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Numerical Analysis and Simulation of Stochastic Partial Differential Equations with White Noise Dispersion

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Akademisk avhandling

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Abstract
This doctoral thesis provides a comprehensive numerical analysis and exploration of several stochastic partial differential equations (SPDEs). More specifically, this thesis investigates time integrators for SPDEs with white noise dispersion.

The thesis begins by examining the stochastic nonlinear Schrödinger equation with white noise dispersion (SNLSE), see Paper 1. The investigation probes the performance of different numerical integrators for this equation, focusing on their convergences, L^2-norm preservation, and computational efficiency. Further, this thesis thoroughly investigates a conjecture on the critical exponent of the SNLSE, related to a phenomenon known as blowup, through numerical means.

The thesis then introduces and studies exponential integrators for the stochastic Manakov equation (SME) by presenting two new time integrators - the explicit and symmetric exponential integrators - and analyzing their convergence properties, see Paper 2. Notably, this study highlights the flexibility and efficiency of these integrators compared to traditional schemes. The narrative then turns to the Lie-Trotter splitting integrator for the SME, see Paper 3, comparing its performance to existing time integrators. Theoretical proofs for convergence in various senses, alongside extensive numerical experiments, shed light on the efficacy of the proposed numerical scheme. The thesis also deep dives into the critical exponents of the SME, proposing a conjecture regarding blowup conditions for this SPDE.

Lastly, the focus shifts to the stochastic generalized Benjamin-Bona-Mahony equation, see Paper 4. The study introduces and numerically assesses four novel exponential integrators for this equation. A primary finding here is the superior performance of the symmetric exponential integrator. This thesis also offers a succinct and novel method to depict the order of convergence in probability.

Keywords
Numerical methods, stochastic partial differential equations, stochastic exponential integrator, convergence in probability.

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