ON RISK PREDICTION

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Abstract

This thesis comprises four papers concerning risk prediction.

Paper [I] suggests a nonlinear and multivariate time series model framework that enables the study of simultaneity in returns and in volatilities, as well as asymmetric effects arising from shocks. Using daily data 2000-2006 for the Baltic state stock exchanges and that of Moscow we find recursive structures with Riga directly depending in returns on Tallinn and Vilnius, and Tallinn on Vilnius. For volatilities both Riga and Vilnius depend on Tallinn. In addition, we find evidence of asymmetric effects of shocks arising in Moscow and in the Baltic states on both returns and volatilities.

Paper [II] argues that the estimation error in Value at Risk predictors gives rise to underestimation of portfolio risk. A simple correction is proposed and in an empirical illustration it is found to be economically relevant.

Paper [III] studies some approximation approaches to computing the Value at Risk and the Expected Shortfall for multiple period asset returns. Based on the result of a simulation experiment we conclude that among the approaches studied the one based on assuming a skewed $t$ distribution for the multiple period returns and that based on simulations were the best. We also found that the uncertainty due to the estimation error can be quite accurately estimated employing the delta method. In an empirical illustration we computed five day Value at Risk’s for the S&P 500 index. The approaches performed about equally well.

Paper [IV] argues that the practise used in the valuation of the portfolio is important for the calculation of the Value at Risk. In particular, when liquidating a large portfolio the seller may not face horizontal demand curves. We propose a partially new approach for incorporating this fact in the Value at Risk and in an empirical illustration we compare it to a competing approach. We find substantial differences.

Key words: Finance, Time series, GARCH, Estimation error, Asymmetry, Supply and demand.
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This thesis consists of a summary and the following four papers:


Paper II is included with permission from the journal.
1 Introduction

The Oxford Advanced Learner’s Dictionary defines risk as ”the possibility of something bad happening at some time in the future”. There are many different types of risks (even in a financial context) and in this thesis the focus is on a type of risk referred to as market risk, which is the risk of adverse price movements. In the lecture delivered in connection with receiving The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel, Robert Engle noted: ”The advantage of knowing about risks is that we can change our behavior to avoid them” (Engle, 2004). Of course, as he further notes, we do not wish to avoid them completely. Rather, we take on risks that we think are worthwhile. This trade-off between return and risk is at the heart of financial economics.

Unlike return, risk is something that we never observe directly and knowing about it is almost synonymous to the proper assessment of it (so that it can be managed). Historically, the standard way of measuring risk has been by the variance of asset returns. This is to a large extent due to the huge impact of the modern portfolio theory of Markowitz (1952). However, what risk measure to use is very much context dependent. For discussions of views on risk and reviews of risk measuring, see Granger (2002) and McNeil, Frey, and Embrechts (2005).

The standard way of measuring market risk in the financial industry today was pioneered by J.P. Morgan’s RiskMetrics with the Value at Risk (VaR), which was unveiled in 1994 (see J.P. Morgan and Reuters, 1996). In fact, in the second Basel framework (often referred to as Basel II) the Basel Committee on Banking Supervision (Basel hereafter) requires banks and financial institutions to set aside capital buffers in order to meet market risks, which are usually measured by VaR’s (Basel, 2006). Thus, accurate VaR’s are crucially important for the stability of the financial system and the measure has received a great deal of attention in the literature (e.g., Jorion, 2007). Essentially, the VaR is defined as a potential portfolio loss that most likely will not be exceeded. In statistical terms it is nothing but a quantile of the return distribution.
The attractive feature of the VaR is that it summarizes the properties of the return distribution into an easily interpreted number. However, a major concern for it is that it is silent about the size of the loss when disaster strikes (see Artzner, Delbaen, Eber, and Heath, 1999, for other concerns and a formal discussion of what constitutes a good risk measure). The Expected Shortfall (ES), on the other hand, gives the expected loss, given that the loss exceeds the VaR. The ES is gaining increasing popularity and Yamai and Yoshiba (2005) and others argue in favor of its use. It is the second risk measure studied in this thesis.

Measuring the market risk essentially boils down to making assumptions about the future outcomes of asset returns and it is often closely related to predicting volatility. Obviously, this renders good volatility predictors crucially important and the most popular framework for it is, without doubt, the ARCH and GARCH of Engle (1982) and Bollerslev (1986). In the first paper, we contribute to this field by proposing a model for the joint evolution of the Baltic stock markets.

