Value at Risk for Large Portfolios

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Abstract
We argue that the practise used in the valuation of the portfolio is important for the calculation of the Value at Risk. In particular, when liquidating a large portfolio the seller may not face horizontal demand curves. We propose a partially new approach for incorporating this fact in the Value at Risk and in an empirical illustration we compare it to a competing approach. We find substantial differences.

Key Words: Demand, Supply, Liquidity Risk, Limit Order Book, Bank, Sweden.
JEL Classification: C22, C51, C53, D40, G00, G10.

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1 Introduction

In this paper we address the question of how to properly assess the risk in large financial portfolios. In risk assessment it is usually assumed that the entire position can be sold at the market price (or mid-price), though one realizes that this can be a quite misleading valuation approach. The reason is that for large enough positions the seller (buyer) of an asset does not face a horizontal demand (supply) curve. Thus, there is an element of liquidity risk involved (see Malz, 2003, for a general discussion of liquidity risk) and this should reasonably be taken into account in risk assessment.

Here, the primary focus is on incorporating the liquidity risk in the Value at Risk (\(VaR\)) measure, which is the standard way of quantifying the risk of adverse price movements. \(VaR\) is defined as the maximum potential portfolio loss that will not be exceeded over a given time horizon for some small probability (see Jorion, 2007, for a survey). We emphasize, as argued by François-Heude and Van Wynendaele (2001) and others, that it is implicitly assumed that the liquidation occurs in one block at the end of the predefined holding period when calculating the \(VaR\). The question of how to incorporate the liquidity risk into the \(VaR\) is a relatively old one and several alternative approaches have been proposed. Bangia, Diebold, Schuermann, and Stroughair (1999) were the first to account for it, with their spread based alternative. Ernst, Stange, and Kaserer (2009) evaluates some measures empirically.

Our proposed approach relies on essentially the same idea as in Giot and Grammig (2006) (GG hereafter). Rather than the mid-price at the end of the horizon they consider the average price per share that would be obtained upon immediate liquidation. Their \(VaR\) is volume dependent and it is based on the difference between the mid-price at the beginning of the horizon and the average price at the end of it. We argue that the relevant initial price is not the mid-price, but that the portfolio should be valued at the average price in the beginning of the period as well. We have assets traded on an order driven markets with a visible limit order book (LOB) (e.g., Gourieroux and Jasiak, 2001, ch. 14) in
mind and the context is intra-day. Though frequently used on a (at least) daily basis, intra-day \( VaR \)'s are of interest as well. For example, Dionne, Duchesne, and Pacurar (2008) argue that the investment horizon for very active agents on the market is typically less than one day.

When it comes to the modelling of the dynamics of the average prices the literature is quite scarce. The model employed in GG is of AR-GARCH type and it is essentially univariate. Other previous attempts include Gourieroux, Le Fol, and Meyer (1998) and Bowsher (2004). The former consider a factor model in transaction time, while the latter proposes a functional signal plus noise time series model in calendar time. Our framework shares features with all three approaches and the resulting multivariate model allows for spatial (in the volume dimension) as well as serial correlation in the time dimension.

The paper is organized as follows. In Section 2 our \( VaR \) framework is presented. Section 3 gives some descriptive statistics of our data set consisting of high-frequency observations on the limit order book of Swedish banking stocks. In Section 4 we propose a time series model for the dynamics of the limit order book. Section 5 contains some empirical results including a comparison with the competing approach of GG.

## 2 The liquidity adjusted \( VaR \)

The object of interest is the conditional \( VaR \) for the horizon \( T \) to \( T + h \) for a univariate portfolio consisting of \( v_T \) shares of a financial asset. We will consider \( VaR \)'s for both long and short portfolios. For the latter we borrow shares today and agree to return them at some future date. Thus, in that case \( v_T \) is negative. We do not allow for portfolio updating, so that \( v_T = v_{T+i} \), \( i = 1, ..., h \), and we denote the value of the portfolio at time point \( t = T, ..., T + h \) by \( V_t \). Following Gourieroux and Jasiak (2001, ch. 16) the \( VaR \) for the position \( v_T \) satisfies

\[
\Pr\{V_{T+h} - V_T < -VaR_{T,h}^{1-\alpha} \mid \mathcal{F}_T\} = \alpha,
\]

