Mixed Taxation, Public Goods and Transboundary Externalities: A Model with Large Jurisdictions*

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Abstract

This paper concerns income taxation, commodity taxation, production taxation and public good provision in a multi-jurisdiction framework with transboundary environmental damage. We assume that each jurisdiction is large in the sense that its government is able to influence the world-market producer price of the externality-generating commodity. The decision-problem facing the government in each such jurisdiction is represented by a two-type model (with asymmetric information between the government and the private sector). We show how the possibility to influence the world-market producer price adds mechanisms of relevance for redistribution and externality-correction which, in turn, affect the domestic use of taxation and public goods. Finally, with the noncooperative Nash equilibrium as a reference case, we consider the welfare effects of policy coordination.

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1 Introduction

In the literature on transboundary environmental problems, it has been recognized that national environmental policies may fail to fully internalize externalities, and that policy cooperation among countries (or regions) is generally required in order to reach a globally optimal resource allocation. There are several sources of inefficiency associated with noncooperative policies; for instance, individual countries are likely to disregard the transboundary component of the environmental damage they cause, since their policy-decisions are typically governed by national objectives, and their policies may also give rise to side effects via changes in the price system. However, despite the existence of certain supranational agreements, there is still substantial room for policies decided upon at the national level or by subgroups of countries such as the EU, suggesting that the incentives underlying decentralized policies are important to understand.

This paper concerns optimal taxation and public good provision at the national level, as well as the welfare effects of policy coordination, in an economy where the aggregate consumption of a particular good, to be called 'dirty good', generates a transboundary environmental problem. Our study is based on a framework with mixed taxation, where each national government faces a nonlinear income tax as well as linear commodity and production taxes. This set of tax instruments provides a reasonably realistic description of the tax system that many national governments have at their disposal. It also implies that the use of distortionary taxes is a consequence of optimization under informational restrictions; it is not a consequence of any (arbitrary) restriction imposed on the set of tax instruments.

Contrary to earlier literature on environmental policy under mixed taxation (see below), we assume that the countries are large in the sense that each national government is able to significantly affect the world-market producer price of the externality-generating good. Such a framework is interesting to consider for at least two reasons. First, although many countries are small enough to make the 'price-taking government' assumption realistic, the environmental policy scene is also characterized by large actors such as the U.S. and some other countries, as well as by subgroups of countries acting together such as the EU, where the price-taking assumption appears to be less realistic. Our study takes this observation to its extreme point by analyzing a world-economy comprising a number of large actors, whose governments recognize (and incorporate into their decision-problems) that their policies will affect the world-market producer prices of externality-generating commodities. Second, our approach integrates earlier literature on the so called 'leakage' phenomenon with the theory of mixed taxation, which makes it possible to compare large and small open economies with respect to the whole tax structure; not just with respect to environmental policy.

The literature on fiscal policy in second best economies with transboundary environmental problems is relatively small by comparison with the corresponding literature dealing with fiscal policy in second best economies with local (i.e. within-jurisdiction) environmental damage[1]. Earlier research in the former category, nevertheless, addresses a variety of issues such as comparisons between noncooperative and cooperative regimes with respect to tax policies [2], labor mobility[3], fiscal competition due to international trade[4] and strategic aspects of public policy in the context of economic federations[5]. However, none of the studies that we are aware of combines transboundary environmental problems and mixed taxation in the context of large open economies. An interesting observation (discussed many times in other contexts) is that there might be emission-leakage associated with the environmental policy decided upon by national governments; for instance, if higher emission taxes in a particular jurisdiction significantly reduces the demand for the externality-generating good, then the producer price will also decrease which, in turn, tends to increase the emissions abroad[6]. This suggests that, if the country is large in the sense that its government can significantly influence the world-market producer price of the externality-generating good, then it may have incentives to modify its use

[1] Earlier literature on fiscal policy under environmental externalities often abstracts from international (or interregional) spillover effects of environmental damage by focusing on 'one-country' model-economies. See the seminal contribution by Sandmo (1975) and the subsequent work by e.g. Pirttilä and Tuomala (1997) and Cremer and Gahvari (2001). See also the related research on environmental policy reforms and so called 'double-dividends', e.g. Bovenberg and de Mooij (1994), Bovenberg and Goulder (1996), Parry et al. (1999) and Aronsson (1999).

[2] See Aronsson and Blomquist (2003).

[3] See Aronsson and Blomquist (2003).

[4] See Cremer and Gahvari (2004, 2005).

[5] See Silva and Caplan (1997), Caplan and Silva (1999) and Aronsson et al. (2006).

[6] Various mechanisms by which emission-leakage may appear have been discussed by e.g. Gurzgen and Rauscher (2000), Conconi (2003) and Lai and Hu (2005). See also the empirical study by Sengupta and Bhardwaj (2004), which is a case study applied to India.

of environmental policy. Our paper incorporates this mechanism into the theory of income and commodity taxation.

As the number of countries is of no particular concern in what follows, the present paper focuses on a two-country model, in which each country is characterized by two-ability types. The countries interact both via the environmental damage they impose on each other and international trade[7]. Our paper contributes to the literature in primarily two ways. The first is by characterizing the income, commodity and production tax structure, as well as the provision of national public goods, in a noncooperative Nash equilibrium, where each country implements its own policy conditional on the policies chosen by the other country. We show how the additional policy incentives associated with the endogenous world-market producer price affect the domestic use of income taxation, commodity taxation and public good provision in a noncooperative Nash equilibrium, relative to the policy rules that would apply with fixed producer prices. Furthermore, our results show that the ability to influence the world-market producer price provides an incentive for each national government to implement a production tax (in addition to the income and commodity taxes) as well as to deviate from production efficiency in the public production. The second is by analyzing the welfare effects of policy coordination, where the noncooperative Nash equilibrium constitutes the reference case. Although the welfare effects of policy coordination are typically ambiguous in the general case, we show for a special case of the model that welfare improving policy coordination may include, e.g., an increase in the production tax or commodity tax accompanied by increased public production, or increased average income taxation accompanied by increased public production. The intuition is that each such reform contributes to reduced environmental damage (either directly or indirectly via the world-market producer price).

The outline of the study is as follows. In section 2, we present the model, the outcome of private optimization and market equilibrium conditions, whereas the decision-problem of the government is discussed in section 3. Section 4 concerns optimal taxation and public good provision at the national level, while policy

[7] We abstract from international factor mobility throughout the paper. Therefore, although the policy incentives discussed below would also appear in a more general framework (with factor mobility being yet another source of interaction between the countries), allowing for factor mobility is clearly an important extension for future research.

coordination is dealt with in section 5. We summarize and discuss the results in section 6.

2 The Model

Consider an economy comprising two jurisdictions, which will be called 'countries' in what follows. We begin by describing the consumers in each such country. Having done that, we continue with the production side and market equilibrium conditions.

2.1 The Consumers

In each country, i (where $i = 1, 2$), there are two types of consumers; a low-ability type (denoted by l) and a high-ability type (denoted by h). The distinction between ability-types refers to productivity, meaning that the high-ability type is more productive and faces a higher before-tax wage rate than the low-ability type. Since the number of indivudals of each such ability-type is not important for the analysis to be carried out below, it will be normalized to one for notational convenience.

The preferences of ability-type j $(j = l, h)$ in country i are described by the utility function $U^{i,j} = U(C^{i,j}, X^{i,j}, Z^{i,j}, G^i, E)$, where $C^{i,j}$ denotes (the consumption of) an environmentally clean good, $X^{i,j}$ an environmentally dirty good, $Z^{i,j}$ leisure, G^i a national public good and E the environmental damage. We assume that $C^{i,j}$ and $X^{i,j}$ are normal goods. Leisure is defined as $Z^{i,j} = H - L^{i,j}$, where H is a time endowment and $L^{i,j}$ the hours of work. The function $U(\cdot)$ is increasing in $C^{i,j}$, $X^{i,j}$, $Z^{i,j}$ and G^i, decreasing in E and strictly quasiconcave. We also assume that the environmental damage is caused by the aggregate consumption (measured over all countries) of the dirty good (see below), and that the consumers treat E as exogenous. The clean good is untaxed and its price is normalized to one. The consumer price of the dirty good is given by $Q^i = P + t^i$, where P is the producer price and t^i the commodity tax decided upon by the government in country i. Therefore, as both commodities are subject to international trade, the producer prices are assumed to be equalized across countries.

The consumer chooses $C^{i,j}$, $X^{i,j}$ and $L^{i,j}$ to maximize utility subject to the budget constraint,

$$w^{i,j}L^{i,j} - T^i\left(w^{i,j}L^{i,j}\right) - C^{i,j} - Q^i X^{i,j} = 0, \tag{1}$$

where $T^i(\cdot)$ is the income tax decided upon by the government in country i. Since the optimal tax and expenditure problem below will be defined in terms of conditional indirect utility functions, it is convenient to follow Christiansen (1984) by solving the consumer's optimization problem in two stages. In the first stage, the utility maximization problem is solved conditional on the hours of work. This problem is written

$$\max_{C^{i,j},X^{i,j}} U\left(C^{i,j}, X^{i,j}, Z^{i,j}, G^i, E\right)$$

subject to

$$B^{i,j} = C^{i,j} + Q^i X^{i,j}$$

in which $B^{i,j}$ is treated as a fixed post-tax income. The solution to this problem gives the conditional demand functions

$$X^{i,j} = X\left(Q^i, B^{i,j}, Z^{i,j}, G^i, E\right) \tag{2}$$

$$C^{i,j} = C\left(Q^i, B^{i,j}, Z^{i,j}, G^i, E\right) \tag{3}$$

and the conditional indirect utility function

$$V^{i,j} = V\left(Q^i, B^{i,j}, Z^{i,j}, G^i, E\right). \tag{4}$$

In the second stage, we can derive the hours of work by maximizing the conditional indirect utility function with respect to $L^{i,j}$ subject to the budget constraint

$$B^{i,j} = w^{i,j}L^{i,j} - T\left(w^{i,j}L^{i,j}\right). \tag{5}$$

The first order condition for this problem is written

$$V_B^{i,j} w^{i,j}(1 - T_I^{i,j}) - V_Z^{i,j} = 0 \tag{6}$$

where $V_B^{i,j} = \partial V^{i,j}/\partial B^{i,j}$ and $V_Z^{i,j} = \partial V^{i,j}/\partial Z^{i,j}$ represent the marginal utility of private income and leisure, respectively, while $I^{i,j} = w^{i,j}L^{i,j}$ is the labor income facing ability-type j in country i and $T_I^{i,j} = \partial T^i \left(I^{i,j}\right) / \partial I^{i,j}$ the corresponding marginal income tax rate.

2.2 Production

The production side in each country consists of one public and two private sectors. The public sector production function is written $G^i = F_G^i(L_G^{i,l}, L_G^{i,h})$, where $L_G^{i,j}$ is the amount of labor of ability-type j ($j = l, h$) that the public sector uses. We assume that both inputs are essential, that the marginal product of each factor is positive and diminishing, and that the production technology is characterized by constant returns to scale.

