Umeå University

This is an accepted version of a paper published in *Journal of Plasma Physics*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the published paper:

Shukla, N., Shukla, P., Morfill, G. (2007)

"Amplification of magnetic fields by polaritonic flows in quantum pair plasmas"

Journal of Plasma Physics, 73: 289-293

URL: <http://dx.doi.org/10.1017/S0022377807006551>

Access to the published version may require subscription.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:umu:diva-19902>

<http://umu.diva-portal.org>

Amplification of magnetic fields by polaritonic flows in quantum pair plasmas

Nitin Shukla [a)] ^{*}, P. K. Shukla [b)] [†], and G. E. Morfill [c)] ^{‡1}

¹*Institut für Theoretische Physik IV and Centre for Plasma Science and Astrophysics,
Fakultät für Physik und Astronomie,
Ruhr-Universität Bochum, D-44780 Bochum, Germany*

(Received 9 February 2007)

Abstract

It is shown that equilibrium polaritonic flows can amplify magnetic fields in an ultra-cold quantum electron-positron/hole (polaritons) plasma. For this purpose, a linear dispersion relation has been derived by using the quantum generalized hydrodynamic (QGH) equations for the polaritons, the Maxwell equation, and Faraday's law. The dispersion relation admits purely growing instabilities, the growth rates of which are proportional to the equilibrium streaming speeds of the polaritons. Possible applications of our work to spontaneous excitation of magnetic fields and the associated cross-field transport of the polaritons in micromechanical systems, compact dense astrophysical objects (e.g. neutron stars), and intense laser-plasma interaction experiments are mentioned.

PACS numbers: 03.65.-w, 52.27.Ep, 52.35.Hr, 52.35.Qz

^{*} E-mail: ns@tp4.rub.de

[†] E-mail: ps@tp4.rub.de

[‡] E-mail: gem@mpg.mpe.de

Ultracold quantum pair plasmas containing electrons and positrons/holes (hereafter referred to as polaritons) are ubiquitous in dense astrophysical environments [1], in intense laser-solid matter/atomic system interaction experiments [2–4], in micro and nano-scale objects (e.g. microplasmas [5], quantum dots and nanowires [6], quantum diodes [7], biophotonics [8], and cool vibes [9]), as well as in micromechanical systems and ultrasmall semiconductor devices [10]. Quantum effects [11, 12] in plasmas are important when the de Broglie length of the polaritons is comparable to the dimensions of the system. Here, quantum-mechanical effects (e.g. tunnelling) play an important role [11, 12]. Quantum corrections lead to dispersion [11] that is caused by strong correlations between charged carriers (polaritons). Recent studies [13–19] dealing with collective interactions have incorporated strong electron correlations at quantum scales, and have reported new features of waves and nonlinear structures in dense quantum plasmas. The electron 1/2 spin effect on the dispersion properties of low-frequency magnetohydrodynamic waves has been examined in Ref. [20]

In this Letter, we show that equilibrium polaritonic drifts in an ultracold quantum electron-positron/hole (pair) plasma can generate magnetic fields via a purely growing electromagnetic instability. Physically, the presence of polaritonic drifts can move polaritons in directions opposite to each other in a quantum plasma. As a result, there would appear charge separation and the associated density perturbations. The corresponding polaritonic current densities become the source for magnetic fields. The latter temporally grow since the polaritonic density fluctuations, which are produced by the Lorentz force involving the cross-coupling between the polaritonic flows and the perturbed magnetic field, cannot keep in phase with the magnetic field perturbation.

