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The Degrees of Monotony-Dilemma in Abstract Argumentation

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Abstract. In this paper, we introduce the notion of the degree of monotony to abstract argumentation, a well-established method for drawing inferences in face of conflicts in non-monotonic reasoning. Roughly speaking, the degree of monotony allows us, given an abstract argumentation semantics and an abstract argumentation framework to be *as monotonic as possible*, when iteratively drawing inferences and expanding the argumentation framework. However, we also show that when expanding an argumentation framework several times using so-called normal expansions, an agent may, at any given step, select a conclusion that has the highest degree of monotony w.r.t. the previous conclusion (considering the constraints of the semantics), but end up with a conclusion that has a suboptimal degree of monotony w.r.t. one or several conclusions that precede the previous conclusion. We formalize this observation as the *degrees of monotony*-dilemma.

Keywords: abstract argumentation · non-monotonic reasoning · argumentation dynamics

1 Introduction

When reasoning in dynamic environments, we expect that an intelligent agent can draw inferences in a *non-monotonic* manner, *i.e.* that the agent is able to reject previously inferred statements on the basis of newly acquired beliefs. In research on formal methods of reasoning, the design and analysis of inference approaches that systematically relax monotony of entailment has been an active field of research since several decades, in particular since the emergence of relaxed monotony principles like restricted [11] and rational monotony [15,13]. More recently, similar principles [4,7,12] have been introduced to formal argumentation approaches, which constitute a particularly vibrant research area related to the study of knowledge representation and reasoning [2]. In this paper, we provide a new perspective on relaxed monotony in the context of formal argumentation that we base on a *measure* of monotony, instead of on Boolean conditions. Assuming that we have an agent that iteratively infers possible conclusions (sets of atoms) from a belief base, selects a conclusion, and then expands the belief base to again infer possible conclusions (and so on), the agent measures the *degree of*

monotony of the conclusions she can infer w.r.t. a previous conclusion she selected, and then commits to an inference result that implies a maximal degree of monotony, *i.e.* to a conclusion that rejects as few of the previously inferred statements as possible. This can, however, lead to a dilemma: if the belief base is expanded several times, and the agent draws inferences after all expansions, selecting a conclusion that is as monotonic as possible w.r.t. *one* of the previously inferred conclusions may lead to the selection of a “suboptimally” monotonic conclusion w.r.t. *another* previous inference result, even if our agent always selects the conclusion that is maximally monotonic (considering constraints imposed by the inference function) w.r.t. the immediately preceding inference result. Indeed, this paper shows that this dilemma occurs in the context of several argumentation semantics that Dung defines in his seminal paper on abstract argumentation [10].

Let us introduce the following informal example to further motivate the research this paper presents.

Example 1. We have an agent $\mathcal{A}_1$ that draws inferences from a belief base BB . Over time, the agent acquires new beliefs and adds them to the belief base, *i.e.* in our example, we have the belief bases BB_1 , $BB_2 = BB_1 \cup BB'_1$, and $BB_3 = BB_2 \cup BB'_2$. Any newly added belief may conflict with any other belief; our agent’s inference function *concls* resolves conflicts at run-time and draws new inferences every time the agent updates the belief base. In addition, we have two agents $\mathcal{A}_2$ and $\mathcal{A}_3$ that observe the inferences that agent $\mathcal{A}_1$ draws at different steps.

- $\mathcal{A}_2$ observes $\mathcal{A}_1$ ’s inferences from BB_1 and BB_3 .
- $\mathcal{A}_3$ observes $\mathcal{A}_1$ ’s inferences from BB_2 and BB_3 .

At any point, $\mathcal{A}_1$ ’s belief base may contain conflicting beliefs and hence allows for a range of alternative conclusions (sets of atoms), *e.g.*:

1. $\text{concls}(B_1) = \{C_{1,1}\} = \{\{a, b\}\}$;
2. $\text{concls}(B_2) = \{C_{2,1}, C_{2,2}\} = \{\{a, b, c, d, e\}, \{c, d, e, f\}\}$;
3. $\text{concls}(B_3) = \{C_{3,1}, C_{3,2}\} = \{\{a, b, g\}, \{c, d, e, f\}\}$.

