

**A Division-of-Labor Hypothesis:
Adaptations to Task Structure in Multiple-Cue
Judgment**

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ABSTRACT

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Judgments that demand consideration of pieces of information in the environment occur repeatedly throughout our lives. One professional example is that of a physician that considers multiple symptoms to make a judgment about a patient's disease. The scientific study of such, so called, *multiple-cue judgments* that involve multiple pieces of information (*cues*: e.g., symptoms) and continuous *criterion* (e.g., blood pressure) has been concerned with the statistical modelling of judgment data (see Brehmer, 1994; Cooksey, 1996; Hammond & Stewart, 2001). In this thesis behavioural experiments, cognitive modelling and brain imaging is used to investigate an adaptive division of labor between multiple memory representations in multiple-cue judgment. It is hypothesized that the additive, independent linear effect of each cue can be explicitly abstracted and integrated by a serial, additive judgment process (Einhorn, Kleinmuntz, & Kleinmuntz, 1979). It is further hypothesized that a variety of sophisticated task properties, like non-additive cue combination, nonlinear relations, and inter-cue correlation, are carried implicitly by *exemplar-memory* (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000). Study I and II investigates the effect of additive versus non-additive cue-combination and verify the predicted shift in cognitive representations as a function of the underlying cue-combination rule. The third study is a review that discusses the nature of these representational shifts; are they contingent upon early perceived learning performance instead of automatic and error-driven? Study IV verifies that this shift is evident also in the neural activity associated with making judgments in additive and non-additive tasks.

Key words: multiple-cue judgment, exemplar models, cue abstraction, cue-combination rule

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Umeå, August, 2007

Linnea Karlsson

LIST OF PAPERS

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INTRODUCTION

Repeatedly in life we are engaged in the spontaneous activity of making judgments of objects, persons, situations, and offers. We are weighing the pros and the cons of the lunch of today, deciding whether it is worth its price, or what amount of cream and sugar to add to the coffee to make it taste perfectly. Many times we also engage in judgments of greater importance to us. Being a physician and considering different symptoms when making a diagnosis involves a certain competence of judgment. The notice a pilot makes of the information on the instrument panel and how the pilot's judgment concerning a maneuver is formed on the basis of that information is crucial for the survival of the passengers. Reflecting over the nature of these cognitive efforts, it becomes evident that a person's memory and collected life experience should be involved in some way in judgment situations like these.

The topic of this thesis is to highlight *why* and *how* different memory resources are utilized as humans are confronted with various judgment tasks. The postulation that will be made is that although we perhaps are unaware of it many times, certain judgment task structures will force us to rely on our memories of previous judgments cases, as for example the memory of a previous patient exhibiting a certain disease. Other judgment task structures will enforce analytical thought, combining pieces of information in a rule-like manner, as for example the consideration of how single symptoms are related to the possibility that the patient has the disease. It will be postulated that retrieving previous judgment cases from memory is especially used in situations where the nature of controlled cognitive processes does not allow us to use analytical thinking. In this thesis, I will present evidence for such a *division-of-labor hypothesis* based on psychological experiments, cognitive modeling and brain imaging.

Judgment and decision making research has witnessed a progress in realistic appreciation of cognitive capacities. When the study of judgment and decision making was introduced 1940-1950 roughly two main research programs were initiated (Goldstein & Hogarth, 1997). One of them was principally concerned with how people decide on a course of action and how they choose what to do next. Are people rational when they make these decisions of choice? At the outset the human mind was compared to an "*intuitive statistician*" with power and time to consider all important options and with a cognitive apparatus thought to be well approximated by normative models (Peterson & Beach, 1967). Expected utility theory offered a framework where it was possible to predict what choices people would make by hypothesizing that people maximized the expected utility of the decision (von Neumann & Morgenstern, 1947). However, alternatives to this strong belief in the human cognitive capacities were soon proposed. Simon (1957) argued that we are *bounded* rational, rather than rational. We tend to choose according to what

satisfies us, rather than what can be seen as normatively optimal. Tversky and Kahneman went even further and in a large number of studies starting in the 1970's they argued that mental shortcuts (heuristics) are used for making decisions. Their conclusions have frequently been that humans are as dissimilar as possible to intuitive statisticians; rather, the intuitive nature of heuristics can sometimes lead performance astray. Although minimizing cognitive effort, erroneous judgments based on heuristics can slip through the gate of cognitive control.

The other program, inspired by ideas from perception and the works of a European psychologist named Egon Brunswik, found interest in how a judge relies on external information about the world when making judgments (*multiple-cue judgments*). How important is the nature of the task for the accuracy of a judgment? How do people identify and use information (*cues*) in the real world to make inferences? It is judgment situations within this latter program that is the topic of the current thesis.

BACKGROUND

Related to the Brunswikian tradition of the study of how multiple cues are utilized (multiple-cue judgment) is the study of how people use one single cue to judge or predict a criterion (*single-cue judgments* or *function learning*). Both research programs are introduced below.

The Study of Single-Cue Judgment

In research in the paradigm of single-cue probability learning (single-cue judgments) part of the focus has been to describe learning of the function relating a cue to a criterion (also called *function learning*; see Bott & Heit, 2004; DeLosh et al., 1997; Kalish et al., 2004; Klayman, 1988; Koh & Meyer, 1991; Slovic & Lichtenstein, 1971). Consider the very simple estimation of how much sugar to put in your coffee each morning; too little sugar makes it bitter, some more makes it perfect, but too much makes it too sweet. An inverted U-shaped function may relate the amount of sugar (cue) to the taste of the coffee (criterion).

A number of interesting characteristics of single-cue function learning have been observed (Busemeyer et al., 1997). First, it appears as though it matters if there actually is a function relating the cue to the criterion in a stimulus set encountered in training, as opposed to if the cue is arbitrarily mapped to a criterion value (Carrol, 1963). When the cue is related to the criterion by a function, learning is faster than when there is arbitrary mapping.

It has moreover been found that some function forms are learned faster than others. Linearly increasing functions are learned fastest. It has been proposed that humans are engaged in hypothesis testing, where a positive linear function

relating the cues to a criterion is the first hypothesis that is tested by a function learner (Brehmer, 1994). Indeed, at the beginning of training with a non-linear function the responses were well described by a linear and additive function (Sawyer, 1991). Cyclic, non-monotonic and non-linearly increasing functions have been found to be more difficult to learn than non-cyclic, monotonic, and linearly increasing functions (Busemeyer et al., 1997).

The ability to *transfer* the knowledge gained from observed cue-criterion pairs to unobserved pairs has proven to be different depending on the function form (DeLosh et al., 1996; Bott & Heit, 2004). This issue is of importance since being able to transfer a learned skill to situations outside a trained range could be valuable for good performance. In some circumstances people have been shown to be able to extrapolate the effect of one cue on the criterion beyond the known value range (Bott & Heit, 2004). However, the ability to extrapolate has proven worse than the ability to interpolate to new cue-criterion pairs within a known training range (DeLosh et al., 1996; Carrol, 1963). A recent attempt to address single-cue function learning asserts that learning a non-linear function between a cue and a criterion is mastered through the acquisition of numerous linear slopes, together covering the cue-criterion distribution. This model is called POLE for a “POpulation of Linear Experts” (Kalish et al., 2004).

The Study of Multiple-Cue Judgment

A typical multiple-cue judgment involves making an inference about a criterion on the basis of a number of cues in the external world. A stock broker may rely on different economic indices (*cues*) to make a judgment of the future interest rate (the *criterion*). This paradigm has been under study since the 1950's. Beginning with the works of Brunswik and Herma (1951) the psychology of judgment was concerned with finding the relations between the task system and the cognitive system (Björkman, 1965; Brehmer, 1972; Hammond et al., 1964; Smedslund, 1955). Experimental designs, where the task was to learn to make judgments either in a real, naturalistic judgment task or in a fictitious environment with one or a few cues probabilistically related to a criterion, were used in order to draw conclusions on the nature of these judgments. What has characterized the main focus in this paradigm has been the statistical modeling of judgment data, in relation to the judgment task (Brehmer, 1994; Cooksey, 1996; Hammond & Stewart, 2001).

Brunswik developed a model to understand perception, which was also appropriate to understand multiple-cue judgment (Brehmer, 1988; Brehmer 1994; Cooksey, 1996; Hammond, 1996). In Brunswik's approach, *probabilistic functionalism*, he stressed the need to understand the organism *and* the environment where the organism functions, as well as the relationship between them (Brehmer, 1988). This point was illustrated in the *Lens Model*

(schematically illustrated in Figure 1). Social Judgment Theory, SJT, (developed by Hammond, see Brehmer, 1988; Cooksey, 1996) adopts Brunswik's model to describe the process of judgment. The psychology of judgment should be concerned with finding the relations between the task system and the cognitive system.

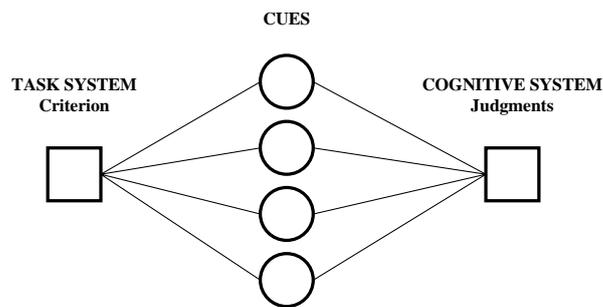


Figure 1. The Lens Model.

SJT adopted the importance of describing the two systems by the same statistical concepts; the lens model equation was introduced. The lens model is based on multiple linear regression models and was suggested as the statistical model that describes the task system, the cognitive system and the relation between them. The basic idea behind multiple linear regressions is to find the weights that optimize each of the different predictors' contribution to an estimate (e.g. Cohen & Cohen, 2003). In terms of multiple-cue judgment, the regression analysis is used to analyze how the different cues are weighted to form a criterion.

Since its introduction, linear multiple regression has been used in a variety of domains to examine multiple-cue judgments. Examples of studies that have used this methodology are numerous and range from weather forecasting (Stewart, Roebber, & Bosart, 1997), to medicine and clinical decision making (see for example Harries & Harries, 2001; Smith, Gilhooly & Walker, 2003; Wigton, 1996), to teachers' decisions about students (Cooksey, Freebody, & Davidson, 1986) and so on. Summarizing the findings gained reveals that the judge usually only considers a few cues, he or she is inconsistent in using them, he or she has poor insight into the judgment process (but see Lagnado, Newell, Kahan & Shanks, 2006), individuals differ greatly when weighting the cues, and the judgment data is often well described by a linear and additive function (Brehmer, 1994; Cooksey, 1996; Hammond & Stewart, 2001).

In sum; within the extensive research paradigms of single- and multiple-cue judgment the focus has mainly been to measure the rate of learning as well as on statistical description of how a person utilizes the cues in the environment to come up with a judgment. However, in many of the studies, neither the

intention nor the designs of the experiments was aimed to allow for detailed cognitive interpretations of the data, until recently in single-cue learning (Bott & Heit, 2004; Busemeyer, Byun, DeLosh & McDaniel, 1997; DeLosh, Busemeyer, & McDaniel, 1997) and multiple-cue judgment (Juslin, Olsson, & Olsson, 2003). For example, even though careful assertions have been made as to how fast a task is learned it is difficult to discern which memory structures that have served the basis for such learning. Moreover, the results from the single-cue function learning literature do not give insight into how abstraction and *integration* of several cues and their relation to the criterion is mastered. The interpretation of the cognitive processes in multiple-cue judgment has nonetheless often been that the process involves consideration of one or several distinct cues (Einhorn, Kleinmuntz, & Kleinmuntz, 1979).

Thus, the cognitive basis for the sort of judgments anticipating the relation between cues and a criterion is still rather unclear. Notice that linear and additive models summarize the output of the judgment process, and do not inform us about how knowledge representations serve as the basis for these judgments. Surprisingly few studies have aimed at mapping how cognitive structures and memory representations may be involved in multiple-cue judgment (see e.g. Dougherty, Gettys, & Ogden, 1999; Juslin, et al., 2003; Olsson, Juslin, & Olsson, 2006; Olsson, Enkvist, & Juslin, 2006).

Cue Abstraction and Exemplar Memory in Multiple-Cue Judgment and Categorization

Recent research on multiple-cue judgment has aimed at identifying the cognitive basis for such judgments (Juslin, Jones, Olsson, & Winman, 2003; Juslin, et al., 2003; Olsson, Juslin, & Olsson, 2006; Olsson, Enkvist, et al., 2006). These authors have identified two different cognitive processes that are at play at the time of judgment which are different both in terms of the computations and the representations that are implied. These are *cue abstraction*: the abstraction and integration of cue-criterion relations, and *exemplar memory*: the consideration of similar examples retrieved from memory.

Cue Abstraction

In cognitive terms the more or less explicit interpretation of the data captured by the linear additive model involves the abstraction and integration of *one* or *several* abstracted cue-criterion relations (Brehmer, 1994; Cooksey, 1996; Einhorn et al., 1979; Hammond & Stewart, 2001; Juslin, Olsson, & Olsson, 2003; Juslin, Jones, et al., 2003). The assumptions are that people explicitly abstract the relationships between the individual cues and the criterion and store knowledge of the weight and sign of this relationship. At the time of

judgment, an integration of this stored knowledge is made in an additive and linear manner. This process is structurally equivalent to an additive linear process model (Einhorn et al., 1979).

Even though linear and additive models *summarize the output* of the judgment process, it may well be that in a significant proportion of multiple-cue judgment studies the actual underlying process has been cue abstraction. Moreover, in binary choice tasks (e.g. *which of these two cities has the largest population?*) where the cities are described by lists of cues, a weighted additive model of information integration predicts participants' performance in some circumstances (see for example Bröder, 2000). A simple heuristic called "*Take-the-Best*" has been observed to be an alternative cognitive process (Gigerenzer et al., 1999). Take-the-best implies that the decision process is guided by the single most valid cue that discriminates between two alternatives, a process that amounts to cue abstraction with the consideration of only one cue.

Processes similar to cue abstraction have been identified in other areas as well (often under the general term 'rule-based processes'), strengthening the viability of cue abstraction as a prominent cognitive process in multiple-cue judgment. In categorization research, it was long believed that rule-based knowledge was the basis for such decisions, representing how single features of objects in a category are related to a category label (Bruner, Goodnow, & Austin, 1956). Indeed, uni-dimensional or conjunctive rules are sometimes found to be abstracted in learning of simple category structures and applied at the time of categorization (see for example Milner, 1963; for a review see Ashby & Maddox, 2005).

Exemplar Memory

Another cognitive process identified as a possible basis for multiple-cue judgment is exemplar memory. The representational and computational assumptions associated with exemplar-based processes are well studied phenomena in cognitive psychology. The rise of exemplar theories in the 1970'ies was in part a reaction to the idea that conceptual knowledge is represented as lists with attributes that define a category (the *classical view* of concepts). It was soon appreciated that this view could not to a satisfactory extent explain the capacities of categorization that is demonstrated by the human species. Instead, exemplar theories were proposed (see Estes, 1994; Medin & Schaffer, 1978; Nosofsky, 1984).

Although different formulations of exemplar theories are amended with different assumptions, the key spirit of exemplar theory is that, when a person is confronted with the task of assessing the criterion value of a probe, the person retrieves similar previously stored exemplars, and their respective criterion values from long term memory (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000). The estimation of the probe's criterion

value is made on the basis of the retrieved criterion values. The *context model* (Medin & Schaffer, 1978) has been one of the most influential mathematical formulations of the exemplar theory. According to the context model, when a judgment is made about a probe, the judge considers the similarity of the probe to all or some of the exemplars encountered within the same and/or similar categories of exemplars. These similarities act as weights on the criterion values. A highly similar exemplar suggests a similar criterion value as the one stored together with that exemplar, whereas a dissimilar exemplar suggests a decreased impact of its stored criterion value. The weighted criterion values are then added and divided by the sum of all similarities considered (see the section below on Cognitive Modeling of Multiple-Cue Judgment for details). Although rarely explicitly discussed, but evident from the mathematical formulation (see below, and also Medin & Schaffer, 1978), this process corresponds to *additive* and *linear* integration, on the basis of exemplars.

What is similarity? Similarity according to the context model is determined by feature overlap. The specific similarity rule (see below) is such that exemplars with large feature overlap are assigned a dominating influence. For similarity computation in environments with continuous cues, the *generalized context model* (GCM: Nosofsky, 1984, 1992) is more appropriate. The similarity between a probe and an exemplar is in the GCM assumed to be a nonlinearly decreasing function of their distance in psychological space.

Exemplar theory has been applied to understand categorization learning and categorization decisions in a great variety of task circumstances (for numerous examples, see Nosofsky & Johansen, 2000). The debate has long focused on whether *one or the other* of *prototype theories* (Reed, 1972) or exemplar theories are the best description of category learning (Nosofsky & Johansen, 2000; Smith & Minda, 2000). Prototype representations involve representations of a category in terms of a prototype; an abstracted prototypical representation of the properties that are typical of the objects within the category. The prototype can also be seen as the most representative concrete exemplar within the category, but it is the abstract prototype version that has received most success in studies of categorization behavior. In this comparison, exemplar theories have received more confirmation, since most of the data that was traditionally used to support the prototype theory could also be used to support the exemplar-based theory (Murphy, 2002).

