of different size categories or finding optimal thinning intensity curves (Pukkala et al. 1998, 2015), which specifies the proportion of removal for different diameters. The consensus of these investigations is that the optimal management plan depends on the initial stand configuration and particularly on the assumptions for the economic parameters, such as discount rate. The optimal number of thinnings varied between 1—6 and in many cases thinning from above was optimal. What has not been fully addressed in the aforementioned investigations are questions regarding the spatial pattern, e.g. how much does spatial configuration after thinning impact the performance of the stand? Spatial patterns are important when for instance comparing selective and systematic thinning. Likewise, the question of devising management plans based on individual trees in order to maximize the economic return has not been considered. Pukkala and Miina (1998) considered a tree-selection algorithm which calculates a growth potential for each tree and uses this as a basis for the individual selection of trees to remove, however this does not guarantee optimality, as the selection of trees will have effect on the growth of remaining individuals.

Chapter 2. Objectives

The objectives of this thesis are listed below. The main objective was to find a method to determine a forest management plan on a single tree-level basis such that the economic return of a given stand is maximized. During the work towards this goal I discovered interrelated goals which were of essence to answer the main question. In a first effort I focused on one of the fundamental questions in individual forest growth modelling, which is the interaction between individual trees or in other words how the growth of a single tree changes depending on its environment. Here, I focus on two aspects, firstly the question of how a tree should allocate its net productivity optimally in a time-invariant environment (paper I), and secondly which so called competition index can be used to approximate the interaction between Norway spruce trees in northern Sweden (paper II). Spatial structures in the forest can easily be implemented in spatially explicit models. With this ability my interest was to investigate how the spatial patterns in the forest following a thinning will impact the performance, in terms of economic revenue and merchantable wood production (paper III). After answering the three previous questions I was able to tackle the main goal of the thesis where I optimize stand management on an individual-tree level, i.e. I try to find the optimal management plan on an individual-tree level such that the economic gain is maximized (Paper IV). To summarize the objectives:

1) How should a tree allocate its biomass optimally in a time invariant environment?
2) Which competition indices can be used to reliably approximate competition for Norway spruce in northern Sweden?
3) How will the spatial evenness, after thinning, between trees impact the merchantable wood production and the net present value of the stand?
4) How will an optimal thinning schedule (selective thinning) impact the economic performance of the stand and what strategy emerges?

Chapter 3. Foundations of forest stand modelling and management

Growth model

In this section I will describe the individual-based model which we constructed in paper III to investigate the effect on spatial evenness on merchantable wood production and economic gain. In paper IV we used the growth model for the individual-based economic optimization. The growth model uses a distance-dependent competition index as a proxy for competition between neighboring trees. In paper II we made a study of different distance-dependent and independent indices. I will start this section with describing the different competition indices before introducing the growth model.

Competition index

Competition indices are computationally efficient models for approximating competition between neighboring trees. Specifically, competition indices model the competition pressure put on a single subject tree by its neighbors. Competition indices can be divided into two categories, distance-dependent indices (indices which uses distance between competing trees) and distance-independent indices. In paper II we compared sixteen indices by how well they can be used to fit to empirical data of diameter increment. Performance of the indices are measured in terms of the adjusted coefficient of determination, R^2_{adj}, and the root mean square error, $RMSE$. I will give a brief overview of the competition indices. The following variable and parameters are used to describe the competition indices:

- The area of the stand A.
- Number of competitors, N.
- The quadratic mean diameter of individuals in the stand, $\bar{d}q$.
- The diameter at breast height for the subject tree, d.
- Diameter at breast height for competitors d_i.
- Distance between subject tree and competitor tree, dist_i.
- Influence area of subject tree, Z.
- Area of overlap between areas of influence of subject tree and competitor, O_i.
- Species of competitor, $\text{sp}(i)$.
- Parameters α, β, λ.

Distance-independent indices
C_1 (Wykoff et al. 1982), measures the competition pressure as the sum of competitors' basal area (cross section area of the tree trunk):

$$C_1 = \frac{\pi}{4}\sum_{i=1}^{N} d_i^2.$$

C_2 (Lorimer 1983), it the sum of ratios between the diameters of the competitor and the subject tree divided by the stand area:

$$C_2 = A^{-1}d^{-1}\sum_{i=1}^{N} d_i.$$

In contrast to C_1, where the size of the subject tree has no impact on the competition pressure, the competition pressure in C_2 is decided by the size of the competitors in comparison to the size of the subject tree.

C_3 (Corona and Ferrara 1989), is the sum of ratios between the basal areas of the competitor and the subject tree divided by the stand area:

$$C_3 = A^{-1}d^{-2}\sum_{i=1}^{N} d_i^2.$$

C_4 (Reineke 1933), is based on the ratio between the number of competitors and the stand area (N/A), i.e. the stem density, and the quadratic mean diameter in the stand:

$$C_4 = 10^{\log(N/A)+1.605\log(\bar{d}g)-1.605}$$

The consequence of the index is that the competition pressure is equal among all individual trees.

