Railway bridge response to passing trains

Measurements and FE model updating

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Abstract

Today’s railway bridges are analysed in more detail for moving loads due to the increase in speeds and axle loads. However, these numerical analyses are very time consuming as they often involve many simulations using different train configurations passing at different speeds and many considerations to take into account. Thus, simplified bridge, track and train models are often chosen for practical and time efficient simulations.

The New Årsta Railway Bridge in Stockholm was successfully instrumented during construction. A simplified 3D Bernoulli-Euler beam element FE model of the bridge was prepared. The FE model was first manually tuned based on static load testing. The most extensive work was performed in a statistical identification of significantly influencing modelling parameters. Consequently, parameters to be included in an optimised FE model updating, with consideration also to synergy effects, could be identified. The amount of parameters included in the optimisation was in this way kept at an optimally low level. For verification, measurements from several static and dynamic field tests with a fully loaded macadam train and Swedish Rc6 locomotives were used. The implemented algorithms were shown to operate efficiently and the accuracy in static and dynamic load effect predictions was shown to be considerably improved.

It was concluded that the complex bridge can be simplified by means of beam theory and an equivalent modulus of elasticity, and still produce reliable results for simplified global analyses. The typical value of an equivalent modulus of elasticity was in this case approximately 25% larger than the specified mean value for the concrete grade in question.

The optimised FE model was used in moving load simulations with high speed train loads according to the design codes. Typically, the calculated vertical acceleration of the bridge deck was much lower than the specified allowable code value. This indicates that multispan continuous concrete bridges are not so sensitive to train induced vibrations and therefore may be suitable for high speed train traffic.

Finally, the relevant area of introducing the proposed FE model updating procedure in the early bridge design phase is outlined.
Sammanfattning

Dagens järnvägsbroar analyseras mer i detalj för rörliga laster till följd av ökade farter och axellaster. Dessa numeriska analyser är dock mycket tidskrävande och innebär ofta många simuleringar med olika tågkonfigurationer vid olika farter och många andra omständigheter att ta hänsyn till. Därför väljs ofta förenklade bro-, spår- och tågmodeller för praktiska och tidseffektiva simuleringar.


Det konstaterades att den konstruktivt komplicerade bron kan förenklands med hjälp av balkteori och en ekvivalent elasticitetsmodul och ändå ge pålitliga resultat för förenklade globala analyser. Det typiska värdet hos en ekvivalent elasticitetsmodul var i detta fall approximativt 25% större än det specifiserade medelvärdet för betongklassen i fråga.

Den optimerade finita elementmodellen användes i simuleringar med höghastighetståg enligt normer. Beräknade vertikala accelerationer hos brobanan var klart lägre än det specifiserat tillåtna normvärdet. Detta tyder på att kontinuerliga flerspannsbetongbroar inte är särskilt känsliga för tåginducerade vibrationer och därför kan vara lämpliga för höghastighetståg.

Slutligen presenteras ett tillämpningsområde för den föreslagna uppdateringsproceduren av en finit elementmodell som verktyg redan vid framtagandet av en brodesign.
The research work presented in this thesis was carried out at the Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH). It was financed by the Railway Group at KTH, the Swedish National Railway Administration (Banverket) and the Division of Structural Design and Bridges at KTH.

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Stockholm, September 2009

Johan Wiberg
List of publications

This thesis consists of a summary and five appended papers.

**Paper A**  

**Paper B**  

**Paper C**  

**Paper D**  

**Paper E**  

Four of the papers were prepared in collaboration with co-authors. The author of this thesis took the following responsibility for the work in those papers:

**Paper A**  
Took part in the theoretical study and the algorithms. Proposed and made changes in writing the report. Presented the work in a pre-version article at The Second International Conference on Structural Engineering, Mechanics and Computation (SEMC 2004).
**Paper B**  Planned the work in collaboration. Performed the operational modal analysis. Wrote the report.