A number of attempts at applying exemplar theory to understand decision making and multiple-cue judgment phenomena exist. Smith and Zarate (1992) described how the exemplar-model could be used as a conceptual framework to understand social judgments, and complemented some of their ideas with model simulations. Social judgments are suggested to be affected by factors such as individual differences, the current social context, the self-image of the perceiver and in-group versus out-group dynamics (Smith & Zarate, 1992). The authors hypothesized that exemplar memories of previous social

encounters guide social judgments and that these factors are captured in the exemplar model by varying the attention weights. Dougherty, Gettys, and Ogden (1999; see also Dougherty, 2001 and Hintzman, 1986) formulated a model where retrieval of memory traces could be used as an explanation for decision making phenomena, including many of the heuristics and biases proposed by Kahneman and others. This model is not explicitly based on exemplar theory, but shares the general idea that memory traces are used in decision making. However, it is mainly based on simulation studies and has not yet undergone much empirical tests (but see Bearden & Wallsten, 2004 and Dougherty, 2001). Juslin & Persson (PROBEX, 2000; see also Nilsson, Olsson & Juslin, 2005) successfully demonstrated how exemplar-memory forms a cognitive basis in decision making. By implementing the generalized context model as a process model, PROBEX (PROBABILITIES from EXEMPLARS) predicted human semantic judgments (e.g. “which city is larger, A or B?”, or “what is the population size of city A?”) as well as confidence judgments. Sieck and Yates (2001) demonstrated that exemplar models could serve as good explanatory candidates for the overconfidence phenomena.

In the last years, it has been demonstrated that *both* cue abstraction and exemplar memory play viable roles in multiple-cue judgment and that certain factors can be identified which induces more reliance on the one or the other process (Juslin, Jones, Olsson, & Winman, 2003; Juslin, et al., 2003; Olsson, Juslin, & Olsson, 2006; Olsson, Enkvist, et al., 2006; the experimental design will be reviewed below). Juslin et al (2003) demonstrated that, when learning to make judgments, the nature of the feedback is one crucial factor. If the feedback is rich enough to allow detecting how the different cues are related to the criterion (i.e. the criterion is continuous) cue abstraction will be favored, but when the feedback is dichotomous (i.e. the criterion is binary) exemplar memory will be spontaneously favored. This was moreover demonstrated in Juslin, Jones et al. (2003). In the latter study it was also shown that exemplar memory might be more important when the relations between the cues and the criterion are deterministic compared to probabilistic.

In multiple-cue judgment, a psychologically plausible model with the aim of predicting a-priori when cue abstraction and exemplar memory will dominate in different situations, is still lacking. The aim of this thesis is to present a model that allows us to make such predictions (called *Sigma*) and provide empirical support for key predictions by the model (Study I-IV).

I now turn to a brief and selective review of different conceptualizations of judgment processes, related to the aim of capturing the essence of such processes. This, in turn, serves as the basis for formulating psychologically plausible assumptions on which to build a process model of judgment.

Overarching Issues: Single- vs Multiple-System Views

At least three themes relevant to a cognitive perspective on judgment can be identified: *a) the automatic vs. controlled distinction; b) multiple vs. single levels of knowledge representation; and c) different judgment strategies vs. a single mechanism*, “fed” with different amounts of knowledge. Although there has of course been some overlap in investigations addressing these and similar questions, it is still possible to discern these as three distinct but interrelated themes.

Automatic vs. Controlled Processes

The distinction between ‘automatic’ and ‘controlled’ cognitive processing comes in many shapes in cognitive psychology, and often as characteristics of modes of thought within *dual-process theories*, or *dual-system theories*. The idea is that there are two cognitive systems, or processing modes, which interact to be in command of our cognitive behavior. One of them is commonly described as holistic, automatic, “quick-and-dirty”, intuitive, associative, and similarity-based. The other one is commonly described as analytical, rational, abstract, rule-based, and controlled. After Aristoteles, William James was one of the first proclaimers of a theory of dual systems (1890/1950). Empirical thinking vs. reasoning was under the scope of his analysis. “Whereas the merely empirical thinker stares at a fact in its entirety, and remains helpless, or gets ‘stuck’, if it suggests no concomitant or similar, the reasoner breaks it up and notices some one of its separate attributes”. (p. 957). In contemporary research, this idea has been elaborated within the fields of social- and personal psychology (e.g. Epstein, Pacini, Denes-Raj, & Heier, 1996; Smith & DeCoster, 2000), and in parallel within the field of cognitive psychology (e.g. Hahn & Chater, 1998; Logan, 1988; Schacter & Tulving, 1994; Shiffrin & Schneider, 1977; Sloman, 1996; Smith, Patalano, & Jonides, 1998). The research has been focused on showing evidence for this dissociation, the factors promoting one or the other processing mode, as well as their operation relative to each other. The general claim of two different processing modes has been argued repeatedly and the term “dual processes” has been widely applied to a number of topic areas (for two comprehensive reviews, see Evans, in press; Smith & DeCoster, 2000).

However, Gigerenzer and Regier (1996) published a critique of the general notion of “dual systems”. They argued that dual process accounts of cognition are seldom defined specific enough to allow for scientific examination. They discussed whether it really can be empirically established that an associative system competes with a rule-based system to control the response in a given situation (cf. Sloman, 1996) instead of, for example, different forms of rule-based processes.

The automatic vs. controlled distinction has its origin in the seminal work on attentional capacities introduced by Shiffrin and Schneider (1977). By studying visual search performance it was observed that when a target in a display was consistently mapped to a certain response, after large amounts of training the search-and-respond process became automatized. These automatic processes are characterized by being especially fast, requiring little effort and allowing for parallel search of the display. To the contrary, when the target's mapping to a response varied from trial to trial, controlled search processes were needed to perform well. The controlled processes are said to be comparably slower, requiring more effort, and search is modulated by serially attending to the items in the display. Later, the authors have implemented their ideas in a computational model, suitable for explaining the distinction also at higher levels of cognition and elaborated on the neurological bases for such a distinction (see e.g. Schneider & Chein, 2003). In a review aiming at an evaluation of the neuroscientific status of the automatic vs. controlled debate, Birnboim (2003) concludes that there is still considerable descriptive power in the distinction, although it still remains to fully elucidate the underlying mechanisms.

In terms of the distinction between "intuition" and "analysis" in judgment and decision making research the term "intuition" is used rather widely, and has been defined in a number of different ways. According to *Encyclopædia Britannica* intuition is: "in philosophy, the power of obtaining knowledge that cannot be acquired either by inference or observation, by reason or experience. As such, intuition is thought of as an original, independent source of knowledge, since it is designed to account for just those kinds of knowledge that other sources do not provide. Knowledge of necessary truths and of moral principles is sometimes explained in this way." As defined by Robin Hogarth "...the essence of intuition or intuitive responses is that they are reached with little apparent effort, and typically without conscious awareness. They involve little or no conscious deliberation." (Hogarth, 2001; p. 14).

In the domain of judgment and decision making a dual systems perspective has been suggested by several researchers (among others; Hammond, 1996; Kahneman & Frederick, 2002; Klein, Cosmides, Tooby, & Chance, 2002; Reyna, 2004; Stanovich, 1999; Tversky & Kahneman, 1983). Tversky and Kahneman (1983; see also Kahneman & Frederick, 2002) distinguished between two systems in decision making, one intuitive and one analytical. It is suggested that many decision making situations will cue the intuitive system, but with deliberate influence of the analytical system occasional errors can be avoided. In fact, the essence of their research program has been to observe the "errors of intuition". Not all intuitive judgments are wrong, Kahneman and colleagues identified situations where the analytical system failed to control the output of the judgment process and let intuitions with erroneous outcomes pass through.

An attempt at a dual systems perspective exists also for multiple-cue judgment theory. In an attempt to provide a cognitive description of multiple-cue judgment, Hammond proposed the Cognitive Continuum Theory (1996), which implies that judgments are based on a cognitive mode that lies on a continuum ranging from intuition, over a mix of intuition and analysis, to judgments based solely on analysis. Even if the cognitive basis for these proposed cognitive modes are rather unclear, and even though they were proposed to be on a continuum rather than discrete processes, the cognitive continuum theory acknowledges the possibility of distinct types of processing underlying multiple-cue judgments.

Related to the dual systems view, but not always discussed with the same terminology or even within the same research community, is the distinction between *explicit* and *implicit* memory and learning. Although probably not strictly dichotomous, and although explicit and implicit cognition might both contribute to behavior even within the same task, authors have aimed at dissociating these different types of cognition. Explicit and implicit *memory* has been defined by Schacter and colleagues (and similarly by others) as: “Implicit memory is revealed when performance on a task is facilitated in the absence of conscious recollection; explicit memory is revealed when performance on a task requires conscious recollection of previous experiences” (Graf & Schacter, 1985, p. 501). Typical studies promoting this distinction have used word-stem completion tasks in which previously studied words are produced more often than they are explicitly remembered.

The distinction between explicit and implicit *learning* (see e.g. Reber, 1993) has in large been studied independently of the above distinction, despite probable theoretical and empirical overlap. It is suggested that there are two dissociable learning systems. One involves the explicit learning of material, that is, learning that is accessible to cognitive control, knowledge that can be communicated to others. Implicit learning is the “silent knowledge” required presumably without explicit involvement and without insight into what has been learnt. Empirical support for this distinction has repeatedly been provided by, for example, Knowlton and colleagues (Knowlton, Ramus, & Squire, 1992) when showing that patients with amnesia, demonstrating damage to the explicit memory system, can perform as well as normal controls in different memory tasks.

As Pothos (2007) points out, “the relation between implicit learning and implicit memory is unclear” (p. 229) but that “there is consensus in that implicit memory and implicit learning both involve the influence of knowledge on a cognitive process without conscious activation of this knowledge” (p. 229).

However, serious doubts on the distinction between explicit and implicit cognition are continuously published (see e.g. Lagnado et al., 2006; Shanks & St. John, 1994; Wilkinson & Shanks, 2004). The main objection to the alleged

dissociation is that the tests used to measure whether there is conscious awareness of the learnt material is often too insensitive and many times do not capture what the participant might actually know.

Knowledge Organization and Representation

The second issue relates to whether there are one or several different types of knowledge representations that underlie judgment behavior. This issue has gained special attention in categorization research. To date, Nosofsky and colleagues are perhaps the most prominent advocates of a single-system view of categorization. For the sake of scientific parsimony they argue that instead of inventing models with several different “modes” it is more fruitful to investigate if one single framework can be used to explain behavior. Indeed, the authors have shown how, with a few amendments and adjustments, the exemplar-based framework can explain perceptual categorization behavior in a wide variety of circumstances (Nosofsky & Johansen, 2000).

However, in categorization research there are reasons to doubt that a single representational system can subserve behavior. That separate memory systems are involved in categorization decisions is an idea consistently reinforced (Ashby & Maddox, 2005; Erickson & Kruschke, 1998; Juslin, et al., 2003; Nomura et al., 2007; Poldrack et al., 2001; Smith et al., 1998). Proponents of multiple representations argue that there are reasons for us to be able to store different types of knowledge. We sometimes need to be able to recall a specific instance guiding our judgments; sometimes we need the facts and arguments giving us that judgment, allowing us to communicate our knowledge to other people. It is argued that no single system for storing and retrieving knowledge can meet these functional demands. Today there exist a number of models of categorization learning that attempts to provide a mechanism for multiple systems in categorization. *COVIS* (Competition Between Verbal and Implicit Systems; Ashby, Alfonso-Reese, Turken, Waldron, 1998) is a neuropsychological theory of category learning. Two separate systems, with different neural correlates, are supposed to compete with each other when category boundaries are learned. It is proposed that we have a bias for rules, in that the verbal system will be more strongly activated than the implicit, in the beginning of learning. Another example is *RULEX* (RULE-plus-EXception model; Nosofsky, Palmeri, McKinley, 1994). In category learning, people are proposed to be learning simple dimensional rules. As soon as an exception to those simple rules is experienced, it will be stored in memory. *ATRIUM* (Erickson & Kruschke, 1998) is an extension of the connectionist model *ALCOVE* with a rule module. A gating mechanism serves to decide whenever the rule-based or exemplar-based module is to provide the correct answer for a categorization task. These models are examples that attempts to acknowledge how different knowledge representations serve as input to a categorization

decision. However, it can be discussed whether the specific mechanism governing the interplay between the different representations, minimization of judgment error on a trial-by-trial basis, is a psychologically plausible mechanism in all circumstances. Moreover, to date, these models are not explicit with any mechanisms for predicting a priori when one or the other representation will come to dominate categorization decisions.

Data perhaps most challenging to a single-system view of categorization are the findings that different neural correlates have been suggested to be recruited as a factor of the characteristics of different categorization tasks (see Ashby & Maddox, 2005 for a review; Nomura et al., 2007).

Different Strategies or One Single Mechanism?

How do we know if people use different judgment strategies and why they use them? Regarding the binary-choice decision-making paradigm (e.g. “which of these cities has the largest population?”), a challenging view is held by Gigerenzer and colleagues (1999), proposing that people have a tool-box equipped with “fast-and-frugal” heuristics. The use of these heuristics is triggered by the structure of the environment. The heuristics are different in terms of the computations they imply. It has recently been proposed that people are able to learn which of these different strategies that should be selected in different task settings (Rieskamp & Otto, 2006).

Proponents of a single strategy view of decision making, on the other hand, posit that the evidence for an adaptive tool-box is scarce (see e.g. Newell & Shanks, 2003). They question the idea that there are different strategies available when making judgments. Instead they propose an “adjustable spanner hypothesis”; decisions are based on different *amounts* of information instead of different strategies (Newell, 2005). It is proposed that sequential sampling of information can explain behavioral data equally well as a model assuming that different strategies are implemented (Lee & Cummins, 2004; Newell, 2005). Although interesting as such, strong empirical tests of such a unified model is still needed before it can be viewed as a plausible account of multiple-cue judgment data.

These lines of research appear to tackle issues that are related to each other. Concerning the relation between automatic and controlled processing and knowledge representations it is tempting to associate controlled/explicit processes to rule-based representations and automatic/implicit processing to exemplar-based representations (Hintzman, 1986; Logan, 1998). However, the extent to which exemplar memory is completely implicit can be questioned; authors have argued that exemplar memory and explicit recognition rests on the same underlying representations (see e.g. Nosofsky & Zaki, 1998). Moreover, the majority of work in the decision making domain seem to assume rule-based representations as the major input to the processes, since the

research has long concerned how people use cues when making judgments (Brehmer, 1994; Cooksey, 1996; Gigerenzer et al., 1999; Hammond & Stewart, 2001; Lee & Cummins, 2004; Newell, 2005), failing to acknowledge other representational formats.

In sum; these issues appear important and intertwined in different ways. The cognitive process model of multiple-cue judgment that will be proposed in this thesis takes its stand in relation to these issues.

The Additive and Sequential Character of Controlled Thought

What other characteristics of the cognitive architecture are worth highlighting when considering a cognitive model of multiple-cue judgment? There is extensive support in diverse fields of psychology for the pervasiveness of *additivity* as a feature of cognitive processing. Research on mental arithmetic in children lends support for a predisposition for additive information integration. Repeated addition has found empirical support as the process underlying children's multiplication behavior (Fischbein, Deri, Nello, & Marino, 1985). Moreover, while addition is demonstrated to involve both a calculation algorithm and retrieval of an answer from memory, multiplication is dominated solely by retrieval of declarative chunks stating the product as a fact (Roussel, Fayol, & Barrouillet, 2002; c.f. Andersson, 1983).

Also in the judgment and decision making literature the additive nature of processing has been stressed. First, in N.H. Anderson's approach to understand judgment behavior (*Information Integration Theory*; 1981), judgments are described as the act of integrating relevant information from the environment. This integration is supposed to follow simple algebraic rules, where empirical data supports different versions of additive functions as the most prominent integration rule. Second, the so often demonstrated good fit of linear multiple regression models to multiple-cue judgment (i.e. through the lens model framework) demonstrates that judgment data is well described by linear and additive weighting of the cues (Brehmer, 1994; Cooksey, 1996; Hammond & Stewart, 2001). Third, exemplar models (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000) have been successful models for describing categorization learning and to some extent judgment (Dougherty, Gettys, & Ogden, 1999; Juslin et al., 2003; Juslin & Persson, 2001; Smith & Zarate, 1999). Exemplar models encompass an additive and linear integration of retrieved criterion values (see Study I).