C_5, is the generalized form of indices C_2—C_3:

$$C_5 = d^{-\alpha}\sum_{i=1}^{N} d_i^{\alpha}.$$

Distance-dependent indices

C_6 (Rouvinen and Kuuluvainen 1997). The competition pressure is determined by the sum of horizontal angles, around the subject tree, which are covered by the sight of the competitors, thus the angles increases with the size of the tree and decreases with the distance to the subject tree. The angles are weighted by the ratio between the diameter of the competitor and the subject tree:

$$C_6 = d^{-1}\sum_{i=1}^{N} d_i \times \arctan(d_i/\mathrm{dist}_i).$$

C_7 (Rouvinen and Kuuluvainen 1997). Same as C_6 but with the exception that the angles are not weighted:

$$C_7 = \sum_{i=1}^{N} \arctan(d_i/\mathrm{dist}_i).$$

C_8 (Bella 1971). The idea for this competition is that every tree has a zone of influence and the area increases with the size of the tree. Two trees start competing once there is an overlap of influence zones. The competition pressure is determined by the area of the overlap in relation to the zone of influence. The overlaps are weighted by the ratio of diameter between the subject tree and the competitor:

$$C_8 = d^{-\alpha}\sum_{i=1}^{N} (O_i \times d_i^{\alpha})/Z.$$

C_9 (Hegyi 1974), approximates the competition pressure as the sum of inverse distance between the subject tree and the competitors, i.e. shorter distance means higher competition pressure. The inverse distance is weighted by the diameter ratio of the competitor and the subject tree:

$$C_9 = d^{-1} \sum_{i=1}^{N} d_i / \mathrm{dist}_i.$$

C_{10} (Hegyi 1974), an alternative version of C_9:

$$C_{10} = d^{-1} \sum_{i=1}^{N} d_i / (\mathrm{dist}_i + 1).$$

C_{11} (Martin and Ek 1984), is a weighted version of C_2. The weights are calculated by an exponential function of dist_i, and the diameter of the competitor and the subject tree. The weights increase with the diameters and decreases with the distance:

$$C_{11} = d^{-1} \sum_{i=1}^{N} d_i \exp[-16\mathrm{dist}_i / (d + d_i)].$$

C_{12} (Canham et al. 2004). Like C_9, the competition pressure is measured as the sum of invers distance. The inverse distance is weighted by the diameter of the competitors. The inverse distance is raised to the power of β and the diameter by α. The competition index makes a distinction of what tree species the competitors are. This is reflected by multiplying the diameter-distance ratio with the species-specific parameter $\lambda_{\mathrm{sp}(i)}$:

$$C_{12} = \sum_{i=1}^{N} \lambda_{\mathrm{sp}(i)} \frac{d_i^{\alpha}}{\mathrm{dist}_i^{\beta}}.$$

C_{13}, is the same index as C_{12} with the exception that the inverse distance is weighted by the ratio of diameters between competitor and subject tree:

$$C_{13} = d^{-\alpha} \sum_{i=1}^{N} \lambda_{\mathrm{sp}(i)} \frac{d_i^{\alpha}}{\mathrm{dist}_i^{\beta}}.$$

C_{14}, is a modified version of C_9, but the inverse distance is no longer weighted, and it is raised to the power of α:

$$C_{14} = \sum_{i=1}^{N} \mathrm{dist}_i^{-\alpha}.$$

C15 (Lin 1974), is an alternative version of C_7:

$$C_{15} = 2\sum_{i=1}^{N} \arctan(0.5 \times d_i/\text{dist}_i).$$

C_{16} (Gerrard 1969), is the unweighted version of C_8:

$$C_{16} = \sum_{i=0}^{N} O_i/Z.$$

The model used for estimating the log-diameter increment, $\ln(d_{\text{inc}})$, is a linear combination of predictors plus an intercept,

$$\ln(d_{\text{inc}}) = a_1 \ln(d) + a_2 \ln(d_{\text{dom}}) + a_3\text{CI} + a_4.$$

Here, d is the diameter of the tree, d_{dom} is the dominant diameter and is calculated as the mean diameter of the hundred largest trees per hectare in the stand. CI is the competition index.

Individual growth model

In paper III and IV we used an individual-based model to simulate the growth of each individual tree in the initial plot. The model was fitted to data for Norway spruce stands from the experimental site Flakaliden (Bergh et al. 2014) in northern Sweden. The dataset included diameter time series and the positions (x-y-positions) for individual trees. I will here briefly describe the model. Let $\text{DBH}_{i,t}$ denote the diameter at breast height (diameter at 1,3 m) for tree nr i (i.e. every tree is associated with a unique index), at stand age t (the age of the trees) and let N_{Tree} be the number of trees in the stand, i.e. $i \in \{1,2,\cdots,N_{\text{Tree}}\}$. The annual diameter increment (cm/year), $\Delta\text{DBH}_{i,t}$, is modelled as a modified logistic growth function (Kot 2001),

$$\Delta\text{DBH}_{i,t} = r \cdot \text{DBH}_{i,t} \cdot \max\left(1 - \frac{\text{DBH}_{i,t} + \text{CI}(i, \mathbf{DBH}_t, \mathbf{Dist}, \boldsymbol{\theta})}{\text{DBH}_{\max}}, 0\right).$$