$\mathcal{A}_1$ ’s inference function is non-monotonic: a belief that has been inferred as true may be rejected by a future inference after a belief update. Still, to ensure some degree of consistency in her decision process, $\mathcal{A}_1$ attempts to be *as monotonic as possible*, using a measure that determines the *degree of monotony* of a newly inferred conclusion C' w.r.t. a previously inferred conclusion C . We denote the degree of monotony of C' w.r.t. C by $\text{deg}_{\text{mon}}(C', C)$, and define $\text{deg}_{\text{mon}}(C', C) = \frac{|C \cap C'|}{|C|}$ if $|C| \neq 0$; 1, otherwise. With this, $\mathcal{A}_1$ proceeds as follows:

- $\text{deg}_{\text{mon}}(C_{2,1}, C_{1,1}) = 1$ and $\text{deg}_{\text{mon}}(C_{2,2}, C_{1,1}) = 0$: after having inferred $C_{1,1}$ and having expanded the belief base from BB_1 to BB_2 , $\mathcal{A}_1$ chooses to infer $C_{2,1}$ from BB_2 , because it ensures a higher degree of monotony w.r.t. $C_{1,1}$ than inferring $C_{2,2}$.
- $\text{deg}_{\text{mon}}(C_{3,1}, C_{2,1}) = \frac{2}{5}$ and $\text{deg}_{\text{mon}}(C_{3,2}, C_{2,1}) = \frac{3}{5}$; according to these measures, $\mathcal{A}_1$ should choose to infer $C_{3,2}$, which means $\mathcal{A}_3$ observes that $\mathcal{A}_1$ ’s inference is *as monotonic as possible*.

- However, $\deg_{mon}(C_{3,1}, C_{1,1}) = 1$ and $\deg_{mon}(C_{3,2}, C_{1,1}) = 0$; according to these measures, $\mathcal{A}_1$ should choose to infer $C_{3,1}$, which means $\mathcal{A}_2$ observes that $\mathcal{A}_1$'s inference is as monotonic as possible. We say that $\mathcal{A}_1$ is in a *degrees of monotony-dilemma*, because she can maximize the degree of monotony merely w.r.t. *either* $C_{2,1}$ *or* $C_{1,1}$.

The reader may consider this scenario a (simplified) model of the following example use-case. Our agents represent different IT systems in a complex enterprise system landscape⁵. $\mathcal{A}_1$ is the so-called *leading system*, and provides business insights to $\mathcal{A}_2$ and $\mathcal{A}_3$, which these two systems then use to update their own configurations. From a practical perspective, we can realistically claim that $\mathcal{A}_2$ and $\mathcal{A}_3$ may observe $\mathcal{A}_1$'s inferences with different frequencies; for example, $\mathcal{A}_2$ could be a legacy system, whose synchronization with $\mathcal{A}_1$ runs as an overnight batch job, whereas $\mathcal{A}_3$ is a modern system that *listens* to updates from $\mathcal{A}_1$, approximately in real-time. Every time $\mathcal{A}_2$ or $\mathcal{A}_3$ observe that $\mathcal{A}_1$ has reversed a previously inferred statement, this implies invasive changes to the systems' configurations and operations. For example, active cases might need to get re-routed or even require manual (human) intervention. Hence, for both $\mathcal{A}_2$ and $\mathcal{A}_3$, the inferences the systems observe should be *as monotonic as possible*. However, because of the differences in observation frequencies, an inference that is *maximally monotonic* w.r.t. the last update that $\mathcal{A}_2$ has observed may not be maximally monotonic w.r.t. the last update that $\mathcal{A}_3$ has observed.

We formalize the notion of a degree of monotony for abstract argumentation [10], an approach for drawing inferences from conflicting beliefs (or: arguments) that has gained tremendous attention in the symbolic artificial intelligence community. Then, we show that for most⁶ of Dung's original argumentation semantics (inference functions), the *degrees of monotony-dilemma* in fact occurs as a problem and present an initial mitigation approach.

The rest of this paper is organized as follows. Section 2 provides the theoretical preliminaries of abstract argumentation. Then, Section 3 introduces the notion of the degree of monotony to abstract argumentation, based on which Section 4 formally defines and analyzes the *degrees of monotony-dilemma*, and also outlines a mitigation approach. Subsequently, Section 5 discusses the findings in the light of related and potential future research, before Section 6 concludes the paper.

2 Theoretical Preliminaries

The central model of this paper is an (abstract) argumentation framework, as introduced by Dung [10].

⁵ For an introduction to academic literature on the topic, see Lankhorst [14] or Dumas *et al.* [9], for example.

⁶ Precisely, the problem occurs for three of the four semantics. The exception is grounded semantics, which always infers exactly one extension and is typically regarded as “too skeptical” for many use-cases.

Definition 1 (Argumentation Framework [10]). An argumentation framework AF is a tuple (AR, AT) , such that AR is a set of elements (called arguments) and $AT \subseteq AR \times AR$ (called attacks).