Likewise there is extensive support for the *sequential* nature of controlled thought processes. In Anderson's Information Integration Theory (1981), for example, the idea is that people integrate information in the environment in a sequential way; "In everyday life, information integration is a sequential process. Information is received a piece at a time and integrated into a continuously evolving impression. Each such impression, be it of a theoretical

issue, another person, or a social organization, grows and changes over the course of time. At any point in time, therefore, the current impression looks both forward and back.” (p. 144).

Another example is the *anchoring-and-adjustment heuristic* proposed by Tversky and Kahneman (1974; see Gilovich et al., 2002) which is based on the idea of adjustment from an anchor point. It has been demonstrated that, if people are to make an estimate, some of the information serves as the anchor in that the produced response is inappropriately affected by the value of the anchor point. The anchoring-and-adjustment heuristic has been used to explain biases in judgment.

A third example is the *belief adjustment model*, proposed by Hogarth and Einhorn (1992). In the belief adjustment model, based on anchoring-and-adjustment, judgments are supposed to be updated sequentially as new pieces of evidence are taken into account. This model has been used successfully to predict order effects in judgment (e.g. primacy and recency effects).

There are few published studies that explicitly challenge the observation that controlled integration of information is additive and sequential. In terms of area judgments (i.e. judging the area of a rectangle on the basis of height and width) Norman Anderson (e.g. Anderson & Cuneo, 1978) hypothesized that children added height and width, whereas adults integrated height and width correctly by multiplying them. However, judging the perceived area of a rectangle has inherent perceptual qualities to it, not necessarily involving controlled mental calculation of Height x Width. Gigerenzer has challenged the cognitive algebra approach represented by Anderson (1981) by observing that the perception of an area of a rectangle doesn't always follow an algebraic integration rule at all (Gigerenzer & Murray, 1987; Gigerenzer & Richter, 1989).

Parallel aspects of cognition seem restricted to cognitive processes involving automatic integration of information. Perceptual integration of information may well occur in parallel at a pre-decisional stage. For example, perceptual and attentional processes involved in *visual search* are suggested to involve parallel components. A visual search experiment typically involves a feature search, where the task is to as quickly as possible respond whether there is an item in the display that exhibits Feature X. It is typically compared with a conjunction search, where the task is to respond to items containing Feature X *and* Feature Y. Conjunction searches in general produces larger response times. The parallel components involved seem restricted either to the identification of single features of items in the display (Treisman, 1988) or to the separation of targets from distracters (see e.g. Palmer, Verghese & Pavel, 2000).

Some theories of attention also assume parallel processing components (e.g. Shiffrin & Schneider, 1977). However, to date there exists convincing evidence for a 'cognitive bottleneck' in attentional processes, referring to the controlled decisional stage of an attentional task (e.g. Pashler, Johnston & Ruthroff, 2001;

Sigman & Dehaene, 2005). Parallel attentional processing is thus allowed at the perceptual and motor components of task execution, while there is a strict bottleneck prohibiting more than one response selection decision to be formed at a time. Hence, a dual task experiment almost always induces performance impairment unless one of the tasks can be carried out automatically, is rather easy, or involves a dissimilar sensory modality (e.g. Eysenck & Keane, 2005).

In regard to memory retrieval, items retrieved from memory are presumably the result of parallel activation of different sources of information (Anderson, 1983). However, this does not stand in opposition to the idea that controlled elaboration of those memories must occur in a serial manner (Anderson, 1983; Baddeley, 1986).

In sum; there appear to be strong arguments for considering the additive and sequential nature of controlled thought processes as the basis for a cognitive process model of multiple-cue judgment.

Neural Correlates to Human Judgment

Cognitive neuroscience is important for the understanding of cognitive functions, and can be used as a tool when testing cognitive theories. To date, there are few attempts to study the underlying neural correlates to multiple-cue judgments. However, different aspects of decision making and categorization are witnessing an extensive growth of studies addressing neural correlates.

Neural Correlates to Decision Making

The study of decision making from a neural perspective have mainly concentrated on aspects such as gambling, reward, and decisions under risk and uncertainty. Prefrontal cortex has been established as the most important brain area for decision making (see Krawczyk, 2002 for a review). The prefrontal cortex can in itself be divided into subregions, where three of them are identified as especially important for diverse aspects of decision making. In tasks where reward and punishment are prominent characteristics, such as in gambling situations, the *orbitofrontal cortex* (OFC) is important. The prominent role of this area for tasks involving reward and punishment has been explained by the OFC providing the emotional links to the decision, since there are numerous connections between OFC and the striatum. Influential work in this respect has been made by Damasio and colleagues (e.g. Bechara, Damasio, A.R., Damasio, H., & Anderson, 1994). Their *somatic marker hypothesis* suggests how emotional influences help in choosing between decision alternatives. Also, OFC has been proposed to be especially important for implementing decisions on the basis of prior knowledge and thus to avoid impulsivity. A second important prefrontal area for decision making is the *dorsolateral prefrontal cortex* (DLPFC). This region is commonly highlighted as

important for working memory functions, and its role in decision making is thus understood as involved in decisions where conscious consideration and manipulation of information is crucial. Moreover, DLPFC has been identified as important in decisions with large uncertainty. The third prefrontal area associated with decision making is the *anterior cingulate* (AC). The role of AC in decision making is suggested to be that of conflict resolution between different options and also of monitoring the decision process to detect if changes are necessary (Krawczyk, 2002).

The striatum, especially the dorsal part (see e.g. Balleine, Delgado, & Hikosaka, 2007 for an overview) is increasingly being identified with various decision making competencies. Foremost, the dorsal striatum is important for encoding action-outcome associations and to anticipate the expected reward value of an action available for choice.

Neural Correlates to Categorization

Research on the conceptually similar, but distinct, research field of categorization has witnessed a growth of studies addressing what neural systems that underlie categorization learning and categorization decisions. One idea is consistently reinforced, which is that different memory systems support category learning, both in a competitive manner as learning progresses within a task (see e.g. Poldrack et al., 2001) but also as a factor of the characteristics of the task (see Ashby & Maddox, 2005 for a review; Nomura et al., 2007). Early work in the categorization domain investigating the neural patterns of rule-based and similarity-based categorization decisions was made by Smith, Patalano, and Jonides (1998; see also Patalano, Smith, Jonides, & Koeppe, 2001). They used artificial animal stimuli divided into two categories: living on Venus or on Saturn. The authors showed that, when explicitly instructing participants to follow a rule-based strategy, activation registered by PET was found in *prefrontal, posterior parietal* and *occipital cortex*, as well as in subcortical structures (*thalamus and cerebellum*). The authors also showed that, when explicitly instructing participants to follow a similarity-based strategy, activation was found in *occipital cortex (BA 17, 18, 19)* and *right cerebellum*.

It has been suggested that different neural systems support category learning depending on the specific categorization task (Ashby & Ell, 2001; Ashby & Maddox, 2005). From the outset at least three systems have been suggested as possible contributors to category learning. First there is a *frontal-striatal* network found associated with *rule-based categorization tasks*. Rule-based categorization tasks are constructed to allow good performance through application of rather simple uni-dimensional rules. A typical such task is one where considering the length of a line stimulus is sufficient to categorize the stimuli into one of two categories (Ashby, Alfonso-Reese, Turken, & Waldron, 1998). Rule-based categorization tasks generally are found to involve prefrontal

cortex (Milner, 1963; for a review see Ashby & Maddox, 2005). There are also a number of studies supporting *basal ganglia* involvement in such tasks. Ell, Marchant and Ivry (2006) have recently found that patients with basal ganglia damage to the *left putamen* are impaired during the first part of learning in a rule-based categorization task.

Moreover, it has been suggested that a *perceptual representation system*, involving *sensory cortex*, can be associated with categorization in the *prototype distortion task* (Aizenstein, MacDonald, Stenger, Nebes, Larson, Ursu & Carter, 2000; Ashby & Maddox, 2005, Reber, Stark, & Squire, 1998). A typical prototype distortion task involves a dot-pattern prototype. The category structure is created by distorting this prototype by moving a few of the dots. An additional number of exemplars are created solely by random distribution of the dots, irrespective of the prototype. The task is to learn to classify the different patterns as belonging or not to the prototype distorted category (e.g. A or not-A). Since visual discrimination is crucial in the prototype distortion task, the perceptual representation system is proposed to mediate learning (Aizenstein et al., 2000; Reber et al., 1998).

Third, the *basal ganglia* has been found to be especially involved in tasks where several dimensions need to be integrated at a 'pre-decisional' stage, referred to as *information integration tasks*. An example of an information integration task is quadratic stimuli boxes varying on the background color dimension (blue or yellow), and three dimensions characterizing symbols depicted on the quadratic boxes (one or two symbols; squares or circles; green or red). A blue background, a green symbol, and a symbol number of 2 are each assigned a value of +1, the other features 0. The rule determining category memberships is: "the stimulus belongs to category A if the sum of values on the relevant dimensions is >1.5; otherwise it belongs to category B" (Ashby & Ell, 2001, p. 206). This integration rule is assumed to be too complex to mediate conscious abstraction and integration of the dimensions, and hence some of the computational properties of basal ganglia, for example coding stimulus-response associations, are argued to be suited for learning in these tasks.

An experimental design used to differentiate individual item learning and abstract rule learning has been to contrast the activity changes within (reflecting instance-based learning) and across (reflecting abstract rule-learning) training blocks (Doeller et al., 2006; Fletcher et al., 1999). Fletcher et al. (1999) observed that a right fronto-parietal network was engaged in individual item learning of artificial grammar strings, presumably reflecting the retrieval of declarative memories (Cabeza & Nyberg, 2000) while left PFC was engaged in learning across blocks. Doeller et al. (2006) found that within-block learning of object-position conjunctions of objects produced increased activity in prefrontal cortex and striatum and decreased activity in hippocampus. Across-block learning was not associated with this modulation of hippocampus activity.

In sum; cognitive neuroscience of decision making have highlighted prefrontal cortex as involved in important aspects of decision making, whereas cognitive neuroscience of categorization have emphasized that different task characteristics will invite different memory systems; fronto-striatal, sensory and basal ganglia systems being those most frequently addressed. Thus, cognitive neuroscience appears promising when aiming at a deeper understanding also of multiple-cue judgment.

RESEARCH OBJECTIVES

The scope of the studies reported in this thesis is to present the first steps toward a cognitive theory of multiple-cue judgment that allows a-priori prediction of what cognitive processes that dominate judgments in a given situation. The aim is also to test the most crucial predictions of the theory; people will adapt to different task structures by adopting a representational input that is appropriate for the task at hand. In the four studies presented, this prediction is assessed in a number of ways. In Study I the outlines of *Sigma* - a unified theory of judgment, is presented and the empirical focus is on the relation between the cues and the criterion in a task with binary cues. In Study II, the focus is on a more complex task with continuous cues, where the cue-criterion functions as well as the combination rule is manipulated. In Study III, the results of Study I together with other results supporting, as well as not supporting, adaptive shifts are reviewed to discuss the nature of adaptations to task structure. In Study IV, the aim is to test the predictions by Sigma with brain imaging methodology; do two different tasks, by Sigma claimed to involve two qualitatively distinct cognitive representations, induce different neural patterns?

IDENTIFICATION OF PROCESSES IN MULTIPLE-CUE JUDGMENT

A possible limitation with the early studies of multiple-cue judgment studies was that the authors were not always addressing what cognitive processes that gave rise to the judgments and the experiments were not explicitly designed to capture memory components (see for example Carrol, 1963). The “new” line of multiple-cue judgment research represented by, among others, the present research group (e.g. Juslin et al., 2003) attempt at filling this gap by combining methods that can distinguish between cognitive processes including psychological experiments, cognitive modeling and, more recently, brain imaging. Combined, these methods should prove useful when aiming to discern whether a-priori predictions of judgment processes and representations

are possible. Below, the three chosen strategy identification methods are described.

Experimental Design: a Model Judgment Task

In the past years a special experimental design has proven useful for distinguishing between different processes in a multiple-cue judgment task (cf. Juslin, et al., 2003). The conceptual design of the judgment tasks used in Study I-IV is virtually the same. By having the participants go through a learning phase where outcome feedback is provided, learning of a judgment task is enabled. By considering the judgments made for new untrained exemplars, introduced for the first time in a test phase, it is possible to discern what process that has been the output of participants learning. To what extent do participants use abstract knowledge about the cue-criterion relations and to what extent do they use an alternative process of exemplar memory; a tendency to judge a probe according to the similarity to exemplars encountered in training (Medin & Schaffer, 1978)?

The fictitious cover story of the task has varied. One version is to judge the toxicity of a lethal bug and another to judge the effectiveness of a herb as a medical treatment to a virus on a continuous scale. In each task there are four cues that vary either binary (has/has not a specific trait; Study I, III and IV) or (pseudo-)continuous (a scale ranging from 0-10; Study II). In the binary version, a total of 16 different cue combinations are possible (Table 1). In the continuous version the total number of possible combinations is 11^4 . A random error is added to the criterion in some of the experiments, yielding a probabilistic relation between cues and criterion and transforming the discrete values in Table 1 into a continuous scale.

The idea of the design is as follows: in a training phase the participant is exposed to a constrained set of the original total amount of exemplars. In the binary version, this means that five of the 16 exemplars are omitted from training. In the test phase that follows, these omitted exemplars are introduced. The processes we are interested in distinguishing provide different predictions concerning the judgments of these new exemplars.

With cue abstraction, the basic prediction is that there will be no systematic differences in performance between old and new exemplars in the test phase. Abstract knowledge about the cue-criterion relations should be equally applicable to old and new exemplars, within the same task. With exemplar memory, on the other hand, a systematic difference in performance between old and new exemplars is expected. Old exemplars can benefit from the fact that close matches are available for recall. This benefit is not the case for new exemplars, for which a judgment is dependent on the similarity to the exemplars previously seen. This difference becomes even easier to detect when this experimental design is combined with cognitive modeling.

Table 1

Example structure of the experimental task design. The table shows 16 different exemplars, their cue values, their additive criterion, and whether they were seen in training or introduced at test

Exemplar #	Cues				Criteria Add	Role
	C_1	C_2	C_3	C_4		
1	1	1	1	1	60	<i>E</i>
2	1	1	1	0	59	<i>T</i>
3	1	1	0	1	58	<i>T</i>
4	1	1	0	0	57	<i>O</i>
5	1	0	1	1	57	<i>N</i>
6	1	0	1	0	56	<i>N</i>
7	1	0	0	1	55	<i>N</i>
8	1	0	0	0	54	<i>T</i>
9	0	1	1	1	56	<i>O</i>
10	0	1	1	0	55	<i>O</i>
11	0	1	0	1	54	<i>T</i>
12	0	1	0	0	53	<i>T</i>
13	0	0	1	1	53	<i>T</i>
14	0	0	1	0	52	<i>T</i>
15	0	0	0	1	51	<i>T</i>
16	0	0	0	0	50	<i>E</i>

Note: E = extrapolation exemplar, T = training exemplar, N = interpolation exemplar (new exemplar in the middle region of the distribution) and O = old matching exemplar (old exemplar in the middle region of the distribution, used for comparison with the new interpolation exemplars)

Cognitive Modeling of Multiple-Cue Judgment

The judgment task just described allows us to distinguish what cognitive process and representation that has dominated. Mathematical formulations of cognitive theories are a powerful method to test theories in cognitive psychology and cognitive science (Polk & Seifert, 2002). To be able to test assumptions of distinct cognitive processes in the present multiple-cue judgment task we use two widely assessed mathematical formulations of the rule-based and exemplar-based cognitive processes; the *cue abstraction model* (Einhorn, Kleinmuntz, & Kleinmuntz, 1979) and the *exemplar-based model* (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000). These mathematical formulations (models) give rise to different predictions for the judgments on the new exemplars introduced at test.

The Cue Abstraction Model

The assumptions of the cue abstraction model (CAM) are that people explicitly abstract the relationships between the individual cues and the criterion and store knowledge of the weight and sign of this relationship. At the time of judgment, an integration of this stored knowledge is made in an additive and linear manner. This process is structurally equivalent to an additive linear process model (Einhorn et al., 1979). To make a judgment of a criterion \hat{c} the process of cue abstraction is described as:

$$\hat{c} = k + \sum_{i=1}^4 \omega_i \cdot C_i \quad (1)$$

where k is the intercept, and ω_i is the weight of the cue C_i . Hence, in the cue abstraction model, the process is one of additive and linear integration on the basis of *cues*.

The Exemplar Model

The assumptions of the exemplar-based model (EBM) are that, when a person is confronted with the task of assessing the criterion value of a probe, the person retrieves similar previously stored exemplars, and their respective criterion values, from long term memory (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000). The similarity of the probe to the exemplars retrieved from memory is calculated. The estimation of the criterion value is made on the basis of the retrieved criterion values.