Here, r and $\text{DBH}_{\max}$ are parameters representing the growth rate of the tree and the maximum DBH the tree can reach, respectively. The model uses the modified crowding competition index (C_{13}),

$$\text{CI}(i, \mathbf{DBH}_t, \mathbf{Dist}, \boldsymbol{\theta}) = \sum_{j\neq i} \lambda\left(\text{DBH}_j/\text{DBH}_i\right)^{\alpha} \text{dist}_{i,j}^{-\beta},$$

to model the competition between trees. The competition index parameters α, β, λ are included in the vector $\boldsymbol{\theta}$. The vector $\mathbf{DBH}_t$ is a collection of all diameters, i.e. $\mathbf{DBH}_t = \left(\mathrm{DBH}_{1,t}, \mathrm{DBH}_{2,t}, \cdots, \mathrm{DBH}_{i,t}, \cdots, \mathrm{DBH}_{N_{\mathrm{Tree}},t}\right)$. $\mathbf{Dist}$ is a matrix of all pairwise distances between trees, i.e. $\mathbf{Dist}_{i,j} = \mathrm{dist}_{i,j}$, where $\mathrm{dist}_{i,j} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}$, and the ordered pair (x_i, y_i) denotes the position in x- and y-axis of tree i.

In addition to the diameter we also modeled the annual tree height increment (m/year), denoted $\Delta H_{i,t}$. The diameter increment is modelled as a power law relation to the diameter increment, i.e.

$$\Delta H_{i,t} = \gamma\left(\Delta DBH_{i,t}\right)^{\delta}.$$

γ and δ are model parameters.

To calculate the value and the cost of a harvest we have to estimate the stem volume for each individual tree. The volume is calculated from the stem tapering, i.e. the revolution of the stem tapering gives the volume. In paper III we estimate the stem tapering with the function of Edgren and Nylinder, (1949) and in paper IV we used the function of Lassasenaho (1982).

Resources, such as light and nutrients, are limited. The consequence of this is that the forest area can only sustain a limited number of trees. When this limit is reached there will be a loss of trees, i.e. self-thinning. In the growth model we implement the algorithm outlined by Pukkala et al. (1998) to account for this loss. The algorithm works as following: At the beginning of a growth period we check if the current stem density (stems per hectare) is larger than the current maximum stem density, $N_{\max}$. If this is the case the number of trees is reduced until the density is below $N_{\max}$. $N_{\max}$ is calculated as a function of the current quadratic mean diameter, $\bar{d}q$ (cm), in the stand. The function was estimated from empirical data presented in Elfving (2010),

$$\ln(N_{\max}) = 12 - 1.5\ln\left(\bar{d}q\right).$$

The survival probability will determine if a tree is affected by self-thinning, i.e. the trees with the lowest survival portability are removed first. The survival probability is calculated by using the survival function of Pukkala *et al.* (2013).

Measures for describing thinning

There are a number of ways to measure and characterize different thinning regimens. Below I will list the measures used in the papers presented in this thesis, particularly in paper III and IV.

Basal area reduction

The basal area, BA, of a forest area is defined as the sum cross-section area, at breast height, of all trees in the forest divided by the forest area, A. That is

$$\mathrm{BA} = \frac{\pi \sum_i d_i^2}{4A},$$

where, d_i is the diameter at breast height of the i:th tree. The basal area reduction, BAR, is defined as the proportion of the basal area removed,

$$\mathrm{BAR} = \frac{\mathrm{BA}_{\mathrm{pre-thinning}}}{\mathrm{BA}_{\mathrm{post-thinning}}}.$$

Thinning ratio

The thinning ratio, TR, is a measure where the mean diameter of the harvested trees, $\bar{d}_{\mathrm{Harvested\ tree}}$, is related to the mean diameter of the residual trees, $\bar{d}_{\mathrm{Residual\ tree}}$, in the stand. Specifically, it is the ratio between the two,

$$\mathrm{TR} = \frac{\bar{d}_{\mathrm{Harvested\ tree}}}{\bar{d}_{\mathrm{Residual\ tree}}}.$$

The thinning ratio measures the preference of the size of the tree for harvesting. A TR value of 1 means that even selection of larger and smaller trees are selected for harvesting. TR value greater than 1 implies mainly that larger trees are targeted for harvest whereas a TR value lesser than 1 implies that mainly smaller trees are harvested.