Let us suppose that our ultracold quantum plasma is composed of the electrons and positrons/holes. The latter are streaming with the equilibrium drift velocities $\hat{\mathbf{z}}u_{0\pm}$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis in a Cartesian coordinate system and the subscript $+$ ($-$) stands for the positrons/holes (electrons). The dynamics of one-dimensional mixed mode electromagnetic perturbations in our cold quantum plasma is governed by the QGH equations composed of the continuity equation

$$\frac{\partial n_{\pm}}{\partial t} + n_{0\pm} \frac{\partial v_{x\pm}}{\partial x} = 0, \quad (1)$$

the x component of the momentum equation

$$\frac{\partial v_{x\pm}}{\partial t} = \mp \frac{e}{m_{\pm} c u_{0\pm}} B_y - \frac{\hbar^2}{4m_{\pm}^2 n_{0\pm}} \frac{\partial^3 n_{\pm}}{\partial x^3}, \quad (2)$$

the z component of the Maxwell equation

$$\frac{\partial B_y}{\partial x} = \frac{4\pi e}{c} (n_{0+} v_{z+} - n_{0-} v_{z-}) + \frac{4\pi e}{c} (n_+ u_{0+} - n_- u_{0-}) + \frac{1}{c} \frac{\partial E_z}{\partial t}, \quad (3)$$

where the z component of the polaritonic fluid velocity $u_{z\pm}$ is determined from

$$\frac{\partial v_{z\pm}}{\partial t} = \pm \frac{e}{m_{\pm}} E_z. \quad (4)$$

Here the z component of the wave electric field E_z is related with the wave magnetic field B_y by Faraday's law

$$\frac{\partial E_z}{\partial x} = \frac{1}{c} \frac{\partial B_y}{\partial t}. \quad (5)$$

In (1) and (3) $n_{\pm} (\ll \text{the equilibrium polaritonic number density } n_{0\pm})$ is a small polaritonic density perturbation caused by the non-vanishing of the divergence of the polaritonic flux $n_{0\pm} v_{x\pm}$ in the presence of the equilibrium polaritonic flows. The latter produce the polaritonic velocity perturbation $v_{x\pm}$ due to the Lorentz force involving the cross-coupling between the equilibrium polaritonic flows and the perturbed magnetic field B_y . Furthermore, $\hbar$ is the Planck constant divided by 2π , $m_{\pm}$ is the mass of the polaritons, e is the magnitude of the electron charge, and c is the speed of light in vacuum. We note that the second term in the right-hand side of (2) represents corrections at quantum scales due to interactions between strongly correlated polaritons in a dense quantum plasma.

Combining (1) and (2) we obtain

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\hbar^2}{4m_{\pm}^2} \frac{\partial^4}{\partial x^4} \right) n_{\pm} = \pm \frac{n_{0\pm} u_{0\pm} e}{m_{\pm} c} \frac{\partial B_y}{\partial x}, \quad (6)$$

which shows that finite polaritonic density perturbations exist only if the polaritonic flows, $u_{0\pm}$, are present.

Taking the x - derivative on both sides of (3), and using (5) we obtain

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} \right) B_y + 4\pi e c \frac{\partial}{\partial x} [(n_{0+} v_{z+} - n_{0-} v_{z-}) + (n_+ u_{0+} - n_- u_{0-})] = 0. \quad (7)$$

Taking the time derivative of (7) and using (4) and (5) we have

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} + \omega_p^2\right) B_y + 4\pi e c \frac{\partial}{\partial x} (n_+ u_{0+} - n_- u_{0-}) = 0, \quad (8)$$

where $\omega_p = \left(\sum_{+,-} \omega_{p\pm}^2\right)^{1/2}$ and $\omega_{p\pm} = (4\pi e^2 n_{0\pm}/m_{\pm})^{1/2}$ is the plasma frequency of the polaritons. Equations (6) and (8) form a closed set for our purposes.