The similarity $S(p, x_j)$ between a probe p and a retrieved exemplar x_j is given by:

$$S(p, x_j) = \prod_{i=1}^4 d_i \quad (2)$$

where d_i is an index that takes value 1 if the cue values on cue dimension i coincide (i.e., both are 0 or both are 1), and s_i if they deviate (i.e., one is 0, the other is 1). The four s_i parameters in the interval $[0, 1]$ capture the impact of deviating cues on the overall similarity $S(p, x_j)$. s_i close to 1 implies that a deviating feature on this cue dimension has little impact on the perceived similarity and is considered irrelevant. s_i close to 0 means that the similarity $S(p, x_j)$ is close to 0 if this feature is deviating, assigning crucial importance to the feature.

For the similarity process in the continuous cue version of the task, the distance between a probe p and an exemplar j is,

$$d_{pj} = h \left[\sum_{m=1}^4 w_m |x_{pm} - x_{jm}| \right] \quad (3)$$

where x_{pm} and x_{jm} , respectively, are the values of the probe and an exemplar on cue dimension m , the parameters w_m are the attention weights associated with cue dimension m , and h is a sensitivity parameter (changed from the usual c to avoid confusion with the criterion c) that reflects overall discriminability in the psychological space. Attentional weights vary between 0 and 1 and are constrained to sum to 1. The similarity $S(p, x_j)$ between a probe p and an exemplar j is assumed to be a nonlinearly decreasing function of their distance (d_{pj}),

$$S(p, x_j) = e^{-d_{pj}}. \quad (4)$$

The criterion estimation process is described as:

$$\hat{c} = \frac{\sum_{j=1}^4 S_j \cdot c_j}{\sum_{j=1}^4 S_j} \quad (5)$$

where c is the criterion to be estimated, S_j is the similarity, and c_j is the stored criterion. The output of the equation is the weighted sum of the criteria of similar exemplars, where the similarities are the weights. In the exemplar model, the process is hence one of additive and linear integration on the basis of *exemplars*.

Quantitative Predictions from the Models

These two models imply distinct predictions in the judgment task previously described (see Figure 2). In addition to the additive task described above, a multiplicative task is used in the studies reported in this thesis. The reason for that will be explicated shortly.

The cue abstraction model predicts judgments that are a linear and additive function of the cues (Figure 2, Panel A). Consider the five new exemplars withheld from the training phase and introduced no sooner than in the test phase (Table 1). The new extreme exemplars (#1 and #16 in Table 1) that were introduced in the test phase are given judgments that are more extreme than the judgments for the most extreme old exemplars. For example, the old

exemplar with the highest criterion in the binary task has the values [1, 1, 1, 0] on the four dimensions and a criterion value of 59 (Table 1). In the training phase, the participant may have learned the sign and weight of each of the cues, for example that the fourth cue implies a small increase in the criterion. In the test phase, a person utilizing a cue abstraction process retrieves the “rules” for each cue dimension, concerning the weight and sign of its relationship with the criterion, and produce a judgment stating that [1, 1, 1, 1] has a higher criterion value than [1, 1, 1, 0].

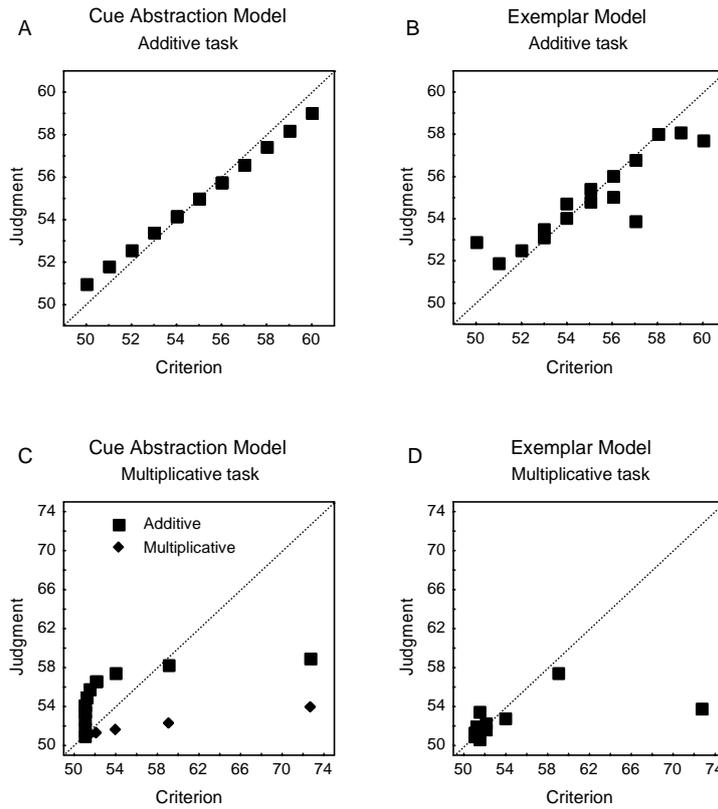


Figure 2. Illustration of model predictions. Panel A: Additive cue-abstraction model (Eq. 1) in the additive task with noisy weights typical of data ($\omega_1=3.2$, $\omega_2=2.4$, $\omega_3=1.6$, & $\omega_4=.8$). Panel B: Exemplar model (Eq. 2 and 5) in the additive task with all similarity parameters $s_i=.1$ (the choice of all $s_i=.1$ is arbitrary and used to compute illustrative predictions). Panel C: Additive and multiplicative cue-abstraction models (the latter is motivated below) in the multiplicative task with noisy weights ($\omega_1=3.2$, $\omega_2=2.4$, $\omega_3=1.6$, & $\omega_4=.8$). Panel D: Exemplar model in the multiplicative task with all $s=.1$.

The exemplar based model, on the other hand, predicts an inability to extrapolate beyond the range of training, since the judgments are based on similarity to previously encountered exemplars (Figure 2, Panel B; Eq. 5). When in a test phase a person is confronted with an exemplar with cue values [1, 1, 1, 1] and a criterion value of 60 (Table 1), no “rules” concerning the relationship between the individual cues and the criterion have been formed, hence the judgment cannot be based on integration of such rules. The judgment is instead based on the perceived similarity between the extreme exemplar and the old exemplars, and the retrieval of previously stored criterion values. Since all stored criterion values are between 51 and 59 (the binary task, Table 1) no other criterion value can be retrieved from memory and integrated into a judgment. Despite the apparent success of these models in diverse fields of cognitive psychology, there is limited knowledge of how they can be a-priori predicted to guide multiple-cue judgment.

Functional Magnetic Resonance Imaging

Brain imaging techniques have proven useful for suggesting what cognitive mechanisms that might be at play in decision making and categorization (see above) and should thus be useful as a complement to behavioral experiments and cognitive modeling. When investigating questions about how cognitive functions relate to the brain there are several different techniques available. One of the most widely used is fMRI (functional Magnetic Resonance Imaging). This technique is often preferred over others since it offers a reasonable trade-off between spatial and temporal resolution (i.e. the ability to provide reliable measures both in space and in time), and also because of its applicability to a wide variety of experimental designs (Huettel, Song & McCarthy, 2004). With fMRI one measures the activity changes in a working human brain by considering the changes in blood oxygenation over time. The brain needs blood to be able to function. The blood contains both glucose and oxygen which are important for the metabolism. The consumption of blood will increase in areas where there is more activity than in a resting state. While a participant performs a cognitive act, the change in blood flow is recorded. Deoxygenated hemoglobin (i.e. hemoglobin that *is not* attached to oxygen) gives rise to a stronger magnetic signal than hemoglobin that is oxygenated (i.e. that *is* attached to oxygen). This difference in magnetic signal is recorded with fMRI through application of strong magnetic fields in close proximity to the brain. By this logic, it is possible to get spatial and temporal information of how the brain works in relation to cognitive tasks.

EMPIRICAL STUDIES

Study I – Information Integration in Multiple-Cue Judgment: a Division-of-Labor Hypothesis

Outline of Sigma

In Study I we propose a unified model of judgment - Sigma. Models trying to summarize behavior in different cognitive domains exist, for example Mental Models Theory of reasoning (Johnsson-Laird, 1999) and Problem-Space Theory of problem solving (Newell & Simon, 1972). General frameworks summarizing behavior in judgment and decision making also exist (Gigerenzer, Todd, and the ABC-group, 1999; Gilovich, Griffin, & Kahneman, 2002; Hammond & Stewart, 2001). However, detailed cognitive theories of judgment are surprisingly few (see Busemeyer et al, 1997; Dougherty, Gettys, & Ogden, 1999).

In Sigma we claim that it is not a mere coincidence that linear and additive models fit judgment data well (Brehmer, 1994; Cooksey, 1996) and that exemplar models involve linear and additive integration of exemplars (Medin & Schaffer, 1978; Nosofsky, 1984; Nosofsky & Johansen, 2000). It is an actuality that stems directly from a proposed additive and sequential structure of our cognitive architecture; when performing spontaneous integration of information we are predestined to *add* or *subtract* information. Controlled processes are generally believed to be distinct from automatic processes (Schiffrin & Schneider, 1977) in that the former are serial, require attention, are slower than automatic processes, and are of limited capacity. Controlled processes should thus not just be limited in terms of the amount of information that can be handled or in terms of time pressure (Gigerenzer et al, 1999; Payne; Bettman & Johnson, 1988); they are also limited when it comes to *how* information is processed.

In the concrete implementation of Sigma provided in Study I it is assumed that the judge considers *implications* δ_n ($n=1\dots N$) of N pieces of evidence to estimate a criterion. The estimate \hat{c}_n after considering piece of evidence n is,

$$\hat{c}_n = \hat{c}_{n-1} + \eta_n \cdot (\delta_n - \hat{c}_{n-1}), \quad (6)$$

where η_n is the *importance* attached to the n th piece of evidence, \hat{c}_{n-1} is the *previous estimate* and δ_n is the *implication* by the newly considered piece of evidence. Eq. 6 is the algorithm level description of the judgment process (Marr, 1982). This equation, through mathematical transformation, is

identical to both the cue abstraction model and the exemplar model (see the Appendix for a numerical example).

When Sigma is driven by cue abstraction the importance attached to each newly considered cue is the predictive weight of the new cue relative to cues considered up to iteration n , as given by

$$\eta_n = \frac{\omega_n}{\sum_{n=1}^n \omega_n}. \quad (7)$$

Further, the implication of the newly considered cue, when Sigma is driven by cue abstraction is

$$\delta_n = \begin{cases} \delta_{\max} & \text{if } C_n = 1 \\ \delta_{\min} & \text{if } C_n = 0 \end{cases}. \quad (8)$$

where δ_{\max} refers to the maximum admissible estimate (e.g., 60 in Table 1; “this cue goes with a high criterion”), and δ_{\min} to the minimum admissible estimate (e.g., 50 in Table 1; “this cue goes with a low criterion”). The limits of the range (δ_{\max} & δ_{\min}) may be known a priori before the task or may be the most extreme values encountered thus far.

When Sigma is driven by exemplar memory the importance attached to each newly retrieved exemplar is the similarity of this exemplar relative to the similarity of all exemplars considered so far, as given by

$$\eta_n = \frac{S_n}{\sum_{n=1}^n S_n}. \quad (9)$$

The implication of the newly considered exemplar, when Sigma is driven by exemplar memory is

$$\delta_n = c_n, \quad (10)$$

the criterion stored with the exemplar considered at iteration n .

The structure of the process as suggested in Sigma through Eq. 6 is central to the argument put forward with this model. First, it implies that the judgment process only involves consideration of two real (or potential) estimates of a criterion in succession. This is psychologically plausible since it adheres to the capacity constraints ascribed to controlled processes (e.g. Schiffrin & Schneider, 1977), to executive functions and working memory (Cowan, 2000; Miller, 1956) and also because it is in line with previous formulations for similar phenomena (Anderson, 1981; Gilovich et al., 2002; Hogarth & Einhorn, 1992).

The consequence of the ability to consider only two real or potential estimates at a time is that the process of cue abstraction becomes strongly biased towards detection of linear slopes between the cues and the criterion. Comparing two similar judgment cases that differ on only one cue enables the detection of that cue's linear relation to the criterion. For example, when inferring the value of the blood pressure of two patients, where the body weight is the only important deviating cue between the patients, the doctor may infer that the body weight causes the difference in blood pressure. Or, in relation to the specific task used in this thesis; sequential consideration of two bugs, differing only in the characteristics of the nose, might enable detecting that a changed nose is associated with a linear change in the criterion of the numerical value 2 (see Figure 3 and Table 1).

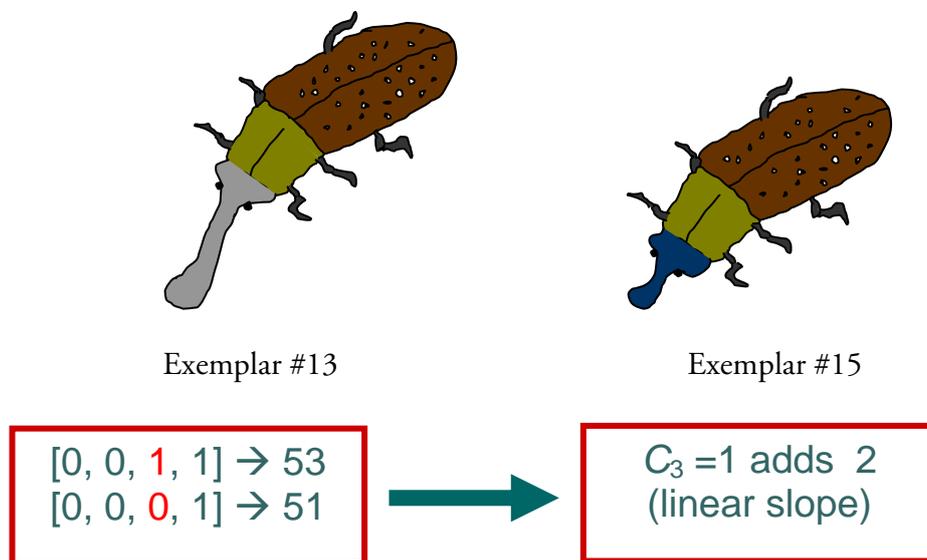


Figure 3. Two examples of the fictitious Death bug, differing only by the nose, enables inferring the linear slope imposed by a change in that cue. If cue # 3 is the only difference between the bugs, this implies that the impact of this cue on the criterion is the difference in the criterion associated with the different cue profiles: $53-51=2$.

If the judgment process is characterized as linear, additive, and sequential, only allowing the successive consideration of two estimates at a time, as suggested by Sigma (Eq. 6), this implies that non-linear relations between a cue and the criterion by necessity can not be inferred with cue abstraction. Detecting non-linear relations would demand simultaneous consideration of at least three different values. For example, in a bivariate plot with only two pairs of x-y values, it is impossible to detect a nonlinear relationship between the two variables. Hence, this implies that inferring the slopes between the cues and the

criterion in judgment tasks where the cue-criterion function is *not* well predicted by a linear function is extremely difficult.

The second reason for why the structure of the process as suggested in Sigma through Eq. 6 has important implications is that it implies successive *addition* or *subtraction* of the linear adjustments ($\eta_n \cdot (\delta_n - \hat{c}_{n-1})$). Thus, the sequential adjustment based on linear slopes can only faithfully emulate additive cue-combination.

We propose that, since there is no doubt that judgment tasks we encounter in our everyday life abound with many different cue combination rules, evolution has equipped us with means to cope also with cue-criterion relations that can not successfully be approximated by an additive and linear function of the cues (for example non-additive and non-linear cue-criterion relations). In Sigma we propose a division of labor between different cognitive representations that allows us to be adaptive and to utilize *exemplar memory*, as the task shifts from being additive to being, for example, non-additive, non-linear, or having inter-correlated cues. The ability to memorize exemplars for later use in judgment is less dependent on the possibility to infer linear slopes and the cue-combination rule. Memorizing exemplars could thus be done independent of the cue-combination rule and thus allow for a flexible complementary learning mechanism.

In sum; the suggested judgment process structure implied by Sigma outlined above conforms to a linear, additive and sequential consideration of information. If the task allows successful performance by inferring the linear slopes between cues and criterion, cue abstraction will dominate the judgments. In other situations, exemplar memory will act as a powerful back-up system. This simple and general model should now allow predictions about when exemplar memory or cue abstraction underlies multiple-cue judgment.

Specific Aim and Results of Study I

Study 1 consisted of three experiments, where the a-priori predictions about cognitive representations in judgment were tested. The main manipulation was contrasting judgments in an *additive task* where the cues combined by an additive function, and a *multiplicative task* where the cues combine by a multiplicative function.

In Experiment 1 we hypothesized that in the additive condition participants would be able to approximate the linear slopes between the cues and the criterion and hence cue-abstraction would be the dominating process. In the multiplicative condition, on the other hand, we hypothesized that participants would not be able to infer the non-additive cue-combination rule, nor would they be able to integrate the cue-criterion knowledge in a non-additive manner. Therefore they would be forced to use their back-up system of exemplar memory to be able to perform well.

