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Preface

The Boost for Mathematics Evaluation Report

This evaluation report was first published in Swedish in 2016[1] and since then, the Boost for Mathematics has been a topic of interest in several international publications. Because of this interest we have decided to have the original comprehensive evaluation translated into English so that it is accessible to a wider audience.

Magnus Österholm, Tomas Bergqvist, Yvonne Liljekvist & Jorryt van Bommel

Karlstad, September 2021

[1] https://www.diva-portal.org/smash/get/diva2:1049895/FULLTEXT01.pdf

Summary

The Boost for Mathematics was a professional development program (PDP) for all mathematics teachers in Sweden. It was conducted between 2012 and 2016. The core of the PDP was collegial learning. Teachers worked together on different modules, each consisting of didactical material to be used in planning, discussing and performing mathematics teaching, as well as collegial reflections and discussions. The modules primarily highlighted four different didactical[2] perspectives: (1) teaching mathematics based on *abilities*[3], (2) assessment for teaching and learning in mathematics, i.e. *formative assessment*, (3) *classroom routines/interactions* and (4*)* classroom norms/*sociomathematical norms*.

This report presents an evaluation of the results of the Boost for Mathematics, examining to what extent the Boost for Mathematics has contributed to the development of a sustainable teaching culture and a sustainable PDP culture. The evaluation also aims to identify factors that benefit or disadvantage the results of the Boost for Mathematics, and is both formative and summative. This means that it aims to draw conclusions on both how the Boost for Mathematics has achieved the goals of developing the teaching and PDP culture, and how support for the planning and implementation of similar initiatives is formulated. The evaluation sample consists of 35 compulsory and upper-secondary schools. At each school, three teachers were randomly selected for inclusion in the evaluation, i.e. 105 teachers in total. The evaluation also includes each school's principal and a representative of the school organiser. The schools were visited on two occasions to investigate changes in the teaching and PDP culture. The data consists of observations of mathematics lessons and of collegial meetings, interviews and surveys with teachers and principals, and interviews with representatives of school organisers. The data also include collected documents (e.g., PDP plans and copies of teaching materials)

[2] "Didactic" is used in the European meaning, mirrored by the Anglo-American "pedagogy".

[3] "Abilities" (Sw: Förmågor) is used instead of Competencies, which is more often used in research, since this is the official translation by the Swedish National Agency for Education.

and background information about the visited schools (from external databases).

This report is the final report of the evaluation of the results of the Boost for Mathematics. The main findings of the evaluation are presented below, together with recommendations for future PDP initiatives of the same type as the Boost for Mathematics.

The Boost for Mathematics has developed teachers' work with their own teaching, but the teachers have not perceived any clear development in the joint work at the school.

Teachers' work with their own teaching is a key aspect of the teaching culture. This aspect of the teaching culture has developed to some extent for each didactical perspective, but there has only been more comprehensive development for some of the perspectives. Interviews with teachers show that the Boost for Mathematics has led them to planning and reflecting based on the didactical perspectives of *abilities*, *routines/interactions* and *sociomathematical norms*. Observations of lessons also show that the Boost for Mathematics has resulted in teachers now teaching more from the didactical perspectives of *abilities*, *formative assessment* and *routines/interactions* compared to before their participation in the Boost for Mathematics.

These changes are sustainable, as the changes observed when teachers take part in the Boost for Mathematics are maintained the year after completing the Boost for Mathematics. However, we cannot draw the same conclusion for the perspective of *sociomathematical norms* because the change does not occur until the year after the Boost for Mathematics. Since we did not collect data later than that, we cannot comment on whether the change in teachers' planning and reflections regarding *sociomathematical norms* is sustainable. Nor can we comment on whether the other changes are sustainable in a time perspective beyond one year after the end of the Boost for Mathematics.

The joint work of teachers and principals in the school to develop mathematics teaching is another key element of the teaching culture. Interviews and surveys with teachers show that they do not perceive

any clear development of this work in schools. On the contrary, there is a tendency for a certain decline in how teachers perceive the school's work to develop teaching. This decline is mainly due to the fact that teachers have *fewer* types of goals for the school's teaching development work after the Boost for Mathematics, e.g. goals only focus on their own role in the development of teaching, and do not include the role of the principal. This may be due to the fact that teachers' perspectives have been limited to the specific type of teaching development that exists within the framework of the Boost for Mathematics.

The Boost for Mathematics has influenced principals and teachers in different ways in relation to the teaching and PDP culture.

With regard to the school's work with the development of teaching (which is part of the teaching culture) and work with teachers' competence development (which is part of the PDP culture), a sustainable development has been seen among principals. When principals take part in the Boost for Mathematics, there is an improvement in their views on the school's work with teaching development and with competence development, and these changes are maintained the year after the Boost for Mathematics. However, there is a tendency for some decline the year after the Boost for Mathematics, which raises the question about whether the changes will be sustained in the longer term.

The developments in the school's work with the development of teaching (which is part of the teaching culture) and work with teachers' competence development (which is part of the PDP culture) that are noted among principals are not found among teachers. This has been described above with regard to the teaching culture, i.e. teachers do not perceive any clear development of the school's work with the development of teaching. Similar results are found for the development of PDP culture, i.e. that teachers do not perceive any clear development of the school's work with the teachers' competence development.

The above results are based on interviews and surveys, where both principals and teachers were asked the same type of questions. Thus, we can show that there is a difference between principals' and teachers' views on how the school works with teaching development and teacher competence development, as well as what types of goals exist for this work. Such a difference between principals and teachers can be an obstacle to sustainable development of the teaching and PDP culture. At the same time, the changes observed in principals, who are the educational leaders of the schools, can be an important component of sustainable development.

The impact of the Boost for Mathematics on the teaching and PDP culture has been independent of many internal and external factors.

The evaluation examined both internal and external factors in relation to the Boost for Mathematics. Internal factors are those which are directly linked to how the Boost for Mathematics has been implemented, such as how modules have been used and how supervision has been carried out in collegial meetings. A total of 13 internal factors have been analysed in the evaluation. External factors are those that are independent of the Boost for Mathematics, such as conditions in the school or municipality. A total of 22 external factors have been analysed in the evaluation.

Few internal factors can reliably explain variations in the changes caused by the Boost for Mathematics. In particular, no internal factors can explain changes in teachers' planning, reflection and implementation of teaching. Thus, when it comes to obtaining the changes observed, it is relatively unimportant *how* the Boost for Mathematics has been implemented. What is important is *that* the Boost for Mathematics has been implemented. However, some variables describing internal factors have relatively little variation across teachers. For example, teachers seem to have generally worked in similar ways with the modules. This can be seen in teachers' ratings of whether they worked with the modules in a satisfactory way according to the instructions provided, and the same applies to ratings of how supervision was carried out. This means that we cannot

determine whether greater variations in variables describing these practical aspects of implementation in the Boost for Mathematics would have produced different effects of the Boost for Mathematics. However, there is considerable variation in other practical aspects of the implementation of the Boost for Mathematics, such as to what extent the work was accommodated within regular working hours and whether substitutes were used to make this possible, as well as whether implementation of the Boost for Mathematics was adapted to local conditions, whether the current status was assessed prior to its implementation and whether teachers sat in on each other's lessons. This variation cannot be linked to different changes caused by the Boost for Mathematics. Within this evaluation, we are therefore not in a position to say which elements of the Boost for Mathematics are critical, i.e. how the results would change if a specific aspect of the Boost for Mathematics was completely eliminated. However, for principals, there is some link between the implementation of the Boost for Mathematics' principal training and changes in work with competence development, but not changes in work with the development of teaching. The training for principals thus seems to have some impact on the results of the Boost for Mathematics.

Few external factors can reliably explain variations in the changes caused by the Boost for Mathematics. Differences among teachers and principals, as well as the different conditions that exist in schools, have not had a major impact on the results of the Boost for Mathematics. However, there is a link between changes in teachers' teaching and certain external factors: teachers in larger municipalities (more pupils) have developed more than teachers in smaller municipalities, and teachers in compulsory schools have developed more than teachers in upper-secondary schools. The results suggest that the Boost for Mathematics somehow seems to have been less suitable for upper-secondary schools. As a result, teachers in upper secondary schools have not developed as well as teachers in compulsory schools in terms of implementation of teaching.

Overall, the structure of the Boost for Mathematics has been sufficiently robust to allow for a large degree of impact independently of the exact implementation of the Boost for Mathematics, and

independently of variations among participating teachers, principals and schools.

The development of teaching and PDP cultures in the Boost for Mathematics was primarily through impact at the individual level and had less to do with impact at the school level

As described above, the work in the Boost for Mathematics based on collegial learning worked well for teachers, as the results show development in teachers' work in planning, reflecting on and implementing teaching. The statistical analyses give the same results whether all teachers were considered as independently selected (randomly selected at the individual level) or the analyses take into account that several teachers came from the same school. This means that the observed effects of the Boost for Mathematics seem to occur mainly at the individual level and not at the school level.

As described above, the impact of the Boost for Mathematics on principals has been clear, as the results show that there has been progress in principals' work with both teaching development and competence development. Teachers do not perceive the same kind of development. Thus, no clear effect has been observed at the school level regarding these aspects of teaching and PDP culture. This conclusion is also supported by a specific analysis at the school level concerning whether there is a greater consensus among teachers and principals in the same school about work with both teaching development and competence development. No clear changes have taken place, but the trends show a decreased consensus in the schools. A decreased consensus among those involved may reduce the conditions for fruitful collaboration, as teachers and principals may have different views on how the school works and what the school's goals should be.

As described above, very few internal and external factors can explain variations in the changes brought about by the Boost for Mathematics. The impact of the Boost for Mathematics has thus been largely unaffected by the context in which teachers and principals found themselves when the Boost for Mathematics was implemented. This

result also shows that changes have primarily taken place at the individual level and not at the school level.

The fact that changes have taken place primarily at the individual level and not more jointly at the school level is considered to be a potential obstacle to sustainable development of the teaching and PDP cultures, as the cultures are based on a collaboration between individuals who are pulling in the same direction.

Recommendations

Based on the findings of this report, we have a number of recommendations that primarily concern similar initiatives in the future. The recommendations also concern what more detailed analyses of the Boost for Mathematics can be done to draw more specific conclusions about which elements of this type of professional development programs are beneficial or detrimental to its outcomes.

The Boost for Mathematics has had a clear sustainable impact on teachers' planning, reflection and implementation of teaching. We therefore recommend continuing to use the approach to teachers' PDP that is currently used in the Boost for Mathematics. However, as there were very few factors that can be clearly linked to these effects, we cannot comment on which elements of the approach are most important. The results show that upper-secondary teachers have not developed their teaching to the same extent as teachers in compulsory schools. There is thus a need for further analysis of the possible effects of how various aspects of the PDP are designed, including different modules, in general, but particularly with regard to differences between compulsory schools and upper-secondary schools. This need relates in particular to *formative assessment* (in relation to teachers' planning and reflections) and *sociomathematical norms* (in relation to teachers' teaching), as no clear effects were noticed for these perspectives.

The Boost for Mathematics has produced clear sustainable effects on principals' work with teaching development and competence development. We therefore recommend continuing to train principals

and to include principals in PDP initiatives like those conducted in the Boost for Mathematics. Since similar effects have not been seen in relation to teachers' work with teaching development and competence development, we recommend exploring whether and how the organisation can be improved in terms of links between the teachers' professional development and principals' training. For example, this could relate to how principals anchor their goals and plans with teachers, and how teachers' competence development could also include work with goals and plans at a broader level, beyond direct work with teaching.

The overall impact of the Boost for Mathematics seems to have been primarily on the individuals and less so on the schools. We therefore recommend that similar initiatives in the future focus more on developing a partnership between teachers and principals at each school. Together with the above suggestion of a link between teachers' PDP and principals' training, we recommend that future initiatives take a clearer overall approach to development at the school level in order to create even better conditions for the development of a sustainable teaching and PDP culture.

Some of the evaluation's findings show a tendency towards decline the year after the Boost for Mathematics, particularly with regard to outcome variables at the principal level. We therefore recommend that further follow-up be carried out to determine with certainty whether the Boost for Mathematics caused sustainable effects from a longer-term perspective. The need for this is also supported by the results showing that the Boost for Mathematics has generally impacted individuals and less so schools, as this poses a risk that the effects will not be sustained.

Overall, we have the following recommendations for similar initiatives in the future:

1. To keep the approach using collegial learning among teachers and training of principals, in order to sustain the positive results observed.
2. To develop professional development programs at a more comprehensive school level in terms of the partnership between

teachers and principals, in order to ensure the development of a sustainable teaching and PDP culture.

3. To carry out more in-depth analyses of the Boost for Mathematics, in order to understand in more detail what has created the observed effects, and to be able to implement similar initiatives that are more effective in the future.
4. To carry out follow-up of results where there is a tendency towards decline after implementation of the Boost for Mathematics, in order to determine with certainty which effects have become more sustainable and to be able to apply such knowledge to implement similar initiatives that are more effective in the future.

1. Introduction

The introductory chapter of the report describes the prerequisites and overall characteristics of the evaluation of the Boost for Mathematics results, i.e. the overall goals of the Boost for Mathematics, the focus of the evaluation, and who participated in the different steps of the evaluation process.

1.1 The Boost for Mathematics

The Boost for Mathematics was a professional development program (PDP) for all mathematics teachers in Sweden, conducted between 2012 and 2016. The core of the PDP was collegial learning. Teachers worked together, supported by supervisors and with different modules, consisting of didactical material to be used in planning and implementing mathematics teaching, as well as reflections and discussions on mathematics teaching. The modules primarily highlighted four different didactical perspective (the short names used in the support for the sake of simplicity are given in parenthesis):

1. teaching mathematics based on abilities (*abilities*)
2. assessment of teaching and learning in mathematics (*formative assessment*)
3. classroom routines/interactions (*routines/interactions*)
4. classroom norms/sociomathematical norms (*sociomathematical norms*)[4]

The Boost for Mathematics also included training of principals and mathematics supervisors.

The Boost for Mathematics had two overarching goals. Firstly, the aim was to develop the *teaching culture*, which refers to a focus on work with pupils' learning, and secondly the aim was to develop the *CDP culture*, which refers to a focus on working with teachers' learning. These two cultures are highly interdependent. In the Boost for

[4] In the evaluation, we needed to limit the analyses to certain aspects of each perspective. E.g., only *sociomathematical* norms were examined. See section 2.6.4.

Mathematics, collegial learning in particular is considered part of, and a prerequisite for, both cultures. A common focus of both cultures is:

- *The design of lessons*: development is based on local needs, conditions are provided for pupils' learning, and planning of and reflections on teaching form a central part of collegial learning.
- *Teachers' subject-specific didactical knowledge*: in terms of both the development and use of this knowledge, and based on research and proven experience.
- *Cooperation between school organizer, principal and teachers*: in terms of both teaching and PDP.

The intention was for these cultures to continue to develop at the schools after implementation of the Boost for Mathematics.

1.2 Focus and questions of the evaluation

This report describes an evaluation of the results of the Boost for Mathematics. In this context, the term "results" does not refer to pupil outcomes (e.g. test performance), but rather to the development of a teaching culture and PDP culture. The evaluation also includes the identification of factors that benefit or disadvantage the results of the Boost for Mathematics.

The evaluation is therefore based on three overarching questions:

A. To what extent has the Boost for Mathematics contributed to the development of a sustainable teaching culture?

B. To what extent has the Boost for Mathematics contributed to the development of a sustainable PDP culture?

C. What factors have influenced how well or less well the Boost for Mathematics contributed to the development of sustainable teaching and PDP cultures?

In the evaluation, the teaching and PDP cultures are examined based on the components of the Boost for Mathematics and how these two cultures are characterized there (see section 1.1). Not all aspects of the teaching and PDP cultures could be examined in this evaluation. The primary focus of the evaluation is the teachers and the teaching culture in relation to the didactical perspectives. This is a subject-specific didactical focus on teachers' work with reflections on, planning and

implementation of teaching. Thus, this evaluation can serve as a complement to other evaluations of the Boost for Mathematics. Other aspects of the teaching culture and aspects of the PDP culture will also be studied. More details on the limitations of this evaluation are described in section 2.7.1.

This evaluation is both summative and formative. This means that it aims to draw conclusions on both how the Boost for Mathematics has achieved the goals of developing the teaching and PDP culture, and how support for the planning and implementation of similar training initiatives is formulated. It is a matter of highlighting both generalising answers about schools and mathematics teachers in Sweden, describing how the Boost for Mathematics has had an impact on individual schools, and creating models for developing future professional development initiatives by focusing attention on what lies behind any effects of the Boost for Mathematics.

1.3 Evaluation team

The evaluation team is divided into a project team, a reference group, and a group of project assistants.

The project team worked on all aspects of the evaluation, with particular responsibility for all planning and reporting, as well as the quality control of work carried out by the project assistants. The project team consisted of[5]:

- Magnus Österholm, Project Manager, Associate Professor of Mathematics Education at Umeå University and Visiting Professor of Mathematics Education at Mid Sweden University
- Tomas Bergqvist, Deputy Project Manager, Associate Professor in Educational Work at Umeå University
- Jorryt van Bommel, Senior Lecturer in Mathematics Education at Karlstad University
- Yvonne Liljekvist, Senior Lecturer in Mathematics Education at Karlstad University and Postdoctoral Researcher at Uppsala University

[5] Name, role, workplace and title dated 2016

The reference group had an advisory role in relation to project planning, analyses and reporting. Their expertise, particularly in mathematics didactics, evaluations, and qualitative and quantitative research methodology, has been used for quality assurance in critical phases of the implementation, i.e. both in planning and reporting of the evaluation. The reference group consisted of:

- Ewa Bergqvist, Senior Lecturer in Mathematics Education at Umeå University. Broad expertise in mathematics didactics, including in particular experience of studies on teaching based on the abilities and on testing and assessment.
- Anna Lind Pantzare, Project Manager for national testing and other assessment support in mathematics and natural science subjects at Umeå University. Expertise in aspects of testing and assessment in mathematics.
- Johan Lithner, Professor in Mathematics Education at Umeå University. Expertise in project management of large evaluation projects in mathematics teaching and in teaching based on abilities in particular.
- Mathias Lundin, Senior Lecturer in Statistics at Umeå University. Expertise in statistical analysis of complex data sets.
- Per Nilsson, Professor in Mathematics at Örebro University. Broad expertise in mathematics education, including experience in large-scale teacher PDP.
- Torulf Palm, Associate Professor in Educational Work at Umeå University. Expertise in formative classroom practice (assessment for teaching and learning).
- Andreas Ryve, Professor in Mathematics Education at Mälardalen University. Expertise in project management of large PDP and evaluation projects in mathematics education.
- Christina Segerholm, Professor in Education at Umeå University. Expertise in evaluation of school activities.

The project assistants carried out data collection and processing. They all have experience as teachers and experience with similar types of data collection in different research projects. Some of the project assistants (flagged with * below), who have at least two years of

doctoral education and documented experience from similar types of methods and analyses, also conducted more detailed analysis of the data. The following project assistants participated:

- Catarina Andersson
- Marie Bergholm
- Anneli Dyrvold*
- Moa Eirell
- Helene Hammenborg
- Jonas Jäder*
- Klara Kerekes
- Leif Maerker
- Jannika Lindvall
- Jan Olsson
- Mattias Pettersson
- Annalisa Rådeström
- Johan Sidenvall*
- Jenny Sullivan Hellgren
- Lotta Vingsle*
- Krister Ödmark

1.4 Focus and layout of the report

The first chapter of this report presents an overview of the purpose of the Boost for Mathematics professional development initiative. It then presents the focus and design of the evaluation and all the participants in all stages of the evaluation. The second chapter describes the methods used in the evaluation. It also describes how the four didactical perspectives (*abilities, formative assessment, roles/interactions* and *sociomathematical norms*) guided the analysis.

Chapters 3–6 present the findings: that the Boost for Mathematics has had an impact on teaching (chapter 3), that principals have changed (chapter 4), that there are indications of a decreased consensus among principals and teachers (chapter 5) and that the changes demonstrated are independent of many factors (chapter 6).

The seventh and final chapter of the report presents the conclusions that can be drawn from the evaluation's findings in relation to the three overarching questions of the evaluation. The chapter concludes with recommendations for future PDP initiatives. The report also contains a list of references and appendices (A–Q). The appendices present in detail, among other things, parts of the methodology for data collection and analysis.

2. Methods of data collection and analysis

This chapter provides a concise description of the methods used in the evaluation. For a more detailed description, see Appendix A.

2.1 General starting points and design

As part of the evaluation of the Boost for Mathematics' results, visits were made to randomly selected schools throughout Sweden. We used repeated data collection, which means that data were collected from the same schools at two points in time. The main reason for this design is that the purpose of the evaluation is largely to investigate *changes* in teaching and PDP cultures, and this is made possible by collecting data at two points in time.

The sample consisted of 35 compulsory and upper-secondary schools. Data collection was done in the same way in all schools, and statistical methods were used for data analysis. The purpose of using statistical analyses was to be able to generalize the results to the population of Swedish mathematics teachers in compulsory and upper-secondary schools. We also visited two compulsory schools for pupils with learning disabilities, one upper-secondary school for pupils with learning disabilities, one school for adults with learning disabilities, and one municipal adult education school, but these are not included in the analysis presented here. Descriptions from these school visits are provided in Appendix Q.

Each school was visited either *before* and *during* implementation of the Boost for Mathematics or *during* and *after* implementation of the Boost for Mathematics. The main part of the data collection was designed in such a way that the same type of data was collected at all school visits in order to characterize the current teaching and PDP culture. This allowed for an examination of changes in the teaching and PDP culture in a direct way.

The design was chosen in order to be able to carry out the evaluation within existing conditions, while retaining the possibility of conducting two types of analyses. It was not possible to visit the schools on three occasions (before, during and after) given the time of the start of the

Boost for Mathematics and the timeline for implementing the evaluation. However, the design allows us to conduct the analyses we wanted to do: firstly, to compare the situation *before* and *during* implementation of the Boost for Mathematics by the schools, which gives answers to whether change occurred with the active phase in the Boost for Mathematics, and secondly to compare the situation *during* and *after* implementation of the Boost for Mathematics by the schools, which gives answers to whether a more sustainable change persists after the end of the PDP initiative.

For each school visit, data collection consisted of observations of three mathematics lessons, interviews with the three teachers whose lessons were observed, an interview with the principal and representatives of the school organizer, and, if the visit took place *during* the school's implementation of the Boost for Mathematics, the observation of a collegial meeting. The evaluation focuses primarily on teachers' practice as part of the teaching culture. This was explored through observations of how teaching was carried out and through interviews dealing with planning of and reflection on teaching. We also examined internal and external factors that might influence the effects of participation in the Boost for Mathematics. Internal factors are the conditions for, and characteristics of, the implementation of the Boost for Mathematics, such as how modules are used and the role of the supervisor in collegial meetings. External factors are those that are independent of the Boost for Mathematics, such as the composition of the group of pupils and staff in the school or in the municipality.

In order to investigate whether the Boost for Mathematics contributed to teachers working more in line with the four didactical perspectives, analyses of the teaching culture were conducted largely from these perspectives. For each didactical perspective there are several different interview questions concerning teachers' planning and reflection. Each interview question was rated on a three or four-point scale (see detailed examples in section 2.6). An outcome variable was then created by calculating an average value across all questions related to the specific didactical perspective for each teacher. All outcome variables were constructed in this way, i.e. by calculating an average value across several different assessments. Thereafter, by carrying out statistical

analyses on the data, we were able to investigate whether there had been any change in these outcome variables as a result of the teachers' and principals' participation in the Boost for Mathematics, and what factors might explain such changes.

All aspects of data collection and analysis were tested during the school visits conducted in the spring of 2014. The first interim report of the evaluation concluded that the data collection and analysis provided sufficient evidence to create a broad and relevant picture of school activity in relation to the purpose of the evaluation. Some adjustments were made to the analysis and data collection tools, but these were only minor. This means that data from these three schools could be included in the evaluation. The bulk of the data collection took place in the spring of 2015 and the spring of 2016. See Appendix A for more information on our quality assurance efforts throughout the evaluation.

2.2 Data selection and non-response

The evaluation involved visits to a total of 35 randomly selected compulsory and upper-secondary schools across Sweden. In addition, two compulsory schools for pupils with learning disabilities, one upper-secondary school for pupils with learning disabilities, one school for adults with learning disabilities, and one municipal adult education school were visited, but these schools are not included in the analysis and are described in Appendix Q. All schools were randomly selected, but stratified by school form, in order to analyse possible differences between school forms. The analysis includes 18 compulsory schools and 17 upper-secondary schools. At each school, three teachers were randomly selected to visit, i.e. 105 teachers in total.

The 35 schools were divided into two groups based on whether the visits took place *before* and *during* the Boost for Mathematics (group IU) or *during* and *after* the Boost for Mathematics (group UE), as shown in Table 1. Each group includes (as close as possible) as many compulsory schools and upper-secondary schools.

Table 1: Timetable for school visits.

Group	Before the Boost for Mathematics	During the Boost for Mathematics	After the Boost for Mathematics
IU (18 schools)	Spring 2015	Spring 2016	
UE (17 schools)		Spring 2014 (3 schools) Spring 2015 (14 schools)	Spring 2016

Table 2 summarises the total number of school visits conducted, thus visualising the extent of non-response for different data types. Overall, the non-response is very small, except for school organisers. Due to the large non-response rate among school organisers, they are excluded from the analyses in this evaluation. When schools were contacted for the first visit, it was relatively common that the school could not or did not want to be included in the evaluation. In such situations, a new school was randomly selected so that we could ultimately conduct visits at a total of 35 schools. A more detailed description of types of non-response and how non-response was handled can be found in Appendix A.

Table 2: Total data sets generated.

		Collected data	
Type of data	Planned number	Visit 1	Visit 2
Teacher, observation	105	98 (93%)	98 (93%)
Teacher, interview	105	101 (96%)	100 (95%)
Teacher, survey	105	95 (90%)	93 (89%)
Principal, interview	35	34 (97%)	31 (89%)
Principal, survey	35	28 (80%)	28 (80%)
Collegial meeting	18 (visit 1) 17 (visit 2)	17 (94%)	15 (88%)

Chair of the school organiser, interview	35	19 (54%)	15 (43%)
Manager of the school organiser, interview	35	20 (57%)	14 (40%)

In some analyses, data from all teachers or principals may be used from a particular visit (i.e. either visit 1 or visit 2). In such analyses, data are thus available from more than 90 teachers and from about 30 principals. We conducted an analysis of the required scope of the data material, i.e. a statistical power analysis. The analysis shows that the number of teachers in the sample is sufficient to detect at least medium effects in regression analyses with at least five explanatory variables, and that the number of principals is sufficient to detect at least large effects in regression analyses with at least two explanatory variables.

Each school was visited twice, one year apart. It was not always possible to visit the same teacher or principal on both occasions. When the same teacher could not be visited, another teacher from the same school was randomly selected. When the principal at the school was replaced, the new principal was interviewed. Table 3 shows the number of teachers and principals who were visited twice, which is the basis for analyses that examine change directly. An analysis of statistical power shows that the number of teachers is sufficient to detect at least large effects in regression analyses with at least three explanatory variables, whereas the number of principals in this type of analysis can only detect very large effects with certainty.

Table 3: Number of people visited twice.

	Visited before and during the Boost for mathematics	Visited during and after the Boost for mathematics
Teachers	36	42
Principals	15	12

Taken together, this means that our data provide very good possibilities for evaluating the effects of the Boost for Mathematics at the teacher level and good possibilities for evaluating the effects at the principal and school level. This is especially true when we have the possibility to

combine different types of analyses. See section 2.7 for more details on these different types of analysis.

2.3 Implementation of data collection

Data collection took place through visits to each school in the sample. During the school visits, a mathematics lesson was observed for the sampled teachers and interviews were conducted with these teachers as well as with the school principal. After the school visit, an electronic survey was sent to the teachers and the principal, and telephone interviews were conducted with representatives of the school organiser. These elements of the data collection are described in more detail below.

Thus, one school visit included lesson observations of three teachers' teaching. After each observed lesson, an interview was conducted with the teacher. In addition, a collegial meeting was observed in the visit if this took place *during* the Boost for Mathematics. An interview was also conducted with the school principal.

2.3.1 Observations

During the lesson observation, the teacher's voice was recorded and during the collegial meeting, the voices of all teachers present were recorded. The audio recordings were supplemented with copies or photographs of teaching materials. In addition, an observation protocol was used that included, for example, notes of what was written on the board and what tasks pupils were working on (see Appendix C).

2.3.2 Interviews

Since many different people conducted the interviews, we used a structured interview format to ensure equivalence in the data collected (the interview questions are included in Appendix B). The interviews were recorded and transcribed for analysis. Interviews with teachers and principals were conducted mainly during the school visit, but could also be conducted by telephone after the visit, if necessary.

Interviews with representatives of the school organiser were conducted by telephone. For each school, a representative was contacted at the Chair level, i.e. the Chair of the education committee or similar when

the organiser is a municipality or the Chair of the Board or equivalent for the owner of an independent school. For those schools where this was possible, a responsible person at management level, i.e. within the administrative unit or equivalent, who was responsible for the school in question was also contacted. Situations where it was not possible to interview a person at management level include, for example, when a corporation consists of a single school.

2.3.3 Documents

In connection with the interviews, documents were collected from teachers, principals and school organisers. This includes any plans, policy documents or similar documents dealing with the (local) development of mathematics education or professional development.

2.3.4 Background information

Background information on the schools visited was collected through the National Agency for Education's SIRIS database. This includes information on the composition of the group of pupils and staff at the school and in the municipality. Appendix D contains a list of the information collected in this manner.

2.4 Ethical considerations

The ethical considerations concern both the evaluation in relation to the respondents and how the evaluation's data collection methods can be optimised to give a fair picture of what is being evaluated. In terms of respondents, written consent was collected from all participating teachers and principals. The interviewed school organisers were asked to give verbal consent during the telephone interview. There is no pupil data in any form included in this evaluation. For example, only the teacher's voice constituted data in classroom recordings. Therefore, no consent was collected from pupils. However, all pupils, as well as the guardians of the younger pupils, were informed via their respective schools that the evaluation of the Boost for Mathematics results would take place at the school and they were given the opportunity to contact the school or the project manager with any questions or comments.

In terms of how the data collection methods provide an accurate picture of teachers' teaching, there is a limitation in only recording

audio and not video during the observations. This is because audio does not capture all of the communication. For some forms of teaching, it is noted that a greater limitation arises when only audio is recorded. An example is when the teaching includes visualisation of geometric concepts and phenomena. Overall, however, the decision was made to only use audio recording due to ethical considerations relative to the pupils and in order to optimise the analysis procedure.