Turning to private production, the clean good is produced in sector c, whereas the dirty good is produced in sector x. Production in each sector is characterized by constant returns to scale. Given these characteristics, the number of firms in each sector is not, itself, important and will be normalized to one. The production functions can be written $F_c^i(L_c^{i,h}, L_c^{i,l})$ and $F_x^i(L_x^{i,h}, L_x^{i,l})$, where $L_c^{i,j}$ and $L_x^{i,j}$ represent the amount of labor of ability-type j used by sector c and x, respectively, in country i. Normalizing with respect to the low-skilled labor type in each sector, we have

$$f_c^i\left(n_c^i\right) = \frac{F_c(L_c^{i,h}, L_c^{i,l})}{L_c^{i,l}} \tag{7}$$

$$f_x^i\left(n_x^i\right) = \frac{F_x(L_x^{i,h}, L_x^{i,l})}{L_x^{i,l}} \tag{8}$$

where $n_c^i = L_c^{i,h}/L_c^{i,l}$ and $n_x^i = L_x^{i,h}/L_x^{i,l}$.

We assume that the government implements a revenue tax in sector x at the rate τ^i, while the good produced in sector c is untaxed. We also assume that the workers are perfectly mobile between sectors (yet immobile between countries), which means that the type-specific wage rates in each country will be the same in both sectors. The first order conditions for profit maximization can be written as

$$w^{i,h} = \frac{\partial f_c^i\left(n_c^i\right)}{\partial n_c^i},$$

$$w^{i,l} = f_c\left(n_c^i\right) - n_c^i \frac{\partial f_c^i\left(n_c^i\right)}{\partial n_c^i},$$

$$w^{i,h} = \left(1-\tau^i\right) P \frac{\partial f_x^i\left(n_x^i\right)}{\partial n_x^i} \text{ and}$$

$$w^{i,l} = \left(1-\tau^i\right) P \left[f_x\left(n_x^i\right) - n_x^i \frac{\partial f_x^i\left(n_x^i\right)}{\partial n_x^i}\right]. \tag{9}$$

2.3 Equilibrium

By using equations (9) together with the identities

$$L^{i,h} - L_G^{i,h} = L_c^{i,h} + L_x^{i,h} \tag{10}$$

$$L^{i,l} - L_G^{i,l} = L_c^{i,l} + L_x^{i,l} \tag{11}$$

we can define $w^{i,j}$, $L_c^{i,j}$ and $L_x^{i,j}$ as functions of $L^{i,h} - L_G^{i,h}$, $L^{i,l} - L_G^{i,l}$ and $\left(1-\tau_x^i\right) P$, i.e.

$$w^{i,j} = w^{i,j}\left(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau_x^i\right) P\right) \tag{12}$$

$$L_c^{i,j} = L_c^{i,j}\left(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau_x^i\right) P\right) \tag{13}$$

$$L_x^{i,j} = L_x^{i}\left(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau_x^i\right) P\right) \tag{14}$$

for $j = l, h$. By substituting equations (13) and (14) into the production functions, we obtain 'the equilibrium supply functions'

$$S_c^i = S_c^i\left(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau_x^i\right) P\right) \tag{15}$$

$$S_x^i = S_x^i\left(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau_x^i\right) P\right). \tag{16}$$

Since the two goods are subject to international trade, the equilibrium condition in the market for the dirty good can be written as

$$\sum_{i=1}^{2} S_x^i - \sum_{i=1,2}\sum_{j=h,l} X^{i,j} = 0. \tag{17}$$

As long as equation (17) is fulfilled, Walras' law implies that the market for the clean good is in equilibrium as well. By using $Q^i = P + t^i$, equation (17) implicitly defines the producer price of the dirty good, i.e.

$$P = P\left(\mathbf{B}^1, \mathbf{L}^1, \mathbf{L}_G^1, t^1, \tau_x^1, G^1, \mathbf{B}^2, \mathbf{L}^2, \mathbf{L}_G^2, t^2, \tau_x^2, G^2, E\right) \tag{18}$$

where $\mathbf{B}^i = \left(B^{i,h}, B^{i,l}\right)$, $\mathbf{L}^i = \left(L^{i,h}, L^{i,l}\right)$ and $\mathbf{L}_G^i = \left(L_G^{i,h}, L_G^{i,l}\right)$ for $i = 1, 2$. To derive equation (18), we have used $L^{i,j} = H - Z^{i,j}$ and suppressed the time endowment.

The environmental damage facing the residents in each country equals the sum of the consumption of the dirty good taken over both countries[8], i.e.

$$E = \sum_{i=1,2}\sum_{j=h,l} X^{i,j}. \tag{19}$$

3 The Public Decision-Problem

We assume that each national government faces a utilitarian social welfare function[9]

$$W^i = \sum_{j=l,h} V^{i,j}. \tag{20}$$

The tax instruments are the production tax, income tax and commodity tax, which are used for purposes of redistribution and public good provision. Therefore, the government budget constraint can be written as

$$\tau^i S_x^i + \sum_{j=h,l} T^{i,j} + t^i \sum_{j=h,l} X^{i,j} - \sum_{j=h,l} w^{i,j} L_G^{i,j} = 0 \tag{21}$$

[8] This formulation can be exemplified by the climate problem.

[9] Alternative approaches would be to assume (as in many comparable studies) that the government is maximizing the utility of one ability-type subject to a minimum utility restriction for the other, or to assume that the government uses a general social welfare function. All qualitative results derived below would hold also under the alternative formulations.

where $T^{i,j} = T^i(w^{i,j}L^{i,j})$.

Since $T^i(\cdot)$ is a general labor income tax, which can be used to implement any desired combination of consumption and hours of work for each ability-type, it is convenient to use $B^{i,h}$, $L^{i,h}$, $B^{i,l}$ and $L^{i,l}$, instead of the parameters of the income tax function, as direct decision-variables in the optimal tax and expenditure problem. Therefore, let us rewrite the budget constraint of the government by combining equation (21) with the individual budget constraints and the zero profit conditions following from the assumption of constant returns to scale in production, i.e.

$$0 = S_c^i + PS_x^i + t^i \sum_{j=h,l} X^{i,j} - \sum_{j=h,l} B^{i,j}. \tag{22}$$

The informational assumptions are conventional; the government observes the income of each individual, although ability is private information. The latter means that, in the absence of appropriate type-revealing mechanisms, the government would not be able to observe whether any given worker is a low-ability or high-ability type. We concentrate on the 'normal' case, where the government wants to redistribute from the high-ability to the low-ability type. Therefore, the relevant aspect of self-selection is to prevent the high-ability type from pretending to be a low-ability type. The self-selection constraint that may bind then becomes

$$V^{i,h} = V\left(Q^i, B^{i,h}, Z^{i,h}, G^i, E\right) \geq V\left(Q^i, B^{i,l}, \hat{Z}^{i,h}, G^i, E\right) = \hat{V}^{i,h} \tag{23}$$

where $\hat{V}^{i,h}$ denotes the utility of a high-ability mimicker and $\hat{Z}^{i,h} = H - \phi^i L^{i,l}$ the amount of leisure consumed by the mimicker. The term $\phi^i = w^{i,l}/w^{i,h} < 1$ denotes the wage ratio (or relative wage rate) in country i. By using equations (12), the wage ratio can be written as $\phi^i = \phi^i(L^{i,h} - L_G^{i,h}, L^{i,l} - L_G^{i,l}, \left(1-\tau^i\right)P)$, in which P is determined by equation (18). Note that the mimicker faces the same before-tax and disposable income as the low-ability type; however, as the mimicker is more productive than the low-ability type, he/she also consumes more leisure than the low-ability type.

The Lagrangean can be written as

$$\mathcal{L}^i = W^i + \rho^i[F_G(\mathbf{L}_G^i) - G^i] + \lambda^i[V^{i,l} - \hat{V}^{i,h}] + \gamma^i[S_c^i + PS_x^i + t^i \sum_{j=h,l} X^{i,j}$$
$$- \sum_{j=h,l} B^{i,j}] + \mu^i[E - \sum_{n=1,2} \sum_{j=h,l} X^{n,j}]$$

for $i = 1, 2$, where W^i, $V^{i,j}$, $\hat{V}^{i,h}$, $X^{i,j}$, S_c^i and S_x^i were defined above, whereas ρ^i, λ^i, γ^i and μ^i are Lagrange multipliers. Note also that P is endogenous to the national government in country i and determined by equation (18). The decision-variables facing the government in country i are $L^{i,l}$, $B^{i,l}$, $L^{i,h}$, $B^{i,h}$, $L_G^{i,l}$, $L_G^{i,h}$, t^i, τ^i and G^i. Note also that equation (19) appears as an explicit constraint in the Lagrangean, meaning that E will be treated as an additional (and artificial) decision-variable. The first order conditions are presented in the Appendix.

4 The Noncooperative Nash Equilibrium

It is convenient to start the analysis by evaluating how an increase in the world-market producer price of the externality-generating good affects the national welfare. In the Appendix, we show that

$$\frac{\partial \mathcal{L}^i}{\partial P} = \frac{\gamma^i}{\beta^i}\left[\frac{\lambda^i L^{i,l}\hat{V}_Z^{i,h}}{\gamma^i}\frac{\partial \phi^i}{\partial P} + NX^i - \frac{\mu^i}{\gamma^i}\sum_{j=h,l}\frac{\partial X^{k,j}}{\partial Q^k}\right] \tag{24}$$

for $k \neq i$, where $\beta^i = 1 + \partial P/\partial t^i > 0$, and $NX^i = S_x^i - \sum_{j=h,l} X^{i,j}$ is the net export of the dirty good.

Equation (24) shows that the welfare effect of an increase in P can be decomposed into three parts. The first term on the right hand side appears because the wage ratio depends on the producer price of the dirty good. If $\partial\phi^i/\partial P > 0$, a higher producer price leads to an increase in the wage ratio, which makes mimicking less attractive and contributes to relax the self-selection constraint. In this case, therefore, the first term within the square bracket contributes to higher welfare. By analogy, if $\partial\phi^i/\partial P < 0$, the first term within the square bracket contributes to lower welfare. The second term, $\gamma^i NX^i/\beta^i$, represents a terms of trade effect. If the country is a net exporter of the dirty good, a higher producer price increases the value of the net exports which, in turn, leads to higher welfare for country i. The opposite argument applies if the country is a net importer of the dirty good.

The final term on the right hand side of equation (24) arises because an increase in the producer price of the dirty good leads to a lower demand for the dirty good in the other country (conditional on the commodity tax implemented by the other country). This effect reduces the environmental damage which, in turn, leads to increased welfare for country i if $\mu^i/\gamma^i > 0$, and decreased welfare for country i if $\mu^i/\gamma^i < 0$. We can interpret μ^i/γ^i as the real shadow price that the government in country i attaches to a reduction in the environmental damage. The determination of this shadow price is the issue to which we will turn next.