where \( \mathcal{F}_T \) is the information available at time \( T \). That is, with the (small) probability \( \alpha \) the change in the value of the portfolio is less
than \(-VaR_{T,h}^{1-\alpha}\). In anticipation of what follows we note that the \(VaR\) depends on how we compute the values \(V_{T+h}\) and \(V_T\). The approach typically adopted in the literature is to assume that the entire portfolio can be sold at one and the same price, e.g., the mid-price, \(\tilde{P}_t\), \(t = T, ..., T+h\) (say). This implies that the portfolio values \(V_T\) and \(V_{T+h}\) in (1) are approximated by \(\tilde{V}_T = \tilde{P}_Tv_T\) and \(\tilde{V}_{T+h} = \tilde{P}_{T+h}v_T\), respectively. The corresponding approximative \(VaR\) for a long position then satisfies

\[
\Pr\{\tilde{V}_{T+h} - \tilde{V}_T < -\tilde{VaR}_{T,h}^{1-\alpha} \mid \mathcal{F}_T\} = \\
\Pr\{(\tilde{P}_{T+h} - \tilde{P}_T)v_T < -\tilde{VaR}_{T,h}^{1-\alpha} \mid \mathcal{F}_T\}.
\]

(2)

For a short position the expression becomes \(\Pr\{(\tilde{P}_{T+h} - \tilde{P}_T)v_T > \tilde{VaR}_{T,h}^{1-\alpha} \mid \mathcal{F}_T\}\). The discussion below is for a long position, but it applies analogously for a short one. Now, for relatively small positions we expect the \(VaR\) as defined by (2) to provide a reasonable approximation. However, as argued in the introduction \(\tilde{V}_T\) does not in general give the correct value of the portfolio. For example, assume that our position consists of 1000 shares and that at time \(T+h\), 500 shares are demanded at the price 2 at the first level of the bid-side of the LOB, and that 1000 shares are demanded at price 1 at the second level. Whereas a marking to the mid-price approach would assign a value of, at least, 2000 we would actually obtain \(500 \times 2 + (1000 - 500) \times 1 = 1500\) upon immediate liquidation. The average price per unit of sold volume for this transaction is 1.5 and it appears that this is the fair price to replace for \(\tilde{P}_{T+h}\) in (2).

Generalizing, we define \(\bar{P}_t(v)\) as the average price as a function of the volume, i.e. the average price per unit of volume that would result from immediately executing a market order of \(v\) shares. In the sequel we let superscripts \(a\) and \(b\) indicate whether the average price is for the ask or the bid side of the LOB. Figure 1 shows demand and supply schedules along with the corresponding average price curves for an observation of one of the stocks (SWB) in our data set.

The question is then how to properly use \(\bar{P}_t(v)\) to compute the relevant change in value and this is where we differ from GG. They consider a one-period setting and in their view the relevant change
in the value of a position of size $v_T$ is given by $\tilde{P}_T = \left[ \tilde{P}_T^b(1) + \tilde{P}_T^b(1) \right]/2$. They specify the dynamics of the log-returns, $p_t^{GG,v} = \ln(\tilde{P}_t(v_T)/\tilde{P}_{t-1})$, on the location-scale form $p_t^{GG,v} = \mu_t^{GG,v} + \sigma_t^{GG,v} \varepsilon_t^{GG,v}$, where $\mu_t^{GG}$ and $\sigma_t^{GG}$ are the conditional mean and standard deviation of $p_t^{GG,v}$, respectively, and $\varepsilon_t^{GG,v}$ is an iid random variable with zero mean and unit variance. Their $VaR$ is\footnote{Actually, their $VaR$ is the quantile of the distribution of the log-returns, but this is the implication for the $VaR$ definition we use.}

$$VaR_{T,1-\alpha}^{GG} = -\tilde{P}_T v_T (\exp(\mu_{T+1}^{GG,v} + \sigma_{T+1}^{GG,v} q_{\alpha}^\eta) - 1),$$

where $q_{\alpha}^\eta$ is the $\alpha$th quantile in the Student’s $t$ distribution with $\eta$ degrees of freedom.

We argue that with the same motivation as we value the portfolio at the average price at the end of the period, we should also value it at the average price in the beginning of it. Thus the relevant one-period change in value is $\tilde{P}_T^b(v_T) v_T - \tilde{P}_T^b(v_T) v_T$. With the corresponding log-return dynamics

$$p_t^v = \mu_t^v + \sigma_t^v \varepsilon_t^v,$$
our VaR alternative is

\[
VaR_{T,1}^{1-\alpha} = -\bar{P}_T^b(v_T)v_T(\exp(\mu_{T+1}^v + \sigma_{T+1}^v q_\alpha) - 1),
\]  

(3)

where \(q_\alpha\) is the \(\alpha\)th quantile of some suitable distribution.