Spatial evenness

In paper III we needed a measure for the spatial distribution of the residual trees. We used Clark-Evans index, R, (Clark and Evans 1954) to quantify the spatial distribution of the trees in the forest, specifically the evenness of the trees. The Clark-Evans index is defined as the ratio between the mean distance to nearest neighbor in the stand and the mean distance to nearest neighbor for a random distribution,

$$R = \frac{\sum_{i=1}^{N_{\text{Tree}}}(r_i)}{\bar{r}N_{\text{Tree}}}.$$

Here, r_i is the distance to nearest neighbor for tree nr i, and $\bar{r}$ is the expected mean distance to nearest neighbor in a random distribution. Edge effects are accounted for by using the edge-corrected $\bar{r}$-value, described by Sinclair (1985),

$$\bar{r} = 0.5\sqrt{A/N_{\text{Tree}}} + \left(0.051 + 0.041/\sqrt{N_{\text{Tree}}}\right)L/N_{\text{Tree}}.$$

A denotes the area of the forest stand and L is the associated circumference. $R = 1$ means the given spatial distribution is indistinguishable from a random distribution. For $R > 1$ (the mean distance in the stand is larger than for the random case) the distribution is more uniform (even) and for $R < 1$ the distribution is aggregated (uneven).

Forest stand performance measure - wood production and economic calculations

In paper III and IV we quantify the performance of a given management plan by 1) the net wood productivity, a measure of the mean annual merchantable wood production, and 2) the two economic measure, net present value (paper III) and the land expectation value (paper IV).

Net production

The net production is calculated as the sum of the volume harvested during the thinning operations and the clear-cut, divided by the rotation time (i.e. stand age at the clear-cut).

Net present value

The net present value of the stand is calculated as the sum of present value of all operations conducted on the stand during one rotation,

$$\text{NVP} = \sum_{i=1}^{N_t}\left(e^{-\ln(1+\text{IR})t_i}V_{t_i}\right) - e^{-\ln(1+\text{IR})t_0}V_{\text{Pre-commercial thinning}} - V_{\text{regeneration}}.$$

Here, IR is the discount rate. t_0 denotes the age of the stand when a pre-commercial thing is conducted and T denotes the chosen stand-age for final felling (the rotation time). The stand ages between t_0 and T where a thinnings was scheduled is denoted by t_i, $i \in \{1,2,\cdots,N_t - 1\}$, and V_{t_i} denotes the net value (income minus costs) that is obtained from a thinning at stand age t_i. The value of the clear-cut is denoted by $V_{t_{N_t}}$. $V_{\text{Pre-commercial thinning}}$ and $V_{\text{Regeneration}}$ denotes the cost for pre-commercial thinning and regeneration respectively. V_{t_i} and $V_{t_{N_t}}$ were determined as the difference between the value of the trees removed and the harvesting, forwarding, and machine relocation costs,

$$V_{t_i} = \sum_{j=1}^{N_R} \left[p\left(\text{DBH}_{j,t_i}\right)\text{Vol}_{j,t_i}\right] - c_{\text{Harvester}} - c_{\text{Forwarder}} - c_{\text{Relocation}}.$$

The number of trees selected for harvesting is denoted by N_R. $p\left(\text{DBH}_{j,t_i}\right)$ is the DBH-specific volume price for tree j, at age t_i, and Vol_{j,t_i} is the stem volume of tree j. $c_{\text{Harvester}}$ and $c_{\text{Forwarder}}$ are the cost for harvesting and forwarding respectively. In paper III and IV we used the cost function by Eriksson and Lindroos (2014) and Nurminen et al. (2006) to estimate the cost for harvesting and forwarding. $c_{\text{Relocation}}$ is the relocation cost for the harvester and forwarder, i.e. the cost to bring the machines to perform the operation.

Land expectation value

The net present value will accurately depict economic gain for the forest stand over the rotation period. However, the drawback of only using the net present value is that it does not account for reuse of the forest stand; meaning after the clear-cut, new forest is planted in a regeneration phase, and the rotation cycle starts over. One way of accounting for this is to calculate the land expectation value, LEV. Here it is assumed that the same forest can be recreated in the regeneration phase, the same management plan is applied, and the cost and income are the same in every rotation. With these assumptions the present value for a second rotation can be calculated as $\text{NPV} \cdot (1 + \text{IR})^{-T}$ and $\text{NPV} \cdot (1 + \text{IR})^{-2T}$ for the third, and so on. The land expectation value is calculated as the sum present value of infinitely rotations,

$$\text{LEV} = \text{NPV} + \text{NPV} \cdot (1 + \text{IR})^{-T} + \text{NPV} \cdot (1 + \text{IR})^{-2T} + \cdots$$

This is a geometric series and the closed form for LEV can be expressed as,

$$\text{LEV} = \frac{\text{NPV}}{1 - (1 + \text{IR})^{-T}}.$$

Chapter 4. Optimization methods

In many of the papers presented in this thesis some form of optimization method was used. In this section I will briefly introduce the most important ones. I start by introducing dynamic programming and Pontryagin's maximum principle, both of which are fundamental for the branch of optimization called optimal control theory. Following this, I will briefly explain the allocation model we derived in Paper I, which employs the previously mentioned methods. For paper III and IV we needed to solve large nonlinear combinatorial optimization problems. In the case of paper III, we wanted to find a selection of trees for harvest such that several conditions were fulfilled. In paper IV we wanted to find the optimal thinning time for each individual tree, such that we maximize the land expectation value. To solve the combinatorial problems we employed two metaheuristic methods, which have been used successfully for other combinatorial problems (Blum and Roli 2003), namely simulated annealing and genetic algorithms.