Following earlier research on environmental policy and mixed taxation[10], it is convenient to define the shadow price of environmental damage over the shadow price of the government's budget contraint, μ^i/γ^i, as this real shadow price will play an important role in the optimal tax and expenditure policy to be analyzed below. One may interpret μ^i/γ^i as the marginal value that the government in country i attaches to reduced environmental damage measured in terms of its tax revenues. Let us define

$$MWP^{i,j}_{E,B} = -\frac{\partial V^{i,j}/\partial E}{\partial V^{i,j}/\partial B^{i,j}}, \quad \widehat{MWP}^{i,h}_{E,B} = -\frac{\partial \hat{V}^{i,h}/\partial E}{\partial \hat{V}^{i,h}/\partial B^{i,l}}$$

as the marginal willingness to pay for a small reduction in the environmental damage by ability-type i and the mimicker, respectively. To simplify the exposition, we will also use the following short notations[11];

[10] See e.g. Pirttilä and Tuomala (1997), Aronsson and Blomquist (2003) and Aronsson et al. (2006).

[11] Note that μ^i/γ^i is calculated by using the first order conditions for E, $B^{i,l}$ and $B^{i,h}$, which explains why compensated derivatives of the demand for the dirty good in country i appear in the expression for μ^i/γ^i along with uncompensated derivatives of the demand for the dirty good in the other country, k. The intuition is that the government in country i only recognizes the domestic budget effects of its own environmental policy.

$$\text{(i) } \alpha = \sum_{i=1,2} \frac{\partial S_x^i}{\partial P} - \sum_{i=1,2} \sum_{j=h,l} \frac{\partial X^{i,j}}{\partial Q^i} > 0$$

$$\text{(ii) } \frac{\partial \tilde{X}^{i,j}}{\partial E} = \frac{\partial X^{i,j}}{\partial E} + \frac{\partial X^{i,j}}{\partial B^{i,j}} MWP_{E,B}^{i,j}$$

$$\text{(iii) } \frac{1}{\sigma^i} = 1 - \sum_{j=l,h} \frac{\partial \tilde{X}^{i,j}}{\partial E} - \sum_{j=l,h} \frac{\partial X^{k,j}}{\partial E}$$

$$\text{(iv) } \frac{1}{\check{\sigma}^i} = \frac{1 + (1-\sigma^i)[\sum_j (\partial X^{k,j}/\partial Q^k)]/\alpha\beta^i}{\sigma^i}$$

which refer to, respectively, (i) the increased net supply of the dirty good caused by an increase in the producer price, i.e. the partial derivative of equation (17) with respect to P, (ii) the change in the compensated demand for the dirty good caused by increased environmental damage, (iii) the environmental feedback effect that would apply under a fixed producer price, and (iv) the full environmental feedback effect. The component $(1-\sigma^i)/\alpha\sigma^i$ of the expression for $1/\check{\sigma}^i$ measures the change in the producer price[12] of the dirty good that would arise from a marginal decrease in E which, if multiplied by $\sum_j (\partial X^{k,j}/\partial Q^k)$, gives the corresponding change in the demand for the dirty good in the other country (country k). Consider Proposition 1;

Proposition 1. *In the noncooperative Nash equilibrium, the shadow price of environmental damage over the shadow price of the government's budget constraint in country i can be written as*

$$\frac{\mu^i}{\gamma^i} = \check{\sigma}^i \left[\sum_{j=h,l} MWP_{E,B}^{i,j} + \lambda^{i,*} \left(MWP_{E,B}^{i,l} - \widehat{MWP}_{E,B}^{i,h} \right) - t^i \sum_{j=h,l} \frac{\partial \tilde{X}^{i,j}}{\partial E} \right]$$
$$+ \frac{\check{\sigma}^i (1-\sigma^i)}{\alpha\beta^i} \left[\frac{\lambda^i L^{i,l} \hat{V}_Z^{i,h}}{\gamma^i} \frac{\partial \phi^i}{\partial P} + NX^i \right]$$

where $\lambda^{i,*} = \lambda^i \hat{V}_B^{i,h} / \gamma^i$.

[12] As the national government recognizes the domestic budget consequences of its environmental policy, this effect is calculated with the domestic utility held constant. See also footnote 11.

Proof: See the Appendix.

The environmental feedback effect, $\breve{\sigma}^i$, captures that a change in the externality - due to a change in the demand for the dirty good - 'feeds back' into the demand equations (both directly and indirectly via the world-market producer price). To guarantee stability of the model, we follow earlier literature[13] by assuming that the environmental feedback effect is positive.

In the first row, all terms within the square bracket are well understood from earlier research (e.g. Pirttilä and Tuomala 1997), and our discussion of each of these components will, therefore, be brief. The consumers' marginal willingness to pay for reduced environmental damage is captured by the first term, which (by the assumptions made earlier) contributes to increase the marginal value that the government attaches to reduced environmental damage. The second term appears because a change in E will affect the self-selection constraint. If the low-ability type is willing to pay more (less) at the margin than the mimicker for reduced environmental damage, the govenment attaches a higher (lower) marginal value to reduced environmental damage than it would otherwise have done, as a reduction in E in this case contributes to relax (tighten) the self-selection constraint. As for the final term, note that a change in the environmental damage influences the revenues from the commodity tax: if a reduction in E leads to increased tax revenues via the demand for the dirty good, ceteris paribus, then the tax revenue effect reinforces the environmental motive behind the public policy and contributes, therefore, to increase μ^i/γ^i. The opposite argument applies if an increase in E leads to higher tax revenues.

The second row of the formula for μ^i/γ^i appears because the world-market producer price of the dirty good is endogenous to the government in country i. Suppose, to begin with, that $1 - \sigma^i < 0$, in which case an increase in the environmental damage (with the utility facing the domestic residents held constant at the optimum) leads to a higher world-market producer price of the dirty good[14], ceteris paribus. In this case, and if $\partial\phi^i/\partial P > 0$ (< 0), which means that a higher (lower) producer price contributes to relax the self-selection

[13] See Sandmo (1980). See also, e.g., Pirttilä and Tuomala (1997) and Aronsson and Blomquist (2003).

[14] Although $1 - \sigma^i$ cannot be signed unambiguously in the general case, we show in the Appendix that it is negative if the environmental damage is weakly separable from the other goods in the utility function.

constraint, the government will attach a lower (higher) value to reduced environmental damage than it would otherwise have done. Note also that country i may either be a net exporter or net importer of the dirty good: if country i is a net exporter, so $NX^i > 0$, the second term on the right hand side contributes to reduce μ^i/γ^i, while it contributes to increase μ^i/γ^i if country i is a net importer of the dirty good. The intuition is, of course, that the higher world-market producer price (caused by increased environmental damage) generates an extra benefit if the country is a net exporter and an extra cost if the country is a net importer. Interpretations analogous to those discussed above - yet with the opposite qualitative effects of the terms in the second row - will follow if $1 - \sigma^i > 0$.

In the interpretations of the optimal tax formulas to be presented below, we will add the (realistic) assumption that the government attaches a positive marginal value to reduced environmental damage, i.e. $\mu^i/\gamma^i > 0$.

4.1 Commodity Taxation

Let us now turn to the commodity tax structure. To simplify the analysis, we shall use the following short notation;

$$\varphi^i = \frac{\sum_{n=1,2} (\partial S_x^n/\partial P)}{\sum_{n=1,2} (\partial S_x^n/\partial P) - \sum_{j=h,l} (\partial X^{k,j}/\partial Q^k)} \in (0,1)$$

for $k \neq i$, for a scale variable that influences the relationship between the commodity tax on the dirty good and the shadow price that the government attaches to reduced environmental damage. This scale variable will be further discussed below. Consider Proposition 2;

Proposition 2. *In the noncooperative Nash equilibrium, the commodity tax on the dirty good implemented by country i can be written as*

$$t^i = \frac{\lambda^{i,*}}{\sum_j \partial \tilde{X}^{i,j}/\partial Q^i} \left(X^{i,l} - \hat{X}^{i,h} \right) - \frac{\lambda^i L^{i,l} \hat{V}_Z^{i,h}}{\alpha \beta^i \gamma^i} \frac{\partial \phi^i}{\partial P} - \frac{NX^i}{\alpha \beta^i} + \varphi^i \frac{\mu^i}{\gamma^i}.$$

Proof: See the Appendix.

The first two terms on the right hand side are due to the self-selection constraint and are analogous to results derived in earlier research (Edwards et al. 1994 and Naito 1999). As $\partial\tilde{X}^{i,j}/\partial Q^i < 0$, the first term on the right hand side is positive if leisure is complementary with the dirty good in the sense that $X^{i,l} - \hat{X}^{i,h} < 0$, and negative if leisure is substitutable for the dirty good in the sense that $X^{i,l} - \hat{X}^{i,h} > 0$. The intuition is that the government may relax the self-selection constraint by implementing a higher (lower) commodity tax on goods that are complementary with (substitutable for) leisure, ceteris paribus[15]. The second term on the right hand side appears because a higher commodity tax reduces the world-market producer price of the dirty good which, in turn, affacts the wage distribution. If $\partial\phi^i/\partial P > 0$ (< 0), an increase in this producer price makes mimicking less (more) attractive, which provides an incentive for the government to implement a lower (higher) commodity tax than it would otherwise have done.

The third term on the right hand side of the tax formula in Proposition 2 represents a terms of trade effect; as such, its qualitative influence on the tax depends on whether country i is a net exporter or a net importer of the dirty good. If the country is a net exporter, meaning that $NX^i > 0$, a higher producer price of the dirty good increases the value of the net export which, itself, is welfare improving and can be accomplished by lowering the commodity tax (recall that $\partial P/\partial t^i < 0$). In this case, therefore, the third term on the right hand side contributes to reduce the commodity tax. The effect would be the opposite, if country i is a net importer of the dirty good.

The desire to correct for the externality imposed on the domestic residents is captured by the fourth term on the right hand side, which reflects the additivity property discussed by Sandmo (1980). However, an important difference by comparison with earlier literature is that this effect is here scaled down with the factor $\varphi^i < 1$. The intuition is that, by implementing a smaller t^i than would be required by full (domestic) externality-correction, it follows that the domestic demand for the dirty good increases. Such a policy leads to a higher world-market producer price of the dirty good which, in turn, reduces the demand for the dirty good by the residents in the other country. The latter contributes to reduce E and is, therefore, welfare improving from the point of view of country i.

[15] Recall that the mimicker faces the same before-tax and disposable income as the low-ability type. Therefore, in the special case where leisure is weakly separable in terms of the utility function, we have $X^{i,l} - \hat{X}^{i,h} = 0$.

