On Aspects of Mathematical Reasoning
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ISSN 1102 -8300
ISBN 978-91-7264-791-6
Typeset by the author using $\LaTeX\,2_\varepsilon$
Printed by Print & Media, Umeå universitet, Umeå, 2009

To my family

On Aspects of Mathematical Reasoning Affect and Gender

DOCTORAL DISSERTATION
by
LOVISA SUMPTER

Doctoral Thesis No. 41, Department of Mathematics
and Mathematical statistics, Umeå University, 2009.

To be publicly discussed in lecture hall MA 121, Umeå University, on Monday, June 1, 2009, at 13:15 for the degree of Doctor of Philosophy.

Abstract

This thesis explores two aspects of mathematical reasoning: affect and gender. I started by looking at the reasoning of upper secondary students when solving tasks. This work revealed that when not guided by an interviewer, algorithmic reasoning, based on memorising algorithms which may or may not be appropriate for the task, was predominant in the students reasoning. Given this lack of mathematical grounding in students reasoning I looked in a second study at what grounds they had for different strategy choices and conclusions. This qualitative study suggested that beliefs about safety, expectation and motivation were important in the central decisions made during task solving. But are reasoning and beliefs gendered? The third study explored upper secondary school teachers conceptions about gender and students mathematical reasoning. In this study I found that upper secondary school teachers attributed gender symbols including insecurity, use of standard methods and imitative reasoning to girls and symbols such as multiple strategies especially on the calculator, guessing and chance-taking were assigned to boys. In the fourth and final study I found that students, both male and female, shared their teachers view of rather traditional feminities and masculinities. Remarkably however, this result did not repeat itself when students were asked to reflect on their own behaviour: there were some discrepancies between the traits the students ascribed as gender different and the traits they ascribed to themselves. Taken together the thesis suggests that, contrary to conceptions, girls and boys share many of the same core beliefs about mathematics, but much work is still needed if we should create learning environments that provide better opportunities for students to develop beliefs that guide them towards well-grounded mathematical reasoning.

Sammanfattning

Den här avhandlingen utforskar två aspekter av matematiska resonemang: affekt och genus. Avhandlingen inleds med en undersökning av gymnasielevers matematiska resonemang. Resultatet visar att eleverna i huvudsak resonerar algoritmiskt baserat på, mer eller mindre adekvata, memorerade algoritmer när de inte blir guidade av intervjuaren. Givet denna brist på matematiskt baserade resonemang, i en andra studie undersöktes vilka argument gymnasieelever lägger fram för deras olika strategival och slutsatser. Denna kvalitativa studie indikerade att vedertagna uppfattningar om säkerhet, förväntningar och motivation spelar en viktig roll när man fattar centrala beslut i problemlösning. Men är dessa uppfattningar och matematiska resonemang könsbundna? Den tredje studien hade som syfte att utforska gymnasielärares syn på genus och studenters matematiska resonemang. I den fann jag att gymnasielärare tillskriver genussymboler som osäkerhet, användandet av standardmetoder och imitativt resonemang till flickor, och symboler som flervalsstragier framför allt på grafräknaren samt att gissa och chansa till pojkar. I den fjärde och sista studien, där vedertagna uppfattningar om säkerhet, förväntningar och motivation undersöktes huruvida dessa är könsdifferenta, bekräftar gymnasieelever lärarnas traditionella syn på maskuliniteter och femininiteter. Anmärkningsvärt upprepades dock ej dessa resultat när elever var tillfrågade att reflektera över sitt eget beteende: det skapades en diskrepans mellan det som eleverna tillskrev flickor och pojkar jämfört med det som flickor och pojkar tillskrev sig själva. Sammantaget föreslår denna avhandling att tvärtom till den traditionella könsdiskursen, delar pojkar och flickor många av de matematiska uppfattningarna som behandlar aspekter av säkerhet, förväntningar och motivation, men att mycket återstår för att skapa lärandemiljöer där studenter kan utveckla uppfattningar som guidar dem till matematiskt grundade resonemang.

Preface

This thesis looks at students' mathematical reasoning especially at questions concerning affect and gender. Why did I choose such questions? First and foremost, my experiences as an upper secondary school teacher gave me lot to think about. How come a student who seemed to be completely unable to add fractions suddenly was able to calculate the shortest path in a graph? Or why did the student who the day before the test solved all the questions correctly and showed what I thought was great understanding, then barely managed to pass the exam? Why did the students only want to know *how* to solve the task and never asked me *why* the algorithm worked? Did I really stress the '*why*-question'? And why were there always so few girls involved whenever the focus was on mathematics?

I also felt that I didn't have any theoretical tools to help me understand situations like these. When I came in contact with research in mathematics education, a whole new world opened up to me. There were theories and frameworks that aimed to give clarification, or at least to provide plausible explanations. The mixed-up thoughts that began to form standing in front of my class became suddenly clear when I entered the world of mathematical reasoning. However, this glimpse of clarity didn't last long as I then struggled to develop the research questions of this thesis. Although I found some answers, there is still so much more left to be investigated. Isn't life wonderful?

Acknowledgements

My journey has been filled with joy and challenges. I like to take this opportunity to show my gratefulness to my co-travellers. First, I like to thank my supervisor Professor Johan Lithner for support and constructive comments. I also like to say thank you to my co-supervisors professor Hans Wallin and Manya Sundström, members of the Umeå Mathematics Education Research Centre (UMERC), colleagues at the Departments of Mathematics and Mathematics Statistics at Umeå University and Professor Terezinha Nunes and her research group at Oxford University. A special thank you to all the students and teachers participating in the studies and to The Graduate School in Mathematics Education of The Bank of Sweden Tercentenary Foundation which funded my research.

Thank you to my friends and family, especially my mum and dad, for allowing me to have a life outside this thesis.

Last, but not least, I turn to my husband David and our children Elise and Henry. Thank you for your support and patience. I love you all so much.

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Chapter 1

Introduction

Ellen is trying to solve following task: Find the largest and smallest values of the function $y = 7 + 3x - x^2$ on the interval $[-1, 5]$. When questioned why she decided to solve it as a quadratic equation, in this case an incorrect method, she replies (Bergqvist et al., 2003):

> I can do two things with a second degree polynomial, I can differentiate or I can solve the equation. Since I am better at using the solution formula for quadratic equations I chose that method.

There is something else than just mathematics that guides Ellen in her reasoning. It could be a question of an incapacity to connect concept images with concept definitions (Tall and Vinner, 1981), but difficulties in mathematics are not only cognitive. Learning how to solve problems in a school situation implies also learning what type of problem you can expect to face, which problems are worth solving and what counts as a solution (Goodnow, 1990). This is a private knowledge, highly informal but still very important since it sets the rules of how school mathematics is done. These rules are often called the didactical contract (Brousseau, 1984; Balacheff, 1990). This knowledge is part of your mathematical world view, a view where you can find beliefs as a sub-domain of the affective domain. Beliefs can be "interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (Schoenfeld, 1992a) (p.358). There have been a large number of studies investigating students' beliefs e.g. Carlson (1999) and Svege (1997). Several researchers have stressed the influence students' beliefs play in problem solving (Lester et al., 1989; Philippou and Christou, 1998; Thompson, 1992), but only a few of them have looked at how beliefs influence students reasoning (Schoenfeld, 1985; Wong et al., 2002).

In order to look at beliefs as an influence on students' reasoning, we first need to find out what type of mathematical reasoning students' perform when solving mathematical tasks. The first study in this thesis aims to characterise upper secondary school students' reasoning when solving school tasks. It is part of a set of empirical studies that contribute to a conceptual framework with the aim to capture relevant reasoning phenomena (Lithner, 2008). The framework also provides the notions needed to communicate this. The conclusion drawn is that imitative reasoning is dominant and creative reasoning is rare and local (ibid.).

The next step is to look at the central decisions and the arguments for these decisions. The assumption is that arguments in decisions are affected by beliefs, and this makes some arguments seen by the students more valid or relevant than others. The second study in this thesis characterises the students' reasoning combined with an analysis of their arguments with beliefs in focus. It aims to connect the view of mathematics with the reasoning performed.