The assumptions made about the outcomes of future asset returns are associated with uncertainty as well. For example, the assumed risk model may be badly misspecified (see Derman, 1996, for a discussion). Typically, the model is specified up to some parameters and even though the model happens to be a good approximation of reality we still have to estimate those parameters based on historical observations, or by an educated guess. Of course, this is associated with uncertainty too, but this uncertainty tends to be neglected in practice. In papers two and three we give it attention, though. In particular, we find that the uncertainty due to the estimation error may be substantial and that it has an effect on the interpretation of the VaR.

Another issue that may arise is that of how to best predict the risk on a particular horizon. Most naturally, one could specify a model for the relevant horizon directly. However, as noted above, a risk model is associated with an estimation error and this is directly related to the size of the dataset. Hence, it may be the case that the data at hand does not suffice for a reliable prediction. The alternative is then to specify a model for a higher frequency and to use this model to get an indirect
prediction. This is easier said than done and in the third paper we consider some approaches for this indirect prediction problem.

Lastly, consider a large portfolio that contains many shares of an asset. A conventional assumption made in the literature is that the entire position can be sold at the same price. This can be a quite misleading valuation approach, since for a large enough position the seller (buyer) of an asset does not face a horizontal demand (supply) curve. In the fourth paper we incorporate this fact in the $\text{VaR}$.

In what follows, the issues indicated above are further developed and the contributions of this thesis are related to the existing literature. First of all, the $\text{VaR}$ and the $\text{ES}$ are formally introduced.

## 2 $\text{VaR}$ and $\text{ES}$

We wish to quantify the risk in a portfolio of financial assets between the times $T$ and $T + k$ and to introduce the $\text{VaR}$ and the $\text{ES}$ we denote by $\mathbf{w} = (w_1, ..., w_M)'$ the time invariant vector of portfolio weights. The log-return (return) between $T$ and $T + k$ for the portfolio is approximately $\mathbf{w}'\mathbf{Y}_{T,k} = \mathbf{w}'(\mathbf{y}_{T+1} + ... + \mathbf{y}_{T+k})$, where $\mathbf{y}_{T+l} = (y_{1,T+l}, ..., y_{M,T+l})'$, $l = 1, ..., k$, is a $M$-dimensional vector of one-period returns. The conditional $\text{VaR}$ for the period $T$ to $T + k$ satisfies

$$\Pr \left( \mathbf{w}'\mathbf{Y}_{T,k} \leq -\text{VaR}^{1-\alpha}_{T,k} | \mathcal{F}_T \right) = \alpha, \quad (1)$$

where $\mathcal{F}_T$ is the information available at $T$ and $\alpha$ is a small probability. The associated conditional $\text{ES}$ is defined as

$$\text{ES}^{1-\alpha}_{T,k} = -E_T \left( \mathbf{w}'\mathbf{Y}_{T,k} \mid \mathbf{w}'\mathbf{Y}_{T,k} \leq -\text{VaR}^{1-\alpha}_{T,k} \right), \quad (2)$$

where $E_T (\cdot)$ is shorthand for expectation conditional on $\mathcal{F}_T$. The minus signs in (1) and (2) stem from the convention of reporting the $\text{VaR}$ and the $\text{ES}$ as positive numbers.

The $\mathcal{F}_T$ typically contains past asset returns and the goal is to use this information in the best possible way to compute predictors $\widehat{\text{VaR}}^{1-\alpha}_{T,k}$ and $\widehat{\text{ES}}^{1-\alpha}_{T,k}$. From (1) it is obvious that the $\text{VaR}$ is a quantile of the
return distribution. Thus, predicting the VaR essentially amounts to employing statistical techniques for quantile estimation. These have been around for a long time and approaches to computing the measures range from non-parametric to fully parametric ones, with lots of hybrids in between. For recent surveys of existing approaches, see Jorion (2007) and McNeil et al. (2005). See also Kuster, Mittnik, and Paolella (2006), for a comparison of some popular alternatives. For example, assuming that the information at hand is a sample of identically and independently distributed (iid) returns a straightforward predictor of the VaR is a suitable order statistic. This approach is referred to as historical simulation in the financial industry.

In this thesis we consider parametric approaches and we assume that the vector process, $y_t$, of the assets returns started in the infinite past and that it is generated in discrete time up through, at least, $T+k$ by

$$y_t = \mu_t + u_t, \quad u_t = H_t^* \varepsilon_t. \tag{3}$$

Conditional on the information available at $t-1$, $\varepsilon_t$ has mean 0 and the identity matrix, $I$, as its variance-covariance matrix. Then, $\mu_t$ is the conditional mean of $y_t$, whereas $H_t = H^*_t H_t^{*'}$ is the conditional variance-covariance matrix. In the next section we discuss specifications of $\mu_t$ and $H_t$.