In cases where photographs were taken in the classrooms, no people were included in the picture. It was only the information on the board or the teaching materials that were photographed.

Ethical considerations have also been made in relation to the link between the Boost for Mathematics and the evaluation of the Boost for Mathematics' results. Neither the project manager nor the deputy project manager have been involved in any part of the Boost for Mathematics. No one in the project team has been involved in the development of any part of the Boost for Mathematics, but Liljekvist has been an external reviewer of the texts in one module (Language in mathematics, compulsory school) and van Bommel carried out supervisor training in the Boost for Mathematics.

2.5 Data processing and analysis procedure

This section describes the elements of data processing and analysis that took place for each type of data. All data, collected during the school visits and via telephone interviews and web surveys, were processed according to specific instructions in order to achieve consistency in the way the project assistants worked during data collection. All project assistants were trained in data collection and initial analysis.

After the school visits, each project assistant transcribed the interviews practically verbatim. What was excluded were statements that were clearly not related to the content of the question at hand. The transcription was done in spreadsheets so that answers to each question could be easily retrieved for further processing and analysis.
The lesson observations were coded by instructional format according to the following table:

Table 4: Categories of instructional formats

X	No mathematics/ teaching	Does not deal with mathematics teaching, but e.g. more practical things, such as information about when and how tests should be held or furnishing the classroom.
HL	Teacher monologue	Primarily the teacher guiding the discussion, with only occasional questions to the pupils.
HE	Pupil monologue	Primarily individual pupils guiding the discussion in front of the whole class.
HD	Dialogue	Teacher and individual pupils having a discussion in front of the whole class.
HA	Everyone can participate	Almost everyone in the whole class has an equal role in participating and (partially) guiding the discussion. However, not everyone has to participate actively, but the *possibility* exists.
G	Group	Explicit instruction to work in groups of at least two people.
I	Individual	Explicit instruction to work individual

For each part of the lesson, the type of pupil activity was also described as one of the following three categories: *one task* where all pupils worked with the same mathematical task, *different tasks* where pupils worked with different tasks specified or delimited by the teacher, or *open* where the activities were not based on mathematical tasks (e.g. discussion or question and answer session).

In addition, each assistant conducted some analysis of the teacher's questions to the pupils and the teacher's evaluations of mathematical statements. These were assessed according to the instructions in Appendix H and Appendix I.

Each project assistant also summarised the collegial meetings and marked which type of activity each part of the discussion consisted of. The alternatives were *Suggestion* (when someone indicates a desired topic of conversation), *Question* (when someone asks for a response of some kind), *Change* (when someone starts talking about something not related to the current topic), *Monologue* (attempted change of topic not picked up by anyone else), and *Unclear* (part of conversation where it is difficult to determine what the topic is). Further, the assistant analysed what characterised the content of the discussions in the collegial meeting as shown in Appendix K. In addition, for each activity it was noted whether the supervisor initiated the activity, and whether the supervisor then was active or passive in it.

When the assistants had completed all initial analyses, the material was passed on to four specially trained project assistants who carried out more detailed analyses. Here, the work was divided in a different way. By using the specific knowledge of these project assistants, a higher quality of analysis could be obtained. For the three didactical perspectives of *formative assessment*, *routines/interactions* and *sociomathematical norms*, the same person therefore carried out all the analyses in the evaluation. For the didactical perspective of *abilities*, one person analysed all data from the first round of data collection and another person analysed all data from the second round. These two project assistants have previously, and collaboratively, worked on the analysis of data relative to *abilities* and therefore have a high degree of commonality in terms of the view of *abilities* and the analysis of data with this focus. In addition, comparability has been tested statistically across the different rounds of data collection, as described in Appendix A.

The overall analyses were conducted by the project team, with the assistance of a specialist in statistical analysis of complex data sets.

2.6 The didactical perspectives as starting points for analysis

Much of the analysis in the evaluation is theoretically grounded in the four didactical perspectives that permeate the PDP initiative, i.e. *abilities* (DP1), *formative assessment* (DP2), *roles/interactions* (DP3) and *sociomathematical norms* (DP4). In the descriptions that follow,

the abbreviations DP1, DP2, DP3 and DP4 will sometimes be used when referring to the four didactical perspectives. Here follows a brief operationalisation of these four theoretical starting points, consisting of a general description of each didactical perspective, a description of the key elements we have chosen to focus on, a justification of these choices, and examples of analysis variables associated with each perspective. The creation of variables based on these types of assessments is described in the next section (2.7).

The coding manuals in sections 2.6.1–2.6.4 were constructed prior to data collection and analysis. The examples are therefore not authentic from the data collection, but statements analogous to most of the constructed examples were found in the analyses of the collected data. A more detailed description of the operationalisations can be found in Appendices E–I.

2.6.1 Abilities

The aim is to investigate the extent of and any changes related to didactical perspective 1, i.e. whether planning, implementation and reflections on teaching are based on *abilities* as described in the syllabuses (cf Bergqvist et al., 2009, 2010; Niss, 2003). The syllabus for compulsory school and syllabus for upper-secondary school state that teaching in the subject should provide pupils with the conditions for developing mathematical skills. Five abilities are applied in both compulsory school and upper-secondary school:

- Problem solving – the ability to solve mathematical problems
- Concepts – the ability to use and analyse mathematical concepts
- Procedures – the ability to choose and use mathematical methods for calculations
- Reasoning – ability to apply and follow mathematical reasoning
- Communication – ability to communicate mathematical ideas

For upper-secondary schools, there are two additional abilities:

- Modelling – the ability to interpret a realistic situation and design a mathematical model
- Relate – the ability to relate mathematics to its meaning in other contexts

A more detailed description and operationalisation of *abilities* can be found in Appendix F.

The evaluation has focused on three key aspects:

1. The teachers' knowledge of the abilities and how it is conveyed in the collegial meeting
2. Which abilities pupils are given the opportunity to develop
3. What understanding teachers show of how classroom activities can provide opportunities for pupils to develop the abilities

When analysing the teacher interviews, 13 separate assessments were made. Nine assessments relate to aspect 1, regarding teachers' knowledge of the abilities (incl. assessments for each ability), one assessment relates to aspect 2, and three assessments relate to aspect 3. Similarly, assessments were made in the analysis of the lesson observations and the collegial meeting. All assessments made for the analysis of the perspective *abilities* are described in Appendix E. The assessments were then used to create variables (see section 2.7). The following are examples of assessments included in the analysis of the teacher interviews and lesson observations regarding abilities. One deals with *abilities* in general and the other with problem-solving ability specifically.

Example 1. Assessment of to what extent the teacher demonstrates competence and willingness to allow abilities to influence lesson planning.

This assessment was carried out for aspect 2, and the value is based on the answers to question 11 and question 16 of the teacher interview guide (see Appendix B). The assessment determined a value according to the following table:

Table 5: Assessment example DP1, teacher interview.

Value	Definition	Example
0	The teacher never talks spontaneously about abilities in connection with lesson planning, and only briefly when asked directly.	*Conceptual ability is one of the abilities that the pupils shall develop during the lessons.*
0.5	The teacher stresses the importance of the abilities influencing lesson planning, but only in general terms. The teacher only talks about cognitive or productive aspects.	*It takes practice to be able to apply a concept, so I usually incorporate a lot of that into the lessons.*
1	The teacher provides examples of how different abilities affect lesson planning. The teacher talks about both cognitive and productive aspects of the abilities.	*To understand a concept, such as variables, requires insights into both what the concept means and how it is handled. Both discussions about the concepts and practice in how to apply them are therefore always included.*

Example 2. Assessment of whether the teacher highlights the problem-solving ability in the teacher-led activity to increase pupils' ability to develop it.

When analysing lesson observations in relation to *abilities*, all activities were assessed based on whether they provided the pupils an opportunity to develop an ability or not. When making this assessment, the analysis was based on the operationalisation of problem-solving ability (see Appendix F). What was analysed were the instructions given by the teacher during the lesson. For each teacher-led activity, to what extent the problem-solving ability was made visible by the teacher

was also analysed. The value of the visibility was determined based on the following table:

Table 6: Assessment example DP1, classroom observation.

Value	Description
0	No visibility
0.5	The ability is mentioned in such a way that the pupils understand that it is the one they will be learning or practicing.
1	The ability is discussed in such a way that the pupils can understand what it means.

Note that an ability may be visible even if pupils are not given the opportunity to develop it, for example if the teacher talks about an ability without linking it to a pupil activity, or if the teacher discusses an activity that was carried out in a previous lesson.

2.6.2 Formative assessment

The aim is to investigate whether a classroom practice is based on *formative assessment* (didactical perspective 2) by studying to what extent and in what way the practice is described, planned for, implemented and discussed based on *the big idea* and *five key strategies* (see Hattie & Timperley, 2007; Wiliam 2011). The big idea is about teachers and pupils collecting, interpreting and using information on pupils' knowledge and performance as the basis for making decisions on the design and implementation of future teaching to achieve desired learning outcomes. The five key strategies are:

- **Goals:** goals for learning and criteria for success are clarified and shared by the teacher.
- **Mapping:** The teacher creates effective classroom discussions and other learning situations that provide evidence of pupils' knowledge.
- **Feedback:** The pupils receive effective feedback that moves them forward in their learning.
- **Resources:** The pupils are activated as resources for each other in the learning.
- **Owner:** The pupils are activated as owners of their own learning.

The four aspects on which the evaluation focused are:

1. Analysis of interviews: To what extent are different parts of the big idea addressed by the teacher?
2. Analysis of interviews: Which of the five key strategies does the teacher say were used or planned for in the classroom?
3. Analysis of lesson observations: Which of the five key strategies are used in the classroom?
4. Analysis of observations of collegial meeting: To what extent and in what way do the discussions focus on and draw on formative assessment?

A total of 26 separate assessments were made. Six assessments were made when analysing the interviews, one for the big idea and one for each of the five key strategies. Each assessment was based on one to three interview questions. Ten assessments were made when analysing lesson observations, two for each key strategy. The first assessment concerned the existence of a key strategy and the second the quality of that key strategy. Similarly, ten assessments were made in the analysis of the peer review, i.e. whether a key strategy was addressed and, if so, the quality with which it was addressed. All assessments made for the analysis of the formative assessment perspective are described in Appendix G. Here are two examples from the coding manual of what was included in the assessments that were made, one for analysis of the teacher interview and one for analysis of the lesson observation.

Example 3. Assessment of how the teacher describes *feedback*; whether pupils receive effective feedback that moves them forward in their learning.

This assessment addresses aspect 2 above, i.e. which key strategies the teacher plans for. The assessment was based on responses to interview questions 4 and 5 in the interview guide (see Appendix B). The assessment determined a value based on the following table:

Table 7: Assessment example DP2, teacher interview.

Value	Description	*Example*
0	Does not describe any plans for feedback to pupils or only general/ambiguous description that results/feedback is (always) given to the pupils.	*Feedback is probably mostly in connection with them getting their test results back.*
0.5	Describes feedback plans that are concrete in terms of a particular approach and/or linked to a particular mathematics content or type of activity.	*The pupils are allowed to correct their own diagnoses. I then talk to them about how it went.*
1	Describes plans for pupils to receive feedforward focused on learning outcomes.	*Once pupils have made their diagnoses, we go through them together and talk about what areas the pupil needs to work on more to achieve the goals.*

Example 4. Assessment of whether the pupils are activated as resources for each other in the learning.

When analysing the lesson observations, the existence of the key strategy and the quality of the activity were determined. The values were determined in two steps based on the following tables:

Table 8: Assessment example DP2, classroom observation, existence.

Yes	Pupil-pupil assessment occurs, and pupil groups are given the opportunity to consult on questions
No	The above does not occur.

Table 9: Assessment example DP2, classroom observation, quality.

Value	Description	Example
0	The pupils are not given any instructions on how to perform assessment.	*The pupils work in pairs and discuss their answers.*
0.5	The pupils are given brief instructions on how to perform assessment.	*The assessment should show whether the task was solved correctly, and what was wrong if it is not correct.*
1	The teacher describes what should be assessed and how assessment is performed, e.g. focus on one or more abilities.	*The pupils may mark in each other's solutions which mathematical concepts were used and how they have been explained.*

2.6.3 Routines and interactions

The aim is to examine the routines and interactions that permeate a classroom (didactical perspective 3) based on three key elements:

- Overarching interaction patterns: monologic – dialogic (Dysthe, 1996).
- Teachers' questions and prompts in terms of type of answer sought (Martino & Maher, 1999):
    - Short answers that can be directly evaluated with right or wrong
    - Questions that are not answered or not expected to be answered
    - Guidance or support
    - "Cognitive questions": Questions which invite students to express their thinking (i.e., to make connections, promoting justification and generalization)
    - "Metacognitive questions": Questions inviting students to monitor, self-regulate and control their own work

- Whole-class discussions of solutions (Stein et al., 2008):
    - Whether the teacher notes what types of mathematical thinking the pupils use
    - How the teacher selects which solutions to present
    - How the teacher comments on connections between different pupil solutions

In the analysis, we focus on the above points as follows:

1. Analysis of interviews: The teacher's view of the overall interaction pattern, on questions and prompts, and on the planning of whole-class discussions of task solutions.
2. Analysis of lesson observations: The overall interaction pattern, the questions and prompts used by the teacher in the classroom, and the organisation of whole-class discussions of tasks.
3. Analysis of collegial meeting: The teachers' discussions of interaction patterns, questions, and whole-class discussions of solutions.

A total of 36 assessments were made regarding routines and interactions, with multiple assessments being made for many situations combined. Twenty-five assessments were made in the analysis of interviews, three in the analysis of classroom observations and eight in the analysis of collegial meetings. In these analyses, the scale for the assessments is in most cases from -1 to 1 because, according to the research on which the evaluation is based, it is negative for pupils' learning if, for example, a monologue interaction pattern is completely dominant in a classroom. All assessments made for the analysis of the perspective *routines and interactions* are described in Appendix H. The assessments were then used to create variables (see section 2.7). Here are two examples of assessments within this didactical perspective. The first is from the analysis of the interviews and the second is from the analysis of the classroom observations.

Example 5. Assessment of whether the teacher favours a dialogic approach in different instructional formats.

For each of the four instructional formats (teacher-led review, whole-class discussion, group work and individual work), the teacher's

description of the considerations made was assessed (interview question 7) with regard to the dialogic approach. If multiple levels in the table could be identified, the one that was dominant in the teacher's response was chosen. *Neutral* was selected if they were favoured about the same. If the teacher rejected or favoured a *monologic* approach, this was also considered neutral. Table 10 is used to assess the answers to each sub-question.

Table 10: Assessment example DP3, teacher interview.

Value	Description	Example (from data)
- 1	Rejects dialogic approach based on transmission view of learning	Pupils cannot learn in group work because there are only pupils with the same level of knowledge.
- 0.5	Generally rejects dialogic approach	It is difficult to create learning in whole-class discussions.
0	Neutral or unclear	
0.5	Generally favours dialogic approach by emphasising the importance of the pupils' voices, communication and/or activity	Group work makes pupils learn better since they get to discuss.
1	Favours dialogic approach more deeply based on pupils' learning.	In whole-class discussions, pupils' thoughts can be challenged in a way that supports their learning.

Example 6. Assessment of whether the teacher asks questions of a more dialogic nature.

A *question* is defined as an occasion when the teacher asks the pupils something with a focus on mathematics. For example, questions of the type "What class do you have after this one?" were excluded. For each lesson, up to 30 questions were analysed in randomly selected parts of

the lesson. For each question, the type was identified according to the following table:

Table 11: Assessment example DP3, classroom observation.

Type	Requirement for a question to be classified as this type. (Note: The same question can be classified as multiple types)
No answer	The teacher does not give the pupil(s) an opportunity to answer the question, e.g. by giving an answer directly or by immediately continuing on and talking (about something else) after asking the question.
Short answer	The pupil is invited to give a short answer, e.g. yes/no or single word/phrase.
Thinking	The pupil is invited to describe, develop or clarify their thinking in some way, e.g. "how did you get that answer?".
Argument	The pupil is invited to reflect on or argue for claims/statements (their own or someone else's), e.g. why questions or "how do you know that...?".

Questions of the types *No answer* and *Short answer* are considered more monologic, while the types *Thinking* and *Argument* are considered more dialogic.

2.6.4 Classroom norms and sociomathematical norms

The aim is to investigate aspects related to classroom norms and sociomathematical norms (didactical perspective 4). In the evaluation, we limited the analysis to *sociomathematical norms*, i.e. how norms specific to the subject of mathematics enter into the social situation of teaching (Yackel & Cobb, 1996). One reason for this delimitation is that several aspects of classroom norms (e.g. forms of teaching) have already been covered by other didactical perspectives, preferably *routines/interactions*. In the analysis, we therefore focus on things that can be linked to "perceptions, attitudes and expectations that

characterise a mathematics teaching situation" (Blomhøj, 2013). These include:

- What considerations teachers show towards pupils' views of mathematics, incl. teaching, knowledge and learning in mathematics.
- How is it determined what is correct and acceptable in the subject mathematics, i.e. values that relate to the subject mathematics and not some general norm for e.g. teaching/school/classroom or social contexts (such as writing clearly or not talking too loud).

In the analysis, we focus on the above two aspects as follows:

1. Analysis of interviews: The teacher's consideration of pupils' views when planning and reflecting on teaching, and the teacher's planning and reflections on who has the authority and control to judge what is correct or acceptable.
2. Analysis of lesson observations: The teacher's response in the discussions with pupils about what is correct or acceptable (we do not analyse the pupils' statements).
3. Analysis of observation of collegial meetings: The consideration of pupils' views when discussing the planning and reflecting on teaching.

A total of five types of assessments were made, all of which were made in many situations. For example, all of the teacher's evaluations in the classroom were assessed in two ways. In these analyses, the scale for the ratings is from -1 to 1, because according to theories of sociomathematical norms, it is negative for pupils' learning if pupils' views are not taken into account, or if it is always the teacher, or some other authority, who decides what is correct or acceptable. All assessments made for the analysis of the *sociomathematical norms* perspective are described in Appendix I. The following is an example of an assessment made in the analysis of both the interviews and the lesson observation:

Example 7. Assessment of who or what determines whether something is correct or acceptable in mathematics.

For interview questions 10 and 18, the teacher's descriptions were assessed in terms of who or what primarily determines whether something is complete, correct, okay or whether it is not. In the analysis, what dominated the teachers' responses was selected, and neutral was selected if things were described about the same (Table 12).

Table 12: Assessment example DP4, teacher interview.

Value	Description	*Example*
-1	Something outside the pupil's control	*The teacher, the textbook key, some expert (like "mathematician").*
0	Neutral or unclear	
1	Something within the pupil's control	*The pupil, incl the pupil in interaction with others*

Example 8. Assessment of who or what determines whether something is correct or acceptable in mathematics.

The same table was used to analyse the teacher's evaluations in the classroom. This included instances when the teacher explicitly stated whether something was complete, correct, okay or not in the subject of mathematics. It was not enough for the teacher to say only "yes" or "no". It had to be an explicit and clearly evaluating statement related to mathematics. Up to 15 evaluations per lesson observation were assessed.

2.7 Variables and statistical analyses

The analysis results in quantitative measures relating to changes in the teaching and PDP culture of schools. By creating outcome variables and comparing between the visits *before* and *during* or *during* and *after*, we can describe to what extent the two cultures have developed. In this section, we therefore describe how the variables used in the statistical analyses were constructed and the statistical methods used. A more detailed description can be found in Appendix A. In order to allow direct comparisons and to provide measures of change, all variables are

constructed in exactly the same way regardless of whether the school visits took place before, during or after the Boost for Mathematics.

First, all outcome variables used in the analyses are described. Next, the methods used to answer the evaluation questions are described.

2.7.1 Outcome variables

Teaching culture refers to a primary focus on pupils' learning. This means that the operationalisation concerns teachers' direct work on teaching (planning, reflection and implementation) as well as joint work at school level on the development of mathematics teaching. This operationalisation thus concerns teaching culture at two different levels: it focuses on individual teachers' teaching in terms of their planning, reflections and implementation, and it also focuses on joint work at the school level in terms of various aspects of collaboration between teachers and principals. *The teaching culture at each school* is described by 14 outcome variables, which are divided into groups of variables as follows:

A. Outcome variables 1–4: For each didactical perspective, one variable is created based on teacher interviews. These 4 variables describe to what extent each individual teacher **plans and reflects** in line with the didactical perspectives.

B. Outcome variables 5–8: For each didactical perspective, one variable is created based on lesson observations. These four variables describe to what extent the **teaching** of each individual teacher is in line with the didactical perspectives.

C. Outcome variables 9–10: Documents, interviews and surveys from teachers and principals are analysed regarding to what extent the **school is working on the development of mathematics teaching**. For each type of actor (teachers and principals), a variable is created to describe this, two variables in total.

D. Outcome variables 11–12: Interviews with teachers and principals are analysed in terms of to what extent plans for developing mathematics teaching include different types of **goals**. For each type of actor (teachers and principals), a variable is created to describe this, two variables in total.

E. Outcome variables 13–14: Variables on the school's work to develop teaching in variable groups C and D are used to analyse to what extent there is a **consensus** among different actors within the same school on the **school's work to develop teaching**. For each school, the standard deviation across actors for variable group C and the standard deviation across actors for variable group D are used to create two variables describing the spread of actors' views on the school's work to develop teaching. A decrease in spread is interpreted as an increase in consensus.

PDP culture refers to a primary focus on teachers' learning. This has been operationalised to refer to schools' work with teachers' competence development. *The PDP culture at each school* is described by 6 outcome variables divided into three groups, where variable groups F, G and H correspond to variable groups C, D and E in teaching culture:

F. Outcome variables 15–16: Documents, interviews and surveys from teachers and principals are analysed regarding to what extent work is done on teachers' **competence development**. For each type of actor (teachers and principal), a variable is created to describe this, two variables in total.

G. Outcome variables 17–18: Interviews with teachers and principals are analysed in terms of to what extent plans for competence development include different types of **goals**. For each type of actor (teachers and principals), a variable is created to describe this, two variables in total.

H. Outcome variables 19–20: The variables on competence development in variable groups F and G are used to analyse to what extent there is a **consensus** among different actors within the same school on the teachers' **competence development**. For each school, the standard deviation across actors for variable group F and the standard deviation across actors for variable group G are used to create two variables describing the spread of actors' views on the school's work to develop teaching. A decrease in spread is interpreted as an increase in consensus.

Each variable used in the analyses is composed of several different measures. The aim is to create a more comprehensive and reliable description of what the variable is intended to measure. For example,

for each didactical perspective, there are several different interview questions concerning teachers' planning and reflections on this didactical perspective. Each interview question is rated on a three or four-point scale (always with a maximum value of 1) in terms of to what extent the teacher's planning and reflections are in line with the didactical perspective. An average is then calculated for all questions concerning the current didactical perspective. All variables are created in this way and are thereby based on several different assessments of specific interview questions, survey questions or aspects observed in the teaching. Appendix D provides an overview of the data used in the creation of all variables, and Appendices E–P describe all assessments in detail.

Teachers' direct work with teaching, in terms of both planning and reflection (variable group A) and implementation (variable group B), is very central to describing the teaching culture (see descriptions of the cultures in section 1.1). Since the four didactical perspectives will inform the teachers' PDP material in the Boost for Mathematics, we have created outcome variables that describe to what extent teachers' teaching is based on these didactical perspectives.

Not all aspects of all didactical perspectives can be examined within this evaluation. For the didactical perspective on classroom norms and sociomathematical norms, we have limited the analysis in the evaluation to *sociomathematical norms*, i.e. how norms specific to the subject of mathematics enter into the social situation of teaching. One reason for this delimitation is that several aspects of classroom norms (e.g. forms of teaching) have already been covered by other didactical perspectives, preferably *routines/interactions*. Another reason for this delimitation is that we primarily focus on subject didactical phenomena within this evaluation. The two didactical perspectives of *abilities* and *formative assessment* refer to delimited and well-defined phenomena. This means that we have been able to base our analysis on existing frameworks. However, this does not apply to the other two didactical perspectives, *routines/interactions* and *sociomathematical norms*, as these perspectives do not as clearly refer to a delimited and well-defined phenomenon, but can cover many different types of phenomena. Moreover, variables related to the didactical perspective

of *sociomathematical norms* are generated from fewer assessments than the other variables. The reason is that the nature of this perspective made it more difficult to operationalise in the context of this evaluation. For example, from the interview data, the variable on *sociomathematical norms* is based on 6 assessments, while the other didactical perspectives use 10–13 assessments each. For observations, the variable for *sociomathematical norms* is based on 2 assessments, while the other didactical perspectives use 5–10 assessments each. These characteristics of the completed assessments mean that, compared to the other didactical perspectives, some more limited aspects are investigated for the didactical perspectives, *sociomathematical norms* and *routines/interactions*.

Assessments underlying the creation of outcome variables on the didactical perspectives (variable groups A and B) are described in section 2.6. Below, we describe the types of assessments underlying the creation of the other outcome variables. More details can be found in the list of variables in Appendix D, and all assessments are described in detail in Appendices J–P.

How teaching development (variable groups C and D) and teachers' competence development (variable groups F and G) is done at school level is seen as very central elements of the teaching culture and the PDP culture, respectively (see descriptions of the cultures in section 1.1). In order to cover both the current situation in the schools and goals for the future, we have created two types of outcome variables.

The interviews and surveys ask about the existence of teaching development (variable group C) and competence development (variable group F) at the school level. Teachers and principals are asked the same types of questions. For example, for the development of teaching, they are asked to assess the quality of the school's work with the development of mathematics teaching and how involved the principal is in this work. For competence development, they are asked, for example, to assess the quality of the teachers' existing learning environment, and how structured the school works with goals and follow-up related to the development of teacher's competencies. In addition, any existing policy documents or similar documents in the

schools are analysed with regard to whether and how they describe goals, plans and follow-up for both teaching development and competence development.

With regard to the breadth of goals for teaching development (variable group D) and competence development (variable group G), questions are asked about this in interviews with both teachers and principals. For these types of variables, they are asked, for example, what support is needed to develop teaching or how collegial learning should be organised. Answers to these questions are assessed according to the types of goals described, for example, whether the focus is on teachers' development, on principals' behaviour, on addressing specific barriers and/or the need for some form of external guidance or support. The focus of these variables is thus on capturing to what extent a holistic perspective exists with regard to teaching development and competence development.

Cooperation between all actors at a school is key to both the teaching and the PDP culture (see description of the cultures in section 1.1). Aspects of cooperation are built into the variables described above, for example with direct questions to teachers and principals on how the cooperation between them works. In addition, we created variables that describe the degree of consensus among teachers and principals at a school, both in terms of teaching development (variable group E) and competence development (variable group H). A consensus among teachers and principals regarding how the school actually works and what types of goals the school should have is seen as central to good collaboration among teachers and principals.

2.7.2 Statistical analyses to answer the questions

In order to answer the first two questions of the evaluation, *To what extent has the Boost for Mathematics contributed to the development of a sustainable teaching culture?* and *To what extent has the Boost for Mathematics contributed to the development of a sustainable PDP culture?* (A and B in section 1.2), the evaluation investigated whether the values of the 20 outcome variables changed as a result of participation in the Boost for Mathematics. We analyse change in two ways. We examine changes in teachers' work who were visited twice

(same teachers over time). For this, group IU *before* the Boost for Mathematics is compared with group IU *during* the Boost for Mathematics, and group UE *during* the Boost for Mathematics is compared with group UE *after* the Boost for Mathematics (see Table 1 in section 2.2). We also examine change by comparing different groups of teachers, i.e. teachers who are at different stages in relation to the Boost for Mathematics, but who were visited during the same semester (different teachers at the same point in time). For this, group IU *before* the Boost for Mathematics is compared with group UE *during* the Boost for Mathematics, and group IU *during* the Boost for Mathematics is compared with group UE *after* the Boost for Mathematics.

These two types of analysis have different advantages and disadvantages, but they complement each other and are used in parallel to test reliability. Results are more reliable if they emerge from both types of analysis. The first type of analysis examines change in a direct way. This means that there is a risk that observed change may be due to something other than the Boost for Mathematics, as other things happen between the two points in time when the data are collected. In the second type of analysis, all the data is collected at the same time, but different groups of individuals are compared. For this analysis to work, the groups must be comparable. A random sample will ensure that the groups are comparable, but we also examine statistically whether this is the case by looking at all available variables describing external factors (see Appendix D where all variables are listed). Among the 29 factors examined, differences between groups are found for 8 of them. Among the factors where there is a difference between the groups, there are five factors related to characteristics at the municipal level (number of inhabitants, number of pupils, proportion of girls among pupils, number of pupils per teacher and number of mathematics grants applied for), two factors related to characteristics at the school level (proportion of pupils with a foreign background and proportion of pupils with parents who have a post-secondary education) and one internal factor related to the collegial meeting (quality of discussions on *routines/interactions*). Only one of these factors can be reliably linked to variation in results from the Boost for Mathematics (see analyses in chapter 6): teachers in larger

municipalities (based on number of pupils) tend to develop their teaching more than teachers in smaller municipalities when implementing the Boost for Mathematics. Since there appears to be no difference in the results of the different methods of analysis regarding teachers' teaching (see analyses in section 3.2), the observed differences between groups of schools do not pose a problem.

The above descriptions of the two methods of analysis show that there is very little likelihood that the differences observed in both methods of analysis arose for reasons other than the influence of the Boost for Mathematics. We can therefore state with a very high degree of certainty to what extent the Boost for Mathematics has contributed to the development of a sustainable teaching and PDP culture.

In order to answer the third evaluation question, *What factors have influenced how well or less well the Boost for Mathematics contributed to the development of sustainable teaching and PDP cultures,* (C in section 1.2), factors are analysed in relation to changes in the outcome variables. This involves examining which factors can statistically explain these changes. We examine both internal factors (describing some aspect of the Boost for Mathematics) and external factors (independent of the Boost for Mathematics). Internal factors are used to investigate whether changes in the teaching or PDP culture are related to the implementation of the Boost for Mathematics. External factors are used to investigate whether certain types of teachers, principals or schools have been most or least affected by the Boost for Mathematics. Data on these factors are collected through interviews and surveys, as well as from databases. A list of all variables can be found in Appendix D.

In the analyses conducted in the evaluation, $p<0.05$ is used as the threshold for statistically significant results. This is marked with * in the graphs in results chapter 3–6. However, for analyses involving smaller groups (principals and schools), the threshold $p<0.1$ is used (marked with ^), because tests with smaller groups have less statistical power (see section 2.2 where we describe data selection in relation to statistical power). Increasing the threshold for what counts as statistically significant increases the risk of obtaining results created by

chance (10% risk for a result with p=0.1). However, since we focus on drawing conclusions from several individual results together, especially using the two different methods of analysis described above, the probability that our conclusions are caused by chance or something other than the Boost for Mathematics is still very low.

For differences marked as statistically significant, a statistical measure of effect size is also described, using Cohen's d. This value of effect size is usually interpreted as small effects being values around d=0.2, medium effects being values around d=0.5, and large effects being values around d=0.8.

The selection of teachers was done through a two-stage sampling process where first schools were randomly selected, and then three teachers were randomly selected from each of the selected schools. In order to take into account that data at the teacher level cannot be considered independent, such data were analysed using linear hierarchical models (linear mixed models). Such models allow schools to have different average values for the variables studied. Since the schools were chosen randomly and independently of each other, data at the principal and school level were analysed using simple methods (i.e. standard t-tests and correlations).

3. The Boost for Mathematics has changed teaching in a sustainable way

This section presents the results of interviews with teachers (section 3.1) and observations of lessons (section 3.2), focusing on the four didactical perspectives. It is the outcome variables in variable groups A and B from section 2.7 that are analysed here to contribute to the answers to the first question: To what extent has the Boost for Mathematics contributed to the development of a sustainable teaching culture?

Results are based on statistical analyses comparing outcome variables at different stages in relation to the Boost for Mathematics. Firstly, the situation *before* and *during* the Boost for Mathematics is compared, which gives answers as to whether there has been a change as a result of the Boost for Mathematics. Secondly, the situation *during* and *after* the Boost for Mathematics is compared, which gives an answer as to whether a more sustainable change has taken place.

All results are reported with figures based on the same structure. Firstly, each figure always contains two diagrams next to each other. The left-hand graph shows comparisons between *before* and *during* the Boost for Mathematics, while the right-hand graph shows comparisons between *during* and *after* the Boost for Mathematics. These comparisons are split into two graphs because they describe analyses of completely separate data sets. Secondly, there are always two figures that contain different types of graphs (line graphs and bar graphs), but present the same type of analysis. The line graphs show changes in a group visited on two different occasions, i.e. either group IU or group UE in Table 1 in section 2.2. The bar graphs show comparisons between two different teacher groups visited during the same time period, but at different stages in relation to the Boost for Mathematics, e.g. group IU and group UE visited in 2015 (see Table 1 in section 2.2).

3.1 Teachers plan and reflect more in line with the didactical perspectives

The figures below present to what extent teachers at the group level plan and reflect on their mathematics teaching in line with the didactical perspectives, based on interviews with teachers, i.e. the graphs show average values across all teachers for each of the outcome variables 1–4 (variable group A in section 2.7).

Figure 1 shows the same pattern as Figure 2 regarding which didactical perspectives show statistically significant change. Thus, the two methods of analysis show the same results, demonstrating the reliability of the results. The only exception to this is *sociomathematical norms*, which are significant in the right-hand graph in Figure 1, but not in Figure 2; however, the difference there is very close to statistical significance.

The graphs show that there is an increase from *before* the Boost for Mathematics to *during* the Boost for Mathematics, in terms of *abilities* and *routines/interactions*. The increase in *formative assessment* is relatively close to statistical significance, so there may be a weak effect for this didactical perspective as well. From *during* the Boost for Mathematics to *after* the Boost for Mathematics, there is an increase in *sociomathematical norms* but no change in the other didactical perspectives. These differences reflect a large effect for *routines/interactions*, a medium effect for *abilities* and a small/medium effect for *sociomathematical norms*.

Thus, *sociomathematical norms* do not change while teachers participate in the Boost for Mathematics, but there is a clear tendency towards an increase in *sociomathematical norms* in the year after the Boost for Mathematics. Otherwise, changes in *abilities* and *routines/interactions* already occur when teachers take part in the Boost for Mathematics and no change occurs one year after the Boost for Mathematics. This indicates that the change is more sustainable. *Formative assessment* shows no clear change, but the tendency is the same as for *abilities* and *routines/interactions*.

*Figure 1: Teachers' planning and reflection. Comparison of the **same teacher on two occasions**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 36 and 42 teachers.*

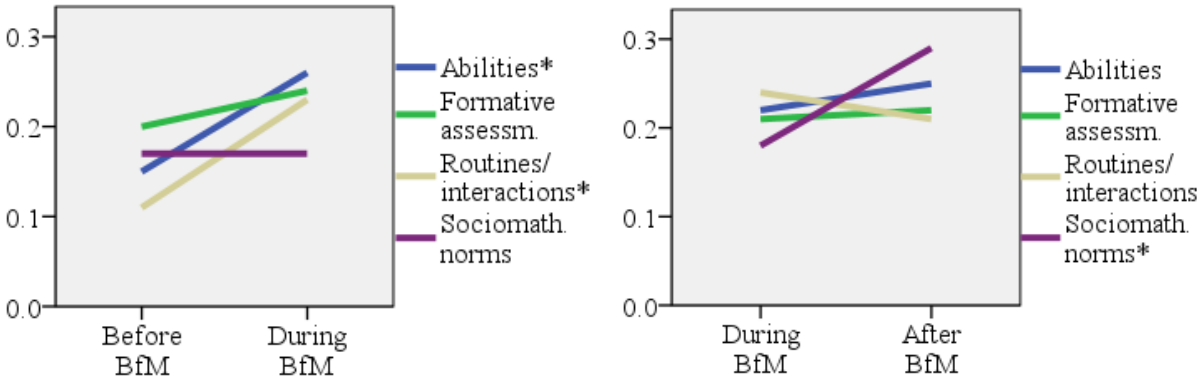
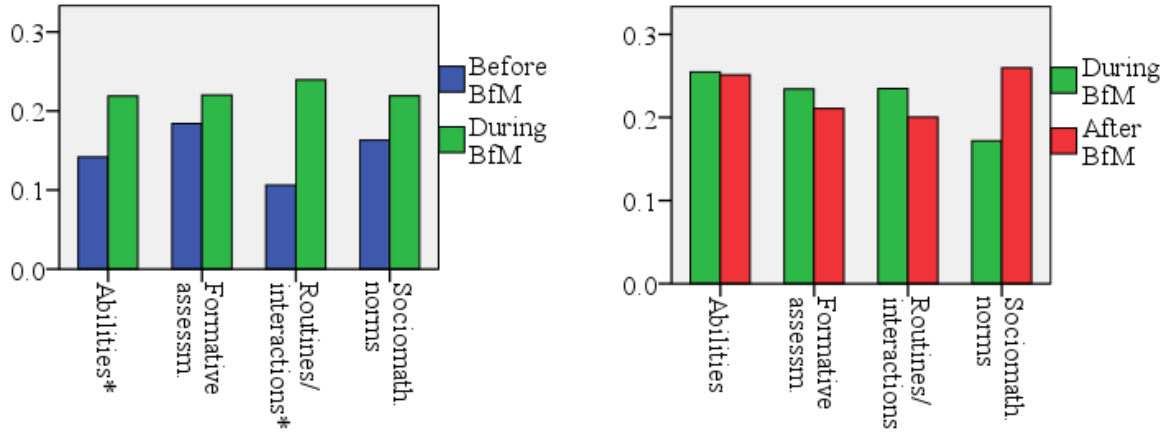


*Figure 2: Teachers' planning and reflection. Comparison of **different teachers at the same point in time**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 98 and 101 teachers.*



3.2 Teachers teach more in line with the didactical perspectives

The figures below present to what extent teachers conduct their mathematics teaching in line with the didactical perspectives based on lesson observations. The results are presented at the group level, i.e. the graphs show averages across all teachers for each of the outcome variables 5–8 (variable group B in section 2.7).

Again, we get the same results from the two different methods of analysis, i.e. statistically significant differences exist for the same didactical perspectives, as shown in Figure 3 and in Figure 4. This

demonstrates the reliability of the results. The graphs show that there is an increase from *before* the Boost for Mathematics to *during* the Boost for Mathematics, in terms of *abilities, formative assessment* and *routines/interactions*. There are no significant changes from *during* the Boost for Mathematics to *after* the Boost for Mathematics, with a tendency towards continued increase for all didactical perspectives. Differences noted reflect a large effect for *abilities* and *formative assessment* and a medium effect for *routines/interactions*.

Thus, there is an increase in three didactical perspectives when teachers take part in the Boost for Mathematics, and these changes are maintained one year after the Boost for Mathematics, indicating that these are more sustainable changes. However, sociomathematical norms do not show a clear increase, but there is a slight tendency towards an increase one year after the Boost for Mathematics, i.e. a similar change as observed for *sociomathematical norms* regarding teachers' planning and reflection, presented in Figure 1.

Figure 3: Teachers' teaching. Comparison of the ***same teacher on two occasions****. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 32 and 40 teachers.*

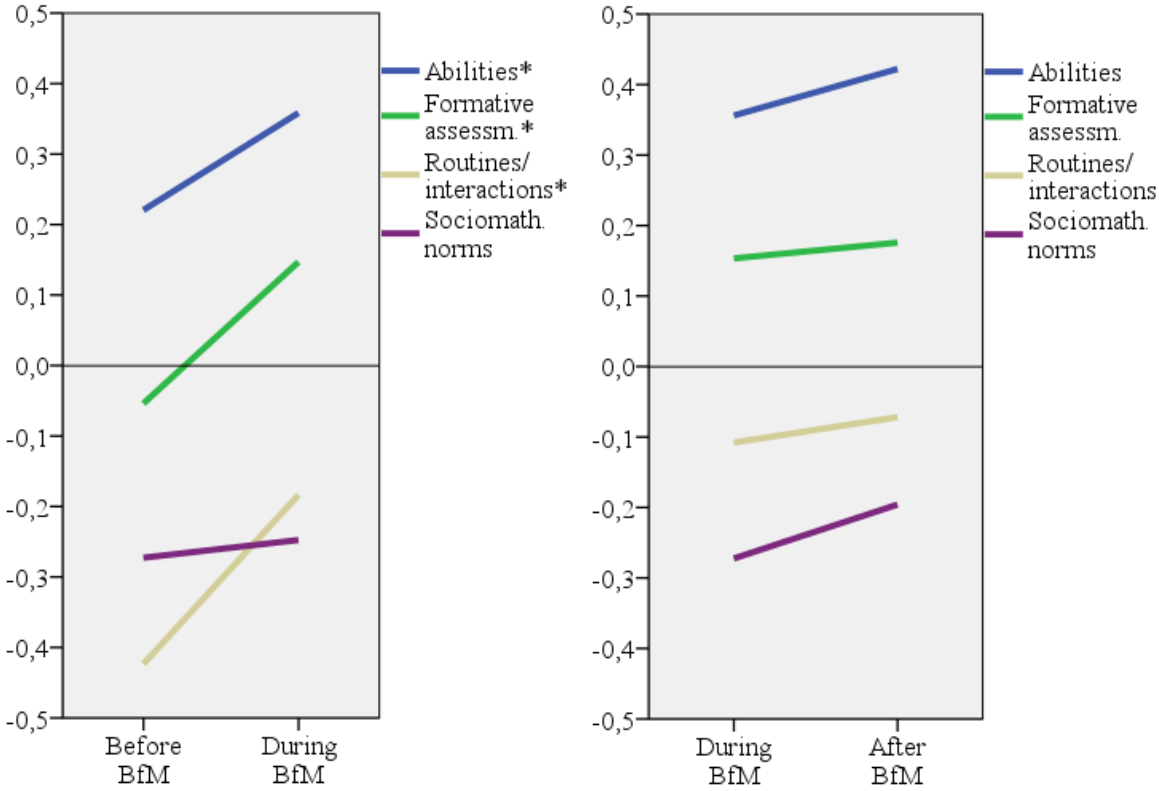
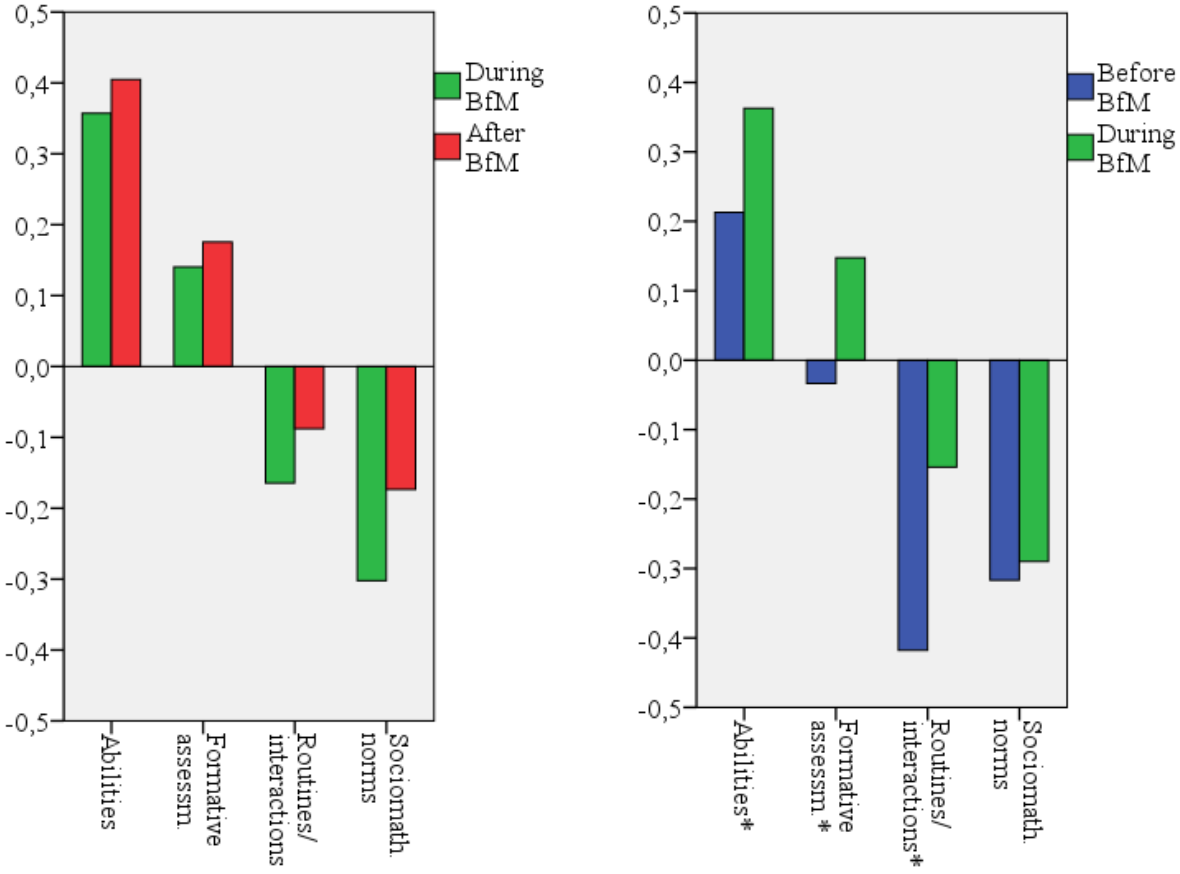


*Figure 4: Teachers' teaching. Comparison of **different teachers at the same point in time**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 90 and 98 teachers.*



3.3 Examples of teacher development

In the examples below, we have selected teachers who have shown significant changes at the individual level for those didactical perspectives that show changes at the group level. Here we provide concrete descriptions of the changes underlying the statistically significant differences visible in the results. The last part of this section also links the magnitude of changes at the individual level to the magnitude of changes at the group level, in order to give a clearer idea of how much teachers have changed as a result of the Boost for Mathematics.

3.3.1 Interviews with teachers

For *abilities*, an example of a change is given here concerning the balance between ability goals and content goals**.** When asked how the

teacher viewed the division of learning outcomes in mathematics into ability goals and content goals, the teacher *before* the Boost for Mathematics responded that "The overarching goals are more ability goals, survival skills. You practise them in all subjects. The content goals are the basics that [the pupils] should take with them". When the question was asked *during* the Boost for Mathematics, the teacher instead responded "The mathematical abilities are linked to which mathematics subject areas I choose to present". How the teacher expresses him/herself about ability goals and content goals in the example *before* the Boost for Mathematics shows a clear focus on the content goals in mathematics teaching. The view that *abilities* is something outside of the subject is clear. In the statement *during* the Boost for Mathematics, the balance between ability goals and content goals is well expressed and the teacher emphasises that this balance influences the teaching. The first quote was given a value of 0, and the second a value of 1 (see Appendix E).

For *routines/interactions*, an example of a change is given here concerning the selection of pupil solutions for presentation on the board. The teacher was asked how selection was done, and *before* the Boost for Mathematics responded that "The selection could probably be very much about democracy [...] everyone should be heard equally". In response to the same question *during* the Boost for Mathematics, the response changed significantly and the teacher said "... I usually try to look for a really good solution; one that is that middle ground [...] and one that is just slightly wrong". Here the quotes show a clear change in line with the principle of selecting examples that represent what is most important for the current learning and mathematics content. The first quote was given a value of -1, and the second a value of 1 (see Appendix H).

For *sociomathematical norms*, an example of a change is given here in terms of who or what determines whether something is correct or acceptable in the subject of mathematics. When asked whether the pupils use the answer key, the teacher's response *during* the Boost for Mathematics was "Don't rely too much on the answer key. [...] Instead, ask me, so I can check it." When the same question was asked *after* the Boost for Mathematics, the teacher responded that the pupils use the

answer key, but also that "They [the pupils] have to rely on themselves". This clearly shows a shift from considering this to be out of the pupil's control to determine whether something is correct (value -1) to considering it to be something within the pupil's control (value 1) (see Appendix I).

3.3.2 Observations of teaching

For *abilities*, an example of a change is given here concerning teaching with a focus on reasoning ability. In the lesson *before* the Boost for Mathematics, no abilities were discussed at all, and the pupils were not given the opportunity to develop their reasoning ability. In the lesson *during* the Boost for Mathematics, the teacher made visible what reasoning ability can be in their introduction to the lesson. In a group exercise, pupils were then given the task "How would you explain the equals sign to a classmate?". After the group exercise, the pupils had to present and argue their explanations to the whole class. Such discussions, where pupils have to argue for their solutions, are central to the development of reasoning skills.

For *formative assessment* an example is given here of a change related to activities linked to different key strategies. In the lesson *before* the Boost for Mathematics, there was evidence of one of the five key strategies (key strategy 5, quality 0). In the lesson *during* the Boost for Mathematics, the teacher went over the lesson's learning outcomes (key strategy 1, quality 0.5), organised group exercises where the teacher could easily get information about the pupils' knowledge (key strategy 2, quality 0.5) and where the pupils were activated as resources for each other's learning (key strategy 4, quality 0.5).

For *routines/interactions*, an example of a change is given here concerning the use of instructional formats that support dialogic interaction. In the lesson observed *before* the Boost for Mathematics, a teacher used teacher monologue for 29 minutes and individual work for 28 minutes. When the same teacher was observed *during* the Boost for Mathematics, teacher monologue was used for 10 minutes and individual work for 22 minutes. The other 38 minutes consisted of group work and whole-class discussion, i.e. instructional formats that promote a dialogic interaction pattern.

3.3.3 Relationships between the magnitude of change at the group and individual level

The graphs in sections 3.1 and 3.2 show changes in average values at the group level of the magnitude 0.10–0.15 for interview results and 0.15–0.20 for observation results. If an assessment for a particular didactical perspective of a teacher changes from 0 to 0.5 (see examples above and all types of assessments in Appendices E–P) and there are a total of 10 assessments for which the average value is calculated for the outcome variable, this corresponds to a 0.5/10 increase in the outcome variable for that teacher, i.e. 0.05. Thus, if all teachers have increased 0.5 on one assessment each, this creates an increase of 0.05 in the average at the group level.

For the didactical perspectives that show the most significant changes at the group level, the value of the variable is based on 10–13 different assessments for interviews and 5–10 assessments for observations. Thus, teachers would need to have increased an average of 0.5 on two to four assessments each to achieve the observed changes in group-level scores from both interviews and observations.

4. The Boost for Mathematics has influenced principals and teachers in different ways in terms of the school's work with teaching development and professional development.

This section presents the results of interviews and surveys with teachers and principals. The focus is on questions about the school's work with the development of teaching, as part of the teaching culture, and work with competence development, as part of the PDP culture. It is the outcome variables in groups C–D and F–G from section 2.7 that are analysed, in order to contribute with an answer to the first two questions regarding to what extent the Boost for Mathematics has contributed to the development of a sustainable teaching culture and a sustainable PDP culture.

The results are based on the same type of analysis as in chapter 3; see the introduction to that chapter for more information. For analyses based on smaller groups (principals), differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05), as described in section 2.7.2.

4.1 Principals but not teachers perceive a sustainable improvement in the school's work with teaching development

The figures below present results describing to what extent teaching development occurs at the school level and to what extent plans for such work include different types of goals. The results are based on interviews and surveys with teachers and principals, as well as analysis of planning or policy documents referenced by teachers or principals. The graphs show averages across all teachers or all principals for each of the outcome variables 9–12 (variable groups C and D in section 2.7).

Figure 5 and Figure 6 show that teachers do not perceive any increasement in goals or in the school’s work concerning developing teaching as a result of the Boost for Mathematics. The only significant change is that teachers tend to see *fewer* types of goals for this work *after* the Boost for Mathematics. However, this difference is only

significant in Figure 5, but is close to statistical significance in Figure 6. The difference reflects a small effect.

*Figure 5: **Teachers'** perspective on the existence of and goals for the **school's teaching development work**. Comparison of the **same teacher on two occasions**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 36 and 42 teachers.*

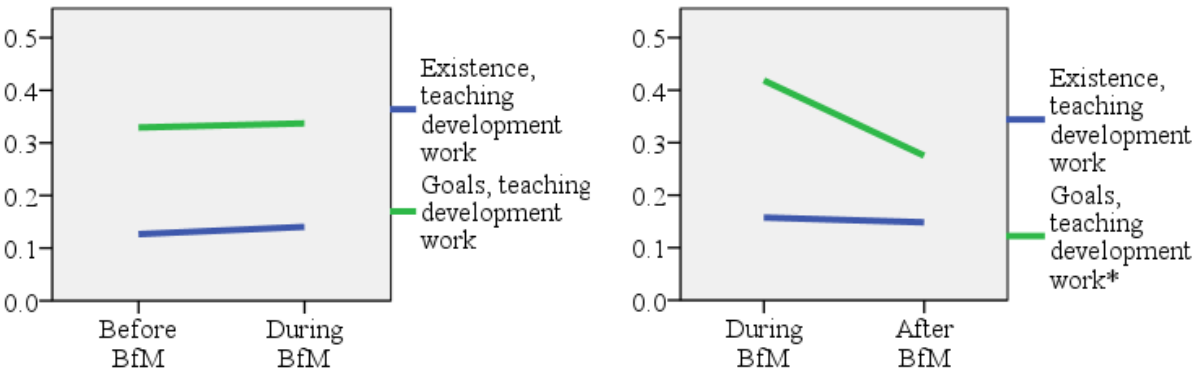


*Figure 6: **Teachers'** perspective on the existence of and goals for the **school's teaching development work**. Comparison of **different teachers at the same point in time**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 98 and 101 teachers.*

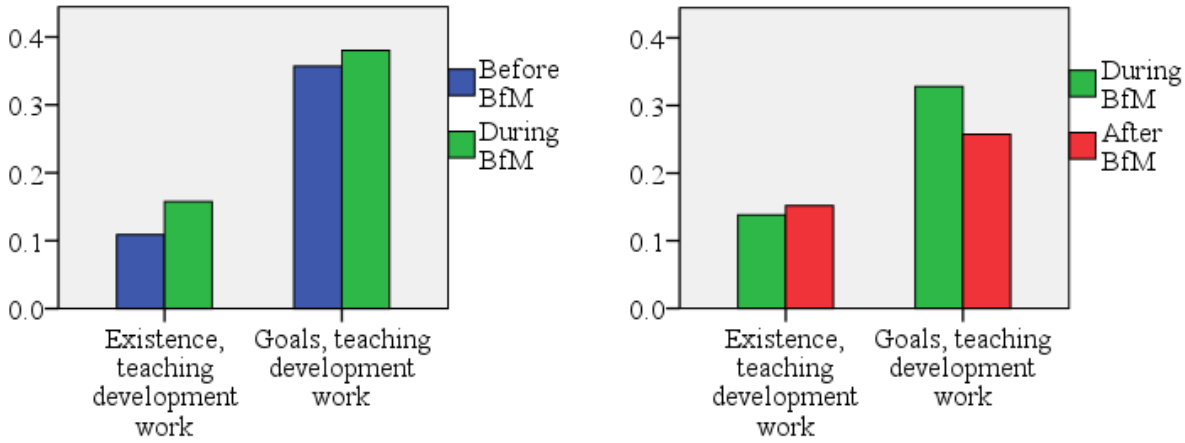


Figure 7 and Figure 8 show a clear increase among principals with the Boost for Mathematics, but no change in the year after the Boost for Mathematics, in terms of both the existence of teaching development at the school level and the goals for this work. The two methods of analysis show the same types of results, which demonstrates the

reliability of the results. Thus, there are positive changes when principals take part in the Boost for Mathematics, but no changes one year after the Boost for Mathematics. This indicates that the changes are more sustainable, although a slight trend is that there is some decline in the year after the Boost for Mathematics. The differences observed reflect medium effects.

Figure 7: ***Principals'*** *perspective on the existence of and goals for the* ***school's teaching development work****. Comparison of the* ***same principal on two occasions****. Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 12 and 15 principals.*

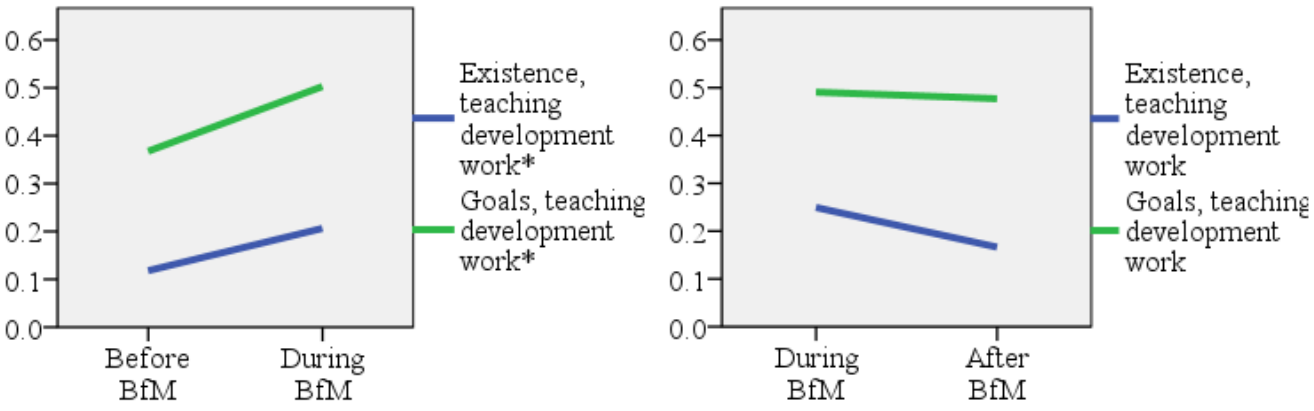
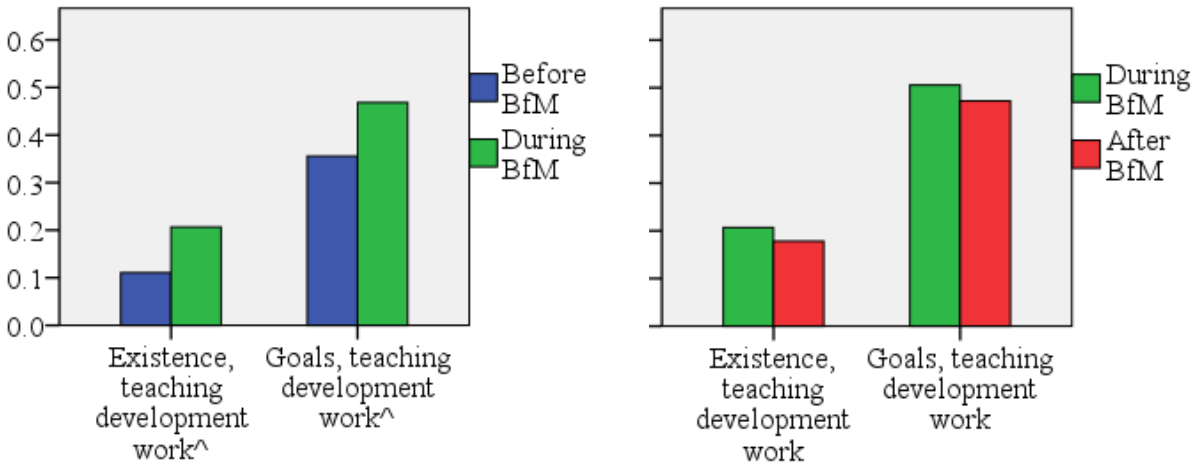


Figure 8: ***Principals'*** *perspective on the existence of and goals for the* ***school's teaching development work****. Comparison of* ***different principals at the same point in time****. Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 31 and 34 principals.*



Teachers and principals are asked the same type of questions regarding the existence and goals of the school's teaching development, but there are thus differences in their perceptions. Among principals there is a clear increase over time, while no such increase is found among teachers.

4.2 Principals have been affected more than teachers in terms of competence development work

The figures below show to what extent competence development occurs and to what extent competence development plans include different types of goals. The results are based on interviews and surveys with teachers and principals, as well as analysis of planning or policy documents referenced by teachers or principals. The graphs show averages across all teachers or all principals for each of the outcome variables 15–18 (variable groups F and G in section 2.7).

All differences in the existence of competence development in Figures 9 and 10 are close to statistical significance (i.e. have a p-value close to 0.05). All differences show a tendency for there to be an increase when teachers start the Boost for Mathematics and then a decrease the year after the Boost for Mathematics. Thus, this possible effect appears to be primarily due to the ongoing implementation of the Boost for Mathematics and does not reflect any more sustainable change. In terms of goals, the tendencies are even weaker (i.e. the p-values are further from 0.05), but all differences show a decrease.