**Paper D**  Planned the work in collaboration. Carried out all statistical and numerical calculations. Wrote the report.

**Paper E**  Planned the work in collaboration. Implemented the optimisation routines and carried out all numerical calculations. Wrote the report.

**Additional relevant publications**


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SUMMARY AND REVIEW OF THESIS
Chapter 1

Introduction

1.1 Background

Today’s railway bridges are analysed in more detail for moving loads due to the increase in speeds and axle loads. Moreover, modern railway bridges have more slender designs and lower energy dissipation (damping), as a consequence of the development in construction and design methods. Consequently, these detailed analyses of the dynamic response of bridges to passing trains are important in order to guarantee the planned lifetime and economical assessment.

Fig. 1.1 illustrates the increase in train speeds for passenger traffic between 1820 and 2040. Following the linear trend line from 1840 for maximum speeds worldwide results in a predictable speed of 350 km/h in 2040. With the advancement in locomotive and control technologies, railway trains that have a design speed of 350 km/h or higher are however not uncommon already nowadays. Considering only the time after the Second World War, the development is much faster. In this case the trend points toward 450 km/h in 2040. Such high speeds call for more realistic bridge, track and vehicle models in the simulations to accurately account for the dynamic load effects.

Willis and Stokes are considered the first to bring the problem of vehicle load impacts to the design desks of bridge engineers. Their historical research contributions on bridge vibrations caused by moving traffic, investigated the collapse of the Chaster Rail Bridge in England in 1847 (Willis, 1849; Stokes, 1849). Swedish pioneers were Lerfors and Hillerborg, but that was first in the 1940s. They investigated the most elementary cases of dynamic influences of moving loads on girders in laboratory tests. Lerfors conducted the first part between 1943 and 1946. From the end of 1946 Hillerborg worked on the problem, which resulted in his doctoral thesis “Dynamic Influences of Smoothly Running Loads on Simply Supported Girders” (Hillerborg, 1951). Hillerborg also referred to Ödman (1948) as another early
Swedish contribution with his differential equation for calculation of vibrations produced in load-bearing structures by moving loads.

A complete literature review of all the research conducted on the vibration of bridges under moving vehicles is too extensive to include here. However, Yang et al. (2004) present a comprehensive list of references, reviewed in short in the following. The previous investigations can be divided into two clearly different categories. Before the advent of digital computers in the 1940s, research were concerned mainly with the development of analytical or approximate solutions for simple, fundamental problems. Typical researchers of this period were Timoshenko (1922), Jeffcott (1929) and Lowan (1935). Inglis (1934) conducted a general treatment on the dynamics of railway bridges, constituting a ground for the development that followed. Digital computers, later followed by workstations, made it possible for researchers to adopt more realistic bridge and vehicle models in the analyses. Timoshenko and Young (1955) and Biggs (1964) presented general structural dynamic works. Frýba (1972) included a detailed review of moving load problem investigations before the 1970s, followed up with a 2nd and later 3rd edition (Frýba, 1999). Other literature that ought to be highlighted are Garg and Dukkipati (1984) and Frýba (1996).

After years of theory development the concept and performance of bridge monitoring drastically increased in the last two decades, due to advanced technology.
1.1. BACKGROUND

As an unique Swedish bridge monitoring project, the New Årsta Railway Bridge in Stockholm was instrumented during construction with sensors placed on the soffit of the track slab and embedded in the bridge deck, see Fig. 1.2. The bridge design is considered complex and the main intention with the instrumentations was a deeper understanding of its structural behaviour. The bridge was not designed for high speed traffic. Still, this instrumentation gave an opportunity to model and study moving load simulations based on measurement validations. One of the typical load tests with a fully loaded macadam train is demonstrated in Fig. 1.3. Fig. 1.4 gives emphasis to the unique but attractive geometry of the New Årsta Railway Bridge. Fig. 1.5 shows the great amount of reinforcement embedded in the concrete section.