Additionally, we tested the hypothesis that, if a distracter of working memory limits the number of informational pieces that can be considered in the judgment process, a dual-task setup in the test phase would affect performance in the additive condition more than in the multiplicative condition. This would be because an exemplar-based process can produce an accurate answer on the basis of just one retrieved exemplar, while a cue-abstraction process demands the integration of several pieces of information. By introducing a working memory distracter task we predicted lower performance in both conditions and also possibly a shift from using cue abstraction to more reliance on exemplar memory, since this process would be less consuming on working memory.

Eighty participants were randomized to each of the two conditions. For additional details concerning the task the reader is referred to Study I. The experiment consisted of a training phase followed by two test phases, one test phase with and one without a working memory distracter. The task was to judge the toxicity of a number of fictitious Death bugs on a continuous range, as described above. Experiment 1 was also extended with a replication with a deterministic criterion (i.e. no random error was added to the criteria provided as feed-back) and equated criterion variance.

The mean judgments in Experiment 1 (Figure 4) reveal that participants in the additive condition have been able to extrapolate their judgments, giving their most extreme responses to the new items at the endpoint of the continuum. On the other hand, in the multiplicative task, great inabilities to extrapolate are shown, indicating that the participants have been using an exemplar-based process.

Performance was better in the additive than in the multiplicative condition when comparing across all exemplars in the test phase, but almost equal when only comparing performance on old exemplars (Panel 4 C). This, together with a more negative *exemplar-index* (a combined measure of ability to extrapolate and interpolate to new exemplars in the test phase, see Study I, p. 16) in the multiplicative condition suggests that in the multiplicative task exemplar-memory was induced (Panel 4 D). In the additive condition the results suggests that cue-abstraction has dominated the judgments. A more negative exemplar-index in the additive condition opposed to a more positive exemplar-index in the multiplicative condition was the result of the working memory distracter (Panel 4 D). Performance in the test phase was negatively affected by the working-memory distracter in both the additive and the multiplicative task.

In Experiment 1, the results from the model fit reveals a poor fit for the cue abstraction model in the multiplicative condition, but a good fit in the additive condition. On the other hand, the exemplar based model fits data best in the multiplicative condition.

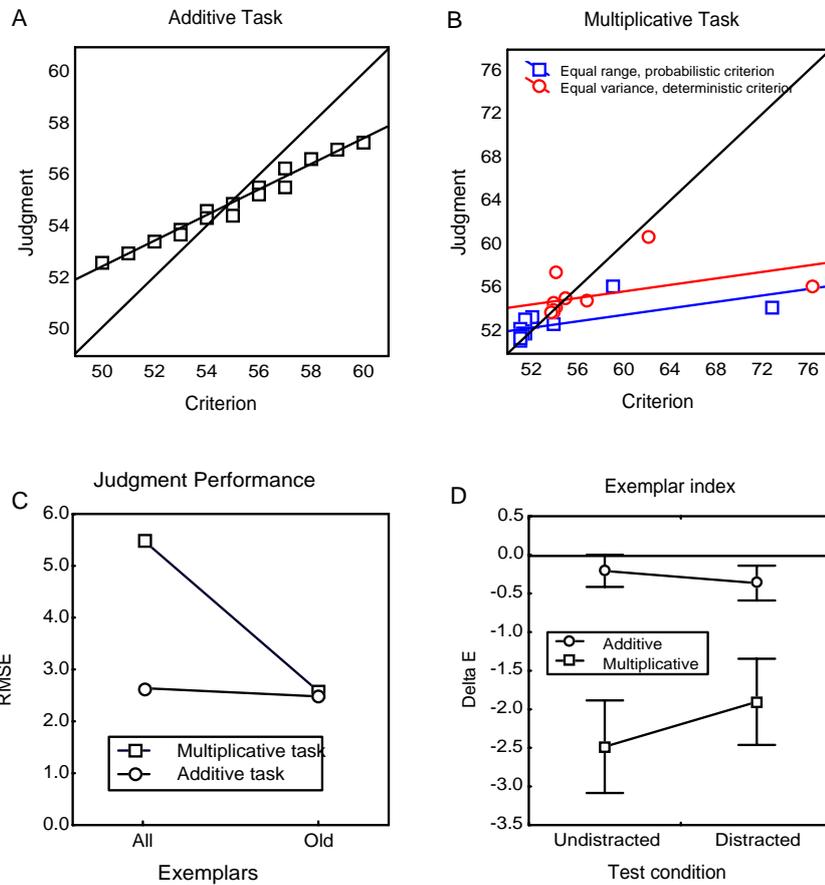


Figure 4. Panel A: Mean judgments in the additive task test phase of Experiment 1 plotted against the criterion with the best-fitting regression line. Panel B: Mean judgments in the multiplicative task test phase of Experiment 1 (“equal range, probabilistic criterion”). The data for “equal variance, deterministic criterion” refers to a replication, as detailed in Study 1. Panel C: The Root Mean Square Error (RMSE) of judgment for old exemplars and all exemplars in the additive and the multiplicative tasks. Panel D: The mean exemplar index ΔE (see Study I, p. 16) for the additive and the multiplicative tasks with undistracted and distracted test phases with 95% confidence intervals.

In Experiment 2 we aimed at verifying that the ability to induce cue-criterion relations by controlled processes in training in the manner specified by Sigma is a crucial difference between the tasks. Sigma implies that cue abstraction involves explicit controlled processes that attempt to identify the cue-criterion relations during training (Brehmer, 1974). Thus, situations

facilitating the identification of the linear slopes between the cues and the criterion should affect learning positively in tasks where cue abstraction is the favored process. In Experiment 2, two differently manipulated training sequences were used to test this prediction. In one, referred to as the *controlled sequence*, exemplars were presented in a sequence where only one cue changed from trial to trial, thus maximizing the chances of inferring the linear slopes. In the other, referred to as the *confounded sequence*, exemplars were presented in a sequence where the number of changing cues from trial to trial was maximized. The hypothesis was that in the additive task learning should be faster in the controlled than in the confounded sequence if cue abstraction is the favored process. On the other hand, in the multiplicative task, if exemplar memory is the dominating process, there should be no such pattern. If anything, the controlled sequence could hinder learning since it could invite futile attempts at cue abstraction.

Forty-eight undergraduate students participated; half of the participants encountered the additive task, half the multiplicative task. The task was exactly the same as in Experiment 1 except that the presentation order of the exemplars in the training phase was manipulated. In the training phase, half of the participants in each task received a *controlled sequence* and half of them received a *confounded sequence*.

In the first two blocks of training (each block consisted of 11 judgments), learning in the additive task is *facilitated* by the controlled sequence, while learning in the multiplicative task is *impaired* by the controlled sequence (Figure 5). This suggests that distinct processes have been at play when trying to learn in the two tasks. The results from the test phase of Experiment 2 replicate the results from Experiment 1: the mean judgments, exemplar-index, and model fits clearly indicate that cue abstraction dominates in the additive task while exemplar memory dominates in the multiplicative task.

In Experiment 3 we investigated the generalizability of knowledge gained in an additive versus a multiplicative task. If participants were specifically, yet implicitly, trained to acquire the appropriate knowledge concerning the underlying cue criterion rules and mode of cue integration in the two tasks, would they be equally able to learn this in an additive and a multiplicative task? By constructing a training phase where the criterion value given by only two cues at a time were shown to the participants, and after this letting the participants generalize their knowledge in a test phase with full (four cue) exemplars, this would reveal whether knowledge consistent with the combination rules had been achieved. Twenty-four undergraduate students participated, and were randomized between the two conditions. The bugs were shown as written propositions on the screen. In a training phase, the task was to judge the amount of toxicity that two cues contributed to the criterion. In a test phase the task was to judge the toxicity of full exemplars.

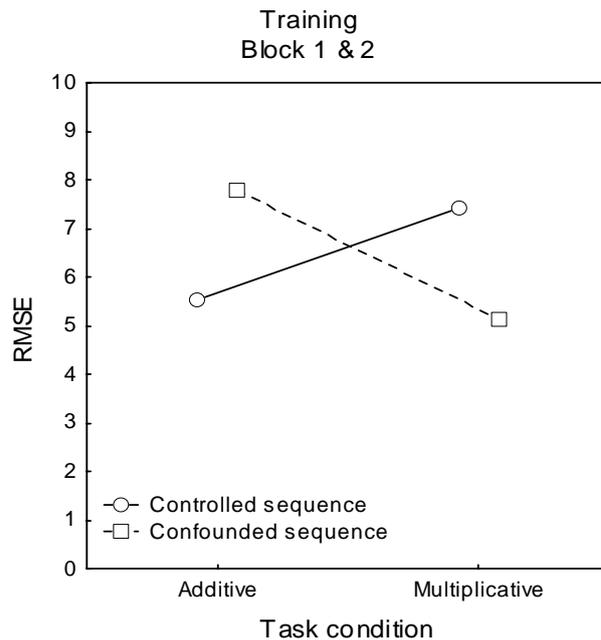


Figure 5. Performance in the first two blocks of training in Experiment 2 measured by Root Mean Square Error (RMSE) between judgment and criterion for the additive and the multiplicative tasks, and for the controlled and confounded sequence conditions.

In Experiment 3, the results strongly suggest that inferring the linear slopes between the cues and the criterion and hence conform to a cue abstraction process was accentuated when the training phase in the additive task was constructed so that the task was to learn to judge the contribution by two cues to the criterion and the test phase demanded judgments of exemplars with all four cues present. Importantly, the results verify that this was not helpful in the multiplicative task, since the participants were not able to abstract knowledge consistent with the multiplicative cue-combination rule. While participants were performing almost equally well after the training phase, their ability to generalize the information differs to a great extent (Figure 6). Even though the judgments in the multiplicative test condition (Panel D) are a positive function of the criterion and even though the range in the multiplicative task is much wider than in the additive task, the most extreme judgments in Panel D are at best the result of a mere addition of whatever cue-criterion knowledge that was gained in training.

Discussion

The hypothesis of a division of labor between distinct cognitive processes in multiple-cue judgment is supported by the experiments in Study I. In

Experiment 1, participants in the additive condition conformed to a cue-abstraction process. In the multiplicative condition, however, cue abstraction was not a viable alternative, and the participants appear to have employed exemplar-memory. Abstraction and non-additive integration of the impact of cues (cue abstraction) was not a viable alternative in environments where the cues were not combined in an additive and linear manner.

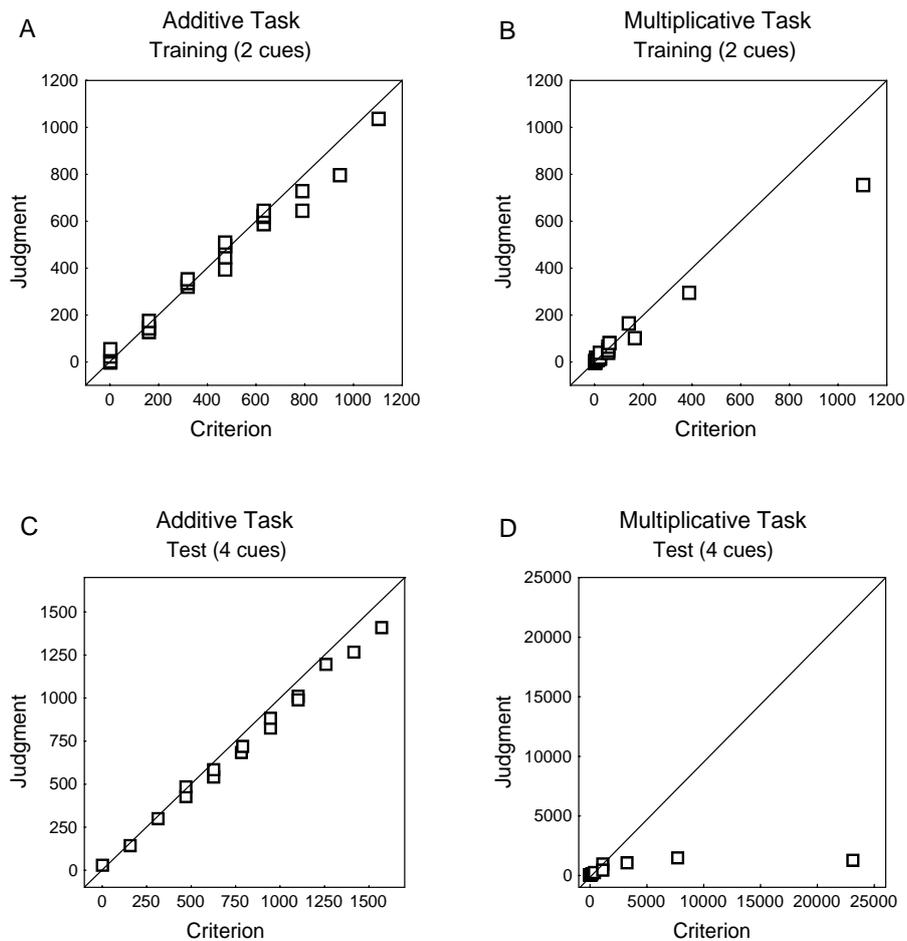


Figure 6. Mean judgments plotted against the criterion. Panel A: the training phase with two cues in the additive task of Experiment 3. Panel B: the training phase with two cues in the multiplicative task of Experiment 3. Panel C: the test phase in the additive task of Experiment 3. Panel D: the test phase in the multiplicative task of Experiment 3.

In Experiment 2, there was an effect of the sequence in which exemplars were presented in training. In the additive task, learning was facilitated by presenting the exemplars in a sequence where only one cue differed from trial to trial, since this presumably encouraged the detection of the linear slopes between the cues and the criterion. This manipulation did not facilitate learning in the multiplicative task because in a multiplicative task the “slope” for one cue always depends on what other cues are present.

In Experiment 3, the results indicate that, even when facilitating the acquisition of knowledge consistent with the underlying combination rule, participants in the multiplicative task fail to abstract knowledge appropriate for generalization to complete exemplars. Since no proper abstraction has been done, no proper integration of information in the test phase could be made. Experiment 3 thus provides a strong support for Sigma.

The working memory distracter impaired performance in both the additive and the multiplicative task in Experiment 1. This was interesting, because it implies that exemplar-memory, as well as cue abstraction, involves working memory and controlled cognitive processing to some extent.

Study II – Adaptive Changes between Cue Abstraction and Exemplar Memory in a Multiple-Cue Judgment Task with Continuous Cues

In Study II we constructed a task more compatible with what is a standard multiple-cue judgment task. Each cue dimension in the model task described above varied on a continuous dimension (0-10) as compared to a binary dimension (0-1; the bugs) and they were presented as written propositions on the screen. There were two main aims with this study. First, to replicate the results from Study I, Experiment 1, where initial support for the division of labor-hypothesis was found, with this more complex task. Is a suggested interplay between exemplar memory and cue abstraction manifested also in a task with continuous cues? Such task could possibly be more inviting of cue abstraction, since it could be more meaningful to consider linear slopes between cues and criterion with continuous cues, when the change in a cue provides more information than a change on a binary cue. Also, a task with continuous cues by necessity involves more possible exemplars. Is exemplar memory still a viable process? The second aim was to manipulate the direction of the cue-criterion function to allow investigation of how people were able to learn the cue-criterion function forms. The majority of empirical data on the issue of learning cue-criterion function forms stems from experiments where only one cue-criterion relation has to be learned (Klayman, 1988; Koh & Meyer, 1991; Slovic & Lichtenstein, 1971). How does this relate to abstraction and integration of multiple cues with different function forms?

In Study II we manipulated the combination rule to be either additive or multiplicative. We also manipulated whether the cue-criterion functions were linear with positive slopes or linear with two cues having positive and two cues having negative slopes. The hypothesis was that in an additive task, whether the cue-criterion functions are all positive (*homogeneous* condition) or whether two of them are positive and two negative (*heterogeneous* condition) there should be evidence for cue abstraction. Cue abstraction should however be more complex in the heterogeneous condition, possibly leading to a performance decrease. In a multiplicative task, this manipulation should have less, since one benefit with exemplar memory is its relative independence of the underlying task structure.

Thirty-two participants participated in the experiment. The task design was roughly the same as in Study I. In a training phase with 300 trials the participants were to judge the effectiveness of a herb as a medical treatment to a lethal virus on a continuous scale ranging from 510-590. Outcome feedback was provided. In a test phase with 44 trials without feedback, exemplars in the extreme ends of the continuum were introduced, as well as some new exemplars in the interpolation range.

Additionally, a replication of the multiplicative task was conducted. The extreme values in the test phases differ as a consequence of the different cue-combination rules. Thus, it can be speculated that an inability to extrapolate is in part due to the higher extreme value in the multiplicative task, through a possible reluctance to give such a high answer. In the replication the high extreme value exemplars at test was exchanged with exemplars constrained to have similar criterion as the extreme exemplars in the additive task ($c=600$).

The results from Study II indeed suggested a qualitative shift between cue abstraction and exemplar memory when the underlying function was shifted from additive to multiplicative, even in this more complex task. Figure 7 shows the mean judgments in this task. In the additive conditions the judgments were a positive linear function of the criterion. In the multiplicative task, inability of extrapolation were evident. Performance was generally poorer in the heterogeneous than the homogeneous conditions.