The next area deals with gender differences. The question arising from the first half of this thesis is whether mathematical reasoning and the beliefs connected to the central decisions are gender specific. Is Ellen's behaviour and her arguments described in the introductory example typical female? There have been several studies looking at gender differences and beliefs where conceptions such as that girls are hardworking and tend to worry are established (Brandell et al., 2007; Brandell and Staberg, 2008), but what about mathematical reasoning? As a starting point, in article 3 I look at teachers' conceptions about students' mathematical reasoning. This study aims to contribute to the understanding of which gender traits are ascribed to girls and boys respectively. The fourth and final study was a follow up study from the second article. Here I focus on upper secondary school students' conceptions about beliefs describing aspects of safety, expectations and motivation. The article tries to highlight the differences between context bound gender traits and personal traits the individuals ascribe themselves. These two articles only look at the potential gender difference in teachers and students' view of mathematical reasoning and the attached beliefs. They have no aim to investigate if there is a factual gender difference in girls and boys' reasoning. The questions are still interesting and valid since there are evidence that girls choose not to pursue a career in mathematics despite good results and a belief saying that mathematics is an important subject (Öhrn, 2002).

Chapter 2

Theoretical Frameworks

In this thesis, three different theoretical frameworks interact: mathematical reasoning, affect and gender perspective. They all have been chosen with the research questions as starting points and main focus. For further discussion about methodological considerations, see section 3.1.

2.1 Mathematical reasoning

The core of this thesis is mathematical reasoning and three of the studies are based on the same, but at this stage extended, theoretical framework concerning mathematical reasoning, extensively presented by Lithner (2008). Reasoning is the line of thought adopted to produce assertions and reach conclusions in task solving. It doesn't have to be based on formal logic, and it may even be incorrect. The choice is to see reasoning as a product that appears in the form of a sequence, starting with a task (e.g. exercises, tests etc.) and ends with an answer. For organisation of data, a four step reasoning structure is used:

1. A *problematic situation* (PS) is met where it is not obvious (for the individual) how to proceed.
2. *Strategy choice* (SC): try to choose (in a wide sense: recall, construct) a strategy that can solve the problematic situation. This could be supported by predictive argumentation.
3. *Strategy implementation* (SI) that can be supported by a verifiable argumentation.
4. A *conclusion* (C) is obtained.

Argumentation is considered to be the substantiation, the part of reasoning that fills the purpose of convincing you or someone else that the reasoning is appropriate. The quality of an argument is determined by three factors: validity, ability to convince, and constructiveness. The validity of an argument is determined by socio-mathematical norms. In order to talk about the content of an argument Lithner (2008) introduce the notion of anchoring of the relevant mathematical properties of the components in the reasoning. These components are: objects (a fundamental entity, e.g. numbers, variables, and functions), transformations (a process to an object where a sequence of these transformations is a procedure, e.g. finding a polynomial maxima) and concepts (a central mathematical idea built on a set of objects, transformations, and their properties, e.g. infinity concept). The division between surface and intrinsic properties aim to capture the relevancy of a property depending on the context. This example provided by Lithner (ibid.) illustrates this (p.261):

> In deciding 9/15 or 2/3 is largest, the size of the numbers (9,15,2,3) is a *surface* property that is insufficient to consider while the quotient captures the *intrinsic* property.

There are two main types of reasoning: imitative and creative mathematical reasoning. Rote learning reasoning is an imitative reasoning, and the opposite is a creative reasoning. Creative Reasoning (CR) is reasoning that is novel, plausible and has a mathematical foundation, all which Imitative Reasoning (IR) does not require.

2.1.1 Imitative Reasoning

This is a family of different types of superficial, from a mathematical point of view, reasoning with two head categories (Lithner, 2008): (1) Memorised reasoning; and, (2) Algorithmic reasoning.

Since all task solving builds on some type of recollection, *Memorised reasoning* (MR) describes an overall strategy with following conditions :

- The strategy choice is founded on recalling a complete answer.
- The strategy implementation consists only of writing (or saying) the answer.

This type of reasoning is useful for school tasks asking for definitions, proofs, and facts, e.g. "How many litres is $100cm^3$?"

The second category is called *Algorithmic reasoning* (AR). A reasoning in a task solution is denoted AR if it fulfils the following conditions:

- The strategy choice is to recall a solution algorithm (a set of rules). The predictive argumentation may be of different kinds, but there is no need to create a new solution.
- The remaining reasoning parts of the strategy implementation are trivial for the reasoner, only a careless mistake can prevent an answer from being reached.

Here, an *algorithm* covers all pre-specified procedures (e.g. the approximation of a maximum value of a function by studying the graph) and it is not restricted to chains of calculations (e.g. polynomial division). The word *trivial* means that it is relative to mathematical courses taken by the reasoner. Following reasoning types are variants of AR:

Familiar AR. The strategy choice is founded on identifying the task as being familiar, which can be solved by a corresponding algorithm. The strategy choice consists of implementing the algorithm.

Delimiting AR. An algorithm is chosen from a set that is delimited through, in relation to the task, surface property by the reasoner. Following the algorithm carries out the implementation of the strategy. If this implementation is not successful, the algorithm is abandoned and a new one is chosen. The argumentation aims to verify is made on surface considerations related to expectations on the solution or the answer.

Guided AR. The main strategy choice is to find external algorithmic guidance from two different sources:

- **Text-guided AR**. The strategy choice is founded on identifying similarities between the task and an object in a text source. These objects could be an example, a definition, a theorem or a rule. This identification does not rely on any intrinsic mathematical properties. Instead it is all done on a surface level. The strategy implementation is to copy the procedure that has been identified, without any verifying argument.
- **Person-guided AR**. All strategy choices that could have been problematic for the solver are made and controlled by someone else. The person who guides gives no predictive argument to support the local and global strategy choices. The strategy implementation is to follow the guidance without any argumentation aimed to verify why this strategy solved the task.

2.1.2 Creative Reasoning

Reasoning is defined as Creative Mathematically Founded Reasoning (CMR), in the articles referred to as CR, if it fulfils following conditions (Lithner, 2008):

- **Novelty**. A new (to the reasoner) sequence of solution reasoning is created, or a forgotten sequence is re- created. To imitate an answer or a solution procedure is not seen as novel.
- **Plausibility**. There are arguments supporting the strategy choice and/ or strategy implementation, motivating why the conclusions are true or plausible. Guesses, vague intuitions and affective reasons are not considered.

- **Mathematical Foundation**. The argumentation is founded on intrinsic mathematical properties of the component involved in the reasoning. Purely experience-based reasons as in keyword strategy are not valid. Intrinsic is considered being central (in comparison to surface properties which have no or little relevance). Mathematical is what is accepted by the mathematical society as being correct. Property is part of a component: objects (fundamental entity, e.g. numbers, functions, graphs); transformations (operations on an object, e.g. calculate a determinant); and concepts (central mathematical idea built on a related set of objects, transformations and their properties, e.g. concept of infinity).

Creative mathematical thinking is not restricted to people with an exceptional ability in mathematics, but might be difficult to produce without sufficient key competencies and a supporting environment. Schoenfeld (1985) called these competencies knowledge and behaviour: resources (basic knowledge); heuristics (rules of thumb for non-standard problems); control (meta-cognition); and beliefs system.

Creative reasoning does not have to be challenging. It also includes rather elementary reasoning. Still, according to previous research imitative reasoning is common and dominant and creative reasoning rare (Lithner, 2008). Imitative reasoning can be thought of as highly productive in the sense that you can solve lots of task in a short amount of time as long as you have the correct algorithms required for the tasks. However, if you come across problematic situations where the standard algorithm does no longer work imitative reasoning cannot longer been viewed as a particular successful strategy from a mathematical standpoint. Nevertheless, according to upper secondary school teachers, creative reasoning is only for high-ability students (Boesen, 2006). Even though the national tests in Sweden has a large proportion of tasks judged to require creative reasoning, this was not reflected in teacher made tests where such tasks were few (ibid.).