We assume that the set of arguments in an argumentation framework is finite. Given an argumentation framework $AF = (AR, AT)$, and $a, b \in AR$, we say that “ a attacks b ” iff $(a, b) \in AT$. For $S \subseteq AR$, we say that “ S attacks a ” iff $\exists (c, a) \in AT$, such that $c \in S$. We say that “ S defends a ” iff $\forall d \in AR$, such that d attacks a , it holds true that S attacks d . To model dynamics in abstract argumentation, Baumann and Brewka introduce the notion of argumentation framework *expansions* [4].

Definition 2 (Argumentation Framework Expansions [4]). Let $AF = (AR, AT)$ and $AF' = (AR', AT')$ be argumentation frameworks.

- AF' is an expansion of AF (denoted by $AF \preceq AF'$) iff $AR \subseteq AR'$ and $AT \subseteq AT'$.
- AF' is a normal expansion of AF (denoted by $AF \preceq_N AF'$) iff $AF \preceq AF'$ and $(AR \times AR) \cap (AT' \setminus AT) = \{\}$.
- AF' is a weak expansion of AF (denoted by $AF \preceq_W AF'$) iff $AF \preceq_N AF'$ and $\forall a \in AR$, $AR' \setminus AR$ does not attack a .

In words, an expansion of an argumentation framework AF adds new arguments and attacks to AF , without removing “old” arguments or attacks, a normal expansion of AF is an expansion of AF that only adds attacks involving at least one new argument, and a weak expansion of AF is a normal expansion of AF in which no new argument attacks an “old” argument. Let us provide an example that illustrates the notions of expansions and normal expansions.

Example 2. Consider the following argumentation frameworks: i) $AF = (AR, AT) = (\{a, b\}, \{(a, b)\})$; ii) $AF' = (AR', AT') = (\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$; iii) $AF'' = (AR'', AT'') = (\{a, b, c\}, \{(a, b), (b, c), (c, a), (b, a)\})$. AF' is a normal expansion of AF ($AF \preceq_N AF'$): $AR \subseteq AR'$, $AT \subseteq AT'$, and $(AR \times AR) \cap (AT' \setminus AT) = \{\}$. This implies that AF' is an expansion of AF ($AF \preceq AF'$). In contrast, $AF' \preceq AF''$, but $AF' \not\preceq_N AF''$: $AR' \subseteq AR''$ and $AT' \subseteq AT''$, but $(AR' \times AR') \cap (AT'' \setminus AT') \neq \{\}$. Also, $AF \preceq AF''$, but $AF \not\preceq_N AF''$. In this example, we do not have any weak expansions. Counter-factually, if AF' (which is a normal expansion of AF) did not feature the attack (c, a) , it would be a weak expansion of AF . Figure 1 depicts the example’s argumentation frameworks.

Fig. 1: $AF \preceq_N AF'$ and $AF' \preceq AF''$, but $AF \not\preceq_N AF''$ and $AF' \not\preceq_N AF''$.

Let us now define the notions of *conflict-free* and *admissible* sets of arguments, which play a crucial role in abstract argumentation.

Definition 3 (Conflict-free and Admissible Sets [10]). Let $AF = (AR, AT)$ be an argumentation framework. A set $S \subseteq AR$:

- is conflict-free iff $\nexists a, b \in S$ such that a attacks b ;
- is admissible iff S is conflict-free and $\forall a \in S$, S defends a .

In his seminal paper that introduces abstract argumentation, Dung defines four *argumentation semantics*, functions that take an argumentation framework and return a set of *extensions*, where each extension is a set of arguments. We say that an argumentation semantics is universally defined iff it yields at least one extension for every argumentation framework.

Definition 4 (Stable, Complete, Preferred, and Grounded Semantics and Extensions [10]). Let $AF = (AR, AT)$ be an argumentation framework. A set $S \subseteq AR$ is a:

- stable extension of AF iff S is admissible and S attacks each argument that does not belong to S . Stable semantics $\sigma_{stable}(AF)$ denotes all stable extensions of AF ;
- complete extension of AF iff S is admissible and each argument that is defended by S belongs to S . Complete semantics $\sigma_{complete}(AF)$ denotes all complete extensions of AF ;
- preferred extension of AF iff S is a maximal (w.r.t. set inclusion) admissible subset of AR . Preferred semantics $\sigma_{preferred}(AF)$ denotes all preferred extensions of AF ;
- grounded extension of AF iff S is the minimal (w.r.t. set inclusion) complete extension of AF . Grounded semantics $\sigma_{grounded}(AF)$ denotes all grounded extensions of AF .