*Figure 9: **Teachers'** perspective on the existence of and goals for **competence development**. Comparison of the **same teacher on two occasions**. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 36 and 42 teachers.*

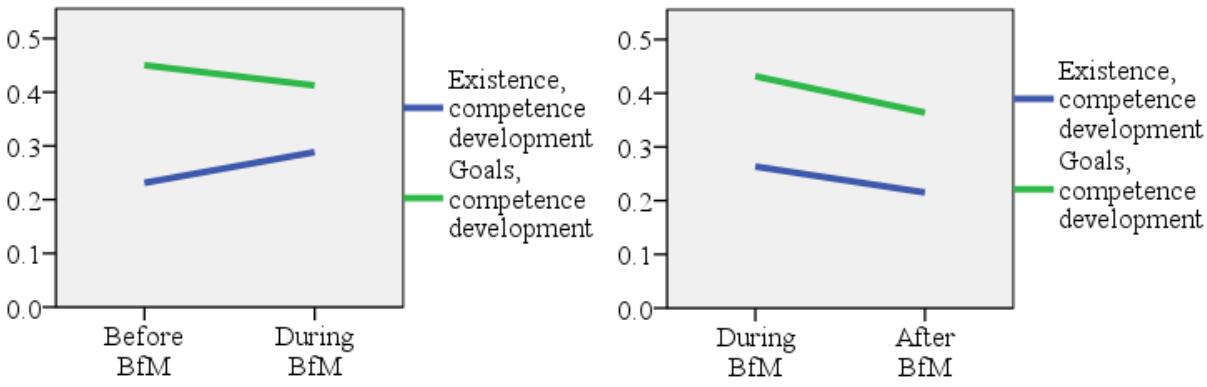
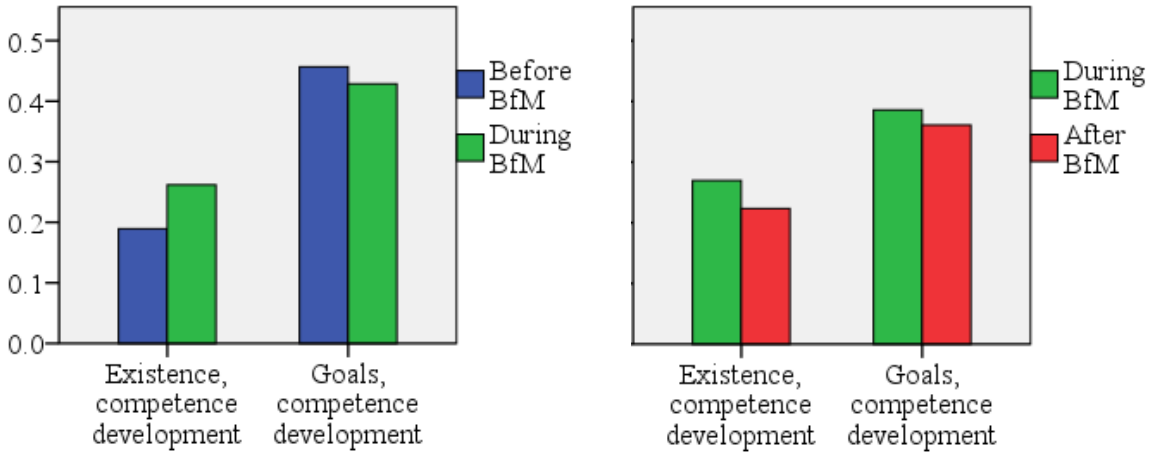


Figure 10: ***Teachers'*** *perspective on the existence of and goals for* ***competence development****. Comparison of* ***different teachers at the same point in time****. Differences that are statistically significant are marked with * (p<0.05). Comparisons include between 98 and 101 teachers.*



Figures 11 and 12 show an increase among principals with the Boost for Mathematics in terms of both the existence of and goals for competence development. The increase for goals is shown in both methods of analysis, which demonstrates the reliability of the results. The increase in existence is more uncertain, as it is only shown in one method of analysis. There is also a risk that this is not a sustainable effect in the case of goals, since there is a tendency for a decrease after the Boost for Mathematics. Differences noted reflect a large effect for goals and a medium effect for existence.

Figure 11: ***Principals'*** *perspective on the existence of and goals for* ***competence development****. Comparison of the* ***same principal on two occasions****. Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 12 and 15 principals.*

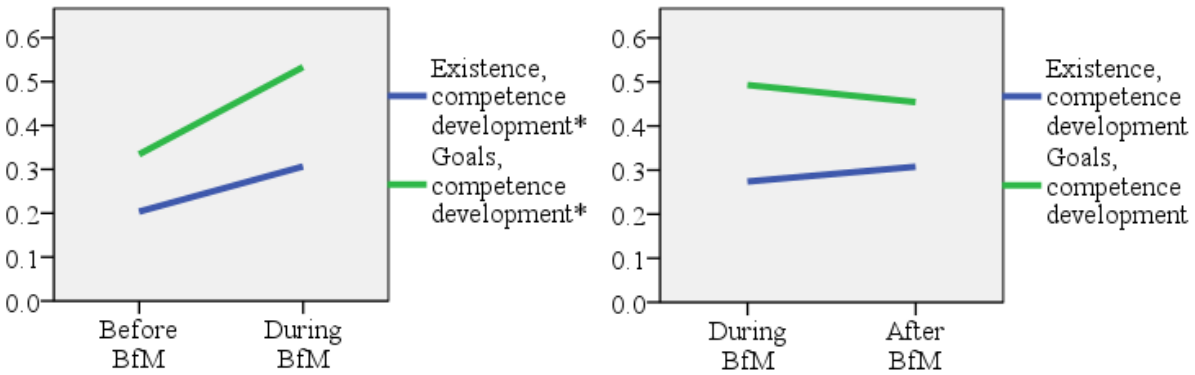
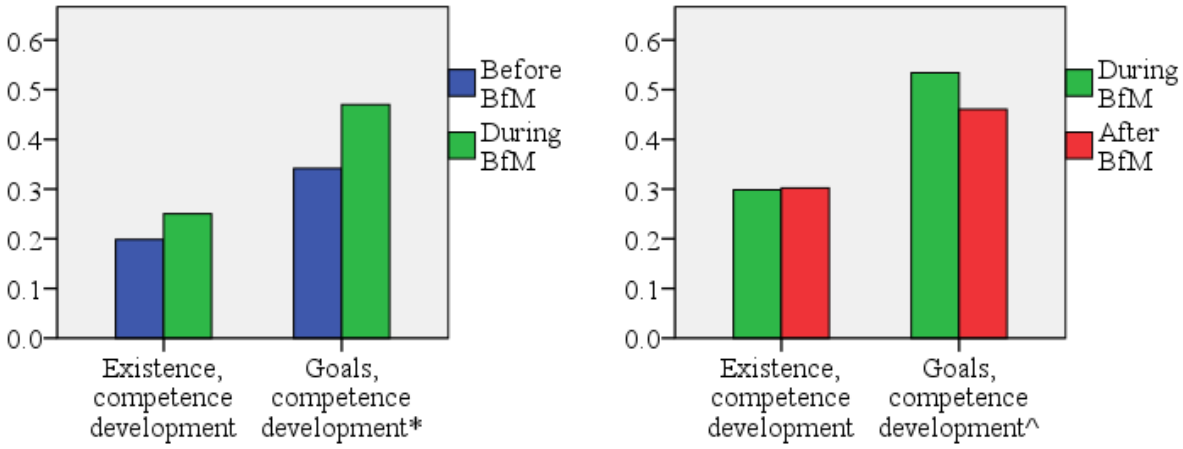


Figure 12: ***Principals'*** *perspective on the existence of and goals for* ***competence development****. Comparison of* ***different principals at the same point in time****. Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 31 and 34 principals.*



Similar to the results on the school's teaching development work, the results for competence development also show a clearer increase among principals compared to teachers.

5. There is a risk that the Boost for Mathematics has created a lack of consensus among teachers and principals regarding the school's teaching development work and competence development work

The results in Chapter 4 show at the group level that principals and teachers have been affected in different ways by the Boost for Mathematics. The results thus indicate a risk of a certain loss of consensus with regard to the school's work with teaching development and competence development. In this Chapter 5, each school is analysed in terms of consensus on the existence of goals concerning the school's work with both teaching development and competence development. This is done to see whether this consensus has changed as a result of the Boost for Mathematics. Standard deviations are examined, with a small standard deviation reflecting a higher level of consensus. In the analyses, we have used outcome variables in variable groups E and H in section 2.7.

Figures 13 and 14 show that the overall trend is for the spread (standard deviation) to increase among teachers and principals, i.e. for consensus to decrease. There are also two differences that are statistically significant (although not for both methods of analysis, and so this is only considered a trend). The first difference observed relates to the school's work with teaching development and the second relates to competence development. The trend is thus not clearly delimited to either the school's work with teaching development or with competence development. The differences observed reflect a large effect for goals related to the school's teaching development work and a medium/large effect for competence development.

*Figure 13: **Spread** among principal and 2–3 teachers from the same school, in relation to the existence of and goals for the school's work with both teaching development and competence development. Comparison of **the same persons on two occasions.** Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 11 and 12 schools.*

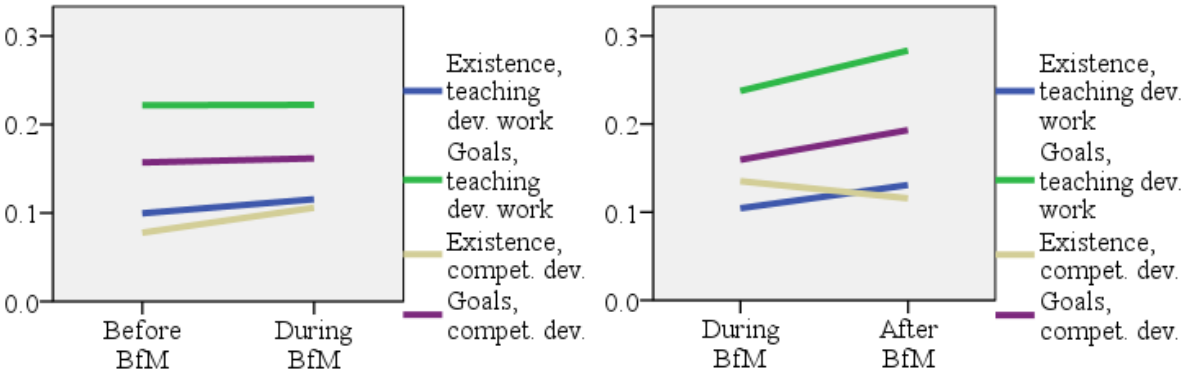
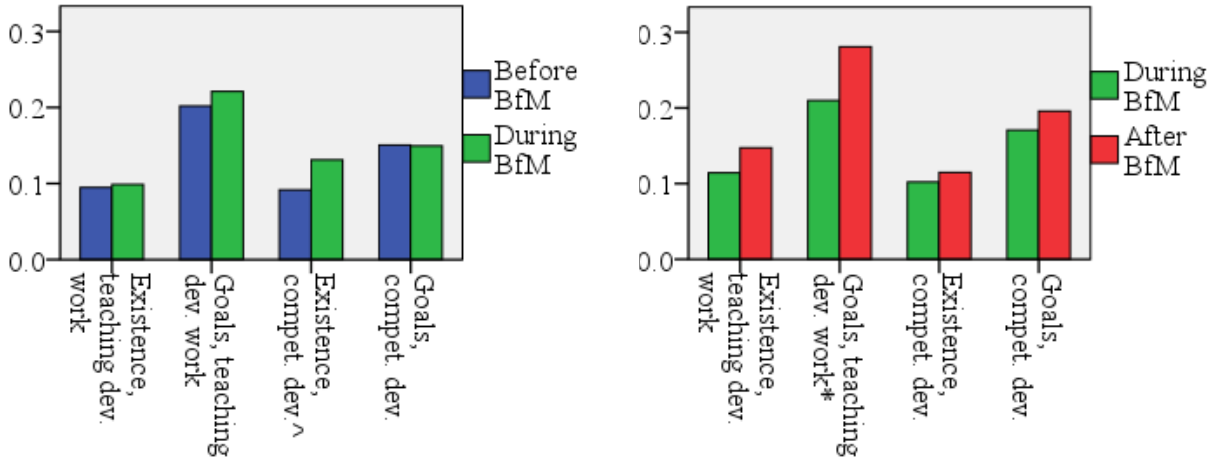


*Figure 14: **Spread** among principal and 2–3 teachers from the same school, in relation to the existence of and goals for the school's work with both teaching development and competence development. Comparison of **different persons and schools at the same point in time.** Differences that are statistically significant are marked with ^ (p<0.1) or with * (p<0.05). Comparisons include between 31 and 34 schools.*



6. The impact of the Boost for Mathematics on the teaching and PDP culture has been independent of many factors

For each outcome variable, two change variables are created: one describing change from *before* to *during* the Boost for Mathematics and one describing change from *during* to *after* the Boost for Mathematics. This section analyses which factors can be statistically linked to the change variables. This is done by examining correlations between each individual factor and each individual change variable. See Appendix D for a list of all factors. Since the internal factors focus on teachers' work in the Boost for Mathematics, such as the use of modules and the role of the supervisor in collegial meetings, these factors are only analysed in relation to the outcome variables from teachers. However, external factors, such as conditions in the school or municipality, are analysed in relation to outcome variables from teachers as well as principals and schools.

Since a large number of correlations are analysed, the focus of the evaluation is not on single corrections that are statistically significant. The focus is instead on those factors that are statistically related to several outcome variables within the different groups of outcome variables. As a result, the results in this section are concerned with reliable effects of factors.

Table 13 shows that there are no internal or external factors that can clearly explain the change in teachers' planning and reflections about teaching.

Table 13: Factors that have statistically significant correlation to several different change variables, regarding ***teachers' planning and reflections on teaching****.*

Factors	Change from before to during the Boost for Mathematics	Change from during to after the Boost for Mathematics
Internal	---	---
External	---	---

Table 14 shows that no internal factors explain change in teachers' teaching, while there are two external factors that have significant correlations. Schools in large municipalities (more pupils) have developed more in terms of teachers' teaching. In addition, the table shows that compulsory schools have developed more than upper-secondary schools in terms of teaching.

Table 14: Factors that have statistically significant correlation to several different change variables, regarding ***teachers' implementation of teaching****.*

Factors	Change from before to during the Boost for Mathematics	Change from during to after the Boost for Mathematics
Internal	---	---
External	School form: Compulsory school/upper-secondary school (negative correlation) Number of pupils in the municipality (positive correlations)	---

Table 15 shows that no external factors but some internal factors have correlations with changes in teachers' views of the school's work with teaching development and competence development. Here there are results that are difficult to interpret due to *negative* correlations between the degree of critical approach in the collegial meetings and changes that take place after the Boost for Mathematics. In addition, the results indicate that the supervisor's leading of the collegial meeting may have influenced teachers to develop their views on the school's work on teaching development and competence development.

Table 15: Factors that have a statistically significant correlation to several different change variables, regarding ***teachers' views on the school's work with teaching development and competence development****.*

Factors	Change from before to during the Boost for Mathematics	Change from during to after the Boost for Mathematics
Internal	Supervisor's initiative in collegial meeting (positive correlations)	Critical approach in collegial meeting (negative correlations)
External	---	---

Table 16 shows that no external factors can explain changes in principals' views on the school's work with teaching development and competence development. An internal factor of specific interest to principals is whether they have fully, partially or not at all completed the principal training within the Boost for Mathematics. This factor has a significant correlation with one of the four change variables (existence of competence development), and there is a tendency towards a link between this factor and one other change variable (goals for competence development). Both of these variables thus relate to competence development, and the training of principals in the Boost for Mathematics thus seems to have had some effect on principals' competence development work. However, there is no tendency towards links to variables related to the school's teaching development work. The changes observed for these variables therefore do not seem to be primarily due to the principal training.

Table 16: External factors that have a statistically significant correlation to several different change variables, regarding ***principals' views on the school's work with teaching development and competence development****.*

Change from before to during the Boost for Mathematics	Change from during to after the Boost for Mathematics
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Table 17 shows that two of the factors have both positive and negative correlations with several different change variables. This makes the results difficult to interpret. It is also difficult to understand why, and how, the proportion of girls and the proportion of pupils with parents with post-secondary education in a school are relevant factors in relation to changes in the spread among principals' and teachers' views of the school's work with teaching development and competence development.

Table 17: External factors that have a statistically significant correlation to several different change variables, regarding the ***spread among principals' and teachers' views*** *on the school's work with teaching development and competence development.*

Change from before to during the Boost for Mathematics	Change from during to after the Boost for Mathematics
---	Proportion of pupils at the school who are girls (both positive and negative correlations) Proportion of pupils at the school who have parents with a post-secondary education (both positive and negative correlations)

It is clear from our discussion above that several correlations were difficult to explain. Since we did not focus solely on isolated significant correlations, the probability that the correlations noted above arose entirely by chance is considered small. However, more in-depth studies than were possible within this review would still be needed to create more detailed explanatory models of the results presented.

There are *no* links between internal factors and changes in teachers' planning, reflection and implementation of teaching. Thus, in order to obtain the changes observed in this evaluation, it does not seem to matter so much *how* the Boost for Mathematics has been implemented. What is important seems to be *that* the Boost for Mathematics has been implemented. However, some variables describing internal factors

have relatively little variation across teachers. For example, teachers seem to have worked in a similar way with the modules overall, if we look at questions concerning teachers' evaluation of whether the modules were worked on in a satisfactory way according to the guidelines. The same applies to supervision. This means that we cannot say whether greater variations in these variables would have produced different effects of the Boost for Mathematics. However, there is considerable variation in other practical aspects of the implementation of the Boost for Mathematics, such as to what extent the work was accommodated within regular working hours and whether substitutes were used to make this possible, as well as whether implementation of the Boost for Mathematics was adapted to local conditions, whether the current status was assessed prior to its implementation and whether teachers sat in on each other's lessons. This variation cannot be linked to different changes caused by the Boost for Mathematics.

Overall, there are relatively few links between external factors and changes in outcome variables. Thus, the effects of the Boost for Mathematics seem to be largely independent of the types of teachers, principals and schools. However, there are links between changes in teachers' teaching and external factors, including school form. The fact that teachers in compulsory schools developed more than teachers in upper-secondary schools in terms of implementation of teaching may be due to the fact that the two types of schools have different modules in the Boost for Mathematics. Ramböll's (2015) evaluation has also shown that teachers from compulsory schools are generally more positive about to what extent the Boost for Mathematics has contributed to a developed teaching culture. In our evaluation, there is also a significant difference between teachers from compulsory schools and upper-secondary schools in terms of how they experienced and implemented the Boost for Mathematics, with teachers from compulsory schools being more positive. However, the results did not show any direct link between such internal factors, describing how teachers experienced and implemented the Boost for Mathematics, and changes in outcome variables. Thus, it does not seem that differences in teachers' experience of or attitudes towards the Boost for Mathematics can explain differences between compulsory and upper-secondary schools. Instead, the Boost for Mathematics seems somehow

to have been less suitable for upper-secondary schools, resulting in less positive developments for teachers in upper-secondary schools than for teachers in compulsory schools in terms of implementation of teaching.

7. Conclusions and recommendations

This section begins with a discussion of the generalisability of the findings of the evaluation. The three questions are then addressed. The section concludes with recommendations for future similar initiatives based on our findings from this evaluation.

7.1 Reliability and generalisability

Throughout the evaluation, two different complementary and parallel types of analysis have been used to test the reliability of the results. If these methods of analysis show the same types of results, it indicates that the results are reliable. This reliability is very clear for results related to teachers, but not always as clear for results related to principals or schools. This difference is probably related to differences in the sample sizes of teachers and schools. The sample size was chosen to make it possible to provide reliable results, but for analyses concerning principals or schools this means that only large, and sometimes only very large, effects can be reliably detected in the analysis. This means that real effects that are small may be missed in the analysis. Medium or large effects can sometimes be detected by one method of analysis and not by the other.

Since the analysis has always focused on combinations of several statistical analyses, there is a high degree of certainty in the results reported. Where different analyses have not been fully consistent, we have instead described trends. The only uncertainty in our results therefore relates to the fact that there may be minor changes that have occurred that we have not been able to detect. This is a direct effect of the sample size.

Thus, based on the design of the evaluation, the results can be considered reliable. The question then is to which population these results can be generalised to, and the answer depends on the characteristics of the sample. The primary purpose of the sample is to be able to generalise results as well as possible to the population of all mathematics teachers in compulsory and upper-secondary schools in Sweden. Results on changes can be generalised in at least two ways: (1) That there have been changes in a certain way thanks to the Boost for

Mathematics can only be generalised to the population that has completed the Boost for Mathematics. (2) The fact that the Boost for Mathematics causes certain changes in mathematics teachers, at the group level, can possibly be generalised to all mathematics teachers (in Sweden). The latter generalisation is important from a formative perspective. It is about being able to make a statement about the appropriateness of continuing with the same type of professional development programs as the Boost for Mathematics.

The random nature of the evaluation sample is an important prerequisite for generalisation. However, it is worth noting that the random sample was drawn from the schools that have chosen to implement the Boost for Mathematics, i.e. there are schools, and thereby teachers and principals, that could not be selected to participate in the evaluation. Such schools could be special in some way. We had no control over this. Overall, however, a very large proportion (about 80%[6]) of Sweden's mathematics teachers have completed the Boost for Mathematics. The sample was thus drawn from a group of teachers that includes a wide variety, and thus results on the effects of PDP can at least be generalised to a very large proportion of Sweden's mathematics teachers.

We have also compared schools participating in the evaluation with schools in Sweden as a whole, with regard to the six external factors for which we were able to obtain data at the national level (see external factors 5–10 in Appendix D). Analyses with t-tests show that the sample of schools differs from schools in Sweden as a whole with respect to two factors: first, schools in the sample are larger in terms of number of pupils (this is to be expected as we actively excluded small schools; see discussion below), and second, schools in the sample have a lower proportion of pupils with parents who have a post-secondary education. It is unclear why such a difference exists, but this factor cannot be reliably linked to variation in results from the Boost for Mathematics; see Chapter 6.

[6] http://www.skolverket.se/kompetens-och-fortbildning/larare/matematiklyftet

Teachers were selected in two stages. First, schools were randomly selected and then three teachers per school were also randomly selected. Thus, the teachers in the evaluation were not selected through an unrestricted random sampling. However, the statistical methods of analysis used take this sampling method into account. Moreover, it turns out that virtually identical results are obtained when statistical methods are used that consider teachers as unrestrictedly randomly selected, indicating that generalisability is not greatly affected by this sampling method.

The comparability between the sample and schools that chose not to participate in the evaluation, as well as between the sample and national data, shows no clear limitations in generalisability. However, we believe that it is reasonable to assume that people with a lack of commitment to the Boost for Mathematics are slightly overrepresented among the schools that actively chose not to participate in the evaluation. It is therefore reasonable to assume that results in such schools would not have been as positive. This means that our results are probably slightly more positive than they would be for the population as a whole.

We actively excluded some schools from participation in the evaluation. These are small schools (where there are fewer than two teachers) and schools where some form of reorganisation was planned. Small schools are therefore not included in the population to which we can generalise. Schools that reorganise frequently have most likely been excluded from the evaluation, and there is a risk that such schools will not achieve the same results as schools that do not reorganise as frequently. However, we estimate that there is not a large number of schools of this type. In addition, there are schools in the evaluation where some reorganisation has taken place between our two visits.

Based on the above considerations, our overall assessment is that our results are generalisable to a very large proportion of Sweden's mathematics teachers in compulsory and upper-secondary schools.

7.2 To what extent has the Boost for Mathematics contributed to the development of a sustainable teaching culture?

Teachers' work with their own teaching is a key aspect of the teaching culture. This aspect of the teaching culture has developed to some extent for each didactical perspective, but there has only been more comprehensive development for some of the perspectives. Interviews with teachers show that the Boost for Mathematics has led them to planning and reflecting more based on the didactical perspectives of *abilities*, *routines/interactions* and *sociomathematical norms*. Observations of lessons also show that the Boost for Mathematics has resulted in teachers now teaching more from the didactical perspectives of *abilities*, *formative assessment* and *routines/interactions* compared to before they participated in the Boost for Mathematics.

These changes are sustainable, as the changes observed when teachers take part in the Boost for Mathematics are maintained the year after completing the Boost for Mathematics. However, we cannot draw the same conclusion for the perspective of *sociomathematical norms* because the change there does not occur until the year after the Boost for Mathematics. Since we did not collect data later than that, we cannot comment on whether the change in teachers' planning and reflections regarding *sociomathematical norms* is sustainable. Nor can we comment on whether the other changes will be sustainable in a time perspective beyond one year after the end of the Boost for Mathematics.

Thus, the Boost for Mathematics has developed a sustainable teaching culture with regard to the didactical perspectives.

Concerning the school's teaching development work, there has been a sustainable development among principals. When principals take part in the Boost for Mathematics, there is an improvement in their views on this work, and the change is maintained the year after the Boost for Mathematics. However, there is a tendency for some decline the year after the Boost for Mathematics, which raises the question about whether the change will be sustained in the longer term.

The development in the school's teaching development work that is noted among principals is not found among teachers. On the contrary, the tendency is that there is a certain decline in how teachers view such work. This decline is mainly due to the fact that teachers have *fewer* types of goals for the school's teaching development work after the Boost for Mathematics, e.g. goals only focus on their own role in the development of teaching, and do not include the role of the principal. This could possibly be due to the fact that teachers' perspectives are limited by the specific type of work with teaching development at the school level that is found in the Boost for Mathematics.

The above results are based on interviews and surveys, where both principals and teachers were asked the same type of questions. Thus, we can show that there is a difference between principals' and teachers' views on how the school works with teaching development, as well as what types of goals exist for this work. Such a difference between principals and teachers can be an obstacle to sustainable development of the teaching culture. At the same time, the changes observed in principals, who are the educational leaders of the schools, can be an important component of sustainable development.

The Boost for Mathematics has thus had a sustainable impact on principals, but not teachers, in terms of the school's work with mathematics teaching development.

7.3 To what extent has the Boost for Mathematics contributed to the development of a sustainable PDP culture?

As regards the school's work with teacher competence development, there has been a sustainable development among principals. When principals take part in the Boost for Mathematics, there is an improvement in their views on the school's work with competence development, and these changes are maintained the year after the Boost for Mathematics. However, there is a tendency for some decline the year after the Boost for Mathematics, which raises the question about whether the changes will be sustained in the longer term.

The development in competence development observed among principals is not found among teachers. On the contrary, the tendency

is that there is a certain decline in how teachers view competence development. This decline is mainly due to the fact that teachers have *fewer* types of goals related to competence development after the Boost for Mathematics. This may possibly be due to the fact that teachers' perspectives have been limited by the specific type of competence development that is found in the Boost for Mathematics.

The above results are based on interviews and surveys, where both principals and teachers were asked the same type of questions. Thus, we can show that there is a difference between principals' and teachers' views on how the school works with teacher competence development, as well as what types of goals exist for this work. Such a difference between principals and teachers can be an obstacle to sustainable development of the PDP culture. At the same time, the changes observed in principals, who are the educational leaders of the schools, can be an important component of sustainable development.

The Boost for Mathematics has thus had a sustainable impact on principals, but not teachers, in terms of the work with teacher competence development.

7.4 What factors have influenced how well or less well the Boost for Mathematics contributed to the development of sustainable teaching and PDP cultures?

The evaluation examined both internal and external factors in relation to the Boost for Mathematics. Internal factors are those which are directly linked to how the Boost for Mathematics has been implemented, such as how modules have been used and how supervision has been carried out in collegial meetings. A total of 13 internal factors have been analysed in the evaluation. External factors are those that are independent of the Boost for Mathematics, such as conditions in the school or municipality. A total of 22 external factors have been analysed in the evaluation.

Few internal factors can reliably explain variations in the changes caused by the Boost for Mathematics. In particular, no internal factors can explain changes in teachers' planning, reflection and implementation of teaching. Thus, when it comes to obtaining the

changes observed, it is relatively unimportant *how* the Boost for Mathematics has been implemented. What is important is *that* the Boost for Mathematics has been implemented. However, some variables describing internal factors have relatively little variation across teachers. For example, teachers seem to have generally worked in fairly similar ways with the modules. This can be seen in teachers' ratings of whether they worked with the modules in a satisfactory way according to the instructions provided, and the same applies to ratings of how supervision was carried out. This means that we cannot determine whether greater variations in variables describing these practical aspects of implementation in the Boost for Mathematics would have produced different effects of the Boost for Mathematics. However, there is more variation in other practical aspects of the implementation of the Boost for Mathematics, such as to what extent the work was accommodated within regular working hours and whether substitutes were used to make this possible, as well as whether implementation of the Boost for Mathematics was adapted to local conditions, whether the current status was assessed prior to its implementation and whether teachers sat in on each other's lessons. This variation cannot be linked to different changes caused by the Boost for Mathematics. Within this evaluation, we are therefore not in a position to say which elements of the Boost for Mathematics are critical, i.e. how the results would change if a specific aspect of the Boost for Mathematics was completely eliminated. However, for principals, there is some link between the implementation of the Boost for Mathematics' principal training and changes in their work with competence development, but not to changes in their work with the development of teaching. The training for principals thus seems to have some impact on the results of the Boost for Mathematics.

Few external factors can reliably explain variations in the changes caused by the Boost for Mathematics. Differences among teachers and principals, as well as the different conditions that exist in schools, have not had a major impact on the results of the Boost for Mathematics. However, there is a link between changes in teachers' teaching and certain external factors: teachers in larger municipalities (more pupils) have developed more than teachers in smaller municipalities, and teachers in compulsory schools have developed more than teachers in

upper-secondary schools. The results suggest that the Boost for Mathematics somehow seems to have been less suitable for upper-secondary schools. As a result, teachers in upper secondary schools have not developed as well as teachers in compulsory schools in terms of implementation of teaching.

Overall, the structure of the Boost for Mathematics has been sufficiently robust to allow for a large degree of impact independently of the exact implementation of the Boost for Mathematics, and independently of variations among participating teachers, principals and schools.

The impact of the Boost for Mathematics on the teaching and PDP cultures has thus been independent of many factors.

As described above, the work in the Boost for Mathematics based on collegial learning worked well for teachers, as the results show development in teachers' planning, reflecting on and implementing teaching. The statistical analyses give the same results whether all teachers were considered as independently selected (i.e. randomly selected at the individual level) or the analyses take into account that several teachers came from the same school. This means that the observed effects of the Boost for Mathematics seem to occur mainly at the individual level and not at the school level.