Importantly, performance on old exemplars was similar in the two tasks, but performance on new exemplars was poorer in the multiplicative task, as expected if exemplar memory is involved, producing systematically poorer judgments for new exemplars. Extrapolation ability was in Study II measured as the proportion of judgments at test falling outside of the training range. An ability to extrapolate beyond the range of training was evident only in the additive homogeneous condition.

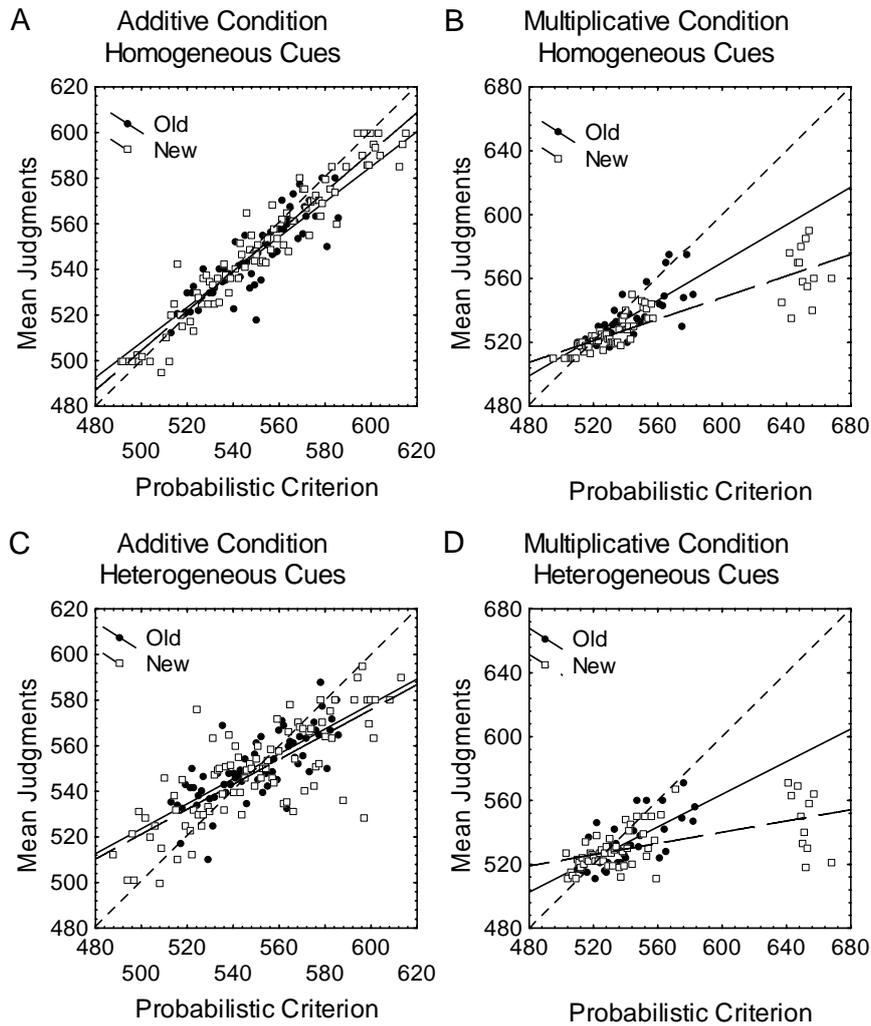


Figure 7. Mean judgments plotted against the criterion for the different conditions. Panel A: additive, homogeneous. Panel B: multiplicative, homogeneous. Panel C: additive, heterogeneous. Panel D: multiplicative, heterogeneous. Best-fitting regression lines are based on a) the old exemplars seen in training or b) the new exemplars introduced at test.

The interaction between the cue abstraction model and the exemplar model hypothesized by Sigma was significant; in the additive tasks the cue abstraction model fits data best, in the multiplicative tasks the exemplar model fits data best. In the multiplicative heterogeneous task, performance was very poor, hence the model fit is difficult to interpret.

The results from the replication of the multiplicative task with the extreme high test exemplar constrained to have a criterion of 600 replicated the results from the original experiment. Importantly, the interaction between

performance on old and new exemplars, and additive (the original data) and multiplicative task was significant even in the replication.

Also in a more typical multiple-cue judgment task, where the cues varied continuously and were presented as written statements, did we find support for the predictions made by Sigma. When the cues are related to the criterion with an additive function participants are able to abstract the underlying relations and rely on cue abstraction. When the cues are related to the criterion with a multiplicative function, participants rely on exemplar-memory. Interestingly, the cue abstraction model does not provide an equally poor fit to this multiplicative task as it did in the task in Study I. One interpretation is that more participants in the continuous multiplicative task was caught in a period of active but futile attempts at hypothesis testing and thus learning with exemplar memory was hindered. Indeed, it is fair to expect that there exist limiting conditions for when a change from cue abstraction to exemplar memory will be observed (Olsson, Enkvist, & Juslin, 2006). Presumably, if it exists a “rule-bias” (Ashby et al., 1998) and as long as cue abstraction provides a good approximation of the task (Dawes & Corrigan, 1974), adoption of exemplar memory may not be considered worth the effort.

Initially, it was expected that heterogeneous cue directions would affect performance negatively in the additive task but not in the multiplicative task. Notably, however, there was no such significant interaction in terms of performance at test. Presumably, supposing that some participants persisted at using CAM as an approximation to the multiplicative task in both conditions, those participants in the heterogeneous condition would possibly perform worse than those doing the same thing in the homogeneous condition.

Additional support exists for the adaptive change between representations demonstrated in Study II (Juslin & Henriksson, 2007). The lens modeling framework, originally implementing a cue abstraction process, was amended with an exemplar version and fitted to the training data of this study. The results show that the exemplar version fits the data better at the end of the multiplicative training phases, while the original cue abstraction implementation fits the data better at the end of the additive training phases.

Study III – Exemplar-Based Inferences in Multiple-Cue Judgment – Contingent, not Automatic, Strategy Shifts?

Study III is a review that aims to discuss the notion of strategy shifts in multiple-cue judgment, and to frame the available data under a tentative, perhaps speculative, but yet plausible interpretation. To what extent should learning to perform in a multiple-cue judgment task be considered involving gradual learning and updating of knowledge, or involving conscious and

controlled judgments of the ability to create stable representations of the ecology? The review focuses on a set of experiments conducted in our lab that provide evidence for representational shifts in multiple-cue judgment, but also experiments that demonstrate circumstances where this shift fails to materialize.

In Juslin, Olsson, & Olsson (2003) three experiments were presented that demonstrated that if the feed-back in the task, described above under Study I, is binary (i.e. the criterion for the bug is “harmless” or “dangerous”) compared to when the feed-back is continuous (i.e. “how poisonous is this bug?”) different processes will dominate the judgments (Experiment 1 & 2 in Juslin et al., 2003). The binary task was dominated by exemplar-memory, but in the continuous version, half of the participants used cue abstraction. Moreover, if the continuous task was probabilistic it invited more cue abstraction than if it was deterministic (Experiment 3, Juslin et al.).

In Juslin, Karlsson, and Olsson (in press; Study I) three experiments verified that if the cue-combination rule was additive, cue-abstraction was induced, whereas if it was multiplicative exemplar-memory was induced. This adaptive choice of representational input to the judgment process was verified also when the training sequence was manipulated, and with regard to generalizability (see previous section on Study I).

In Olsson, Enkvist, & Juslin (2006) a hypothesized representational shift failed to materialize. Sigma predicts that if the cue-combination rule is non-linear it is not possible to abstract and integrate the cues in a manner following the correct function, as has been verified with a non-additive cue-combination rule. Hence, in a non-linear version of the task described above, exemplar memory should be the dominating process. However, with a number of different manipulations, in Olsson et al. this shift did not occur. If the task was deterministic or probabilistic did not affect learning much, neither did an attempt to facilitate learning by presenting the rare exemplars more often. Most surprisingly, a doubling of the amount of training trials, from 220 to 440, did not help. Only when the participants were explicitly instructed to rely on exemplar memory did their learning of the task improve.

The interpretation of the data reviewed in Study III is that the shift between cue abstraction and exemplar memory is strategic and contingent upon early perceived learning performance. In line with the existence of a general “rule-bias” (Ashby et al., 1998; Brehmer, 1994; Juslin, et al., 2003), cue abstraction will be the favored initial process of the majority of participants approaching the non-linear tasks in Olsson, et al., (2006). Participants will attempt at creating stable representations of the cue-criterion relations. Soon, participants will realize that attempts at inferring the non-linear slopes between cues and criterion do not enable them to perform well in the non-linear task (as predicted by Sigma, Study I). Hence, they will shift and try to memorize exemplars instead. However, early in learning this might also prove problematic, since if only a few exemplars have been stored in memory, one

criterion can have been stored together with rather dissimilar exemplars, as given by the non-linear structure of the task. Participants might revert back to try cue abstraction again, because their attempts at creating stable exemplar representations early in learning were not fruitful. This can effectively have hindered learning in the non-linear tasks in Olsson, Enkvist and Juslin (2006). The data is thus suggested to be in line with a *strategic* interpretation of representational shifts.

This interpretation is in opposition to other ways of conceiving of what happens during learning in related task paradigms. Influential accounts of strategy shifts either suggests a gradual accumulation of individual instances that ultimately come to dominate the response output (Logan 1988) or suggests shifts as a side-effect of experience in a general trial-by-trial aim of minimizing error (see e.g. Ashby et al., 1998; Erickson & Kruschke, 1998). Additionally, it has been proposed that learning in multi-attribute decision making is the gradual appreciation of the information confronting the judge, without any reference to a strategy and/or representational shift (Lagnado et al., 2006; Lee & Cummins, 2004; Newell, 2005). It is difficult to see how any of these proposals can a-priori predict the data reviewed in Study III, and especially the failure to learn in the non-linear task.

The issue needs further research before stable conclusions can be drawn. But the idea at least implies that error-driven learning models may not provide a good account of data in all situations. The interpretation moreover implies that utilization of exemplar memory can be effectively controlled, and is thus less in line with the ideas of Logan (1988) which suggest that a memory trace will inevitably be laid down and available for use at a later time.

Study IV – Distinct Neural Correlates Underlie Multiple-Cue Judgment Depending on the Cue-Combination Rule

A theory that allows detailed predictions concerning the cognitive processes at play in different judgment tasks offers a possibility to predict the neural substrates in different tasks, as a function of the underlying cue-combination rule. In Study IV we wanted to investigate whether the additive and the multiplicative task gave rise to different neural substrates and how these substrates were related to what one would expect with the different processes proposed; cue abstraction and exemplar memory. A strive in this direction should furthermore be a complement to the role that psychological experiments and cognitive modeling plays in order to learn more about the underlying cognitive mechanisms in judgment.

With this intent, identifying the processing components that differ between cue abstraction and exemplar memory should be useful. First of all, as proposed

by the division-of-labor hypothesis, per Eq. 6 above, after training there should be non-trivial similarities in process structure given the proposed additive and sequential judgment process. Thus, neural correlates to the processing components subserving the consideration of the previous estimate, the importance and the implication of the newly considered piece of evidence should be identified. Prefrontal cortex especially the dorsolateral prefrontal cortex (DLPFC), has been demonstrated to be largely associated with working memory aspects of decision making (Krawczyk, 2002) and should thus be a candidate area for common activity across the tasks.

In terms of differences, the division-of-labor hypothesis posits differences in the knowledge representations that serve as input to the process. Cue abstraction should demand utilization of working memory related areas to a larger extent than exemplar memory. Integration of the impact of more than one individual cue relative to the criterion is demanded to be able to perform well in the task utilizing cue abstraction. This should be compared with exemplar memory, in principle enabling good performance based on retrieval of only one stored exemplar. This implies that working memory related areas, like *prefrontal cortices* and parts of the *basal ganglia* (Ashby & Maddox, 2005; D'Esposito et al., 1998; Ell et al., 2006; Milner, 1963; Patalano, et al., 2001; Smith & Jonides, 1999) should be found activated to a larger extent with cue abstraction than with exemplar memory.

Exemplar memory should demand utilization of areas related to retrieval of stored previous cases. If this retrieval can be expected to involve explicit retrieval of declarative memories, activation in frontal and temporal areas is to be expected (Cabeza & Nyberg, 2000; Doeller et al., 2006; Kramer et al., 2005; Poldrack et al., 2001). On the other hand, if this retrieval can be considered implicit, activation of areas associated with implicit recall is to be expected. Cerebellum and striatum are thus good candidate areas (Ashby et al., 1998; Bishoff-Grethe et al., 2002; Knowlton, Mangels & Squire, 1996; Maddox & Ashby, 2004; Poldrack et al., 2001).

Is the division-of-labor hypothesis supported also at the neural level, in that *a)* there exists commonality in activity patterns reflecting the proposed unity of the judgment process, *b)* an additive task and a multiplicative task induces different neural activity, And *c)* that the activity pattern induced is in agreement with what can be expected with cue abstraction and exemplar memory, respectively?

Additionally, we tested whether there was a main effect of exemplar type (old vs. new) and an interaction; are trained and untrained exemplars treated differently at the neural level, as a function of the process induced?

In a within subjects design participants learned both the additive and the multiplicative version of the task in Study I, Experiment 1. The design was exactly the same, with binary cues and training- and test phases. Two different cover stories were counterbalanced across the tasks, one involving the toxicity

of the bugs, the other the effectiveness of different herbs, presented as written propositions instead of pictures. If pictorial stimuli would have been used it would have been unavoidable that the stimuli would have differed considerable in terms of perceptual features. By using propositions the perceptual similarity was maximized, and so avoided that any neural differences observed between the tasks were pertaining to superficial characteristics of the stimuli. For logistic reasons, training and test were administered on one day, and a second test phase was administered with an event-related fMRI-design the day after.

The results verified predictions by the division-of-labor hypothesis. The behavioral data supported that cue abstraction was more involved in the additive task (although influences of exemplar memory were also evident), and exemplar memory was involved more than cue abstraction in the multiplicative task. The imaging data also gave evidence for distinct processes in the two tasks. An ANOVA revealed that while *frontal cortex* and *putamen* were more engaged in the additive task, *cerebellum* and the *tail of the caudate* were more engaged in the multiplicative task (Figure 8; see Study IV for additional details concerning the results).

There was also a main effect of old and new exemplar type revealing six differentially activated areas. More involved for old exemplars were the *posterior cingulate*, the *parahippocampal gyrus*, *middle/inferior temporal cortex* and *anterior cingulate*. More involved for the new exemplars were *inferior frontal cortex*, and *superior temporal gyrus*.

Additionally, frontal (right *middle frontal cortex*, left *middle frontal cortex*, *BA 9/48*, *BA 44* and left *inferior frontal cortex*) and visual (right *BA 17* and left *BA 18*) areas showed common activity between the two tasks as evident in a conjunction analysis (Nichols et al., 2005).

The imaging results for the additive task; frontal areas and putamen activity is commonly associated with working memory and rule-based processes (Ashby & Maddox, 2005; D'Esposito et al., 1998; Ell et al., 2006; Milner, 1963; Patalano, et al., 2001; Smith & Jonides, 1999). This interpretation is in line with the possibility that the controlled integration of knowledge of how several cues relate to the criterion consumes more working memory resources than exemplar memory, possibly involving fewer steps of information integration.

The interpretation of the imaging results obtained in the multiplicative task enforces an implicit account of exemplar memory. One interesting interpretation is that the retrieval of exemplars appear to be mastered by implicit memory retrieval, as evident in the activation of cerebellum and the tail of caudate, often associated with implicit processes (Ashby et al., 1998; Bishoff-Grethe et al., 2002; Knowlton, Mangels & Squire, 1996; Maddox & Ashby, 2004; Poldrack et al., 2001). Especially the tail of the caudate is interesting, since it is included as a prominent component of the implicit category learning system suggested by Ashby et al. (1998)¹. These findings are

surprising and offer a challenge to prominent notions of exemplar memory as being related to recognition memory (e.g. Nosofsky & Zaki, 1998).