Note that i) for some argumentation frameworks there does not exist a stable extension (consider the argumentation framework $AF = (\{a, b\}, \{(a, a)\})$; ii) each preferred extension is also a complete extension [1]; iii) for every argumentation framework, there exists exactly one grounded extension [3]. Because this work deals with the selection of extensions from the set of extensions a semantics returns, we focus our analysis on complete and preferred semantics, i.e. the two original Dung-semantics that are universally defined [3] but may return more than one extension. Let us note that the four semantics all rely on the notion of admissibility. Over the past decades, additional abstract argumentation semantics families have been designed that are not admissible set-based, most notably naive set-based semantics [17] and weak admissible set-based semantics [5]. However, for the sake of conciseness, we do not consider these semantics families in our analysis. Finally, let us provide the definition of the *weak cautious monotony* argumentation principle, which describes argumentation semantics that satisfy a notion of relaxed monotony.

Definition 5 (Weak Cautious Monotony in Abstract Argumentation [12]). Let σ be an argumentation semantics. σ satisfies weak cautious monotony iff for every two argumentation frameworks $AF = (AR, AT)$ and $AF' = (AR', AT')$, such that $AF \preceq_N AF'$, and $\forall E \in \sigma(AF)$, it holds true that if $\{(a, b) \mid (a, b) \in AT', a \in AR' \setminus AR, b \in E\} = \{\}$ then $\exists E' \in \sigma(AF')$ such that $E \subseteq E'$.

3 Degrees of Monotony

This section formalizes the notion of the degree of monotony in the context of abstract argumentation and provides a brief theoretical analysis on how normally expanding an argumentation framework may affect the degree of monotony of an extension inferred from the argumentation framework expansion w.r.t. an extension inferred from the original argumentation framework.

Definition 6 (Degree of Monotony). *Let E and E' be two finite sets of arguments. We define the degree of monotony of E' w.r.t. E , denoted by $\deg_{\text{mon}}(E', E)$, as follows:*

$$\deg_{\text{mon}}(E', E) = \begin{cases} 1 & \text{if } |E| = 0; \\ \frac{|E' \cap E|}{|E|} & \text{otherwise.} \end{cases}$$

The degree of monotony tells us to what extent we violate monotony/are non-monotonic in a dynamic inference scenario, in which we draw conclusions from a changing knowledge base. Having such a measure can be useful because in practice, knowledge bases typically change with time, and the “more non-monotonic” (colloquially speaking) we are, the more careful we need to be when assessing the consequences of the changed inference, for example when updating documents and IT system configurations to accommodate the change. The normalization (division by $|E|$) can have practical advantages as it facilitates benchmarking. Let us highlight that potentially, argumentation variants that extend abstract argumentation could assign a weight to each argument in an argumentation framework, and this weight could then inform the *costs* of violating monotony, roughly speaking. However, for the sake of conciseness, we abstain from introducing such a formal argumentation framework variant here. Let us prove the infimum and supremum of the degree of monotony.

Proposition 1. *Given two finite sets of arguments E and E' , the infimum of $\deg_{\text{mon}}(E', E)$ is 0 and the supremum of $\deg_{\text{mon}}(E', E)$ is 1.*

Proof. Let us first prove the infimum. By definition of the degree of monotony (Definition 6), $\deg_{\text{mon}}(E', E)$ is minimal iff $|E' \cap E| = 0$ and $|E| > 0$. Given E and E' , such that $|E| > 0$ and $|E' \cap E| = 0$, it holds that $\deg_{\text{mon}}(E', E) = 0$.

Let us now prove the supremum. By definition of the degree of monotony (Definition 6), $\deg_{\text{mon}}(E', E)$ is maximal iff $E' \cap E = E$. Given E and E' , such that $E' \cap E = E$, it holds that $\deg_{\text{mon}}(E', E) = 1$. $\square$

We define a notion that allows us to select the *monotony-maximizing extensions* w.r.t. a previously inferred set of arguments E from the extensions an argumentation semantics infers from an argumentation framework.

Definition 7 (Monotony-Maximizing Extensions). *Let $AF = (AR, AT)$ be an argumentation framework. Let σ be a universally defined argumentation semantics, and let S be a finite set of arguments. We define the monotony-maximizing σ -extensions of AF w.r.t. S , denoted by $\text{Exts}_{\text{mon}}(S, AF, \sigma)$, as $\{E \mid E \in \sigma(AF), \nexists E' \in \sigma(AF), \text{ such that } \deg_{\text{mon}}(E, S) < \deg_{\text{mon}}(E', S)\}$.*

Let us introduce an example to illustrate the notions of the *degree of monotony* and *monotony maximizing extensions* in the context of an argumentation framework and one of its normal expansions.

Example 3. Let us consider the following argumentation frameworks (AF and AF' , such that $AF \preceq_N AF'$):

- $AF = (AR, AT) = (\{a, b, c, d\}, \{(a, b), (b, a), (b, c), (d, c)\})$.
- $AF' = (AR', AT') = (\{a, b, c, d, e\}, \{(a, b), (b, a), (b, c), (d, c), (a, e), (e, d)\})$.