Optimal control

Dynamic programming

Dynamic programming is a mathematical and computational method to solve sequential decision problems. Dynamic programming was develop in the 1950's by Richard Bellman (Bellman 1954). The general problem is formulated as a system at state y_i, at the discrete time-point i, and an agent who must make consecutive decisions (sometime referred to as an action). The decision taken at time i is denoted α_i. α_i is also known as the control variable of the system. Each time a decision is made a reward, R_i, is awarded to the agent. R_i is determined by a function, R, of the current state and the decision made,

$$R_i = R(y_i, \alpha_i).$$

The consequence of the decision will not only affect the reward but also the state. Thus, the state is changed according to a transition function, Y,

$$y_{i+1} = Y(y_i, \alpha_i).$$

As a consequence, the value of all future rewards is affected by the decision made in the current state. The goal is to find a sequence of decisions, $\{\alpha_0, \alpha_1, \dots, \alpha_N\}$, that maximizes the sum of all rewards,

$$V_N = \sum_{i=0}^{N} \beta^i R_i.$$

Here, $\beta > 0$ denotes the discount rate which indicates the decay of relevance of future rewards. If $\beta < 1$ and $|R_i| \leq R_{\max}$, for some positive number $R_{\max}$, then V_N is bounded for $N \to \infty$,

$$-\frac{R_{\max}}{1-\beta} \leq \lim_{N\to\infty} V_N \leq \frac{R_{\max}}{1-\beta}.$$

Bellman solved the problem by reformulating the problem and breaking it down into several sub-problems. This is clarified by Bellman's principle of optimality:

> *"Principle of Optimality: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."* (Bellman 1954)

If we denote $V(y_i)$ as the maximum value of V we can obtain by starting form state y_i,

$$V(y_i) = \max_{a_i, a_{i+1}, \ldots, a_N} \sum_{j=0}^{N-i} \beta^j R\left(y_{j+i}, \alpha_{j+i}\right)$$

Using this definition, we can reformulate the initial problem into a recursive form,

$$V(y_i) = \max_{a_i} \{R(y_i, a_i) + \beta V[Y(y_i, a_i)]\}.$$

This equation is known as Bellman's equation. The reason for rewriting the problem in this form is because we have a special case when we consider $V(y_N) = \max_{a_N} R(y_N, a_N)$. This is a conventional optimization and if solved, the value $V(y_N)$ can be used to calculate $V(y_{N-1})$ in the same manner. Thus, we solve the problem by recursively solving the sub-problems backwards in time. This can be formulated into the following algorithm (Kirk 2004):

Step 1. For every feasible end-state y_N try all possible control decisions, a_N, and calculate $V(y_N, a_N) = R(y_N, a_N)$. Save the maximum value of $V(y_N) = \max_{a_N} V(y_N, a_N)$ and the control decision associated with this value for every state y_N.

Step 2. For $i = N-1, N-2, \ldots, 0$ do:
For every feasible state, y_i, try all possible control decisions, a_i, and calculate $V(y_i, a_i) = R(y_i, a_i) + \beta V(y_{i+1})$, where $y_{i+1} = Y(y_i, a_i)$. use the previously calculated values of $V(y_{i+1})$. Save the maximum value of $V(y_i,) = \max_{a_i} V(y_i, a_i)$ and the control decision, a_i, associated with this value, for every state, y_i.

In this formulation I have only considered deterministic problems. The framework can easily be extended for solving stochastic problems, so called Markov decision problems (Powell 2011).

Pontryagin's maximum principle

In the forgoing sub-chapter, we studied the Bellman equation for solving discrete decision problems. There is an equivalent equation for continuous problems which is known as the Hamilton—Jacobi—Bellman equation (HJB),

$$\dot{V}(y,t) + \max_a \{\nabla V(y,t) \cdot \dot{y} + R(y,a)\} = 0.$$

Here, t denotes time, y is the state of the system, a are the controls, R, is the reward function, as before, and ∇V is the gradient of V. Both y and a are now continuous and time dependent. The dynamics of y are given by a function Y,

$$\dot{y} = Y(y,a).$$

Solving the partial differential equation, i.e. finding the controls $a(t)$ that satisfies the HJB, subjected to the terminal condition $V(y,T) = D(y)$ and initial condition $y(0) = y_0$, will maximize the continuous value function V,

$$V(y_0, 0) = \int_0^T R\big(y(t), a(t)\big)dt + D\big(y(T)\big).$$

Here, $D(y(T))$ represents a terminal reward and T is the terminal time. It should be noted that y, a, are assumed to be vector valued functions. Solving the HJB is not easy. In the 1950's Lev Pontryagin formulated necessary (but not sufficient) conditions for optimal control (Pontryagin 1962). Using these conditions one can derive useful information easier than by studying the HJB. The conditions are derived by using theory from the field of calculus of variations, analogue to the derivation of the Euler—Lagrange equation. The main condition is that the Hamiltonian for the system must be maximized at every point in time along the optimal trajectory, with respect to the controls. The Hamiltonian for the system is defined as,