2.2 Affect and knowledge

One of the major issues in the research of affect is to understand the relationship between affect and cognition. There are many different theoretical frameworks which are dealing with this question, complementing each other but not covering the whole field (Hannula et al., 2004). There is still a need to continue the discussion.

2.2.1 Conceptions

Thompson (1992) describes conceptions as "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences" (p. 132). I follow this description and conceptions are defined as abstract or general ideas that may have both affective and cognitive dimensions, inferred or derived from specific instances. Consequently, teachers and students' conceptions consist of their beliefs system, values and attitudes reflecting their experiences.

However, in order to be able to do fine grain analysis I need to separate the concepts emotion and knowledge from beliefs. This is particular important since there is no coherence in the field of affect (Furinghetti and Pehkonen, 2002). Goldin (2002) provides a good description of the distinction between the different sub-domains of affective representation (p.61):

> (1) *emotions* (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context), (2) *attitudes* (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition), (3) *beliefs* (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured, and (4) *values, ethics and morals* (deeply-held preferences, possibly characterized as 'personal truths', stable, highly affective as well as cognitive, may also be highly structured).

Goldin (2004) then adds one more level when he talks about meta- affect: affect about affect, affect about cognition about affect and affective context of affect (dependent on cognition). I accept the description above on a general level, but the concepts have to be further investigated.

2.2.2 Beliefs

Beliefs have components in both the cognitive area and the affective area and that is why it could be difficult to separate an emotion or attitude from a belief. They also appear to be in symbiosis, such that beliefs often interact with and, at times, shape attitudes and emotions (Lester et al., 1989; Cobb et al., 1989). Beliefs are highly cognitive and, relative to emotions, developed over a long period of time (McLeod, 1992). One's self- image or self- concept is part of beliefs (Hannula, 2004).

There is no single definition of beliefs in the field (Furinghetti and Pehkonen, 2002) but depending on the research question different theoretical framework will provide different sets of analytical tools. Since the research questions in the studies presented here deal with mathematical reasoning and more specifically central decisions, the theoretical framework concerning beliefs must take these aspects to account. Therefore, I turn to Schoenfeld's (1992a) definition to use as a starting point. Beliefs are "interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (p.358). However, the same belief could be shared by two people but be connected to two different sets of emotions (Hannula, 2004).

Therefore I like to separate emotions from the definition of beliefs, although they are closely linked to each other.

The next step is to identify the timeframe of interest to allow a discussion with the correct focus. This helps us to find a definition that will capture the right aspect of beliefs. Hannula (2006) uses three different timeframes when discussing different aspects of cognition, emotion and motivation. These are, from shortest to longest,

Rapid self-regulation of actions and thoughts (e.g. solving a given mathematical task)

State aspect which is an intermediate timeframe that regards self and psychological traits as stable constructs, but allowing manipulation of context (e.g. a student solving the problem may start collaborating with a peer)

Trait aspect where psychological traits are constructed and reconstructed (e.g. the student may become more confident through a series of successful problem solving episodes)

Hannula (ibid.) defines cognition in the state aspect as thoughts in mind. With the focus in the state aspect, I define beliefs as an individual's understandings that shape the ways that the individual conceptualizes and engages in mathematical behaviour generating and appearing as thoughts in mind. In this sense beliefs are primarily cognitive. They are neither objective knowledge (see Section 2.4) nor an emotion (e.g. mathematical tasks about physics are more difficult to solve than others).

In terms of separating between 'professed' and 'attributed' beliefs, I'll turn to Speer's (2005) claim that all beliefs are attributed by researchers. To say that professed beliefs is a proper representation of students' cognitive thinking would neglect issues such as the impact of the social context, methods of data collection, interpretation of the data and so forth. However, the attribution is helpful in predicting and explaining behaviour as long it is consistent with the students' behaviour. It is crucial to triangulate with different types of data to strive for accuracy of the attribution.

Beliefs could be categorized in several ways (Op't Eyende et al., 2002), but Schoenfeld's (1985) division into four different categories suits my purposes with my studies: beliefs about mathematics (e.g. mathematics is based on rules); beliefs about self (e.g. I am able to solve problems); beliefs about mathematics teaching (e.g. teaching is telling); and beliefs about the social context (e.g. learning is competitive). When categorising beliefs it is important to remember that an individual's beliefs are linked to each other in a system, and the individual defines the links in this system itself. Schoenfeld (1985; 1992a) describes this belief system as 'the individual's mathematical world view' and it is often created without the person being aware of it. The notion of a belief system is a metaphor used to describe how one's beliefs are organized (Green, 1971). It consists of three dimensions: quasi- logicalness, psychological centrality and cluster structure. Each of an individual's beliefs is dependent on their other beliefs. They are connected and the relationship between different beliefs is not necessarily logical, since it is the individual herself who arranges them from how she sees these connections. There is a *quasi- logical structure* with primary beliefs and some derivative beliefs. How convinced an individual is about something depends on the *psychological strength* of the belief. A belief could be central and strongly held, or peripheral and likely to change. This dimension doesn't exist in a knowledge system (Furinghetti and Pehkonen, 2002). If you *know* something, you are not likely to accept any contradiction to this. Beliefs are held in *clusters*. These clusters don't necessarily have any relationship to each other and therefore can be kept more or less isolated. The reasons for seeing beliefs as a part of a system are because beliefs are not isolated and they are context/situation bound. They function in operational terms as a part of a model of cognition.

2.2.3 Emotions, attitudes, values

An emotion is a pure affective reaction, negative or positive. Affective response to a stimuli can be produced without any conscious awareness of the cognitive process (Eysenck and Keane, 2003), and it is plausible to accept the idea that beliefs triggers emotions and vice versa. They are the most intense and the least stable of beliefs, attitudes and emotions. With the focus in the state aspect, I follow Hannula (2006) and define emotion as an emotional state, which includes mood. Examples of emotions are fear, panic or joy.

Attitudes are not as stable as beliefs. They have more of an emotional side to them in comparison with beliefs. If you contrast them to emotions they are more stable, and combine a balance between the affective and the cognitive side. They are often about one's self or mathematics as a subject. Examples of attitudes are 'I like calculation' and a dislike of geometric proof. Because attitudes and beliefs share some of the same area, they integrate with each other. The following example shows this integration: 'I am not good in mental calculation' is a belief about myself doing mathematics, but also could be interpreted as an attitude towards mathematics (Pehkonen and Pietilä, 2003).

Hart (1989) distinguishes the three concepts by saying that beliefs are used "to reflect certain types of judgements about a set of concepts" (p.44) whereas attitudes are an emotional reaction towards an object and emotions are a hot, gut level reaction. McLeod (1992) shares the same idea when he says "we can think of beliefs, attitudes and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (p.579). These two descriptions give a good picture of the relative relationship of affective intensity and cognitive stability between the concepts.

The fourth concept in Goldin's (2002) description is values. Values are not in the focus of this thesis and therefore this concept has not been not explored to the same extent as beliefs. One of the definitions is provided by Goldin (2004), where he says that values are including ethics and morals “the deep personal “truths” held by individuals that help to motivate priorities; values are stable, usually highly affective as well as cognitive, and may also be highly structured” (p.112). Values can, just as beliefs, be connected with different emotions. It is hard to distinguish beliefs from values, or define the relationship between these two concepts. One way of doing the latter is to say that beliefs is about principles and propositions and values is about choices, priorities and actions. Goldin (ibid.) says that beliefs have the possibility of being true or false, whereas values can’t be measured in that way. They both serve to shape what we focus on, consciously and unconsciously. Green (1971) has another way of looking at values. He says that as soon as someone holds a certain beliefs, an attitude towards that belief is taken, which creates new beliefs. It becomes a belief about a belief and sometimes such beliefs are described as values (ibid.). I choose to see values as a stable construct based on motivation (Hannula, 2006), and therefore values are about choices and priorities in a trait aspect.

2.2.4 Motivation

Motivation can be described as the engine, the driving force, during decision-making. Each individual sets their own goals, by the influence of their beliefs, and these goals functions as directions for our behaviour. One approach in order to distinguish between different needs and goals is to divide motivation into extrinsic and intrinsic motivation (Ryan and Deci, 2000). In this thesis the focus lays in the state aspect and motivation is seen as active goals, either intrinsic (e.g. I want to solve this task because I want to feel good about myself) or extrinsic (e.g. I want to pass this course).

