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Introduction

1 Background

1.1 Evolution and development of statistics

Statistics has originated as a science of statehood. As a result, in the olden days statistics was confined to only state affairs. Modern statistics may be defined as the science of collection, presentation, analysis and interpretation of numerical data. It is basically a discipline of the twentieth century. In the early stage of its development it was intensely involved with problems of interpreting and analyzing empirical data originating in agricultural and biological sciences. The vernacular of experimental design in use today bears evidence of the agricultural connection and origin of this body of theory. Terms such as block or split-plot, emanated from descriptions of blocks of land and experimental plots in agricultural field designs. The theory of randomization in experimental designs was developed by Fisher to neutralize the spatial effects among experimental units or equivalently field plots. There are reports that indicate the development of sampling procedures in forestry practice; about 1850 the first national forest survey was conducted in Denmark and strip survey introduced in Burma, linear strip samples were used as early as in the 1840s in Sweden and in the 1920s first national forest survey conducted in Sweden using strip sampling (Gregoire and Köhl, 2000). Moreover, several other sampling techniques including random sample survey and initiatives of first experimental design though without replication are also believed to be introduced in the forestry practice (cf. Gregoire and Köhl (2000), Matern (1982), and Schreuder *et al.* (1993)).

Hence, even though Statistics is rooted in Mathematics, it is developed in biological and agricultural sciences. Statistical methods are now realized to be essential to many studies to sustain and develop as a science. Neyman (1955) described statistics as a servant of all sciences .

Among the few prominent founders of modern statistics Karl Pearson (1857–1936) and R. A. Fisher (1890–1962) are widely recognized. Pearson played a leading role in the creation of modern statistics of which one of his most significant contribution is known to be the chi-square test. Pearson built up a biometric laboratory and founded the journal Biometrika in 1901, the first journal of modern statistics. Walker (1968) sums up Pearson's importance as follows.

> Although Pearson made contributions to statistical technique that now appear to be of enduring importance, these techniques are of less importance than what he did in arousing the scientific world from a state of sheer interest in statistical studies to one of eager effort by a large number of well-trained persons, who developed new theory, gathered and analyzed statistical data from every field, computed new tables, and re-examined the foundations of statistical philosophy.

Fisher who was hired in 1919 as statistician at Rothamsted Experimental Station (England), pioneered the application of statistical procedures to the design of scientific experiments among many other of his contributions. Fisher is regarded by many as the most influential statistician of all ages (Johnson and Kotz, 1997). In the modern world of computers and information technology, the importance of statistics is becoming more important in all disciplines.

1.2 Forest Biometry

Statistics has been the most fertile field for the development of new scientific disciplines (Mosteller, 1988). Owing to the indisputable role of statistics in many fields and hence the tendency to nurture statisticians and statistics in these areas create several new applied statistical fields. Among many of them, Forest biometry or Forest statistics is widely known with a long tradition as a discipline in the forest sciences. Forest biometry can be defined as statistical or quantitative study in forestry aimed to manage forestry resources wisely. It is believed to have been existing about 100 years ago in forestry education programs in North America and since then the field of forestry has changed critically (Temesgen *et al.*, 2007). Gregoire and Köhl (2000) reported that several sampling procedures and the first long term experiment without replication were introduced in forestry practice and research before sound statistical theory was developed for these methods.

Initially, the purpose of forest biometry was estimation of wood volume and economically important characteristics for a forest area. Later, with the wider concept of forest management coupled with new technology of forest inventory tools and demand for precise estimation, creates new challenges and opportunities for the development of forest biometrics. Schumacher (1945) wrote an article about statistical methods in forestry in which he discussed the role of statistics in forestry practice and management. In light of the importance of statistics in forestry, the International Union of Forest Research Organization has initiated the international advisory group of forest statisticians in 1959 on its 25th session (Nair, 1967).

1.3 Forest models

A model is an abstraction of reality, utilized to help us understand how reality works, to aid in the making of complex decisions and to understand the implications of decisions. Modeling has been an important tool in forest management for a long time. Models have been constructed for a host of management and research objectives. Forest management decision making is predicated on accurate forecasting of growth and yield. According to Burkhart (2003) primary uses of models can be categorized into inventory updating, management planning, evaluation of silvicultural alternatives and harvesting scheduling. However, modern society has come to expect a comprehensive accounting of the forest's structure, function, products, services, sustainability and response to changing conditions (Amateis, 2003). Such expectation is a driving force for more complex models that are effective across spatial and temporal scales. Today, complex forest models utilize computer programs to simulate forest growth and dynamics to undertake timber supply analysis, prepare strategic level plans and to answer other forest management, planning and monitoring issues. Model parametrization using the ordinary least squares method was a common practice. In recent years an effort is shown to use the mixed effects modeling approach in order to accommodate a wide range of data structures.