# 3 GARCH

In line with the hypothesis of efficient markets, asset prices are widely taken to be random walks (Gourieroux and Jasiak, 2001) and the effort in terms of modeling is often made on the variance part of (3). Thus, for the conditional mean function various ARMA specifications are routinely adopted (e.g., McAleer and Da Veiga, 2008). An interesting alternative is the use of the asymmetric moving average model of Wecker (1981) in Brännäss and De Gooijer (2004). Brännäss and Soltanaeva (2006) later extended it to include explanatory variables.

The most popular framework when it comes to the modeling of the conditional variance is the GARCH. Since Engle’s seminal paper,
the ARCH-literature has exploded with extensions of the basic model; adapting it to different stylized facts of financial asset returns (see Cont, 2001, for an account of stylized facts). In fact, the GARCH models were originally developed to cope with the stylized fact of volatility clustering. For a survey on GARCH models and other volatility predictors, e.g., models of stochastic volatility, see Andersen, Bollerslev, Christoffersen, and Diebold (2006).

The workhorse specification (cf. Hansen and Lunde, 2005) in univariate situations, i.e. $H_t = h_t$, is the GARCH(1,1) model

$$h_t = \omega + \alpha u^2_{t-1} + \beta h_{t-1}, \quad (4)$$

where $u_t = y_t - \mu_t = \sqrt{h_t} \varepsilon_t$, i.e. the one-period ahead prediction error. A stylized fact that has proved highly relevant empirically is the so-called leverage effect, i.e. that negative returns are followed by higher volatility than positive ones. The leverage effect was first acknowledged by Black (1976) and it has been incorporated in the GARCH framework by Glosten, Jagannathan, and Runkle (1993), Nelson (1991) and many others. The model of the former appears to be the most popular one in empirical work and it extends (4) by the term $\gamma \min(0, u_t) u_t$, thus allowing positive and negative shocks to affect future volatility asymmetrically.

In financial contexts we usually deal with portfolios, i.e. we are interested in the joint evolution of several assets or markets. Consequently, multivariate models with variance specifications of the GARCH-type have been developed (see Bauwens, Laurent, and Rombouts, 2006, for a survey). The important feature that multivariate models wish to capture is that of how shocks transmit across assets and markets (e.g., Karolyi, 1995; Bonfiglioli and Favero, 2005). Understanding the nature of this transmission is of great practical interest, as it may have consequences for, e.g., risk management decisions (Fleming, Kirby, and Ostdiek, 1998).

The intra-day literature suggests that information processing is very fast (e.g., Engle and Russell, 1998). Hence, for models specified on a (say) daily frequency it may be important to incorporate simultaneous effects. Indeed, structural VAR models have quite recently been
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employed to study the joint behavior and contemporaneous interaction among asset returns (e.g., Rigobon and Sack, 2003; De Wet, 2006; Lee, 2006). Obviously, and perhaps more interestingly, there is also reason to expect simultaneous effects in volatilities. Gannon and Choi (1998) and Gannon (2004, 2005) have addressed this question in terms of realized volatilities, i.e. squared returns. However, in a multivariate GARCH context it seems natural to allow for simultaneity in conditional variances. In the first paper of this thesis we propose the, to our knowledge, first model with this particular feature. We apply the model and study the joint evolution of returns and volatilities of the stock exchanges in the Baltic cities Riga, Tallinn and Vilnius.