2.2.5 Knowledge

I choose to separate subjective knowledge (e.g. beliefs) from objective knowledge (formal knowledge). The first one is personal and is the result of an individuals own experiences and understanding, and the latter one in mathematics means what is generally accepted as a structure that is a compound of mathematicians’ work (Pehkonen and Pietilä, 2003). This structure is based on what is considered logically true by a wider audience, whereas subjective knowledge is purely individual. Objective knowledge, sometimes referred to as resources, always has this truth property. A belief, on the other hand, is only true to the person holding it.

2.3 Gender perspective

To enable a discussion about gender and reasoning, I choose a perspective where the focus is on the result of having a specific gender in a specific situation. This is in contrast than for instance to see boys and girls as different independent of the context, or to see sex- differences as a biological difference.

Gender is then thought of as an "analytic category which humans think about and organize their social activity rather than as a natural consequence of sex difference" (p. 17) (Harding, 1986). People have through history assigned gender to non-humans entities such as ships, countries and hurricanes. I see this assignment in two ways: (1) you can attribute a gender to an object, characteristics or an action e.g. a ship is female; or, (2) you can attribute an object, characteristics or an action to a gender e.g. boys are more likely to use the graphic calculator. In both these cases, an element (object, characteristics or action) is identified and picked out as typical with the assignment to a specific gender. A gender trait is ascribed.

Gender is asymmetrical; human thought, social organisation and individual identity and behaviour are categorised in an order making some more a 'boy-thing' or a 'girl- thing'. Harding (1986) emphasises the ranking within the asymmetrical organisation of gender saying "part of what it means to become gendered as masculine is to become that kind of social person who is valued more highly than woman" (p. 104). This view shares the same idea as the theory about gender and power where what is considered male is the norm (A) and what is thought of as female is the exception (non- A) (Hirdman, 1990).

This is a fundamental structure that constantly reproduces and changes. It has three aspects (Harding, 1986): (1) gender symbolism (or gender totemism); (2) gender structure; and (3) individual gender. By using these three aspects, it is easier to separate what is related to the structure (e.g. younger children are taught by women, the majority of the professors in mathematics are men), to the symbols in thoughts, word and pictures (e.g. males are considered being more logic than females) and to the individual gender. The structure confirms the symbolism, which then supports the structure. Both will influence the individual's choices. So even though most teachers at a lower level are females, and girls will perform as good or better than boys at compulsory school, the overall system through text books, teacher education and teaching practice will affect the students' view not only of mathematics as a subject but also of who could be a mathematician. For instance, negative perceptions held by underachieving girls was considered a product of the type of school mathematics that was taught in the UK and USA (Boaler, 1997). Harding concluded (1986) (p.53):

> The proponents of equity recommended a variety of affirmitative action strategies and resocialization practices for female children in order to increase the representation of women in science. But these

> critiques often fail to see that the division of labor by gender in the larger society and the gender symbolism in which science participates are equally responsible for the small number of women in science and for the fact that girls usually do not want to develop skills and behaviors considered necessary for success in science.

Previous research indicates that boys more often than girls perceive themselves as being good in mathematics, a perception shared by teachers (Kimball, 1994; Li, 1999). The gender difference is pro-male at higher levels in problem solving and pro-female at lower levels in arithmetic. This is connected to the valuation of success where girls' success in mathematics is downgraded and thought of as rule-following and rote-learning and where the picture of the ideal pupil is still a boy who can reason (Walkerdine, 1998). But most recent research shows little gender differences if any at all, especially when looking at grades (Kimball, 1995; Öhrn, 2002). So why this focus on gender?

In Sweden, the gender differences in mathematics amongst young school children are minor but increases at the higher levels (Öhrn, 2002). Especially at university level the inequalities are big and explicit (Brandell and Staberg, 2008). The students participating in my studies are at upper secondary school level, most of them from the Natural Science programme. They are the most likely ones to perceive mathematics as a male domain (Brandell et al., 2005) which means that the structure, the symbols and the identity within this environment are thought of as pro-male. Such an environment could be connected to a potential underperformance by women since they are under a stereotype threat (Cadinu et al., 2005; Spencer et al., 1999). A stereotype threat is a model of a situation where the activation of a negative stereotype will influence with the performance. According to this model increasing anxiety reduces cognitive resources and leads to performance deficits (Spencer et al., 1999).

When developing your own gender identity you have to deal with the norm of gender equality that exists in the Swedish society but also with the traditional discourses of masculinity and femininity in mathematics education that coexist at the same time. These different discourses function side by side. It is in this context that boys and girls are developing their gender identities by facing, often contradictory, images and negotiate them to a personal identity (Volman and Ten Dam, 1998).

Öhrn (2002) concludes that there is a transaction taking place in the class room where new feminities are taking more space in the educational context. But, this is not a case of new female gender identities replacing the old ones. Rather, a new type of girl, a more bold and out-spoken one, appears with the quiet girls still existing in the classroom. However, the male norm is still present and re-inforced adding more pressure to girls to be both 'the new type of girl' but with the 'old feminine attributes' (ibid.).

Chapter 3

On methods and methodological considerations

The research in this thesis is *problem driven*. One can of course develop research questions from an educational perspective or a specific theoretical framework, but in this thesis the research questions have been central. The advantage with problem driven research is that you are not restricted to a specific method or theory. The difficulty, on the other hand, is that often challenging questions do not have an easy method or theory to apply in order to obtain answers. To quote Schoenfeld (1992b), "the relationship between problems and theory is by no means straightforward: The pursuit of what one finds "challenging and interesting" can lead to an ad hoc empiricism that is theoretically vacuous" (p.180-181).

It is therefore crucial that every research question has appropriate theoretical frameworks, methods and analysis. To follow Schoenfeld (ibid.), the context has to be established, and enough data has to be provided in order to (a) allow the reader to analyse it on their own terms and (b) allow for the reader to employ the methods and see if the analyses are reproductable. Finally, for every study a methodological discussion has to be offered in order to raise questions about the scopes and limitations of the method(s) used and the issues of validity and reliability.

What problems are addressed here? Niss (1999) brought forward the *dual nature* of mathematics education when defining and discussing the subject of research. One can either ask the question 'what is the case' aiming at description, or one can ask the question 'why is it so' aiming at explanation. The research questions in this thesis are of the 'what is the case' kind making it a *descriptive* thesis. For each of these research questions, definitions and theoretical frameworks have been explored and developed, a process described in the following sections, in order to avoid what Schoenfeld (1992b) describes as empty ad hoc empiricism.

3.1 Methodological considerations

This thesis is written within the qualitative research paradigm where the intended outcome is "illuminative subjective understandings" and the interest it to "understand and make sense of the world" (Ernest, 1998) (p.37). Since the research is problem driven, I do not profess any specific theory or methods within this paradigm. Instead, I have used the definition and the context as starting points from which I have determined or developed the methods and theories that are appropriate for the problem. I will now describe this process.

The purpose of the first article is to describe and characterise different types of mathematical reasoning produced by upper secondary school students when solving school tasks in a test like situation. The research question posed was 'In what ways do students manage or fail to engage in CR, MR, and AR as a means of making progress in problematic situations?' This question was already connected to a theoretical framework (i.e. Lithner (2008)) and the study was a part of a bigger project that aimed to understand more about students' mathematical reasoning. In this framework, reasoning is defined as the line of thought adopted to produce assertions and reach conclusions in task solving. It doesn't have to be based on formal logic and it may even be incorrect. The methodological choice is to see reasoning as a product that appears in the form of a sequence, starting with a task (e.g. exercises, tests etc.) and ends with an answer. This definition sees reasoning as the individual's construction. It is this construction that is described and is characterised.