Let us consider complete, preferred, and grounded semantics⁷.

- Complete semantics: $\sigma_{\text{complete}}(AF) = \{\{d\}, \{a, d\}, \{b, d\}\}$ and $\sigma_{\text{complete}}(AF') = \{\{\}, \{a, d\}, \{b, e\}\}$. Given $E_c = \{d\}$, $E_c \in \sigma_{\text{complete}}(AF)$, we have $Ext_{\text{mon}}(E_c, AF', \sigma_{\text{complete}}) = \{\{a, d\}\}$ and $deg_{\text{mon}}(\{a, d\}, E_c) = 1$.
- Preferred semantics: $\sigma_{\text{preferred}}(AF) = \{\{a, d\}, \{b, d\}\}$ and $\sigma_{\text{preferred}}(AF') = \{\{a, d\}, \{b, e\}\}$. Given $E_p = \{b, d\}$, $E_p \in \sigma_{\text{preferred}}(AF)$, we have $Ext_{\text{mon}}(E_p, AF', \sigma_{\text{preferred}}) = \{\{a, d\}, \{b, e\}\}$ and $deg_{\text{mon}}(\{a, d\}, E_p) = deg_{\text{mon}}(\{b, e\}, E_p) = \frac{1}{2}$.
- Grounded semantics: $\sigma_{\text{grounded}}(AF) = \{\{d\}\}$ and $\sigma_{\text{grounded}}(AF') = \{\{\}\}$. Given $E_g = \{d\}$, $E_g \in \sigma_{\text{grounded}}(AF)$, we have $Ext_{\text{mon}}(E_g, AF', \sigma_{\text{grounded}}) = \{\{\}\}$ and $deg_{\text{mon}}(\{\}, E_g) = 0$.

Figure 2 depicts the example's argumentation frameworks.

Fig. 2: Example argumentation frameworks to illustrate degrees of monotony.

Intuitively (and roughly speaking), the maximal degree of monotony may be 0 even in case of a small expansion of a large initial argumentation framework from which we infer a large set of arguments.

Proposition 2. *Let σ_x be an argumentation semantics, such that $x \in \{\text{stable}, \text{complete}, \text{preferred}, \text{grounded}\}$, and let $n \in \mathbb{N}, n > 1$. There exist two argumentation frameworks $AF = (AR, AT)$, $AF' = (AR', AT')$, such that $|AR| = n + 1$, $AF \preceq_N AF'$, $|AR' \setminus AR| = 1$, $|AT' \setminus AT| = 1$, and $\exists E \in \sigma(AF)$, such that $|E| = n$ and $\forall E' \in Ext_{\text{mon}}(E, AF', \sigma_x)$, $deg_{\text{mon}}(E', E) = 0$.*

⁷ We do not consider stable semantics for the sake of conciseness, and because stable semantics is not universally defined.

Proof. Let us consider the following argumentation framework constructions:

- $AF = (AR, AT)$, such that $AR = \{a_1, \dots, a_n, a_{n+1}\}$ and $AT = \{(a_{n+1}, a_i) | 1 \leq i < n\} \cup \{(a_n, a_{n+1})\}$;
- $AF' = (AR', AT')$, such that $AR' = AR \cup \{b\}$, $b \notin AR$ and $AT' = AT \cup \{(b, a_n)\}$.

Note that $AF \preceq_N AF'$ and $|AR' \setminus AR| = 1$, $|AT' \setminus AT| = 1$. We observe that $\sigma_x(AF) = \{E\}$, such that $E = \{a_1, \dots, a_n\}$, $|E| = n$ and $\sigma_x(AF') = \{\{b, a_{n+1}\}\}$. Hence, $\forall E$ such that $E \in \sigma_x(AF)$, $\forall E' \in \text{Ext}_{\text{mon}}(E, AF', \sigma_x)$, $\text{deg}_{\text{mon}}(E', E) = 0$. This proves the proposition⁸. $\square$

In contrast, if a normal expansion of an argumentation framework does not add any attacks to a specific extension E that has been inferred from the original argumentation framework, using complete, preferred, or grounded semantics, then we can infer an extension E' from the expansion such that the degree of monotony of E' w.r.t. E is 1. To prove this, let us first introduce the notion of (normal) E -weak expansions. Note that this definition draws upon the theoretical preliminaries provided by Definitions 2 and 5.

Definition 8 ((Normal) E-Weak Expansions). Let $AF = (AR, AT)$ and $AF' = (AR', AT')$ be argumentation frameworks and let $E \subseteq AR$.