To further highlight the interpretation of the fourth term on the right hand side of the tax formula in Proposition 2, i.e. in order to focus solely on externality-correction, we consider the following special case;

Corollary 1. *In a symmetric noncooperative Nash equilibrium ($NX^i = 0$), and if the self-selection constraint does not bind ($\lambda^i = 0$), the commodity tax on the dirty good reduces to*

$$t^i = \varphi^i \frac{\mu^i}{\gamma^i} < \frac{\mu^i}{\gamma^i}.$$

The corollary means that the commodity tax falls short of the marginal value that each national government attaches to reduced environmental damage; in other words, the externality-correcting tax falls short of the tax that would follow from a standard Pigouvian tax formula for the domestic economy, i.e. μ^i/γ^i. Therefore, and by comparison with a globally optimal resource allocation, each national government does not only neglect that the environmental damage generated by the domestic residents affects the well-being of the residents in the other country; it also reduces the tax below the marginal value it attaches to the domestic externality in order to increase the world-market producer price of the externality-generating good, which reinforces the inefficient use of environmental policy in the Nash equilibrium.

4.2 The Production Tax

In a standard model for mixed taxation with fixed producer prices, in which the environmental damage depends on the aggregate consumption of the dirty good, production taxes would be redundant. In our framerwork, on the other hand, the production tax is not redundant. We can derive the following result;

Proposition 3. *In the noncooperative Nash equilibrium, the production tax implemented by country i is given by*

$$\tau^i = \frac{\chi^i}{\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P}$$

where

$$\chi^i = -\frac{\partial P/\partial \tau^i}{\left(\partial F_x^i/\partial L_x^{i,h}\right) \sum_j (\partial L_x^{i,j}/\partial \tau^i) P} > 0$$

in which $\partial P/\partial \tau^i > 0$ *and* $\sum_j \partial L_x^{i,j}/\partial \tau^i < 0$.

Proof: See the Appendix.

Proposition 3 has a simple interpretation: if an increase in the world-market producer price leads to higher (lower) domestic welfare, then the government in country i will implement a positive (negative) production tax on the dirty good. Note also that, although the endogeneity of the world-market producer price is the only mechanism via which the production tax is operative, the production tax will, nevertheless, serve multiple purposes. This is seen from equation (24), where the national welfare effect of an increase in the world-market producer price is shown to depend on (i) a component relating to the wage distribution (which is due solely to the self-selection constraint), (ii) a terms of trade effect and (iii) a component representing externality-correction.

To illustrate the corrective role of the production tax more thoroughly, consider once again the special case of a symmetric equilibrium, in which we also add the assumption that the self-selection constraint does not bind;

Corollary 2. *In a symmetric noncooperative Nash equilibrium ($NX^i = 0$), and if the self-selection constraint does not bind ($\lambda^i = 0$), the production tax in country i reduces to*

$$\tau^i = -\frac{\mu^i}{\gamma^i}\frac{\chi^i}{\beta^i}\sum_{j=h,l}\frac{\partial X^{k,j}}{\partial Q^k} > 0$$

for $k \neq i$.

In this case, therefore, the only reason for implementing a (positive) production tax is to increase the world-market producer price of the dirty good, which provides environmental benefits to country i as the consumption of the dirty good in the other country decreases. The motivation for using the corrective production tax highlighted by the corollary also relates to a more general result: if the government has fewer effective policy instruments than the number of variables it wishes to control, then the commodity tax on the dirty good no longer constitutes a perfect environmental policy instrument. The intuition is that the domestic government cannot use the commodity tax to control both the domestic and foreign consumption of the dirty good. Therefore, the government also uses other policy instruments - in this case the production tax - for the explicit purpose of externality-correction.

4.3 Labor Income Taxation

The arguments behind the use of commodity and production taxation also carry over, in a natural way, to the incentive structure underlying marginal income taxation. To shorten the tax formulas to be discussed below, let

$$MRS_{Z,B}^{i,j} = \frac{V_Z^{i,j}}{V_B^{i,j}} \text{ and } \widehat{MRS}_{Z,B}^{i,h} = \frac{\hat{V}_Z^{i,h}}{\hat{V}_B^{i,h}}$$

denote the marginal rate of substitution between leisure and private (disposable) income for ability-type j and the mimicker, respectively. We also use the short notations

$$\frac{\partial \tilde{X}^{i,j}}{\partial Z^{i,j}} = \frac{\partial X^{i,j}}{\partial Z^{i,j}} - MRS_{Z,B}^{i,j} \frac{\partial X^{i,j}}{\partial B^{i,j}}$$

$$\Psi^{i,j} = -\frac{\partial S_x^i / \partial L^{i,l} + \partial \tilde{X}^{i,l} / \partial Z^{i,l}}{\alpha}$$

for how the conditional compensated demand for the dirty good changes in response to an increase in the use of leisure by ability-type j, and how the world-market producer price responds to a utility-compensated increase in the labor supply by ability-type j, respectively. Consider the following result;

Proposition 4. *In the noncooperative Nash equilibrium, the marginal income tax rates implemented by country i can be written as*

$$T_I^{i,l}\left(w^{i,l}L^{i,l}\right) = \frac{\lambda^{i,*}}{w^{i,l}}\left(MRS_{Z,B}^{i,l} - \phi^i \widehat{MRS}_{Z,B}^{i,h}\right) - \frac{\lambda^{i,*}L^{i,l}}{w^{i,l}} \widehat{MRS}_{Z,B}^{i,h} \frac{\partial \phi^i}{\partial L^{i,l}}$$
$$+ \left(t^i - \frac{\mu^i}{\gamma^i}\right) \frac{1}{w^{i,l}} \frac{\partial \tilde{X}^{i,l}}{\partial Z^{i,l}} - \frac{\Psi^{i,l}}{w^{i,l}\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P}$$

$$T_I^{i,h}\left(w^{i,h}L^{i,h}\right) = -\frac{\lambda^{i,*}L^{i,l}}{w^{i,h}} \widehat{MRS}_{Z,B}^{i,h} \frac{\partial \phi^i}{\partial L^{i,h}}$$
$$+ \left(t^i - \frac{\mu^i}{\gamma^i}\right) \frac{1}{w^{i,h}} \frac{\partial \tilde{X}^{i,h}}{\partial Z^{i,h}} - \frac{\Psi^{i,h}}{w^{i,h}\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P}.$$

Proof: See the Appendix.

The tax formulas in Proposition 4 distinguish between three basic motives for influencing the hours of work; (i) to relax the self-selection constraint (via channels not directly linked to the world-market producer price), (ii) to compensate the consumer for distortions created by the (less flexible) commodity tax, and (iii) to influence the world-market producer price of the dirty good.

We start by discussing the marginal income tax rate implemented for the low-ability type. The first motive for using income taxation mentioned above is captured by the first row on the right hand side, where both terms are analogous to results derived in earlier research (Stiglitz 1982). As the mimicker needs to forego less leisure than the low-ability type to accomplish a given increase in the before-tax income, one can show that $MRS_{Z,B}^{i,l} - \phi^i \widehat{MRS}_{Z,B}^{i,h} > 0$, which means that the first term on the right hand side contributes to increase the marginal income tax rate. The second term on the right hand side in the tax formula for the low-ability type, and the first term on the right hand side in the tax formula for the high-ability type, reflect that a change in the hours of work affects the wage distribution. If (as in Stiglitz 1982) $\partial\phi^i/\partial L^{i,l} < 0$ and $\partial\phi^i/\partial L^{i,h} > 0$, a decrease in the hours of work supplied by the low-ability type and an increase in the hours of work supplied by the high-ability type will contribute to reduce the wage inequality and, therefore, relax the self-selection constraint. As a consequence, these effects contribute to increase the marginal income tax rate of the low-ability type and decrease the marginal income tax rate of the high-ability type.

The first part of the second row in each tax formula in Proposition 4 serves to compensate the consumer for distortions created by the commodity tax[16]. To see the intuition behind this result, note that the government has no <u>direct</u> motive besides externality-correction to distort the consumption of the dirty good; in other words, the self-selection component, the terms of trade effect and the producer price effect only appear in the commodity tax formula in Proposition 2 because the government lacks direct tax instruments to relax the self-selection constraint and/or fully control the world-market producer price.

[16] This motive for using marginal income taxation was also addressed by Aronsson et al. (2006); let be in a simplified model without asymmetric information.

Therefore, if $t^i \neq \mu^i/\gamma^i$ at the optimum, the government may (in part) use marginal income taxation to compensate the consumers for the distortionary effect caused by the commodity tax. For instance, if $t^i > \mu^i/\gamma^i$, the commodity tax is interpretable as being 'too high' from the perspective of pure externality-correction, in which case it is welfare improving to stimulate the consumption of the dirty good. This constitutes, in turn, an incentive for the government to implement a higher marginal income tax rate if leisure is complementary with the dirty good ($\partial \tilde{X}^{i,j}/\partial Z^{i,j} > 0$), and a lower marginal income tax rate if leisure is substitutable for the dirty good ($\partial \tilde{X}^{i,j}/\partial Z^{i,j} < 0$), than it would otherwise have done. The argument will be the opposite if $t^i < \mu^i/\gamma^i$.

The second part of the second row in each tax formula in the proposition depends on the joint effect of two mechanisms; how the world-market producer price of the dirty good changes in response to an increase in the hours of work (captured by $\Psi^{i,j}$), and how an increase in the world-market producer price affects the domestic welfare (captured by $\partial \mathcal{L}^i/\partial P$). If, as one would normally expect, the world-market producer price decreases in response to an increase in the hours of work supplied domestically in the sense that $\Psi^{i,j} < 0$, and if an increase in the world-market producer price leads to higher domestic welfare, there is an incentive for the government to implement a higher marginal income tax rate for ability-type j than it would otherwise have done. The intuition behind other possible sign-combinations for $\Psi^{i,j}$ and $\partial \mathcal{L}^i/\partial P$ is analogous. Once again, note that the sign of $\partial \mathcal{L}^i/\partial P$ reflects a desire to reduce the wage inequality (which relaxes the self-selection constraint) <u>and</u> a desire to correct for the environmental externality; therefore, the incentive created by the producer price effect is a mixture of several underlying motives for tax policy.