There are several reasons for using Lithner's (ibid.) framework. One is that the framework helps to distinguish between what is creative mathematically well-founded reasoning and what is not. Another reason is the view of reasoning as the line of thought. There is therefore no restriction to deductive mathematical reasoning or proofs. It highlights the argumentation for the decisions made while solving problematic situations. The characterisation of imitative and creative reasoning enables a discussion about the origins and consequences for such reasoning. In comparison to other frameworks, Lithner (2008) provides not only notions but also defines them and places them in a conceptual framework (Bergqvist, 2006). Since the framework is well-defined and uniform means that it can be used to communicate key phenomena to other teachers and researchers.

The research question for the second study was 'How do beliefs influence the central decisions students make in their reasoning while solving problematic situations?' Influence is interpreted as the arguments given for central decisions when solving school tasks. It is important to note that there is more to influence a person's behaviour than just beliefs. For this work in affect, the was no available definition of, or theoretical framework about, beliefs that matched the research question completely from the start. Most theoretical frameworks did not allow a focus on decisions made in single, isolated problematic situations. The article was not intended to be about affective pathways nor was it a study of dynamics. Neither did I wish to compare individuals' behaviour in different contexts. Rather, I wanted compare individuals' arguments in the same context and in the same problematic situations since their arguments for the different choices should be compared to their different mathematical reasoning.

On this basis I chose Schoenfeld's (1992a) definition a starting point, a definition that was developed for studying problem solving strategies. Beliefs are "interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (p.358). However, there were convincing arguments for separating emotions from beliefs (Hannula, 2004) which meant that the concept emotion had to be defined as well. Then it was a matter of defining which timeframe the study took place in. Different time frames provide a different focus on and different definitions of the three concepts of cognition, motivation and emotions. In the second article the focus is in the state aspect of cognition (Hannula, 2006) and beliefs are defined as an individual's understandings that shape the ways in which the individual conceptualizes and engages in mathematical behaviour generating and appearing as thoughts in mind. In this sense beliefs are primarily cognitive. Affect is seen as a subjective experience and the focus is on the individual constructs. But if beliefs are highly cognitive and at the same time a subjective experience, i.e. a subjective knowledge, beliefs then had to be separated from objective knowledge.

Using the above definition and theoretical framework does not diminish the impact that the social settings have on the individuals and their performances when doing mathematics. It is just not the focus for this thesis. Instead, the results have to be interpreted in the following context: 'beliefs as an influence' is a theoretical model to describe a specific phenomena, i.e. the type of arguments given by students when solving school tasks in a lab setting. The results are still interesting if this model can produce attributions that can help predict and explain behaviour.

When talking about gender the focus shifts to social constructs. The research questions for article 3 and 4 were 'Which gendered symbols are attributed by the teachers to students' reasoning?', 'How do upper secondary school students gender stereotype beliefs about safety, expectations and motivation?' and 'How does the gender stereotyping differs from the traits the students ascribe to themselves?' In these questions there is a view of gender as something that is context bound. Gender differences are not general but specific to cultural and situational contexts. It is the relation between the gender and the different conceptions of what is considered male and female that are emphasised. Therefore, I follow Harding's (1986) definition and gender is defined as the "analytic category which humans think about and organize their social activity rather than as a natural consequence of sex difference" (p. 17). This view of gender is a fundamental and dynamic structure. The focus is on the traits ascribed by teachers or students to a specific group (boys or girls). The two articles list and characterise the most common gender traits ascribed by the students and the teachers. There is no implication that all students or teachers make exactly the same attributions nor is it assumed that by ascribing gender traits implies that boys and girls form homogenous groups.

3.2 Methods

This thesis consists of both quantitative and qualitative studies with the emphasis on the latter kind. The studies have all been made within the qualitative research paradigm. Different methods have been used in order to answer the different research questions, and the methods have been chosen with the research questions and the theoretical frameworks in mind. The process of deciding what methods to use has developed through time and included several pilot studies. The process has also included flexibility which accounts for earlier results. For instance, in the third study a semi-structured interview was developed from the questionnaire in order to find explanatory factors. The results were then connected back to the questionnaire.

The data in the first two articles comes from video observations and interviews. Article 2 also uses a Likert-type attitude questionnaire however, due to the the small number of respondents, this is interpreted in qualitative terms. Both articles use case studies to illustrate the specific phenomena of interest and it is important that enough data has been provided to the reader. There is a need for rich descriptions of the cases. Especially for article 2 there is a strong reliance on qualitative, high inference measures. However, as stated in section 3.1, the results should be interpreted as a description of a phenomena within the theoretical model 'Beliefs as an influence'.

A tricky part when investigating a factor as a potential influence is data collection. It involves a balance between interfering and leaving the students alone. Much depends on the skilfulness of the interviewer. Therefore, several ways of collecting data is necessarily not just for grasping more but also for triangulation. For future studies, I note that in article 2 a second camera recording facial expression could have been an additional way to accomplish this.

Another challenge is the limitations of various methods. Since the second article wasn't designed with interrelations of cognition, emotion and motivation in mind some of the methods don't deal with this issue. For instance, the questionnaire only gives fragments of information about individual and isolated beliefs. A different type of questionnaire, or at least a broader one in terms of covering these three concepts, might be helpful for investigating these issues further.

Semi-structured interviews were used in article 1, 2 and 3. The benefit with this type of interview is that one can prepare key-questions but also allow time to following up threads which develop during the interview. Again, the quality of the interviews depends on how skilful the interviewer is. In the first article we used video-based stimulated recall interviews. The students were asked to reflect on and further describe their decisions when solving the school tasks. In the second article, the same type of interview was used and similar questions were posed but there was an additional emphasis on why decisions were made. In the third article, teachers were asked to describe which gender symbols they picked out as typical in the different cases of mathematical reasoning.

The third and fourth studies both use a specific Likert-type attitude scale. The inspiration to these questionnaires comes from Leder and Forgasz' instruments *Who and mathematics* and *Mathematics as a Gendered Domain* that were developed from Fennema-Sherman's *Mathematics Attitudes Scales, MAS* (Leder and Forgasz, 2002). These instruments give the opportunity to see mathematics not just as a male domain but also as female or neutral. The *Who and mathematics* instrument, both used in article 3 and 4, measures if and to what extent the students and teachers stereotype statements as gendered. In article 3 this instrument was mainly applied to qualitative data. The second questionnaire, *Mathematics as a Gendered Domain*, was here renamed *Me and mathematics* following Brandell et al. (2005). It measures how true the students rank the statements from a personal standpoint making it possible to compare boys and girls.

There are several cases of studies using these non-parametric questionnaire, but when making their analysis they chose parametric statistical methods (e.g Leder and Forgasz (2002) and Brandell et al. (2007)). I would instead argue for non-parametric tests. It is not clear that the respondents value the response options with the same scale as the researchers and the assumption of normally distributed data is not justified. One way to develop the questionnaire towards parametric tests is to mark each response category with a given number (a value) so it becomes obvious how big the steps are between them. In the fourth article I use two types of non-parametric tests: sign-test for *Who and mathematics* and Fisher's exact test for *Me and mathematics*. The sign test was made to test the null hypothesis that the respondents are equally likely to choose boy or girl (i.e. if the gender stereotyping was significant). A comparison of boys and girls' responses is then made making it possible to analyse similarities and differences between the two groups. This test does not measure to what extent the stereotyping is made, only if it exists and if it is statistically significant. In order to compare girls and boys' responses to *Me and mathematics*, Fisher's exact test was made. In contrast to the sign test, which identifies whether answers tend towards particular stereotyping, Fisher's exact test explicitly tests whether statements differ between girls and boys. The p-value is the probability that this outcome would have occurred given the null hypothesis that there is no difference between boys and girls. The limitation of this test is that it does not measure to what extent the ranking is made, only if there is a difference between the two groups.

3.3 Method of analysis

In this section, for each article, I will describe the type of data that was collected, the type of phenomena I wanted to capture and give an indication of the type of analysis that was made. For more elaborated descriptions of the analysis methods I refer to the four articles.

For article 1, the data was transcriptions of video observations and interviews. What we wanted to capture was the different type of mathematical reasoning, or more precisely, the different key characteristics of the different type of mathematical reasoning the students produced when solving school tasks. The data was organised using a four step reasoning structure. Each sequence of reasoning of interest was described in this structure. To categorise the students' reasoning we focused on the the different characteristics of mathematical reasoning. Lithner's (2008) framework worked as the classification tool for the sequences of reasoning.