- AF' is an E -weak expansion of AF (denoted by $AF \preceq_{W(E)} AF'$) iff $AF \preceq AF'$ and $\{(a, b) \mid (a, b) \in AT', a \in AR' \setminus AR, b \in E\} = \{\}$.
- AF' is an E -weak normal expansion of AF (denoted by $AF \preceq_{NW(E)} AF'$) iff $AF \preceq_N AF'$ and $\{(a, b) \mid (a, b) \in AT', a \in AR' \setminus AR, b \in E\} = \{\}$.

Note that colloquially speaking, we can say that cautious monotony (Definition 5) is monotony in the face of normal E -weak expansions.

Now, we can introduce and prove the proposition.

Proposition 3. Let σ_x be an argumentation semantics, such that $x \in \{\text{complete, preferred, grounded}\}$. For every argumentation framework $AF = (AR, AT)$, $\forall E \in \sigma_x(AF)$ and for every argumentation $AF' = (AR', AT')$, such that $AF \preceq_{NW(E)} AF'$, $\exists E' \in \sigma_x(AF')$ such that $\text{deg}_{\text{mon}}(E', E) = 1$.

Proof. Let us provide a proof by contradiction. Let us assume that $\exists E \in \sigma_x(AF)$, such that $\nexists E' \in \sigma_x(AF')$ and $\text{deg}_{\text{mon}}(E', E) = 1$.

1. By definition of the degree of monotony (Definition 6), the following statement holds true:

$$\begin{aligned} & \forall E \in \sigma(AF), \\ & \text{if } \nexists E' \in \sigma(AF'), \text{ such that } \text{deg}_{\text{mon}}(E', E) = 1 \\ & \text{then } \nexists E' \in \sigma(AF'), \text{ such that } E \subseteq E' \end{aligned}$$

⁸ Note that the fact that stable semantics is not universally defined is not relevant for this proof, as we prove the *existence* of a phenomenon, roughly speaking.

2. Because $AF \preceq_N AF'$, by definition of σ_x (Definition 4, σ_x is universally defined and each σ_x -extension contains all arguments it defends; consider that each preferred/grounded extension also is a complete extension), the following statement holds true:

$$\begin{aligned} & \forall E \in \sigma(AF), \\ & \text{if } \nexists E' \in \sigma(AF'), \text{ such that } E \subseteq E' \\ & \text{then } \exists a \in E, \text{ such that } a \text{ is attacked by } AR' \setminus AR \end{aligned}$$

3. From 1. and 2., it follows that the following statement holds true:

$$\begin{aligned} & \forall E \in \sigma(AF), \\ & \text{if } \nexists E' \in \sigma(AF'), \text{ such that } \deg_{mon}(E', E) = 1 \\ & \text{then } \exists a \in E, \text{ such that } a \text{ is attacked by } AR' \setminus AR \end{aligned}$$

4. From 3. it follows that $\forall E \in \sigma_x(AF)$ if $\nexists E' \in \sigma(AF')$ such that $\deg_{mon}(E', E) \neq 1$ then $AF \not\preceq_{NW(E)} AF'$. Contradiction! This proves the proposition. $\square$

Let us claim this proof entails the proof that complete, grounded, and preferred semantics satisfy cautious monotony and note that there exist argumentation semantics for which an analogous proposition does not hold true, consider *e.g.*, stable semantics and the argumentation frameworks $AF = (\{a\}, \{\})$, $AF' = (\{a, b\}, \{(b, b)\})$. This claim can be extended to other semantics like stage semantics [17], by considering the argumentation frameworks $AF = (\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$, $AF' = (\{a, b, c, d\}, \{(a, b), (b, c), (c, a), (d, a)\})$.

4 The “Degrees of Monotony”-Dilemma

This section formally introduces the notion of the degrees of monotony-dilemma and defines a possible mitigation approach.

Definition 9 (The Degrees of Monotony-Dilemma). *Let σ be an argumentation semantics. σ is affected by the degrees of monotony-dilemma iff there exist argumentation frameworks $AF = (AR, AT)$, $AF' = (AR', AT')$, and $AF'' = (AR'', AT'')$, such that $AF \preceq_N AF'$, $AF' \preceq_N AF''$ and the following statement does not hold true:*

$$\begin{aligned} & \forall E \in \sigma(AF), \forall E' \in \sigma(AF'), \text{ such that } E' \in \text{Exts}_{mon}(E, AF', \sigma), \\ & \exists E'' \in \sigma(AF''), \text{ such that } E'' \in \text{Exts}_{mon}(E', AF'', \sigma) \\ & \text{and } E'' \in \text{Exts}_{mon}(E, AF'', \sigma) \end{aligned}$$

Let us note that we constrain the definition to normal expansions because arguments can be *defeated* instead of removed to keep track of history and the addition of new attacks between existing arguments can be emulated by adding new arguments as well (for an analogy, think of immutable variables in some programming languages). Moreover, the constraint allows us to show that the degrees of monotony-dilemma occurs

even under these somewhat strict conditions. Complete and preferred semantics (the two universally defined Dung-semantics that may return more than one extension for a given argumentation framework) are affected by the degrees of monotony-dilemma.

Proposition 4 (Complete and Preferred Semantics are Affected by the Degrees of Monotony-Dilemma). *Let σ_x be an argumentation semantics, such that $x \in \{\text{complete}, \text{preferred}\}$. σ_x is affected by the degrees of monotony-dilemma.*

Proof. We provide a proof by counter-example. Let us consider the following argumentation frameworks: i) $AF = (AR, AT) = (\{a, b, c\}, \{(a, b), (b, a), (b, c)\})$; ii) $AF' = (AR', AT') = (\{a, b, c, d, e, f\}, \{(a, b), (b, a), (b, c)\})$; iii) $AF'' = (AR'', AT'') = (\{a, b, c, d, e, f, g\}, \{(a, b), (b, a), (b, c), (b, g), (g, d), (g, e), (g, f)\})$. Let us observe that $AF \preceq_N AF'$ and $AF' \preceq_N AF''$ and that: i) $\sigma_{\text{preferred}}(AF) = \{\{b\}, \{a, c\}\}$ and $\sigma_{\text{complete}}(AF) = \sigma_{\text{preferred}}(AF) \cup \{\{\}\}$; ii) $\sigma_{\text{preferred}}(AF') = \{\{b, d, e, f\}, \{a, c, d, e, f\}\}$ and $\sigma_{\text{complete}}(AF') = \sigma_{\text{preferred}}(AF') \cup \{\{d, e, f\}\}$; iii) $\sigma_{\text{preferred}}(AF'') = \{\{b, d, e, f\}, \{a, c, g\}\}$ and $\sigma_{\text{complete}}(AF'') = \sigma_{\text{preferred}}(AF'') \cup \{\{\}\}$.

Let us consider $E = \{a, c\}$, $E \in \sigma_x(AF)$. $\text{Exts}_{\text{mon}}(E, AF', \sigma_x) = \{\{a, c, d, e, f\}\}$. Given $E' = \{a, c, d, e, f\}$, $E' \in \text{Exts}_{\text{mon}}(E, AF', \sigma_x)$, we have $\text{Exts}_{\text{mon}}(E', AF'', \sigma_x) = \{\{b, d, e, f\}\}$. However, $\text{Exts}_{\text{mon}}(E, AF'', \sigma_x) = \{\{a, c, g\}\}$. Hence, we can conclude that given $E = \{a, c\}$, the following statement does not hold true:

$$\forall E' \in \sigma_x(AF'), \text{ such that } E' \in \text{Exts}_{\text{mon}}(E, AF', \sigma_x), \exists E'' \in \sigma_x(AF''), \\ \text{such that } E'' \in \text{Exts}_{\text{mon}}(E', AF'', \sigma_x) \text{ and } E'' \in \text{Exts}_{\text{mon}}(E, AF'', \sigma_x)$$

By definition of the degrees of monotony-dilemma (Definition 9), this proves the proposition. $\square$

Fig. 3: Complete and preferred semantics are affected by the degrees of monotony-dilemma.

Figure 3 illustrates the argumentation frameworks that are used in the proof of Proposition 4. The proposition highlights that the degrees of monotony-dilemma that we have colloquially described in the introduction does indeed occur in the context of

abstract argumentation. Let us observe that grounded semantics is not affected by the degrees of monotony-dilemma, as it is universally uniquely defined [3]⁹. In contrast, given an argumentation framework and its normal expansion, stable semantics may yield an extension for the original framework, but return no extension for the expansion (consider $AF = (\{a\}, \{\})$, $AF' = (\{a, b\}, \{(b, b)\})$); *i.e.*, even the more basic problem of maximally monotonic extension selection cannot be guaranteed for stable semantics.

To mitigate the degrees of monotony-dilemma, we can, for instance, compute a weighted average of several degrees of monotony, considering the results of past inferences. Let us first define the notion of the *weighted average degree of monotony*.

Definition 10 (Weighted Average Degree of Monotony). *Let E' be a finite set of arguments and let $ES = \langle E_0, \dots, E_n \rangle$ be a finite sequence of finite sets of arguments. We define the weighted average degree of monotony (denoted by $deg_{\gamma, wmon}(E', ES)$) as follows:*

$$deg_{\gamma, wmon}(E', ES) = \frac{\sum_{i=0}^n \gamma^{(n-i)} deg_{mon}(E', E_i)}{|n+1|},$$

where $0 < \gamma \leq 1$.

Note that in the definition, γ can be considered the *discount factor*, and a smaller γ gives a higher importance to recent inferences (relative to less recent inferences), roughly speaking.

Now, we can define the mitigation approach.

Definition 11 (Weighted Average Monotony-Maximizing Extensions). *Let $AF = (AR, AT)$ be an argumentation framework, let σ be an argumentation semantics, and let $ES = \langle E_0, \dots, E_n \rangle$ be a finite sequence of finite sets of arguments. We define the weighted average monotony-maximizing σ -extensions of AF w.r.t. ES , denoted by $Exts_{\gamma, wmon}(ES, AF, \sigma)$ as $\{E | E \in \sigma(AF), \nexists E' \in \sigma(AF), \text{ such that } deg_{\gamma, wmon}(E, ES) < deg_{\gamma, wmon}(E', ES)\}$, where $0 < \gamma \leq 1$.*

Let us illustrate this approach using an example.

Example 4. We consider preferred semantics and the argumentation frameworks depicted by Figure 3. We start by “selecting” $E_0 = \{a, b\}$, $E_0 \in \sigma_{\text{preferred}}(AF)$. Then, we have $Exts_{mon}(E, AF', \sigma_{\text{preferred}}) = Exts_{\gamma, wmon}(\langle E \rangle, AF', \sigma_{\text{preferred}}) = \{E_1\}$, such that $E_1 = \{a, c, d, e, f\}$. We assume $\gamma = 0.9$. Now, we want to select an extension from $\sigma_{\text{preferred}}(AF'')$, considering E_0 and E_1 . $\sigma_{\text{preferred}}(AF'') = \{\{a, c, g\}, \{b, d, e, f\}\}$. We have $deg_{\gamma, wmon}(\{a, c, g\}, \langle E_0, E_1 \rangle) = \frac{13}{20}$ and $deg_{\gamma, wmon}(\{b, d, e, f\}, \langle E_0, E_1 \rangle) = \frac{3}{10}$. Hence, we select $\{a, c, g\}$.

5 Discussion

This paper contributes to the continuation of a long line of research on systematically relaxing monotony, see for example the works by Gabbay [11] and Makinson [15] that

⁹ However, grounded semantics is typically considered a simplistic approach to abstract argumentation, roughly speaking because it is too skeptical.

introduce the restricted monotony and rational monotony principles. In the context of formal argumentation research, this paper adds to the body of research on *dynamics* in formal argumentation (see [8] for a survey), and in particular relies on work by Baumann and Brewka that presents monotony results in abstract argumentation [4]. Let us highlight that the work presented in this paper is different from (but has a similar motivation to) our recent results that show how to ensure consistent preferences over powersets of arguments in the context of normal expansions and naive set-based semantics [12], taking principles of economic rationality as a starting point (Dung’s admissible set-based semantics violate the corresponding argumentation principle). The results this paper provides raise the following questions.

1. Can we specify more bounds to the degree of monotony when expanding a belief base (argumentation framework), given some constraints to the nature of the expansion and the inference function (argumentation semantics)?
2. What additional approaches can mitigate the *degrees of monotony*-dilemma and according to which criteria can these approaches be formally analyzed?
3. What roles can degrees of monotony play in formal argumentation variants that extend abstract argumentation, such as value-based argumentation [6]?

Future research can set out to answer these questions for abstract argumentation, other formal argumentation variants, and indeed for any method of non-monotonic reasoning in the face of uncertainty. In the context of abstract argumentation, the analysis provided in this paper can, for example, be extended to other argumentation semantics (see [1] for an overview, and also see the weak admissible set-based semantics family as recently introduced by Baumann *et al.* [5]), and consider additional constraints, *e.g.* other notions of expansions [4], constraints to the structure of an argumentation framework, or formal argumentation principles [16] that constrain semantics behavior.

6 Conclusion

In this paper, we have defined the notion of the degree of monotony to abstract argumentation and then introduced the *degrees of monotony*-dilemma, which can occur when repeatedly first determining the extensions of an argumentation framework and then normally expanding the argumentation framework. The *degrees of monotony*-dilemma highlights that selecting extensions from every argumentation framework that maximize the degree of monotony w.r.t. an extension that has been inferred by the immediate predecessor may lead to a suboptimal degree of monotony of any extension inferred from an argumentation framework and any “indirect” predecessor. With this, the findings shed new light on the challenge of drawing “consistent” inferences from an expanding base of conflicting beliefs, and provide an increment to the point of departure for work on relaxed notions of monotony in formal argumentation and beyond.

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