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Generation of magnetic field fluctuations in relativistic electron-positron magnetoplasmas

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Abstract

It is shown that magnetic field aligned equilibrium relativistic plasmas flows can excite electromagnetic fluctuations in a magnetized electron-positron (e-p) plasma. For this purpose, a new dispersion relation is derived by using a relativistic two-fluid model and the Maxwell equations. The dispersion relation admits purely growing instabilities of electromagnetic perturbations across the ambient magnetic field direction. The results have relevance

for understanding the origin of magnetic field fluctuations in cosmological and laser-produced plasmas.

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Understanding the origin of magnetic fields in cosmic and laser produced plasmas is of current interest [2–4]. It is widely thought that magnetic fields in plasmas are spontaneously generated by the Biermann battery [5] and via the Weibel instability [6, 7]. In order for the Biermann battery to work, there should be a non-parallel density and temperature gradients, so that the curl of the electric field, which balances the electron pressure gradient, remains finite. On the other hand, the Weibel instability, which generates purely growing magnetic fields, is triggered by electron temperature anisotropy or by beams of counterstreaming electron streams in plasmas. The importance of the Weibel instability has been recently recognized in the context of the magnetic field generation in cosmological [8–10] and laboratory [11] plasmas. Computer simulations [12] conclusively demonstrate the generation of magnetic fields due to colliding electron clouds in an unmagnetized electron-ion plasma. In the presence of an external magnetic field and non-relativistic electron flows, we can have counterstreaming instabilities involving the ordinary, extraordinary and Bernstein waves [13].

However, in the early universe [14–16] and in astrophysical objects [17] such as pulsars (e.g. Crab Nebula) [18, 19] and dense neutron stars, supernova remnants and active galactic nuclei [20], as well as in gamma-ray bursts [23] and in micro-quasars [21], there exist relativistic electron-positron pairs [22], and the pair plasma is typically magnetized. The electron-positron pairs may be created by collisions between charged particles that are accelerated by electromagnetic and electrostatic waves, and/or by gravitational forces. In pulsar environments, there is also a possibility of pair creation via high-energy curvature radiation photons that are triggered by charged particles streaming along the curved magnetic field [24], with a resulting collection of positrons at the polar caps of the pulsar [25, 26]. High-energy laser-plasma interactions [27–29] and magnetic fusion devices [30] on Earth also constitute a source of electron-positron plasmas. Experiments with Petawatt laser experiments ($> 10^{20}$ W/cm²) have demonstrated the production of MeV electrons and evidence of positron production via electron collisions [28, 29]. Positrons are also believed to be created in post-disruption plasmas in large tokamaks through collisions between MeV electrons and thermal particles [30].

In this Letter, we present a simple picture of the magnetic field generation in a magnetized e-p plasma, which contains equilibrium relativistic electron and positron flows along the ambient magnetic field direction. In such a situation, the Lorentz force, arising from the

coupling between the magnetic field-aligned equilibrium plasma flows and an infinitely small sheared magnetic field perturbation, acts like a source to drive electrons and positrons apart. The resulting parallel (along the ambient magnetic field direction) electric field due to the charge separation and the associated magnetic field-aligned e-p currents are, in turn, responsible for the generation of enhanced magnetic fields in a magnetized e-p plasma. It should be stressed that the present theory should help to understand the initial phase of the magnetic field structures that are observed in particle-in-cell simulations of Fonseca *et al.* [31].

Let us consider a two-component plasma composed of relativistic streams of electrons and positrons in an external magnetic field $B_0 \hat{\mathbf{z}}$, where B_0 is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit-vector along the z axis in a Cartesian coordinate system. The equilibrium velocity and density of relativistic electrons (positrons) are $\hat{\mathbf{z}}u_{0-}$ ($\hat{\mathbf{z}}u_{0+}$) and n_{0-} (n_{0+}), respectively. We now study the instability of our equilibrium against electromagnetic perturbations that are propagating along the x axis. We suppose that the electromagnetic perturbations are mixed modes, so that there is a finite density perturbation due to the cross-coupling between the equilibrium magnetic field-aligned e-p flows and the magnetic field perturbation $\hat{\mathbf{y}}B_y$, where $\hat{\mathbf{y}}$ is the unit-vector along the y axis and B_y is the sheared magnetic field perturbation. The dynamics of mixed modes is governed by the continuity equation

$$\frac{\partial n_{\mp}}{\partial t} + n_{0\mp} \frac{\partial u_{\mp}}{\partial x} = 0, \quad (1)$$

the momentum equations

$$\frac{\partial u_{\mp}}{\partial t} = \pm \frac{eu_{\mp}}{m_{\mp}\gamma_{\mp}} B_y \pm \frac{\omega_{c\mp}}{\gamma_{\mp}} v_{\mp}, \quad (2)$$