The data in the second article was transcriptions of video observations and interviews and a quantitative questionnaire. In this study the type of phenomena I wanted to capture was plausible reasons for why the students acted in the way they did. I did this by looking for consistent behaviour and possible underlying causes, and attributed possible beliefs as these causes. As a tool for the analysis of dynamics of mathematical thinking, Belief Indication (BI) was introduced. Belief Indication is data carrying information about the person's belief as defined earlier (see section 2.2.2). The result of the analysis was the characterisation and description of BI as an influential factor to the central decision made in reasoning.

In the third article the data consisted of teachers' responses to a questionnaire: (1) their ranking of cases describing reasoning as gendered; and, (2) written arguments for their option selected. The results generated from these two types of data were a summary from the quantitative section and a summary and categorisation of the written arguments. From these results, semi-structured interviews were constructed and executed. The data from the interviews was transcriptions. I wanted to find out which gender traits were ascribed by the teachers to students' mathematical reasoning, or more specifically which gender symbols were assigned to girls respectively to boys. Gender symbols are the attributions made by the teachers when picking out elements, parts of the reasoning, as 'typical' with the specific association to gender (this feels like a 'boy thing' or a 'girl thing') or considered neutral. The different types of reasoning were used, not just as stimuli, but also for organisation of data. In the teachers' argument, both from the written questionnaire and in the transcriptions, gender symbols were identified and characterised. These symbols were summarised for each of the cases of reasoning.

The fourth article differs from the first three by using quantitative data and methods. The data was responses to pre-formulated statements. The participants were asked to either indicate one of the two different gender or neutral for each statement, or to indicate degree of agreement with each statement. The three types of phenomena I wanted to capture were (1) what types of statements were ranked as gendered by the students, (2) if ranking differs between the genders, and (3) if there was a difference between the traits that were gender stereotyped and the traits the students ascribe themselves. Appropriate statistical tests were chosen for the analysis and the results from these tests were summarised, compared and discussed from different theoretical standpoints.

3.4 Summary

Here I have described the methodological grounds and reflected upon the methods used in this thesis. To summarise, I describe the research in this thesis as descriptive and problem-driven research with the main focus on individual constructs when talking about affect and social constructs when looking at gender traits. This research is then conducted within the qualitative research paradigm.

Chapter 4

Summary of the results of the articles

4.1 Article 1

In this article, the theoretical framework about mathematical reasoning was further developed through the exploration of upper secondary school students' reasoning (Bergqvist et al., 2007). The aim of the study was to investigate what types of reasoning upper secondary students perform when solving school tasks. The research posed question was: In what ways do students manage or fail to engage in CR, MR, and AR as a means of making progress in problematic situations?

Imitative reasoning was dominant; mainly Familiar Algorithmic Reasoning (FAR) and Delimiting Algorithmic Reasoning (DAR) were used. The characterisation of the latter one was one of the results of this study. Creative Reasoning (CR) was absent with a few exceptions (e.g. analysing the shape of a graph and deducing where to find the smallest value). It appears that the students don't make any attempt to reason creatively. Sometimes this could be a result from lacking conceptual understanding, although in relation to the mathematical course taken this would be relatively elementary.

Several situations, mainly with students from the pre-university programmes, were classified as Person-guided Algorithmic Reasoning. Here the students relied heavily on the interaction with the interviewers which is probably not an uncommon situation within the classroom context.

4.2 Article 2

This study aimed to investigate the possible impact beliefs have in problem solving (Sumpter, 2008). The research question posed was 'How do beliefs influence the central decisions students make in their reasoning while solving problematic situations?' Three major themes stand out: expectations (e.g. 'I've got one answer and I expected two from this algorithm.'), safety/security (e.g. 'a well- known algorithm is safer.'), and motivation (e.g. 'I can't reason to a solution myself .'), the latter one consisting of motivational beliefs and active goals. These appear to be rather dominant, especially when compared to the students' use of mathematical knowledge. Even when progress would have been not far away, these types of beliefs have influenced the student to take another route. For instance Paul, who concludes a correct answer is wrong because his expectations says otherwise even though he has solved the task earlier and got the same answer. He refers to mathematical properties in his argumentation, but his expectation of how the algorithm should behave is stronger.

Another result is that the concepts cognition, emotion and motivation are intertwined with each other. In most of the decisions, beliefs are combined with the students' active goals and/ or emotional state and vice versa. This was illustrated by one student who stood out with her negative emotional reactions that affected her reasoning. One of the conclusions drawn, just as Meyer and Turner (2002), is that students' emotional state cannot be neglected.

4.3 Article 3

In this article I explored upper secondary school teachers' conceptions about gender and students' mathematical reasoning (Sumpter, 2009a). This was investigated by looking at how the teachers stereotypes aspects of students' task solving as gendered. The research question was 'Which gendered symbols are attributed by the teachers to students' reasoning?'.

62 upper secondary school teachers answered a questionnaire consisting of eight cases describing students solving mathematical tasks. These eight cases represented four different type of reasoning highlighting different aspects of these: Familiar Algorithmic reasoning (FAR); Delimiting Algorithmic Reasoning (DAR); Memorised Reasoning (MR); and Creative Reasoning (CR). Six teachers were interviewed with the aim of clarifying what elements of the reasoning that were considered gendered and why. A post-questionnaire was made making it possible to triangulate the results.

Summarising the results in a table, these are the gender symbols most frequently used in the arguments made by the teachers:

Boys	Neutral	Girls
(use the) calculator	good student	safety
multiple strategies	standard solution	use the standard method
chance- taking, guessing	guess within context	imitative reasoning
make a mess, not careful		insecurity
quick solution		long reflection time
graphic solution		wants *the* correct answer
explore		think

Table 1: Gender traits ascribed by teachers

In the post-questionnaire, most teachers confirm that Table 1 gives a valid description of teachers' conceptions of male and female behaviour but at the same time recognising that boys and girls do not form homogeneous groups.

4.4 Article 4

This study aimed to investigate secondary school students' gender stereotyping of beliefs about aspects of safety, expectations and motivation, and if this gender stereotyping differs from the traits the students ascribe to themselves (Sumpter, 2009b). The research questions posed are: (1) How do upper secondary school students gender stereotype beliefs about safety, expectations and motivation?; and, (2) How does the gender stereotyping differs to the traits the students ascribe to themselves?

Two questionnaires were used: *Who and mathmatics* and *Me and mathematics*. They are two forms of one instrument (Leder and Forgasz, 2002). Both questionnaires consist of 24 statements, all given as arguments for decisions made during school tasks solving by year two students from the Natural Science programme (Sumpter, 2008). 180 students (102 boys and 78 girls) participated.

Most statements in the first study (*Who and mathematics*) were by the majority of students considered neutral, but for seven of the statements some gender differences were identified. Among the statements that were considered gendered, girls seem to be connected to beliefs about aspects of expectations and safety: what you are expected to do and what is considered a safe strategy. Boys were assigned beliefs about what you can expect from the graphic calculator. They could work as a motivational belief in terms of why you should use it. The results from this study indicated partly similar masculinities and femininities as previous research (Brandell et al., 2005; Sumpter, 2009a). The second study (*Me and mathematics*) did not confirm these conceptions. Girls did not rank statements about safe strategies more true than boys, and boys were not more likely to find the graphic calculator more useful than girls. There was a gap between what was gender stereotyped and what was ascribed as personal traits by boys and girls.

Also, girls at the Natural Science programme were as confident about their own capacity as the boys although not evaluating their performance as high.

Chapter 5

Discussion

In this section I discuss the results and possible implications of the four articles. I start with mathematical reasoning and beliefs and then move on to the gender perspective.

5.1 The first part: Reasoning and beliefs

What can we say about student's mathematical reasoning and beliefs? First of all, imitative reasoning dominates and creative reasoning is rare and local. By using the results from the second article, some of the central decisions made by the students in the first article can be further discussed.