To take the interpretation of the second row of each tax formula in Proposition 4 a bit further, we may use the expression for t^i in Proposition 2 and substitute into the expressions for the marginal income tax rates. The second row of the expression for the marginal income tax rate may then be rewritten as

$$-\left(t^i - \frac{\mu^i}{\gamma^i}\right)\frac{1}{w^{i,j}}\frac{\partial \tilde{X}^{i,j}}{\partial Z^{i,j}} - \frac{\Psi^{i,j}}{w^{i,j}\gamma^i}\frac{\partial \mathcal{L}^i}{\partial P} \tag{25}$$

$$= \frac{\lambda^{i,*}\left(X^{i,l} - \hat{X}^{i,h}\right)}{w^{i,j}[\sum_j \partial \tilde{X}^{i,j}/\partial Q^i]}\frac{\partial \tilde{X}^{i,j}}{\partial Z^{i,j}} + \frac{1}{\alpha w^{i,j}\gamma^i}\frac{\partial S^i_x}{\partial L^{i,j}}\frac{\partial \mathcal{L}^i}{\partial P}.$$

In equation (25), the first term on the right hand side shows how a self-selection component familiar from the commodity tax formula (see Proposition 2) reappears in the expression for the marginal income tax rate. If $X^{i,l} - \hat{X}^{i,h} < 0$, in which case this self-selection component contributes to increase the commodity tax (and, therefore, reduce the consumption of the dirty good, ceteris paribus), there will be an incentive for the government to increase the consumption of the dirty good via the income tax. With $\partial \tilde{X}^{i,j}/\partial Z^{i,j} > 0$ (< 0), this mechanism means that the government implements a higher (lower) marginal income tax rate than it would otherwise have done. Incentive effects opposite to those just described will apply if $X^{i,l} - \hat{X}^{i,h} > 0$. The second term on the right hand side is also straight forward; if $\partial \mathcal{L}^i/\partial P > 0$, the government has an incentive to increase P, meaning that it will try to decrease the supply of the dirty good. This may, in turn, be accomplished by discouraging the labor supply via a higher marginal income tax rate. Again, the intuition is analogous if $\partial \mathcal{L}^i/\partial P < 0$.

Finally, note that Proposition 4 does not presuppose that the production tax is suboptimal from the perspective of the domestic government. In other words, as the national government is not able to perfectly control the world-market producer price of the externality-generating good by any single tax instrument, all tax instruments will be used, in part, for the purpose of exercising (let be imperfect) control of the world-market producer price.

As we did before, let us also here briefly address the corrective role of taxation by considering the special case of a symmetric equilibrium, in which we also add the assumption that the self-selection constraint does not bind. We can then derive the following corollary to Proposition 4;

Corollary 3. *In a symmetric noncooperative Nash equilibrium ($NX^i = 0$), and if the self-selection constraint does not bind ($\lambda^i = 0$), the marginal income tax rates reduce to*

$$T_I^{i,j}\left(w^{i,,j}L^{i,j}\right) = \frac{\mu^i}{\gamma^i}\varepsilon^{i,j} > 0 \text{ for } \frac{\mu^i}{\gamma^i} > 0$$

for $j = l, h$, *where*

$$\varepsilon^{i,j} = -\frac{1}{\alpha\beta^i w^{i,j}} \frac{\partial S_x^i}{\partial L^{i,j}} \sum_{j=h,l} \frac{\partial X^{k,j}}{\partial Q^k} > 0 \text{ for } k \neq i.$$

Therefore, in the special case, the government implements (positive) marginal income tax rates in order to increase the world-market producer price. With t^k held constant, this will reduce the foreign consumption of the dirty good, which leads to higher domestic welfare.

4.4 Public Good Provision

Define

$$MRS_{G,B}^{i,j} = \frac{V_G^{i,j}}{V_B^{i,j}} \text{ and } \widehat{MRS}_{G,B}^{i,h} = \frac{\hat{V}_G^{i,h}}{\hat{V}_B^{i,h}}$$

to be the marginal rate of substitution between the public good and private disposable income for ability-type j and the mimicker, respectively, and

$$MRT_{C,G}^{i} = \frac{\partial F_c^i / \partial L^{i,l}}{\partial F_G^i / \partial L_G^{i,l}}$$

to be the marginal rate of transformation[17] between the numeraire private good and the public good. The provision of the public good is characterized by the following optimality condition from the perspective of country i;

[17] A cost benefit rule equivalent to that in Proposition 5 can be derived, if the first order condition for $L_G^{i,h}$ (instead of the first order condition for $L_G^{i,l}$) is used to calculate the marginal rate of transformation between the numeraire private good and the public good. This is seen by combining equations (A7) and (A8), which gives

$$\frac{\partial F_c^i / \partial L^{i,l}}{\partial F_G^i / \partial L_G^{i,l}} - \frac{\partial S_x^i / \partial L^{i,l}}{\alpha\gamma^i \left(\partial F_G^i / \partial L_G^{i,l}\right)} \frac{\partial \mathcal{L}^i}{\partial P}$$

$$= \frac{\partial F_c^i / \partial L^{i,h}}{\partial F_G^i / \partial L_G^{i,h}} - \frac{\partial S_x^i / \partial L^{i,h}}{\alpha\gamma^i \left(\partial F_G^i / \partial L_G^{i,h}\right)} \frac{\partial \mathcal{L}^i}{\partial P}.$$

Proposition 5. *In the noncooperative Nash equilibrium, the provision of the national public good in country i is characterized by*

$$\sum_{j=l,h} MRS^{i,j}_{G,B} = MRT^{i}_{C,G} - \lambda^{i,*}[MRS^{i,l}_{G,B} - \widehat{MRS}^{i,h}_{G,B}]$$

$$- [t^i - \frac{\mu^i}{\gamma^i}] \sum_{j=l,h} \frac{\partial \tilde{X}^{i,j}}{\partial G^i} - \frac{1}{\alpha\gamma^i}[\sum_{j=l,h} \frac{\partial \tilde{X}^{i,j}}{\partial G^1} + \frac{\partial S^i_x/\partial L^{i,l}}{\partial F^i_G/\partial L^{i,l}_G}] \frac{\partial \mathcal{L}^i}{\partial P}.$$

Proof: See the Appendix.

Let us start by interpreting the first row of the formula in Proposition 5. The left hand side represents the sum of marginal rates of substitution between the public good and private consumption (i.e. the consumers' marginal willingness to pay for the public good), whereas the right hand side contains the marginal rate of transformation between the public good and the numeraire as well as a direct effect created by the self-selection constraint. Therefore, if the terms in the second row were absent, and if leisure is substitutable for (complementary with) the public good in the sense that $MRS^{i,l}_{G,B} - \widehat{MRS}^{i,h}_{G,B} > 0$ (< 0), we may relax the self-selection constraint by overproviding (underproviding) the public good relative to the Samuelson rule. This result is well understood from Boadway and Keen (1993).

The second row of the formula in Proposition 5 is, in a sense, analogous to the corresponding effects in the expressions for the marginal income tax rates (given by the second row of each formula in Proposition 4). To see this more clearly, suppose first that $t^i > \mu^i/\gamma^i$, in which case the commodity tax on the dirty good is 'too high' from the perspective of pure externality-correction. This mechanism constitutes, in turn, an incentive for the government to stimulate the consumption of the dirty private good by adjusting its provision of the public good. Accordingly, if the public good is complementary with (substitutable for) the dirty private good in the sense that $\partial\tilde{X}^{i,j}/\partial G^i > 0$ ($\partial\tilde{X}^{i,j}/\partial G^i < 0$), the government will provide more (less) of the public good than it would otherwise have done. The interpretation is analogous - yet with the opposite policy incentives - if $t^i < \mu^i/\gamma^i$. Once again, the intuition is that the government uses its other policy instruments, in this case the public good, at least in part to compensate the consumers for the distortionary effect created by the commodity tax[18].

Turning finally to the second part of the second row, which measures the policy incentives associated with the world-market producer price of the dirty good, there are two channels via which the provision of the public good can influence P. The first is via the demand for the dirty good. To illustrate, suppose first that an increase in the world-market producer price increases the domestic welfare (due, for instance, to decreased consumption of the dirty good abroad and/or that country i is a net exporter of the dirty good). In this case, and if the public good is complementary with (substitutable for) the dirty good in the sense that $\partial \tilde{X}^{i,j}/\partial G^i > 0$ (< 0), an increase (a decrease) in G^i leads to a higher world-market produce price and, therefore, higher domestic welfare. The second channel by which the government may influence the world-market producer price of the dirty good is via the supply side. If the government increases the supply of the public good, then the resources available to the private sector will decrease which, in turn, reduces the supply of the dirty private good. Again, the policy incentives will be the opposite to those just described if an increase in the world-market producer price leads to lower domestic welfare.

The assumption that the world-market producer price is endogenous for each national government is also important from the point of view of public production. We can derive the following production-inefficiency result[19];

Corollary 4. *In the noncooperative Nash equilibrium, the production of the public good is, in general, characterized by production inefficiency in the sense that*

$$\frac{\partial F_c^i/\partial L^{i,l}}{\partial F_G^i/\partial L_G^{i,l}} \neq \frac{\partial F_c^i/\partial L^{i,h}}{\partial F_G^i/\partial L_G^{i,h}}.$$

The intuition behind this result is, of course, that a reallocation of low-ability

[18]Another possible interpretation of the first term in the second row is that it captures a tax revenue effect of the public good; see e.g. Edwards et al. (1994) for such an interpretation of a corresponding term in a model without environmental externalities (i.e. where $\mu^i/\gamma^i = 0$). This interpretation is, perhaps, less obvious in our framework, since $t^i - \mu^i/\gamma^i$ times the change in the compensated conditional demand only measures part of the associated effect on the tax revenues.

[19]This is analogous to Naito (1999).

labor from private to public production does not, in general, affect the world-market producer price in the same way as a corresponding reallocation of high-ability labor from private to public production. Therefore, this production-inefficiency result is solely due to the ability of the national government to influence the world-market producer price: if this price were fixed (as in the context of small open economies), the public production would be characterized by production-efficiency.

Let us finally consider the special case with a symmetric equilibrium with a non-binding self-selection constraint, which we also did in connection to the optimal tax structure. The following result is a direct consequence of Proposition 5;

Corollary 5. *In a symmetric noncooperative Nash equilibrium ($NX^i = 0$), and if the self-selection constraint does not bind ($\lambda^i = 0$), the optimal provision of the public good is characterized by*

$$\sum_{j=l,h} MRS^{i,j}_{G,B} = MRT^i_{C,G} + \frac{\mu^i}{\gamma^i}\psi^i < MRT^i_{C,G} \text{ for } \frac{\mu^i}{\gamma^i} > 0$$

where (for $k \neq i$)

$$\psi^i = \frac{\sum_{j=l,h} \partial S^i_x / \partial L^{i,j}}{\sum_{j=l,h} \partial F^i_G / \partial L^{i,j}_G} \frac{\sum_{j=l,h} \left(\partial X^{k,j} / \partial Q^k\right)}{\sum_{n=1,2} \left(\partial S^n_x / \partial P\right) - \sum_{j=h,l} \left(\partial X^{k,j} / \partial Q^k\right)} < 0.$$

In the special case exemplified by the corollary, the government in country i attempts to push up the world-market producer price of the dirty good in order to reduce the environmental damage created abroad. It does so by overproviding the public good relative to the Samuelson rule, which reduces the amount of resources available to the private sector and, therefore, the supply of the dirty private good.