$$H(y(t), a(t), \lambda(t), t) = R(y, a) + \lambda \cdot \dot{y}.$$

Here, λ is vector valued and denotes the so called costate variables. These can be viewed as Lagrange multipliers associated with the state y, that is, λ reflects the constraints of the optimization problem. Depending on the problem, extra conditions must be fulfilled (Kirk 2004). For the problem presented in paper I, we have the case that $D(x) = 0$ and $T \to \infty$, this is an infinite-horizon problem. The necessary constraints for this type of problem are the following (Halkin 1974),

$$\begin{gathered} H(s(t), a^*(t), \lambda(t), t) \geq H(s(t), a(t), \lambda(t), t), \qquad 0 \leq t \leq T \\ \dot{\lambda} = -\nabla H \end{gathered}.$$

Here, $a^*(t)$ denotes the optimal controls, i.e. the controls maximizing the Hamiltonian.

Future allocation model

In paper I we created a growth model for a single tree, based on the model by Mäkelä (1986). We considered the tree to be divided into several compartments: fine root, course root, stem, branches, and foliage. The relationship between the sapwood area and the foliage mass is assumed to be proportional, i.e. it obeys the pipe model (Shinozaki et al. 1964). For our model the tree could allocation its net production into stem-height and crown-size, and seed production. The dynamics of the stem-height and crown-size has the following general form,

$$\frac{dy}{dt} = \big(a(t) \circ C(y)\big)P(y).$$

Here, y is vector valued and denotes the size of the stem-height and crown-size, P is a scalar function which denotes the net productivity of the tree (gross production from photosynthesis minus the growth and maintenance respiration) and a is the vector-valued control which dictates the proportion of the net production invested into the two quantities. C is a vector-valued conversion function which translates the invested biomass into the appropriate units and guarantees that the pipe model and other size relationships are fulfilled and $\circ$ is the Hadamard product (entry-wise product). In paper I we investigated how the allocation should be carried out when a small tree starts growing under a shaded canopy. The shaded canopy is assumed to create a time-invariant light environment for the tree to grow in. We defined optimal as the allocation schedule, i.e. finding the function $a(t)$, which maximize the fitness proxy V,

$$V = \int_0^{\infty} e^{-\int_0^t m(y(s))ds} b\big(y(t), a(t)\big)dt,$$

where $b\big(y(t), a(t)\big)$ is the biomass invested into reproduction, i.e. the portion proportion of the net production not invested in either stem-height or crown-size. m is the size-dependent mortality rate. Analyzing this system with Pontryagin's maximum principle we derived that at any given time it is optimal to invest all the net production into either stem-height or crown-size. Simultaneous investment should only take place when both quantities have equal effect on the net productivity to mortality ratio ($P{:}\, m$). When simultaneous investment takes place, the net production should be distributed such that effects remain equal.

Metaheuristic methods

The term metaheuristic was first used by Glover (1986). There is no formal definition for metaheuristic methods (MM), however they are grouped together by a couple of characteristics. For one, they use a high-level strategy (heuristics) to "guide" the search through the vast search space for the (near-) optimal solution. The search is conducted iteratively and in many cases they employ some form of stochastic component. The search mechanics can be divided into two different parts; diversification and intensification (Blum and Roli 2003). Diversification refers to the exploration of the search space, intensification refers to the act of utilizing knowledge from previous exploration. Most of these methods do not guarantee finding the global maximum/minimum, but they have shown to give near optimal solutions in relatively short computation time (Blum and Roli 2003). Metaheuristic methods can be divided into to two categories, those who use a population of solutions and those using a single-point solution. As the name hints at population-based methods starts with a number of solutions (a population) and "evolves" this population through the search process, while single-point based methods describes a trajectory in search space. I will briefly describe two metaheuristic methods which we used in Paper III and IV, namely simulated annealing (single-point method) and genetic algorithms (population-based method).

Simulated annealing

As the name implies, this stochastic optimization method is inspired by annealing procedures in metallurgy, where metal is systematically cooled and reheated to change the physical properties of the metal. The method itself is used to find the minimum energy in a system (minimization of a function) by changing the temperature of the system. The method starts with an initial state, i.e. initial guess for the solution of the optimization problem. A candidate state (a new solution guess) is drawn randomly from a proposal distribution conditional on the current state. If the candidate state has lower energy (lower function value) the candidate is accepted as the new solution. However, if the energy for the candidate is higher then there is still a chance that the candidate is accepted as the new solution. The chance of acceptance is determined by the difference in energy and the current temperature of the system. If the temperature is high the chance of acceptance is higher. The temperature is systematically lowered, making the acceptance of suboptimal candidates less likely. Letting the temperature approach zero, results in a greedy algorithm (only candidates with lower energy are accepted). If the temperature is lowered too quickly there is a risk for the method to converges into a sub-optimal local minimum. On the other hand, if the temperature lowered too slow the method converges slowly, thus temperature scheduling is crucial. It has been proven that the method converges to the global minimum (Granville et al. 1994), however the temperature scheduling required is unfeasible for most problems as it takes too long to converge. In paper III we assumed a constant temperature as this was sufficient to solve our problem. When the temperature is constant the method is called Metropolis—Hastings algorithm (Hastings 1970). The problem in Paper III was to find the selection of trees for harvest which were closest to the preset targets for basal area reduction, spatial evenness, and thinning ratio.

