FORMATIVE ASSESSMENT AND PROBLEM SOLVING IN MATHEMATICS

Sharmin Söderström
To my mom, who has always inspired her two daughters
to accomplish higher degree.

There are not many things which I profess to know,
but this is most certainly one of them.

-Socrates (Meno)
# Table of Contents

Abstract.......................................................................................................................................................... iii

List of papers.................................................................................................................................................. v

Enkel sammanfattning på svenska .............................................................................................................. vi

1. Introduction.................................................................................................................................................. 1
   1.1 Aims and purposes............................................................................................................................... 4
   1.2 Outline of the thesis............................................................................................................................. 5

2. Research framework.................................................................................................................................. 6
   2.1 Problem-solving in mathematics....................................................................................................... 7
   2.2 Reasoning in mathematics.................................................................................................................. 9
   2.3 Formative assessment.......................................................................................................................... 9
   2.4 Theory of Didactical Situations in Mathematics............................................................................. 13
   2.5 Feedback perception........................................................................................................................... 14
   2.6 Self-efficacy....................................................................................................................................... 15
   2.7 Types of Motivation............................................................................................................................ 16
   2.8 Goal orientation.................................................................................................................................. 17
   2.9 Summary............................................................................................................................................ 18

3. Materials and Methods............................................................................................................................ 20
   3.1 Research method for Study 1............................................................................................................... 21
      3.1.1 Data collection.............................................................................................................................. 22
      3.1.2 Data analysis................................................................................................................................. 23
   3.2 Research method for Study 2............................................................................................................... 24
      3.2.1 Data collection.............................................................................................................................. 24
      3.2.2 Data analysis................................................................................................................................. 25
   3.3 Research method for Study 3............................................................................................................... 26
      3.3.1 Data collection:............................................................................................................................ 26
      3.3.2 Data analysis................................................................................................................................. 30
   3.4 Research method for Study 4............................................................................................................... 31
      3.4.1 Data collection.............................................................................................................................. 32
      3.4.2 Data analysis................................................................................................................................. 33
   3.5 Research credibility and ethical considerations.................................................................................. 34
      3.5.1 Reliability ................................................................................................................................... 34
      3.5.2 Research validity .......................................................................................................................... 35
      3.5.3 Ethical considerations .................................................................................................................. 36
4 Results ......................................................................................................................... 37
  4.1 Article 1................................................................................................................. 37
  4.2 Article 2................................................................................................................. 38
  4.3 Article 3................................................................................................................. 39
  4.4 Article 4................................................................................................................. 41

5 Discussion .................................................................................................................. 43
  5.1 Lack of research in some feedback types......................................................... 43
  5.2 The potential of providing formative assessment that supports reasoning .......................................................... 44
  5.3 Motivation for using feedback ........................................................................ 47
  5.4 Contributions ..................................................................................................... 48
  5.5 Limitations and Implications ........................................................................... 51

Acknowledgment ....................................................................................................... 53

References ................................................................................................................... 55
Abstract

In this thesis, the focus is on how reasoning in problem solving can be supported and which factors are associated with this support. In four studies, I investigated four aspects which address the overarching aim of the thesis. I report on two non-empirical studies and two intervention studies of formative assessment that deal with problem solving in mathematics.

Study 1 proposes a model based on different characteristics of feedback in mathematics education that have been studied in the literature. Study 2 addresses the effectiveness of different feedback types in mathematics. Study 3 investigates the usefulness of formative assessment in supporting students who engage in problem solving. Study 4 examines the relationship between a student’s self-efficacy, national test grade, motivation type, learning goal orientation, task-solving success, and the perceived usefulness of feedback.

I have used the concept of devolution of problem from Brousseau’s (1997) theory of didactical situations in mathematics to design a computer-based formative assessment support tool. The students were not provided with any solution method template to solve the tasks; instead, they were given the responsibility of constructing their own solution method with self-diagnosis and feedback support from the computer. The students determined where they had struggled and chose the diagnosis, and feedback was designed corresponding to each diagnosis. The feedback for each task starts at a relatively general metacognitive level; if it is insufficient, feedback is then provided in the form of general heuristic strategy suggestions.

Thematic analysis and systematic literature reviews were used in the first two studies. Participants in Intervention Study 3 were 17 first-year university students, whereas 134 students from upper secondary high school participated in Intervention Study 4. Think-aloud protocols have been used in this thesis analysis along with computer log files. In Study 4, structural equation model analyses were used.

The first study’s proposed model identified in which ways the characteristics of feedback both between and within feedback levels may be very different and thereby might affect students’ responses and learning differently. The results from Study 2 indicated that effective feedback provides to students sufficient motivational and cognitive support to use the feedback to engage in thinking about the mathematical learning targets. Such feedback characteristics are more often found in process-level feedback and self-
regulation feedback than in task-level feedback. Study 3 showed how the use of computer-based formative assessment, including self-assessment and metacognitive and heuristic feedback, can support students to overcome difficulties in problem-solving by their own reasoning. The results from Study 4 showed that students’ mastery goals had a direct effect on the perceived usefulness of the feedback, but no such effects were found for students’ national test grades, self-efficacy beliefs, performance goals, intrinsic or extrinsic forms of motivation.
List of papers


II. Söderström, S., & Palm, T. (Manuscript to be submitted). A review of feedback characteristics that affect student achievement in mathematics.


IV. Söderström, S., Palm, T., & Granberg, C. (Under Review). The effects of mathematical ability and motivational beliefs on students’ perceptions of feedback usefulness.

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**Enkel sammanfattning på svenska**

I denna avhandling fokuseras på hur resonemang i problemlösning kan stödjas och vilka faktorer som är associerade med detta stöd. I fyra studier undersökte jag fyra aspekter inom det övergripande syftet med avhandlingen. Jag redovisar två icke-empiriska studier om formativ bedömning i matematik och två interventionsstudier med datorbaserad formativ bedömning som stöd till elevers problemlösning i matematik.

Studie 1 föreslår en modell baserad på olika egenskaper av återkoppling inom matematikundervisning som har studerats i litteraturen. Studie 2 behandlar effektiviteten av olika återkopplingstyper inom matematik. Studie 3 undersöker användbarheten av formativ bedömning för att stödja eleverna att engagera sig i problemlösning. Studie 4 undersöker förhållandet mellan elevers tro på sin egen förmåga, nationella provbetyg, motivationstyp, målorientering, uppgiftslösningsframgång och upplevd användbarhet av återkoppling.

Jag har använt begreppet ”devolution of problem” (delegering av ansvar för problemlösning) från Brousseaus (1997) teori om didaktiska situationer inom matematik för att utforma ett datorbaserat formativt bedömningsstöd. Eleverna försågs inte med någon lösningsmetodsmall för att lösa uppgifterna, istället fick de ansvaret att konstruera sin egen lösningsmetod med självdiagnostik och återkopplingsstöd från datorn. Eleverna identifierade var de hade svårigheter och de valde diagnos, och återkoppling var förutbestämd i enlighet med varje diagnos. Återkopplingen började på en relativt allmän metakognitiv nivå, och om den var otillräcklig gavs återkoppling som allmänna heuristiska strategiföreslag.

Tematisk analys och systematiska litteraturöversikter användes i de två första studierna. Deltagare i interventionsstudie 3 var 17 förståårs universitetsstudenter och 134 elever från gymnasieskolan deltog i interventionsstudie 4. Tänka-högt-protokoll har använts i studie 3 tillsammans med datorloggfiler. I studie 4 användes strukturella ekvationsmodellanalyser.

Den första studiens föreslagna modell identifierade på vilka sätt egenskaperna för återkoppling både mellan och inom återkopplingsnivåer kan vara olika och därmed kan påverka elevernas svar och lärande. Resultaten från studie 2 indikerade att effektiv återkoppling ger eleverna tillräckligt motiverande och kognitivt stöd för att använda återkopplingen för att tänka på de matematiska
inlärningsmålen. Sådana återkopplingsegenskaper återfinns oftare i återkoppling på processnivå och självregleringsnivå än i återkoppling på uppgiftsnivå. Studie 3 visade att användningen av formativ bedömning från datorn, baserat på självdiagnos och metakognitiv och heuristisk återkopplingen, kan användas på ett formativt sätt som ett stöd för att övervinna en svårighet under problemlösning. Resultaten från studie 4 visade att elevernas bemästrandemål (lärandemål) hade en direkt effekt på den upplevda användbarheten av återkopplingen, men inga sådana effekter hittades för elevernas nationella provbetyg, tron på sin egen förmåga, prestationsmål, inre eller yttre former av motivation.
1. Introduction

How can we assist students when they are stuck when solving a mathematics problem? Should we provide the solution method or help them to solve the problem by themselves? If we provide the solution method, we run the risk that the student will only have the opportunity for rote learning. The question, then, is: how can we assist students without giving them a solution method? Learning mathematics requires students to tackle math problems by creating (parts of) the method by themselves rather than applying memorized procedures. Therefore, if we want students to solve problems by themselves, they will need guidance when they encounter difficulties. This thesis suggests that in order to provide guidance for supporting students during problem solving, there is a need to first diagnose what kind of problematic situation students are facing and then provide feedback accordingly. However, the usefulness of guidance and feedback can be perceived differently depending on the student’s characteristics.

Let’s assume that a student encounters difficulty solving a problem and that the difficulty could be anywhere in their problem-solving process. They might, for example, have difficulties in interpreting the problem, choosing a useful problem-solving strategy, or justifying whether their method is right or wrong. To support the student, a computer environment can be designed to offer the student the ability to identify (self-diagnose) their difficulty among different diagnostic statements presented on the screen. Then, depending on what diagnostic statement they choose, the computer will provide feedback that matches their difficulty. The feedback the student receives is designed to support them so that they can resume their problem-solving without being provided the method. For example, if the student’s difficulty could be described as: “I am not sure my solution method is correct or not,” then the feedback that is intended to trigger the student’s reasoning could be: “Try to convince a friend in your mind: why do you think that your solution strategy is correct.” However, the question remains whether all students perceive this kind of feedback as useful or not. The perceived usefulness of feedback and how it is used might vary depending on a student’s characteristics, such as, their motivation, self-efficacy, etc. In this thesis, I will provide some insight into the usefulness of computer-provided formative assessment, including both diagnostic (the student will identify their difficulty) and feedback strategies (supporting students’ reasoning to overcome a particular difficulty). In addition, this thesis considers how a student’s characteristics (e.g., abilities, type of motivation) is associated with success in using feedback, as well as...
how useful they perceive feedback intended for supporting mathematical reasoning. In the paragraphs below, I will present the rationale for why I have undertaken this thesis.

Researchers have pointed out that students need to construct problem-solving methods by themselves in order to learn mathematics (e.g., Brousseau, 1997; Schoenfeld, 1985). This argument is supported by various studies showing that students’ learning outcomes are better when they practice at creating solution methods by themselves compared to students who practice using prescribed methods (e.g., Jonsson et al., 2020; Jonsson et al., 2014; Kapur, 2014; Norqvist, 2018; Olsson & Granberg, 2019). However, studies have also revealed that many students encounter difficulties when they are solving mathematics problems (Verschaffel et al., 2020), and that students also encounter difficulties they cannot overcome on their own (e.g., Jonsson et al., 2020; Jonsson et al., 2014; Norqvist, 2018; Olsson & Granberg, 2019). How to support the students, however, is not clarified as Lester and Cai (2016) pointed out:

*In fact, although there is a great deal of consensus within the mathematics education community that the development of students’ problem-solving abilities should be a primary goal of classroom instruction...there is no consensus about what we teachers should do in classrooms to reach this goal* (p.118).

Thus, students might need support when they are solving mathematics problems, but helping students with their thinking process without providing a solution method is not easy for teachers (Lee & Cross Francis, 2018; Teuscher et al., 2016). To support students, it is first necessary to identify the specific difficulty students are facing, and then support should be provided according to the diagnosed difficulty. This kind of teaching-learning support could be described as formative assessment (Black & Wiliam, 2009; Stiggins, 2006; Wiliam & Thompson, 2008)

In formative assessment, feedback is considered one of the most important components for supporting students (e.g., Brookhart, 2018). Several studies have shown, for example, that feedback can support students' reasoning during the problem solving process (e.g., Kapa, 2007). However, the research on feedback has largely focused on how teachers can provide effective feedback and pays less attention to how to effectively increase student involvement with feedback (Van der Kleij et al., 2019; Winstone et al., 2017). Moreover, students’ self-regulation influences the extent to which students monitor their progress while problem solving and seek and use feedback for
the purpose of enhancing their learning outcomes (Black & Wiliam, 2009; Brown & Harris, 2013; Clark, 2012; Hattie & Timperley, 2007). However, researchers claim that there are still unanswered questions regarding self-regulation and feedback. There needs to be “a more explicit investigation of the dynamics of self-regulation and engagement with feedback to advance theoretical understandings of how feedback may or may not result in learning improvements” (Van der Kleij et al., 2019, p. 321).