5 Policy Coordination

As the noncooperative Nash equilibrium analyzed in section 4 is inefficient from the perspective of both countries, policy coordination becomes interesting to consider. Here, we do not interpret the concept of 'coordination' such that the countries pool their resources in order to implement a cooperative equilibrium (even if this is a common approach in earlier literature). It is more realistic to assume that they agree upon smaller projects, the purposes of which are to improve the resource allocation by comparison with the noncooperative Nash equilibrium analyzed above. We will not discuss the conditions under which such international agreements are likely to be formed; only the welfare consequences if they arise.

Note that all public decision-variables have already been optimally chosen on a national basis in the noncooperative Nash equilibrium. Therefore, a coordinated infinitessimal change in one or several policy instruments only affects welfare because changes in the public decision-variables in country i give rise to welfare effects in country k and vice versa. Let us begin by characterizing the cost benefit rule. By observing that the national welfare function facing any country, i, equals the national Lagrangean in the noncooperative Nash equilibrium, i.e. $W^i = \mathfrak{L}^i$, a straight forward application of the Envelope Theorem implies

$$dW^i = \sum_{j=l,h} \theta_L^{i,j} dL^{k,j} + \sum_{j=l,h} \theta_G^{i,j} dL_G^{k,j} + \theta_t^i dt^k + \sum_{j=l,h} \theta_B^{i,j} dB^{k,j} + \theta_\tau^i d\tau^k \quad (26)$$

for $k \neq i$, where (for $j = l, h$)

$$\theta_L^{i,j} = \mu^i \frac{\partial X^{k,j}}{\partial Z^{k,j}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L^{k,j}}$$

$$\theta_G^{i,j} = \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L_G^{k,j}}$$

$$\theta_t^i = -\mu^i \sum_{j=l,k} \frac{\partial X^{k,j}}{\partial Q^k} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial t^k}$$

$$\theta_B^{i,j} = -\mu^i \frac{\partial X^{k,j}}{\partial B^{k,j}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial B^{k,j}}$$

$$\theta_\tau^i = \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial \tau^k}.$$

Equation (26) implies that a policy reform may influence welfare via two channels; a direct effect on the foreign demand for the dirty good and an indirect effect via the world-market producer price. To simplify the analysis slightly, let us add the (relatively common) assumption that the public good and the environmental damage are both weakly separable from the other goods in terms of the utility function. The simplification gained by this assumption is that neither the national public goods nor the environmental damage will directly affect the world-market producer price. Then, by assuming that the noncooperative Nash equilibrium described in the previous section constitutes the prereform equilibrium, and if each national government attaches a positive marginal value to reduced environmental damage (as we assumed above), we can immediately derive the following result from equation (26);

Proposition 6. *If $U^{n,j}$ is weakly separable in G^n and E and if $\partial\mathcal{L}^n/\partial P > 0$ (< 0) for $n = 1, 2$, a coordinated increase (decrease) in the production tax accompanied by a budget-balancing increase (decrease) in the resources spent on public production - with the hours of work and disposable income of each ability-type and the commodity tax held constant - leads to increased welfare. Furthermore, if $\partial\mathcal{L}^n/\partial P > 0$ and $\partial\phi^n/\partial P \leq 0$ for $n = 1, 2$, and if the net export is small enough not to be a dominant source of welfare change following increased commodity taxation, then a coordinated increase in the commodity tax accompanied by a budget-balancing increase in the resources spent on public production - with the hours of work and disposable income of each ability-type and the production tax held constant - leads to increased welfare.*

Proof: See the Appendix.

The intuition behind the first part of Proposition 6 is straight forward. A higher production tax in, say, country k contributes to increase the world-market producer price, ceteris paribus, which is desirable (undesirable) from the perspective of country i if $\partial\mathcal{L}^i/\partial P > 0$ ($<$). In addition, if country k uses the additional tax revenues to increase the public production - while the income tax is adjusted in such a way that the hours of work and private disposable income are held constant - the world-market producer price will increase even further, since the supply of the dirty good becomes smaller when resources are reallocated from the private to the public sector. In the second part of the proposition, the condition imposed on the net export is to avoid that that the sign of the national welfare change caused by an increase in the other country's commodity tax becomes dependent on whether the net export is positive or negative. The intuition behind the second part is that an increase in the commodity tax in country k decreases the demand for the dirty good in country k; this is welfare improving for country i as long as the associated decrease in the world-market producer price does not give rise to more wage inequality. Increased public production then plays the same role as in the first part of the proposition.

To take the analysis a step further, and by analogy to Corollaries 1, 2, 3 and 5, let us consider policy coordination in the special case where the prereform Nash equilibrium is symmetric and the self-selection constraint does not bind. As before, this special case enables us to address the corrective role of the tax and expenditure policies in a framework simple enough to derive several unambiguous results. By focusing on pairwise changes, which is a minimum requirement for budget balance for each national government, we can generalize Proposition 6 as follows;

Proposition 7. *If the prereform resource allocation is a symmetric noncooperative Nash equilibrium, if the self-selection constraint does not bind, and given the separability assumption in Proposition 6, it is welfare improving to;*

(i) increase the production tax - while the hours of work and private disposable income of each ability-type and the commodity tax are held constant - and then use the additional tax revenues to increase the public production,

(ii) increase the commodity tax - while the hours of work and private disposable income of each ability-type and the production tax are held constant - and then use the additional tax revenues to increase the public production,

(iii) reduce the hours of work and private disposable income simultaneously for each ability-type with the commodity and production taxes held constant, such that the tax revenues remain fixed, provided that leisure is not a strong enough substitute for the dirty good to completely offset the increase in the world-market producer price caused by a decrease in the supply of the dirty good, or

(iv) reduce the private disposable income - while the hours of work as well as the commodity and production taxes are held constant - and use the additional tax revenues to increase the public production.

The intuition behind policy reforms (i) and (ii) is the same as in the context of

Proposition 6 above; the only difference here is that the domestic welfare effect for country i of an increase in the world-market producer price reduces to

$$\frac{\partial \mathcal{L}^i}{\partial P} = -\frac{\mu^i}{\beta^i} \sum_{j=h,l} \frac{\partial X^{k,j}}{\partial Q^k} > 0$$

for $\mu^i > 0$, $i = 1, 2$ and $k \neq i$.

From the perspective of any country, i, policy reform (iii) is interpretable to mean $dL^{k,j} < 0$, $dB^{k,j} < 0$ and $dt^k = d\tau^k = dL_G^{k,j} = 0$ (for $k \neq i$ and $j = l, h$). Increased marginal income taxation in country k (which reduces $L^{k,j}$) is here accompanied by increased average income taxation (which reduces $B^{k,j}$). A smaller number of work hours leads to reduced supply of the dirty private good (recall that the resources used for public production are held constant), which gives rise to an increase in the world-market producer price of the dirty good. Furthermore, reduced private disposable income leads to lower demand for the dirty good in country k which is, in turn, welfare improving from the perspective of country i.

Finally, and again from the perspective of country i, we may interpret policy reform (iv) to imply $dB^{k,j} < 0$, $dL_G^{k,j} > 0$ and $dt^k = d\tau^k = dL^{k,j} = 0$ (for $k \neq i$ and $j = l, h$). This can be accomplished by a combination of higher marginal and average income taxation in such a way that the hours of work are held constant. As we mentioned above, a reduction in the private disposable income is, itself, welfare improving, as it leads to reduced demand for the dirty good abroad. Spending the additional tax revenues on public production, then means a reallocation of labor from the private to the public sector, which contributes to increase the world-market producer price of the dirty good and, therefore, decrease the foreign consumption of the dirty good.

6 Summary and Discussion

This paper concerns optimal taxation and public good provision in a two-country economy, where each country is characterized by two-ability types and asymmetric information between the government and the private sector. We assume that one of the consumption goods, referred to as a 'dirty' good, gives rise to transboundary environmental damage. Each national government faces a mixed tax problem, where the set of tax instruments consists of a nonlinear income tax as well as linear commodity and production taxes on the dirty good. We also assume that each country is large in the sense that its government may significantly influence the world-market producer price of the externality-generating commodity via its tax and expenditure policies. The idea is to capture the incentives facing large actors on the environmental policy scene; an issue neglected in earlier comparable literature on mixed taxation.

We would like to emphasize the following results;

• In the noncooperative Nash equilibrium, the corrective component of the commodity tax falls fall short of the marginal value that the national government attaches to reduced environmental damage.

• If, in the noncooperative Nash equilibrium, an increase in the world-market producer price leads to higher (lower) domestic welfare - which, in turn, depends on the properties of the wage distribution and whether or not the country is a net exporter of the dirty good - the national government will implement a positive (negative) production tax on the dirty good.

• If, in the noncooperative Nash equilibrium, an increase in the world-market producer price leads to higher (lower) domestic welfare - and by comparison with the situation where the world-market producer price of the dirty good is fixed - the public policy also reflects a motive to reduce (increase) the hours of work and/or increase (decrease) the hours of work spent in public production relative to the hours of work spent in private production. As a consequence, the endogenous world-market producer price also affects the incentives underlying marginal income taxation and public good provision.

• The public production is characterized by production-inefficiency.

• In the noncooperative Nash equilibrium, the national goverment also (in part) uses the income tax and provision of the public good to compensate the consumers for distortions created by the commodity tax.

• Welfare improving policy coordination - where the noncooperative Nash equilibrium constitutes the prereform equilibrium - may include increased commodity and/or production taxation with the additional tax revenues spent on public production.

As a complement to the more general model, we have also analyzed the corrective role of taxation and public provision in a special case, where the noncooperative Nash equilibrium is symmetric and the self-selection constraint does not bind. In the noncooperative Nash equilibrium, we can then show; (i) that the commodity tax (which is, in this case, a pure environmental tax) falls short of the marginal value that each national government attaches to reduced environmental damage, (ii) that each national government implements a positive production tax, and (iii) that the marginal income tax rates and level of the public good are higher than they would have been had the government perceived the world-market producer price of the dirty good as fixed. In addition to the policy coordination result mentioned above, we show in the special case that (iv) a simultaneous coordinated increase in the marginal and average tax rates (in both countries), and (v) a simultaneous coordinated increase in the average tax rate and the provision of the public good, can be designed to increase welfare by comparison with the noncooperative Nash equilibrium.

Possible extensions of the analysis carried out here would be to consider a model that contains both small and large open economies (which differ with respect to the perceived endogeneity of the world-market producer price) and by incorporating the individual jurisdictions into an economic federation. We leave these and other extensions for future research.