$$\frac{\partial v_{\mp}}{\partial t} = \mp \frac{\omega_{c\mp}}{\gamma_{\mp}} u_{\mp}, \quad (3)$$

$$\frac{\partial w_{\mp}}{\partial t} = \mp \frac{e}{m_{\mp}\gamma_{\mp}} E_z, \quad (4)$$

and the y component of the Maxwell equation and Faraday's law, respectively,

$$\frac{\partial B_y}{\partial x} = \frac{4\pi e}{c}(n_{0+}w_+ - n_{0-}w_-) + \frac{4\pi e}{c}(n_+u_{0+} - n_-u_{0-}) + \frac{1}{c} \frac{\partial E_z}{\partial t}, \quad (5)$$

$$\frac{\partial B_y}{\partial t} = c \frac{\partial E_z}{\partial x}. \quad (6)$$

Here $n_{\mp} (\ll n_{0\mp})$ is a small density perturbation, $u_{\mp}, v_{\mp}$ and $w_{\mp}$ are the x, y , and z components of the e-p fluid velocities, $m_{\mp}$ is the rest mass, $\gamma_{\mp} = (1 - u_{0\pm}^2/c^2)^{-1/2}$ is the relativistic gamma factor [32], c is the speed of light in vacuum, e is the magnitude of the electron charge, $\omega_{c\mp} = eB_0/m_{\mp}c$ is the gyrofrequency, and E_z is the magnetic field-aligned (z component) electric field perturbation. The minus (plus) sign stands for the electrons (positrons).

Eliminating $v_{\mp}$ from (2) by using (3), we have

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_{c\mp}^2}{\gamma_{\mp}^2} \right) u_{\mp} = \mp \frac{eu_{0\mp}}{m_{\mp}\gamma_{\mp}} \frac{\partial B_y}{\partial t}, \quad (7)$$

which shows that the perpendicular (to $\hat{\mathbf{z}}$) gyro-motion of the electrons and positrons are driven by the Lorentz force involving the equilibrium magnetic field-aligned e-p flows and the perturbed magnetic field B_y . According to Eq. (7), the electrons and positrons would have transverse (to $\hat{\mathbf{z}}$) speeds in the opposite directions. Hence, there would result charge separation and the associated parallel electric field E_z and density perturbations $n_{\mp}$. The latter are obtained from (1) and (7). We have

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_{c\mp}^2}{\gamma_{\mp}^2} \right) n_{\mp} = \mp \frac{n_{0\mp} eu_{0\mp}}{m_{\mp}\gamma_{\mp}} \frac{\partial B_y}{\partial x}. \quad (8)$$

Equations (4), (5) and (8) exhibit that the electromagnetic fields, B_y and E_z , are coupled with the density perturbations $n_{\mp}$, if there exist equilibrium magnetic field-aligned e-p flows in plasmas.

Supposing that B_y, E_z and $n_{\mp}$ are proportional to $\exp(ikx - i\omega t)$, where k and ω are the wavenumber and frequency, respectively, we Fourier transform ($\partial/\partial t = -i\omega$ and $\partial/\partial x = ik$) Eqs. (4), (5) and (8) and combine the resultant equations to obtain the new dispersion relation

$$1 - \frac{\omega^2}{k^2 c^2} + \frac{\left(\sum_{-,+} \omega_{p\mp}^2 / \gamma_{\mp} \right)}{k^2 c^2} = - \sum_{-,+} \frac{\omega_{p\mp}^2 u_{0\mp}^2}{\gamma_{\mp}^2 (\omega^2 - \omega_{c\mp}^2 / \gamma_{\mp}) c^2}, \quad (9)$$

where $\omega_{p\mp} = (4\pi n_{0\mp} e^2 / m_{\mp})^{1/2}$ is the plasma frequency.

Several comments are in order. First, in the absence of the equilibrium e-p flows, viz. $u_{0\mp} = 0$, we have from (9) the ordinary electromagnetic wave frequency

$$\omega = \left[k^2 c^2 + \left(\sum_{-,+} \omega_{p\mp}^2 / \gamma_{\mp} \right) \right]^{1/2} \equiv (k^2 c^2 + \omega_p^2)^{1/2}. \quad (10)$$

Second, in the absence of the external magnetic field, we have from (9)

$$\omega^4 - \omega^2(k^2 c^2 + \omega_p^2) - k^2 \sum_{-,+} \frac{\omega_{p\mp}^2 u_{0\mp}^2}{\gamma_{\mp}} = 0, \quad (11)$$

which has the solutions

$$\omega^2 = \frac{1}{2}(k^2 c^2 + \omega_p^2) \pm \frac{1}{2} \left[(k^2 c^2 + \omega_p^2)^2 + 4k^2 \left(\sum_{-,+} \omega_{p\mp}^2 u_{0\mp}^2 / \gamma_{\mp} \right) \right]^{1/2}. \quad (12)$$