2 Mixed effects models

2.1 Theory and estimation

Mixed effects models appear to be a confusing concept at least for beginners as it is known under different names in different applications. Generally, it is used to analyze repeated measurements data, longitudinal data, cluster and panel data. In some literatures it is also known as hierarchical and multilevel models. This statistical method was developed in the health sciences in the 1970's and in these days it enjoys popularity in many application areas. In his book, Demidenko (2004) noted that mixed models methodology brings statistics to the next level. Unlike other main reference books (Davidian and Giltinan, 1995; Vonesh and Chinchilli, 1997; Pinheiro and Bates, 2000; Verbeke and Molenberghs, 2000) in mixed effects models, Demidenko (2004) discussed further applications and uses of mixed effects models particularly in shape and image analysis.

Another confusing concept in the mixed effects models area is with the term longitudinal data which historically connotes to data collected over time. However, these models are broadly applicable to any kind of repeated measurement data. Similarly, even though repeated measurements most often takes place over time, this is not the only way that measurements may be taken on the same unit. Thus, in the application of mixed effects models, the term longitudinal data indicates to data collected in the form of repeated measurements that may well be over time but may also be over some other set of dimensions. For example, for each tree in the forest, measurements of the diameter of the tree are made at several different points along the bole of the tree. Thus the tree is measured repeatedly over positions along the bole.

Classical statistics assumes that observations are independent and identically distributed. In contrast, the mixed effects model assumes that observations in a given cluster (subject or unit) are dependent and need to be addressed likewise. Accordingly, it acknowledges the variation between subjects and within subjects and incorporates these sources of variation in the modeling process. Using multilevel random effects, mixed effects models facilitate a unified analysis of multilevel data structures.

Mixed effects model contains both fixed and random effects, where the fixed effects complete characterization of the systematic part of the response and random effects account among unit variation. The general form of linear mixed effects model can be presented as

$$\boldsymbol{y}_i = \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i \tag{1}$$

where $\boldsymbol{X}_i$ is $(n_i \times p)$ design matrix for the fixed effects, $\boldsymbol{\beta}$ is a $(p \times 1)$ vector of fixed effects parameters, $\boldsymbol{Z}_i$ is $(n_i \times k)$ design matrix for the random effects, $\boldsymbol{b}_i$ is a $(k \times 1)$ vector of random effects, and $\boldsymbol{\epsilon}_i$ is a $(n_i \times 1)$ vector of within unit deviations. The model components $\boldsymbol{b}_i$ and $\boldsymbol{\epsilon}_i$ characterize the two sources of random variation, among and within-units and it is assumed that $\boldsymbol{\epsilon}_i \sim N_{n_i}(\boldsymbol{0}, \boldsymbol{R}_i)$ and $\boldsymbol{b}_i \sim N_k(\boldsymbol{0}, \boldsymbol{D})$. $\boldsymbol{R_i}$ is a $(n_i \times n_i)$ covariance matrix that characterizes variance and correlation within-unit sources and $\boldsymbol{D}$ is a $(k \times k)$ covariance matrix that characterizes variation among unit sources, assumed the same for all units. The assumptions of $\boldsymbol{\epsilon}_i$ and $\boldsymbol{b}_i$ imply that

$$\boldsymbol{y}_i \sim N_{n_i}(\boldsymbol{X}_i\boldsymbol{\beta}, \boldsymbol{\Sigma}_i) \tag{2}$$

where $\boldsymbol{\Sigma}_i = \boldsymbol{Z}_i\boldsymbol{D}\boldsymbol{Z}_i' + \boldsymbol{R}_i$.