It has also been argued that there is a lack of understanding of the mechanisms behind feedback due to a lack of systematic research on how students interpret and respond to feedback (e.g., Leighton, 2019). There is a need for more in-depth knowledge regarding the principles governing how to facilitate effective student engagement with feedback (Brooks et al., 2021). There is also a lack of research on how students interpret and respond to feedback aimed at supporting their mathematical reasoning and the role a student’s characteristics plays in how they respond to feedback. For example, students with certain characteristics may not be motivated to use feedback while problem solving. In one review, Van der Kleij and Lipnevich (2021) identified 164 research papers published up to and including 2018 addressing the issue of students’ perceptions of assessment feedback from primary school up to university. However, merely 27 reported on how perceived feedback might depend on a student’s characteristics, and only one of the 27 concerned mathematics.

Moreover, interest in computer-assisted formative assessment has been constantly growing in educational research (Attali & van der Kleij, 2017; Sullivan et al., 2021), especially for providing feedback. Hattie and Timperley’s (2007) meta-analysis showed that computer-assisted feedback can provide cues or reinforcement for improving learning, and that it is one of the most effective modes of feedback. Further research is also needed on computer-given feedback, which can, for example, offer students to choose the feedback they think they need. Van der Kleij et al. (2015) suggested, from their meta-analysis of studies focused on the effectiveness of feedback in computer-based assessment environments, that future research in “digital learning environments could become even more powerful if the feedback included a diagnostic component” (p.504). Finally, there are researchers arguing that additional research is necessary to further our knowledge of how students can be supported in carrying out problem solving activities (e.g., Lester & Cai, 2016; Liljedahl & Cai, 2021). Therefore, despite the considerable research on feedback, there are still questions that need to be addressed (Bangert-Drowns et al., 1991; Kluger & DeNisi, 1996).
Considering the situation and gap described above, the aims and research questions of this thesis are formulated in the following section.

1.1 Aims and purposes

The main aim of this study is to investigate how a computer-based formative assessment design can support students’ reasoning during the problem-solving process. To design this formative assessment, attention was first given to investigating what kinds of feedback were provided to students in the research literature on mathematics education. The second focus of research was to distinguish what kinds of feedback are effective for student achievement from a research framework perspective. The third focus was to design a formative assessment and investigate the usefulness of this design for supporting student reasoning during the problem-solving process. The final focus of research was to investigate to what extent a student’s characteristics influence the use of feedback and its perceived usefulness. The research questions guiding this thesis are:

1. Which characteristics of feedback to students are studied in the mathematics education research literature?
2. Which feedback characteristics affect student achievement in mathematics?
3. How can formative assessment from a computer assist students with problem-solving when they encounter difficulties?
4. How are students’ different characteristics associated with task-solving success using feedback and the perceived usefulness of feedback intended to support students’ reasoning in problem-solving?

This thesis consists of four studies that collectively address the overarching aim of my thesis. For the first study, a model was developed from a systematic literature review on feedback provided to students in mathematics education (Article 1). The second study is a systematic literature review focusing on the effectiveness of feedback provided to students in mathematics education (Article 2). Then a third study was conducted (Article 3) to investigate how a formative assessment design can support student’s reasoning process while problem solving. Lastly, a fourth study (Article 4) investigated the association between different student characteristics and the students’ task-solving success using feedback and the perceived usefulness of feedback. The first two studies are non-empirical investigations, and first two articles coauthored with Torulf Palm. The last two studies are empirical where fourth article is coauthored with Carina Granberg and Torulf Palm.
1.2 Outline of the thesis

This thesis consists of two parts. The first part is a comprehensive summary (Kappa\textsuperscript{1}), and the second part consists of four articles. Since I mention the articles in the comprehensive summary, I would suggest reading them first. The comprehensive summary comprises five chapters, which provide a central setting for the articles.

In Chapter 1 (Introduction), I have provided an introduction that presents the overall aims of the thesis and this outline of the thesis’s sections.

In Chapter 2 (Research framework), I describe relevant research that this thesis draws upon and expands upon. The chapter starts with a presentation of the research on problem-solving. Then, I describe the part of the theory addressing the didactical situation that guided my thesis. I also explain the significance of formative assessment. Finally, I elaborate upon the different psychometric components (e.g., type of motivation, self-efficacy, etc.) that have been used in this thesis.

In Chapter 3 (Material and Methods) I describe the methodological concerns. I start with the research design and then describe the issues concerning data collection and analysis methods. Thereafter, I present the issues related to ethical considerations and the studies’ validity and reliability.

In Chapter 4 (Results), I present a summary of each of the four articles, focusing on the articles’ results and conclusions.

In Chapter 5 (Discussion), I discuss the main results of the studies as well as the contributions connected to these results. I end the chapter with thoughts on further research.

\textsuperscript{1} In Swedish, it is called Kappa.
2. Research framework

In this chapter, I describe the research framework within which the research was conducted. According to Lester (2005), a research framework is “a basic structure of the ideas (i.e., abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated” (p. 458). Thus, the aim of this chapter is not to discuss a holistic literature review or provide a whole theory, but rather to provide the reader with sufficient information about the theoretical choices made to design the four studies, particularly the interventions in Study 3 and Study 4. Moreover, to guide the design of the formative assessment used in this thesis, it is necessary to consider the research framework in order to comprehend the logic behind the designing of the research studies as well as how to make sense of the data (Lester, 2005).

The aim of this thesis is how a computer-based formative assessment design can support mathematical reasoning and problem-solving for the learning of mathematics. More precisely, four aspects were studied: (i) what research says about feedback (i.e., one main component of formative assessment) in mathematics education, (ii) what research says about the effectiveness of feedback on student achievement, (iii) how helpful a particular design of formative assessment is for reasoning in problem-solving, and (iv) what impact psychometric factors have on using this particular formative assessment design. To study these aspects, I first clarify learning through and about problem solving (Section 2.1) and define reasoning (Section 2.2) since these aspects are considered important in learning mathematics effectively. Second, I discuss formative assessment (Section 2.3) because students might need help when engaged in problem-solving. In contrast to working with routine tasks, formative assessment that identifies the specific difficulty and provides suitable feedback is necessary to support problem solving. Third, I specify which part that was used from the theory of didactical situations (Section 3.4) since designing a computer tool for problem solving requires that the student take responsibility for constructing the solution. Fourth, I define the concept of perceived usefulness of feedback (Section 3.5); this is necessary to analyze how the student weighs the usefulness of the provided feedback. Then I describe the concepts of different student characteristics such as self-efficacy (Section 3.6), types of motivation (Section 3.7), and achievement goals (Section 3.8). At the end of this, I provide a summary. The following section begins by introducing the concepts that have been used to design the studies.
2.1 Problem-solving in mathematics

Problem-solving is defined as “engaging in a task for which the solution method is not known in advance” (NCTM, 2000, p. 51). Similar definitions of problem-solving are provided by others which sometimes specify that the task must be a challenge (Schoenfeld, 1985) or that it should require exploration (Niss, 2003). In this thesis, problem solving has been conceptualized as tasks where the student does not know a solution method in advance (see 3.3.2).

When it comes to the process of solving problems, there are various arrangements of the problem-solving process documented in the research literature (see Figure 1). For example, Dewey (1910), described five phases in the problem-solving process: “(i) encountering a problem (suggestions), (ii) specifying the nature of the problem (intellectualization), (iii) approaching possible solutions (the guiding idea and hypothesis), (iv) developing logical consequences of the approach (reasoning (in the narrower sense)), and (v) accepting or rejecting the idea by experiments (testing the hypothesis by action)” (cited in Rott et al., 2021, p. 738). Similarly, Polya (1945) suggested that the problem-solving process involved: (i) understanding the problem, (ii) developing a plan, (iii) carrying out the plan, and (iv) looking back. In Pólya’s model, the problem-solving process is a linear progression from one phase to the next.

![Figure 1: Different phase models of problem-solving processes (Rott et al., 2021).](image)

Nevertheless, when we are stuck on a problem and try to solve it, we know that this is not a linear process since we usually go back and forth (in thought process) to solve the puzzle. Researchers have also provided non-linear
models for problem-solving processes (e.g., Mason et al., 1982; Schoenfeld, 1992; Wilson et al., 1993; Yimer & Ellerton, 2010) (see Figure 1). A six-phase model was provided by Schoenfeld (1985) that divided Pólya’s four-phases model into six² possible phases (i.e., (i) reading the task, (ii) analyzing the task, (iii) exploring methods, (iv) making a plan, (iv) implementing the plan, and (vi) verifying the results). In this model, Pólya’s “devise a plan” phase is broken down into “exploration” and “planning” phases. One may also note that all these phases do not need to be present during the problem-solving process. For instance, the analysis phase of a given task could be enough to plan how to progress, and thus the exploration phase would be not needed (Schoenfeld, 1985). Pólya’s four phases and Schoenfeld’s six phases were considered and modified into five phases (see Section 3.3), indicating problematic situations during problem solving that guided the design of the interventions used in Study 3 and Study 4. A diagnosis template was created that characterizes problematic situations students might encounter while problem solving.

Furthermore, Schoenfeld (1985) describes four different knowledge characteristics that are required to be a good problem solver. Success or failure in problem-solving relates to one or more of the following four categories (Schoenfeld, 2014): (i) Beliefs: the students' personal views on mathematics, which are important for how the student tackles novel tasks (Schoenfeld, 1985); (ii) Metacognitive monitoring and control: this includes evaluating progress and initiating effective searches for strategies and resources to select and use. These are the executive processes of reflecting on and controlling one’s own thinking, and these procedures are essential for the self-regulation needed to choose wisely when making problem-solving decisions (e.g., Schraw et al., 2006). A student with good control tries to make the most of their resources and heuristic knowledge so that they can solve novel tasks or problems in a more efficient way. (iii) Heuristics: the selection and implementation of suitable rules of thumb for mathematical problem-solving. In other words, these are the strategies and techniques needed to solve unfamiliar problems. These are “general suggestions that help an individual understand a problem better or to make progress towards its solution” (Schoenfeld, 1985, p. 23). Examples of more specific heuristics principles are (e.g., ‘draw a figure’, ‘try a few examples’, ‘try to solve a simpler task’, etc., see Schoenfeld, 1985). (iv) Resources: the student’s previously acquired knowledge that is necessary to solve a particular task, “the set of relevant facts known by the individual” (Schoenfeld, 1985, p. 17). Some

² Please note that the figure from Rott et al. mentioned only five phases.
examples would be knowing how to subtract or multiply, what a rectangle is, etc. In this thesis, these four characteristics were considered when designing the feedback to support students engaged in problem solving (see Section 3.3.2).

2.2 Reasoning in mathematics

Reasoning is defined as “the explicit act of justifying choices and conclusions by mathematical arguments” (Lithner et al., 2010, p. 161). The definition of reasoning is in part based on NCTM’s (2000) definition of reasoning: “to develop and evaluate mathematical arguments and proofs” (p. 55), but it also includes the idea from (Niss, 2003) that reasoning can be seen as a juridical aspect in problem solving and modeling. Reasoning can be considered as a thinking process, or the product of the thinking processes, or both. Furthermore, mathematically founded arguments “motivate why the conclusions are true or plausible and are anchored in intrinsic properties of the mathematical components (objects, transformations, and concepts) involved in the reasoning” (Lithner et al. 2010, p.161). Reasoning does not in this definition have to be strict but at least plausible, for example in line with (Polya, 1954). The relation between reasoning and proofs in this definition is that proofs are a sequence of reasoning where the mathematical arguments are logically strict. Lithner (2008) further explained students’ thinking process on task-solving creativity, which is based on “thinking processes that are flexible, fluently admit different approaches and adaptations to the situation, and are not hindered by fixation” (p. 267). In this thesis, reasoning is conceptualized as students justifying choices and conclusions while problem solving, and this has been utilized in Study 3’s data analysis (see Section 3.3.3).