Equation (12) admits a purely growing instability ($\omega = i\Gamma_1$), the growth rate of which is

$$\Gamma_1 = \left(-\frac{1}{2}(k^2 c^2 + \omega_p^2) + \frac{1}{2} \left[(k^2 c^2 + \omega_p^2)^2 + 4k^2 \left(\sum_{-,+} \omega_{p\mp}^2 u_{0\mp}^2 / \gamma_{\mp} \right) \right]^{1/2} \right)^{1/2}. \quad (13)$$

Furthermore, for $\omega \ll (k^2 c^2 + \omega_p^2)^{1/2}$ we have from (11)

$$\omega^2 = -\frac{k^2}{k^2 c^2 + \omega_p^2} \left(\sum_{-,+} \frac{\omega_{p\mp}^2 u_{0\mp}^2}{\gamma_{\mp}} \right), \quad (14)$$

which also admits a purely growing instability $\omega = i\Gamma_2$. The growth rate is

$$\Gamma_2 = \frac{k}{(k^2 c^2 + \omega_p^2)^{1/2}} \left(\sum_{-,+} \frac{\omega_{p\mp}^2 u_{0\mp}^2}{\gamma_{\mp}} \right)^{1/2}. \quad (15)$$

The above results may account for the generation of magnetic fields in electron-positron plasmas, which are observed in particle-in-cell simulations [31].

Third, we examine the influence of the ambient magnetic field on the purely growing instabilities, as discussed above. In the low-frequency limit, viz. $\omega \ll \omega_{c\mp} / \gamma_{\mp}$, we obtain from (9)

$$\omega^2 = 1 + \frac{\omega_p^2}{k^2 c^2} - k^2 \sum_{-,+} \frac{\gamma_{\mp} \omega_{p\mp}^2 u_{0\mp}^2}{\omega_{c\mp}^2}, \quad (16)$$

which depicts a purely growing instability ($\omega = i\Gamma_3$) if

$$\sum_{-,+} \frac{\gamma_{\mp} \omega_{p\mp}^2 u_{0\mp}^2}{\omega_{c\mp}^2} > \frac{k^2 c^2 + \omega_p^2}{k^4 c^2}. \quad (17)$$

Furthermore, for $\omega \ll kc$, Eq. (9) reduces to

$$1 = -\frac{k^2}{(k^2c^2 + \omega_p^2)} \sum_{-,+} \frac{\omega_{p\mp}^2 u_{0\mp}^2}{\gamma_{\mp} (\omega^2 - \omega_{c\mp}^2/\gamma_{\mp}^2)}. \quad (18)$$

Supposing that $u_{0+} = u_{0-} = u_0 \equiv c(\gamma_0^2 - 1)/\gamma_0$, $n_{0-} = n_{0+} = n_0$, $m_- = m_+ = m_0$, $\omega_{c-} = \omega_{c+} = \omega_{c0}$, and $\gamma_- = \gamma_+ = \gamma_0$, we can express (18) as

$$\omega^2 = \frac{\omega_{c0}^2}{\gamma_0^2} - \frac{k^2}{(k^2c^2 + 2\omega_{p0}^2/\gamma_0)} \frac{2\omega_{p0}^2 u_0^2}{\gamma_0}, \quad (19)$$

where $\omega_{p0} = (4\pi n_0 e^2/m_0)^{1/2}$. Equation (19) reveals that a purely growing ($\omega = i\Gamma_m$) electromagnetic instability sets in if

$$u_0^2 > \frac{\omega_{c0}^2 (k^2c^2 + 2\omega_{p0}^2/\gamma_0)}{2\omega_{p0}^2 k^2 \gamma_0}. \quad (20)$$

Equation (19) exhibits that the growth rate (Γ_m) of the instability is reduced in the presence of the ambient magnetic field.

To summarize, we have presented a simple picture of purely growing electromagnetic instabilities in a magnetized e-p plasma that contains equilibrium relativistic e-p flows. We have found that the cross-coupling between the magnetic field-aligned e-p flows and the perturbed magnetic field gives rise to the Lorentz force that separates charges. Since the resulting density fluctuations cannot keep in phase with the magnetic field perturbation, the latter would grow at the expense of the free energy stored in the equilibrium e-p plasma flows. In conclusion, we stress that the present results should help to understand the origin of nonthermal magnetic field fluctuations in the early universe, in astrophysical objects (e.g. active galactic nuclei, micro-quasars, pulsar magnetospheres, dense neutron stars), as well as in intense laser-plasma interaction experiments, where plasmas contain relativistic e-p flows and the ambient magnetic field. Finally, it is anticipated that enhanced magnetic fields may also cause cross-field transport of e-p pairs in a magnetized pair plasma.

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