During the process of this thesis work, different numerical models of the bridge were constructed and evaluated. These are, in chronological order, the early stage results of the 3D beam element FE model implemented by the author in Wiberg (2006), the finite difference method approach in Hannewald (2006), the 2D beam FE model approach in Gonzalez Silva (2008) and the volume element approach in Poisseroux (2009). In addition, there is also an ongoing structural health monitoring research project coupled to this bridge instrumentation, where modern fibre optic sensors are implemented, see e.g. Enckell (2006).
CHAPTER 1. INTRODUCTION

Figure 1.3: A fully loaded macadam train positioned on the bridge during a static load test.

1.2 Outline

This thesis consists of a summary and five appended papers. All results are presented in the papers.

The summary part has the intention of making the reader familiar with the subject. It also explains the research structure, the relation between the papers and highlights the role and objective of each paper in the overall research work. As the summary includes additional references, not included in the appended papers, a reference list is given. The summary also includes two algorithms as appendices. Appendix A presents the Yates’s algorithm, implemented in Paper D. Appendix B explains the optimisation solver based on the Nelder-Mead simplex algorithm, used in Paper E.

The reader may read the papers independently but is encouraged to take part of Chapter 3 for an overview and then continue with the papers.

1.3 Aim and scope

Detailed numerical analyses of passing trains on bridges are very time consuming as they involve many simulations using different train configurations at different speeds and has lots of considerations to take into account. Thus, simplified bridge and train models are often chosen for time efficient simulations.
1.3. AIM AND SCOPE

Figure 1.4: The smoothly shaped red concrete bridge.

Figure 1.5: A part of the bridge is being prepared with reinforcement and tendon tubes before casting.
CHAPTER 1. INTRODUCTION

The main objectives of the present work were:

- Instrument the New Årsta Railway Bridge for monitoring static and dynamic bridge behaviour.
- Implement a simplified FE model of the bridge for sufficiently accurate and time efficient moving load simulations.
- Implement, test and demonstrate the potential in combining a simplified model with modern system identification techniques and FE model updating routines.
- Verify the damping ratio in design codes for a prestressed continuous bridge.
- Study the possible influence of modelling parameters, separately and jointly, on load effects and dynamic characteristics of the bridge.
- Make dynamic simulations of high speed trains to investigate the sensitivity of the bridge to train induced vibrations.

Notice, it was not the intention to reflect the physical quantities of the bridge most accurately in the FE model, but to facilitate practical moving load simulations for large and complicated bridges. A more detailed and/or complex FE model system would probably render more accurate load effect predictions, but would at the same time be impractical in time efficient moving load simulations.

The present work is based on the following main assumptions and limitations:

- Linear material properties.
- A bridge model with 3D Bernoulli-Euler beam elements.
- Accurately known positions of measuring sensors and vehicle loads in load testing.
- Vehicle axle loads represented as point forces.

1.4 Thesis contribution

The concept of simplified FE modelling, in combination with optimised FE model updating, based on measurements, is presented in this thesis.

From a scientific point of view, the main contributions consist of:

1. Instrumenting the bridge and performing measurements over a long period of time.
2. Investigating the usefulness of measured data for bridge as well as vehicle identification.

3. Introducing and testing the concept of an equivalent modulus of elasticity approach, instead of in detail considering reinforcement, tendons, boundary conditions and other contributors to the overall bridge stiffness.

4. Demonstrating the reliability of a simplified FE bridge model by using actual load effects and operational modal analysis.

5. Highlighting the potential and value of using statistical methods, i.e. herein Design and analysis Of Experiments (DOE), as a first step in FE model updating. To the author’s knowledge, the adopted method of factorial experimentation has not been used before for discovering significant modelling factors of the bridge system.
Chapter 2

Train-bridge modelling aspects

Due to the successful implementation and operation of high-speed railways worldwide, the dynamic response of railway bridges is receiving the most attention ever. These structural systems consist of the vehicle, the track and the bridge. In the following, some modelling aspects are presented briefly to introduce the reader to the subject.