As described above, the impact of the Boost for Mathematics on principals has been clear, as the results show that there has been progress in principals' work with both teaching development and competence development. Teachers do not perceive the same kind of development. Thus, no clear effect has been observed at the school level regarding these aspects of teaching and PDP culture. This conclusion is also supported by a specific analysis at the school level concerning whether there is a greater consensus among teachers and principals in the same school about work with both teaching development and competence development. No clear changes have taken place, but the trends show a decreased consensus in the schools. A decreased consensus among those involved may reduce the conditions for fruitful

collaboration, as teachers and principals may have different views on how the school works and what the school's goals should be.

As described above, very few internal and external factors can explain variations in the changes brought about by the Boost for Mathematics. The impact of the Boost for Mathematics has thus been largely unaffected by the context in which teachers and principals found themselves when the Boost for Mathematics was implemented. This result also shows that changes have primarily taken place at the individual level and not at the school level.

The fact that changes have taken place primarily at the individual level and not more jointly at the school level is considered to be a potential obstacle to sustainable development of the teaching and PDP cultures, as the cultures are based on a collaboration between individuals who are pulling in the same direction.

Several elements of the results thus show that the Boost for Mathematics' development of teaching and PDP cultures has primarily taken place through impact at the individual level, and to a lesser degree through impact at the school level.

7.5 Recommendations

Based on the findings of this report, we have a number of recommendations that primarily concern similar initiatives in the future. The recommendations also concern what more detailed analyses of the Boost for Mathematics can be done to draw more specific conclusions about which elements of this type of professional development programs are beneficial or detrimental to its outcomes.

The Boost for Mathematics has had a clear sustainable impact on teachers' planning, reflection and implementation of teaching. We therefore recommend continuing to use the approach to teachers' PDP that is currently used in the Boost for Mathematics. However, as there were very few factors that can be clearly linked to these effects, we cannot comment on which elements of the approach are most important. The results show that upper-secondary teachers have not developed their teaching to the same extent as teachers in compulsory

schools. There is thus a need for further analysis of the possible effects of how various aspects of the PDP are designed, including different modules, in general, but particularly with regard to differences between compulsory schools and upper-secondary schools. This need relates in particular to *formative assessment* (in relation to teachers' planning and reflections) and *sociomathematical norms* (in relation to teachers' teaching), as no clear effects were noticed for these perspectives.

The Boost for Mathematics has produced clear sustainable effects on principals' work with teaching development and competence development. We therefore recommend continuing to train principals and to include principals in PDP initiatives like those conducted in the Boost for Mathematics. Since similar effects have not been seen in relation to teachers' work with teaching development and competence development, we recommend exploring whether and how the organisation can be improved in terms of links between the teachers' professional development and principals' training. For example, this could relate to how principals anchor their goals and plans with teachers, and how teachers' competence development could also include work with goals and plans at a broader level, beyond direct work with teaching.

The overall impact of the Boost for Mathematics seems to have been primarily on the individuals and less so on the schools. We therefore recommend that similar initiatives in the future focus more on developing a partnership between teachers and principals at each school. Together with the above suggestion of a link between teachers' PDP and principals' training, we recommend that future initiatives take a clearer overall approach to development at the school level in order to create even better conditions for the development of a sustainable teaching and PDP culture.

Some of the evaluation's findings show a tendency towards decline the year after the Boost for Mathematics, particularly with regard to outcome variables at the principal level. We therefore recommend that further follow-up be carried out to determine with certainty whether the Boost for Mathematics caused sustainable effects from a longer-

term perspective. The need for this is also supported by the results showing that the Boost for Mathematics has generally impacted individuals and less so schools, as this poses a risk that the effects will not be sustained.

Overall, we have the following recommendations for similar initiatives in the future:

1. To keep the approach using collegial learning among teachers and training of principals, in order to sustain the positive results observed.
2. To develop professional development programs at a more comprehensive school level in terms of the partnership between teachers and principals, in order to ensure the development of a sustainable teaching and PDP culture.
3. To carry out more in-depth analyses of the Boost for Mathematics, in order to understand in more detail what has created the observed effects, and to be able to implement similar initiatives that are more effective in the future.
4. To carry out follow-up of results where there is a tendency towards decline after implementation of the Boost for Mathematics, in order to determine with certainty which effects have become more sustainable and to be able to apply such knowledge to implement similar initiatives that are more effective in the future.

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Appendices

Appendix A: Methodology

This appendix describes in more detail how the data were collected and analysed.

Data selection and non-response

The types of non-response varied. They include when schools chose not to participate or were unable to participate, for example due to illness, or when an individual teacher or principal chose not to respond to the electronic survey. It has been generally difficult to get in touch with representatives of school organisers at all, and many of those we contacted declined to participate. Data from school organisers are therefore not included in the analyses.

In cases where a school was selected but, for reasons including those mentioned above, could not be visited, a new school was randomly selected and contacted with a request to participate in the evaluation in order to include the planned 35 schools. These changes of schools are therefore not included in the table in section 2.2, which only shows non-response that occurred after receiving approval from the principal and teachers to visit the school. Thus, when schools were contacted, visits were sometimes not possible, for example because the principal or teachers declined to participate. Practical circumstances may also be a reason for not visiting schools. For example, we did not visit very small schools because we needed three teachers per school. We also did not visit schools where there was a plan for some kind of restructuring or major change in the school (including cases where the principal was to be replaced or the school closed), because two visits were needed and we did not want to risk such changes affecting the results of the evaluation.

Prior to the first school visit, there were 44 changes of school in total, 14 of which were because the principal or teachers declined to participate (usually due to time constraints), 12 due to major changes planned at the school, 10 due to the school only having a few mathematics teachers to visit, 3 times because the school had already started working with the Boost for Mathematics modules before they formally started their participation in the Boost for Mathematics, and

3 times because the school had dropped out of the Boost for Mathematics.

Prior to the second school visit, 3 schools declined to participate, i.e. among the schools that participated in the first school visit. New schools to visit were then selected randomly. For the second visit there were therefore 9 changes of school, of which 3 were because a stable contact could never be established with the contact person at the school, 4 were because the principal or teachers declined to participate, and 2 were because there was only one mathematics teacher to visit at the school.

There is a risk that individuals who chose not to participate or selected schools where visits could not take place for whatever reason differ from the individuals and schools that did participate. If this were the case, it would create limitations in the generalisability of the findings presented in the evaluation. We have therefore compared the schools that participated with the schools where visits could not be carried out. We analysed all variables describing external factors, the values of which were retrieved from databases (see Appendix D for list of variables). Analyses with t-tests show that the group of schools that was contacted but not visited does *not* differ from the group of visited schools with respect to the 20 variables examined. As always, there may also be other differences between schools or differences between individuals that we cannot examine, but which may cause schools not to be clearly equivalent. This may relate to factors such as the attitude of the principal or teachers towards professional development programs (PDP) for teachers or towards mathematics. However, the schools that could not be visited are in any case not clearly different from the schools that were visited.

Quality assurance

To obtain values for all variables, we used multiple methods to ensure good quality of our data. First, a thorough procedure was used in the creation of very detailed data collection plans and assessment templates. This work was based on knowledge gained from existing research, including research carried out by members of the project team and the reference group. The plans and assessment templates

were also reviewed by the reference group, and tested through visits to three schools. Secondly, data collection was carried out using audio recordings, and a documentation process was used that allows for the processing and analysis of data to be checked and redone if necessary. Thirdly, data processing and analysis was done in several steps and by different persons. The handling at a particular step identifies any shortcomings in the previous step, e.g. by making it difficult to complete the current step of the analysis procedure with sufficient quality. Where shortcomings have been identified, more detailed checks were carried out and, where necessary, some processing and analyses were redone. Fourthly, key assessments used in the creation of outcome variables were carried out in a consistent manner, as the same person performed these analyses across all data for a given year.

When assessments are carried out over a long time span, which in this evaluation refers to assessments carried out at one-year intervals, it is not unusual for so-called *drift* to occur. This refers to the assessments on average becoming more "generous" or more "restrictive", which can happen even when the same person is performing the analyses. The schools in Group A and Group B were both visited during the Boost for Mathematics, but the visits were conducted in different years. Since the schools are randomly selected, the values of the outcome variables from the visits during the Boost for Mathematics should be comparable between Group A and Group B. This is corrected for by adjusting all values from one year by the difference in average value between the groups. Among all 20 outcome variables, drift exists for 3 of them: assessment of teaching for abilities and for formative assessment, and assessment of the existence of teaching development work by principals. The values for these variables were thus adjusted, and the adjusted values were then used in analyses.

Note that the problem of drift only affects analyses comparing the same individuals over time and not analyses comparing the different groups, since the latter type of analysis is based on assessments carried out over the same period of time.

Appendix B: Interview questions

This appendix contains the questions that were included in the teacher and principal interviews. Questions in bold were mandatory. Other questions were follow-up questions that could be asked depending on the answer to the mandatory question.

Teacher interview

1. **How do you think the lesson went?**
2. **What did you want the pupils to learn during the lesson?**
    i. *If the goal described only relates to core content*: Ask if there were also goals aimed at abilities.
    ii. *If the goal described only relates to abilities*: Ask if there were also goals aimed at core content.
3. **To what extent do you think the pupils know what they will be learning during the lesson?**
    i. **Why do you think they do (do not) know this?**
    ii. Ask about pupils' awareness of the focus of the lesson as part of a larger picture of their learning (such as several weeks or a whole semester), and why the teacher thinks they do (do not) know this.
4. **In what ways did you assess, or will you assess, what pupils would learn during the lesson?**
    i. **How will you use the results of that assessment?**
    ii. *If the use only concerns the teacher's own use*: Ask about possible use for feedback to the pupils.
    iii. *If the use is only for feedback to the pupils*: Ask about possible use for the teacher himself/herself, e.g. to plan further teaching.
5. **In what ways have you worked with assessment during this school year?**
    i. **Why have you worked in these ways?**
    ii. **How do you usually give feedback to pupils on the results of assessments?**

iii. **How do you use the results of assessments to plan further teaching?**

6. **People sometimes talk about summative and formative assessment. What function do you think the different forms of assessment can have in the teaching of mathematics?**
7. **Before your lesson, what considerations did you make when planning which instructional settingswould be used?**
    i. Ask whether (and if so, how) considerations were made about these four forms: teacher-led review, whole-class work, group work and individual work. Point out that (a) these four categories are broad and do not necessarily cover all instructional settings, (b) with teacher-led review, pupils can also be active, but the focus is on presenting something to the pupils, such as theory, and (c) with whole-class work, the focus is on working on and discussing something together, particularly solving tasks.
8. **What advantages and disadvantages do you see in the following instructional settings in mathematics education: teacher-led review, whole-class work, group work and individual work?** If necessary, point out again that (a) these four categories are broad and do not necessarily cover all instructional settings, (b) with teacher-led review, pupils can also be active, but the focus is on presenting something to the pupils, such as theory, and (c) with whole-class work, the focus is on working on and discussing something together, particularly solving tasks.
    i. *Make sure that the teacher touches on both the advantages and disadvantages of all four instructional settings.*
    ii. **How much have you used these four different instructional settings during this school year?** Point out that no exact amount needs to be given, but that a relative distribution of time between the four

instructional settings, such as which has been used most and least, will suffice.

iii. **Do you use other instructional settings that you think are not covered by these four different instructional settings?** Ask the teacher for examples.

9. **How do you usually plan for whole-class discussion work on the solution for a task?**
   i. **What do you think is most important to have a good discussion of this type?**
   ii. **Anything else that comes to mind, e.g. based on examples of lessons you have had?**
   iii. *If the teacher has not already commented on the choice of examples of pupil solutions:* **Once all pupils have solved the same task and you want to choose some examples of pupil solutions to show on the board/OH/projector, what do you usually think about when choosing such examples? In other words, how do you usually choose examples of pupil solutions to show?**
10. **When pupils work on tasks independently or in groups, how do the pupils decide when they are finished?** Explain to the teacher that it does not have to be tasks/exercises from a textbook, but any type of activity or assignment given to the pupils.
   i. **Is the answer key in the book used?**
   ii. **Do you collect the pupil's solutions and check them?**
   iii. **Do you use other approaches?**
   iv. **Which approach do you think works best and why?**
11. **Different mathematical abilities are described in the syllabus. How do you think these abilities affect your lesson planning in general?**
   i. If necessary, give examples of how this might affect (a) how the lesson is structured, (b) the order of the content, (c) the choice of tasks, (d) other?
   ii. Ask the teacher for examples.

iii. For each ability, ask the teacher to briefly describe (making sure that both are covered):

- what the teacher considers the core of the ability in terms of what pupils need to learn
- an example of how the teacher worked in any lesson to give pupils the opportunity to develop this ability

12. In what way do you think pupils are aware of the abilities?

i. **Why do you think they are (are not) aware of this?**

13. Have you actively worked to make the abilities clear to the pupils?

i. **Why (not)?**

ii. *If the teacher has worked to clearly communicate the abilities*: **How was this done?**

14. What do you think pupils should learn in mathematics education?

15. Learning goals are sometimes divided into content goals and ability goals. How do you view this division?

i. If necessary, clarify that these are learning outcomes related to the core content and learning outcomes related to abilities in the syllabus.

16. If you think about learning outcomes in lessons in relation to the syllabus: Do you usually take lesson goals from any part of the syllabus?

i. *If goals are taken from the syllabus*: **What part of the syllabus and how?**

ii. **Do you use learning outcomes from other sources, such as the textbook, any local syllabus, or collegial agreements?**

iii. **Do you use other approaches to learning outcomes for lessons?**

17. **How do you think you can best find out whether pupils have learned what they need to learn?**
    i. Ask the teacher to provide the reasoning behind their answer.
    ii. **Are there other good ways?**
    iii. **Do you think of different ways for core content and abilities?**
    iv. **Why (not)?**
18. **Have you used pupils to assess their own or other pupils' knowledge and learning?**
    i. *Please make sure that the teacher touches on both self-assessment and assessing other pupils, including in the questions below, where appropriate.*
    ii. **Why (not)?**
    iii. **Is this done often?**
    iv. **In what way is it done?**
19. **How would you describe the existing learning environment at the school for mathematics teachers' own learning.**
    i. **What do you think is good about that learning environment, and what needs improvement?**
    ii. Ask the teacher to comment: What is the principal's role in this? What is the municipality's/corporation's role in this?
20. **How does the school work to enable the development of mathematics teaching?**
    i. **What works well, and what needs improvement?**
    ii. Ask the teacher to comment: What is the principal's role in this? What is the municipality's/corporation's role in this?

21. Do you have any work plans, policy documents or similar regarding mathematics teaching?

i. Ask if documents exist within the teacher team, the school or the municipality/corporation/corporate group.

ii. **What do you think is good about these documents, and what needs improvement?**

iii. Ask for a copy of such documents.

22. Do you have any work plans, policy documents or similar regarding professional development for teachers?

i. Ask if documents exist within the teacher team, the school or the municipality/corporation/corporate group.

ii. **What do you think is good about these documents, and what needs improvement?**

iii. Ask for a copy of such documents.

23. What support would you need to be able to develop as a mathematics teacher and to develop mathematics teaching?

i. Make sure that the teacher touches on both PDP of teachers and teaching development.

ii. **How do you think this was met by the Boost for Mathematics?**

24. Ideally, how do you think collegial learning should be organised among mathematics teachers in order to work as well as possible?

i. **Why this organisation?**

ii. **What do you consider the principal's role in this organisation?**

iii. **What obstacles, if any, do you see to achieving this ideal?**

Principal interview

1. **How would you describe the existing learning environment at the school for mathematics teachers' own learning.**
    i. **What do you think is good about that learning environment, and what needs improvement?**
    ii. Ask the principal to comment: What is your own role in this? What is the municipality's/corporation's role in this?
2. **How do you at the school work to enable the development of mathematics teaching?**
    i. **What works well, and what needs improvement?**
    ii. Ask the principal to comment: What is your own role in this? What is the municipality's/corporation's role in this?
3. **What type of support do the mathematics teachers at the school need to develop as a teacher and develop their mathematics teaching?**
    i. Make sure that the principal touches on both PDP of teachers and teaching development.
    ii. **How do you think this was met by the Boost for Mathematics?**
4. **How do you at the school work to highlight and follow up on mathematics teachers' own learning?**
    i. **What works well, and what needs improvement?**
    ii. Ask the principal to comment: What is your own role in this? What is the municipality's/corporation's role in this?
5. **Do you have any work plans, policy documents or similar regarding mathematics teaching?**
    iii. Ask if documents exist within the teacher team, the school or the municipality/corporation/corporate group.
    iv. **What do you think is good about these documents, and what needs improvement?**
    v. Ask for a copy of such documents.

6. **Do you have any work plans, policy documents or similar regarding professional development for teachers?**
    i. Ask if documents exist within the teacher team, the school or the municipality/corporation/corporate group.
    ii. **What do you think is good about these documents, and what needs improvement?**
    iii. Ask for a copy of such documents.
7. **What framework do you have to work within or what guidance do you have from others regarding development work for mathematics teaching and regarding PDP for mathematics teachers?**
    i. Make sure that the principal touches on both teaching development and PDP of teachers.
    ii. Ask about any type of control from the municipality/corporation/corporate group.
    iii. Ask about the type of control, such as financial control, control of the type of activity or other control.
8. **Ideally, how do you think collegial learning should be organised among mathematics teachers in order to work as well as possible?**
    i. **Why this organisation?**
    ii. **What do you consider your own role in this organisation?**
    iii. **What obstacles, if any, do you see to achieving this ideal?**
9. **What do you consider the most important goals to achieve after the Boost for Mathematics at your school?**
    i. **Any additional goals?**
10. **In what way have you been involved in building understanding at your school of what the Boost for Mathematics aims to achieve?**
11. **In what way will the work on teachers' own learning continue after the Boost for Mathematics at your school?**

i. Ask about the principal's own role in this work.

12. In what way will the work on the development of mathematics teaching continue after the Boost for Mathematics at your school?

i. Ask about the principal's own role in this work.

13. What support do you need to be able to develop as a pedagogical leader at your school, especially with regard to the development of mathematics teaching and PDP of mathematics teachers?

i. **Anything else?**

ii. **How do you think this was met by the Boost for Mathematics?**

Appendix C: Observation protocol

This appendix contains the instructions followed by each observer, and the record that was filled in both before and during the lesson.

Observation of the lesson

- Note the following in the observation protocol (pages 3–5):
    - The start and end time of the lesson.
    - Information on the division of the lesson into different elements. However, this can be done rather roughly and more intuitively during the lesson, but then refined in the post-lesson work.
        - For each element where pupils work in groups or individually: Note (approximately) what proportion of the pupils/groups the teacher visits and either interacts with or just observes their work.
- On a separate paper (number/name this paper so it can be easily linked to the lesson in question):
    - A simple sketch of the layout of the classroom desks and chairs, incl. the teacher's desk/podium.
    - What is shown on the board/OH/PowerPoint (note both the content and structure as thoroughly as possible) during the lesson – regularly make note of the time.
    - If and how any other artefacts are used, such as laboratory materials, electronic tools – regularly make note of the time.
    - Any numbers/names of tasks/exercises that the pupils are asked to work on, such as exercise number in the textbook or the name of the worksheet.
- Ask the teacher for permission to take photos of the material or if you can receive copies/printouts (even if only electronic versions are available) – alternatively, describe them in your own notes.
    - Any lesson plan or weekly plan (mark the lesson in question in the weekly plan) – indicating e.g. what tasks,

textbook pages or subject content are to be covered – also note any clear deviations from such plan during the lesson.

- Any worksheets/information sheets **that are used during the lesson** and are not taken from the textbook.
- Any other artefacts **used during the lesson**, such as laboratory materials or electronic tools.

- Ask the teacher what tasks the pupils were meant to work on if this was not clear at any part of the lesson, e.g. if the teacher only stated that the pupils can continue working in the textbook (or with some other material) – note the teacher's answer.
  - If possible: Note the numbers of tasks/exercises, pages or sections.
  - If the pupils are working with many different things, this can be described more generally, e.g. if several different learning materials are used, then it is not necessary to indicate the individual numbers of tasks/exercises the different pupils are working on, but pages/sections can instead be indicated.

Cover page

Date & time (start of recording):	
Teacher's name:	
Module (if the lesson is being used as Element C in the Boost for Mathematics) – indicate school level, name and number of the module:	
Type of class (grade level, any specialisation, etc.):	
Number of pupils in the classroom:	
Name of the textbook, including any version number:	
Audio recording file (number/name):	
Photos/copies/notes (number/name):	

Observation

Start time (hh:mm)	Teacher -led review	Whole-class work	Group work	Individual work	Not mathematics	Comments/notes on the activity *For group work or individual work in particular: Indicate (approximately) what proportion of the pupils/groups the teacher visits and either interacts with or just observes.*
(:)						
(:)						
(:)						
(:)						
(:)						
(:)						
(:)						
(:)						

Appendix D: List of variables

This appendix contains lists of all of the variables used in the evaluation. It also contains brief descriptions and references to additional information in other appendices on how the variables were created. The list is broken down as follows:

- outcome variables related to the didactical perspectives, i.e. groups A–B in section 2.7
- other outcome variables, i.e. groups C–H in section 2.7
- variables describing internal factors
- variables describing external factors.

Outcome variables related to the didactical perspectives (groups A–B)

Each outcome variable relates to a didactical perspective and is created by averaging the various sub-variables associated with different aspects of each didactical perspective (see section 2.6 describing these aspects). Each sub-variable, in turn, is created by averaging the different assessments made. Details of how all these assessments were carried out can be found in Appendices E, G, H and I for the four didactical perspectives.

Group A contains 4 variables describing to what extent each individual teacher **plans and reflects** in line with the didactical perspectives. These variables are created based on analyses of **teacher interviews**.

Outcome variable	Didactic perspective	Sub-variable	Interview questions
1	Competencies	Knowledge of abilities	2, 11, 15
		Lesson planning based on abilities	11, 16
		Reflections on teaching based on abilities	1, 2
2	Formative assessment	Planning based on the big idea	6
		Planning based on key strategy 1: Objectives	3, 12, 13
		Planning based on key strategy 2: Mapping	4, 17

		Planning based on key strategy 3: Feedback	4, 5
		Planning based on key strategy 4: Resources	7, 8, 18
		Planning based on key strategy 5: Owner	10, 18
3	Routines/ interactions	Approach to monologic/dialogic interaction patterns	7, 8
		Planning of whole-class discussions	9
4	Norms	Planning with respect to pupils' views	2, 7, 9, 11
		Approach to who decides what is correct	10, 18

Group A contains 4 variables describing to what extent **the teaching** of each individual teacher is in line with the didactical perspectives. These variables are created based on analyses of **lesson observations**.

Outcome variable	Didactic perspective	Sub-variable
5	abilities	Proportion of the lesson dealing with more than procedure
		Number of abilities covered
		Number of abilities highlighted
		Number of abilities discussed
6	Formative assessment	Existence of formative assessment strategies
		Quality of formative assessment strategies
7	Routines/ interactions	Balance between monologic/dialogic interaction patterns
		Dialogic or monologic questions to pupils
8	Norms	Teacher's values: Who decides what is correct and how is it decided

Other outcome variables (groups C–H)

Groups C–E contain variables (no. 9–14) related to the **development of teaching** in terms of existence and goals, at the teacher level, the principal level and the school level. These variables are created based on analyses of interviews and surveys with teachers and principals, and analyses of documents from teachers and principals. Each variable is created by averaging different assessments. Details of how these assessments were carried out can be found in Appendices J, L, M and P.

Outcome variable		Interview questions	Survey questions	Documents
9 (teachers)	Existence of development work	20, 21	18, 19, 21, 22	Plans
10 (principals)		2, 5	9, 10, 12, 13	Plans
11 (teachers)	Objectives of development work	23, 24		
12 (principals)		3, 8, 13		
13	Dissemination of views on the existence of development work	Outcome variables 9 and 10		
14	Dissemination of views on the goals of development work	Outcome variables 11 and 12		

Groups F–H contain variables (no. 15–20) related to the **competence development** in terms of existence and goals, at the teacher level, the principal level and the school level. These variables are created based on analyses of interviews and surveys with teachers and principals, and analyses of documents from teachers and principals. Each variable is created by averaging different assessments. Details of how these

assessments were carried out can be found in Appendices J, L, M and P.

Outcome variable		Interview questions	Survey questions	Documents
15 (teachers)	Existence of competence development	19, 22	18, 19, 23	Plans
16 (principals)		1, 4, 6, 7	9, 10, 14	Plans
17 (teachers)	Objectives of competence development	23, 24		
18 (principals)		3, 8, 13		
19	Dissemination of views on the existence of competence development	Outcome variables 15 and 16		
20	Dissemination of views on the goals of competence development	Outcome variables 17 and 18		

Variables describing internal factors

Internal factors are factors that constitute the conditions for, and characteristics of, the implementation of the Boost for Mathematics. Variables describing internal factors are created from analyses of teacher surveys and collegial discussions. Each variable is created by averaging different assessments. Details of how assessments were carried out on survey responses can be found in Appendix P. See below for references to other appendices.

Internal factor		Basis for variable
1	Quality/degree of implementation	Teacher survey 2, 4, 15 e–h, 16 a–b, 17
2	Quality/degree of module use	Teacher survey 7, 8, 9, 10, 11 a–g
3	Quality/degree of supervision	Teacher survey 11 h–i, 12, 13
4	Principal's activity	Teacher survey 1 b, 5 b, 15 a–d, 16 c–d
5	School organiser's activity	Teacher survey 1 a, 5 a
6	Supervisor's experience	Information on participation, see Appendix K
7	Supervisor's initiative in collegial discussion	Analysis of collegial discussion, see Appendix K
8	Supervisor's activity in collegial discussion	Analysis of collegial discussion, see Appendix K
9	Discussions of abilities in collegial discussion	Analysis of collegial discussion, see Appendix E
10	Discussion of formative assessment in collegial discussion	Analysis of collegial discussion, see Appendix G
11	Discussions of routines/interactions in collegial discussion	Analysis of collegial discussion, see Appendix H
12	Discussions of norms in collegial discussion	Analysis of collegial discussion, see Appendix I
13	Critical approach in collegial discussion	Analysis of collegial discussion, see Appendix K

Variables describing external factors

External factors are factors that are independent of the Boost for Mathematics. Variables describing external factors are created from analyses of surveys and from values obtained from the database on schools. Variables based on surveys are created by averaging different assessments. Details of how these assessments were carried out can be found in Appendix P.

External factor		Basis for variable
1	Teacher's experience	Teacher survey 26, 28, 29, 30
2	Principal's experience	Principal survey 17, 21–26
3	School form: Compulsory school/upper-secondary school	Information on participation
4	School type: Independent/municipal school	Information on participation
5	Number of pupils at the school	SIRIS database
6	Proportion of pupils at the school who are girls	SIRIS database
7	Proportion of pupils at the school with a foreign background	SIRIS database
8	Proportion of pupils at the school who have parents with a post-secondary education	SIRIS database
9	Proportion of teachers at the school with a university degree in education	SIRIS database
10	Number of pupils per teacher at the school	SIRIS database
11	Number of inhabitants in the municipality	SCB database
12	Number of pupils in the municipality	SIRIS database
13	Proportion of pupils in the municipality who are girls	SIRIS database
14	Proportion of pupils in the municipality with a foreign background	SIRIS database
15	Proportion of pupils in the municipality who have parents with a post-secondary education	SIRIS database
16	Proportion of teachers in the municipality with a university degree in education	SIRIS database
17	Proportion of teachers in the municipality with a degree in special education	SIRIS database

18	Number of pupils per teacher in the municipality	SIRIS database
19	Costs per pupil in the municipality	SIRIS database
20	Number of mathematics grants applied for in the municipality	SIRIS database
21	Proportion of awarded mathematics grants in the municipality	SIRIS database
22	Proportion of pupils who reached the requirement level in national mathematics testing in the municipality	SIRIS database

Appendix E: Operationalisation of didactical perspective 1 – Teach based on the abilities

This appendix shows how the didactical perspective *abilities* was operationalised. The appendix presents the assessment guidelines that were used regarding teaching based on abilities, based on interview responses from teachers, classroom observations, and observations of collegial discussions. The interview questions can be found in Appendix B.

Analysis of teacher interview

Assessment: Knowledge of abilities. One assessment for each ability, i.e. 5 abilities (compulsory school) and 7 abilities (upper-secondary school).

Questions 2 and 11

0	The teacher mentions the ability by name, but cannot specify what it means. They only repeat the description from the syllabus.	*Conceptual ability is the ability to use and understand concepts.*
0.5	The teacher can give examples of what the ability involves, but to a limited extent.	*Being able to apply mathematical reasoning is e.g. being able to give arguments for a relationship.*
1	The teacher can give a detailed explanation of what the ability involves. The teacher talks about both cognitive and productive aspects of the ability.	*Understanding the concept of area meaning understanding both how area relates to e.g. length and how to calculate areas of specific surfaces.*

Assessment: Knowledge of content goals.

Questions 2 and 15

0	The teacher never talks spontaneously about content goals and only briefly when asked directly.	*Of course they should know percentages.*

0.5	a) The teacher can explain what the content goals involve, but to a limited extent. b) The teacher mentions several content goals, but does not comment on them.	*a) Arithmetic is core knowledge for the pupils.* *b) There is geometry, statistics and probability in the course.*
1	The teacher demonstrates good knowledge of the content goals and can talk about them both generally and specifically.	*Algebra is a central concept in the core content of year 8. Particular emphasis is placed on algebraic expressions, formulas and equations.*

Assessment: Balance between ability goals and content goals.
Questions 2 and 15

0	The teacher focuses entirely on only one of the two types of goals.	*Problem solving is at the heart of mathematics.* *or* *Percentage calculation is the most important thing in year 7.*
0.5	The teacher emphasises one type of goal more than the other, and uses e.g. content goals only as examples.	*The pupils shall learn how to apply reasoning to determine e.g. percentages.*
1	The teacher considers ability goals and content goals equally important, and they should be balanced against each other.	*It is important that the pupils show their reasoning by explaining their solutions with clear arguments when solving problems in geometry, e.g. area calculations.*

Assessment: Competence and willingness to let abilities influence lesson planning.
Questions 11 and 16.

0	The teacher never talks spontaneously about abilities in connection with lesson planning, and only briefly when asked directly.	*Conceptual understanding is one of the abilities that the pupils shall develop during the lessons.*
0.5	The teacher stresses the importance of the abilities influencing lesson planning, but only in general terms. The teacher only talks about cognitive *or* productive aspects.	*It takes practice to be able to apply a concept, so I usually incorporate a lot of that into the lessons.*
1	The teacher provides examples of how different abilities affect lesson planning. The teacher talks about both cognitive and productive aspects of the abilities.	*To understand a concept, such as variables, requires insights into both what the concept means and how it is handled. Both discussions about the concepts and practice in how to apply them are therefore always included.*

Assessment: a) Ability to reflect. Consistency – what the teacher says and what the teacher does in the classroom, with respect to the abilities.
Questions 1 and 2.