4 Estimation error

The task of computing the VaR and the ES is predictive in nature and it is clearly subject to uncertainty. Hendry (2000) discusses various error sources in prediction. Here, we focus on the error that is due to the fact that the parameters of the hypothesized model of the data-generating process are unknown and must be estimated. The additional uncertainty from this error source should be of concern to risk managers. Surprisingly little work has been done on it, though. In fact, Lan, Hu, and Johnson (2007) report that the research on the uncertainty of VaR predictors only amounts to about 2.5 percent of the VaR literature. Jorion (1996) was the first to attempt to formally quantify it, but following his paper this research area appears to have rested for some time and regained interest quite recently. For example, Christoffersen and Gonçalves (2005) used resampling techniques to study the uncertainty of VaR and ES predictors in a GARCH framework. The obvious disadvantage of their method is that it is time consuming since it amounts to repeated estimation of a possibly complicated model. Analytical expressions (when sufficiently accurate) to quantify the uncertainty are obviously preferred. For this purpose Chan, Deng, Peng, and Xia (2007) and others consider the conventional delta method, which is done here as well.
In what follows, we will take as given a consistent and asymptotically normally distributed estimator, $\hat{\theta}$, that is centered at the true parameter vector. When $\mu_t$ and $H_t$ in (3) are correctly specified, one such estimator is the traditional (conditional) maximum likelihood (ML) estimator with a normality assumption on $\varepsilon_t$. This assumption does not fare well with the stylized fact of conditionally leptokurtic and sometimes conditionally skewed asset return distributions. However, as shown by Weiss (1984, 1986) and Bollerslev and Wooldridge (1992), the estimator remains consistent and asymptotically normal even if the distribution of $\varepsilon_t$ is non-normal and it is then known as the Quasi-Maximum Likelihood (QML) estimator. Of course, ML estimation has been employed with other distributional assumptions as well. For example, Bollerslev (1987) considers the Student’s $t$ distribution.

Early attempts (e.g., Schmidt, 1974) to quantify the effect on prediction of errors in parameters relied on the asymptotic distribution of the parameter estimator, assumed to be independent of the conditioning information. In the notation set out above, the predictors are functions of $\mathcal{F}_T$ both directly and indirectly through $\hat{\theta}$. Denote this (continuous) function by $u[\mathcal{F}_T, \hat{\theta}(\mathcal{F}_T)]$. The approach then amounts to conditioning the first argument of $u(\cdot)$ on a realization of $\mathcal{F}_T$ and viewing randomness to arise through the random $\mathcal{F}_T$ in the second argument. This approach now appears to be the conventional (see Kaibila and He, 2004, for a recent discussion). Indeed, Hansen (2006) takes this route and shows asymptotic normality for $\hat{V}aR_{T,1}$.

Now, the question a practitioner naturally poses is how uncertainty in the $VaR$ affects risk management, i.e. does it in some way change what value to report. Indeed, Tsay (2005, ch. 7) points out that the $VaR$ should be computed using the predictive distribution of returns, and it should take into account the parameter uncertainty in a properly specified model. In the second paper we accept this challenge and demonstrate a way of incorporating the estimation error in a $VaR$ predictor. The key insight is that, in practice, we do not use the $VaR$ that satisfies (1), i.e. the true $VaR$. Instead, we use a random predictor of it and the relevant probability is $\Pr(w'Y_{T,k} \leq -\hat{V}aR_{T,k} | \mathcal{F}_T)$. Clearly,
Figure 1: \( VaR \) density and return density refers to the conditional densities of the \( VaR \) predictor and the return, respectively.

this probability is not necessarily equal to \( \alpha \). In Figure 1 we depict the situation.

Related discussions appear in Schaller (2002) and Escanciano and Olmo (2008). The latter is given in a back-testing\(^1\) context, though. We emphasize that the situation is not bias in the conventional sense, i.e. that the expected value of the \( VaR \) predictor is different from the true value. For studies of conventional bias, see Bao and Ullah (2004), Gomes and Pestana (2007) and Hartz, Mittnik, and Paolella (2006).

\(^1\) Back-testing is the blanket term for statistical techniques of \( VaR \) predictor validation (e.g., Campbell, 2005).
5 Horizon

It is sometimes of interest to measure the risk on horizons longer than (say) one day. An important example when this is the case is for the market risk charge in Basel II, that is based on an horizon of 10 trading days. It is then natural to specify a risk model for the relevant horizon directly. Indeed, this is the recommendation put forth by Diebold, Hickman, Inoue, and Schuermann (1997). However, as noted above \( \text{VaR} \) and \( \text{ES} \) predictors are subject to an estimation error, which is directly related to the sample size. Thus, it may be the case that the available sample size is not large enough for reliable predictions. The alternative is then to specify a model for a higher frequency and iterate on this model to obtain predictions for the relevant horizon. This corresponds to the case \( k > 1 \) in (1) and (2) and the properties of the multiple period returns are thus of interest.

Now, assume that the one-period portfolio return is normally and independently distributed (nid) with zero mean and variance \( \sigma^2 \). Then, the \( k \)-period return is nid with zero mean and variance \( k\sigma^2 \). In this case the task of computing the \( \text{VaR} \) and the \( \text{ES} \) for the multiple period returns is trivial and they are simply obtained by scaling the one-period measures by a factor \( \sqrt{k} \). This is the so-called Root-\( k \) approach and it is allowed in Basel II. However, it is safe to say that asset returns are not normally distributed and certainly not independent in time and it is well known that this approach may give erroneous \( \text{VaR} \)'s (see Brummelhuis and Guégan, 2005; Brummelhuis and Kaufmann, 2007, for discussions). Thus, alternative approaches are called for and this is the focus of the third paper in this thesis.