The nonlinear mixed effects (NLME) model is a generalization of the linear mixed effects model in which some or all of the fixed and random effects enter nonlinearly in the model function. The NLME model can be written in two stages (Lindstrom and Bates, 1990; Davidian and Giltinan, 1995; Vonesh and Chinchilli, 1997; Pinheiro and Bates, 2000; Demidenko, 2004). The first stage of this model can be formulated in a matrix form as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{f}_i(\boldsymbol{\phi}_i, \boldsymbol{v}_i) + \boldsymbol{\epsilon}_i, \\ \boldsymbol{\epsilon}_i &\sim N_{n_i}(\boldsymbol{0}, \boldsymbol{R}_i) \qquad \text{for } i = 1, \ldots, M \end{aligned}$$

Similarly the second stage can be written as

$$\begin{aligned} \boldsymbol{\phi}_i &= \boldsymbol{A}_i\boldsymbol{\beta} + \boldsymbol{B}_i\boldsymbol{b}_i, \\ \boldsymbol{b}_i &\sim N_k(\boldsymbol{0}, \boldsymbol{D}) \qquad \text{for } i = 1, \ldots, M \end{aligned}$$

In the NLME model setting, $\boldsymbol{f}_i$ is a vector of nonlinear functions, $\boldsymbol{\phi}_i$ is a vector of subject specific parameters, and $\boldsymbol{v}_i$ is a vector of covariates. The matrices $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are design matrices for the fixed and random effects, respectively. $\boldsymbol{R_i}$ is a $(n_i \times n_i)$ covariance matrix that characterizes variance and correlation due to within-unit sources and $\boldsymbol{D}$ is a $(k \times k)$ covariance matrix that characterizes variation among unit sources. As in the LME model, $\boldsymbol{\epsilon}_i$ is assumed to be independent for different i and to be independent of the random effects $\boldsymbol{b}_i$.

The problem with the estimation of the NLME model is that the likelihood function involves an integral, which does not have a closed-form solution. This is because the random effects enter the model in a nonlinear form. Several suggested methods for avoiding the integration problem include the two-stage estimator (Steimer *et al.*, 1984), first-order approximation around $\boldsymbol{b}_i = 0$ (Beal and Sheiner, 1982), first-order conditional methods

(Lindstrom and Bates, 1990; Vonesh and Carter, 1992), Laplace approximation (Wolfinger, 1993), Adaptive Gaussian Quadrature (Pinheiro and Bates, 1995), EM-related methods (Walker, 1996), and Stochastic approximation EM (Delyon *et al.*, 1999; Kuhn and Laveille, 2005). More methods or versions of the methods and detailed discussions are given in several sources (cf. Demidenko (2004), Davidian and Giltinan (2003), Pillai *et al.* (2005, Table II), Pinheiro and Bates (1995), Pinheiro and Bates (2000), Davidian and Giltinan (1995), Vonesh and Chinchilli (1997)).

2.2 Subject specific prediction

One of the important objectives of modeling in general and in forestry in particular is to predict future or unvisited values of a specific characteristic of individual unit. Assuming some form of dependency between the sample data and the unobserved data for which prediction is desired, Hall and Clutter (2004) demonstrated use of nonlinear mixed effects models for prediction of timber volume (yield). However, there are many single tree models such as tree volume, taper and height models in which dependency between observed and unobserved data is not realized. In such situations, existing theory suggests (cf. Vonesh and Chinchilli (1997), Lappi (1991), Trincado and Burkhart (2006)) use of additional information from the unobserved data desired for prediction. This method practically imposes to visit every subject for some measurements required for prediction of corresponding random effects $\boldsymbol{b}_k$. Using observed information (some measurements, say some few diameter measurements for tree taper model) from the subject desired for prediction, $\boldsymbol{y}_k$, it is possible to estimate the individual's subject specific random effect, $\boldsymbol{b}_k$ and then incorporate this directly to the estimated NLME model to predict individual specific values. To illustrate, consider that both fixed effects parameters and random parameters are known from the developed NLME model. Since the individual is new, we must first estimate the random effect, $\boldsymbol{b}_k$. To predict a new observation y_{kj} at a given υ_{kj}, we may use first order expansion of $f(\boldsymbol{\phi}_k, \boldsymbol{\upsilon}_k)$ about $\boldsymbol{b}_k = 0$ where $\boldsymbol{\phi}_k = \boldsymbol{A}_k\boldsymbol{\beta} + \boldsymbol{B}_k\boldsymbol{b}_k$. Hence

$$\begin{aligned}
\boldsymbol{y}_k &= f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k) + \left.\frac{\partial f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k)}{\partial \boldsymbol{\beta}_k'}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} (\boldsymbol{B}_k\boldsymbol{b}_k - \boldsymbol{B}_k\mathbf{0}) \\
&= f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k) + \left.\frac{\partial f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k)}{\partial \boldsymbol{\beta}_k'}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \boldsymbol{B}_k\boldsymbol{b}_k \\
&= f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k) + \boldsymbol{Z}_k\boldsymbol{b}_k
\end{aligned}$$

where

$$\boldsymbol{Z}_k = \left.\frac{\partial f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k)}{\partial \boldsymbol{\beta}_k'}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \boldsymbol{B}_k$$