7 Appendix

The first order conditions for country i can be written as

$$B^{i,h} : 0 = \left(1+\lambda^i\right) V_B^{i,h} + \gamma^i \left(t^i \frac{\partial X^{i,h}}{\partial B^{i,h}} - 1 \right) - \mu^i \frac{\partial X^{i,h}}{\partial B^{i,h}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial B^{i,h}} \tag{A.1}$$

$$\begin{aligned} L^{i,h} : 0 = &- \left(1+\lambda^i\right) V_Z^{i,h} + \lambda^i \frac{\partial \phi^i}{\partial L^{i,h}} L^{i,l} \hat{V}_Z^{i,h} \\ &+ \gamma^i \left(w^{i,h} - t^i \frac{\partial X^{i,h}}{\partial Z^{i,h}} \right) + \mu^i \frac{\partial X^{i,h}}{\partial Z^{i,h}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L^{i,h}} \end{aligned} \tag{A.2}$$

$$B^{i,l} : 0 = V_B^{i,l} - \lambda^i \hat{V}_B^{i,h} + \gamma^i \left(t^i \frac{\partial X^{i,l}}{\partial B^{i,l}} - 1 \right) - \mu^i \frac{\partial X^{i,l}}{\partial B^{i,l}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial B^{i,l}} \tag{A.3}$$

$$\begin{aligned} L^{i,l} : 0 = &- V_Z^{i,h} + \lambda^i \hat{V}_Z^{i,h} \left(\phi^i + \frac{\partial \phi^i}{\partial L^{i,h}} L^{i,l} \right) \\ &+ \gamma^i \left(w^{i,l} - t^i \frac{\partial X^{i,l}}{\partial Z^{i,l}} \right) + \mu^i \frac{\partial X^{i,l}}{\partial Z^{i,l}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L^{i,l}} \end{aligned} \tag{A.4}$$

$$\begin{aligned} t^i : 0 = &- \left(1+\lambda^i\right) X^{i,h} V_B^{i,h} - X^{i,l} V_B^{i,l} + \lambda^i \hat{X}^{i,h} \hat{V}_B^{i,h} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial t^i} \\ &+ \gamma^i \left[X^{i,h} + X^{i,l} + t^i \left(\frac{\partial X^{i,h}}{\partial Q^i} + \frac{\partial X^{i,l}}{\partial Q^i} \right) \right] - \mu^i \left(\frac{\partial X^{i,h}}{\partial Q^i} + \frac{\partial X^{i,l}}{\partial Q^i} \right) \end{aligned} \tag{A.5}$$

$$\begin{aligned} G^i : 0 = &\left(1+\lambda^i\right) V_G^{i,h} + V_G^{i,l} - \lambda^i \hat{V}_G^{i,h} - \rho^i + \gamma^i t^i \left(\frac{\partial X^{i,h}}{\partial G^i} + \frac{\partial X^{i,l}}{\partial G^i} \right) \\ &- \mu^i \left(\frac{\partial X^{i,h}}{\partial G^i} + \frac{\partial X^{i,l}}{\partial G^i} \right) + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial G^i} \end{aligned} \tag{A.6}$$

$$L_G^{i,l}: \rho^i \frac{\partial F_G^i}{\partial L_G^{i,l}} - \gamma^i \frac{\partial F_c^i}{\partial L_c^{i,l}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L_G^{i,l}} = 0 \tag{A.7}$$

$$L_G^{i,h}: \rho^i \frac{\partial F_G^i}{\partial L_G^{i,h}} - \gamma^i \frac{\partial F_c^i}{\partial L_c^{i,h}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L_G^{i,h}} = 0 \tag{A.8}$$

$$\begin{aligned} E: 0 = &\left(1+\lambda^i\right) V_E^{i,h} + V_E^{i,l} - \lambda^1 \hat{V}_E^{i,h} + \gamma^i t^i \textstyle\sum_j \dfrac{\partial X^{i,j}}{\partial E} \\ &+ \mu^i \left[1 - \textstyle\sum_n \sum_j \dfrac{\partial X^{n,j}}{\partial E}\right] + \dfrac{\partial \mathcal{L}^i}{\partial P} \dfrac{\partial P}{\partial E} \end{aligned} \tag{A.9}$$

$$\begin{aligned} \tau^i: 0 = &\gamma^i \left[\left(P \frac{\partial F_x^i}{\partial L_x^{i,h}} - \frac{\partial F_c^i}{\partial L_c^{i,h}}\right) \frac{\partial L_x^{i,h}}{\partial \tau^1} + \left(P \frac{\partial F_x^i}{\partial L_x^{i,l}} - \frac{\partial F_c^i}{\partial L_c^{i,l}}\right) \frac{\partial L_x^{i,l}}{\partial \tau^1}\right] \\ &+ \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial \tau^1}. \end{aligned} \tag{A.10}$$

Note also that

$$\begin{aligned} \frac{\partial \mathcal{L}^i}{\partial P} = &- \left(1+\lambda^i\right) X^{i,h} V_B^{i,h} - X^{i,l} V_B^{i,l} + \lambda^i \hat{X}^{i,h} \hat{V}_B^{i,h} + \lambda^i \frac{\partial \phi^i}{\partial P} L^{i,l} \hat{V}_Z^{i,h} \\ &+ \gamma^i \left[\textstyle\sum_j X^{i,j} + NX^i + t^i \sum_j \dfrac{\partial X^{i,j}}{\partial Q^i}\right] \\ &- \mu^i \textstyle\sum_n \sum_j \dfrac{\partial X^{n,j}}{\partial Q^i} \end{aligned} \tag{A.11}$$

in which we have used that $S_x^i = \sum_j X^{i,j} + NX^i$, where NX^i is the net export of the dirty good. To simplify the expression for $\partial \mathfrak{L}^i / \partial P$, define

$$\begin{aligned} A^i = &- \left(1+\lambda^i\right) X^{i,h} V_B^{i,h} - X^{i,l} V_B^{i,l} + \lambda^i \hat{X}^{i,h} \hat{V}_B^{i,h} \\ &+ \gamma^i \left[\textstyle\sum_j X^{i,j} + t^i \sum_j \dfrac{\partial X^{i,j}}{\partial Q^i}\right] - \mu^i \textstyle\sum_j \dfrac{\partial X^{i,j}}{\partial Q^j} \end{aligned} \tag{A12}$$

and note that the first order condition for t^i can be written as

$$A^i + \left[A^i + \lambda^i \frac{\partial \phi^i}{\partial P} L^{i,l} \hat{V}_Z^{i,h} + \gamma^i NX^i - \mu^i \textstyle\sum_j \dfrac{\partial X^{k,j}}{\partial Q^k}\right] \dfrac{\partial P}{\partial t^k} = A^i + \dfrac{\partial \mathcal{L}^i}{\partial P} \dfrac{\partial P}{\partial t^k} = 0 \tag{A.13}$$

for $k \neq i$ (note that k is used here to denote the 'other country'). Therefore, by using equations (A11) and (A13), we have

$$\frac{\partial \mathcal{L}^i}{\partial P} = \lambda^i \frac{\partial \phi^i}{\partial P} L^{i,l} \hat{V}_Z^{i,h} + \gamma^i N X^1 - \mu^i \textstyle\sum_j \frac{\partial X^{k,j}}{\partial Q^k} - \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial t^k}.$$

Rearrangement gives

$$\frac{\partial \mathcal{L}^i}{\partial P} = \frac{\lambda^i L^{i,l} \hat{V}_Z^{i,h} (\partial \phi^i / \partial P) + \gamma^i N X^i - \mu^i \sum_j \partial X^{k,j} / \partial Q^k)}{1 + \partial P / \partial t^i}. \tag{A.14}$$

Proof of Proposition 1

Define

$$\frac{\partial \tilde{X}^{i,j}}{\partial E} = \frac{\partial X^{i,j}}{\partial E} + \frac{\partial X^{i,j}}{\partial B^{i,j}} MWP_{E,B}^{i,j}. \tag{A.15}$$

Substitute $V_E^{i,j} = -V_B^{i,j} MWP_{E,B}^{i,j}$ for $j = l, h$ into equation (A9). Next, use equations (A1) and (A3) to derive expressions for $V_B^{i,h}$ and $V_B^{i,l}$, respectively, and substitute into equation (A9). By using equation (A15), we can then derive

$$\frac{\mu i}{\gamma^i} = \sigma^i \left[\sum_j MWP_{E,B}^{i,j} + \frac{\lambda \hat{V}_B^{i,h}}{\gamma^i} \left(MWP_{E,B}^{i,l} - \widehat{MWP_{E,B}^{i,h}} \right) - t^i \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial E} \right]$$

$$- \frac{\sigma^i}{\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P} \left[\frac{\partial P}{\partial E} + \sum_j \frac{\partial P}{\partial B^{i,j}} MWP_{E,B}^{i,j} \right] \tag{A.16}$$

where

$$\frac{\partial P}{\partial E} = \frac{1}{\alpha} \sum_n \sum_j \frac{\partial X^{n,j}}{\partial E}$$

$$\frac{1}{\sigma^i} = \left[1 - \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial E} - \sum_j \frac{\partial X^{k,j}}{\partial E} \right]$$

for $k \neq i$.

Note that the expression within the square bracket in the second row of equation (A16) can be written as

$$\frac{\partial P}{\partial E} + \sum_j \frac{\partial P}{\partial B^{i,j}} MWP^{i,j}_{E,B} = \frac{1}{\alpha}\left(\sum_j \frac{\partial \tilde{X}^{i,j}}{\partial E} + \sum_j \frac{\partial X^{k,j}}{\partial E}\right)$$

$$= -\frac{1}{\alpha}\left(1 - \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial E} - \sum_j \frac{\partial X^{k,j}}{\partial E} - 1\right)$$

$$= -\frac{1}{\alpha}\left(\frac{1}{\sigma^i} - 1\right).$$

Therefore,

$$\frac{\mu^i}{\gamma^i} = \sigma^i \left[\sum_j MWP^{i,j}_{E,B} + \frac{\lambda \hat{V}^{i,h}_B}{\gamma^i}\left(MWP^{i,l}_{E,B} - \widehat{MWP}^{i,h}_{E,B}\right) - t^i \sum_{j=h,l} \frac{\partial \tilde{X}^{i,j}}{\partial E}\right]$$
$$+ \frac{1-\sigma^i}{\alpha\gamma^i}\frac{\partial \mathcal{L}^i}{\partial P}. \tag{A.17}$$

Finally, by observing that the expression for $\partial \mathcal{L}^i/\partial P$ in equation (24) directly depends on μ^i/γ^i, we can derive the formula in Proposition 1.■

Note also that if E is weakly separable in terms of the utility function, $1-\sigma^i$ reduces to

$$1 - \sigma^i = -\sigma^i \sum_j \frac{\partial X^{i,j}}{\partial B^{i,j}} MWP^{i,j}_{E,B} < 0. \tag{A.18}$$