For a horizon of \(h\) periods the VaR satisfies \(\Pr\{[\bar{P}_{T+h}(v_T) - \bar{P}_T(v_T)]v_T \leq -VaR_{T,h}^{1-\alpha} | \mathcal{F}_T\}\). However, the dynamics of the \(h\)-period returns do not follow easily from that of the one-period returns (cf. Lönnbark, 2009). Note also that our VaR and the VaR in Giot and Grammig (2006) are related by

\[
VaR_{T,h}^{1-\alpha} = VaR_{T,h}^{GG,1-\alpha} - (\bar{P}_T(v) - \bar{P}_T)v_T.
\]

Hence, given one of the VaR’s it is possible to obtain the other through an additive transformation that is known at time \(T\). Note also that the difference between the two measures grows with an increasing volume.

The VaR in (3) implicitly assumes that we own the portfolio at \(T\). If it is to be purchased at \(T\) we use \(\bar{P}_T^a(v_T)\) for the initial price and the VaR becomes

\[
VaR_{T,1}^{1-\alpha} = \bar{P}_T^a(v_T)v_T - \bar{P}_T^b(v_T)v_T \exp(\mu_{T+1}^v + \sigma_{T+1}^v q_\alpha).
\]

We end this section by giving the corresponding VaR’s for a short position. They are, respectively, given by

\[
VaR_{T,1}^{1-\alpha} = -\bar{P}_T^a(v_T)v_T(\exp(\mu_{T+1}^v + \sigma_{T+1}^v q_{1-\alpha}) - 1),
\]

(4)

\[
VaR_{T,1}^{1-\alpha} = -\bar{P}_T^b(v_T)v_T \exp(\mu_{T+1}^v + \sigma_{T+1}^v q_{1-\alpha}) - \bar{P}_T^b(v_T)v_T.
\]

(5)

Note that \(q_{1-\alpha}\) instead of \(q_\alpha\) appears in (4) and (5).

3 Data and descriptives

Our dataset consists of time series for the four largest banks in Sweden (Nordea NRD, Skandinaviska Enskilda Banken SEB, Handelsbanken SHB, Swedbank SWB) and covers the period May 3 – August 8, 2005.\(^3\)

\(^2\)Föreningssparbanken in the sample period.

\(^3\)For technical reasons the period June 7–10 is missing for all banks, and additionally May 27–June 1 for SWB and NRD.
Table 1: Descriptive statistics for the number of traded shares and closing prices in individual transactions for the four banks in the first trading month.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Nr of Traded Shares</th>
<th>Closing Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>StDev</td>
</tr>
<tr>
<td>NRD</td>
<td>9921.6</td>
<td>95424.5</td>
</tr>
<tr>
<td>SEB</td>
<td>5341.5</td>
<td>1.12·10^6</td>
</tr>
<tr>
<td>SHB</td>
<td>3963.1</td>
<td>33227.2</td>
</tr>
<tr>
<td>SWB</td>
<td>3644.2</td>
<td>22917.7</td>
</tr>
</tbody>
</table>

Table 1 gives a few descriptive statistics for the trading patterns in the four banking stocks for the first trading month (21 days) of the data. The number of traded shares distributions are quite skewed with a long upper tail and the largest transactions in each month are quite large. The largest transaction was in SEB and amounted to about 1653 million SEK using the average price. This corresponds to about 17 percent of total transactions during the month. For the other stocks the corresponding percentages are about 4 percent. Trading is most frequent in NRD with about 900 daily transactions or about 2 per minute.

The sampling frequency is chosen to be 30 minutes, such that the records immediately preceding the given half-hour are chosen. The daily records cover 1000–1700, i.e. there are 15 observations during the day and the total time series length is \( T = 936 \) for SEB and SHB and \( T = 861 \) for NRD and SWB.

For the empirical modelling we can obtain time series of average prices for any chosen volume level. For the analyses reported later we have chosen five volume levels \( v = 1, 100000(50000)300000 \) and all results are based on log-returns \( p_t^v = \ln(\bar{P}_t^v) - \ln(\bar{P}_{t-1}^v) \). As an illustration of the spatial/volume correlations within stocks we consider log-returns for the ask side of SHB, cf. Table 2. As expected from the smoothness of the average curve in Figure 1, we find that correlations between log-returns at the different volume levels are close to 1. Obviously, the correlations are weaker for lagged log-returns. The autocorrelation function closely matches the cross correlation, except for the first volume level.
Table 2: Cross correlations for log-returns (ask) in SHB across volume levels with \( v = 200000 \) as a base.