The vehicles are complex structures that can be modelled using masses, springs and dashpots, see Fig. 2.1. However, the most complex train models should only be used if e.g. acceleration levels in the train need to be checked and if the track is not well maintained (i.e. the rail condition is poor and therefore rail irregularities need to be considered in the analyses), see e.g. Karouni (1998) and Yang et al. (2004). For normal vehicle speeds, most of the excitation of the dynamic system is then caused by the roughness of the rail surface and very little is caused by the elastic displacement of the bridge itself. Majka and Hartnett (2009) studied this effect of random track irregularities, using the complex 3D vehicle model in Fig. 2.2.

![Vehicle model alternatives](image)

Figure 2.1: Vehicle model alternatives.
CHAPTER 2. TRAIN-BRIDGE MODELLING ASPECTS

By neglecting the inertia effect of the vehicle, it only symbolises moving forces according to Fig. 2.1(a). This kind of most simple vehicle model was for example adopted in the studies of simple beams by Timoshenko (1922) and extended to continuous beams by Chen (1978), or more recently by Dugush and Eisenberger (2002). In cases when the inertia effect of the vehicle is not regarded small, the moving mass model can be used, see Fig. 2.1(b). Such a vehicle model was implemented by Stanišić (1985), resulting in an exact closed form solution for a simple beam carrying a single moving mass. A vehicle model that considers the elastic and damping effects of a suspension system is referred to as the sprung mass model in Fig. 2.1(c). Vehicle models of this more sophisticated kind can make the simulations more realistic but also result in divergence or slow convergence in the iteration process of searching a large number of contact forces due to the interaction between vehicle and bridge (Yang et al., 1996). Instead, the use of simplified vehicle models make the identification of key parameters, dominating the dynamic response of the bridge, much easier (Humar and Kashif, 1993).

Theoretically, the difference between the vehicle models in Fig. 2.1 is probably best demonstrated by reviewing the equation of motion for a simple beam under a moving train. This is presented here as in Yang et al. (2004) but can also be found elsewhere, see e.g. Karoumi (1998) or Olsson (1986).

Consider a simply-supported beam of length $L$ travelled by a train at speed $v$, consisting of a number of identical cars of length $d$. The train is assumed to travel along the centerline of the beam, i.e. no torsional action is present. Approximating the train as a series of lumped loads $p$, the corresponding load function can be given as

$$ F(t) = \sum_{i=1}^{N} p \cdot U_i(t, v, L) $$

(2.1)
where

$$U_i(t, v, L) = \delta[x - v(t - t_i)] \cdot \left[ H(t - t_i) - H \left( t - t_i - \frac{L}{v} \right) \right]$$  \hspace{1cm} (2.2)$$

Here, $\delta$ denotes the Dirac delta function, $x$ the coordinate of the beam, $H(\bullet)$ a unit step function, $t_i$ the arrival time of the $i$th load at the beam, $t_i = (i - 1)d/v$, and $N$ the total number of moving loads. Consequently, the effect of the $i$th moving load is turned on by the term $H(t - t_i)$ when it enters the beam and is turned off by the term $H(t - t_i - L/v)$ when it leaves the beam.

Normally, each car is supported by two bogies, each of which consists of two wheel sets. In the following formulas however, each bogie is simplified and considered as one single component, a wheel assembly. By then letting $L_c$ denote the distance between the two wheel assemblies of a car and $L_d$ the distance between the rear wheel assembly of a car and the front wheel assembly of the following car, it follows that the car length $d$ is equal to the sum of $L_c$ and $L_d$ and that a train can be represented as a sequence of wheel loads $p$ with alternative intervals $L_c$ and $L_d$.

For a better representation of the load configuration, it is realised that each train consists of two wheel load sets, the first set representing the wheel loads of all the front bogies and the second set the rear ones. Consequently, the distance between any two consecutive wheel loads in each set is then $d$. