THE ROLE OF IMPLICIT THEORETICAL ASSUMPTIONS IN EMPIRICAL RESEARCH

Ewa Bergqvist[1] and Magnus Österholm[1,2]

[1]Department of Science and Mathematics Education, Umeå Mathematics Education Research Centre (UMERC), Umeå University, Sweden
[2]Department of Mathematics and Science Education, Mid Sweden University

There is much research on the role of theory in mathematics education research, at least from more overarching or theoretical perspectives. Micro analyses of the role of theory in particular research studies are rarer. We contribute by analysing one empirical study to allow for in-depth analyses and discussions around the role of theory in a specific case, concerning relationships between mathematics and reading. Our results show that studies that do not use an explicit theoretical model can still be strongly influenced by implicit theoretical assumptions. We conclude that it is important to identify existing theoretical assumptions in an empirical research study and try to convey them as clearly as possible, and we discuss specific issues concerning research on relationships between mathematics and reading.

INTRODUCTION

Theory is often considered to play an important, and sometimes crucial, role in mathematics education research. For example, it is sometimes described that theory should influence all parts of the research process. For example, this is done in the description of characteristics of a high quality JRME manuscript (NCTM, 2021), concerning influence of theory on "the study's design; its instrumentation, data collection, and data analysis; and the interpretation of its findings." However, there are also researchers that question whether theory should be so influential as described by NCTM. For example, Lester (2005) describes some shortcomings in relation to the use of theoretical frameworks and suggests that the use of *conceptual* frameworks is more suitable for mathematics education research. Furthermore, Niss (2019, p. 2) is critical towards an "ideal-typical research paper", which JRME (NCTM, 2021) can be said to describe, since this "represents a far too narrow and rigid understanding of mathematics education research". He describes different aspects of this "ideal-typical research paper", where theory is a key component.

We agree with critics concerning a type of over-reliance on theory in mathematics education research and a purpose with this paper is therefore to contribute to the discussion about the role of theory. We do this by analysing and discussing if and how certain aspects of theory have a role in certain parts of specific empirical research. We do not presume that theory is always needed in all parts of all types of empirical research, but we address this issue from an empirical standpoint, by examining the (potential) role of theory in specific research studies. We choose to focus on a specific research study, since much has been written about the role of theory in mathematics

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education research from more overarching or theoretical perspectives, as discussed more below, while less work has been done concerning more micro analyses of the role of theory in research studies.

THE CONCEPT OF THEORY AND ITS ROLE IN EMPIRICAL RESEARCH

The notions of "theory" and "theoretical" are used in different ways and can be considered vague and ill-defined (cf. Niss, 2019). In addition, it can often be unclear if and how theory actually has been used in a research study, "when some theoretical framework is being referred to in the beginning or at the end of the publication without having any presence in between" (Niss, 2007b, p. 1309). Therefore, we need to clarify both the meaning of "theory", or similar notions, such as "framework", and the potential roles of theory in empirical research. This work has been initiated by some researchers. Niss (2007a) has suggested a type of definition of "theory", as consisting of an organized network of concepts and claims, where the concepts are linked in a connected hierarchy. He also presents different roles theory can have in research, for example, to predict or explain phenomena, to organize observations and interpretations into a coherent whole, and to give a methodology for empirical studies. Radford (2008) also suggests a definition of "theory", which has much in common with the definition from Niss, but he also stresses that the use of theory does not only include explicitly formulated theoretical perspectives, but also "implicit views" (Radford, 2008, p. 320). Lester (2005) presents different types of research frameworks; theoretical, practical, and conceptual, and discusses their different roles in research, in relation to some general purposes of using a research framework; to give structure to a research study, to make sense of data, to come further than common sense, and in order not to be limited to finding answers to local problems.

These above perspectives on issues of theories show a complexity concerning relationships between theory and empirical research. There are different types of theories/frameworks that can function in different ways in relation to empirical research; there are different parts of theories, such as concepts, claims, and methodology, which can be more or less prominent (or explicit) in empirical research; and there are different parts of empirical research, such as purpose and research questions, and collection and interpretation of data, which can be affected by explicit or implicit theory in different ways. Therefore, when we want to discuss and analyse the role of theory in empirical research more specifically, we need to specify what type of theory and what parts of theory are addressed in relation to what parts of empirical research. In this paper, we focus on the implicit use of theoretical aspects in this situation.

PREVIOUS EMPIRICAL RESEARCH ON THE ROLE OF THEORY

Above, we discuss research that concerns the concept of theory and role of theory in empirical research on a general level, without addressing specific theories or specific empirical research. This type of research is important, but we also need more empirically based research on the role of theory in empirical research.

Most relevant for this paper is empirical research that analyses how theory is used (or not used) in specific empirical studies, which is the type of research presented in the present paper. Some studies highlight how different parts of empirical studies are dependent on which theory is chosen. For example, Gellert (2008) focuses on empirical data of students' collaborative problem solving where he shows how two different theoretical perspectives lead to different interpretations. A similar conclusion is drawn by Bergsten (2008) when he focuses on three empirical studies on limits of functions, in an analysis of how the use of different frameworks relate to the questions, methods, evidence, conclusions, and implications within these studies. Despite this type of conclusion, both authors address a potential of, but also a difficulty in, combining results from studies on the same topic that use different theories.

Other studies also highlight differences between theories in empirical research but at the same time see a potential of "translating" between these theories, which gives evidence that theories sometimes do not necessarily have a strong influence on (some parts of) empirical research. For example, Rodríguez et al. (2008) focus on empirical research on issues of metacognition in relation to problem solving. Their analyses show that it was not possible to do a "simple translation" of concepts concerning metacognition from one perspective to another. Instead, the problematic question that was the origin in one perspective could be "reformulable" in terms used in another perspective, which was also the case for some key aspects of metacognition (such as monitoring and self-regulation). Österholm (2011) comes to a similar conclusion when he compares two empirical studies about beliefs, where a main difference between these studies, concerning some specific aspects of theory, can be seen as a change of wording.

In summary, it is important to scrutinize the use of theories, including implicit assumptions regarding theoretical aspects, in empirical research. There is also a need for further studies of the relation between particular theoretical aspects and specific empirical studies, to understand how these can be related.

PURPOSE AND METHOD

The main purpose of this study is to deepen the scientific understanding of the role of theory in mathematics education research. We contribute to the line of research that analyse the role of theory in specific empirical studies, in particular when the theory is implicit. We analyse one empirical study (Caponera et al., 2016) that examines relationships between students' achievements in mathematics and reading, without explicitly relying on a theory or theoretical framework regarding the central concepts. We delimit our analyses to this study to allow for more in-depth analyses and discussions around the role of theory in a specific case, in particular, concerning if and how more implicit theoretical aspects can be of relevance in empirical research. The results can be added to previous similar type of research and allow for comparisons and cumulation of research results. However, as part of our analyses of this one study,

we also relate to other studies concerning the issues that come up from the analysis, concerning relations between achievements in mathematics and reading.

In line with the argumentation presented above, our analysis first focuses on identifying the implicit aspects of theory used in the article. These aspects concern the central *concepts* used and the *claims about relationships* between these concepts (cf. Niss, 2007a). Since the theoretic perspective is implicit, we will base our claims on how data is interpreted and how conclusions are drawn (cf. Radford, 2008). The implicit theory used in the study will be compared to other (implicit) theoretical perspectives used in research in the same area. We will also discuss the consequences of the chosen theoretical perspectives.

ANALYSIS OF THE EMPIRICAL STUDY

The article we primarily analyse is "The influence of reading literacy on mathematics and science achievement" by Caponera, Sestito, and Russo (2016), which has the aim "to evaluate the influence of students' reading literacy, measured by the PIRLS (Progress in International Reading Literacy Study) test, on their performance in the TIMSS (Trends in International Mathematics and Science Study) mathematics and science tests" (p. 197). Below we focus only on reading and mathematics, since mathematics and science are treated similarly. The article analyses correlations between students' achievements in reading and mathematics for 4,125 Italian students in Grade 4. The correlations between achievements were high and the authors conclude that the students' reading literacy influenced their mathematics achievement. Caponera et al. (2016) do not present a theoretical model or explicit definitions of the central notions of reading and mathematics and do not state explicit assumptions regarding the relationships between these notions. However, the study relies on implicit assumptions about the concepts and their relation, namely that reading and mathematics have nothing in common, as we specify in the following.

First, Caponera et al. (2016), as many others, interpret the correlation between mathematics achievement and reading literacy as a causal relation, since they state that "results confirmed the influence of reading literacy on mathematics achievement" (p. 197). Here, it is the word "influence" that signals causality. The authors do not (explicitly) consider that the influence could exist in the other direction, which is another possible conclusion. For example, such a conclusion has been drawn in another empirical study, where "mathematical performance predicted subsequent reading comprehension during the first year rather than vice versa" (Lerkkanen et al., 2005, p. 121). Furthermore, Caponera et al. do not consider that the correlation could be created by a common feature of these variables (e.g., when there is a third confounding variable). If a correlation implies that one variable influences the other, there is an underlying assumption that the variables have nothing in common, except what has been controlled for. In this case, it is assumed that *achievements in mathematics and reading have nothing in common*, except that both depend on students' socioeconomic status, which is controlled for in the study.

Second, the analyses by Caponera et al. (2016) show that good readers in general perform better than not so good readers on mathematics tasks. Based on this, the authors draw the conclusion that a "good reader had some advantages [...] independently on their mathematics ability" (Caponera et al., 2016, p. 202). The study does not control for mathematics ability in the analysis of the effect of reading ability, and therefore this conclusion is based on an implicit assumption that an effect of reading ability on mathematics performance cannot at the same time be an effect of mathematics ability. That is, any connection between reading ability and results on mathematics tasks is interpreted as saying something only about the influence of reading. The implicit assumption is that *reading ability and mathematics ability have nothing in common*, and therefore any connection to reading ability is interpreted as only an effect of reading ability.

Third, in the final statement of the article by Caponera et al. (2016), the authors make a connection between level of readability and validity of mathematics (and science) tests: "Our study seemed to indicate that the readability level of the mathematics and science test is a crucial aspect to consider to *correctly* assess mathematics and science achievement" (Caponera et al., 2016, p. 203, emphasis added). In the study, tasks with low and high reading demand are analysed, and a result is that "bad readers performed better on the mathematics low reading demanding scale than on the mathematics high reading demand scale" (Caponera et al., 2016, p. 201). Therefore, the authors' conclusion implies the implicit assumption that mathematics tasks with high reading demand do not "correctly" assess mathematics achievement. This is only reasonable if *reading and mathematics have nothing in common*, because then any effects of reading demands of tasks on students' performance on these tasks would be interpreted as a sign of lower validity for these tasks.

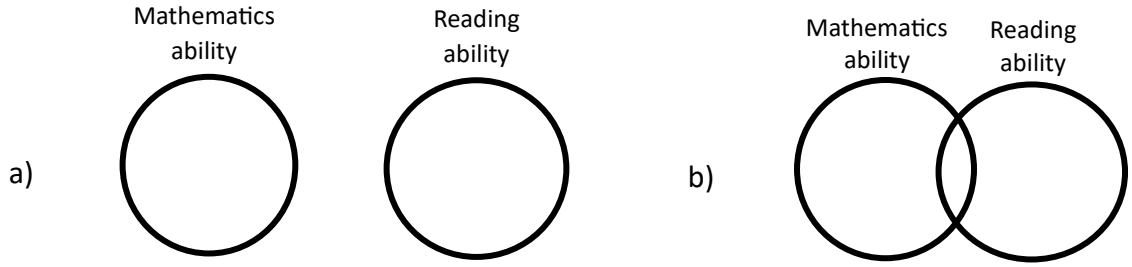

Figure 1: Two basic models of relationships between mathematics ability and reading ability.

We conclude that the study relies on an implicit theoretical model where mathematics and reading are separated. This includes a separation between mathematics ability and reading ability as well as between measures of achievement in mathematics and reading. We here suggest a simple theoretical model (see Figure 1a) that could be the current basis for conclusions by Caponera et al. (2016). Based on this model, any connection to issues of reading when focusing on mathematics tasks is unwanted, since reading ability has nothing in common with mathematics ability.

DIFFERENT THEORETICAL ASSUMPTIONS IN RELATED RESEARCH

Much research in mathematics education, as seen in different frameworks describing school mathematics, convey another theoretical model of the relation between mathematics and reading. For example, the framework of PISA (OECD, 2016) includes aspects of communication as important parts of mathematics. Mathematics performance is then not just *influenced* by reading ability but reading and interpreting mathematics text is one relevant and central *aspect or part of* mathematics ability. Such theoretical perspectives would better be illustrated using Figure 1b.