<table>
<thead>
<tr>
<th>Volumes (thousands)</th>
<th>( 1 \cdot 10^{-3} )</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.77</td>
<td>0.94</td>
<td>0.99</td>
<td>1.00</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>1</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>2</td>
<td>-0.02</td>
<td>-0.13</td>
<td>-0.13</td>
<td>-0.13</td>
<td>-0.13</td>
<td>-0.13</td>
</tr>
<tr>
<td>3</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.06</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td>4</td>
<td>-0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3: Parameter estimates and descriptive statistics for MA(1) models and their residuals of the ask/bid (a and b) average log-return series of the four banks at volume level \( v = 200000 \). \( p \)-values are used for the Ljung-Box statistics, LB.

<table>
<thead>
<tr>
<th>Bank</th>
<th>MA(1)</th>
<th>( t )</th>
<th>LB(_{10})</th>
<th>LB(_{10}^2)</th>
<th>Skew</th>
<th>Kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRD</td>
<td>a</td>
<td>0.088</td>
<td>2.59</td>
<td>0.86</td>
<td>0.02</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.141</td>
<td>4.15</td>
<td>0.95</td>
<td>0.03</td>
<td>-0.04</td>
</tr>
<tr>
<td>SEB</td>
<td>a</td>
<td>-0.030</td>
<td>-0.90</td>
<td>0.83</td>
<td>0.00</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-0.010</td>
<td>-0.32</td>
<td>0.82</td>
<td>0.00</td>
<td>-0.38</td>
</tr>
<tr>
<td>SHB</td>
<td>a</td>
<td>0.044</td>
<td>1.34</td>
<td>0.05</td>
<td>0.00</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.063</td>
<td>1.94</td>
<td>0.60</td>
<td>0.00</td>
<td>-0.08</td>
</tr>
<tr>
<td>SWB</td>
<td>a</td>
<td>0.088</td>
<td>2.60</td>
<td>0.31</td>
<td>0.63</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.018</td>
<td>0.54</td>
<td>0.63</td>
<td>0.00</td>
<td>-0.64</td>
</tr>
</tbody>
</table>

Based on the SWB series the autocorrelation functions suggest that MA(1) models will account for most of the serial correlation in the time series. Table 3 gives estimated models and some descriptive statistics for the residuals of the models. In all but one case there is significant autocorrelation in squared residuals, suggesting that ARCH effects are of major importance. For the ask series there is positive skewness and weak but negative for the bid series. For most series there is substantial kurtosis.
4 A time series model for the average price curves

We specify the dynamics of the average price curves in terms of log returns. Stock prices are widely taken to be random walks with drift and for returns various autoregressive and/or moving average extensions of the basic model seem to empirically surface. Based on some initial specification searches on the SWB stock we take as a reasonable model

\[
\begin{align*}
    p_t^{v_1} &= \alpha_{v_1} + \beta' d_t + \varepsilon_t^{v_1} + \theta_0 \varepsilon_{t-1}^{v_1} \\
    p_t^{v_i} &= \alpha_{v_i} + \beta' d_t + \gamma_{v_i} p_{t-1}^{v_i-1} + \varepsilon_t^{v_i} + \theta_{v_i} \varepsilon_{t-1}, \quad i = 2, \ldots, m,
\end{align*}
\]

where \( p_t^{v_i} = \log[\tilde{P}_t(v_i)] - \log[\tilde{P}_{t-1}(v_i)] \). The parameters \( \gamma_{v_i} \) and \( \theta_{v_i} \) are volume dependent; \( \gamma_{v_i} = \gamma_0 + \gamma_1 v_i - 1 \) and \( \theta_{v_i} = \theta_0 + \theta_1 v_i, \quad i = 2, \ldots, m \). The \( d_t \) is a vector of dummy variables to catch overnight impacts on the first observation of the day and time of day effects. In addition, the models of different volume levels may be correlated such that

\[
E(\varepsilon_{t,v_i} \varepsilon_{s,v_j}) \neq 0, \quad \text{for all } v_i, v_j \text{ and also for } t \neq s.
\]