Suppose now that the risk manager wants to assess the \( k \)-period risk in the portfolio and decides to employ the iterating approach within the GARCH framework. A problem that arises is then that the properties of the multiple period return distribution may not follow easily from the one-period model. For example, even though the multiple period conditional variance implied from a one-period GARCH model with normal innovations is tractable, less so is the distribution of the correspond-
ing innovation (e.g., Boudoukh, Richardson, and Whitelaw, 1997). Two ways to go about it are to use simulations (e.g., Christoersen, 2003) or to consider some other (than the Root-\(k\)) analytical approximation.

To explain the simulation based approach, we first assume that the model (3) have been estimated based on observations available up through \(T\). Based on some assumption on the distribution of \(\varepsilon_{T+l}, l = 1, ..., k\), we then simulate future \(k\)-period portfolio returns and compute the \(VaR\) and the \(ES\) as empirical counterparts.

Refinements of this basic setup include for example the use of kernel functions for increased efficiency (Scaillet, 2004; Chen and Tang, 2005; Chen, 2008) and extreme value theory (McNeil and Frey, 2000). As for the distributional assumption it is of course natural to maintain the one used for estimation in a maximum likelihood framework. However, an approach that has gained popularity is the so-called filtered historical simulation, that was proposed in a univariate context by Barone-Adesi, Bourgoion, and Giannopoulos (1998), Diebold, Schuermann, and Stroughair (1998) and Hull and White (1998). It involves estimating (3) by QML and the distribution of \(\varepsilon_{T+l}, l = 1, ..., k\), is approximated by the empirical distribution of the standardized residuals (see also Christoersen, 2009, for a multivariate extension).

For the analytical approximations we assume that the conditional mean, \(\mu_{T,k}\), and the conditional variance-covariance matrix, \(H_{T,k}\), of \(Y_{T,k}\) are tractable, and that the \(k\)-period portfolio return admits the scale-location representation

\[
\mathbf{w}'\mathbf{Y}_{T,k} = \mathbf{w}'\mu_{T,k} + \varepsilon_{T,k}\sqrt{\mathbf{w}'H_{T,k}\mathbf{w}},
\]

where \(\varepsilon_{T,k}\) has zero mean, unit variance, and (intractable) conditional density function \(g_{T,k}(\cdot)\). The problem then boils down to that of finding a suitable approximation for \(g_{T,k}(\cdot)\), and in the third paper we study two alternatives for this. The first approach was originally proposed by Wong and So (2003, 2007), and it involves a fully parametric assumption. The second approach employs a Gram-Charlier expansion (e.g., Baillie and Bollerslev, 1992; Jondeau and Rockinger, 2001).

Alternative approaches based on (5) include Fan and Gu (2003) and
Cotter (2007). The former employ non-parametric techniques on the standardized residuals, while the latter scales the one-period VaR relying on an extreme value theory argument. Taylor (1999, 2000) propose a regression quantile approach that may be viewed as a combination of the direct and the iterating approach.

6 Valuation

In the computation of the VaR and the ES it is often assumed that the assets in the portfolio may be traded at mid-prices\(^2\). For small positions and with tight spreads\(^3\) this may work fine, but it is not a fair valuation approach in general. For example, trading typically does not occur at mid-prices, but at the best bid and ask prices. Consequently, early adjustments to the VaR focused on incorporating adverse movements in the spread (e.g., Bangia, Diebold, Schuermann, and Stroughair, 1999). However, the seller (buyer) of large enough positions does not face horizontal demand (supply) curves. Hence, the liquidation of a position may give rise to an adverse price impact that goes beyond the spread. The question of how to incorporate this fact in the VaR is a relatively old one and several approaches have been proposed (see Ernst, Stange, and Kaserer, 2009; Stange and Kaserer, 2009, for overviews). In particular, the way to go about it depends on what type of market the asset in question is traded on (see Gourieroux and Jasiak, 2001, ch. 14, for an account of the characteristics of quote-driven and order-driven markets). On quote-driven markets one or several market makers set a bid and an ask price, and the additional information available is essentially transaction data. On order-driven markets (with visible limit order books), on the other hand, it is possible to infer the actual price per share that would be obtained upon immediate liquidation. Indeed, Giot and Grammig (2006) use this information and propose an adjusted VaR. This approach appears to us as the most sound of the existing ones, but, of course, it is of limited applicability on quote-driven mar-

\(^2\)The mid-price is the average of the best bid price and the best ask price.
\(^3\)The spread is the difference between the best bid price and the best ask price.
markets. In the fourth paper we build on the approach in Giot and Grammig (2006) and we give our views on how to adjust the \textit{VaR} with limit order book data at hand.