The second model is a slightly more complex model for the relationship between mathematics and reading ability. Here, the two circles of mathematics and reading ability are overlapping (see Figure 1b). The overlap symbolizes not merely the empirical results showing correlations between achievements in mathematics and reading, but also signifies that the two subjects have much in common *by definition*. This model is still quite simple but makes analyses a bit more complex. Any empirical connection between achievements in mathematics and reading (e.g., through correlation analyses) could be unwanted, if the result reflects an effect of the area in Figure 1b that lies outside mathematics but inside reading. At the same time, such a connection could also be highly relevant and nothing to avoid, if the result reflects an effect of the overlapping area in Figure 1b, which is part of both mathematics and reading.

For example, nominalizations are often described as making texts more difficult to read. Therefore, one would expect mathematics tasks with more nominalizations to have stronger connection to reading ability, so that students with lower reading ability would perform worse on such tasks compared to tasks without nominalizations. Caponera et al. (2016), as well as other studies, interpret this type of empirical result as a sign of lower validity for such mathematics tasks. However, nominalizations are not just surface features of a text that can be avoided without changing the meaning of the text, since "a nominalisation, by transforming a process into an object, opens up the possibility of a higher complexity of generalization" (Morgan, 2006, p. 233). For example, by transforming the *process* of adding into the *object* of addition, it becomes possible to talk about more advanced properties of addition, including that addition is commutative and that subtraction is the inverse operation to addition. Thus, tasks with more nominalizations could very well be more difficult to solve, since the language is used to describe more complex mathematics, in which case they also *should* be more difficult. Of course, there could as well be uses of nominalizations that are unnecessary and make the text more difficult to read without being part of mathematics.

DISCUSSION AND CONCLUSIONS

The present study aims to contribute to the scientific understanding of the role of theory in mathematics education research, in particular regarding the role of implicit theory in empirical studies. The article we have analyzed as a case (Caponera et al., 2016) has no explicit theory, but has implicit assumptions regarding the theoretical relation

between mathematics and reading, and these assumptions differ from what is assumed in other research in the same area. Our analyses show that studies that do not use an explicit theoretical model can still be strongly influenced by implicit theoretical assumptions. As mentioned in the background, we argue that it is not necessary to always use a theory in all parts of empirical research, which is also supported by previous empirical research (e.g., Rodríguez et al., 2008; Österholm, 2011). Therefore, it is important to identify which theoretical assumptions that are essential for the analyses in empirical studies and try to convey these assumptions as clearly as possible.

In particular, empirical studies on relationships between achievements in mathematics and reading should be explicit about assumptions regarding the relationship between these domains. Without relating to any theoretical model, it is difficult to compare and combine conclusions from different studies. For example, it is difficult to decide how to combine a conclusion that achievement in reading influences achievement in mathematics (from Caponera et al., 2016) with a conclusion that achievement in mathematics influences achievement in reading (from Lerkkanen et al., 2005).

Since connections between mathematics and reading *can be* relevant and wanted but *can also be* irrelevant and unwanted, we cannot rely on too simplistic models for analyses of these issues. It can make us draw unfounded conclusions and lead us to practical recommendations that are not helpful. For example, let us say that we have a study showing a correlation between the number of nominalizations in mathematics tasks and task difficulty, and the study is based on a model that separates reading from mathematics (Figure 1a). The authors of this study might then recommend teachers and other task creators to avoid nominalizations, perhaps primarily for students with lower reading ability. That recommendation could lead to fewer opportunities for these students to become familiar with objectifications in mathematics, which would be negative for their learning of mathematics. Therefore, we suggest that a theoretical model takes the overlap between reading and mathematics into account (Figure 1b), that is, assumes that some part of reading ability is also a part of mathematics ability, by definition.

Furthermore, studies only focusing on associations between the existence of certain linguistic features of mathematics tasks and students' results on these tasks are not relevant since these studies are not informative. It is not possible to draw any meaningful conclusions based only on such an association, since it is not possible to know if the association is relevant or irrelevant, as described above. We encourage literature reviews of empirical studies to examine what types of conclusions and recommendations that have been made that are not valid when placed within a more relevant model.

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