2.3 Formative assessment

Formative assessment involves collecting information on students’ achievement and then using this information to make decisions about how to provide feedback or instructions to support students’ learning (Bennett, 2011; Black & Wiliam, 1998, 2009; Wiliam & Thompson, 2008). Black and Wiliam (2009) provided a definition of formative assessment based on their earlier works (i.e., 1998, 2002) as:

Practically in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better,
Black and Wiliam (2009) clarified that instruction is any activity that creates learning where the decision-making is made by any agent (i.e., individual student, peer, teacher). Furthermore, Wiliam and Thompson (2008) provided a framework for formative assessment consisting of five key strategies (see Table 1). From this framework, two aspects of formative assessment that I emphasized in my thesis are (1) students’ involvement in their learning processes in the form of self-assessment, and (2) providing feedback to students by agents (e.g., teachers, computers, etc.) that advance students’ learning. This choice was made since the aim of the thesis was to support students’ problem solving without providing a solution method and support the students’ own thinking in creating solution methods. To support students in this way, it is necessary to diagnose the student’s problematic situations (i.e., the difficulty that is hindering the student when trying to improve their task-solving abilities) and provide feedback on those problematic situations in order to help students create solutions (Lithner, 2017).

Table 1: Aspects of formative assessment (Wiliam & Thompson, 2008, p. 15)

<table>
<thead>
<tr>
<th></th>
<th>Where the learner is going</th>
<th>Where the learner is right now</th>
<th>How to get there</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher</td>
<td>1) Clarifying learning intentions and sharing and criteria for success</td>
<td>2) Engineering effective classroom discussions, activities and tasks that elicit evidence of learning</td>
<td>3) Providing feedback that moves learners forward</td>
</tr>
<tr>
<td>Peer</td>
<td>Understanding and sharing learning intentions and criteria for success</td>
<td>Activating learners as instructional resources for one another</td>
<td></td>
</tr>
<tr>
<td>Learner</td>
<td>Understanding learning intentions and criteria for success</td>
<td>Activating learners as the owners of their own learning</td>
<td></td>
</tr>
</tbody>
</table>

Activating students as the owners of their own learning is one of the focuses of this thesis. The purpose is make students responsible for creating their solution methods (Brousseau, 1997) for solving problems, support self-assessment, and motivating students to take responsibility for their learning.
Self-assessment is the first step in the student’s self-regulation and ability to manage, monitor, redirect, and control their own learning (Zimmerman, 2000, 2006). Self-regulatory processes have three cyclical phases: (i) the forethought phase (i.e., indicates processes and beliefs that occur before efforts to learn; (ii) the performance phase (i.e., indicates processes that occur during behavioral implementation, and (iii) self-reflection (indicates processes that occur after each learning effort). Self-regulation influences the extent to which students monitor their progress in terms of seeking and using feedback while problem solving (Black & Wiliam, 2009; Brown & Harris, 2013; Clark, 2012; Hattie & Timperley, 2007). A recent meta-analysis also indicated that formative assessment interventions are effective when focused on providing student-initiated self-assessment (i.e., medium-sized impact; d = .61) (Lee et al., 2020). Therefore, when designing this thesis intervention, the self-assessment approach was considered as advocated by researchers (e.g., Van der Kleij et al., 2019). The design of the student’s self-assessment (i.e., diagnosis) is further detailed in Section 3.3.2.

The importance of providing feedback has been emphasized in formative assessment (e.g., Black & Wiliam, 1998, 2009; Sadler, 1989; Sadler, 2010). The three main questions regarding formative assessment represented in Table-1, “Where is the learner going?”, “Where is the learner right now?”, “How does the learner get there?”, mainly reflect the feedback questions of Hattie and Timperley (2007). Hattie and Timperley (2007) also provided a feedback model that focuses on feedback effectiveness. In their feedback model, they described four levels at which the feedback can be focused: (1) Feedback on the task includes information about how well the task is being accomplished or performed, (2) Feedback on the processing of the task pertains to feedback focusing on the task-solving process or aims to provide the students with the information they need to understand or to solve the task, (3) Feedback about self-regulation addresses the way students monitor, direct, and regulate actions concerning their learning objectives, and (4) Feedback on “the self”, which is personal and expresses general positive or negative comments about the student’s work or the student as a person. In Study 1 (see Section 3.1.2) and Study 2 (see Section 3.2.2), Hattie and Timperley’s (2007) four levels of feedback were adopted in the analysis to categorize the feedback examined in the included articles.

There are different kinds of feedback studied in the feedback literature, such as elaborated feedback, bug-related feedback, corrective feedback, worked example feedback, hints, prompts, metacognitive feedback, etc. (see Shute,
Some feedback is more effective for learning and supporting reasoning in problem solving. Hattie and Timperley (2007) addressed the significance of metacognitive feedback, which concerns “the way students monitor, direct, and regulate actions toward the learning goal” (p. 93). At the same time, it fulfills the following basic metacognitive and motivational functions outlined by Narciss (2008). Metacognitive functions: these provide information about metacognitive strategies, provide specifications by offering metacognitive strategies for the solution, provide information that can be used to correct erroneous strategies, and provide guidance by conveying help for monitoring, evaluating, or assessing the suitability of the students’ solution strategies. Motivational functions: these provide incentives by rendering the result of a task visible, facilitate task completion by offering suggestions to overcome difficulties, enhance self-efficacy by providing information that makes it possible to master tasks, and contribute to mastery practice that can be related to personal causation.

In self-regulated learning, metacognition is one of the unique components along with the other two components, cognition and motivation (Schraw et al., 2006). Feedback via metacognitive questioning may lead students through the problem-solving process and perhaps also help them reason better when constructing their own solution methods. Moreover, feedback during the problem-solving process must help the learner see where their strategy failures or information gaps are (Mory, 2004). In this thesis, therefore, metacognitive feedback has been designed to support the students’ reflections and mathematical reasoning (see Section 3.3.2) in problem solving.

A computer environment that can support metacognition is “one that is designed for instructional purposes and uses technology to support the learner in achieving the goals of instruction” (Azevedo, 2005, p. 194) and may provide an opportunity for self-directed learning. It is the students who are often responsible for determining which support/guidance from the computer to use while problem solving. Research on the role of self-regulated learning in computer-based learning environments is described by Winters et al. (2008). They concluded from their review that “different learner and task characteristics (e.g., prior knowledge, goal orientation, learner control) and types of learner support are related to students’ self-regulated learning when using a computer-based learning environment.” Azevedo (2005) further categorized characteristics of the computer-based learning environment as a cognitive tool, where “any environment where the learner’s use and deployment of key metacognitive and self-regulatory processes prior to, during, and following learning are critical for successful learning.” (p. 194).
Furthermore, Perrenoud (1998) advocated using a digital learning environment for formative assessment, and that “one element of regulation can be delegated to the didactic apparatus itself, to interaction between pupils, to technologies, to various forms of metacognition” (p.90). Hence, one aim of this thesis is to utilize the educational potential of the computer environment for formative assessment. The features provided by the computer environment in this thesis are diagnoses and feedback (metacognitive & heuristic). The environment does not for example offer any interactions such as drawing, or dynamic tools such as formulas-graphs in GeoGebra, etc. A more detailed explanation on how this was done is provided in Section 3.3.2.

2.4 Theory of Didactical Situations in Mathematics

One aim of this thesis is to demonstrate how a computer-based formative assessment tool can be designed to support mathematical reasoning in mathematical problem-solving. The design of the computer environment was guided by Brousseau’s (1997) Theory of Didactical Situations in Mathematics (TDS), which claimed that students should construct their own knowledge. Theory of didactical situations in mathematics offers a model which is inspired by the mathematical theory of games. Theory of didactical situations in mathematics seeks, in a scientific way, the problems related to the teaching of mathematics and a basis for designing more effective teaching methods in which students actively create knowledge. Brousseau’s belief is that students are given the responsibility of the task-solving process, which helps them to understand the intended knowledge. The theory of didactical situations in mathematics is a detailed and comprehensive theory, and only the aspect regarding the “devolution of a problem” is used in this thesis to guide the design of the task, the diagnoses, and the feedback in the computer environment (see 3.3.2). To accomplish this, a situation or environment is created for students by teachers (in this study, by the researcher) and presented in a student-computer interaction format as a tool in the teaching process (Brousseau, 1997).

The devolution of a problem signifies delegating responsibility for a situation to students, thus allowing them to produce a response. This means that students are given the responsibility of creating (parts of) the solution method. This notion of devolution of problem is used in this thesis to select tasks when students are given the responsibility of creating solution strategies.
Problem-solving (see Section 2.1) tasks were thus provided to students in order to lead them into a productive struggle without demonstrating the solution procedure. The idea is that the students should take responsibility for the logical effort when solving the tasks. Since “doing mathematics does not consist only of receiving, learning and sending correct, relevant (appropriate) mathematical messages” (Brousseau, 1997, p. 15), the tasks were designed in such a way as to provoke the expected learning by engaging in problem solving. This idea has been embraced and incorporated into the design of the tasks (see Section 3.3.2) presented in the interventions of Study 3 and Study 4. The idea is that if devolution takes place, the problem is owned by the student even though the problem is initially provided by the instructor (in this thesis case, provided by computer).

For all the tasks presented in Study 3 and Study 4, feedback from the computer was first provided as metacognitive feedback, then the option of receiving heuristic feedback was given. No solution method was provided as feedback. This signifies that metacognitive feedback guiding students’ own reflection was provided first, and if the metacognitive feedback was insufficient, somewhat more specific heuristic feedback was provided. By doing this, if a student successfully creates a solution method with metacognitive feedback, they have thus taken more responsibility for solving the task (details in Section 3.3.2).

2.5 Feedback perception

Seeing feedback as useful is one of the fundamental aspects of feedback perception (e.g., Brett & Atwater, 2001). It has been argued that students need to perceive the feedback as useful in order to help them solve tasks and enhance their learning (Mouratidis et al., 2010). Ilgen et al. (1979) identified the central role of perception in feedback processing and proposed four stages in understanding how the feedback process works: Feedback is (i) perceived, (ii) accepted as accurate, (iii) perceived as useful, and (iv) used as intended. Studies have found empirical evidence for the facilitating role of perceived usefulness (e.g., Brett & Atwater, 2001; Harks et al., 2014; Rakoczy et al., 2013). For example, Harks et al. (2014) found that students perceived process-oriented feedback (which consisted of general performance ratings, the weaknesses, and strengths of inner mathematics task information, and motivational statements) as more useful than grade-oriented feedback. Process-oriented feedback perceived as useful appeared to help students recognize their mistakes and identify strategies to proceed (Narciss, 2008; Shute, 2008). Therefore, this thesis investigates how students perceive the
metacognitive and heuristic feedback described above and the relation between the students’ characteristics and perceived usefulness (see Section 3.4.2).

2.6 Self-efficacy

Self-efficacy is “the beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1997, p. 3). Self-efficacy has an influence on how one makes choices and acts (Schunk & Pajares, 2009). The student with higher self-efficacy is more active in their efforts (Bandura, 1977; Schunk & Pajares, 2009). The students with higher self-efficacy will expend more effort on an activity, will persevere when confronting obstacles, and will be resilient in adverse situations (Schunk & Pajares, 2009). Students were found to be more willing to persist in working on difficult tasks as well as regulate their learning when they possessed a high sense of self-efficacy (Pintrich & De Groot, 1990). Studies have also indicated that students with higher self-efficacy use more learning strategies, such as elaboration and metacognitive strategies, than students with low self-efficacy (Berger & Karabenick, 2011; Pintrich & De Groot, 1990; Wang & Wu, 2008). Skaalvik et al. (2015) conducted a study on Norwegian middle school students studying mathematics subjects and found that students’ self-efficacy was positively associated with their help-seeking behavior, effort, and persistence.

Thus, when it comes to using feedback, students’ self-efficacy might be a factor. Self-efficacy might account for how successfully students use feedback when they are stuck in a problematic situation during problem solving as well as how positively they perceive the feedback. For example, students with low self-efficacy may not believe they will be able to successfully use feedback that requires them to engage in a productive struggle and will therefore not find it useful. On the other hand, students with high self-efficacy might not be willing to use feedback, and they might try to solve the task without asking for help. If they did use feedback, however, they might be recognizing its benefits and thus have a positive perception of its usefulness. However, students with low self-efficacy might also perceive the feedback as useful since they might use feedback more often and might be successful in solving at least some of the tasks. This might influence their perceived usefulness. Therefore, this thesis investigated relations between students’ self-efficacy, use of feedback, and perceived usefulness (see Section 3.4).
2.7 Types of Motivation

In Self-determination theory, Deci and Ryan (2000) distinguished between two major categories of motivation based on the different reasons or goals that give rise to an action: (1) intrinsic motivation (i.e., doing something because it is inherently interesting or enjoyable), and (2) extrinsic motivation (i.e., doing something because it leads to a separable outcome). Motivation is a reason for acting or behaving in a particular way. One “can be motivated by intrinsic and by varied types of extrinsic motivations, often simultaneously” (Ryan & Deci, 2017, p. 16). Extrinsic motivation differs in the extent to which the reasons for student actions are self-determined or autonomous. There are four subtypes (i.e., external, introjected, identified, and integrated) within the extrinsic motivation category. Externally motivated students engage in a task because of external rewards or to avoid discomfort or punishment (this is the least autonomous form of extrinsic motivation). Students may also engage in an activity to avoid feeling guilt or to attain ego-enhancement or pride. With such introjected regulation, the students’ reasons for engaging in an activity are a bit more autonomous, but they still experience the reasons as being imposed on them. Identified regulation is a type of extrinsic motivation in which a person values to some extent the importance of a task, and thus accepts the process of regulation as their own. Students may also have more autonomous forms of extrinsic motivation and engage in a problem-solving activity because they find it valuable and have identified its regulation as their own (identified and integrated extrinsic motivation). Students with less autonomous forms of extrinsic motivation (external and introjected motivation) may find feedback requiring that they search for a solution method by themselves less useful than students with intrinsic motivation, or identified, or integrated extrinsic motivation because it requires substantial effort, and they do not consider engaging in task solving as personally valuable or enjoyable. The overall positive relationships between intrinsic motivation and learning outcomes is supported by a recent meta-analysis (Howard et al., 2021). Thus, one of this thesis’s objectives is to investigate whether the use of feedback and its perceived usefulness are affected by the students’ different types of motivation. Based on self-determination theory (Deci & Ryan, 2000), it has been anticipated in the study that the usefulness of feedback and its perceived usefulness might be positively related to intrinsic motivation and identified regulation, and probably negatively related to introjected and external regulation (see 3.4.2).
2.8 Goal orientation

Goal orientation theory (also known as achievement goal theory) defines students’ reasons for engaging in various achievement behaviors for learning. This theory also focuses on the reasons why and how students learn (Dweck, 1986; Dweck & Leggett, 1988). Mastery goals and performance goals are considered as the two primary goal orientations in this theory (e.g., Dweck, 1986). Studies have presented how students respond to task difficulty and failure based on these two types of goal orientations (e.g., Dweck & Leggett, 1988). For example, students who apply a mastery goal orientation will participate in a task for reasons such as challenge, curiosity, and mastery (Pintrich, 1991). They will have the desire to increase their competence by developing new skills and mastering new situations (Dweck, 1986; Dweck & Leggett, 1988; Shute, 2008). Students who have a performance goal orientation will participate in tasks for reasons such as grades, awards, and performance (Pintrich et al., 1991). They will focus on demonstrating their competence to others and receiving a positive evaluation from others (Dweck, 1986; Shute, 2008). Furthermore, these two goals orientations are divided into four types of goals based on approach and avoidance: mastery-approach, mastery-avoidance, performance-approach, and performance-avoidance (Elliot, 1999; Elliot & Thrash, 2001). Avoidance and approach orientation differ in that “approach motivation, behavior is instigated or directed by a positive or desirable event or possibility, whereas in avoidance motivation, behavior is instigated or directed by a negative or undesirable event or possibility” (Elliot, 1999, p. 170). However, a trichotomous model subdividing only performance goals is frequently used in educational research (Murayama & Elliot, 2009; Vedder-Weiss & Fortus, 2012). Thus, students who pursue performance-approach goals are focused on attaining favorable appraisals of their competence, but students who pursue performance-avoidance goals are focused on avoiding unfavorable appraisals of their competence (Elliot, 1999; Vedder-Weiss & Fortus, 2012).

Ford et al. (1998) found that learning goal orientation is associated positively with metacognitive activity, while performance goal orientation has no significant relationship with metacognitive activity. In addition, studies have found that the performance-avoidance goal associated negatively with metacognitive activity (e.g., Schmidt & Ford, 2003). Since performance goals have been shown to be less adaptive than mastery goals when solving challenging tasks (Linnenbrink-Garcia et al., 2008), students with performance goals may perceive feedback that requires productive struggle and new thinking as less useful. However, the desire to avoid a negative
attainment, for example, and the fear of failure, which is assumed to prompt the adoption of performance avoidance goals (Elliot, 1999), might lead to task engagement and applied effort, and thus might be associated positively with task-solving success and the perceived usefulness of feedback. Nevertheless, very few studies report on students’ perception of provided feedback and whether it might depend on the students’ achievement goals. However, students in a study by (Rakoczy et al., 2013) perceived process-oriented feedback as more useful than social-comparative feedback, and mastery goal orientation moderated the effect of feedback on perceived usefulness. Thus, exploring the relationship between students’ different achievement goals and the use of feedback and perceived usefulness might offer some insights into how students learning via problem solving may be assisted. How students’ achievement goals may moderate how they respond to feedback is further detailed in Section 3.4.2.

2.9 Summary

In chapter 2, central notions regarding the process of problem solving, reasoning, and formative assessment have been derived from the literature and used to formulate guiding principles for the intervention in Study 3 and Study 4. These two studies intend to study feedback that could help students continue reasoning during the problem-solving process, while at the same time ensuring that responsibility for solving the task remains with the students (Brousseau, 1997). Therefore, I have chosen metacognitive feedback as a first step in providing feedback. In Study 3 and Study 4, the students solved tasks with the help of a computer-assisted formative assessment tool that encouraged them to assess their difficulties so that they could then receive suitable feedback from the computer based on their self-diagnoses.
Figure 2: Research framework of this thesis.

Figure 2 provides an overview of this thesis research framework. First, the figure denotes the components that have been used to build the computer tool. That is, two components of formative assessment (see Section 2.3) were utilized; feedback and self-assessment. Three types of feedback were used in the computer tool, feedback on task level, process level, and self-regulation level. As part of process level and self-regulation level, feedback questions targeting students’ metacognition and use of heuristics were offered to students as support for their problem solving. Self-assessment was facilitated based on problem solving process (see Section 2.1). To design metacognitive feedback and problem-solving tasks, the theory of didactical situation (TDS) (see Section 2.4) was used as framework. Then it was assumed that task-solving success using the tool and the perceived usefulness of the feedback might depend on student’s self-efficacy, types of motivation, goal orientation and national test grade. It is also assumed that perceived usefulness of the feedback might mediate by task-solving success using the tool.
3. Materials and Methods

This thesis includes four studies. In the first study, the goal was to propose an elaborated model of the well-used and highly cited feedback model by Hattie and Timperley (2007). The elaborated model focuses specifically on feedback in mathematics. While conducting the first study, it was noticed that not all studies presented information concerning the effectiveness of the given feedback. That led to a second study in which my goal was to investigate feedback characteristics that affect student achievement in the subject of mathematics. Based on what I learned from these two studies, such as the characteristics of feedback provided and its effectiveness, I designed a computer application to offer students engaged in problem solving the opportunity to self-diagnose the difficulties they are having and receive feedback based on that diagnosis. The third study showed, among other things, that participating students perceived the usefulness of the provided feedback very differently. This result led me to conduct Study 4, which focused on associations between the students’ different characteristics and feedback perception. Table 2 provides an overview of the studies research methods as well as the data sources. In the following sections, I describe the research method for each study; the final part (Section 3.5) concerns research validity and ethical consideration.
Table 2. An overview of the studies’ research methods

<table>
<thead>
<tr>
<th>Research component</th>
<th>Study 1</th>
<th>Study 2</th>
<th>Study 3</th>
<th>Study 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td>Model development</td>
<td>Review study</td>
<td>Intervention study</td>
<td>Intervention study</td>
</tr>
<tr>
<td><strong>Sample</strong></td>
<td>110 papers</td>
<td>13 papers</td>
<td>17 students</td>
<td>134 students</td>
</tr>
<tr>
<td><strong>Data Collection Methods</strong></td>
<td>Systematic literature review</td>
<td>Systematic literature review</td>
<td>Math test, TAP, interview, computer log, audio recording</td>
<td>Math test, computer log, surveys</td>
</tr>
<tr>
<td><strong>Data Analysis Methods</strong></td>
<td>Thematic analysis (deductive and inductive)</td>
<td>Systematic Review/Meta Synthesis</td>
<td>Content analysis</td>
<td>SEM</td>
</tr>
<tr>
<td><strong>Outcome</strong></td>
<td>Conceptual model construction</td>
<td>Synthesize d findings from research literatures</td>
<td>Identify whether and how the tool is helpful or not</td>
<td>Identify the association between students’ characteristics, use of tool and perception</td>
</tr>
</tbody>
</table>

Notes: SEM = structural equation modelling; TAP = Think-aloud protocols.

### 3.1 Research method for Study 1

The first study addresses the first research question: Which characteristics of feedback to students are studied in the mathematics education research literature? To answer this research question, a thematic analysis methodology was conducted (Braun & Clarke, 2006) with a systematic literature review to elaborate the Hattie & Timperley’s (2007) feedback model (see Section 2.3) and to identify the characteristics of feedback that are addressed in studies on feedback in mathematics education research. Such an elaborated
feedback model may be valuable in several ways. It may contribute to the recognition of feedback as a complex and differentiated construct that has different effects on student learning, depending on the many facets of the feedback characteristics, which has been argued to be an important recognition (Wisniewski et al., 2020). It may also be used for decisions on how to categorize feedback in reviews and meta-analyses of the effects of feedback, and in analyses and syntheses of research findings. Moreover, it may be used in more in-depth studies of how different feedback characteristics work to improve learning. Finally, it may be used to identify the landscape of feedback characteristics the research community focuses on, and as an inspiration for new ideas for future research. Insights into the feedback characteristics that have been addressed in mathematics education research may, for example, be used to identify those characteristics that are in particular need of more research. To ensure methodological thoroughness, the procedures for the first study took into consideration typical systematic reviews as prescribed by Cooper et al. (2018) and Gough et al. (2017). The data collection and data analysis methods are described in the following sections.

3.1.1 Data collection

To locate literature in the review process, terms were searched using the terms feedback, formative assessment, and assessment for learning in the title, abstract, and keywords of journals listed in the database. The feedback term was complemented by the two assessment terms since feedback is an integral part of these practices, and research on formative assessment and feedback for learning is likely to include research on feedback as well. To narrow down the search results more precisely, I applied filters such as language (English), publication type (academic journal), etc. An initial search for publications from before 2016 was conducted using the MathEduc (former MATHDI) database that specialized in mathematics education publications. While MathEduc closed permanently in 2019, a second search for publications from 2016 to 2021 was done using the ERIC (Education and Resources in Education) and Academic Search Premier databases. The search resulted in 658 distinct articles in MathEduc database published before 2016. In addition to this, 204 records were identified from the ERIC and Academic Search Premier databases published from 2016 to 2021.

Next, studies were identified that were eligible for a complete review by reading each abstract. The references retrieved were exported to Endnote version X8.2, and the full texts of the studies were retrieved. Only 110 papers were eligible for analysis, while 752 articles were excluded for any one of the
following reasons: (i) the studies did not focus on feedback to students (instead they focused, for example, on feedback to teachers about classroom dynamics, students’ perceptions or perspectives on mathematics or formative assessment), (ii) the topics addressed were other than mathematics, such as computer programming courses, etc., and (iii) the studies did not provide sufficient details about the feedback.

3.1.2 Data analysis

Text data can be analyzed in different ways, for example by thematic analysis, content analysis, open coding etc. (Hsieh & Shannon, 2005). In thematic analysis, there are two main ways of identifying themes (e.g., categories of feedback); deductive approach or inductive approach (Braun & Clarke, 2006). In a deductive approach, data is scrutinized in a top-down process where codes or themes are driven by particular theoretical or analytic interests (Boyatzis, 1998; Braun & Clarke, 2006). On the other hand, the inductive approach is “a process of coding the data without trying to fit it into a pre-existing coding frame” (Braun & Clarke, 2006, p.83). This can be described as a bottom-up process where the identified themes or codes are strongly connected to the data themselves (Patton, 1990; Braun & Clarke, 2006).

To answer the first research question concerning what types of feedback that are described in the research literature, both a deductive and an inductive approach was followed in analyzing the data (see Section 3.1). First, coding was conducted for all feedback descriptions from the literature using the four levels of feedback described by Hattie & Timperley (2007): feedback about the task (FT), about the processing of the task (FP), about self-regulation (FR), and about the self (FS) (see Section 2.3). That is, categories were derived from existing research and theory as our initial guidance, which fits the description of a deductive approach. Secondly, a search was conducted for similarities and differences in the characteristics of the feedback categorized as belonging to each feedback level so as to identify different categories within each level, and then compared these categories to identify possible characteristics that occurred at all feedback levels. The analysis ended up identifying several level-independent feedback variables as well as a number of level-specific feedback categories. This second approach was inductive since we coded the feedback descriptions without trying to fit it into a preexisting coding frame. Furthermore, Braun and Clarke (2006) provided an outline guide through the six phases which is a recursive process. This guideline was followed in the analysis of the data.
In total, 110 studies were read, summarized, and entered into a database. To do so, an Excel file was created to summarize details about the methods and findings in each study. The 110 selected studies were summarized in an alphabetically ordered table containing general information (author, country, year of publication, title, journal) and specific information (the student sample analyzed in the study (e.g., number of participants, age or grade level), feedback types and subtypes, mathematics aspects, research types, the intervention (e.g., the specific instructional improvement strategy used), the measuring method used (e.g., properties of tests), and the results (e.g., effect sizes), etc.

3.2 Research method for Study 2

The second study addresses the second research question: Which feedback characteristics affect student achievement in mathematics? To answer this research question, we conducted a systematic literature survey approach. Existing reviews on the effects of feedback on student achievement have mainly focused on feedback independently of the subject, and they most commonly do not go into detail about the characteristics of the feedback (e.g., Hattie & Timperley, 2007; Shute, 2008; Wisniewski et al., 2020). Existing reviews focusing on the effects of feedback in the specific subject of mathematics usually focus on a particular learning goal or a particular type of feedback. For example, Fyfe and Brown (2018) studied only one mathematics component (i.e., the mathematical equivalence problem) and one characteristic of feedback (i.e., the correctness of the student’s responses). Therefore, there is a need to identify which characteristics of feedback that have an effect on student achievement in the specific subject of mathematics.

3.2.1 Data collection

The figure 3 below shows the PRISMA (Haddaway et al., 2022) flow chart describing how papers in the study were identified, screened, analyzed, and included. Data were collected in the same way as for Study 1, but to identify studies investigating feedback efficiency, two additional inclusion criteria were added to Study 1’s criteria: (1) only quantitative results are included in the analysis, and (2) results on feedback efficiency were considered only when based on students’ test performance (e.g., not the results of students perceived usefulness about feedback, the impact of feedback on interest development, etc.). As a result, 13 empirical studies were identified for the final analyses.
3.2.2 Data analysis

The 13 identified articles were recorded in an Excel file containing publication information (author, country, year of publication, title, journal), student information (e.g., number of participants, age, or grade level), feedback characteristics (see coding scheme below), type of intervention (e.g., instructional strategy), the format of feedback delivery (teacher/computer), the measuring method used for student achievement (e.g., properties of tests), as well as the results in terms of effects on student achievement (e.g., statistical significance levels, effect sizes), etc.

Since available studies are few and the feedback investigated in these studies differs with respect to the characteristics and conditions in which they were provided, we did not conduct a meta-analysis using Cohen’s d. Instead, we identified the characteristics of the feedback given in each of the 13 studies,
and whether the feedback had been found to have a (positive or negative) statistically significant effect on student achievement. The four levels of feedback described by Hattie and Timperley (2007) were used as the main categories to identify the feedback characteristics (see Section 2.3). However, even though feedback pertaining to each level has the same main characteristics in relation to the level’s definition, they may still differ in other characteristics. Therefore, similarities and differences both within and between these categories were identified. Comparisons were then made between feedback on the four levels concerning the effects on student achievement. In addition, the similarities and differences in feedback characteristics within and between these four levels were used to analyze why some feedback affected achievement while other did not.

3.3 Research method for Study 3

The third study addresses the third research question: How can formative assessment from a computer assist students with problem-solving when they encounter difficulties? To answer this research question, I needed to design a study that would collect detailed information on the difficulties the students encountered, their self-diagnosis, and how they used, perceived, and possibly were helped by the feedback they received. The focus of this study was on how to help students solve problems when they encounter difficulties by emphasizing the use of computer-based formative assessment as an example of an approach for providing such support. To accomplish this, a web application was designed that includes problems to solve, diagnoses to choose from (i.e., self-diagnosis by identifying one’s difficulty), and feedback designed to fit each diagnosis. I present the details in the following sections.

3.3.1 Data collection:

**Web application:** A web application was constructed specifically for this study that includes tasks designed as problems (see Section 2.1), diagnosis statements, and feedback. The web application was designed to be autonomous. After logging in and reading the instructions on how to choose among the diagnosis statements and follow the feedback provided, the students click “Start” to initiate the problem-solving session. The web application then presents the first problem along with a box to submit the answer, as well as a timer counting down from 10 minutes, and five diagnosis statements (see Figure 4). When a student encounters any difficulty, they can click on the diagnosis statement that best corresponds to their difficulty, and metacognitive feedback (see Section 2.3) is shown. If the metacognitive
feedback does not help, the student can choose “I need more help,” and heuristic feedback is shown (see Figure 4). Thus, if the student is helped by metacognitive feedback, they can decide not to ask for heuristic feedback. The students have 10 minutes to solve each problem, and if their answer is correct, the next problem is presented. If the answer is incorrect, they can try again as many times as they need within a time frame of 10 minutes. The diagnoses statements and feedback are the same for all problems.

Figure 4: The web application: example of a problem and the five diagnosis statements

**Task design:** The goal of the task design was in line with theory of didactical situations in mathematics (see Section 2.3). That is, the tasks were designed as problems, which means that since the solution method was not already known by the students, they instead needed to create (parts of) the method by themselves. Ten problems requiring students to create methods were chosen for this study. Care was taken to minimize the calculation and linguistic challenges. The problems were designed to be solved using different kinds of methods.
**Formative assessment design:** To design a computer-based formative assessment tool, two aspects were considered: (i) diagnosis, and (ii) feedback (see Section 2.2). In other words, to support the student when they encountered difficulties during problem-solving, each one of the problems were accompanied by five diagnosis statements in the form of suggestions as to what kind of difficulty they might have. In line with theory of didactical situations in mathematics, the key idea here is to support the student’s problem-solving process without providing the solution method. To design the diagnosis, Schoenfeld’s (1992) six phases (see Section 2.1) of problem solving were considered and transformed into five phases representing problematic situations in which students might encounter difficulties. For example, Schoenfeld’s reading phase was transformed into “Interpretation” of the task because students not only read the task, but also need to interpret the meaning of the task. In addition, I merged Schoenfeld’s analysis and exploring phases into one phase—the “Exploration/Analysis” of the task—because they do not necessarily occur sequentially. Schoenfeld (1985) planning phase was transformed into the “Hypothesis” phase, as inspired by Lampert (1990), because students need to hypothesize a solution method and justify their hypothesis before actually applying the method. Thus, the “Justification” and “Application” phases were adopted instead of Schoenfeld’s implementation and verification phases. Hence, the five phases—(1) Interpretation, (2) Exploration/Analysis, (3) Hypothesis, (4) Justification, and (5) Application—were implemented in the design of the diagnostic statements used to identify student difficulties. The interpretation of the task includes any actions involved in reading the task and understanding the meaning of the task—in other words, what the task is asking for. As a student begins clarifying the meaning of the task, the Exploration/Analysis phase occurs, which includes actions to break down the task into its parts, or searching for relevant information that can be used later in the problem-solving process. During the hypothesis phase, the student constructs a method that has the potential to solve the problem. Afterwards, the student attempts to justify the hypothesis, and when this is done, the student will attempt to apply the method. As a result, five diagnosis statements were developed targeting the difficulties students might face during the problem-solving process (see Table 3).
Table 3: Diagnosis table

<table>
<thead>
<tr>
<th>Diagnosis</th>
<th>Diagnosis statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation</td>
<td>I have difficulty understanding the task information or</td>
</tr>
<tr>
<td></td>
<td>what is asked for.</td>
</tr>
<tr>
<td>Exploration/Analysis</td>
<td>I don’t know how to begin solving the task.</td>
</tr>
<tr>
<td>Hypothesis</td>
<td>I have tried different approaches but still can’t find a</td>
</tr>
<tr>
<td></td>
<td>method to solve the task.</td>
</tr>
<tr>
<td>Justification</td>
<td>I have found a possible solution method but am not sure</td>
</tr>
<tr>
<td></td>
<td>if it is suitable or not.</td>
</tr>
<tr>
<td>Application</td>
<td>I know for sure that my solution method is correct but</td>
</tr>
<tr>
<td></td>
<td>the answer is still wrong.</td>
</tr>
</tbody>
</table>

The purpose of these diagnosis statements is to provide suitable feedback based on the students’ difficulty. To design suitable feedback, Schoenfeld’s (1985) four characteristics of problem solving are considered (see Section 2.1). When students self-diagnosed their difficulty by choosing any of the statements, metacognitive feedback was provided (i.e., to initiate monitoring and control; in other words, self-regulation level feedback). If the students decided they needed more help, they could then click on “More Help” and heuristic feedback was provided (i.e., concerning general suggestions for solution strategies). Metacognitive and heuristic feedback was the same for all problems. Additionally, the characteristics of the metacognitive and heuristic components (see Section 2.3) were employed to design suitable feedback. Feedback regarding beliefs and resources was excluded. Belief concerns one’s worldview of mathematics, which takes a long time to develop and thus might be difficult to change and measure during the short intervention of this study. Resource feedback is always task-specific and concerns basic knowledge and tools (e.g., procedures, facts, definitions, concepts). However, this study was designed to suggest and evaluate general feedback instead of task-specific feedback, and therefore resource feedback was excluded.

Participants: The study participants were first-year university students enrolled in the basic year (basår in Swedish) program’s mathematics course. More than 180 students were contacted by email. In addition, students were contacted at lectures held at the University campus. Many of these students were positive about participating. Nonetheless, a vast majority finally chose not to participate. Seventeen students, however, ended up volunteering.
Think-aloud protocols: Think-aloud protocols have been used as a data gathering tool since it has been argued that the reasoning process “may contain more data than an ordinary written solution, which may be supplemented by, for example, think-aloud protocols and interviews” (Lithner, 2008, p.257). Since the aim was to get detailed information on how diagnosis and feedback offer opportunities for the students to overcome the difficulties they encountered, the students were asked to think aloud during their problem solving. The students’ think-aloud protocol was documented as audio recordings. Additionally, smart pen and screen recordings were used to record when and what kinds of feedback students asked for during the problem-solving process. All data were logged in the computer. The student’s problem-solving method was observed following an observation protocol.

Interview: Semi-structured interviews were conducted with the participants after the problem-solving session. The aim of the interviews was to understand the students’ points of view, why they chose certain diagnoses, and whether and how the received feedback helped them (or not) continue solving the problem. During the interviews, open-ended questions were asked to give the students the opportunity to share, in their own words, their experiences using feedback (Creswell, 2012). This was done to address more specific issues (Bryman, 2012; Kvale, 2007), such as whether feedback helped them to progress with the task or not, and how or whether they faced difficulty locating their encountered problem in the diagnosis. The interviews were audio recorded.

3.3.2 Data analysis

To answer the research questions, all data from the screen recordings and audio recordings were analyzed task by task for each student using a content analysis approach (Mayring, 2014). According to Hsieh and Shannon (2005), there are three ways one can analyze text data: conventional content analysis (i.e., codes are derived directly from text data), directed content analysis (i.e., codes are derived from theory and/or relevant research), and summative content analysis (i.e., keywords are derived from content and/or reviewed literature). In Study 3, I derived categories from existing research/theory by Schoenfeld (1985) as my initial guidance which fits the description of a directed approach of content analysis. Additionally, I categorized different characteristics from the data, which fits the description of a conventional content analysis. The analysis was carried out following the two steps below:

The first step consisted of identifying problematic situations when students encountered difficulties, and how they interacted with the computer: all
problematic situations were noted for each task. These were identified as any situation in which a student intermitted their problem-solving process and asked for help from the computer by choosing one of the suggested diagnoses (see Table 3). Thereafter, each problematic situation was categorized according to the student's diagnosis. Each problematic situation was divided into two sub-categories under five diagnosis levels: receiving metacognitive feedback and receiving heuristic feedback. Here I used quantitative coding of the data; this included how many times students received certain feedback and what they did after receiving the feedback (skipped the feedback, followed the feedback, gave up, ran out of time, etc.).

The second step consisted of identifying if and how the chosen feedback helped students to continue their problem solving: Data from the thinking-aloud protocol and interviews were examined to determine whether the provided feedback helped the students to resume the problem-solving process. Resuming the problem-solving process is considered the result of receiving feedback based on three aspects: (i) if there were signs the students resumed the problem-solving process or made progress right after receiving/reading feedback, (ii) if their reasoning (i.e., thinking aloud) referred to the received feedback, and (iii) they were positive about received feedback during the interview (for more details see Article 3).

### 3.4 Research method for Study 4

The fourth study addresses the fourth research question: How are students’ different characteristics associated with task-solving success using feedback and the perceived usefulness of feedback intended to support students’ reasoning in problem-solving? Since the purpose of this research is to measure the associations between variables, a factorial experiment (Bryman, 2012; Cohen, 2017) was conducted. This type of experiment was chosen since our aim was to assess the relationship between independent variables (e.g., self-efficacy beliefs, external motivation, intrinsic motivation, mastery goals), and dependent variables (i.e., the perceived usefulness of feedback and the proportion of successfully solved tasks when receiving feedback). The intervention design of Study 4 was similar to Study 3; the students solved similar problems and were provided the same computer-based support for diagnoses and feedback.
3.4.1 Data collection

Participants: The participants (N = 134), 82 females and 52 males, were upper secondary school students enrolled in the Business program at a public high school in a mid-sized city in Sweden. Required sample sizes were estimated using a power analysis with R-package semPower (Moshagen & Erdfelder, 2016). Assuming a minimum achieved power of 0.8, a priori power analysis, conducted with semPower (Moshagen & Erdfelder, 2016), indicated that with an alpha of 0.05, an RMSEA of 0.08, 129 students were needed. The average age of the participants was 17.3 years old (SD = 0.76). Participation in the study was voluntary, and the students had given their informed consent to participate. All participants received a gift card (20 euros) as their participation incentive.

Problem-solving session: In Study 4, the students solved six problem-solving tasks during a problem-solving session similar to the one described in Study 3. The data were logged in computer for collecting information about students’ choice of feedback and whether or not students were able to solve the problem after receiving feedback.

Survey data: Two surveys were developed and distributed digitally. The first survey included items about the students’ national test grades in mathematics, self-efficacy, achievement goals, and intrinsic and extrinsic forms of motivation. The items used in the first survey were adopted from a study by Hofverberg et al. (2022). The second survey, taken after the task-solving session, was designed to measure the students’ overall perceived usefulness of feedback. The students were asked to rate three items on a 5-point scale ranging from 1 (completely disagree) to 5 (completely agree). The key item was “I would consider this feedback useful”. Cronbach’s alpha was calculated at 0.937, which suggests a good internal consistency of the scale. These items are adopted from a study by Strijbos et al. (2021).

Study procedure: After a brief introduction, the students logged in to a web survey to take the first survey. The students could thereafter choose when to start the web application and begin the problem-solving session (this is the same as described in Study 3). The students had 10 minutes to solve each of the six problems. After completing the problem-solving session, the second survey appeared automatically. They received compensation for their participation at the end.
3.4.2 Data analysis

All the collected data (e.g., test results, survey data) in Study 4 were quantitative, so the data were statistically analyzed. The computer log file was analyzed to determine whether the students’ received feedback or not (details are provided in Article 4). All statistical analyses were conducted in SPSS\(^3\), Amos and R-package. The structural equation modelling (SEM) analytical framework was chosen because SEM excels in analyzing relationships among observed and latent variables (Bollen, 1989; Byrne, 2012; Kline, 2015). Byrne (2012) summarized four aspects of SEM that distinguish it from a number of other statistical approaches (e.g., multivariate): First, SEM adopts a confirmatory as opposed to an explanatory approach; this provides the opportunity to evaluate the existing theory by conducting hypothesis testing. Second, SEM allows for measurement errors in order to obtain an unbiased estimation, in contrast with most other multivariate measuring methods that largely disregard their existence. Third, SEM can integrate both observed and unobserved variables while the other multivariate methods use only observed variables. Fourth, SEM is able to measure indirect effects which other statistical methods do not generally measure. Given these advantages over other multivariate methods, SEM was used to explore the association among the variables in Study 4.

The analysis process begun with analyzing computer logged data from each student’s task-solving activities to determine (a) whether students received feedback, and (b) whether students managed to solve the tasks for which they received feedback. After that, the proportion of successfully solved tasks for which feedback was received (PSTF) was computed for each student. Then, the descriptive statistics (e.g., mean, standard deviation, Pearson correlation, etc.) were computed for all variables using R-package. After that, structural equation modelling (SEM) analysis were computed in Amos 28 via three different models to assess the relationships between independent variables (i.e., grade, self-efficacy beliefs, external motivation, introjected motivation, identified motivation, intrinsic motivation, performance avoidance goals, performance approach goals, and mastery goals) and dependent variables (i.e., perception of usefulness of feedback and the proportion of successfully solved tasks when receiving feedback). To test for the significance of indirect effects, 1000 bootstrap samples with Monte Carlo simulation and estimated 95% confidence intervals (CIs) were used. Additionally, to test the effect of

\(^3\) Statistical Package for the Social Sciences
the sample size for each model, the R-package semPower (Moshagen & Erdfelder, 2016) was used.

3.5 Research credibility and ethical considerations

In this section, first, the included studies’ validity and reliability are explained and discussed. Then, ethical considerations are discussed.

3.5.1 Reliability

Reliability refers to “dependability, consistency and replicability over time, over instruments and over groups of respondents” (Cohen, 2017, p. 199). The purpose of reliability is to reduce the errors and biases in a study (Yin, 2009). Throughout my thesis, I increased the reliability and validity of my findings by providing transparent, detailed information on the different stages of the research process, which is regarded as an important aspect of research (Creswell, 2012). To enhance reliability, different types of data collection procedures were used to capture audio data as well as visual data from screen recordings. When analyzing data in Study 3, student excerpts were used from these recordings to make data accessible to the reader. Moreover, reproducibility and stability were achieved by explicitly presenting the analysis methods, and analyses were continuously discussed and developed in collaboration with the other members of the research group. I have also kept records of all the data that I gathered for enhancing the reliability of the research findings (Bryman, 2012; Yin, 2009). Additionally, pilot testing of the design was performed, and interviews were conducted and modified accordingly to enhance the reliability of the research tools in both studies. To develop the computer tool for Study 3, students were asked what they found problematic, such as, for example, if some words were unclear. The students’ suggestions were taken into consideration to further develop the tool. Additionally, the pilot testing results were used to modify the interview questions; this was done to ensure that the questions were clear and asked for the correct thing.

Overall, the PhD project is partially applying quantitative methods to data. Therefore, the issue of reliability in quantitative research was addressed by providing statistical test results; for internal consistency, the Cohen’s kappa d (Bryman, 2012) was calculated.
3.5.2 Research validity

The term validity refers to the rigor of research (Creswell et al., 2011), and there are different kinds of validity (Cohen et al., 2017). For example, Construct validity denotes the agreement between a theoretical construct and the operationalization, clarifying what we mean when we measure these constructs (Cohen et al., 2017; Shadish et al., 2002). To increase the construct validity of my research, all the studies provide theoretical frameworks and interpretations of the constructs of interest; for example, the diagnosis and feedback design for data collection conform to the theoretical context from Schoenfeld’s (1992) work.

Internal validity seeks to demonstrate whether causal inferences can be drawn from the covariation between predictor and outcome variables (Shadish et al., 2002), that is, “the explanation of a particular event, issue, or set of data which a piece of research provides can actually be sustained by the data” (Cohen et al., 2017, p.183). In Study 4, path coefficients were used to show the relation between the independent and dependent variables. Study 3 achieved internal validity by providing a detailed description of the data collection and data analysis methods, and it tried to minimize bias through structured interviews. The analysis in Study 3 consisted of detailed descriptions of students’ difficulties as indicated in excerpts from think-aloud protocols. In addition, I have continuously discussed my data analyses with my three supervisors to reduce individual bias. The analytical process was also discussed in different research groups to strengthen the credibility of my research. Moreover, student quotes from the interviews are included in the analysis to make the data transparent to the reader.

External validity indicates “the degree to which the results can be generalized to the wider population, cases, settings, times, or situations, i.e., to the transferability of the findings” (Cohen et al., 2017, p.186). In Study 3, which is qualitative research, external validity was attained by providing a detailed and in-depth description of the research process so that others can assess whether transferability is possible (Cohen et al., 2017). Both for Study 3 and Study 4, the specific problem-solving tasks used as data-gathering tools were developed and tested in other studies (e.g., Jonsson et al., 2014). These tasks were designed to lead students into problem-solving activities and were pilot-tested in a couple of cycles.

To strengthen the validity of my results, different methods in triangulation are included (Creswell, 2012). Study 3 is grounded on three types of data:
audio recordings of thinking aloud, interview data, and screen recordings of the test.

3.5.3  Ethical considerations

Study 3 and Study 4 conform to the Swedish Research Council’s Ethical Principles. The involvement of the participants in the studies was in line with the obligation to respect integrity, freedom, and the right to participate. Informed consent was provided by participants after specifying the research objectives. In addition, I explained the structure of the tests and interviews to participants, and they were given the opportunity to ask questions and request more information. All participants in the studies participated voluntarily. All data is presented in anonymous format.
4. Results

In this chapter, I present a summary of each of the four articles included in this thesis that answer the four research questions (Section 1.1) posed in this thesis. In previous chapters, I elaborated on the research’s framework and methods, so this chapter mainly focuses on the summary of the studies’ results.

4.1 Article 1

Article 1 addresses the first research question: Which characteristics of feedback to students are studied in the mathematics education research literature? Which characteristics of feedback to students are studied in the mathematics education research literature? To answer this research question, a model of feedback was developed based on a model by Hattie & Timperley (2007), and from the analysis of the 110 publications found in the literature search. The model consists of six feedback level-independent variables (see Table 4), and a number of level-specific feedback categories (see below). Characteristics of feedback can be described in relation to these variables and categories.

The analysis also showed that most studies addressing feedback do so for feedback on task level (66 studies) and process level (47 studies). Fewer studies have investigated feedback on self-regulation level (13 studies) and feedback on self-level (13 studies).

Table 4: Feedback level-independent variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means of communication</td>
<td>computer, face-to-face, written, oral.</td>
</tr>
<tr>
<td>Sign system</td>
<td>words, pictures, colors.</td>
</tr>
<tr>
<td>Timing</td>
<td>during task solving, after one task, after several tasks.</td>
</tr>
<tr>
<td>Frame of reference</td>
<td>students’ own responses, correct task solutions, other students’ responses.</td>
</tr>
<tr>
<td>Explicitness</td>
<td>explicit, e.g., “correct”; implicit, e.g., “nice.”</td>
</tr>
<tr>
<td>Opportunities of using feedback</td>
<td>Prompts to modify a task response, possibility of using the feedback for subsequent tasks or on other occasions.</td>
</tr>
</tbody>
</table>
**Level-specific feedback categories:**

At the task level, the analysis identified the following categories of feedback: (i) information about the correctness or quality of the answer, (ii) explanations as to why the answer is correct or not, and (iii) extra mathematical information added to the solution.

At the process level, the analysis identified the following categories of feedback: (i) information about (mis)understanding and errors in task solutions, (ii) information about how to proceed in task solving, and (iii) information about learning.

At the self-regulation level, feedback characteristics are identified that target: (i) students’ self-confidence, (ii) students’ answers to tasks, (iii) students’ task-solving processes, and (iv) students’ learning. Feedback was provided mainly via prompts, metacognitive questions etc.

For feedback at the self-level, the analysis identified four categories that differ in relation to the targets of the evaluations made in the feedback: feedback targeting students’ work, targeting students’ effort, and targeting students’ conduct.

In the discussion, the potential of proposed model, the similarities and differences with other research are made. Suggestions for how the model can be used in future research are also proposed.

**4.2 Article 2**

Article 2, reports on a literature review and addresses the second research question: Which feedback characteristics affect student achievement in mathematics? The literature review included 13 papers that investigated the effects of feedback on student achievement in mathematics. Four main categories were used to characterize feedback: feedback on task level, process level, self-regulation level, and self-level. In addition, similarities and differences both within and between these categories were analyzed to identify further feedback characteristics. Feedback characteristics that did and did not affect achievement were identified and described in greater detail.

The results showed that the effectiveness of feedback depends on feedback characteristics that give students sufficient motivational and cognitive support to use the feedback to engage in thinking about the mathematical
learning targets. Process and self-regulation level feedback were more effective for student achievement than no feedback and task-level feedback. For example, task-level feedback such as providing correct answers did not have a positive effect on student achievement. The effectiveness also depends on how feedback is provided. For example, process level feedback such as providing information about the correctness of the solution strategies did not have a positive effect on student achievement, while feedback at the process level was found to be effective when it was given, for example, in the form of tips on how to solve the tasks, explanations of correct or incorrect answers together with worked examples, strategic information for error correction, the correctness of each solution step, a hint on how to proceed with the next step, or a working example that shows all the solution steps simultaneously. Lastly, feedback at the self-regulation level was found effective when it was provided as metacognitive questions during and/or after the problem-solving process.

In the discussion, the results of the review were compared with the results from subject-independent reviews and suggestions for future research are provided. For example, more research regarding how different feedback characteristics influence students’ achievement in mathematics as well as studies conducted both in the contexts of classroom and laboratory are needed.

4.3 Article 3

Article 3 reports on an intervention study and addresses the third research question: How can formative assessment from a computer assist students with problem-solving when they encounter difficulties? The intervention study focused on computer-based formative assessment for supporting problem-solving and reasoning in mathematics. A formative assessment design was created to be able to assist students who are experiencing difficulties while solving problems. A web application was made for this purpose in which tasks are accompanied by diagnoses (suggested descriptions of the encountered difficulty the students could choose from) and feedback. The feedback was provided as general metacognitive and heuristic feedback. 17 first-year university students participated in this study.

Results showed that the problematic situations in which students most often asked for feedback were Exploration/Analysis (27.3%), Justification (27.3%), and Hypothesis (21.5%). Students asked comparatively fewer times for feedback on Application (10.7%) and Interpretation (13.2%). Students asked
for metacognitive feedback in 74.4% of cases and heuristic feedback in 25.6% of cases. It was found that students followed the feedback provided in 59.5% of the problematic situations (in the remaining situations, the students gave up in 3.3% of cases, ran out of time in 9.1% of cases, skipped feedback in 26.4% of cases, and chose another diagnosis in 1.7% of cases). When the students followed the feedback provided and did not run out of time, in most cases they were able to resume the problem-solving process (See Figure 5).

Figure 5: The success rate for students following feedback and not running out of time, in each of the five types of problematic situations. (MF = metacognitive feedback, HF = heuristic feedback.)

The findings provide insight into how the feedback guided students in justifying, monitoring, and controlling their problem-solving process. The student interviews indicated that the students, who managed to solve tasks after following feedback, in general perceived provided feedback as helpful. The feedback made them try again instead of giving up, motivated them to go back to the problem-solving process again, and suggested ways and strategies on how to proceed. The students stated that the feedback helped them to argue with themselves about why their method was right or wrong and to make judgments and go forward. Students who did not perceive feedback as useful pointed out that they would have preferred task-specific
feedback for some tasks, especially on those tasks they found difficult to solve.

In the discussion, the potential of providing general metacognitive and heuristic feedback during the problem-solving process was brought up based on the results found. From the study, it was observed that some students skipped metacognitive feedback, so a future study recommendation was provided regarding student perception of general metacognitive and heuristic feedback.

4.4 Article 4

Article 4 reports on an intervention study and addresses the fourth research question: How are a student’s different characteristics associated with the task-solving success using feedback and the perceived usefulness of feedback intended to support students’ reasoning in problem-solving? The study investigated how students' mathematical ability and motivational characteristics might influence how useful students perceive feedback that aims at supporting mathematical reasoning. In the study, students worked with mathematics problems and could choose metacognitive and heuristic feedback provided by a computer in the same way as in Article 3. Structural equation modeling (SEM) was used to analyze the association between the variables.

The results showed that the students’ type of motivation, achievement goal, national test grade, and self-efficacy had no statistically significant direct effect on the proportion of solved tasks when receiving feedback. It was found that students’ mastery goals had a direct effect on the perceived usefulness of the feedback, but no such effects were found with regard to the students’ national test grades, self-efficacy beliefs, performance goals, and intrinsic or extrinsic forms of motivation. The proportion of solved tasks when receiving feedback did not mediate effects on perceived usefulness. Table 5 shows the measurement models of the Confirmatory factor analysis (CFA) and structural models. The standardized factor loadings of the items ranged from 0.6 to 0.90 for each model.
Table 5. Measurement and structural model tests for the motivation type, self-efficacy, goal orientation, and grade.

<table>
<thead>
<tr>
<th>Model</th>
<th>Measurement model fit indexes</th>
<th>SEM fit indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^2$</td>
<td>CMI</td>
</tr>
<tr>
<td>Self-efficacy: Model 1</td>
<td>17.3</td>
<td>1.33</td>
</tr>
<tr>
<td>Goal orientation: Model 2</td>
<td>96.1</td>
<td>1.14</td>
</tr>
<tr>
<td>Motivation: Model 3</td>
<td>30.4</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: $\chi^2$ = chi-square statistic; RMSEA = robust root mean square error of approximation; CFI = robust comparative fit index; TLI = Tucker Lewis index; CMIN/df = chi-square/ degree of freedom.

In the discussion, the possible explanations of the results and implications for teaching were discussed.
5. Discussion

The overarching aim of this thesis was to gain knowledge about how students’ reasoning during problem solving processes can be supported. In this thesis, this has been studied through four studies. In this section, I will discuss the findings for each of the four studies (5.1-5.3). Afterwards, I will discuss the contributions of this thesis (5.4). Finally, I will end the chapter discussing the limitations and implications (5.5) of the current thesis.

5.1 Lack of research in some feedback types

Study 1 showed what characteristics of feedback are provided to students in mathematics education research. It was found that there is less research on some types of feedback. For instance, the studies addressing feedback in self-regulation are certainly informative for the mathematics education community; regardless, relatively few papers analyzed feedback on self-regulation in mathematics. Designing an effective learning environment requires a comprehensive understanding of where and how the knowledge and skills are generated. Therefore, if there is less research in certain areas (e.g., on regulation), it can lead us to significant disadvantages in understanding methods, theory, and applications through which research is conducted as well as restrain us due to the limited options to use research results in practice. Limited research on feedback in self-regulation is also a disadvantage for generating the knowledge and skills needed in teaching; This is also true for supporting individuals in education as well as understanding how teachers can best support their students’ learning. It is important for educators to be aware of the most important feedback mechanisms in order to transmit these skills in the classroom effectively. Moreover, mathematics, learners require an understanding of the rationale behind the different problem-solving methods and strategies to employ learned knowledge that is relevant to a new problem. Hence, feedback on self-regulation helps students to understand problem solving and take control of, and responsibility for, their own reasoning.

The results from Study 1 also highlight that, instead of providing progress feedback which focuses on the learner’s improvement compared to their previous performance, the feedback examined in this research focuses mostly on verification feedback at task level. A potential explanation for this finding may lie in the nature of using digital feedback tools for providing feedback. For instance (Keuning et al., 2018) have found from their systematic literature
review that, in general, the feedback that tools (i.e., automated learning platforms) generate is not very diverse, and mainly focuses on identifying mistakes. While doing the systematic analysis in Study 1, it has been noticed that 2/3 of studies have used digital feedback tools for providing feedback.

Despite many hundreds of studies published on feedback, we found very few studies that measured the effectiveness of different feedback for learning mathematics, as detailed in Study 2. Not surprisingly, Study 2 provides evidence that there is wide variation in the type and impact of feedback on student achievement. The results also demonstrate that the type of feedback is vital for furthering achievement in mathematics when comparing the efficiency of different kinds of feedback. Providing feedback on task level and providing ‘no feedback’ do not differ in achievement. The results highlight that providing feedback on processing the tasks was found to be superior to giving feedback on the task itself. Similarly, some researchers have suggested that feedback on the process (strategy feedback) may be more beneficial than feedback on the task (outcome-feedback) (e.g., Luwel et al., 2011). Providing feedback on how to proceed is helpful for both fixing problems and progressing towards a solution when stuck, whereas just pointing out an error may not help. However, a main contribution of Study 2 is that the effect of feedback cannot be determined solely by the task level it is at, it is decisive that the feedback provides students with the opportunities and incentives to use the feedback to think about and reflect on the mathematics they are to learn through the feedback.

In terms of mathematics achievement, the superiority of feedback on the process compared to feedback on the task may be greater than the results presented in this review because most of the studies were of short duration, and there are no studies on the long-term effects of feedback. For example, there is evidence that feedback on the process significantly increases student interest compared to feedback on the task (e.g., Rakoczy, et al., 2013). Therefore, the effects of process feedback on achievement may be even greater in the long run since interest increases a student’s engagement in learning.

5.2 The potential of providing formative assessment that supports reasoning

In Studies 3 and 4, a digital tool based on formative assessment was designed in which self-diagnosis was structured according to the problem-solving
process, and feedback consisted of metacognitive and heuristic support. Instead of being provided with specific feedback after each task as in other feedback studies (e.g., Narciss & Huth, 2006), the students received feedback that was general in this thesis. Now the question is whether general metacognitive and heuristics feedback is useful or not in supporting problem-solving.

In this thesis, the purpose of providing general metacognitive and heuristic feedback was to make students take primary responsibility for creating the task solution (Brousseau, 1997) (see Section 3.5). The feedback consisted of guidance in situations that engaged students while solving tasks without providing the solution method. Although the students only succeeded in solving half of the tasks to which they received feedback, the results of Studies 3 and 4 showed that students can implement successful strategies with the help of general metacognitive and heuristic feedback even though they were not provided task-specific feedback. That is, many of the students can construct a solution by engaging in problem-solving. This is in line with earlier research (e.g., Kapa, 2007), which has shown that metacognitive feedback is helpful for the long-term development of problem-solving abilities because of the self-reflection mechanism, which is an essential part of problem-solving skills.

Results from Study 3 showed that students could successfully use metacognitive feedback for exploration, strategy choices, justification, and verifying their mathematical ideas. In Study 3, it was also observed that students who received feedback in the first few tasks were using the feedback to solve subsequent tasks without explicitly reading the feedback again, thereby extending their understanding or learning strategies successfully when solving new tasks. Similar promising results have been obtained by other studies through the use of educational software (e.g., GeoGebra) for supporting students’ reasoning and problem solving (e.g., Kuzle, 2017; Olsson, 2018). In Studies 3 and 4, students spent only an hour on problem-solving, but it takes time to substantially develop problem-solving and reasoning skills. Giving students more responsibility (Brousseau, 1997) and authority in their learning of mathematics has previously been shown to be important in learning mathematics (Granberg, 2016; Kapur, 2014; Schoenfeld, 2020).

Brousseau and Warfield (2014) argued that the first step toward promoting mathematics learning is developing a didactical situation in which the student should take responsibility as the owner of their own knowledge construction.
Furthermore, one of the strengths of problem-solving research is that it forms a common ground wherein teachers, students, and researchers work together. Thus, the analysis of self-assessment and feedback supporting student reasoning in problem solving (Studies 3 and 4) can be seen as a gateway for supporting student learning. In this thesis, the general metacognitive feedback was provided based on the students’ self-diagnosis when they encountered difficulties. As a consequence, self-diagnosis was employed to support students’ problem-solving reasoning by requiring students to make individual choices. Thus, the design of this thesis’s diagnoses and feedback prompted students to engage cognitively in the learning process. This implies that the students were required to be involved in their own problem-solving process by clarifying/understanding the problem they were facing, and as such the process was characterized by a strong emphasis on the self-reflection step in the self-regulated learning cycle (Zimmerman, 2000). Zimmerman (2002) further emphasized that to help students become self-regulatory, they need to be asked to self-evaluate, reflect on, and self-monitor their work. In Study 3, some students mentioned that the self-diagnosis worked as a reminder for them to look back on the strategies and reflect on the encountered difficulties.

To encourage students to engage in problem-solving, the feedback should be designed in such a way that students need to think by themselves and justify their reasoning when solving the task. At the same time, some students might need heuristic strategy support as feedback. The results of this thesis showed that in many cases, metacognitive and heuristic feedback was sufficient for the students to resolve the problematic situations that occurred while problem-solving. In Study 3, it was observed that with the assistance of feedback, students were engaged in reasoning by creating arguments to support their solution (Lithner, 2008).

Students need to deal with the problem in order to gain new knowledge (Brousseau, 1997). Therefore, feedback should not provide students with the solution template. In Studies 3 and 4, it was clear that students could many times resolve the problematic situation with the help of feedback, but this was not always the case. Sometimes the students asked for task-specific feedback and showed some frustration at not getting the solution method as feedback. A possible explanation could be that the students were not used to getting this kind of feedback in their everyday mathematics lessons. This is, may be, because the way students engage in problem-solving represents an agreement grounded on the long-term interaction between the teacher and the student (Brousseau, 1997). Of course, general metacognitive and heuristic
feedback is not an optimal option for all tasks and for all students. Sometimes students might need task-specific feedback, such as, if it is too difficult for students to construct the solution method using their own reasoning. Nevertheless, this thesis showed that there is potential in providing general metacognitive and heuristic feedback to support student reasoning when creating a solution method (or part of the method).

5.3 Motivation for using feedback

I have not found any studies focusing on the effects of intrinsic or extrinsic forms of motivation on students’ perception of the usefulness of feedback. However, students in the study by Rakoczy et.al. (2013) perceived process-oriented feedback as more useful than social-comparative feedback, and mastery goal orientation moderated the effect of feedback on perceived usefulness (Rakoczy et al., 2013). With regard to the feedback investigated in Study 4, the students’ mastery goals played a significant role in their perception of feedback as useful. Similarly to Study 4, the students’ self-efficacy did not affect the perception of the usefulness of the process-oriented feedback investigated in the study by Rakoczy et al. (2019).

The only student characteristic that had a statistically significant effect on the perceived usefulness of the feedback was the students’ mastery goals. While mastery goals had a positive direct effect on perceived usefulness, performance-approach goals had a nearly significant but negative effect. These results can be understood by considering the properties of these goals. Students with mastery goals are driven by the goal of learning and are evaluating goal attainment in terms of task-based or intra-personal standards. Thus, feedback that focuses on conceptual issues and which requires the students to think instead of providing solution methods may be consistent with the mastery-goal students’ ideas of feedback usefulness. In contrast, students with performance goals who are driven by demonstrating competence to others may not perceive feedback as useful if it makes it difficult for them to continuously demonstrate their competence. These results are also consistent with studies that have found performance goals to be less adaptive than mastery goals when solving challenging tasks (Linnenbrink-Garcia et al., 2008). This means that to support students’ perception that the metacognitive and heuristic feedback is useful, it may be productive for teachers to support the students’ mastery goals and not their performance-approach goals. This would include teaching behavior focusing on each student’s learning in relation to themselves and specific standards,
and not focusing on competition and student learning in relation to each others.

There was a difference in setting between Study 3 and Study 4. In Study 3, students participated one-to-one with only the researcher sitting by their side. In Study 4 students were working individually in a classroom setting outside their lesson time but with no interaction with the researcher (or teacher). This might be the reason why feedback wasn’t used as much in Study 4. In Study 3, it might be that students were more polite to the researcher and tried to use feedback when they were stuck, whereas in Study 4, they were more independent and could choose whatever they wanted to do (i.e., they were not as closely observed by the researcher as in Study 3). Perhaps the students chose to not use feedback because of the autonomy they were given.

In Study 4, it was found that regardless of the students’ characteristics, they could often use the feedback they received successfully. Study 3 showed that the students were able to resolve their problem-solving difficulties when they received and followed the feedback. This indicates that feedback motivated the students to engage in effortful struggle with tasks, which is essential for mathematical understanding (Hiebert & Grouws, 2007).

### 5.4 Contributions

One of the primary aims of this thesis is to contribute to the empirical understanding of using computer-based formative assessment for supporting reasoning in problem-solving in mathematics. The design of the formative assessment, the analysis of how and to what extent feedback was helpful during the problem-solving process (Study 3), the association between different student characteristics, and the use of received feedback as well as students’ perception towards the given feedback (Study 4) provided insights into using formative assessment during problem-solving in mathematics. In addition, the thesis mentions contributions linked to the use of the theory of didactical situations, general metacognitive feedback, the problem-solving process, and formative assessment. Moreover, insight into the aspects of feedback characteristics for mathematics was gained by reviewing literature in a systematic way (Study 1 and 2). These contributions are discussed further below.

The empirical contribution of this thesis is that it increases our knowledge about formative assessment performed interactively by computers during the problem-solving process. Even though concepts of formative assessment and
problem-solving have gained a position in mathematics (e.g., Burkhardt & Schoenfeld, 2019) the findings of this thesis suggest that it is not an easy and straight-cut formula for all students, and a student’s characteristics play a role in using formative assessment. The use of formative assessment in the two studies seemed to be useful for students since many of them seem to successfully use self-assessment and apply the received feedback when they were stuck in the problem-solving process. As pointed out by researchers (e.g., Lester & Cai, 2016; Liljedahl & Cai, 2021; Lithner, 2017; Schoenfeld, 1992), students need to struggle when solving problems in mathematics tasks, and this thesis concerns formative assessment in problem solving.

Over the last couple of decades, mathematics curricula worldwide have stressed the importance of developing reasoning and problem-solving competencies among students (e.g., Hiebert, 2003; Niss, 2003; Palm et al., 2011). This thesis contributed to this by showing how formative assessment may be used to assist student reasoning during the problem-solving process.

Another finding addresses the association between a student’s characteristics and feedback perception. The findings in the two empirical studies suggest that students might not be used to getting this kind of formative assessment in their mathematics lessons; thus, they are not favorable towards receiving metacognitive and heuristic feedback while problem solving. This thesis provided evidence that general metacognitive and heuristic feedback can help if we want to support students’ problem solving without providing solution methods, which may lead to rote learning (e.g., Hiebert, 2003; Schoenfeld, 1985).

Another aim of this thesis was to investigate the existing studies that examine under which feedback conditions a student’s learning processes might lead to better learning outcomes and which might not (Study 1 and 2). One might ask what these studies add to the extensive body of knowledge about feedback in mathematics education. They reveal which characteristics of feedback researchers have considered when reporting on feedback to students in mathematics education contexts and which feedback characteristics that affect learning. That is, the proposed elaborated feedback model and the results of the review of which feedback characteristics that affect student achievement offer additional insight by breaking down how feedback characteristics can vary in mathematics and still have an effect on achievement.
Since the devolution of problems is a fundamental aspect of the theory of didactical situations, the student must take responsibility in the problem-solving process. To accomplish this, a suitable didactic situation was created by the instructor (i.e., through the computer) in the form of a problem with support. The instructor (computer) refrained from showing how to solve the task (but assisted with feedback), and the students took responsibility for solving the problem (details in Section 3.3). This thesis contributes by showing concrete examples of how this devolution can be implemented by providing general metacognitive and heuristic feedback instead of complete solution methods.

An additional contribution relates to the applicability of the formative assessment design. While the design was initially used to analyze mathematical problem solving, the results from Studies 3 and 4 suggest that teachers can also use the design to provide feedback on mathematical reasoning. Thus, the use of this design could support teachers in grasping notions of formative assessment and problem solving and in implementing this design when planning lessons and teaching approaches.

A substantial correlation between a student’s characteristics and the perceived usefulness of the provided feedback was observed. Mastery goal has a positive association with perceived usefulness (reported in Study 4). Being aware of these associations is important both from a research perspective, for example, when investigating metacognitive feedback for problem-solving empirically, and a teaching perspective, for example, when planning mathematics teaching and learning activities to provide metacognitive and heuristic feedback to students that helps with problem-solving.

This thesis demonstrates the possibility of using formative assessment in problem solving that is useful for student learning in mathematics. As there has been a call for more empirical evidence to ensure how feedback characteristics and problem-solving work (Lester & Cai, 2016) this thesis shows that explorative qualitative analysis and the SEM analysis approach could potentially be one such source of empirical evidence and could provide valuable information for developing an understanding of how formative assessment can support problem-solving.
5.5 Limitations and Implications

The main aim of this thesis has been to contribute knowledge regarding how formative assessment for problem solving in mathematics can be used for student learning. The findings in the last two studies suggest that computer-based formative assessment that helps with reasoning can assist in problem solving. When interpreting this conclusion, one needs to consider that there are limitations to the studies. With regard to the formative assessment tool design, the goal of this thesis was to investigate how student support can be provided by a computer combining two aspects: self-diagnosis and feedback. One limitation is that the usefulness of formative assessment is studied only for the particular design used in this thesis, and the results might vary if we changed the design or content of the feedback. Another limitation is that the one-time interventions in both studies were relatively short, lasting approximately 100 minutes (Study 3) and 60 minutes (Study 4) respectively. Other results may have been obtained from longer interventions lasting perhaps one week or one month. In addition, other types of data such as pretests, posttests, transfer tests, or comparisons with control groups could have provided more information about the effects of the intervention. This could be a direction for future research.

While formative assessment and problem solving in mathematics are emphasized in curriculum reforms worldwide as part of an enriched notion of mathematical understanding, this does not appear to be echoed in everyday classroom practices (Boesen et al., 2014; Burkhardt & Schoenfeld, 2019; Hiebert, 2003; Lee et al., 2020; Niss et al., 2016; Palm et al., 2011). One reason might be that teachers do not have much time to reflect on and design formative assessment supporting the students’ own reasoning processes while problem solving. Thus, the teacher can use the design of this thesis’s formative assessment and computer-based metacognitive and heuristic feedback to help students take more responsibility for solving the task. Since this design is not task-specific, it can be the first step in supporting students needing help to solve a problem. Teachers then may give more task-specific feedback depending on the students’ needs.

This design can also be used when teachers feedback/help is not available. The design of this thesis’s digital interactive diagnoses and general metacognitive and heuristic feedback can be included in a digital textbook. The thesis includes an example of how to implement formative assessment to support problem solving. Further development of the formative assessment (e.g., task-specific feedback can be included) and other aspects of
a student’s characteristics (e.g., prior knowledge, emotions) could be interesting for further research in problem solving. It would be important for future research to consider how a student’s cognitive load, emotions etc., affects self-assessment and receiving general meta cognitive feedback and heuristic feedback while solving problems.

Based on the investigation, there seems to be a need for some further steps on the road towards how different student characteristics influence the use of feedback. I have not found any study reviewing feedback types in the mathematics subject area, and the results of this study might be helpful to mathematics education research and to the teaching practice. For example, we could promote learning goal feedback because students who have learning goals or mastery goals achieve higher grades than students who focus on their performance goals (Dishon-Berkovits, 2014; Latham & Brown, 2006), and are also much more likely to enjoy learning than are those who are focused primarily on getting good grades or avoiding failure (Dweck & Leggett, 1988; Kover & Worrell, 2010). Shute (2008) concluded from her research review that teachers should give feedback to promote learning goals. Teachers’ feedback strategies must include learning goal orientation feedback as well as their didactic analysis of the pupils’ learning needs so that teacher feedback can transform performance goals into learning goals.

Nonetheless, more research is needed on teacher implementation and student engagement in formative assessment to support problem solving. Further research is needed to determine whether teachers’ practices successfully enhance learning, such as with problem solving skills. For instance, when presenting and discussing notions of providing feedback for formative assessment in teacher education and professional development programs, it would be important to consider student characteristics. Teachers can use the findings of this thesis to apply formative assessment during instruction to support creative reasoning. Thus, there are empirical arguments that general metacognitive and heuristic feedback may be considered in the teaching practice during problem-solving. This does not mean that this study suggests replacing teachers’ roles with a computer environment, rather that computer support can be used as a complement.
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Looking back on past thoughts and memories,
The glimpse of the green sunny land, where life is vivid.
Settle in this north and surrounded by cold white.
I never thought that I’d across this path.
No matter how hard I get pushed to the limit,
Never willing to give in without putting up a fight.
Probably it is not possible to reach those dizzy heights,
Still, the endless dream out there on the long road.
I feel the emptiness,
I knew the moment would arrive,
and I can’t find the words to say goodbye.

Umeå, February 2023
Sharmin Söderström
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