0	Essentially no agreement between the teacher's account and the observation analysis.	*The pupils worked a lot with communication* *vs.* *Virtually no communication during the lesson.*
0.5	Some agreement. Some abilities were mixed up and the descriptions are somewhat unclear.	*The pupils worked mostly with reasoning, as they are allowed to talk almost the whole time.* *vs.*

		Communication dominated the lesson.
1	Good agreement. The teacher's description is more or less the same as the results of the observation analysis.	*The pupils mainly worked with problem solving, reasoning and communication as they solved problems together, and were encouraged to argue the reasoning behind their solutions.* vs. *Problem solving, reasoning, communication and procedure handling are covered during the lesson.*

Assessment: b) Evaluation. To what extent can the teacher evaluate how the activities during the lesson worked in relation to the ability goals?

Questions 1 and 2.

0	Essentially no evaluation, or only evaluation not linked to the ability goals.	*The activity went well; the pupils were working the whole time.*
0.5	Some evaluation. The teacher touches superficially on how the activities were linked to the ability goals.	*The pupils worked mostly with problem solving, as they had to solve some mathematical problems.*
1	Extensive evaluation. The teacher relates the activities to the ability goals, and can discuss the advantages and disadvantages of the activities.	*The pupils worked mostly with problem solving, but the first problem was too easy which meant that too few pupils were challenged because they already knew how to do it. The second problem worked well and, since it also required*

		cooperation, the pupils were able to practice argumentation, which is part of the reasoning ability.

Assessment: c) Adaptation. To what extent can the teacher reflect on how future teaching could be adapted?

Questions 1 and 2.

0	Essentially no reflection.	*It went well. I will probably do it again.*
0.5	Some reflection. The teacher touches superficially on how the activities could be adapted.	*Perhaps some better problems could be found for the next time.*
1	Extensive reflection. The teacher discusses how the activities could be developed to give the pupils better opportunities to develop their abilities.	*The pupils mainly worked with problem solving, but next time I will start by letting all of the pupils have the opportunity to think on their own before working in pairs. This, I think, will help them to get a better grasp of the problems.*

Analysis of classroom practice (observation)

Analysis of the observations is done in four steps: 1) Divide into sequences, 2) Determine whether the pupils are given the opportunity to develop the abilities (based on operationalisation of the abilities in the document *Ramverk förmågorna.docx [ability framework])* 3) Mark whether the abilities are made visible 4) Answer the question as to whether an activity is carried out based on abilities.

1. First divide the lesson into parts based on instructional settings and activities as described in section 2.5.
2. Analyse each part by means of operationalisation of the abilities. For each of the abilities, indicate whether the pupils are given the opportunity to develop it by it being covered by a part of the lesson.

For each ability:

No	The teaching is not carried out based on the ability.
Yes	The teaching is carried out in such a way that the pupils are given the opportunity to develop the ability.

For parts led by the teacher, analyse the instructions given by the teacher. For parts where pupils work alone or in groups, analyse the task(s) (documentation of the task(s)). If the pupils work with many tasks that differ greatly in nature, do not analyse that part of the lesson.

3. For each part that is teacher-led and each ability, determine to what extent the teacher has made the ability visible to the pupils.

0	No visibility.
0.5	The ability is mentioned in such a way that the pupils understand that it is the one they will be learning or practicing.
1	The ability is discussed in such a way that the pupils can understand what it means.

4. For each lesson, indicate to what extent the teaching is based on abilities and to what extent abilities are made visible:
Assessment: Teaching based on abilities.
Proportion of the lesson (in terms of time) in which an ability other than routine/procedure was covered.

Number of abilities covered: 0, 1, 2, 3, >3 (assessed as 0 - 0.25 - 0.5 - 0.75 - 1)
Assessment: Visibility of abilities.
Number of abilities with some visibility: 0, 1, 2, 3, >3 (assessed as 0 - 0.25 - 0.5 - 0.75 - 1)
Number of abilities discussed: 0, 1, 2, 3, >3 (assessed as 0 - 0.25 - 0.5 - 0.75 - 1)

Analysis of collegial discussion

Divide the collegial discussions into parts based on activities and sequences as indicated in a separate document. For each part dealing with the abilities, determine to what extent the discussion focuses on or deviates from these. This is determined based on operationalisation of the abilities (see separate document). Create two variables – one that shows the proportion of all parts that deal with the abilities and one that determines the qualitative way in which the abilities are covered in each such theme.

Assessment: The abilities in the collegial discussion – depth
Analyse each part dealing with abilities according to the table below. Calculate averages across all parts and then across all abilities as an overall measure of the depth of the discussion regarding abilities.

0	ability is named or meaning of the ability is mentioned, but only in passing and not central to the discussion	*The pupils worked very well in small groups. They were very focused on the tasks and talked the whole time.*
0.5	The ability is discussed to some extent, but its meaning is not a central aspect of the discussion	*The pupils worked in pairs and it went quite well. They wanted to divide into groups themselves, so they were allowed to do so. The problem-solving task went quite well, and they cooperated and communicated well in the groups.*
1	The meaning of the ability is a central aspect of the discussion	*The lesson focused on getting the pupils to understand the concept of growth factor and being able to explain it to another pupil. The aim was for them to also be able to apply the concept when solving the problems. The pupils were quite successful in solving the problems, but had trouble explaining the concept to other pupils.*

Appendix F: Framework of abilities

This document contains a description of the framework that was the theoretical starting point for the operationalisation carried out later (see Appendix E) and used for analysis in relation to didactical perspective 1: Teaching based on *abilities*.

The abilities are based on those presented in the curriculums LGR11 (compulsory school) and LGY11 (upper-secondary school). As support for our interpretations, we also use the commentary material for LGR11.

abilities in the syllabuses

Five abilities are applied in both compulsory school and upper-secondary school:

1. Problem solving
   - The ability to formulate and solve problems, and to evaluate chosen strategies and methods (US: also analyse problems and evaluate results).
2. Concepts
   - The ability to use and analyse (US: describe the meaning of) mathematical concepts and the relationships between concepts
3. Procedures
   - Select and use mathematical methods (US: manage procedures) to perform calculations and solve routine tasks (US: tasks of a standard nature)
4. Reasoning
   - The ability to create and follow mathematical reasoning (US: also assess)
5. Communication
   - CS: The ability to use forms of mathematical expression to discuss, argue and explain questions, calculations and conclusions
   - US: The ability to communicate mathematical ideas verbally and in writing and action

In addition to these, there are two further abilities in upper-secondary school:

6. Modelling
    - The ability to interpret a realistic situation and design a mathematical model, and to use and evaluate the properties and limitations of the model.
7. Relevance
    - The ability to relate mathematics to its significance and use in other subjects, in a professional, social and historical context.

What types of activities can be linked to the abilities?

One possible aspect to consider is the types of activities that can be linked to a specific ability. One way of breaking this down comes from Niss (2003) and concerns what is referred to there as *Competence-related activities*, or ability-related activities. This breakdown highlights the two-fold nature of abilities, namely a cognitive aspect that is concerned with understanding, interpreting, investigating and evaluating a phenomenon. The second aspect is the productive part, performing processes of various kinds (Niss, 2003).

A. Cognitive aspect.
    - Identifying, interpreting, understanding, etc. Acquiring information about an ability, identifying and interpreting how something related to an ability works or relates to other abilities.
    - Evaluating, reflecting on, assessing. This refers to consideration at a meta level, reflecting over aspects of an ability, evaluating activities based on an ability, and formulating perceptions and drawing conclusions in relation to the ability.
B. Production aspect
    - Performing, solving, using, practicing, choosing, answering, etc. Using one's knowledge to solve tasks (in a broad sense).

What does it mean to have developed an ability?

The definitions above provide general descriptions of what a particular ability means. It is not always clear what it means to have a particular ability. A more detailed explanation of each ability is therefore found below, focusing on what a pupil who has developed an ability can do in relation to the two aspects A and B.

Problem solving

"Mathematical problems, unlike pure routine tasks, are situations or tasks where the pupils do not immediately know how to solve the problem" (Commentary material to curriculum LG11, page 9). Problem solving is therefore engaging in tasks of such a nature that the method of solution is not known in advance.

A. Cognitive aspect.

Being able to identify different components of a problem, being able to see alternative ways of solving a problem and understanding the methods, tools and goals of problem solving. Being able to evaluate and assess solutions, strategies and methods. Assess the reasonableness of results in relation to the problem. The commentary material states "being able to assimilate the mathematical content in an everyday or mathematical problem situation, interpret the content, and then formulate a mathematical question" (page 9).

B. Production aspect.

Using mathematics to solve problems that arise in mathematics and in different contexts. Using and adapting problem-solving strategies and methods. Formulating and specifying different types of mathematical problems.

NOTE: When analysing work with tasks, both A and B are always activated if the task requires problem solving. (Solving the problem involves the cognitive aspects, e.g. seeing alternative ways of solving it.)

Comment: The commentary material for LGR11 adds modelling to problem solving and states that part of problem solving is "interpreting everyday and mathematical situations", which according to the text can also mean "formulating a simple mathematical model which is a general description of a real situation". This is included in the upper-secondary school syllabus in the modelling competence.

Concepts

The relevant concepts are specified in the *Core content* section of the syllabus.
Comment: There is almost nothing about concepts or conceptual ability in either the syllabus or the commentary material.

A. Cognitive aspect.

Knowing and understanding the meaning, use, validity and limitations of a given concept. Being able to relate different concepts to each other and understand their relationships.

B. Production aspect.

Being able to use and apply concepts and the relationships between concepts. Being able to describe the meaning of a concept and its relationship to other concepts.

NOTE: When analysing work with tasks, both A and B are always activated if the task requires conceptual ability. (You cannot use a concept without knowing its meaning.)

Procedures

A method or procedure is a set of mathematical actions that is an accepted way of solving a task (task in a broad sense). As this can include everything done in a mathematics classroom, we limit ourselves to two main types:

a. A standard procedure used to solve a standard problem and
b. A given procedure when the pupil is given a solution method or algorithm to follow to solve a specific problem.

Methods (GR) and procedures (GY) include mental arithmetic, written calculations, calculations with a calculator or computer. According to the commentary, this also includes "routine procedures such as taking measurements or constructing tables and coordinate systems" (page 10).

Comment: What is a "routine procedure"? It is not the same as in the upper-secondary school curriculum, but more a focus on a task that does not require much thinking.

A. Cognitive aspect.

Understanding and interpreting one's own and others' procedures. Being able to describe a procedure. Being able to evaluate one's own and other people's procedures and to assess the outcome of a procedure. It also includes a more general ability to reflect on the role of procedures in mathematics teaching.

B. Production aspect.

Being able to choose and use procedures to obtain a result. In order to benefit from procedures, one must be able to do so in a flexible and efficient way.

Reasoning

Reasoning is explicit justification of choices made and conclusions drawn using mathematical arguments. To be considered mathematical arguments, they must be based on mathematical properties that are central to the components of the reasoning. A proof is an argument in which the arguments are logically rigorous.

Comment: The commentary material also states that "Part of being able to reason involves developing an understanding that mathematical relationships are constructed, and that they can therefore also be 'rediscovered' by reasoning." This is included in the cognitive aspect.

A. Cognitive aspect.

Being able to take in and interpret the arguments of others. Understanding that through reasoning one can "rediscover" mathematical connections. Evaluating and assessing one's own and others' arguments.

B. Production aspect.

Using mathematical arguments to justify choices and conclusions. Using reasoning to interpret information.

Communication

Communicating mathematically means "exchanging information with others about mathematical ideas and ways of thinking" (commentary

material page 11). This is done orally and in writing using precise mathematical language and expressions of mathematics. In order to avoid trivial aspects ("everything that happens in the classroom is communication"), the content must not be trivial or immediately familiar.

Comment: The commentary material talks about mathematics' forms of expression*, but this is not defined. Our interpretation is that, in addition to text, it includes anything that is not in textual form, such as tables, diagrams, graphs and symbols.*

A. Cognitive aspect.

Being able to receive and interpret mathematical communication. Being able to evaluate and assess one's own and others' communication

B. Production aspect.

Being able to formulate and convey information to a recipient. Being able to use oral, written and other forms of communication. Being able to switch between different forms of expression.

NOTE: In relation to tasks, communication is activated when the task specifically asks for explanations or descriptions of how the tasks was solved.

Modelling

Modelling involves interpreting a realistic situation and translating this into a mathematical language, i.e. designing a mathematical model. Modelling also involves using, adapting and evaluating the properties and limitations of a model.

Comment: In the compulsory school syllabus, modelling is part of problem solving and is described as follows: "To develop a model is to translate a situation from a field other than mathematics into a mathematical symbolic language".

A. Cognitive aspect.

Being able to interpret a realistic situation and translate it into mathematical language in the form of a mathematical model.

Being able to evaluate the properties and limitations of a model.

B. Production aspect.

Being able to translate a realistic situation into mathematical language in the form of a mathematical model.
Being able to use a mathematical model.
Being able to adapt a mathematical model.

Relevance

Relevance ability means being able to relate mathematics to its significance and use in other subjects, in a professional, social and cultural history context.

Comment: The commentary material for LGR11 contains a section on historical context and the relevance of mathematics. Here, it states that mathematics plays an important role in the development of society, and that one aim of teaching is to provide pupils with the opportunity to reflect on the significance, use and limitations of mathematics in everyday life, in other school subjects and in historical events.

A. Cognitive aspect.

Being able to relate something to its meaning outside of mathematics.
Being able to reflect on the significance of mathematics outside of mathematics.

B. Production aspect.

Being able to give examples of how the mathematics in an activity or course is linked to its use in other subjects, society or historical contexts.
Being able to reason about the relevance of such examples.

Appendix G: Operationalisation of didactical perspective 2 – Formative assessment

This appendix shows how the didactical perspective *formative assessment* was operationalised. The appendix presents the assessment guidelines that were used regarding teaching based on formative assessment, based on interview responses from teachers, classroom observations, and observations of collegial discussions. The interview questions can be found in Appendix B.

Analysis of teacher interview

Assessment: To what extent are different parts of the big idea addressed by the teacher?
Question 6. Assess whether the teacher touches on the following properties of formative assessment:

- Collecting information about pupils' knowledge and performance.
- Using information about pupils' knowledge and performance as a basis for revising planned teaching and shaping future teaching. Either at the micro level (e.g. conversations with pupils) or at the macro level (e.g. planning an activity over several lessons).
- Working towards set learning outcomes.
- Relevant distinction between formative and summative assessment.

State the value as the proportion of these four points that are addressed on a scale of 0–1 (e.g. if three out of four points are addressed, the value is 0.75).

Assessment: NS1 – Objectives for learning and criteria for success are clarified and shared.
Questions 3, 12 and 13.

Value	Description	Example
0	No answer to the Why question or no link to the teacher's own activity.	*The teacher notes that "it is difficult for the pupils to know".*

0.5	Relates to a willingness/ambition to communicate goals to pupils.	*Of course there is a desire for the pupils to know what they will be learning.*
1	States that goals in the form of "where we are heading" are clarified/shared with pupils.	*We occasionally talk about what they are going to learn during the lesson.* *We have a semester planning that includes learning goals, which the pupils are made aware of.*

Assessment: NS2 – The teacher creates situations that provide evidence of pupils' knowledge.
Questions 4 and 17.

Value	Description	Example
0	Does not describe any plan, or is only very general/ambiguous, for using assessment to gather information about the pupils' knowledge.	*I make assessments all the time.*
0.5	Relates to a willingness or concrete plans to use assessment to gather information on the state of the pupils' knowledge.	*I try to do this continuously by noting the types of tasks pupils get stuck on.*
1	Describes in more detail features of lesson design or lesson content in terms of how this relates to generating evidence of pupils' knowledge.	*When we have group work, I make sure that each group has something to present on the board so that I can get an idea of what they know.*

Assessment: NS3 – The pupils receive effective feedback that moves them forward in their learning.
Questions 4 and 5.

Value	Description	Example
0	Does not describe any plans for feedback to pupils or only general/ambiguous description that results/feedback is (always) given to the pupils.	*Feedback is probably mostly in connection with them getting their test results back.*
0.5	Describes feedback plans that are concrete in terms of a particular approach and/or linked to a particular mathematics content or type of activity.	*The pupils are allowed to correct their own diagnoses. I then talk to them about how it went.*
1	Describes plans for pupils to receive feedforward focused on learning outcomes.	*Once pupils have made their diagnoses, we go through them together and talk about what areas the pupil needs to work on more to achieve the learning goals.*

Assessment: NS4 – The pupils are activated as resources for each other in the learning.
Questions 7, 8 and 18.

Value	Description	Example
0	Does not describe any benefits to working in pairs/groups.	*The pupils may work in pairs occasionally.*
0.5	Describes benefits of pupils working in pairs/groups.	*It is good that pupils work in pairs because then they can help each other.*

1	Describes benefits of pupils working in pairs/groups in relation to their learning.	*When pupils work in pairs, they can give each other feedback on their communication ability in order to understand each other.*

Assessment: NS5 – The pupils are activated as owners of their own learning.
Questions 10 and 18.

Value	Description	Example
0	Does not describe anything about pupils' work with reflecting on/assessing knowledge.	*I collect and correct their exercise books or the answer key is used for this.*
0.5	Describes that pupils are allowed to reflect on and assess the qualities and abilities of their own or other pupils' knowledge.	*They are sometimes allowed to look at each other's work, with the task of explaining what is good about a solution.*
1	In addition to what is stated for 0.5, the purpose/benefit of this type of activity is also described in relation to e.g. pupils' learning or the teacher's use of information from this type of activity.	*I sometimes pair pupils/groups up to look at each other's solutions and reach an agreement on which pupil to present, so that they can argue why different solutions work.*

Analysis of classroom observation

The aim is to determine to what extent teaching is based on the five key strategies, designated NS1 – NS5. Existence and quality are determined for each key strategy.

Assessment: NS1 – Learning goals and criteria for success are clarified and shared

Existence:

Yes	Goals in the form of "where we are heading" are clarified and shared with pupils. This can be in writing, on the board, orally, in discussion with pupils. Questions such as "what do you think we will learn from this task?". Goals can also be included in semester planning or weekly planning if they are used together with the pupils.
No	Indicated if such goals are not clarified and shared.

Quality: If existence is assessed as "Yes", quality should be assessed as follows:

Value	Description	Example
0	The goals are presented only: no discussion, no link to previous/future goals.	*The teacher writes "today's goal is..." on the board*
0.5	The goals are presented and linked to previous/future goals.	*The teacher explains what the goals of today's lesson are, and how they relate to the weekly planning*.
1	Clear sub-goals linked to tasks and activities are presented. The goals are linked to previous/future goals.	*Describes continuation from the previous lesson in relation to learning what a function is, and that today they will be able to describe functions in different ways.*

Assessment: NS2 – The teacher creates situations that provide evidence of pupils' knowledge.

Existence:

Yes	The teacher uses specific activities aimed at obtaining evidence of more than one pupil's knowledge, e.g. whiteboards or exit passes.

No	No specific activities as described above are included in the lesson.

Quality: If existence is assessed as "Yes", quality should be assessed as follows:

Value	Description	Example
0	Response to the task or activity in the form of right or wrong only.	*The teacher announces that the correct answer to the question on the whiteboard is 13. Teacher states that "Most seem to have the right answer".*
0.5	Follow-up questions are asked.	*Why was it 13, Carl?*
1	The pupils are encouraged to explain their thinking and present their arguments to both the teacher and other pupils	*The teacher encourages two pupils who came to different answers to discuss and explain to each other how they arrived at their different answers.*

Assessment: NS3 – The pupils receive effective feedback that moves them forward in their learning.

Existence:

Yes	Effective feedback is given, i.e. with a focus on the next step in the assessment.
No	The feedback given is primarily that the answer is right or wrong.

Quality: If existence is assessed as "Yes", quality should be assessed as follows:

Value	Description	Example

0	The pupils receive feedforward at the task level.	*The feedback the pupil receives informs them of how the next task can be solved in comparison to the first.*
0.5	The pupil receives feedforward on general aspects of the task.	*The pupil is told how key it is to draw figures when trying to solve problems in geometry.*
1	The pupil receives feedforward focused on explaining how feedback can be given and how the pupils themselves can assess their progress.	*"When you look at your own task, you can see how you handled mathematical concepts, and then you can judge for yourself whether you have succeeded. It is important to practice assessing your own work in this way."*

Assessment: NS4 – The pupils are activated as resources for each other in the learning.

Existence:

Yes	Pupil-pupil assessment occurs, and pupil groups are given the opportunity to consult on questions
No	The above does not occur.

Quality: If existence is assessed as "Yes", quality should be assessed as follows:

Value	Description	Example
0	The pupils are not given any instructions on how to perform assessment.	*The pupils work in pairs and discuss their answers.*

0.5	The pupils are given brief instructions on how to perform assessment.	*The assessment should show whether the task was solved correctly, and what was wrong if it is not correct.*
1	The teacher describes what should be assessed and how assessment is performed, e.g. focus on one or more abilities.	*The pupils may mark in each other's solutions which mathematical concepts were used and how they have been explained.*

Assessment: NS5 – The pupils are activated as owners of their own learning.
Existence:

Yes	The pupils are allowed to reflect on and assess qualities and abilities of their knowledge, e.g. by being given the opportunity to • formulate feedback/feedup/feedforward • keep a log • maintain focus on change in their knowledge • have access to each other's work and reasoning and give feedback on it
No	None of the above activities take place.

Quality: If existence is assessed as "Yes", quality should be assessed as follows:

Value	Description	Example
0	Instruction to perform an activity	*"Keep a log of the tasks you have done. Mark in a table on a page in your notebook."*
0.5	Instructions with brief examples of what the activity could lead to	*"A log can help you keep track of how you are working with the tasks".*

1	Instructions with qualitative discussion on what could be included in the activity.	*"Why do you think you should keep a log? How does this benefit your own learning?"*

Based on the above assessments, two sub-variables were created:

- Existence of formative strategies: The proportion of key strategies (NS) that existed during the lesson.
- Quality of formative strategies: First, the average across all parts of the lesson is calculated as a measure of quality for each key strategy (NS).

Analysis of collegial discussion

Assessment: NS1 – Learning goals and criteria for success are clarified and shared.

Existence: Indicate whether the discussion addresses in what way learning goals are clarified/shared with the pupils

No	NS1 is not addressed in the activity.
Yes	NS1 is addressed in the activity.

Quality: For each discussion activity where NS1 is addressed, the depth of the discussion is assessed with a value from 0 to 1 as shown in the table.

Value	Description	Example
0	The discussion only concerns the presentation of the goals: no discussion, no link to previous goals/future goals	*The day's goals are often written on the board.*
0.5	Discussion concerns whether/how the goals are	*If pupils have a book in which they write down the* goals*, they*

	presented and linked to previous goals	*can see how they are connected.*
1	The discussion concerns sub-goals related to tasks/activities, and how the goals are linked to previous goals and future goals	*The* goals *should be written on a flipchart where one can point out the progression. For each activity, you can then point out where we are on the flipchart.*

Assessment: NS2 – The teacher creates learning situations that provide evidence of pupils' knowledge.

Existence: Indicate whether the discussion addresses any of the following points:

- in what way a baseline assessment can be made
- how to ask questions that allow for a number of possible wrong answers
- how to follow up with developed follow-up questions
- how to assess what knowledge pupils have acquired during the course

No	None of the points were addressed in the activity.
Yes	At least one of the points was addressed in the activity.

Quality: For each discussion activity where NS2 is addressed, the depth of the discussion is assessed with a value from 0 to 1 as shown in the table.

Value	Description	Example
0	The discussion is only about how to know whether the pupils are doing things right or wrong, with no focus on pupils' understanding/thinking, such	*If you use whiteboards, you can see how many are answering correctly.*

	as focusing on factual/procedural questions with no follow-up questions that challenge/develop	
0.5	The discussion touches on different types of understanding/thinking in pupils, such as focusing on questions for pupils where they are challenged/need to develop their answers	*But how do you know whether pupils have actually applied the right thought pattern when everyone just writes answers on their whiteboards?*
1	The discussion concerns pupils' knowledge in relation to a holistic perspective on knowledge in mathematics, such as in relation to governing documents or other literature	*If you listen to pupils discussing in groups, you can often hear whether they make connections to several of the knowledge requirements in the syllabus.*

Assessment: NS3 – The pupils receive effective feedback that moves them forward in their learning.

Existence: Indicate whether the discussion touches on how to work with effective feedback.

No	NS3 is not addressed in the activity.
Yes	NS3 is addressed in the activity.

Quality: For each discussion activity where NS3 is addressed, the depth of the discussion is assessed with a value from 0 to 1 as shown in the table.

Value	Description	Example
0	The discussion touches on how pupils can receive feedforward, but without	*You have to point out to the pupils what is wrong so that they fix the next task.*

	processing: feedback is aimed at the next activity	
0.5	The discussion concerns feedforward aimed at learning goals	*You have to point out to the pupils what they are actually supposed to learn from the tasks.*
1	The discussion concerns feedforward, directed towards the learning goals, and space to process this feedback (e.g. by processing answers)	*Once the pupils know what they are supposed to learn and how the activity they are working on relates to this, they need time to think about how to achieve the goals.*

Assessment: NS4 – The pupils are activated as resources for each other in the learning.

Existence: Indicate whether the discussion touches on how to work with pupil-pupil assessment or pupil groups who are given the opportunity to work together on questions

No	NS4 is not addressed in the activity.
Yes	NS4 is addressed in the activity.

Quality: For each discussion activity where NS4 is addressed, the depth of the discussion is assessed with a value from 0 to 1 as shown in the table.

Value	Description	Example
0	The discussion contains nothing other than the mention of the activity.	*You need to let the pupils assess each other's work sometimes.*
0.5	The discussion touches on how the activity is done.	*When the pupils assess each other's work, you can have them use two stars and a wish.*
1	The discussion touches somewhat on the benefits	*When the pupils are allowed to discuss each other's work in a*

	of the activity, e.g. how the activity can help the pupils become better resources for each other.	*group setting, they can learn from others in the group what is constructive when commenting on task solutions.*

Assessment: NS5 – The pupils are activated as owners of their own learning.

Existence: Indicate whether the discussion touches on how to get the pupils to reflect on and assess the qualities and abilities of their knowledge, e.g. by having them

- formulate feedback/feedup/feedforward
- keep a log
- focus on change in their knowledge
- have access to each other's work and reasoning and give feedback on it

No	NS5 is not addressed in the activity.
Yes	NS5 is addressed in the activity.

Quality: For each discussion activity where NS5 is addressed, the depth of the discussion is assessed with a value from 0 to 1 as shown in the table.

Value	Description	Example
0	The discussion concerns only one of the activities in the checklist. The discussion does not address how the activity can be used as part of the work with follow-up and planning of teaching	*My pupils keep a log, where they write down what tasks they have done.*
0.5	The discussion concerns several of the activities in the checklist, but without addressing how the activity	*I have tested activities where pupils practice giving feedback to themselves, with*

	can be used as part of the work with follow-up and planning of teaching	*a focus on what happened in their own knowledge.*
1	The discussion addresses how activities can be used as part of the work with follow-up and planning of teaching as well as planning of future teaching, based on individual or multiple activities	*The pupils' self-assessment is a good resource to see what the teaching should focus on*

Appendix H: Operationalisation of didactical perspective 3 – Routines and interactions

This appendix shows how the didactical perspective *routines and interactions* was operationalised. The appendix presents the assessment guidelines that were used regarding the routines and interactions that permeate teachers' teaching, based on interview responses from teachers, classroom observations, and observations of collegial discussions. The interview questions can be found in Appendix B.

Analysis of teacher interview

The aim is to examine the routines and interactions that permeate a classroom based on three key elements:

1. Overarching interaction patterns: monologic – dialogic.
2. Teachers' questions and prompts in terms of type of answer:
    a. Short answers that can be directly evaluated with right or wrong.
    b. Questions that are not answered or not expected to be answered
    c. Guidance or support
    d. Cognitive questions
    e. Metacognitive questions
3. Whole-class discussions of solutions based on
    a. Whether the teacher notes what types of mathematical thinking the pupils use
    b. How the teacher selects which solutions to present
    c. How the teacher comments on connections between different pupil solutions

Assessment: Dialogic approach in different instructional settings.
For each of the four instructional settings(teacher-led review, whole-class discussion, group work and individual work), assess the teacher's description of the considerations made (question 7) with regard to the dialogic approach.

Question 7. This table is used to assess the answers to each sub-question.

Value	Description	Example
- 1	Rejects dialogic approach based on transmission view of learning	Pupils cannot learn in group work because there are only pupils with the same level of knowledge.
- 0.5	Generally rejects dialogic approach	It is difficult to create learning in whole-class discussions.
0	Neutral or unclear	
0.5	Generally favours dialogic approach by emphasising the importance of the pupils' voices, communication and/or activity	Group work makes pupils learn better since they get to discuss.
1	Favours dialogic approach more deeply based on pupils' learning.	In whole-class discussions, pupils' thoughts can be challenged in a way that supports their learning.

Assessment: Nuanced approach.
Assess how the teacher's description has a nuanced approach as shown in the table.
Question 7. Assess the answers using the table below.

- One-sided approach: *Either* almost exclusively favours dialogic and rejects monologic *or* almost exclusively favours monologic and rejects dialogic.
- Nuanced approach: Not one-sided approach.
- In relation to pupil learning: Relevant connections are made to the pupils' learning.
- Generally: No relevant connections are made to the pupils' learning.

Value	Description
- 1	One-sided approach in relation to pupil learning.
- 0.5	Generally one-sided approach.
0	Neutral or unclear.
0.5	Generally nuanced approach.
1	Nuanced approach in relation to pupil learning.

Assessment: Dialogic approach in different instructional settings.
Question 8. Use the same table as for the same assessment aspect for question 7 (above).

Assessment: Nuanced approach.
Question 8. Use the same table as for the same assessment aspect for question 7 (above).

Assessment: Use of nuanced, dialogic interaction patterns.
Question 8. Assess to what extent the teacher's use of different instructional settings is nuanced, based on the table below.

- Monologic instructional settings: Teacher-led review and individual work.
- Dialogic instructional settings: Discussions and group work.

Value	Description
- 1	One-sided interaction pattern with almost exclusively monologic instructional settings.
- 0.5	One-sided interaction pattern with almost exclusively dialogic instructional settings.
0.5	Nuanced interaction pattern with mostly monologic instructional settings
1	Nuance interaction pattern with mostly dialogic instructional settings or balance between monologic and dialogic instructional settings.

Assessment: Alignment with each of the five strategies for whole-class discussions. If the teacher's description touches on multiple levels of the table below, choose the one that is dominant in the teacher's response. Choose neutral if they are each described about the same.

1. Anticipating pupils' mathematical responses: Focus on various types of mathematically-relevant approaches and solutions, and how these relate to what the teacher wants the pupils to learn.
2. Monitoring pupil responses: Note which types of mathematical thinking pupils use, in relation to what the teacher wants the pupils to learn.
3. Selecting pupil responses for public display – Purposefully: Choose examples of approaches and solutions among pupils that are most relevant to the learning and mathematical content at hand and can best be used in discussions to provide the best possible learning in the classroom.
4. Sequencing pupil responses – Purposefully: Choose what order to present the different examples to maximise the chances that the desired learning will occur.
5. Connecting pupil responses: Compare what different or same mathematical ideas exist in different solutions, and what possibilities or limitations different solution methods have.

Question 9. Use the table below for each of the five strategies.

Value	Description	Example
- 1	Goes against the principle of the strategy.	I leave the pupils alone while they are working on the task (Strategy 2).
0	Neutral or unclear	
0.5	Generally favours the strategy (or part of the strategy)	I try to think through in advance what solutions the pupils might come up with (Strategy 1).
1	Favours the essence and the whole of the strategy in a more detailed way.	I choose solutions to discuss that address common misunderstandings as well as an approach that I want to highlight as desirable (Strategy 3).

Analysis of classroom observation

Assessment: Balance between monologic/dialogic interaction patterns.
Divide each lesson into parts according to the instructional setting described in section 2.5. Monologic interaction pattern includes the instructional settingsTeacher monologue (HL), Pupil monologue (HE) and Individual. Dialogic interaction pattern includes the instructional settingsDialogue (HD), Everyone can participate (HA) and Group (G). Based on the proportion of lesson time using a monologic or dialogic interaction pattern, assess the balance based on the table below:

Proportion of monologic interaction pattern (%)	Value
90-100	-1
0-10	-0.75
80-90	-0.5
10-20	-0.25
65-80	0.25
20-35	0.5
50-65	0.75
35-50	1

Assessment: Teacher's questions.
Questions: A *question* is when the teacher asks the pupil(s) something or prompts the pupil(s) to do something. Only questions/prompts that focus on mathematics should be included. For example, questions of the type "what class do you have after this one" or prompts of the type "open your book to page 23" are excluded. Assess up to 30 questions from each lesson.

For each question, identify the type of question using the table below:

Type	**Requirement for a question to be classified as this type** (Note: The same question can be classified as multiple types)

No answer	The teacher does not give the pupil an opportunity to answer the question, e.g. by giving an answer directly or by immediately continuing on and talking (about something else) after asking the question.
Short answer	The pupil is invited to give a short answer, e.g. yes/no or single word/phrase.
Thinking	The pupil is invited to describe, develop or clarify their thinking in some way, e.g. "how did you get that answer".
Argument	The pupil is invited to reflect on or argue for claims/statements (their own or someone else's), e.g. why questions or "how do you know that...".

Determine a value by calculating the proportion of dialogic questions (Thinking or Argument) in relation to the total number of questions.

Analysis of collegial discussion

Assess each part of the discussion dealing with interaction patterns, question patterns or whole-class discussions as follows:
Assessment: Dialogic approach.

If multiple levels in the table can be identified, choose the one that is dominant in the analysed part of the discussion. Choose *neutral* if they are each favoured about the same. If the *monologic* approach is rejected or favoured, this is considered neutral in the table.

Value	Description	Example
- 1	Rejects dialogic approach based on transmission view of learning	Pupils cannot learn in group work because there are only pupils with the same level of knowledge.
- 0.5	Generally rejects dialogic approach	It is difficult to create learning in whole-class discussions.
0	Neutral or unclear	

0.5	Generally favours dialogic approach by emphasising the importance of the pupils' voices, communication and/or activity	Group work makes pupils learn better since they get to discuss.
1	Favours dialogic approach more deeply based on pupils' learning.	In whole-class discussions, pupils' thoughts can be challenged in a way that supports their learning.

Assessment: Nuanced approach.
Assess in what way the analysed part of the discussion has a nuanced approach based on the table. If multiple levels apply to the discussion, choose the one that is dominant or choose *neutral* if they each apply about the same. Assess the answers based on the table below using the following descriptions:

- One-sided approach: *Either* almost exclusively favours dialogic and rejects monologic *or* almost exclusively favours monologic and rejects dialogic.
- Nuanced approach: Not a one-sided approach.
- In relation to pupil learning: Relevant connections are made to the pupils' learning.
- Generally: No relevant connections are made to the pupils' learning.

Value	Description
- 1	One-sided approach in relation to pupil learning.
- 0.5	Generally one-sided approach.
0	Neutral or unclear.
0.5	Generally nuanced approach.
1	Nuanced approach in relation to pupil learning.

Assessment: Focus on dialogues.
If the focus in the discussion touches on multiple levels of the table below, choose the one that is dominant in the discussion, or choose neutral if they each apply about the same.

-1	Favours aspects of IRE interaction (short pupil responses evaluated as right/wrong)	Quick yes/no questions are most effective for knowing whether pupils understand or not
0	Neutral/unclear	
+1	Favours open-ended questions to pupils; explore thinking and encourage argumentation	Ask pupils to develop their thinking and then build on it in the desired direction

Assessment: Strategies for whole-class discussions.
Alignment with each of the five strategies for whole-class discussions. If the teacher's description touches on multiple levels of the table below, choose the one that is dominant in the teacher's response. Choose neutral if they are each described about the same.

1. Anticipating pupils' mathematical responses: Focus on various types of mathematically-relevant approaches and solutions, and how these relate to what the teacher wants the pupils to learn.
2. Monitoring pupil responses: Note which types of mathematical thinking pupils use, in relation to what the teacher wants the pupils to learn.
3. Selecting pupil responses for public display – Purposefully: Choose examples of approaches and solutions among pupils that are most relevant to the learning and mathematical content at hand and can best be used in discussions to provide the best possible learning in the classroom.
4. Sequencing pupil responses – Purposefully: Choose what order to present the different examples to maximise the chances that the desired learning will occur.
5. Connecting pupil responses: Compare what different or same mathematical ideas exist in different solutions, and what possibilities or limitations different solution methods have.

Use the table below for each of the five strategies.

Value	Description	Example
- 1	Goes against the principle of the strategy.	I leave the pupils alone while they are working on the task (Strategy 2).
0	Neutral or unclear	
0.5	Generally favours the strategy (or part of the strategy)	I try to think through in advance what solutions the pupils might come up with (Strategy 1).
1	Favours the essence and the whole of the strategy in a more detailed way.	I choose solutions to discuss that address common misunderstandings as well as an approach that I want to highlight as desirable (Strategy 3).

Appendix I: Operationalisation of didactical perspective 4 – Classroom norms and sociomathematical norms

This appendix shows how the didactical perspective *classroom norms/sociomathematical norms* was operationalised. The appendix presents the assessment guidelines that were used regarding classroom norms and sociomathematical norms in teaching, based on interview responses from teachers, classroom observations, and observations of collegial discussions. The interview questions can be found in Appendix B.

Analysis of teacher interview

Assessment: Reference to pupils' views of mathematics
Questions 2, 7, 9 and 11

Value	Description	*Example*
0	Pupils' views are never mentioned	
0.5	Pupils' views are mentioned, but without explicit link to teaching.	*Pupils often think this way. It can be problematic. Would be better if they thought…*
1	Explicit link between pupils' views and planning of or reflection on teaching	*Pupils think – that's why I organise the lesson…* *If we organise the lesson like this, the pupils may start to see math as…*

Assessment: Who or what decides whether something is correct or acceptable in mathematics.
Questions 10 and 18.

Value	Description	*Example*
-1	Something outside the pupils' control	*The teacher, the textbook key, some expert (like "mathematician").*

0	Neutral or unclear	
1	Something within the pupils' control	*The pupil, including the pupil in interaction with others*

Analysis of classroom observation

The classroom analysis focuses primarily on the teacher's response about what is correct or acceptable in the teacher's discussions with pupils (we do not analyse the pupils' statements).

Evaluations: An e*valuation* is when the teacher explicitly states whether something is correct/okay or not in the subject of mathematics, i.e. when the teacher makes a clear evaluation as to whether something is correct/okay or not, and this evaluation concerns the subject of mathematics. It is therefore not enough for the teacher to just say yes/no. There must be an explicit and clear evaluation regarding the subject of mathematics. Up to 15 evaluations per lesson are assessed.

Assessment: The teacher's evaluations – who decides whether something is correct or not? Acceptable or not?

Value	Description	*Example*
-1	Something outside the pupils' control	*The teacher, the textbook key, some expert (like "mathematician").*
0	Neutral/unclear	
+1	Something within the pupils' control	*The pupil, including the pupil in interaction with others.*

Assessment: The teacher's evaluations – how is it decided what is correct or acceptable?

Value	Description	*Example*
-1	Only defined	*What does the answer key say? Okay, then it was wrong.*

		You (the pupil) know that is the case.
0	Neutral/unclear	
+0.5	General reference to thinking/activity of some kind	*Mathematicians have shown that…* *If you think about it, you will see that…* *Flawed argument: "You can only do that if…"*
+1	Logical argument/reasoning	*No, since x=2, I (the teacher) see that…* *Exactly, since it was like this, you can determine…*

A value is calculated by first averaging all instances, separately for the two assessments, and then calculating the average value of these averages.

Analysis of collegial discussion

Assessment: The pupils' views of mathematics are addressed in the discussion.
Existence: Indicate with "Yes" or "No" whether pupils' views of mathematics are addressed in the discussion, i.e. if the pupils' opinions or perceptions of mathematics are addressed.
Value: For each part of the discussion where existence is assessed with "Yes", the value is determined based on the table.

Value	Description	*Example*
0	The pupils' views on mathematics are touched on, but it is not a central part of the discussion.	*Pupils often think this way. It can be problematic.*

0.5	The pupils' views of mathematics are discussed, but without explicit link to planning of or reflections on teaching	*Pupils often think this way. It can be problematic. It would be better if they thought like this…*
1	There is an explicit link between pupils' views of the subject of mathematics and planning of or reflection on teaching	*Pupils think… – that's why I organise the lesson…* *If we organise the lesson like this, the pupils may start to see math as…*

Appendix J: Analysis of documents

This appendix describes the assessments that were carried out to clarify what support is available (locally) for the teachers'/principals' work with teaching development and professional development programs (PDP).

Assess the existence of plans for the development of mathematics teaching according to the table below, i.e. whether there is any type of plan for what the teaching should achieve or what form the teaching should take.

0	No plans exist; this includes if plans only exist specifically for subjects other than mathematics.
+0.5	Plans generally exist for all teaching, but not specifically for mathematics.
+1	Plans exist specifically for mathematics.

The degree of specification of plans for mathematics teaching development is assessed with respect to:

- Objectives: Whether or not there is a stated "measurable" goal, i.e. some characteristic that can be used to gauge whether it is fulfilled or not. (0/1)
- Implementation: Whether concrete plans are given for how something will be implemented. (0/1)
- Timetable: Whether a specific point in time is given for something to be completed or achieved. (0/1)

Assessment: Extent and quality of the teaching culture in the documents**:** Calculate the average value of all four of the above assessments for each document. Then calculate the maximum value of all documents.

Existence of plans for **PDP of mathematics teachers** is assessed according to the table below.

0	No plans exist; this includes if plans only exist specifically for subjects other than mathematics.
+0.5	Plans generally exist for all teachers, but not specifically for mathematics.
+1	Plans exist specifically for mathematics.

The degree of specification of plans for PDP of mathematics teachers is assessed with respect to:

- Objectives: Whether or not there is a stated "measurable" goal, i.e. some characteristic that can be used to gauge whether it is fulfilled or not. (0/1)
- Implementation: Whether concrete plans are given for how something will be implemented. (0/1)
- Timetable: Whether a specific point in time is given for something to be completed or achieved. (0/1)

Assessment: Extent and quality of the PDP culture in the documents: Calculate the average value of all four of the above assessments for each document. Then calculate the maximum value of all documents.

Appendix K: Analysis of collegial discussion beyond the didactical perspectives

This appendix describes the assessments carried out beyond the didactical perspectives in the analysis of collegial discussion observations. The variable related to the supervisor's experience is also described.

What characterises the discussion?

For each part of the discussion, count the number of occasions when different events occur. Record these in columns E to H in a separate document.

Column	Description	*Example*
E	Someone makes an argument (i.e. not just an opinion) against a previous statement by a colleague or against a text that was read.	*I do not agree with X because…* *But if it is like this… it is actually more likely that…*
F	Someone changes their opinion based on a statement of a colleague or the content of a text that was read.	*I now think this way instead…* *That seemed to be a better way to look at it…*
G	Someone asks a question to deepen/clarify a previous statement made by a colleague or text that was read, i.e. not questions simply asking for a description of e.g. a chain of events or opinion, but rather questions that build on or delve deeper into something that was said or read.	*How can you think that this is related to this…* *When you tested it in your teaching, how did the pupils react to…*

H	Someone uses research findings or proven experience to argue for a particular position.	*So many pupils I taught have… so therefore…* *The fact that a particular teaching method has proved to be effective means that…*

Supervisor's experience

These values are taken from information about participation in the Boost for Mathematics.

Supervisor's background: Experience
Teaching experience in mathematics (number of years):

0	3 years or less
0.25	4–9 years
0.5	10–15 years
0.75	16–21 years
1	22 years or more

Whether mainly taught in the grade levels in which supervision is now taking place

1	If the grade levels that the supervisor mainly taught are within the school level indicated in the form for the collegial discussion
0	If the above is not met

Previous supervision experience
1: Yes
0: No

Assessment: Norm each individual assessment above to the interval 0–1 and then calculate the average value of all assessments.

Appendix L: Analysis of teacher interviews beyond the didactical perspectives

This appendix describes the assessments carried out beyond the didactical perspectives in the analysis of teacher interviews. The variable related to the supervisor's experience is also described.
The interview questions referenced in the text can be found in Appendix B.

Assessment: The Boost for Mathematics' organisation in practice. Question 23.

Value	Description	Example
-1	Clear and exclusively negative assessment of the Boost for Mathematics.	I am doubtful. There has been a lot of time-consuming work.
-0.5	Predominantly negative assessment of the Boost for Mathematics.	Good to discuss teaching, but we already do this. It has been too extensive.
0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	It has given me a lot of new knowledge, but it is difficult to incorporate into the classroom.
0.5	Predominantly positive assessment of the Boost for Mathematics.	It has been good to have collegial discussions and to try new things in the classroom, even if it has taken time away from other things.
1	Clear and exclusively positive assessment of the Boost for Mathematics.	It has been very good in every way.

Assessment: Quality of teachers' existing learning environment. Question 19.

Value	Description	Example
-1	Clear and exclusively negative assessment of the learning environment.	There is virtually no support for us to develop.
-0.5	Predominantly negative assessment of the learning environment.	We receive professional development on an ongoing basis, but it is almost always short and meaningless.
0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	We have had several good lecturers here, but it would be good to have longer projects.
0.5	Predominantly positive assessment of the learning environment.	It is a bit hard to keep up, but the principal and my colleagues provide good support.
1	Clear and exclusively positive assessment of the learning environment.	It is very good in every way.

Assessment: The role of the principal and school organiser for the learning environment.
Question 19. The role of the principal and the role of the school organiser are assessed separately according to the table.

Value	Description	Example
0	Considered to have a passive role, incl. cases where it is unclear what the role is, or only "don't know" type answers are given.	**Principal** leaves everything to us to plan. I have never seen anything from the **school organiser** for our professional development.

0.5	Considered to have a somewhat active role, but unclear what this entails.	**The principal/municipality** is important to give us the right conditions for development.
1	Considered to have a clearly active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	**The principal** holds development dialogues each year, where professional development is addressed. **The municipality** has invested money in bringing in lecturers from the university.

Assessment: Existence of documents for professional development. Question 22. Each level (teacher team, school, municipality/ corporation) is assessed separately according to the table.

Value	Description
0	Documents as listed below do not exist, incl. cases of "don't know" response or if there is only an unclear reference to a document without being able to describe this document.
1	Some sort of work plan, policy document or similar document exists regarding professional development.

Assessment: Proposed support for development as a teacher. Question 23. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed support is linked to personal learning, i.e. concerns the development of personal knowledge or abilities.	I need to take courses to learn about...

1	Proposed support involves requirements for action by others, i.e. the principal, school organiser or another party needs to implement or arrange something.	Reorganisation is needed to create the conditions for good professional development.
1	Proposed support is linked to resources, i.e. teaching materials, computers or the like.	We need more lecturers who can inspire us.

Assessment: Proposed organisation of collegial learning.
Question 24. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed organisation is linked to the teacher's own development, i.e. concerns the development of personal knowledge or abilities.	Working together to use each other's knowledge to learn new things.
1	Proposed organisation is linked to action by the principal, i.e. principal's role is described rather than stating "the role is important/large".	The principal should have a long-term strategy for the whole school.
1	Proposed organisation is linked to perceived obstacles, i.e. obstacles are stated and linked to something in the described ideal.	Money is needed to be able to use substitutes so we can have collegial meetings.
1	Proposed organisation is linked to supervision, i.e. an outside resource in the collegial work is part of the proposal.	We need input from outside the school so a fresh pair of eyes looks at things.

Assessment: Quality of the school's work with the development of mathematics teaching.
Question 20.

Value	Description	Example
-1	Clear and exclusively negative assessment of the school's development work.	Nothing is done. We just carry on as usual.
-0.5	Predominantly negative assessment of the school's development work.	New equipment is purchased, but it is not used. We do not have time.
0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	There is no clear strategy, but we use results from national tests productively.
0.5	Predominantly positive assessment of the school's development work.	Teaching is developed collegially, but more time is desired for this.
1	Clear and exclusively positive assessment of the school's development work.	We invest a lot into this in terms of both time and money.

Assessment: The role of the principal and school organiser in the development of mathematics teaching.
Question 20. The role of the principal and the role of the school organiser are assessed separately according to the table.

Value	Description	Example
0	Considered to have a passive role, incl. cases where it is unclear what the role is, or only "don't know" type answers are given.	**Principal** leaves everything to us to develop. I have never seen anything concrete from the **school organiser** for teaching.

0.5	Considered to have a somewhat active role, but unclear what this entails.	**The principal/municipality** is important to give us the right conditions for development.
1	Considered to have a clearly active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	Each year, **the principal** focuses on an area or subject to develop. **The municipality** has purchased better teaching materials.

Assessment: Existence of different approaches in the school's teaching development.

Question 20. For each of the three aspects, give a value of 1 if this aspect is indicated as existing, and 0 if it is not:

Value	Description	Example
1	Time is allocated for development work.	Each teacher has one hour per week.
1	There are groups for development work, such as some form of work team.	All math teachers meet every other week.
1	External resources exist for development work, such as lecturers, maths coaches, etc.	A researcher is invited each semester for inspiration.

Assessment: Existence of documents for mathematics teaching.

Question 21. Each level (teacher team, school, municipality/ corporation) is assessed separately according to the table.

Value	Description
0	Documents as listed below do not exist, incl. cases of "don't know" response or if there is only an unclear reference to a document without being able to describe this document.

1	Some sort of work plan, policy document or similar document exists regarding mathematics teaching.

Assessment: Proposed support for teaching development.
Question 23. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed support is linked to personal learning, i.e. concerns the development of personal knowledge or abilities.	I need to be able to create better tasks to enable pupils to develop their abilities more broadly.
1	Proposed support involves requirements for action by others, i.e. the principal, school organiser or another party needs to implement or arrange something.	The principal should have a clearer strategy on which areas or subjects to focus on.
1	Proposed support is linked to resources, i.e. teaching materials, computers or the like.	We need better computers to be able to discuss forms of expressions better.

Assessment: Impact on teaching based on proposed organisation of collegial learning.
Question 24. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	The impact on teaching is linked to the teacher's own development.	I need support to be able to create better tasks to enable pupils to develop their abilities more broadly.

1	The impact on teaching is linked to actions of the principal.	The principal needs to give us more time to plan our teaching together, so that there is more variety in the classrooms.
1	The impact on teaching is linked to perceived obstacles.	There are too few of us teachers, but if there were more of us, we could collaborate in planning for more thoughtful lesson planning.
1	The impact on teaching is linked to supervision.	An outsider can provide suggestions for new ways of working in teaching.

Appendix M: Analysis of interview with principal

This appendix describes the assessments carried out on the interviews with principals. The interview questions referenced in the text can be found in Appendix B.

Assessment: Description of goals for the Boost for Mathematics. Question 9. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Objective includes teacher learning, i.e. something about the development of teachers' abilities.	Increased subject-specific didactical ability of teachers.
1	Objective includes development of teaching, i.e. something about the improvement of teaching itself or pupils' learning.	All pupils should be considered based on their own merits.
1	Objective includes the role of the principal, i.e. something about the development of the principal's abilities or improved working methods.	I will be given tools to work more closely with the teachers.
1	Objective includes local conditions, i.e. something about specific characteristics or problems at their own school.	Teachers are spread out in different locations, so we want to work more closely together.

Assessment: The role of the principal in the Boost for Mathematics. Question 10.

Value	Description	Example

0	Described as having a passive role, incl. cases where it is unclear what the role is, or only "don't know" type of answers.	I leave most things to the teachers.
0.5	Considered to have a somewhat active role, but unclear what this entails.	I meet with the teachers continuously and discuss things.
1	Described as having a clearly active role, i.e. if something is described regarding what this role is, not just the statement "has been very involved".	I have had meetings with the teacher team before we started the Boost for Mathematics to go over the goals.

Assessment: The Boost for Mathematics as support.
Question 3.

Value	Description	Example
-1	Clear and exclusively negative assessment of the Boost for Mathematics.	I am doubtful. There has been a lot of time-consuming work.
-0.5	Predominantly negative assessment of the Boost for Mathematics.	Good to discuss teaching, but we already do this. It has been too extensive.
0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	It has given me a lot of new knowledge, but it is difficult to incorporate into the classroom.
0.5	Predominantly positive assessment of the Boost for Mathematics.	It has been good to have collegial discussions and for teachers to try new things in the classroom, even if it has taken time away from other things.

1	Clear and exclusively positive assessment of the Boost for Mathematics.	It has been very good in every way.

Assessment: Quality of teachers' existing learning environment.
Question 1.

Value	Description	Example
-1	Clear and exclusively negative assessment of the learning environment.	We are trying, but it is hard to find time for professional development.
-0.5	Predominantly negative assessment of the learning environment.	We have professional development, but it is often short with a risk of being fragmented.
0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	We have had several good lecturers here, but it would be good to have longer projects.
0.5	Predominantly positive assessment of the learning environment.	It is a bit hard to keep up, but we focus on one area to develop each year.
1	Clear and exclusively positive assessment of the learning environment.	It is very good in every way.

Assessment: The role of the principal and school organiser in the learning environment.
Question 1. The role of the principal and the role of the school organiser are assessed separately according to the table.

Value	Description	Example
0	Considered to have a passive role, incl. cases where it is	**I** leave a lot to the teachers.

	unclear what the role is, or only "don't know" type answers are given.	We have never received anything from the **school organiser** for professional development.
0.5	Considered to have a somewhat active role, but unclear what this entails.	**I** am/**the municipality** is important to give the teachers the right conditions for development.
1	Considered to have a clearly active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	**I** hold development dialogues each year, where professional development is addressed. **The municipality** has invested money in bringing in lecturers from the university.

Assessment: Role of principal, school organiser and teachers for the teachers' own learning.
Question 4. The roles of the principal, the school organiser and the teachers are assessed separately according to the table.

Value	Description	Example
0	Considered to have a passive role, incl. cases where the role is not described at all, or only "don't know" type answers are given.	**I** leave a lot to the teacher team. We have no support from the **school organiser** for professional development. **The teachers** are informed about initiatives.
0.5	Considered to have a somewhat active role, but unclear what this entails.	**I** am/**the municipality** is/**the teachers** are important to create the right conditions for development.

1	Considered to have a clearly active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	**I** hold development dialogues each year, where the need for professional development is addressed. **The municipality** has started a programme to document competence development. **The teachers** get to assess their own needs.

Assessment: Characteristics of the school's work with teachers' own learning.
Question 4. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Part of quality work and is reported, i.e. goals are documented and followed up on.	We report our competence development needs to the municipality each year.
1	Follow-up takes place at development dialogues, i.e. competence development is discussed at that time.	I hold development dialogues each year, where the need for professional development is addressed.
1	Written competence development plans exist, i.e. something is documented for individuals.	The previous year's professional development plan is followed up on at the performance review.

Assessment: Existence of documents for professional development
Question 6. Each level (teacher team, school, municipality/ corporation) is assessed separately according to the table.

Value	Description

0	Documents as listed below do not exist, incl. cases of "don't know" response or if there is only an unclear reference to a document without being able to describe this document.
1	Some sort of work plan, policy document or similar document regarding professional development exists.

Assessment: Control from the school organiser in relation to professional development.
Question 7. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	The school organiser controls through resource allocation, i.e. money is allocated to the school for specific activities.	We receive funding for professional development based on how well we used last year's funding.
1	The school organiser actively controls the direction, i.e. specifies the type of activities to be carried out.	The municipality has decided that all teachers shall take at least one university course within two years.
1	The school organiser actively controls content, i.e. specifies what is to be developed.	Formative assessment is an initiative at all schools in the municipality.

Assessment: Plans for continued work with PDP after the Boost for Mathematics.
Question 11. How detailed these plans are.

Value	Description	Example
0	No plans, incl. cases that only indicate a willingness to continue, or only "don't know" type answers.	It would be good to be able to continue with the same approach.

0.5	Superficial plans, i.e. there is a plan, but it is not yet complete.	We have received money from the municipality to be able to build on the Boost for Mathematics.
1	Detailed plans, i.e. more advanced or completed plans exist.	We will work with module X at half speed.

Assessment: Plans for continued work with PDP after the Boost for Mathematics.
Question 11. Role of the principal in these plans.

Value	Description	Example
0	Considered to have a passive role, incl. cases where the role is not described at all, or only "don't know" type answers are given.	I play an important role, but the teachers themselves plan how the work is organised.
1	Considered to have an active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	Together with the teachers, I chose the module to use that can give us the best effect.

Assessment: Proposed support for development as a teacher.
Question 3. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed support is linked to teachers' learning, i.e. concerns the development of the teachers' personal knowledge or abilities.	The teachers need to take courses to learn about...

1	Proposed support is linked to the actions by the principal, i.e. the principal needs to do or organise something.	I need to plan some reorganisation to create conditions for good professional development.
1	Proposed support is linked to resources, i.e. teaching materials, computers or the like.	We need more lecturers who can inspire us.

Assessment: Proposed organisation of collegial learning.
Question 8. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed organisation is linked to the teachers' personal development, i.e. concerns the development of the teachers' personal knowledge or abilities.	The teachers work together to use each other's knowledge to learn new things.
1	Proposed organisation is linked to actions by the principal, i.e. principal's role is described rather than stating "the role is important/large".	I need to have a long-term strategy for the whole school.
1	Proposed organisation is linked to perceived obstacles, i.e. obstacles are stated and linked to something in the described ideal.	Money is needed to be able to use substitutes so we can have collegial meetings.
1	Proposed organisation is linked to supervision, i.e. an outside resource in the collegial work is part of the proposal.	We need input from outside the school so a fresh pair of eyes looks at things.

Assessment: Proposed support for the principal's development as a pedagogical leader.
Question 13. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed support is linked to the principal's development, i.e. concerns the development of personal knowledge or abilities.	I need to take courses to be better at…
1	Proposed support is linked to actions by the school organiser, i.e. the school organiser's role is described rather than stating "the role is important/large".	The municipality needs to organise exchanges between principals of different schools.
1	Proposed support is linked to resources, i.e. money, equipment or the like.	Our facilities would need to be adapted to…
1	Proposed support is linked to administrative support, i.e. administrative staff or administrative systems or tools of some type.	Our administrators would need to get more involved in documentation and follow-up of PDP initiatives.

Assessment: Quality of the school's work with the development of mathematics teaching.
Question 2.

Value	Description	Example
-1	Clear and exclusively negative assessment of the school's development work.	Nothing special is done. We carry on as usual.
-0.5	Predominantly negative assessment of the school's development work.	New equipment is purchased, but it is not used. The teachers do not have time.

0	Highlights both negative and positive aspects, which are given roughly equal weight/space.	There is no clear strategy, but we use results from national tests productively.
0.5	Predominantly positive assessment of the school's development work.	Teaching is developed collegially, but more time is needed for this.
1	Clear and exclusively positive assessment of the school's development work.	We invest a lot into this in terms of both time and money.

Assessment: The role of the principal and school organiser in the development work for mathematics teaching.
Question 2. The role of the principal and the role of the school organiser are assessed separately according to the table.

Value	Description	Example
0	Considered to have a passive role, incl. cases where it is unclear what the role is, or only "don't know" type answers are given.	**I** leave a lot to the teachers. I have never seen anything concrete from the **school organiser** for teaching.
1	Considered to have an active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	Each year, **I** focus on an area or subject to develop. **The municipality** has purchased better teaching materials.

Assessment: Existence of different approaches in the school's teaching development work.
Question 2. For each of the three aspects, give a value of 1 if this aspect is indicated as existing, and 0 if it is not:

Value	Description	Example

1	Time is allocated for development work.	Each teacher has one hour per week.
1	There are groups for development work, such as some form of work team.	All math teachers meet every other week.
1	External resources exist for development work, such as lecturers, maths coaches, etc.	A researcher is invited each semester for inspiration.

Assessment: Existence of documents for mathematics teaching.
Question 5. Each level (teacher team, school, municipality/corporation) is assessed separately according to the table.

Value	Description
0	Documents as listed below do not exist, incl. cases of "don't know" response or if there is only an unclear reference to a document without being able to describe this document.
1	Some sort of work plan, policy document or similar document exists regarding mathematics teaching.

Assessment: Plans for continued work to develop mathematics teaching after the Boost for Mathematics.
Question 12. How detailed these plans are.