The discussion above is viewed as a source of liquidity risk in the literature and it is very relevant in practise (e.g., Malz, 2003). Liquidity risk has received interest from the regulatory side as well (see Basel, 2008).

7 Summary of the papers

Paper [I]: Simultaneity and Asymmetry of Returns and Volatilities in the Emerging Baltic State Stock Exchanges

The paper suggests a nonlinear time series model framework that enables the study of simultaneity in returns and volatilities, as well as asymmetric effects arising from shocks. Using daily data 2000-2006 we study the joint evolution of returns and volatilities in the indices of the Baltic state stock exchanges. Shocks from the Moscow stock exchange enters the model through exogenous explanatory variables. As a motivation for the study we take the potential presence of cross market linkages and information spillovers in international investment and risk management decisions. It appears reasonable to expect that these features are of importance for the markets under study, as they are geographically close and share other common features.

The estimation results indicate recursive structures with Riga directly depending in returns on Tallinn and Vilnius, and Tallinn on Vilnius. For volatilities both Riga and Vilnius depend on Tallinn. In addition, we find evidence of asymmetric effects on both returns and volatilities of shocks arising in Moscow and in the Baltic states.

The practical use of the model is outlined and studied. In particular, we compare portfolio allocations and \textit{VaR}'s obtained from our model to those implied by univariate models. We find substantial differences.
Paper [II]: A Corrected Value-at-Risk Predictor

We argue that the additional uncertainty due to the estimation error matters for the interpretation of VaR predictors. In particular, we demonstrate that reported VaR’s may be too small, in the sense that the probability that a portfolio loss exceeds the predicted VaR is higher than desired. A simple way of correcting a VaR predictor to give the correct interpretation is proposed. The approach relies on the so-called delta method of computing the approximative variance of the sampling distribution of the VaR predictor. In numerical and empirical illustrations we verify statistical and economic significance, respectively.

Paper [III]: Uncertainty of Multiple Period Risk Predictors

The focus of this paper is on predicting the VaR and the ES for multiple period asset returns. In general, the properties of the conditional distribution of multiple period returns do not follow easily from the one-period data generating process. This renders computation of the VaR and the ES for multiple period returns a non-trivial task and we consider some approaches to approximating these measures. The first one is the Root-k approach that simply scales the one-period measures by the square root of the number of periods. The second one targets the measures by means of simulations. The third and the fourth approaches derive the measures from analytical approximations to the conditional density of the multiple period returns. We consider a skewed $t$ distribution and a Gram-Charlier expansion. In addition, we view the additional uncertainty due to the estimation error as important and keep it an integral part of the paper. In particular, we study the usefulness of the so-called delta method.

Based on the result of a simulation experiment we conclude that among the approaches studied the one based on assuming a skewed $t$ distribution for the multiple period returns and that based on simulations were the best. The predictors based on the Gram-Charlier expansion and the Root-k approximation showed positive and negative bias, respectively. Except for the Root-k approach and in some cases for the
Gram-Charlier approach we found that the uncertainty due to the estimation error can be quite accurately estimated employing the delta method.

In an empirical illustration all predictors performed about equally well in predicting five day $VaR$’s for the S&P 500 index.

**Paper [IV]: Value at Risk for Large Portfolios**

In this paper we address the question of how to properly assess the risk in large positions of financial assets. We argue that the practise used in the valuation of the portfolio is of importance for the calculation of the $VaR$. Commonly, it is assumed that the entire position can be sold at the market price (or mid-price), though one realizes that this can be a quite misleading valuation approach. The reason is that for a large enough position the seller of an asset does not face a horizontal demand curve. Instead, we argue, a portfolio should be valued at the actual prices that would be obtained upon immediate liquidation. Based on a model for the dynamics of the limit order book we propose a partially new approach for incorporating the argument in an intra-day $VaR$. In an empirical illustration we found substantial differences between our $VaR$ and a competing alternative.
References


Introduction and summary