For all volume levels, \( \mathbf{v} = \{v_1, v_2, \ldots, v_m\} \), we write

\[
\begin{pmatrix}
    p_t^{v_1} \\
    \vdots \\
    p_t^{v_m}
\end{pmatrix} =
\begin{pmatrix}
    \alpha_{v_1} \\
    \vdots \\
    \alpha_{v_m}
\end{pmatrix} + \beta' \mathbf{d}_t + \begin{pmatrix}
    \gamma_{v_2} & \cdots & 0 \\
    0 & \gamma_{v_3} & \cdots \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & \gamma_{v_m}
\end{pmatrix} \begin{pmatrix}
    0 & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots \\
    0 & \cdots & \cdots & \cdots
\end{pmatrix}
\]

\[
\begin{pmatrix}
    0 & \cdots & 0 \\
    0 & \cdots & \cdots & \cdots \\
    0 & \cdots & \cdots & \cdots
\end{pmatrix}
\begin{pmatrix}
    \varepsilon_{t,v_1} \\
    \vdots \\
    \varepsilon_{t,v_m}
\end{pmatrix} + \begin{pmatrix}
    \theta_{v_1} & 0 & \cdots & 0 \\
    0 & \theta_{v_2} & \cdots & \cdots \\
    \vdots & \ddots & \vdots & \vdots \\
    0 & \cdots & 0 & \theta_{v_m}
\end{pmatrix} \begin{pmatrix}
    \varepsilon_{t,v_1} \\
    \vdots \\
    \varepsilon_{t,v_m}
\end{pmatrix}
\]

or compactly
\[ p^v_t = \alpha + \beta^t d_t + \Gamma v p^v_{t-1} + \varepsilon_t + \Theta v \varepsilon_{t-1}, \]  
(6)

where \( \varepsilon_t \) is a vector of ones and \( \varepsilon_t \) has zero mean and conditional covariance matrix \( \Sigma_t \). Thus, the model is of VARMAX type and has both a time series and volume/spatial dimension. The \( \Sigma_t \) may contain nonzero off-diagonal elements and is also indexed by \( t \) to allow for ARCH-effects. For the conditional variances we employ a version of the asymmetric GARCH specification of Glosten, Jagannathan, and Runkle (1993)

\[ h^v_i = \omega_{v_i} + \delta h^v_i + \eta (\varepsilon^{v_i}_{t-1})^2 + \lambda (\varepsilon^{v_i}_{t-1})^2 1(\varepsilon^{v_i}_{t-1} < 0), \]  
(7)

where \( 1(\cdot) \) is the indicator function. Note that \( \omega_{v_i} \) is the only parameter that changes across \( v_i \). As a full baseline model for \( \Sigma_t \) we consider (7) together with constant off-diagonal elements

\[ \Sigma_t = \Omega + \delta \text{ diag}(h^v) + \eta \text{ diag}(\varepsilon^{2, v}_{t-1}) + \lambda \text{ diag}(\varepsilon^{2-v}_{t-1}), \]

where \( h^v, \varepsilon^{2, v}_t \) and \( \varepsilon^{2-v}_t \) have elements \( h^v_i, (\varepsilon^{v_i}_t)^2 \) and \( (\varepsilon^{v_i}_t)^2 1(\varepsilon^{v_i}_t < 0), \) \( i = 1, ..., m \), respectively. The \( \text{diag}(\cdot) \) operator returns a matrix with the vector argument on the diagonal and zeros elsewhere. Hence, the conditional expectation and the conditional variance of the log returns are, respectively, given by

\[ E(p^v_t | \mathcal{F}_{t-1}) = \alpha + \beta^t d_t + \Gamma v p^v_{t-1} + \Theta v \varepsilon_{t-1} \]
\[ V(p^v_t | \mathcal{F}_{t-1}) = \Sigma_t. \]  
(8)

These expressions are useful both for estimation and forecasting over time. From (6) it is straightforward to obtain the corresponding price levels as \( \tilde{P}^v_{i} = \tilde{P}^v_{t-1} \exp(p^v_t), \) \( i = 1, ..., m \). The conditional expectation and variance of \( \tilde{P}^v_t \) may be obtained by taking first order expansions of the exponential function and (8).

With respect to the spatial aspects of the model note that this is an unusual context of observation availability for all volume levels. However, for low levels the volume curves are typically flat and for very large levels linear. Therefore, it appears reasonable to focus the modelling exercise on the intermediate levels, where the curvature is most...
pronounced. The way we choose $v$ and $m$ in the estimation phase impacts the precision of the estimates, but as our model is not able to predict in the volume direction, the choice is also practically related to the model’s end use for VaR calculations.