From the multivariate normal distribution theory where $\boldsymbol{b}_k$ and $\boldsymbol{y}_k$ are jointly distributed normal random variables, the conditional expectation of $\boldsymbol{b}_k$ is (see Johnson and Wichern (2002, page 160))

$$\boldsymbol{E}[\boldsymbol{b}_k|\boldsymbol{y}_k] = \boldsymbol{E}[\boldsymbol{b}] + \text{cov}[\boldsymbol{b}_k, \boldsymbol{y}_k][\text{var}(\boldsymbol{y}_k)]^{-1}[\boldsymbol{y}_k - \boldsymbol{E}(\boldsymbol{y}_k)].$$

Since $\text{cov}(\boldsymbol{b}_k, \boldsymbol{y}_k) = \boldsymbol{D}\boldsymbol{Z}_k'$, $\text{var}(\boldsymbol{y}_k) = \boldsymbol{Z}_k\boldsymbol{D}\boldsymbol{Z}_k' + \boldsymbol{R}_k$ and $\boldsymbol{E}(\boldsymbol{b}_k) = \mathbf{0}$ and $\boldsymbol{E}(\boldsymbol{y}_k) = f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k)$, then the best linear unbiased predictor is

$$\boldsymbol{b}_k = \boldsymbol{D}\boldsymbol{Z}_k'(\boldsymbol{Z}_k\boldsymbol{D}\boldsymbol{Z}_k' + \boldsymbol{R}_k)^{-1}(\boldsymbol{y}_k - f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k))$$

Hence, with $\boldsymbol{e}_k = \boldsymbol{y}_k - f(\boldsymbol{A}_k\boldsymbol{\beta}, \boldsymbol{\upsilon}_k)$, an approximate Bayes estimator can be given as

$$\hat{\boldsymbol{b}}_k \approx \boldsymbol{D}\hat{\boldsymbol{Z}}_k'(\hat{\boldsymbol{Z}}_k\boldsymbol{D}\hat{\boldsymbol{Z}}_k' + \boldsymbol{R}_k)^{-1}\hat{\boldsymbol{e}}_k \tag{3}$$

In (3), it must be clear that the new information (observed measurements) is used to estimate $\boldsymbol{Z}_k$ and $\boldsymbol{e}_k$ while others are readily picked from the NLME model developed from the sample data.

Recently, Trincado and Burkhart (2006) used additional upper stem diameter measurements to estimate random effects using equation (3) to localize stem profile curves for individual trees. They conclude that the inclusion of random effects for a new tree based on upper stem diameter measurements and an approximate Bayes estimator increased the predictive performance of the model. They also noted that advances in technology will make upper-stem diameter measurements increasingly accurate and feasible. Indeed, technology may facilitate accuracy of upper stem diameter measurements but the drawback of this approach is the necessity to visit each tree desired for prediction for some measurements. This is a huge drawback to use mixed effects models for prediction in the context of the single tree models mentioned above since the cost and time will be unbearable.

In the last decade, an interest in the mixed effects models application in forestry is much motivated (Fang and Bailey, 2005; Jordan *et al.*, 2005; Garber and Maguire, 2003; Zhao *et al.*, 2005; Trincado and Burkhart, 2006). In this thesis, Papers I and II deal with mixed effects models in forestry that will be also briefly summarized later in this introductory section.

3 Optimum Experimental Design

Statistical models have become increasingly important and unavoidable in all spheres of man's activities to explore and utilize resources in a sustainable and efficient manner. In this context the precision of parameter estimates and prediction from models is a cental issue in a study of statistical methods. The variances of parameter estimates and predictions depend upon the experimental design and should be as small as possible. The unnecessary large variances and imprecise predictions resulting from a poorly designed experiment waste resources.