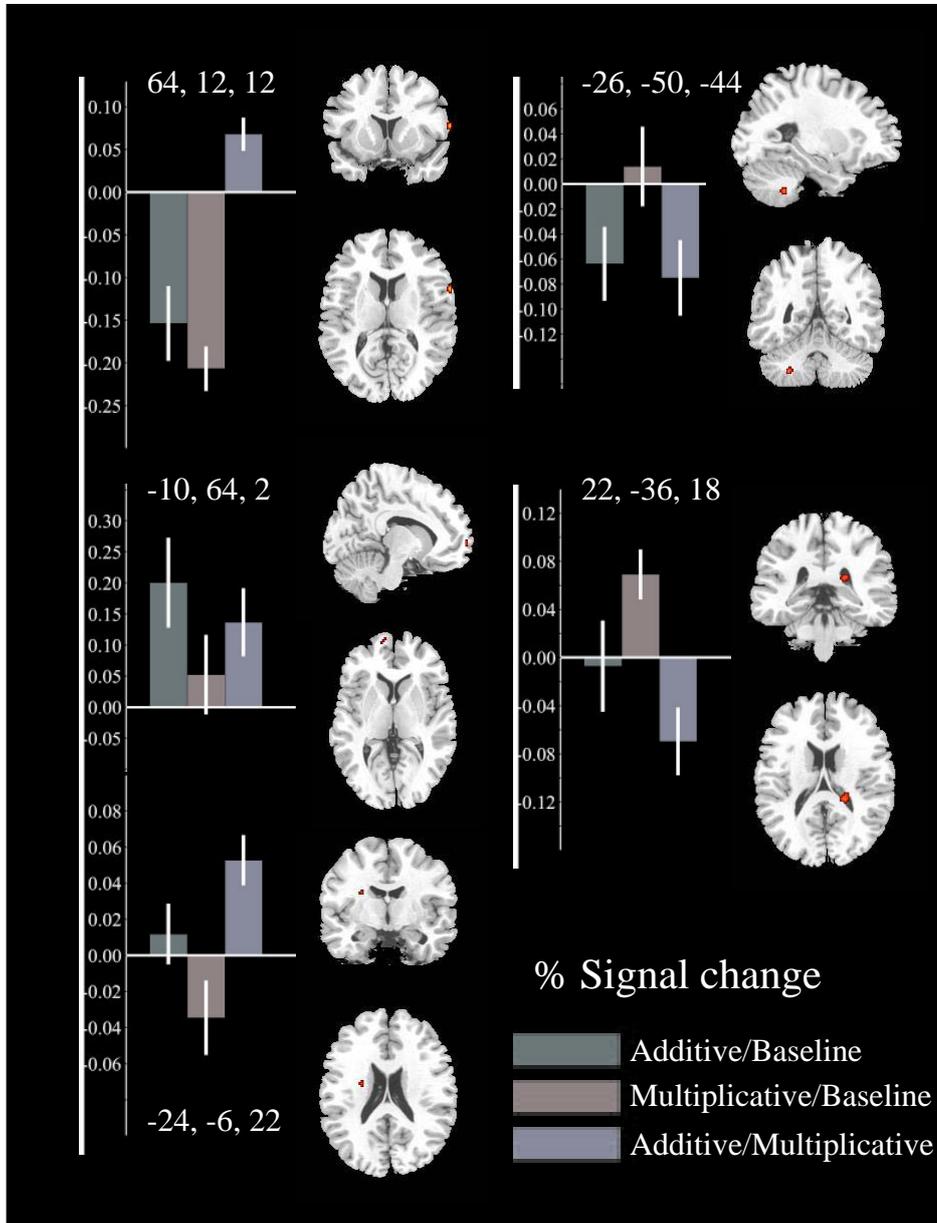


Figure 8. Percent signal change and MNI-coordinates for areas more activated in the additive than in the multiplicative task (left part) and in the multiplicative more than in the additive task (right part) as evident as a main effect of task in the ANOVA; $p < 0.001$; cluster size $k > 4$. Green staples = Additive/Baseline, brown staples = Multiplicative/Baseline, blue staples = Additive/Multiplicative. Error bars denote 1 standard error from the mean.

The main effect for old vs. new exemplar type is interpreted as a beneficial memory effect for old exemplars. The areas uniquely associated with old exemplars are in the literature assigned importance in declarative memory processes (Cabeza & Nyberg, 2000; Thompson-Schill, 2003). The interaction effects were small. Presumably, old and new exemplars are treated similarly at the neural level, and it is the computational aspects of cue abstraction and exemplar memory, respectively, that give rise to the systematic differences in performance (i.e. judgment is a mean of stored criterion values with the exemplar model).

The fact that there was overlap in frontal areas, in line with the role of prefrontal cortex in decision making (Krawczyk, 2002), moreover provides tentative support that there exists a commonality in the judgment process; controlled consideration of information. The emergent interpretation of the results in Study IV is that a general judgment process structure (with its neural basis evident in the conjunction analysis) acts on different types of memory representations depending on the task: explicit working memory processing in the additive task, with cue abstraction, and implicitly retrieved memory traces in the multiplicative task, with exemplar memory. Further studies are needed to more specifically investigate these claims.

GENERAL DISCUSSION

This thesis presents convergent evidence from three different methods (psychological experiments, cognitive modeling, and brain imaging) supporting the division-of-labor hypothesis in multiple-cue judgment: people adapt to different task structures by exploiting different memory representations. Utilizing representations of the relation between cues and criterion (*cue abstraction*) has proven viable in tasks where the criterion is well-predicted by linear and additive combination of the cues. Representing knowledge in tasks where the criterion is *not* well-predicted by linear and additive combination of the cues is done by means of concrete memories of previous judgment cases (*exemplar memory*). Even though cue abstraction and exemplar memory may not represent a complete inventory of different representations used in multiple-cue judgment, the data and theory presented in this thesis have highlighted how the structure of the task determines what representations that drive the judgments.

Support for a Division-of-Labor Hypothesis in Multiple-Cue Judgment

A model of judgment with detailed cognitive processing assumptions, enabling a-priori predictions of cognitive processes, should be a contribution to the field. In Study I such model is proposed; *Sigma*. Sigma rests on the assumption that the judgment process is constrained to additive and sequential updating of a judgment estimate, and hence to consideration of only two estimates at a time. This implies that a cue abstraction process, attempting to infer the relations between the cues and the criterion, can only infer the linear slopes. Sigma predicts that in a multiple-cue judgment task where the cue-combination rule is additive, cue abstraction will dominate the judgments, but in a task where the cue-combination rule is non-additive, the slopes can not be inferred, and exemplar memory will be induced.

Indeed, the results from Study I supported these claims. In additive tasks, abstraction of the cue-criterion relations as well as integration of that knowledge consistent with additive cue-combination was evident in the judgment data. On the other hand, in multiplicative tasks, no evidence for abstraction and integration of cue-criterion knowledge consistent with multiplicative cue-combination was found. These tasks instead seem to have induced reliance on exemplar memory.

Also in a more typical multiple-cue judgment task of greater informational complexity, the main prediction of Sigma could be verified (Study II). In a task with continuous cues and a large number of different exemplars in training, evidence was found that cue abstraction had dominated judgments in an additive task and exemplar memory in a multiplicative task. The results from Study II thereby strengthen the validity of the division-of-labor hypothesis.

One limiting condition for the proposed representational shift is when the cue-combination rule in this model task is non-linear. Participants, although given as much as 440 trials to learn to judge 11 different exemplars, blatantly failed to demonstrate successful shifts to exemplar memory (Olsson, Enkvist, & Juslin, 2006). In Study III, data from our lab demonstrating shifts as well as no shifts were reviewed, and this challenge in relation to the predictions of Sigma was discussed. The reviewed results were interpreted in favor of a strategic notion of representational shifts; shifts do not always occur as a byproduct of minimizing error (e.g. Ashby et al., 1998; Erickson & Kruschke, 1998). Shifts are perhaps mediated by the process of attempting to fulfill a metacognitive need on behalf of the participant. A strategy that is initially applied is further pursued only if the judge perceives that he or she can create stable and meaningful representations of the judgment task.

Sigma predicts that there will be both common and different neural underpinnings to human multiple-cue judgment. This prediction was verified in Study IV: common frontal and visual areas were observed, possibly

implementing consideration of the new piece of evidence in light of the previous. Different neural underpinnings were also observed, as a factor of the cue-combination rule. While the additive task induced more working memory related areas than the multiplicative task (frontal cortex together with putamen) the multiplicative task instead induced more reliance on cerebellum and the tail of the caudate, areas implicated in studies of implicit cognition (Ashby et al., 1998; Bishoff-Grethe et al., 2002; Knowlton, Mangels & Squire, 1996; Maddox & Ashby, 2004; Poldrack et al., 2001). Study IV not only provided support for the division-of-labor hypothesis. It also rendered interesting insights into a neuropsychological theory of exemplar memory.

Taken together, the four studies converge on a deeper understanding of the cognitive basis of multiple-cue judgment by providing initial support for the key predictions of Sigma. Thereby the outlines of a cognitive process model of judgment can be proposed.

Implications for Related Theory

The theory and data reported in this thesis provides a first step towards a unifying framework for the interplay between different representations in multiple-cue judgment. The theoretical implications are discussed below.

Implications Challenging a “Single-Systems View” of Judgment

The theory and data presented in this thesis poses obvious challenges to a single-systems perspective on judgment.

Automatic vs. controlled processes

Some aspects of the dual-system accounts of cognition, and in particular of judgment and decision making, are captured by the theory and results reported in this thesis. The additive and sequential character of controlled judgments hypothesized in this thesis naturally encompasses the claims repeatedly made about controlled processes (Shiffrin & Schneider, 1977), of the rule-based system (Sloman, 1996; Smith & DeCoster, 2000), and of the analytical mode (Hammond, 1996). The theory and data presented in this thesis highlights the controlled aspects of multiple-cue judgments. Hence, the theory does not take into consideration judgments that may be automatic. Consider the difference between a judgment about the amount of money to invest in a stock portfolio and the judgment of when to put on the turn signal before driving up to your house. The latter presumably involves automatic processing, repeating an action carried out several hundred times before. Nevertheless, the cognitive basis underlying these automatic decisions may well be retrieved memory traces, as stated by Logan (e.g. 1988).

The former presumably involves controlled processing, in that it is nothing you do without thinking. Even so, the cognitive basis for this decision may be the abstracted knowledge of how to integrate the different economic indices (cue abstraction) as well as the similarity of this particular portfolio to one you invested in yesterday (exemplar memory).

In terms of the distinction between intuition and analysis (e.g. Hammond, 1996) the theory and data presented in this thesis conforms to the general idea, that there are (at least) two qualitatively different basis for a judgment. It is tempting to interpret them as equivalent to exemplar memory and cue abstraction, respectively. However, this should be done with caution since the generous use of the term ‘intuition’ sometimes lead to the interpretation of it being *everything we are not aware of* (Hogarth, 2001). In terms of the related distinction between implicit and explicit cognition, Study IV implies that there are possibly identifiable implicit components in controlled judgment, presumably pertaining to implicit recall of information in the multiplicative task. This, however, should not be confused with the notion of implicit *learning* (e.g. Reber, 1993; Pothos, 2007), implying that there is no conscious knowledge about what has been learnt. Rather, one interpretation is that at the time of controlled judgment in the multiplicative task, exemplars are retrieved “in the absence of conscious recollection” (cf. Graf & Schacter, 1985).

In sum; Sigma is aimed to account for explicit judgment processes. To the extent that implicit judgments actually exist (for a discussion see e.g. Lagnado et al., 2006; Shanks & St. John, 1994; Wilkinson & Shanks, 2004) amendments to this theory might prove appropriate.

Knowledge Organization and Representation

For each of the experiments presented it is difficult to interpret the data without assuming differences in process or representation. It is hard to see how an exemplar-based account (Nosofsky & Johansen, 2000) could provide adequate explanations of the data, specifically *a)* the fact that the majority of participants in the additive tasks were able to interpolate as well as extrapolate and made the most extreme judgments to the most extreme exemplars, although they had never received feedback on them; *b)* the fact that the controlled and confounded training sequences (Study I, Experiment 2) had a differential effect on performance in the additive and the multiplicative task strongly suggests that different processes were at play; *c)* the observation that without explicit instructions participants did not spontaneously adopt an exemplar-based strategy in the non-linear tasks (in Olsson, Enkvist & Juslin, 2006; reviewed in Study III) and *d)* a very challenging result for a single-systems account is the fact that the neural underpinnings to the two tasks could be dissociated (Study IV).

Different Strategies or One Single Mechanism?

Is the data and theory presented in this thesis in line with the “adjustable spanner hypothesis” (Lee & Cummins, 2004; Newell, 2005)? That hypothesis posits that there is in reality one single process that is at play but what differs in behavior from task to task and between different participants is the amount of information that is considered. At one end the adjustable spanner hypothesis is in broad agreement with the division-of-labor hypothesis. The process that implements judgments in Sigma is described by one algorithm, imposing additive and linear constraints on the judgment process. Judgments can thus be described as obeying the same process. On the other hand, the adjustable spanner does not acknowledge the fact that not just the amount of information differs between tasks and participants; also the type of information can differ. According to Sigma, what differs between tasks is also the information input; knowledge about *cues* or of *exemplars*. The spanner hypothesis seems to regard all input in relation to the number of cues that is considered. Moreover, the spanner hypothesis is not in agreement with the strategic hypothesis about the nature of representational shifts, proposed in Study III.

Implications for an Ecological Perspective of Psychology

Considering the results put forward in this thesis, it appears that the structure of the task is a strong predictor of what cognitive processes people will use in judgment. These findings thus add to the body of research highlighting the relation between the task and the organism (Brehmer, 1988; Brunswik, 1956; Cooksey, 1996; Gibson, 1979; Gigerenzer et al., 1999; Hammond, 1996; Simon, 1957). The findings explicates the importance of not studying cognitive performance in isolation, against a norm, and the importance of analyzing the task structure when aiming at a-priori predictions of what cognitive process that will dominate in a judgment situation. The assumptions regarding how people master multiple-cue judgments needs to be tested as a function of different task properties.

Should multiple-cue judgment research aim at even more ecological influences, in line with the Gibsonian perspective on perception (e.g. Vicente, 2003)? One intriguing question is to what extent the structure of the task acts as “affordance” (Gibson, 1979) to judgment. Affordance is one of the corner stones of Gibson’s view of perception. The input to our perceptual system pertaining to for example an object comes in a way that makes it obvious regarding what action that object can be used for. For example, the perceptual input from a chair in a modern western culture affords “sitting” (but see Marr, 1982; Pylyshyn, 1981). To what extent is it unavoidable that certain task characteristics will trigger certain judgment strategies (e.g. Gigerenzer, 1999)? Following the main theme of this thesis, it is probable that a judgment task

affords certain cognitive processes to a large extent. On the other hand, also following the main theme of this thesis, the processes that are at use are not mainly triggered by affordances in the task, but also by an a-priori hypothesis order (Brehmer, 1994).

Implications for a Cognitive Neuroscience of Judgment

Neuroimaging did prove successful in terms of addressing differences in cognitive processes in multiple-cue judgment. In light of the fact that multiple-cue judgment is such a prominent feature of our daily cognitive activities there exists surprisingly few neuroscientific studies of multiple-cue judgment; the consideration of pieces of information in the external world in order to make a continuous estimate. This should be compared to the large quantity of studies addressing category learning, and the growing body of studies addressing decision making in relation to, for example, reward, punishment and risk. Thus, the cognitive neuroscience of multiple-cue judgment is in its commence. Why can it be important? Theory driven imaging studies could inspire the development of biologically plausible models of judgment. Even if the experiment presented in Study IV is to be seen as first preliminary support for the processing components involved, it indicates a general judgment process, acting on different types of memory as a factor of the task structure. It has thus strengthened conclusions from the theoretical proposals. Further research should concentrate on more direct comparisons between signal change in the brain, and the fit of exemplar models; are exemplar memory in other circumstances associated with explicit memory processes?

It is an intriguing possibility that different cognitive processes reveal different neural patterns. This implies that it could be possible to “diagnose” the judge in respect to what process that has been used. This moreover renders the possibility to learn more about naturalistic judgments. Virtual reality techniques exist for usage in a fMRI-scanner. “Naturalistic” multiple-cue judgment scenes could thus be implemented in virtual reality and participants performing judgments in such situations could be scanned – and “diagnosed”.

Another possibility within a cognitive neuroscience of judgment is to study the issue of strategic vs. automatic strategy choice, as discussed in Study III. As far as can be found in the literature to date, there are not many systematic studies addressing the neural substrate of strategy choice in judgment. Is learning to use different judgment strategies the relative strengthening of their respective underlying networks (Poldrack et al, 1999; 2001) *and/or* is learning to use different decision strategies requiring a meta-cognitive neural basis (for example *striatum*, Poldrack et al, 1999; *and/or IPS*, (Marklund & Nyberg, in press) to enable a shift before learning can proceed with another strategy?

Limitations

In this thesis the division-of-labor hypothesis is presented together with empirical support for the idea. Due to sequential and additive constraints of controlled cognitive processes people adopt exemplar memory when they are confronted with judgment tasks that are non-additive. The ecological relevance of the simple experimental tasks used in this thesis can be questioned. The stimuli are less complex than many real world situations, the training and test design is artificial and the choice of cue-combination rules may appear arbitrary. Nevertheless, if a division-of-labor can be observed even within this limited complexity in comparison with real life, there are reasons to believe that this machinery we are equipped with supports judgments also in real life. For example, why would a tendency to use exemplar memory only be visible in cognitive experiments? That is somehow equal to saying that our episodic or procedural memories never aid us in cognitive activities. Moreover, since participants were unable to abstract and integrate non-additive cue-criterion relations in these simple tasks, with feed-back after each judgment, there are reasons to doubt that they can do it in more complex real-world situations. The data in this thesis provides the initial support for the division-of-labor hypothesis, and thus a ground for further pursuits in this direction. Without this ground naturalistic experiments with the same aim would seem risky.