4.1 Estimation

When it comes to predicting the VaR we use a multivariate version of a popular methodology known as filtered historical simulation (FHS) in the literature (e.g., Christoffersen, 2009). To explain the approach we first collect all model parameters in the vector $\psi$ and consider the prediction error $e_t = p_t^\psi - E_\psi(p_t^\psi|\mathcal{F}_{t-1})$, where we subindex the expectation operator to emphasize that it is to be taken under $\psi$. Assuming that the standardized prediction errors $\tilde{e}_t = (\Sigma_t^{1/2})^{-1} e_t$, $t = 1, ..., T$, is an iid sequence we may approximate the conditional distribution of $p_{T+1}^\psi$ with the sequence $p_{T+1,j}^\psi = E_\psi(p_{T+1}^\psi|\mathcal{F}_T) + \Sigma_{T+1}^{1/2} \tilde{e}_j$, $j = 1, ..., T$. The predictors of the one-period VaR’s are then trivially obtained from suitable empirical quantiles of the $p_{T+1,j}^\psi$ sequences.

The FHS is a two-step procedure that in the first step estimates the underlying model parameters employing some estimator, $\hat{\psi}$. In the second step it filters out the $\tilde{e}_t$ sequence.

A natural choice for $\hat{\psi}$ is the quasi maximum likelihood estimator. Given observations up til time $T$ it involves finding the $\psi$ that maximizes the log-likelihood function

$$
\ln L = -\frac{1}{2} \sum_{t=2}^{T} (\ln |\Sigma_t| - e_t' \Sigma_t^{-1} e_t).
$$

For practical estimation we use the RATS 6.0 package and employ robust standard errors.

5 Empirical results

The empirical results are summarized in terms of VaR measures in Table 4 for the case when we own the portfolio at the horizon origin. Parameter estimates may be found in Table 5. The measures are calculated for the
Table 4: VaR estimates for $\alpha = 0.01$.

<table>
<thead>
<tr>
<th>Volume</th>
<th>NRD Short</th>
<th>NRD Long</th>
<th>SEB Short</th>
<th>SEB Long</th>
<th>SHB Short</th>
<th>SHB Long</th>
<th>SWB Short</th>
<th>SWB Long</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.59</td>
<td>0.38</td>
<td>1.29</td>
<td>1.33</td>
<td>1.30</td>
<td>0.53</td>
<td>1.63</td>
<td>0.86</td>
</tr>
<tr>
<td>100 000</td>
<td>57 698</td>
<td>37 733</td>
<td>136 107</td>
<td>119 240</td>
<td>125 733</td>
<td>74 944</td>
<td>175 379</td>
<td>92 403</td>
</tr>
<tr>
<td>150 000</td>
<td>86 138</td>
<td>48 532</td>
<td>225 120</td>
<td>161 578</td>
<td>198 631</td>
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<td>341 902</td>
<td>606 274</td>
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first post sample time period, i.e. 5PM of August 8 to 10AM of August 9, 2005. The numbers reported for a short position are throughout larger than the ones for the corresponding long position. This is a consequence of, at least, the asymmetry in average cost curves.

Figure 2 gives $VaR$’s per share for SWB. With some exceptions, there is a modest growth in all measures. If we take the view that we own the portfolio at the horizon origin, our $VaR$’s are smaller than those calculated as in GG. If the portfolio is to be purchased, they are larger. Noteworthy is also that for the latter view our $VaR$’s rise more sharply with volume. There is a growing difference between our $VaR$’s and the ones as in GG, starting from one half of a tick (0.25 SEK) at volume 1 to exceeding 2 ticks for the largest position of $v = 300000$ shares. Obviously, these differences will have substantial consequences for how to set the required capital for large financial institutions.
Table 5: Parameter estimates. Bold facing indicates significance at 5 percent level. The Ljung-Box statistics are evaluated at the volume level $v = 200000$.

<table>
<thead>
<tr>
<th>Param</th>
<th>NRD</th>
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<th>SWB</th>
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<td>Ask</td>
<td>Bid</td>
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<td>$-8.15 \cdot 10^{-5}$</td>
<td>$1.31 \cdot 10^{-4}$</td>
<td>$1.58 \cdot 10^{-4}$</td>
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<td>$\alpha_3$</td>
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### VaR for large portfolios

**continued**

<table>
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<th>SWB</th>
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Figure 2: VaR per share vs volume for long and short positions in the SWB stock. GG refers to the VaR as given by the approach in Giot and Grammig (2006). LHB1 and LHB2 are our VaR’s for a portfolio owned and purchased at \( T \), respectively.


References