In both studies a rather negative motivational belief is highlighted. There is no strive to investigate intrinsic mathematical properties. The focus is on which algorithm goes with which task. In the second article this was emphasized by beliefs such as 'I can't reason myself to a solution' and 'Mathematical tasks should be solved in a specific way'. Creative reasoning is not an alternative even in fairly easy situations[1]. The following example from article 1 illustrates this behaviour. A student tries to verify an equation using a procedure he applied part of in a previous task, but this time he fails. The connection is not made and there is no insight into the transformations.

This behaviour is further illustrated by students using Global Delimiting Algorithmic reasoning (DAR) where the commonality is a lack of consideration of the intrinsic mathematical properties. The motivation to even consider these properties seems not to exist, and creative reasoning does not seem to be an option. With this absence of supporting beliefs and/or motivation to look at the intrinsic mathematical properties maybe the lack of Creative mathematically

[1]This is relative to the mathematics course taken.

founded reasoning (CMR) together with the dominance of Imitative reasoning (IR) is not a peculiar finding. These results give an indication that CMR is not an alternative, and it might even be thought of as 'unsafe'. There is an algorithmic view presented combined with an algorithmic reasoning.

What type of beliefs is this algorithmic view based on? The first type is *expectations*. In the second study this is illustrated for instance by Sam who shows indication of having preconceived beliefs about the numerical number 47. He is trying to find the maximum and minimum value for a function in a fixed interval, $[-1, 5]$, by calculating the function values for the end points of the interval, an incorrect method. After he got $y(5) = 47$, he assumes that this must be the maximum value without knowing the function value for the $x = -1$. Sam might have these beliefs because in school tasks, maximum values often are between 10 and 50. Another example is Paul's expectations about a local step of an algorithm overriding his own reasoning. Paul is also trying to decide the maximum and minimum value for a function in a fixed interval, $[-1, 5]$, using a correct method: to differentiate the function and put the differentiation equal to zero. Even though he has solved the task previously arriving with the correct answers, seen the graph and uses the right references to intrinsic mathematical properties, his expectation that the differentiation of a second degree function should result in two x-values makes him conclude that his mathematical reasoning is wrong. Expectations are not a new or strange phenomena, we all have them, but the problem arises when expectations become the dominating factor compared to mathematical knowledge, and replaces a more creative and productive mathematical reasoning.

Like previous research (Kloosterman, 2002; Op't Eyende et al., 2006), I find *motivational beliefs* an important subcategory of beliefs. They could be in the shape of external motivation (e.g. short term motivation as in solving a specific task or a long term goal of doing well in mathematics courses) or intrinsic motivation (e.g. a belief saying that I will feel good about myself if I learn to understand this mathematical concept). Intrinsic motivation can constrain students' mathematical reasoning through beliefs that say that 'the only way' for students to solve tasks is by using specific algorithms for specific tasks. Hence, creative reasoning is not an option. In the second article this type of beliefs is illustrated by Ella who says that her own mathematical reasoning is not a safe option. Motivation is then combined with the third category of beliefs, *safety/security*. Ella's solution is to focus on memory, to remember which algorithm to use to which task, and it becomes her safety net. Up to a point this safety net proves successful, but she has now reached a stage where it is hard to remember all the local steps of specifics algorithms. There is a limit, which differs from person to person, how far you can get with this type of strategy. These results increase the knowledge about why there is a dominance of imitative reasoning and how beliefs functions as a influential factor in decision making that guides the students into a specific type of reasoning.

Another result is that the concepts cognition, emotion and motivation are intertwined with each other, and there are important interrelations between them. We have seen this in previous research in trait aspect, e.g. Hannula (2004) talks about a positive change of beliefs and behaviour when a student sets a new performance goal. But even in a state aspect, the impact is noticeable. In most of the decisions in the second article, beliefs depend on student's active goal and/or emotional state and vice versa.

What can we do about the algorithmic view and algorithmic behaviour presented above? Repeating the same type of tasks would form these types of expectations and this would then lead to imitative reasoning simply because the student faces the same problematic situations repeatedly. Why would I as a student change a successful behaviour if (a) I'm not forced to, (b) it feels safe, and (c) I know what I'm expected to do? If students are not exposed to different types of problems that provide a range of difficulties, it is hard to stimulate the intellectual development (Pólya, 1945). Op't Eyende et al. (2006) present another aspect of this dilemma. They concluded that "Teaching students how to solve mathematical problems then implies that we have to teach them also how to cope effectively with feelings of frustration or sometimes anger" (s. 204). It should be a positive feeling being frustrated when solving mathematical problems (Hannula et al., 2004). The combination of these two arguments emphasizes a need for stepping away from what is created being safe or what you can expect mathematics to be e.g. what answers usually look like, the look of an equation, or a strategy choice based on 'we normally do that'. These all have little or nothing to do with intrinsic mathematical properties. Since good problem solvers spend their time differently when solving problems compared to less successful ones (Schoenfeld, 1985), stimulating planning, analysis and evaluation of a task would be one way escaping a dominating focus on algorithms and instead help to highlight questions such as 'what is this task really about?' and 'what does this mean?'.

There is therefore a need for students to work with different types of problems so they are able: (1) to strengthen their knowledge coping with negative emotions and relaxed attitude to *not* knowing exactly how to solve a task, or to be aware of ones meta- affect; (2) to explore and stimulate their creative reasoning; and (3) to try to change the didactical contract so you as a student can't rely on what is normally being done and also trying to avoid what Vinner (1997) refers to as pseudo-learning. Teaching students how to solve mathematical problems then also implies teaching a wide variety of different problems providing different type of problematic situations with the focus on the different intrinsic mathematical properties of each specific task. As a result of this, teaching mathematics would then require a good, solid background in mathematics.

5.2 The second part: The gender aspect of reasoning and beliefs

Starting with the gender aspect of reasoning, previous research supports the claim that some gender differences exist in problem-solving strategies (Carr and Jessup, 1997; Fennema et al., 1998; Gallagher and DeLisi, 1994). In article 3 girls seem to be linked to gender symbols such as imitative reasoning, in particular Familiar Algorithmic Reasoning (FAR), and the use of standard methods. Familiar Algorithmic Reasoning (FAR) can be considered an effective way of working if you want to be sure that you solve certain tasks just as long as you are careful, but it is also highly restrictive when facing a genuine problem. When combining these gender symbols with the view that girls are diligent and successful because of their hard work (Brandell et al., 2005) and not because of their creative mathematical thinking, this combination leads to the same implications as Walkerdine's (1998) conclusion: female success is based on properties (such as rule-following, rote-learning and carefulness) that can then be used to downgrade girls performances. The high-ability boys, on the other hand, are successful because of them being bright kids (Brandell et al., 2005).

Some of the gender symbols most frequently attributed to boys in the third article are multiple strategies (especially on the calculator), and to guess/ to take a chance. This goes with results claiming that boys are gamblers and risk-takers (Ben-Shakhar and Sinai, 1991); they try different methods to increase the probability to of coming across something that might be right. Such behaviour was illustrated by the following comment: "Boys press all the buttons [on the calculator] and hope it will help [Article 3, Q34]". This is not a strategy choice based on intrinsic mathematical properties of the task. In Delimiting Algorithmic reasoning (DAR), the choices and the argumentation for these choices are made on surface reasons and in Memorised reasoning (MR) the strategy choice is simply trying to recall an answer and writing it down without any other consideration.

This rather traditional stereotyping was echoed in the first study of article 4, where upper secondary school students were asked to reflect upon if mathematical beliefs about aspects of safety, expectation and motivation are gendered. The students reported in some measure a similar view of masculinity and femininity as the teachers, confirming the traditional gender discourses. However, theses results were not as straightforward as they initially looked. The results from the second study of the fourth article indicate there is a discrepancy between the gender traits and the personal traits ascribed by boys and girls individually. The masculinities and femininities indicated in article 3 and the first study in article 4, *Who and mathematics* were indeed in line with previous research e.g. Brandell et al. (2005), but the second study of article 4, *Me and mathematics*, did not confirm these conceptions. Girls did not rank statements about safe strategies more true than boys, and boys were not more likely to find the graphic calculator more useful than girls. Also, girls at the Natural Science programme were as confident about their own capacity as the boys although they did not evaluate their performance as high.