The hypothetical cue-combination rules in the tasks were chosen with the restriction that they should *a)* be non-additive, since we wanted to isolate the non-additive property in a judgment task; *b)* be a simple mathematical transformation of the existing additive task equation (Juslin et al., 2003); and *c)* not be too easily approximated by linear and additive cue-combination. Naturally, the exponential component of the multiplicative cue-combination rule gives rise to a somewhat skewed distribution of criteria. But the results witness that learning has taken place despite this. It can be discussed whether the exact cue-combination rules chosen abound in real world judgment tasks (e.g. the exponential component). (However, an exponential function can be expressed as a multiplicative function, for example $e^{1.1} = 2.71 * 2.71$, and is thus as “realistic” as other multiplicative relations.) Moreover, it is worth highlighting that the apparent ease with which we can identify judgment tasks well approximated by linear and additive combination of the cues can be a reflection of the inherent bias to consider cues in an additive and linear manner.

A second main limitation of the theory and data presented in this thesis is the lack of verbal reports. How much insight into their own processes do the participants demonstrate? Is it more difficult to demonstrate insight into the judgment process if it has been dominated by exemplar memory? If exemplar memory has a dominating implicit component to it, as suggested in Study IV,

the ability to verbalize the process might be low (Ashby et al., 1998; but see Lagnado et al., 2006).

A Tentative Real-Time Implementation

Since the presented theory and data should be considered as the first step towards the cognitive process model Sigma, a number of details of the theory still need to be worked out. For example, it remains to implement a real-time version of the judgment process. Since Sigma makes strong predictions about the cognitive processes also in training an implementation of a learning model is also a natural next step. As a first attempt to more explicitly test the process-model assumptions of Sigma, a preliminary implementation of Sigma in ACT-R (Anderson et al., 2004) has been made. ACT-R is a cognitive architecture that exploits an integrative approach to cognition. Implementing models in ACT-R provides an opportunity to take advantage of already existing knowledge of psychological phenomena. It would enable a cognitive process model of multiple-cue judgment to be implemented on the basis of plausible psychological assumptions. Moreover, since ACT-R rests on assumptions about underlying neural systems, an implementation in ACT-R should render biologically plausible models as well. Specifically, we were interested in taking advantage of the well-studied memory structures built into the system. For example, in ACT-R a plausible mechanism for information retrieval is offered. The probability that a certain item is retrieved from memory is a function of both how recently and frequently that item has been used in the past, as well as its similarity to the situation in which it is recalled.

By implementing Sigma in ACT-R the opportunity arises to test assumptions about memory sampling within a psychologically plausible process model. For example, what is the effect of sampling one, several, or “all” cues/exemplars from memory in relation to the task? Implementing Sigma in ACT-R also enables us to investigate the process as it is done on-line by means of reaction time data. Are there different predictions about reaction time data depending on the sort of representational input (cues or exemplars) as well as in relation to the task?

In the first preliminary implementation of Sigma in ACT-R idealized knowledge representations at a state of asymptotic training was implemented as chunks in the model. This was done since Sigma is first of all a judgment model (although it has implications also for real-time modeling of the course of learning) and since we wanted to be able to test assumptions regarding knowledge input. The production rules firing to appropriate chunk input implemented the basic steps of Sigma. The rules were held as equal as possible even though they handled different chunk types (cues or exemplars). The preliminary simulations are intriguing. When both the exemplar model and the cue abstraction model were implemented within Sigma in ACT-R they gave

rise to identical predictions as the original models, for both an additive and a multiplicative judgment task. Implementing the two models in ACT-R within Sigma moreover resulted in testable predictions about reaction times: reaction times should be longer for new exemplars in the test phase if exemplar memory has been at play but not if cue abstraction has been at play. This pattern is also evident in the reaction time data of Experiment 1, Study I (although plagued by individual variance since the time to complete the experiment was self paced). This implementation illustrates the first steps of one approach to an on-line process-model of multiple-cue judgment.

Additional Suggestions for Future Research

Additional to the suggestions for future studies already mentioned previously in the thesis, future research based on the themes investigated can be associated with judgment phenomena in general, the fast-and-frugal program proposed by G. Gigerenzer and the ABC research group (Gigerenzer et al., 1999), individual differences, and a developmental perspective.

One further direction from the work presented in this thesis could be to relate observed judgment phenomena, like *base-rate neglect*, the *conjunction fallacy*, and so on, to the constraints imposed by Sigma. Considering the possibility that there are some validity in the data supporting Sigma, and that there are psychological constraints concerning what cognitive representations that can be formed during learning of a judgment task, it could be interesting to investigate how these different phenomena can be related to such constraints. Can the assumptions behind Sigma be used to explain other well-known judgment phenomena?

In the fast-and-frugal program proposed by Gigerenzer et al. (1999) rather “simple” heuristics have been identified and proved successful in simulations of a variety of circumstances. The fast-and-frugal program posits that simple heuristics often outperform more complex strategies and that people in many situations rather uses the simple rules of thumb than integrates information. Not only is it shown in this thesis that the majority of participants actually seem to integrate information from a number of cues, whenever they have the possibility to do so. It is also shown that the adaptive toolbox should be amended with yet another tool; the exemplar-based strategy. The review presented in Study III is highly suggestive of a notion of exemplar-memory as a strategy that people choose to use, among other strategies. Future research could aim at integrating the study of ecological rationality with the issue of how people decide what process to adopt. The implications of the current thesis is that the characteristics of the judgment process structure, together with the characteristics of the task and the metacognitive need to create meaningful knowledge representations are promising grounds.

It seems to exist evidence in the judgment literature that people are able to switch adaptively between strategies as a function of what the task requires (Juslin, et al., 2003; Study I-IV in this thesis). An important question, however, is to which extent such task-dependent shifts occur spontaneously, or whether they are controlled strategy shifts contingent on early perceived learning performance (see Study III). Are the shifts both task-dependent but also dependent on the other strategies that have been tried earlier in the hierarchy? In essence: when can we expect strategic strategy shifts and when can we expect automatic strategy shifts? Or, is it always the one or the other?

How do individual differences characterize the judgment process? One approach to understand more about the influence of individual differences in multiple-cue judgment is to consider how *executive functions* are related to multiple-cue judgments. Surprisingly little research has been dedicated at relating cognitive concepts from research on executive functions to multiple-cue judgment research. For example, how are the specific constructs *shifting*, *updating* and *inhibition* (Miyake et al., 2000) involved in multiple-cue judgment? In that paper, updating is characterized as “*monitoring and coding incoming information for relevance to the task at hand and then appropriately revising the items held in working memory by replacing old, no longer relevant information with newer, more relevant information*”, shifting is characterized as “*shifting back and forth between multiple tasks, operations, or mental sets*” and inhibition is characterized as “*one’s ability to deliberately inhibit dominant, automatic, or prepotent responses when necessary*”. The definition of these constructs suggests that they should be important in multiple-cue judgment. Can we see that persons who perform well on a typical task addressing ‘shifting’ ability also perform well in a multiple-cue judgment task that demands shifts between different judgment strategies? In the typical multiple-cue judgment task used in the studies in this thesis it has proven possible to measure a) performance and b) process. If we also collect data on typical tasks tapping ‘shifting’, ‘updating’ and ‘inhibition’ we would be able to pinpoint if and how these constructs are involved in different tasks, as well as if they are differently important for different strategies. Training of executive functions (Dahlin et al, 2007), of working memory (Olesen et al, 2004) and of focus of attention (Verhaeghen et al, 2004) have proven successful. Do people trained on executive functions have an advantage to controls in judgment tasks demanding strategy choice as an exploitation of the task structure?

There are examples showing that there may be non-trivial relations between working memory and strategy choice (Gaissmaier, Schooler & Rieskamp, 2006) and intelligence and strategy choice (Bröder, 2003). What remains to be more carefully tested is how working memory capacity relates to the assumptions in Sigma. Sigma predicts that working memory is needed both when integrating cues as well as integrating exemplars while performing a judgment.

The theory and results presented in this thesis implies that performing multiple-cue judgments is a matter of a division-of-labor between different representations. Given that the frontal lobes, to a great extent involved in executive functions and working memory, are not fully developed until the age of 25 (Paus, 2005) implementing more complex decision strategies like cue abstraction should be more difficult among children (Sloutsky & Fischer, 2004). Hence the division-of-labor might not even take place among children and experiments where a division-of-labor is manifested among adults might show a different pattern for children. This also implicates that tasks where strategic division-of-labor seem to actually have hindered adults from performing well, like the non-linear task in Olsson et al., 2006 (reviewed in Study III) could be mastered better by children, only having the similarity strategy to choose.

Practical implications

The theory and results presented in this thesis are first of all intended to provide a deeper understanding of the cognitive mechanisms sub-serving multiple-cue judgment. Even so, at this early stage, it is possible to identify a number of tentative future practical implications.

The demonstrated possibility to predict when judgments are based on generalizable rules and when it is based on memory for previous judgment cases, by considering the structure of the task in experimental settings, implies that it should be possible to do similar predictions also in naturalistic settings. Why should knowledge about what cognitive processes and representations that are the basis of judgments in different domains be valuable? There are at least three tentative applications.

First, it suggests how teaching and training could be designed. To date, the results presented in this thesis, together with results of other studies from our lab, have identified factors more prone at promoting cue abstraction and others at promoting exemplar memory, as suggested by the division-of-labor hypothesis. Within the multiple-cue judgment literature there is a tradition of exploring how the use of different forms of *cognitive feedback* can be helpful in terms of improving judgment performance in both laboratory and natural experimental settings (for a review see Doherty & Balzer, 1988; for more recent studies see e.g. Denig et al., 2002; Gattie & Bisantz, 2006; Leung & Trotman, 2005; O'Connor, Remus, & Lim, 2005; Youmans & Stone, 2005). What is interesting is that this line of research to a large extent has focused on feedback where the (subjective and/or objective) cue weights have played the major role, together with the function forms with which the individual cues are related to the criteria. The feedback under investigation is often given in numerical, pictorial and sometimes verbal formats, with both continuous and binary cue data. Hence, it could be inviting of both intuition and analysis in terms of

Hammond's Cognitive Continuum Theory (1996). Nevertheless, the focus has been on the individual cues. The research presented in this thesis suggests that there should be circumstances where other forms of feedback, promoting the use of exemplar memory, should be valid. A study by Leung and Trotman (2005) seems promising in this respect. They have shown that in an auditor judgment task that demanded *configural cue processing*, enhancing that there is an interaction among the cues improved judgment performance. Hence, by thorough task analysis in occupations where judgments form an important part, the feedback utilized for training those professionals could be constructed to properly acknowledge the use of exemplar memory.

In terms of teaching, speculations would infer that teaching people in situations suggested to induce cue abstraction the superior method should be to use formal instructions, helping the person to create stable representations of how cues are related to the criterion. On the other hand, teaching people in situations suggested to induce exemplar memory, a fruitful method should be to utilize case-based or problem-based methodologies, providing the person with examples that are available for storage in memory. Through quasi-experimental designs in naturalistic settings this issue could be investigated.

Secondly, and related, it has implications for the design of effective judgment support systems. If it is known under which judgment situations persons utilize cue abstraction or exemplar memory it also indicates how to best support the judgments. For example, judgments based on cue abstraction might be best supported by highlighting the most valid cues in the task environment and perhaps also present them in a serial order to enhance the sequential integration of the impact of distinct cues. Are judgments based on memory for previous judgment cases it is perhaps good to provide recollection aid, such as cues helping to recall valid examples from memory, and also databases where previous cases are stored, facilitating memory storage.

Third, it should be possible to construct valid methods for identifying and predicting judgment errors. Given what has been suggested in this thesis concerning limiting conditions for information integration we should be able to identify real-world circumstances that are far from optimal when it comes to taking cognitive constraints into consideration. Presenting information that needs to be integrated in a non-additive or non-linear fashion and still assume that people will be good at doing these integrations might not be useful in all circumstances. We should be able to pinpoint formulations in preparations of matters to be taken up for consideration among decision makers in a variety of domains that could invite judgment errors. For example, decision problems involving the conjunction rule ($P(A+B) < P(B)$) have proven especially difficult to solve, since people mistakenly judge the conjunction as more probable than each of the separate conjuncts (e.g. Tversky & Kahneman, 1983; but see e.g. Fiedler, 1988, for a discussion). Presumably, a decision problem involving the conjunction rule may best be presented in a format diminishing the need to be

forced to do the actual multiplication, and instead highlighting other calculation possibilities.

CONCLUSIONS

The research reported in this thesis does not only integrate research on multiple-cue judgment with methods and inquiries from categorization and memory research. It should also be seen as a contribution to the lack of knowledge about psychologically plausible means of a priori predicting what processes and representations that are used in different multiple-cue judgment situations. The structure of the task is one important factor when predicting the relative contribution of qualitatively distinct representational systems to behavior.

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Footnotes

1. Additional preliminary support for the relation between exemplar memory and the signal changes within the tail of the caudate is found when correlating the fit of the exemplar model in the multiplicative task (RMSD) with the relative signal change in the tail of the caudate (% signal change Additive/Multiplicative; see Figure 8). RMSD for the exemplar model is significantly correlated to the change of activation within this area ($r=.77;p=.009$). The same is not true for the cue abstraction model ($r=-.35;p=.316$).

Appendix

Numerical example illustrating that Sigma yields identical output as the cue abstraction model and the exemplar model, respectively.

Cue Abstraction

Cue 1:	weight $\omega_1 = 4$,	Cue value $C_1 = 1$
Cue 2:	weight $\omega_2 = 3$,	Cue value $C_2 = 0$
Cue 3:	weight $\omega_3 = 2$,	Cue value $C_3 = 0$
Cue 4:	weight $\omega_4 = 1$,	Cue value $C_4 = 1$

Cue abstraction

$$\hat{c} = k + \sum_{i=1}^4 \omega_i \cdot C_i$$

$$k = 50 + .5 \cdot (10 - \sum \omega_i)$$

Sigma (process)

$$\hat{c}_0 = \text{default}$$

$$\hat{c}_n = \hat{c}_{n-1} + \eta_n \cdot (\delta_n - \hat{c}_{n-1})$$

$$\eta_n = \frac{\omega_n}{\sum_1^n \omega_i}$$

$$\delta_n = \begin{cases} 60 & \text{if } C_n = 1 \\ 50 & \text{if } C_n = 0 \end{cases}$$

Cue abstraction for all four cues

$$50 + .5 \cdot (10 - (4 + 3 + 2 + 1)) + 4 \cdot 1 + 3 \cdot 0 + 2 \cdot 0 + 1 \cdot 1 = 55$$

Sigma for the first two cues:

$$60 + \left(\frac{3}{4+3} \right) \cdot (50 - 60) = 55.7$$

Sigma for the first three cues:

$$55.7 + \left(\frac{2}{4+3+2} \right) \cdot (50 - 55.7) = 54.4$$

Sigma for all four cues:

$$54.4 + \left(\frac{1}{4+3+2+1} \right) \cdot (60 - 54.4) = 55$$

Exemplar memory

Exemplar 1:	similarity $S_1 = .75$,	criterion $c_1 = 58$
Exemplar 2:	similarity $S_2 = .5$,	criterion $c_2 = 56$
Exemplar 3:	similarity $S_3 = .25$,	criterion $c_3 = 54$
Exemplar 4:	similarity $S_4 = .5$,	criterion $c_4 = 57$

The context model

$$\hat{c} = \frac{\sum_1^N S_j \cdot c_j}{\sum_1^N S_j}$$

Sigma (process)

$$\hat{c}_n = \hat{c}_{n-1} + \eta_n \cdot (\delta_n - \hat{c}_{n-1})$$

$$\eta_n = \frac{S_j}{\sum_1^n S_j}$$

$$\delta_n = c_j$$

The context model and Sigma over the first two exemplars:

$$\frac{.75 \cdot 58 + .5 \cdot 56}{.75 + .5} = 57.2$$
$$58 + \left(\frac{.5}{.75 + .5} \right) \cdot (56 - 58) = 57.2$$

The context model and Sigma over the first three exemplars:

$$\frac{.75 \cdot 58 + .5 \cdot 56 + .25 \cdot 54}{.75 + .5 + .25} = 56.7$$
$$57.2 + \left(\frac{.25}{.75 + .5 + .25} \right) \cdot (54 - 57.2) = 56.7$$

The context model and Sigma over the first four exemplars:

$$\frac{.75 \cdot 58 + .5 \cdot 56 + .25 \cdot 54 + .5 \cdot 57}{.75 + .5 + .25 + .5} = 56.8$$
$$56.7 + \left(\frac{.5}{.75 + .5 + .25 + .5} \right) \cdot (57 - 56.7) = 56.8$$