Genetic algorithms

Genetic algorithms (GA) are metaheuristic algorithms which draws inspiration from evolution and natural selection. In Paper IV we used a built-in genetic algorithm optimizer in *MATLAB,* based on the algorithm of Deep et al. (2009), to find the optimal (i.e., maximizing land expectation value) number of thinnings, the timing for these and the clear-cut, and for every individual tree decide in which thinning it should be harvested. The algorithm starts with an initial population (set of solution suggestions) and with each iteration creates a new generation of populations. Each individual in the population has a genetic representation and a fitness value can be calculated for each individual. The goal is to evolve the population, such that fitness (in our case the land expectation value) increases. The individuals in the new population is generated in three ways. The simplest way is by choosing elite individuals (i.e. high fitness value) from the previous population and adds them to the next generation. The second way is called recombination or crossover, where two or more parents are selected from the old population. The offspring (new individuals) are created by recombining parts of the genetic representation of the parents. The last way of creating new individuals is mutation. Here the new individuals are created by making slight changes to the genetic representation of individuals from the previous population.

Chapter 5. Results

Recall the four objectives posted in the objectives section. Below, I will summarize the results from Paper I-IV and present them under these four headlines.

How should a tree allocate its biomass optimally in a time invariant environment?

Using the result from the analysis of the multi-compartment tree-growth model (see the optimal control section), we constructed in Paper I a heuristic allocation model. The allocation model is consisted of a few rules. First, the tree invests all resources into crown-size, next follows stem-height growth, and finally crown-size investment or a gradual shift from stem-height investment to crown-size growth. The gradual shift in investment appears when stem-height and crown-size investments have equal impact on the net productivity-to-mortality ratio ($P{:}m$). The tree continues to grow until an optimal size is reached, at which growth ceases and reproduction starts. The shifts in allocation are dependent on the light environment. To summarize the optimal allocation problem has been transformed into a two-dimensional allocation problem. The heuristic model was tested against the optimal solution, as determined by dynamic programming and found that indeed the heuristic model approximated the optimal trajectories well.

Which competition indices can be uses to reliably approximate competition for Norway spruce in northern Sweden?

In Paper II we found that the best competition indices where the distance dependent indices. The best performing distance-dependent index indices are C_8 (R^2_{adj}= 0.41—0.66), C_{12} (R^2_{adj}= 0.43—0.66), C_{16} (R^2_{adj}= 0.36—0.62) and C_{11} (R^2_{adj}= 0.41—0.63). In contrast the best distance-independent indices perform worse. The best distance independent indices where C_1 and C_4 with R^2_{adj} between 0.39 to 0.59 (depending on the stand treatment). The result also showed that the ranking between the indices was fairly consistent between the different treatments.

How will the spatial evenness, after thinning, between trees impact on the merchantable wood production and the net present value of the stand?

From the results of Paper III, we observe that the net production and net present value increased with the spatial evenness. The mean net production increase was about 2% for all plots and the mean net present value increase was 2.5%. We detected an even greater impact on net production and net present value when we changed the thinning ratio in conjunction with the spatial evenness. The results from our investigation showed that the maximum increase in mean net production and net present value are 5.7% and 8.0%, respectively. The maximum increase depended on the configuration of the initial stand. The general trend was: higher thinning ratio and spatial evenness result in greater performance.

How will an optimal thinning schedule (selective thinning) impact the economic performance of the stand and what strategy emerges?

The results form the optimal thinning investigation in paper showed that for all tested initial stand configurations, a single thinning was scheduled at a stand age between 40-50 years and a final felling at a stand age of 65-75 years. In all cases the basal area reduction was high compared to a conventual thinning. The results showed that thinning ratio should be high, meaning preferably larger trees should be selected for harvesting (thinning from above). When compared with the conventional strategy the net production was around 20% higher and the land expectation value was around 25% higher. The optimal thinning strategy changed dependent on the economic assumptions. Decreasing the relocation cost increased the number of scheduled thinnings to two or three. Increased interest rate prompted earlier scheduling for both thinning and final felling.

Chapter 6. Conclusions and discussions

The main goal of this thesis was to present new methods for optimizing, from an economic perspective, stand management on an individual tree basis. Towards this goal, my coauthors and I investigated different facets of tree growth and forest management. In the first two paper we looked at different ways for modelling the impact of the biotic and abiotic environment on an individual tree.

In paper I we looked the impact from a theoretical point of view and asked how a tree should optimally invest its resources in a shaded light environment. We demonstrated that the optimal investment follows two sequential strategies, one for shaded, or understory, conditions and one for canopy conditions. Thus, we reduced the computationally heavy task of solving the Bellman equation into a two-dimensional optimization problem, by assuming the tree possess full knowledge of the light environment. In comparison to more commonly used allocation models, where static allometric relationships are assumed, our allocation method has advantages of, 1) eliminates the need for species specific allometric parameters and 2) it automatically integrates a more realistic plastic response to its environment.