This is a situation where the gender stereotyping follows traditional discourses creating a gap to boys and girls' own standpoints. This discrepancy seems to occur independent of age and programme (Brandell and Staberg, 2008; Volman and Ten Dam, 1998) and it is likely that the gap is about the differences between the personal gender identity and perceived masculinity and femininity (Volman and Ten Dam, 1998).

Different theoretical frameworks offer different explanations to why this gap is created and maintained. You can see it as a case of different *belief clusters*. This would mean that this is not about irrational behaviour since according to the theory about belief systems, the relationship between different beliefs do not have to be logical (Green, 1971). The arrangement is made by the individual herself according to how she sees and values them. It is a subjective experience. The different beliefs could even have the same *psychological strength* but since one is about myself and the other one is about my gender group in general, i.e. two different clusters, it does not affect my gender identity. The clusters do not necessarily have any relationship to each other and can be kept more or less isolated. Therefore, the discrepancy between what the student gender stereotypes and the personal traits that she ascribes herself does not have to produce a conflict.

It could also be explained as a *coping strategy* (Volman and Ten Dam, 1998). Given this view of mathematics as a male domain, in order to avoid the risk of being a victim in the female identity with all the negative gender symbol attached to it, the personal identity is separated from the traditional discourses. As a part of a coping strategy girls identify themselves with non-female attributions such as graphic calculator and being confident and by asymmetry reject traditional female symbols such as insecurity and rule-following. It would be plausible to assume that this identification would affect the belief system but the question arising is, to which extent are one's self-beliefs affected? This is where self-evaluation offers an additional tool to look at students' view of themselves. This is not just about the actual grade or performance; it is about how *you* value your own performance. Just as in previous findings (Brandell et al., 2005; Kimball, 1994; Jakobsson, 2000) the female students participating in the fourth study show indications of evaluating their own performance lower relative to boys despite the pro-female responses to *Me and Mathematics*.

An additional way to view this gap is by using the *system justification* theory. Its main idea is that it is possible that stigmatized group adapt the negative assumptions that are stereotyped to them by the dominant group (Jost and Banaji, 1994). Given such stereotypic beliefs, women may feel a pressure to conform to gender norms and downgrade their own performances. In the long run their gender identity can be changed. So even though the girls might feel, think or experience there is no gender difference in the bigger society, they could incorporate the gender stereotyping projected in the sub-domain. A result of this incorporation could be that girls rate their ability lower – that activating gender stereotypes can lower self-evalutation (Guimond and Roussel, 2001).

Boys behaviour could be explained as a case of *legitimizing myths*. In a group-based social hierarchy, according to the social dominance theory (Sidanius and Pratto, 1999), the dominant and hegemonic group (here males) would enjoy the social power and privilege just because of a particular membership in a socially constructed group. This hierarchy is affected by legitimizing myths that "consist of attitudes, values, beliefs, stereotypes, and ideologies that provide moral and intellectual justification for the social practices that distribute social values within the social system" (p.45). Events such as an experience in the classroom could then be interpreted as supporting evidence when holding gender stereotypic views (Eccles and Jacobs, 1986).

The discrepancy between the two questionnaires in the fourth article could also suggest that in a complex area such as 'Mathematics as a gendered domain', one quantitative questionnaire does not provide enough data in order to make broad conclusions. We cannot simply conclude that 'mathematics is a male domain' or 'girls are confident'. Looking at the bigger picture implies that further research is required.

5.3 The two parts together

Taken together the two parts of this thesis suggests that, contrary to the traditional discourses of feminities and masculinities, girls and boys at the Natural Science programme share many of the same mathematical beliefs about aspects of safety, expectations and motivation. However, the results from the first two studies indicate that much work is still needed if we are to create learning environments that provide better opportunities for students to develop beliefs that guide them towards mathematical reasoning based on intrinsic mathematical properties. The work towards this goal should include not only teaching problem solving skills, such as control and use of heuristics, but also how to cope with negative emotions when facing difficult problematic situations. To reason mathematically creatively is not about producing the right behaviour for the right occasion; it is about allowing oneself cognitive flexibility when solving problems.

The results of this thesis suggest that when investigating complex areas, such as affect and gender, discrepancies can occur due to choice of methods. Even though each of these studies in this thesis is carefully conducted with appropriate methods and methods of analysis, when combining the results the limitations of these methods and the corresponding results become more clear. It seems that one can obtain different result whether asking about personal trait or gender stereotyping, e.g. when comparing the result from article 3 and *Who and mathematics* with the result from article 4 and *Me and mathematics*. The same tendencies are indicated in the case of self-evalution and self-concept where it seems that despite girls pro-female responses when reflecting upon beliefs about expectations, safety, and motivation from an individual perspective, girls self-evaluation do not nescessarily follow the same path. There are also indications that discrepancies can occur when looking at a concrete situation compared to an abstract one. The algorithmic view that was the result of the arguments given by the students when solving school task in article 2 was not replicated to the same extent in *Me and mathematics* in article 4. Since this question was not the focus of article 4, more research is required in order to investigate this issue further.

A possible implication of these results of the thesis could be that girls and boys do not need different types of mathematics education: they both need help developing problem solving skills to be able to produce creative mathematical reasoning when facing problematic situations where imitative reasoning is fruitless. As claimed above, teaching mathematics requires a good solid background in mathematics combined with a knowledge about problem solving skills and an awareness of how affective factors can influence. However, when considering self-concept and the impact of gender stereotyping e.g. the effect of stereotype threats (Cadinu et al., 2005; Spencer et al., 1999), teaching mathematics would also include an active consciousness about the traditional gender discourses.

5.4 Further research

The first area of interest is creative reasoning. Lithner (2008) shows that is sometimes possible for students to, with little stimuli, change task solving strategy from inefficient imitative reasoning to constructive creative reasoning. What type of beliefs would support a Creative Mathematically founded Reasoning (CMR) instead of Imitative Reasoning (IR)? Here it is not necessarily a question about changing beliefs, but activating the right supporting beliefs. Research supports the claim that successful students are industrious and flexible, the latter one including being prepared to make many attempts when solving a task (Carlson, 1999). These students see the obstacles as something challenging, with a positive tone. Would this be reflected in their mathematical reasoning? Are there specific beliefs connected to CMR and if so, what type of beliefs? In addressing this problem, we should consider that different areas of mathematics might produce different types of reasoning and maybe therefore bring different type of beliefs to the surface. Would the same behaviour be recorded when working in arithmetic compared to discrete mathematics? Are some areas in mathematics dominated by a culture focusing on memorizing rules, whereas others might even invite students to be creative?

A second area for further research focuses on younger pupils. This thesis looks at upper secondary school students and their reasoning, beliefs and gender stereotyping but what about pupils at pre-school and compulsory school? What type of reasoning do younger children produce and what would be their arguments for their reasoning? Is their mathematical world view gendered? How would a seven year old describe a mathematician? By studying younger childrens' mathematical reasoning and their beliefs we would better understand why Sweden as a nation performed in such a way as we did in the latest international tests e.g. TIMMS 2007 (Skolverket, 2008) where Swedish grade 8 pupils' results were significantly lower compared to TIMMS 1995. One of the conclusions made in the Swedish report for TIMMS 2007 (2008) was there is a need to move away from a mathematics education focusing on procedures. The results from this thesis indicate that a dominant focus on imitative reasoning seems to be one of the major limiting factors when solving problematic situations. By studying younger pupils we might also be able to know more about if and in what way mathematical reasoning is gender different.

A third area deals with a more specific issue and would work as a follow up study to article 4. In this article it was indicated that different results can be obtained when asking in a concrete situation (such as a school task solving situation) compared to an abstract one (such as a discussion about problem solving in general). One of the results from article 2 was an algorithmic view based on the statements that were given by students when solving school tasks as arguments for central decisions. In *Me and mathematics* in article 4 students were asked to rank the truthfulness of these statements. the results of this ranking did not repeat to the same extent this algorithmic view. Since this was not the focus of article 4, complementing data collection (e.g. interviews) and appropriate analysis are required in order to investigate this issue further.

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