The theory of optimum experimental design is a statistical method that we use to design experiments. The pioneering effort in this regard is due to Smith (1918) who studied the problem of minimizing the worst-case prediction error in the construction of polynomial models. In her paper she introduced the concept that is known to the optimal design theory as G-optimality design coined by Kieifer and Wolfowitz (1959). G-optimality is defined as a design that minimizes the maximum standardized prediction variance over the design space. Kieifer and Wolfowitz (1959) also coined the name of D-optimality to the design criterion that concerns on the precision of parameter estimates as introduced by Wald (1943). The D-optimal design minimizes determinant of the inverse of the information matrix, hence minimizing the volume of the confidence ellipsoid of any unbiased parameter estimates.

Another important concept in the optimal experimental design theory is the General Equivalence Theorem. This equivalence theorem originally given in Kiefer and Wolfowitz (1960) for linear models and extended to nonlinear models in White (1973) establishes the equivalence of the D- and G-optimal designs. It states that for continuous designs G-optimum are also D-optimum designs and that the maximum value q (number of parameters in the model) of the standardized prediction variance function occurs at the points of support of the design. The standardized variance function makes the design problem easier to understand from a graphical point of view. Equivalence theorems of G- and D-optimality provide algorithms for the construction of designs and methods of checking the optimality of designs. Compared to linear models, the theory of optimum experimental design for nonlinear models was lately developed. Box and Lucas (1959) studied locally D-optimal designs for nonlinear models in which they also uncovered the dependency of such designs on prior knowledge of parameter values.

In forestry, despite that models play a vital role in the management and planning of forest activities, no effort is shown sofar to utilize the optimal experimental design theory to develop efficient designs. In this thesis we study the D_s-optimality design (a version of the D-optimality design) that optimize a design for precise estimation of the subset of parameters of a statistical model. In particular based on the best tree taper model obtained from the taper models study in Paper III, the D_s-optimality design for the Kozak (1988) tree taper model is investigated in Paper IV. More details on the optimal design theory related to this study and the Kozak (1988) tree taper model is also given as theoretical ground in Paper IV.

4 Summary of Papers

This thesis work consists of four papers. Paper I presents briefly the historical developments of forest estimation methods, tree sample data structures, an overview of mixed effects models in general and in particular nonlinear mixed effects (NLME) models in forestry. This overview work creates an insight into the state of the art of the modeling process of the mixed effects models and growing interest of this statistical method in forestry. Moreover, it enabled to identify statistical concerns in the modeling process that may require attention for valid inferences. Paper II demonstrates the nonlinear mixed effects modeling process motivated by tree merchantable volume data and introduces four phases of modeling process for clarity and simplicity for practitioners in the area. It also compares the inferential consequences of assuming simplified covariance structure vis-a-vis the covariance structures revealed by the data.

Paper III compared tree taper models on the basis of several statistical criteria and recommended the best model for use of forest management and planning activities of the Shashemene Forest Industry Enterprise in Ethiopia. On the basis of the best tree taper model (Kozak, 1988) recommended for use in paper III, an effort of optimal experimental design study is made in Paper IV for the Kozak (1988) tree taper model. In the following sections a more detailed summary of the papers is given.

4.1 Paper I: An overview of Mixed Effects Models in Forestry

In about six decades, forest quantification methods have evolved from a simple graphical approach to complex regression models with stochastic structural components. Forest growth and yield data is of a typical longitudinal data type where repeated measurements over time or some other condition are acquired from the same tree or plot. Currently, mixed effects models methodology is receiving attention in the forestry literature. In particular nonlinear mixed effects models have shown importance as tools for growth and yield modeling . However, the review indicates a tendency to use simple covariance structures in the NLME modeling process and this may lead to mis-specification of the covariance structures and hence affect subsequent inferences.

4.2 Paper II: Mixed effects models in forestry: Modeling covariance structures

The literature review work in Paper I indicates a tendency in forestry research to overlook the within-group covariance structures in the modeling process. Motivated by this statistical concern, a nonlinear mixed effects modeling process is demonstrated using *Cupressus lustanica* tree merchantable volume and compared several models with and without covariance structures. For simplicity and clarity of the nonlinear mixed effects modeling, four phases of modeling process are introduced. The nonlinear mixed effects *C. lustanica* tree merchantable volume model with the covariance structures for both the random effects and within group errors has shown significant improvement over the model with simplified covariance matrix.