Proof of Proposition 2

By using equations (A1) and (A3) to derive expressions for $\left(1+\lambda^i\right) V^{i,h}_B$ and $V^{i,l}_B$, respectively, substituting into equation (A5) and using the Slutsky condition gives

$$0 = -\lambda^i \hat{V}^{i,h}_B [X^{i,l} - \hat{X}^{i,h}] + \gamma^i t^i \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial Q^i} - \mu^i \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial Q^i}$$
$$+ \frac{\partial \mathcal{L}^i}{\partial P}\left[\frac{\partial P}{\partial t^1} + \sum_j X^{i,j} \frac{\partial P}{\partial B^{i,j}}\right]. \tag{A.19}$$

Note that equation (17) implies

$$\frac{\partial P}{\partial t^i} = \frac{\sum_j \partial X^{i,j}/\partial Q^i}{\alpha} < 0 \tag{A.20}$$

$$\frac{\partial P}{\partial B^{i,j}} = \frac{\partial X^{i,j}/\partial B^{i,j}}{\alpha} > 0 \text{ for } j = l, h. \tag{A.21}$$

Therefore, the expression within the square bracket in the second row of equation (A19) can be written as

$$\frac{\partial P}{\partial t^i} + \sum_j X^{i,j} \frac{\partial P}{\partial B^{i,j}} = \frac{1}{\alpha} \sum_j \frac{\partial \tilde{X}^{i,j}}{\partial Q^i} < 0.$$

Substituting into equation (A19) gives

$$t^i = \frac{\mu^i}{\gamma^i} + \frac{\lambda \hat{V}_B^{i,h}}{\gamma^i \Delta^i} \left(X^{i,l} - \hat{X}^{i,h} \right) - \frac{\partial \mathcal{L}^i}{\partial P} \frac{1}{\alpha \gamma^i} \tag{A.22}$$

where $\Delta^i = \sum_j \partial \tilde{X}^{i,j}/\partial Q^i < 0$. Finally, by using the expression for $\partial \mathcal{L}^i/\partial P$ in equation (24) and substituting into equation (A22), we obtain the formula in Proposition 2.■

Proof of Proposition 3

By using equations (9), we can rewrite equation (A10) to read

$$\tau^i P \frac{\partial F_x^i}{\partial L_x^{i,h}} \underbrace{\left(\frac{\partial L_x^{i,h}}{\partial \tau^i} + \frac{\partial L_x^{i,l}}{\partial \tau^i} \right)}_{-} + \frac{1}{\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial \tau^i} = 0 \tag{A.23}$$

where $\partial P/\partial \tau^i = -(\partial S_x^i/\partial \tau^i)/\alpha > 0$. Rearrangement gives the tax formula in the proposition.■

Proof of Proposition 4

Consider first the marginal income tax rate of the low-ability type. By combining equations (A3) and (A4), we have

$$MRS_{Z,B}^{i,l} \left[\lambda^i \hat{V}_B^{i,h} - \gamma^i \left(t^i \frac{\partial X^{i,l}}{\partial B^{i,l}} - 1 \right) + \mu^i \frac{\partial X^{i,l}}{\partial B^{i,l}} - \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial B^{i,l}} \right] \tag{A.24}$$

$$= \lambda^i \hat{V}_Z^{i,h} \left(\phi^i + \frac{\partial \phi^i}{\partial L^{i,l}} L^{i,l} \right) + \gamma^i \left(w^{i,l} - t^i \frac{\partial X^{i,l}}{\partial Z^{i,l}} \right) + \mu^i \frac{\partial X^{i,l}}{\partial Z^{i,l}} + \frac{\partial \mathcal{L}^i}{\partial P} \frac{\partial P}{\partial L^{i,l}}.$$

By using equation (6), equation (A24) can be rewritten as

$$T_I^{i,l} = \frac{\lambda^{i,*}}{w^{i,l}} \left(MRS_{Z,B}^{i,l} - \widehat{MRS}_{Z,B}^{i,h} \phi^i \right) - \frac{\lambda^i}{w^{i,l}\gamma^i} L^{i,l} \hat{V}_Z^{i,h} \frac{\partial \phi^i}{\partial L^{1,h}}$$
$$+ \frac{1}{w^{i,l}} \left(t^i - \frac{\mu^i}{\gamma^i} \right) \left(\frac{\partial X^{i,l}}{\partial Z^{i,l}} - MRS_{Z,B}^{i,l} \frac{\partial X^{i,l}}{\partial B^{i,l}} \right)$$
$$- \left(\frac{\partial P}{\partial L^{i,l}} + MRS_{Z,B}^{i,l} \frac{\partial P}{\partial B^{i,l}} \right) \frac{1}{w^{i,l}\gamma^i} \frac{\partial \mathcal{L}^i}{\partial P}. \tag{A.25}$$

Now,

$$\frac{\partial P}{\partial L^{i,l}} = -\frac{\partial S_x^i / \partial L^{i,l} + \partial X^{i,l} / \partial Z^{i,l}}{\alpha} \tag{A.26}$$

$$\frac{\partial P}{\partial B^{i,l}} = \frac{\partial X^{i,l} / \partial B^{i,l}}{\alpha} \tag{A.27}$$

$$\frac{\partial \tilde{X}^{i,l}}{\partial Z^{i,l}} = \frac{\partial X^{i,l}}{\partial Z^{i,l}} - MRS_{Z,B}^{i,l} \frac{\partial X^{i,l}}{\partial B^{i,l}}. \tag{A.28}$$

Equations (A26) and (A27) imply

$$\frac{\partial P}{\partial L^{i,l}} + MRS_{Z,B}^{i,l} \frac{\partial P}{\partial B^{i,l}} = -\frac{\partial S_x^i / \partial L^{i,l}}{\alpha} + \frac{1}{\alpha} \left(-\frac{\partial X^{i,l}}{\partial Z^{i,l}} + MRS_{Z,B}^{i,l} \frac{V_Z^{i,l}}{V_B^{i,l}} \frac{\partial X^{i,l}}{\partial B^{i,l}} \right). \tag{A.29}$$

By substituting equations (A28) and (A29) into equation (A25), we obtain the expression for the marginal income tax rate of the low-ability type in the proposition. The procedure to derive the marginal income tax rate of the high-ability type is analogous.■

Proof of Proposition 5

The starting point here is equation (A6). Note also that $V_G^{i,j} = V_B^{i,j} MRS_{G,B}^{i,j}$, which means that we can replace $V_G^{i,j}$ by $V_B^{i,j} MRS_{G,B}^{i,j}$. Then, use equations (A1) and (A3) to derive expressions for $V_B^{i,h}$ and $V_B^{i,l}$, respectively, substitute into the modified equation (A6) and use the Slutsky-like condition

$$\frac{\partial X^{i,j}}{\partial G^i} = \frac{\partial \tilde{X}^{i,j}}{\partial G^i} + MRS_{G,B}^{i,j} \frac{\partial X^{i,j}}{\partial B^{i,j}} \text{ for } j = l, h.$$

Rearrangement gives

$$\sum_j MRS^{i,j}_{G,B} = \frac{\rho^i}{\gamma^i} - \lambda^{i,*}\left(MRS^{i,l}_{G,B} - \widehat{MRS^{i,h}_{G,B}}\right) - \left(t^i - \frac{\mu^i}{\gamma^i}\right)\sum_j \frac{\partial \tilde{X}^{i,j}}{\partial G^i}$$
$$- \frac{1}{\gamma^i}\left(\frac{\partial P}{\partial G^i} - \sum_j MRS^{i,j}_{G,B}\frac{\partial P}{\partial B^{i,j}}\right)\frac{\partial \mathcal{L}^i}{\partial P}. \tag{A.30}$$

Note that

$$\frac{\partial P}{\partial G^i} = \frac{\sum_j \partial X^{i,j}/\partial G^i}{\alpha}$$
$$\frac{\partial P}{\partial B^{i,j}} = \frac{\partial X^{i,j}/\partial B^{i,j}}{\alpha}.$$

Therefore,

$$\frac{\partial P}{\partial G^i} - \sum_j MRS^{i,j}_{G,B}\frac{\partial P}{\partial B^{i,j}} = \frac{1}{\alpha}\sum_j \frac{\partial \tilde{X}^{i,j}}{\partial G^i}. \tag{A.31}$$

Substituting equation (A31) into equation (A30), while using

$$\frac{\rho^i}{\gamma^i} = MRT^i_{C,G} - \frac{\partial S^i_x/\partial L^{i,l}}{\alpha\gamma^i\left(\partial F^i_G/\partial L^{i,l}_G\right)}\frac{\partial \mathcal{L}^i}{\partial P}$$

from equation (A7), gives the formula for public good provision in the proposition.■

Proof of Proposition 6

The first part of the proposition follows by observing that the welfare effect is derived from the following welfare differential

$$dW^i = \sum_j \theta^{i,j}_G dL^{k,j}_G + \theta^i_\tau d\tau^k \tag{A.32}$$

where $\partial P/\partial L^{k,j}_G > 0$ and $\partial P/\partial \tau^k > 0$, so

$$\theta^{i,j}_G = \frac{\partial \mathcal{L}^i}{\partial P}\frac{\partial P}{\partial L^{k,j}_G} > 0\ (<0) \text{ if } \frac{\partial \mathcal{L}^i}{\partial P} > 0\ (<0)$$

$$\theta^i_\tau = \frac{\partial \mathcal{L}^i}{\partial P}\frac{\partial P}{\partial \tau^k} > 0\ (<0) \text{ if } \frac{\partial \mathcal{L}^i}{\partial P} > 0\ (<0).$$

Therefore, a simultaneous increase (decrease) in the production tax accompanied by a corresponding adjustment of the public production is welfare improving if $\partial\mathcal{L}^i/\partial P > 0$ (< 0).

In the second part, the corresponding welfare differential is

$$dW^i = \sum_j \theta_G^{i,j} dL_G^{k,j} + \theta_t^i dt^k \tag{A.33}$$

where

$$\begin{aligned}\theta_t^i &= -\mu^i \sum_{j=l,k} \frac{\partial X^{k,j}}{\partial Q^k} + \frac{\partial\mathcal{L}^i}{\partial P}\frac{\partial P}{\partial t^k} \qquad\qquad (A.34)\\ &= -\mu^i \left[1 + \frac{\sum_j \partial X^{k,j}/\partial Q^k}{\sum_n \partial S_x^n/\partial P - \sum_j \partial X^{k,j}/\partial Q^k}\right] \sum_j \frac{\partial X^{k,j}}{\partial Q^k}\\ &+ \frac{\gamma^i}{\beta^i}\left[\frac{\lambda^i L^{i,l}\hat{V}_Z^{i,h}}{\gamma^i}\frac{\partial\phi^i}{\partial P} + NX^i\right]\frac{\partial P}{\partial t^k}.\end{aligned}$$

Since the term within the square bracket in the second row of equation (A34) is between zero and one, $\partial\phi^i/\partial P \leq 0$ by assumption and $\partial P/\partial t^k < 0$, it follows that $\theta_t^i > 0$ if the net export (which can take any sign) is sufficiently small. The proof is then analogous to the proof of the first part of the proposition.■

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