In paper II we focused on an empirical approach and tested several competition indices for how well they can model growth data. Competition indices are computationally-efficient to approximate tree-to-tree interactions. As in other similar investigations (Stadt et al. 2007; Contreras et al. 2011) we found that the best performing indices were distance-dependent. This indicates the importance of not only accounting for individual tree size but also spatial information.

The importance of spatial information was further demonstrated in paper III. From the results, we can see that a higher spatial evenness, after thinning, will improve the overall performance of the stand, both in terms of net present value and net production. This means that when a specific basal area reduction is chosen the tress selected for thinning should be chosen such that the residual trees are evenly distributed over the area. A higher spatial evenness means that, on average, we increase the distance to the nearest neighbor. This results in more space for most trees. However, if the distance to nearest neighbor is too high (i.e. the basal area reduction is too high), it can have a negative impact on net production (Wallentin 2007). Our finding agrees with the empirical studies by Baldwin et al. (1989) and Mäkinen et al. (2006). We also found that by selecting the optimal thinning ratio and spatial evenness we obtain a maximum increase in net production and net present value of 8.0%. However, the magnitude of improvement depends on the initial stand characteristics. Specifically, the initial stands with higher heterogeneity, i.e. stands with greater initial variation in tree sizes and spatial evenness, had higher magnitude of improvement.

The main goal of this thesis was to device methods for optimizing the stand management on an individual-tree level. In paper III we restricted the number of choices on the thinning to three (spatial evenness after thinning, thinning ratio, and basal area reduction). The timing of and the number of thinnings (one) where already predetermined and we only had implicit influence on which individual trees should be harvested. However, in paper IV we took full advantage of the individual-based model and addressed the main goal of this thesis. We devised new procedure for optimizing the thinning scheduling for each individual tree. The results showed that for all stands, one scheduled thinning was optimal, the basal area reduction should be high, and the larger trees should be targeted, i.e. thinning from above. This is in agreement with other similar investigations (Pukkala and Miina 1998; Cao et al. 2006). The reason being to get as much revenue from the first thinning (larger trees are more valuable) and leaving the smaller trees, which have the greatest growth potential (in terms of income). The accuracy of the economic analysis and the optimal thinning is greatly depending on the choice of cost estimates, timber prices, interest rates, etc. which also has been observed by other studies (Solberg and Haight 1991; Valsta 1992; Hyytiäinen et al. 2004). We observed an earlier timing for both thinning and final-felling when increasing the interest rate. This is due to the decreased importance of future incomes. The biggest change appeared when the relocation cost was decreased. This resulted in an increase in the number of scheduled thinning, because it was then profitable to harvest timber that would have otherwise been lost in self-thinning.

There are several limitations and room for improvement to the individual-based growth model and the optimization in general. For one, the results from paper IV show a significant difference in net production between thinning from below, the thinning form employed in conventional thinning, and thinning from above, the thinning form suggested by the optimal strategy, which deviates from observation in empirical studies (e.g. Nilsson et al. 2010). This deviation might be attributed to limitations in the data for fitting or the limitation in the growth model.

Another limitation is that we assumed that all trees could be selected for harvesting. However, in practical forestry corridor planning needs to be considered in the optimization to ensure that the trees selected for thinning can also be reached by the harvesters. Moreover, calculations of costs for forwarder and harvester where done on a stand-level basis. In order to realistically estimate the cost for a thinning one would need to employ an individual tree-based cost function.

In the current growth we account for local variation in terms of competition (i.e. the growth of a trees is effected by the size of its neighbors and the distance between them). However, we neglect to account for variation in terms of soil condition and the growth potential (i.e. genetic variation), which affects the growth rate (r) and maximal diameter of the trees ($\mathrm{DBH}_{\mathrm{max}}$). We assume that the soil condition is uniform over the whole forest area and that all trees have the potential to reach the same maximum size. In a realistic setting these would vary and could be accounted for in our model by letting the growth rate and maximum diameter be individually determined for each tree by the local soil condition and the inherited genes of the trees.

The sensitivity analysis in paper IV was limited to study how the optimal thinning strategy changed with the assumed economic parameter (i.e. relocation cost and discount rate). This analysis can be extended and test sensitivity in regard of the initial stand data (i.e. individual sizes of the trees and distance between trees). This is of interest because in a practical use one would start with data for the initial stand, gather by remote sensing techniques, and these can contain errors. To test this one would introduce random errors, drawn from predetermined distribution, to the initial stand configurations.

To conclude, although we could not derive a universal optimal strategy for forest management we have presented and demonstrated a new procedure which is capable of find near optimal forest planning for several scenarios. The procedure has many possibilities for enhancements and extensions for future use in precision forest management. In this thesis most of my attention was focused on optimizing forest management from an economic stand point, but the same procedure can readily be applied for optimization of other or additional values, such as environmental and social values.

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