4.3 Paper III: Tree taper models for *Cupressus lusitanica* plantations in Ethiopia

To provide a taper model for planning and management purpose of *Cupressus lusitanica* plantation in Ethiopia, seven taper models were compared. Several performance indicator statistics were used for comparing the models in their ability to estimate tree diameter and volume. For the selected species, Kozak (1988) was found to be the best, followed by modified Lee *et al.* (2003) and Kozak (2004) as second and third best taper models, respectively. Both the Kozak (Kozak, 1988, 2004) and the modified Lee *et al.* (2003) models were very flexible in capturing the different shapes of trees. Particularly Kozak (2004) proved to be best of all models in diameter estimation even though found to be inferior to Kozak (1988) and modified Lee *et al.* (2003) for total and merchantable volume estimation. Further investigation of the

Kozak (1988) tree taper model was carried out using Monte Carlo and mixed effects modeling mainly to understand the influence of the the parameter p in the performance of the model. The study seems to indicate the need to estimate p from the data.

4.4 Paper IV: D_s-optimal design for the Kozak (1988) tree taper model

The optimal experimental design for the Kozak (1988) tree taper model is investigated in this work. The important advantage of the optimal experimental designs is to reduce both cost and time for data collection by significantly reducing the number of necessary measurements. Moreover, optimal experimental designs yield more precise parameter estimates. The (near) D_s-optimal designs obtained for the Kozak (1988) model were found not to be dependent on tree size and hence used as a reference to construct a general replication-free D_s-optimal (sub) design for use in the tree taper data collection. The limitation of this method is that it requires good preliminary estimates of the true parameter values. Hence, it is important to set good prior parameter estimates either from preliminary data or from results of similar studies.

5 Concluding Remarks

Interest in statistical applications is not only an increasing demand in many scientific areas at this age but there is also an increasing demand and challenge for new or tailored statistical methods to solve problems stimulated by man's ever growing quest to wisely exploit scarce resources. Likewise, statistical methods have a wide application in forestry which otherwise could have been difficult to imagine existing development in forest sciences. A long tradition of the forest biometrics as a discipline in forest sciences is a test of the truth. In this thesis, issues related to statistical modeling and design in single tree models are explored.

In paper I, the contribution is basically a review of the development of forest quantification methods, and mixed effects models in application to forestry. This overview work creates an insight into the state of the art of the modeling process of the mixed effects models and the growing interest of this statistical method in forestry. Moreover, it enabled to identify statistical concerns in the modeling process that may require attention for valid inferences. On the basis of the paper I recommendation, paper II demonstrates a nonlinear mixed effects modeling process motivated with *Cupressus lustanica* tree merchantable volume and compared several models with and without covariance structures. The nonlinear mixed effects *C. lustanica* tree merchantable volume model with the covariance structures for both the random effects and within group errors has shown significant improvement over the model with simplified covariance matrix. However, the statistical significance gained from the complex covariance structures over the simplified covariance matrix has little to explain in the prediction performance of the nonlinear mixed effects *C. lustanica* tree merchantable volume model. Hence this dilemma requires more attention for further investigation before coming to conclusion in favor of simple covariance structures.

In paper III, using several performance indicator statistics, tree taper models were compared in an effort to propose the best model for the forest management and planning purpose of the Shashemene Forest Industry Enterprise in Ethiopia. As noted in subsection 2.2, the practicality of the mixed effects models for prediction purpose in this particular taper models is thought to be limited and hence focused on comparing fixed effects model in this paper. Since the best and recommended Kozak (1988) tree taper model is a complex model with several parameters and may be also with collinearity concerns, further study for improvement of the model or better parsimonious tree taper model is open. Based on the Kozak (1988) tree taper model, D_s-optimal experimental design study is carried out in Paper IV. Hence, the forestry models seem to be yet an open area to explore the theory of optimal experimental design. In this study, D_s-optimal (sub) replication free designs are suggested for the Kozak (1988) tree taper model. The limitation of this method is that it requires good preliminary estimates of the true parameter values. Thus, it is important to set good prior parameter estimates either from preliminary data or from results of similar studies. With this regard, as a further work, it may be recommended to study a maximin design procedure to incorporate uncertainty with respect to the parameters in the construction of optimal designs.

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