We now Fourier transform (6) and (8) by supposing that $n_{\pm}$ and B_y is proportional to $\exp(-i\omega t + ikx)$, where ω and k are the frequency and wavenumber, respectively. The resultant equations are then combined to obtain the dispersion relation

$$\omega^2 - k^2 c^2 - \omega_p^2 - k^2 \sum_{+,-} \frac{\omega_{p\pm}^2 u_{0\pm}^2}{(\omega^2 - \hbar^2 k^4/4m_{\pm}^2)} = 0. \quad (9)$$

Two comments are in order. First, for $\omega^2 \ll k^2 c^2 + \omega_p^2$, we have from Eq. (9)

$$1 + \frac{k^2}{\Omega_{em}^2} \sum_{+,-} \frac{\omega_{p\pm}^2 u_{0\pm}^2}{(\omega^2 - \hbar^2 k^4/4m_{\pm}^2)} = 0, \quad (10)$$

where $\Omega_{em} = (k^2 c^2 + \omega_p^2)^{1/2}$ is the frequency of the electromagnetic wave in an electron-positron/hole plasma. In a pair plasma with equal polariton masses ($m_+ = m_- = m$) and equal equilibrium polaritonic flows ($u_{0+} = u_{0-} = u_0$), we have from (10)

$$\omega^2 = \frac{\hbar^2 k^4}{4m^2} - \frac{2k^2 u_0^2 \omega_{p0}^2}{(k^2 c^2 + 2\omega_{p0}^2)}, \quad (11)$$

where $\omega_{p0} = (4\pi n_0 e^2/m)^{1/2}$ and $n_0 = n_{0+} = n_{0-}$. Equation (11) admits a purely growing instability ($\omega = i\gamma$, where γ is the growth rate), if

$$u_0^2 > \frac{\hbar^2 k^2 (k^2 c^2 + 2\omega_{p0}^2)}{8m^2 \omega_{p0}^2}. \quad (12)$$

The growth rate above the threshold is

$$\gamma = \frac{\sqrt{2} k u_0 \omega_{p0}}{\sqrt{k^2 c^2 + 2\omega_{p0}^2}}. \quad (13)$$

Second, in the quantum dominated limit, $\omega^2 \ll \hbar^2 k^4/4m_{\pm}^2$, we have from (9)

$$\omega^2 = \Omega_{em}^2 - \sum_{+,-} \frac{4m_{\pm}^2 u_{0\pm}^2 \omega_{p\pm}^2}{\hbar^2 k^2}, \quad (14)$$

which also admits a purely growing instability, if

$$\sum_{+,-} m_{\pm}^2 u_{0\pm}^2 \omega_{p\pm}^2 > \hbar^2 k^2 \Omega_{em}^2 / 4. \quad (15)$$

The growth rate above the threshold is

$$\gamma = \frac{2}{\hbar k} \left(\sum_{+,-} m_{\pm}^2 u_{0\pm}^2 \omega_{p\pm}^2 \right)^{1/2}. \quad (16)$$

To summarize, we have shown that the equilibrium polaritonic flows in an ultracold quantum electron-positron/hole plasma can spontaneously create magnetic fields. Physically, the Lorentz force arising from the cross-coupling between the polaritonic flows and an infinitely small magnetic field perturbation can move electrons and positrons/holes in directions opposite to each other. Hence, there would appear space charge electric field and polaritonic density perturbations. Since the latter cannot keep in phase with the magnetic field perturbation, one would encounter purely growing instabilities via which magnetic fields are spontaneously created in a quantum pair plasma. Spontaneously excited magnetic fields can produce cross-field transport of the polaritons at quantum scales. Thus, we expect that the present results would be relevant for understanding the origin of magnetic fields and the associated polaritonic transport at quantum scales in semiconductors and in dense pair plasmas, such as those in astrophysical objects and forthcoming intense laser-plasma interaction experiments.

This research was initiated while the authors were attending the International Workshop on Frontiers of Plasma Science at the Abdus Salam ICTP, Trieste, Italy. This work was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591 entitled “Universal behavior of plasmas in non-equilibrium state: Heating, transport, and structure formation”.