Value	Description	Example
0	No plans, incl. cases where only indicate a willingness to continue, or only "don't know" type answers.	It would be good to be able to continue with the same approach.
0.5	Superficial plans, i.e. there is a plan, but it is not yet complete.	We have received money from the municipality to be able to build on the Boost for Mathematics.

1	Detailed plans, i.e. more advanced or completed plans exist.	We will work with module X at half speed.

Assessment: Plans for continued work to develop mathematics teaching after the Boost for Mathematics.
Question 12. Role of the principal in these plans.

Value	Description	Example
0	Considered to have a passive role, incl. cases where the role is not described at all, or only "don't know" type answers are given.	I play an important role, but the teachers themselves plan how the work is organised.
1	Considered to have an active role, i.e. if something is described regarding what this role is, not just the statement that "the role is important/large".	Together with the teachers, I chose the module to use that can give us the best effect.

Assessment: Proposed support for development of teaching.
Question 3. For each of the three aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	Proposed support is linked to teachers' learning, i.e. concerns the development of the teachers' personal knowledge or abilities.	Teachers need to be able to create teaching that develops pupils' abilities more broadly.
1	Proposed support is linked to the actions by the principal, i.e. the principal needs to do or organise something.	I need to have a clear strategy on which areas or subjects to focus on.

1	Proposed support is linked to resources, i.e. teaching materials, computers or the like.	We need better computers to use in teaching.

Assessment: Impact on teaching based on proposed organisation of collegial learning.
Question 8. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example
1	The impact on teaching is linked to the teachers' development.	Teachers need to work together to be able to create teaching that develops pupils' abilities more broadly.
1	The impact on teaching is linked to actions of the principal.	I need to organise more time for teachers to jointly plan teaching, so there is more variation in the classrooms.
1	The impact on teaching is linked to perceived obstacles.	We have too few teachers, but if there were more, they could collaborate more in planning for more thoughtful lesson planning.
1	The impact on teaching is linked to supervision.	An outsider can provide suggestions for new ways of working in teaching.

Assessment: Proposed support for the principal's development as a pedagogical leader.
Question 13. For each of the four aspects, give a value of 1 if the following is fulfilled, and 0 if it is not:

Value	Description	Example

1	Proposed support is linked to the principal's development, i.e. concerns the development of personal knowledge or abilities.	I need to take courses to be better at…
1	Proposed support is linked to actions by the school organiser, i.e. the school organiser's role is described rather than stating "the role is important/large".	The municipality needs to organise exchanges between principals of different schools.
1	Proposed support is linked to resources, i.e. money, equipment or the like.	Our facilities would need to be adapted to…
1	Proposed support is linked to administrative support, i.e. administrative staff or administrative systems or tools of some type.	Our administrators would need to get more involved in documentation and follow-up of pupils' results.

Appendix N: Survey questions for teachers

This appendix describes the content of the survey that was sent (electronically) to all teachers.

This survey will take max. 20 minutes to complete. Thank you for your participation!

Question 1. To what extent was each of the following involved in the decision to participate in the Boost for Mathematics?

	Very high	Quite high	Quite low	Very low	Don't know
School organiser					
Principal					
Teaching faculty					
Individual teachers					

Question 2. To what extent has the implementation of the Boost for Mathematics been adapted to local conditions?
[ ] Very high [ ] Quite high [ ] Quite low [ ] Very low [ ] Don't know

Question 3. How has the implementation of the Boost for Mathematics been adapted to local conditions?

Question 4. Was an assessment of the current situation carried out prior to the Boost for Mathematics?
[ ] Yes [ ] No [ ] Don't know

Question 5. If an assessment of the current situation was carried out: To what extent was each of the following involved in this assessment?

	Very high	Quite high	Quite low	Very low	Don't know
School organiser					
Principal					
Teaching faculty					
Individual teachers					

Question 6. Any comments or clarifications of your answers to the above questions

Question 7. How good or bad do you think the material in the Boost for Mathematics modules has been to work with?
[ ] Very good [ ] Pretty good [ ] Neither good nor bad [ ] Pretty bad [ ] Very bad [ ] Don't know

Question 8. Which compulsory school modules have you worked with so far?
[ ] Number sense and use of numbers
[ ] Algebra
[ ] Geometry
[ ] Probability and statistics
[ ] Correlations and change
[ ] Problem solving
[ ] Mathematics instruction with ICT
[ ] Language in mathematics

[ ] I have not worked with any of these modules
[ ] Don't know

Question 9. Which upper-secondary school modules have you worked with so far?
[ ] Teaching mathematics based on problem solving
[ ] Teaching mathematics based on abilities
[ ] Assessment for teaching and learning in mathematics
[ ] Teaching mathematics in vocational programmes
[ ] Mathematics instruction with ICT
[ ] Language in mathematics
[ ] Teaching mathematics in higher education preparatory programmes
[ ] I have not worked with any of these modules
[ ] Don't know

Question 10. When you worked with a module, did you work with all 8 elements of the module?
[ ] Yes [ ] No [ ] Don't know

Question 11. When we worked with a module…

	Always	Usually	Sometimes	Never	Don't know
…I worked with element A (individual preparation)					
…I worked with element B (collegial work)					
…I worked with element C (activity in the classroom)					
…I worked with element D (joint follow-up)					
…I was able to set aside 45–60 minutes for element A (individual preparation)					
…I was able to set aside 90–120 minutes for element B (collegial work)					
…I was able to set aside 45–60 minutes for element D (joint follow-up)					
…a supervisor was involved in the work with element B (collegial work)					
…a supervisor was involved in the work with element D (joint follow-up)					

Question 12. How good or bad do you think the arrangement with a supervisor has worked in the Boost for Mathematics?
[ ] Very good [ ] Pretty good [ ] Neither good nor bad [ ] Pretty bad [ ] Very bad [ ] Don't know

Question 13. Did the supervisor come from the same school as you, or a different school?
[ ] Same school [ ] Different school [ ] Don't know

Question 14. Any comments or clarifications of your answers to the above questions

Question 15. To what extent…

	Very high	Quite high	Quite low	Very low	Don't know
… has your principal demonstrated commitment to the Boost for Mathematics?					
… has your principal actively participated in the Boost for Mathematics's implementation?					
… has your principal contributed to increasing your commitment to the Boost for Mathematics?					

… has your principal justified the value of your school's participation in the Boost for Mathematics?					
… has the work of the Boost for Mathematics fit into your normal working week?					
… has the work of the Boost for Mathematics fit into your normal annual working time?					
… have substitutes been used to enable you to participate in the Boost for Mathematics?					
… has participation in the Boost for Mathematics required you to de-prioritise other work tasks?					

Question 16. Within the Boost for Mathematics, how many times…

	Number of times	Don't know
… have you visited your colleagues' mathematics lessons?		
… have your colleagues visited your mathematics lessons?		
… has your principal visited your mathematics lessons?		
… has your principal visited your collegial discussions?		

Question 17. How good or bad has your work with colleagues been during the Boost for Mathematics?
[ ] Very good [ ] Pretty good [ ] Neither good nor bad [ ] Pretty bad [ ] Very bad [ ] Don't know

Question 18. Have you participated in any other professional development programs or other development projects this school year, other than the Boost for Mathematics?
[ ] Yes [ ] No [ ] Don't know

Question 19. If you participated in any other professional development programs or other development projects: Briefly describe the project's focus.

Question 20. Any comments or clarifications of your answers to the above questions

Question 21. How close do you perceive that the cooperation is between your principal and the teachers concerning development of mathematics teaching?
[ ] Very close cooperation
[ ] Pretty close cooperation
[ ] No close cooperation
[ ] Don't know

Question 22. If you think about the cooperation between your principal and teachers in the development of mathematics teaching, how good or bad do you think this cooperation works?
[ ] Very good [ ] Pretty good [ ] Neither good nor bad [ ] Pretty bad [ ] Very bad [ ] Don't know

Question 23. This school year, how often have you discussed mathematics teaching collegially among teachers (outside of the Boost for Mathematics) by…
(Formal meetings are meetings with a specially designated meeting time, while informal meetings are meetings that are not formal, such as over lunch or a coffee break.)

	Often	A few times	Never	Don't know
… formal collegial meetings with mathematics teachers only				
… formal collegial meetings with teachers from different subjects				

… informal collegial meetings (regardless of teacher composition)				

Question 24. Last school year, how often have you discussed mathematics teaching collegially among teachers (outside of the Boost for Mathematics) by…
(Formal meetings are meetings with a specially designated meeting time, while informal meetings are meetings that are not formal, such as over lunch or a coffee break.)

	Often	A few times	Never	Don't know
… formal collegial meetings with mathematics teachers only				
… formal collegial meetings with teachers from different subjects				
… informal collegial meetings (regardless of teacher composition)				

Question 25. Any comments or clarifications of your answers to the above questions

Question 26. How many years have you been working as a teacher?

Question 27. How many years have you been working as a teacher at your current school?

Question 28. Do you have teaching certification?
[ ] Yes, in mathematics, for all grade levels I am teaching this year
[ ] Yes, in mathematics, for some of the grade levels I am teaching this year
[ ] Yes, in mathematics, but not for any of the grade levels I am teaching this year
[ ] Yes, but not in mathematics
[ ] No
[ ] Don't know

Question 29. What university studies have you completed in education/didactics?
[ ] None
[ ] Freestanding courses equivalent to less than 1 year of full-time studies
[ ] Freestanding courses equivalent to at least 1 year of full-time studies
[ ] Teacher Education, started but not completed
[ ] Teacher Education, completed
[ ] Other programme, please specify:

Question 30. What university studies have you completed in mathematics?
[ ] None
[ ] Freestanding courses equivalent to less than 1 year of full-time studies
[ ] Freestanding courses equivalent to at least 1 year of full-time studies
[ ] Teacher Education, started but not completed
[ ] Teacher Education, completed
[ ] Other programme, please specify:

Question 31. Any comments or clarifications of your answers to the above questions

Appendix O. Survey questions for principals

This appendix describes the content of the survey that was sent (electronically) to all principals.

This survey will take max. 15 minutes to complete. Thank you for your participation!

Question 1. To what extent was each of the following involved in the decision to participate in the Boost for Mathematics?

	Very high	Quite high	Quite low	Very low	Don't know
School organiser					
Principal					
Teaching faculty					
Individual teachers					

Question 2. To what extent has the implementation of the Boost for Mathematics been adapted to local conditions?
[ ] Very high [ ] Quite high [ ] Quite low [ ] Very low [ ] Don't know

Question 3. How has the implementation of the Boost for Mathematics been adapted to local conditions?

Question 4. Was an assessment of the current situation carried out prior to the Boost for Mathematics?
[ ] Yes [ ] No [ ] Don't know

Question 5. If an assessment of the current situation was carried out: To what extent was each of the following involved in this assessment?

	Very high	Quite high	Quite low	Very low	Don't know
School organiser					
Principal					
Teaching faculty					
Individual teachers					

Question 6. Any comments or clarifications of your answers to the above questions

Question 7. To what extent…

	Very high	Quite high	Quite low	Very low	Don't know
… have you actively participated in the Boost for Mathematics's implementation?					
… have you planned the Boost for Mathematics's implementation together with the supervisor?					
… have you actively looked into how the teachers' work with the Boost for Mathematics was progressing?					
… have you justified the value of your school's participation in the Boost for Mathematics to the teachers?					
… have you actively worked to strengthen the teachers' commitment to the Boost for Mathematics?					
… has the work of the Boost for Mathematics fit into the teachers' normal working week?					
… has the work of the Boost for Mathematics fit into the teachers' normal annual working time?					
… have substitutes been used to enable the teachers to participate in the Boost for Mathematics?					
… has participation in the Boost for Mathematics required the teachers to de-prioritise other work tasks?					
… has participation in the Boost for Mathematics made it necessary to de-prioritise other continuing professional development initiatives?					

Question 8. Within the Boost for Mathematics, how many times have you…

	Number of times	Don't know
… visited the teachers' mathematics lessons?		
… visited the teachers' collegial discussions?		

Question 9. Other than the Boost for Mathematics, have the school's mathematics teachers participated in any other professional development programs or other development projects this school year?
[ ] Yes [ ] No [ ] Don't know

Question 10. If the schools' mathematics teachers participated in any other professional development or other development projects: Briefly describe the focus of the project.

Question 11. Any comments or clarifications of your answers to the above questions

Question 12. How close do you consider the cooperation between you and the teachers when it comes to the development of mathematics teaching?
[ ] Very close cooperation
[ ] Pretty close cooperation
[ ] Non-existent cooperation
[ ] Don't know

Question 13. In relation to the cooperation that exists between you and the teachers when it comes to the development of mathematics teaching – how good or bad do you think this cooperation is working?
[ ] Very good [ ] Pretty good [ ] Neither good nor bad [ ] Pretty bad [ ] Very bad [ ] Don't know

Question 14. This school year, how often have you at the school discussed mathematics teaching collegially among teachers (outside of the Boost for Mathematics) by…

(Formal meetings are meetings with a specially designated meeting time, while informal meetings are meetings that are not formal, such as over lunch or a coffee break.)

	Often	A few times	Never	Don't know
… formal collegial meetings with mathematics teachers only				
… formal collegial meetings with teachers from different subjects				
… informal collegial meetings (regardless of teacher composition)				

Question 15. Last school year, how often have you at the school discussed mathematics teaching collegially among teachers (outside of the Boost for Mathematics) by…
(Formal meetings are meetings with a specially designated meeting time, while informal meetings are meetings that are not formal, such as over lunch or a coffee break.)

	Often	A few times	Never	Don't know
… formal collegial meetings with mathematics teachers only				
… formal collegial meetings with teachers from different subjects				
… informal collegial meetings (regardless of teacher composition)				

Question 16. Any comments or clarifications of your answers to the above questions

Question 17. How many years have you been working as a principal?

Question 18. How many years have you been working as a principal at your current school?

Question 19. Have you undergone the training for principals in the Boost for Mathematics?
[ ] Yes, the entire training
[ ] Yes, parts of the training
[ ] No
[ ] Don't know

Question 20. If you have undergone principal training in the Boost for Mathematics: Where did this training take place?
[ ] Umeå or Stockholm Arlanda
[ ] Stockholm City
[ ] Örebro
[ ] Gothenburg
[ ] Malmö
[ ] Karlstad
[ ] Don't know

Question 21. Have you taken the National School Leadership Training Programme (30 credits at university level)?
[ ] Yes, the entire programme
[ ] Yes, parts of the programme
[ ] No
[ ] Don't know

Question 22. What university studies have you completed in education/didactics?
[ ] None
[ ] Freestanding courses equivalent to less than 1 year of full-time studies
[ ] Freestanding courses equivalent to at least 1 year of full-time studies

[ ] Teacher Education, started but not completed
[ ] Teacher Education, completed
[ ] Other programme, please specify:

Question 23. What other types of studies in education/didactics have you completed?

Question 24. What university studies have you completed in mathematics?
[ ] None
[ ] Freestanding courses equivalent to less than 1 year of full-time studies
[ ] Freestanding courses equivalent to at least 1 year of full-time studies
[ ] Teacher Education, started but not completed
[ ] Teacher Education, completed
[ ] Other programme, please specify:

Question 25. Other than the above-mentioned studies, what other types of studies relevant to your position as principal have you completed?

Question 26. Have you worked as a teacher, as a preschool teacher, at a leisure-time center or other similar profession? If so, please describe your professional experience, including the number of years you held such a position.

Question 27. Any comments or clarifications of your answers to the above question

Appendix P: Analysis of survey responses

This appendix describes how the overall assessments were carried out, based on the responses to the surveys in Appendix N (teachers) and O (principals). Most of the values were coded so that the given scale in the survey were recalculated and divided evenly between 0 and 1. Where the division was done differently (e.g. in case of responses to open-ended answers), this is indicated below.

Teacher

Assessment: The role and function of the school organiser in the Boost for Mathematics

Survey question: 1a, 5a. Value according to survey template.

Assessment: The role and function of the principal in the Boost for Mathematics

Survey question: 1b, 5b, 15a-d. Value according to survey template.
Survey question: 16c. Number of visits: 0, 1, 2, >2 (0 / 0.33 / 0.67 / 1).
Survey question: 16d. Number of visits: 0, 1, 2, 3, >3 (0 / 0.25 / 0.5 / 0.75 / 1).

Assessment: The Boost for Mathematics's organisation in practice

Survey question: 2. Value according to survey template.
Survey question: 4. Value according to survey template (1 / 0 / 0).
Survey question: 15 e-g. Value according to survey template.
Survey question: 15 h. Value according to survey template.
Survey question: 16 a-b. Number of classroom observation visits: 0, 1, 2, 3, >3 (0 / 0.25 / 0.5 / 0.75 / 1).
Survey question: 17. Value according to survey template.

Assessment: Role and function of supervision in the collegial discussions

Survey question: 12. Value according to survey template.
Survey question: 11 h-i. Value according to survey template.
Survey question: 13. Value according to survey template (0 / 1 / 0).

Assessment: Role and function of the modules

Survey question: 7. Value according to survey template.
Survey question: 8. 0, 1, >1 number of modules (0 / 0.5 / 1).
Survey question: 9. 0, 1, >1 number of modules (0 / 0.5 / 1).
Survey question: 10. Value according to survey template (1 / 0 / 0).
Survey question: 11 a-g. Value according to survey template.

Assessment: Quality of teachers' learning environment

Survey question: 23a. Value according to survey template (1 / 0.5 / 0 / 0).
Survey question: 23b. Value according to survey template (1 / 0.5 / 0 / 0).
Survey question: 23c. Value according to survey template (1 / 0.5 / 0 / 0).
Survey question: 18+19. Value according to survey template (1 / 0 / 0). (19 = open-ended question.)

Assessment: Quality of the work with the development of mathematics teaching

Survey question: 21. Value according to survey template.
Survey question: 22. Value according to survey template.
Survey question: 18+19. Value according to survey template (1 / 0 / 0). (19 = open-ended question.)

Assessment: Teacher's background in terms of experience and competence

Survey question: 26. Number of years as teacher: <4, 4-9, 10-15, 16-21, >21 (0 / 0.25 / 0.5 / 0.75 / 1).
Survey question: 28. Value according to survey template.
Survey question: 29. Value according to survey template (0 / 0.33 / 0.67 / 0.67 / 1) – other deemed equivalent to something
Survey question: 30. Value according to survey template (0 / 0.5 / 1 / 0.5 / 1) – other deemed equivalent to something

Principal

Assessment: Quality of teachers' learning environment

Survey question: 14a. Value according to survey template.

Survey question: 14c. Value according to survey template.
Survey question: 14a. Value according to survey template.
Survey question: 9+10. Value according to survey template (1 / 0 / 0). (10 = open-ended question.)

Assessment: Quality of the work with the development of mathematics teaching

Survey question: 12. Value according to survey template (1 / 0.5 / 0 / 0).
Survey question: 13. Value according to survey template.
Survey question 9+10 Value according to survey template (1 / 0 / 0). (10 = open-ended question.)

Assessment: Principal's background in terms of experience and competence

Survey question: 17. Number of years as principal: <4, 4-9, 10-15, 16-21, >21 (0 / 0.25 / 0.5 / 0.75 / 1)
Survey question: 21. Value according to survey template.
Survey question: 22+23+25. Value according to survey template (0 / 0.33 / 0.67 / 0.67 / 1) – other deemed equivalent to something (23 and 25 = open-ended question)
Survey question: 24+25. Value according to survey template (0 / 0.5 / 1 / 0.5 / 1) – other deemed equivalent to something (25 = open-ended question)
Survey question: 26. Educational experience: 0, <4, 4-9, 10-15, >15 (0 / 0.25 / 0.5 / 0.75 / 1)

Appendix Q: Examples from other school forms

The evaluation also visited four other school forms: two compulsory schools for pupils with learning disabilities, one upper-secondary school for pupils with learning disabilities, one school for adults with learning disabilities, and one municipal adult education school. These five schools were not analysed the same way as standard compulsory and upper-secondary schools. The aim was instead to highlight the teaching culture and professional development culture of these schools. In this section, we therefore provide examples from these schools based on observations of teaching and interviews with teachers, principals and school organisers.

Special needs schools[7] in Sweden have their own curriculum and syllabuses for mathematics that differ from the governing documents of compulsory and upper-secondary schools. For example, the abilities in the mathematics syllabus for compulsory schools for pupils with learning disabilities are *1) solve mathematical problems, 2) use mathematical methods to perform calculations and solve routine tasks, 3) reflect on the reasonableness in situations with a mathematical connection,* and *4) use subject-specific words, concepts and symbols*. For upper-secondary schools for pupils with learning disabilities, there are also four abilities, with the third ability expanded to include mathematical reasoning. Municipal adult education at the upper-secondary level has its own curriculum, with the same syllabuses as upper-secondary school.

During the school visits, we saw different ways of organising the teaching. For example, we saw situations with two teachers and one pupil, two teachers with four pupils, and one-on-one teaching. In adult education schools in Sweden, there are both schools with small classes and schools with large classes, as well as schools with a mix of large and small classes. In the municipal adult education school visited, pupils and teachers worked in small classes and through distance learning.

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[7] For short, we use the term *special needs schools* when referring to the compulsory schools for pupils with learning disabilities, the upper-secondary school for pupils with learning disabilities, and the school for adults with learning disabilities that were visited as part of the evaluation.

The teaching culture in the schools studied

Special needs schools

According to the teachers interviewed, teaching at the special needs schools differs in several ways from teaching at standard compulsory and upper-secondary schools. One aspect mentioned by the teachers was that the abilities are not expressed in the syllabus in the same way as in compulsory or upper-secondary school. The focus instead is on core content:

> It is more that you should be able to count a bit, and so on. It does not say "For maths, you have to learn this". It is part of a block (Teacher in school for adults with learning disabilities)

Teachers in the visited special needs schools also pointed out that there are other goals that are central, with associated instructional settings, e.g. theatre, allowing pupils to make video recordings, and creating a poster exhibition. Thus, experiencing mathematics or participating in an activity can be goals in their own right in special education teaching. In the context of a discussion about watching a video in a mathematics lesson, one teacher described: "Doing an activity together, [...] seeing an experience of mathematics in the videos!" as an example of such an goal.

Another clear difference between special needs schools and other school forms highlighted by the teachers is that the pupils cannot perceive long-term goals or ability goals, as the pupils to a large extent "live in the present", which should not be interpreted as meaning they cannot develop the abilities. Mathematical concepts were made clear in the lessons studied and were central in many of the teacher-led discussions between teachers and pupils. The concepts touched on in observed lessons included the equals sign, days of the week, small/big and long/short. Even when there was no mathematical content defined in advance, for example in a lesson that consisted of a discussion about pictures in a newspaper, key mathematical concepts came up in the discussion, such as the concept of *pairs*.

Formative assessment was described by the teachers in the special needs schools observed as very much integrated in the teaching. During

the observations of the teachers, they were perceived to be in a constant flow of formative assessment practice. They were constantly ready to change and adjust the teaching based on pupil reactions, which was made possible by the size of the teaching groups: "We have so few pupils, so it is quite easy to see where they are and what they need to do next." (Teacher in compulsory school for pupils with learning disabilities.) One effect of this that was highlighted in the interviews was that the content of the lessons is adjusted to the pupils' capacities and current state. The teachers also suggested that one reason is that pupils are in great need of positive feedback, and cannot handle being told that they have not done well:

> For these pupils, it is important to constantly let them know what is missing, but in a positive way. You failed – those words do not exist. You are about to – motivation is the mother of learning, I believe. Then they need a positive response... (Teacher in school for adults with learning disabilities.)

The interviews revealed that teaching is highly individualised, which allows for the very type of feedback that the teacher in the quote above is talking about. Furthermore, in classes where there was more than one pupil, it was observed that pupil solutions were shared between pupils, both verbally and in writing. For example, one pupil went around to the other pupils and showed their solution.

Teachers in the special needs schools visited stressed that clear routines are important for pupils (often specific routines at the individual level). This was also evident in the lesson observations where the routines were clearly defined. Dialogic communication was very dominant in the special needs schools, which is partly a direct consequence of the size of the groups. An example of this emerged in one of the collegial interviews, where a teacher suggested that one should be more sensitive to gestures from pupils in order to better capture their conversations.

Adult education school

The teaching of mathematics in regular adult education is very similar to that in upper-secondary school, with identical syllabuses, for

example. The teachers at the observed adult education school described how the different abilities are presented in the lessons and how the abilities are interconnected:

> Take today's lesson. When I know that this is the content that we are going to work with today, how do I want to communicate the key concepts, how do I want them to be communicated, how do we reason about them, what images do I have, forms of representation to describe the concepts, what common images do I want us to create together for them? It then becomes clear that it is these concepts, communication and reasoning that get the focus in that lesson, as it is such a concept-filled area that we are working with. (Teacher in adult education school.)

It is also clear from the lesson observations that the aim of developing pupils' abilities governs how the core content is presented. One teacher expressed the close link as follows: "It's the abilities I use to be able to handle the content."

Teachers felt that assessment and feedback are different in adult education compared to other school forms, and one argument put forward was that pupils "know what they are supposed to learn". Unlike upper secondary school, peer assessment cannot be used and there is a constant need to be careful about how one expresses oneself, as according to the teachers there are so many pupils with very low self-confidence that the slightest setback can cause them to give up. At the same time, the teachers felt that because the pupils are adults, they can more easily articulate what they can and cannot do.

The teachers interviewed pointed out that they clarify the goals (both abilities and core content) that are available both in the plans that the pupils have available and on the learning platform used for communication with the pupils. Because of the more limited teaching time and largely individual work, feedback is usually provided in writing on the learning platform used:

> Then I always write a similar comment, in our learning platform, which is formative – you need to think about this, this is the level you are at, but for a higher one you need these and these parts. (Teacher in adult education school)

Teachers also described using the assessment of pupils' activities and achievements to develop teaching, in that the assessment influences the design or content of the next lesson. Such influence is a key element of formative assessment.

In the adult education school, a dialogic approach was observed, but there were also elements of monologic interaction patterns in teachers' presentations of material. The teachers also highlighted different pupil solutions during the observed lessons. The selection of pupil solutions to show was often done in such a way that several different types of solutions were produced:

> Then I usually make sure that, depending on what I'm wanting to score this time, either that they're different, or that they're similar but with variations. (Teacher in adult education school.)

Professional development culture and the Boost for Mathematics in the studied schools

What was most often highlighted by teachers as valuable about the Boost for Mathematics was the collegial learning, particularly classroom observation visits and other classroom visits. Teachers also expressed the wish to continue such collaborations and that they would also like to meet other teachers in special needs schools, possibly in other municipalities, as there are rarely multiple special needs schools in the same municipality. The principals also underscored this and that they are satisfied with how the Boost for Mathematics has worked.

However, the teachers in the special needs schools visited were not satisfied with the Boost for Mathematics' modules. The special needs schools visited had worked with the module for special needs schools in the autumn semester and another module aimed at compulsory schools in the spring semester. The teachers interviewed felt that the modules had not been sufficiently adapted for pupils in special needs schools. They also described the difficulty of not working in the same type of school if they want the collegial discussion to be fruitful. Similar wishes were expressed by teachers at the adult education school, who asked for an adult education perspective in the collegial discussions.

Three of the five municipalities had plans for further work with the Boost for Mathematics, probably in some other form. One example is that they were exploring the possibility of conducting professional development at half speed. However, there were examples of a discrepancy between the description of goals and planning, for example where it was described that "the Boost for Mathematics will be long-lasting", but there were no plans for the coming school year.

One of the characteristics of the PDP culture at the schools visited was that both teachers and principals spoke of the importance of classroom observation visits and collegial discussions to develop as a teacher. This may be an indication of a change in the schools' PDP culture initiated by the Boost for Mathematics. The fact that three of the municipalities intended to continue with the Boost for Mathematics in some form is also an indication of this change. At the same time, the teachers at the visit one year after the Boost for Mathematics said that it had been difficult to keep up the work with the Boost for Mathematics because other professional development efforts had taken priority.

The contribution of the Boost for Mathematics to the teaching and PDP cultures

In the interviews with teachers and principals, two internal factors from the Boost for Mathematics emerged as particularly positive for the development of both the teaching and the PDP culture. The first is Boost for Mathematics' encouragement of classroom observation visits and other visits to other teachers' classrooms. Teachers highlighted the importance of gaining insight into the activities of others for their own development. The second factor is the collegial discussion, meetings that give teachers the opportunity to talk about mathematics under supervision, discuss activities, exchange experiences and learn from each other. In the interviews after the Boost for Mathematics, teachers expressed that they would like to have more time in the future to continue similar collegial learning. However, teachers expressed concern that other work and PDP activities would take over and that there was a risk of the Boost for Mathematics work being reduced.

At the same time, according to the teachers and principals interviewed, there are five internal factors that have counteracted the potential for

good effects from the Boost for Mathematics at the schools visited. The first is that, according to the teachers, the modules have not been sufficiently adapted in relation to the pupils in special needs schools, with the exception of the specific module for special needs schools. Similarly, the teachers at the adult education school felt that the modules lacked the distance learning aspect, which is relatively common in their type of school. The second factor relates to the time spent on the module and is something that was mainly commented on by the principals. Two of the principals felt that the modules for special needs schools are too extensive and should have been spread over a longer period of time, for example two semesters. However, during implementation of the Boost for Mathematics at the national level, both the special needs school module and adult education module were revised to address criticism on these particular aspects. The third factor is that the principals interviewed experienced difficulties in finding supervisors with expertise in the field of special education. In order to have functioning professional development, the principals said that it is very important to have a good supervisor, but this has been a problem in special needs schools, according to the principals, because there have been so few supervisors with the specific competence required in special education. For similar initiatives to work, a conscious effort to train supervisors with special education competence is therefore probably needed. The fourth factor concerns collegial learning groups. Teachers said that this has been a challenge because the groups in which the collegial discussions took place were sometimes mixed from different school forms. For example, teachers from the compulsory school for children with severe learning disabilities have worked with teachers who work with pupils with learning disabilities integrated in compulsory school and special education teachers, which has made it difficult to create focused discussions. The fifth factor relates to the main responsibility for the special needs school. Three of the four special needs school principals pointed out that special needs schools do not have a clear place among school organisers. At the same time, school organisers at four of the five schools visited described that their responsibility is at a general level, and it was not their responsibility to manage how different schools arrange the Boost for Mathematics. At the same time, school organisers said that they have good confidence in the teachers and principals.

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The Boost for Mathematics Evaluation Report

This evaluation report was first published in Swedish in 2016 and since then, the Boost for Mathematics has been a topic of interest in several international publications. Because of this interest we have decided to have the original comprehensive evaluation translated into English so that it is accessible to a wider audience.

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