-
- [a)] Permanent address: Department of Physics, K. N. Government Postgraduate College, Gyanpur, Bhadohi 221304, U. P., India; Also at Department of Physics, Umeå University, SE-90187 Umeå, Sweden and GoLP/Centro de Física de Plasmas, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal.
- [b)] Also at the Max-Planck Institut für extraterrestrische Physik, D-45741 Garching, Germany; Centre for Nonlinear Physics, Department of Physics, Umeå University, SE-90187 Umeå, Sweden; CCLRC Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, U.K.; SUPA Department of Physics, University of Strathclyde, Glasgow G4 0NG, U. K.; GoLP/Centro de Física de Plasmas, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal; School of Physics, University of KwaZulu-Durban, 4000 Durban, South Africa.
- [c)] Permanent address: Max-Planck Institut für extraterrestrische Physik, D-85740 Garching, Germany.
- [1] D. Kremp, Th. Bornath, M. Bonitz, and M. Schlanges, *Phys. Rev. E* **60**, 4725 (1999).
- [2] Y. F. Jung, *Phys. Plasmas* **8**, 3842 (2001); M. Opher, L. O. Silva, D. E. Danger, V. K. Decyk, and J. M. Dawson, *ibid.*, **8**, 2454 (2001).
- [3] M. Marklund and P.K. Shukla, *Rev. Mod. Phys.* **78**, 591 (2006).
- [4] Y. I. Salmin, S. X. Hu, and K. Z. Hatsagortsyan, and C. H. Keitel, *Phys. Rep.* **427**, 41 (2006).
- [5] K. Becker, A. Koutsospyros, S. M. Yin *et al.*, *Plasma Phys. Control. Fusion* **47**, B513 (2005); K. H. Becker, K. H. Schoenbach, and J. E. Eden, *J. Phys. D* **39**, R55 (2006).
- [6] G. Shpatakovskaya, *J. Exp. Theor. Phys.* **102**, 466 (2006); J. A. van Dam, Y. V. Nazarov, E. P. A. M. Bakkers *et al.*, *Nature (London)* **442**, 667 (2006).
- [7] L K. Ang, W. S. Koh, Y. Y. Lau, and T. J. T. Kwan, *Phys. Plasmas* **13**, 056701 (2006).
- [8] W. L. Barnes, A. Dereux, and T. W. Ebbesen, *Nature (London)* **424**, 824 (2003).
- [9] T. C. Killian, *Nature (London)* **441**, 298 (2006).
- [10] P. A. Markovich, C. A. Ringhofer, and C. Schmeister, *Semiconductor Equations* (Springer, Berlin, 1990).
- [11] D. Pines, *J. Nucl. Energy: Part C: Plasma Phys.* **2**, 5 (1961).
- [12] C. L. Gardner and C. Ringhofer, *Phys. Rev. E* **53**, 157 (1996).

- [13] F. Haas, G. Manfredi, and M. Feix, Phys. Rev. E **2763**, (2000); G. Manfredi and F. Haas, Phys. Rev. B **64**, 075316 (2001).
- [14] D. Anderson, B. Hall, M. Lisak, and M. Marklund, Phys. Rev. E **65**, 046417 (2002).
- [15] F. Haas *et al.*, Phys. Plasmas **10**, 3858 (2003); F. Haas, *ibid.* **12**, 062117 (2005); G. Manfredi, Fields Inst. Commun. **46**, 263 (2005).
- [16] L. G. Garcia, F. Haas, L. P. de Oliveira, and J. Goedert, Phys. Plasmas **12**, 012302 (2005); M. Marklund, *ibid.* **12**, 082110 (2005).
- [17] P. K. Shukla, Phys. Lett. A **352**, 242 (2006); L. Stenflo, P. K. Shukla, and M. Marklund, Europhys. Lett. **74**, 844 (2006).
- [18] P. K. Shukla and B. Eliasson, Phys. Rev. Lett. **96**, 245001 (2006).
- [19] P. K. Shukla and L. Stenflo, Phys. Plasmas **13**, 044505 (2006).
- [20] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).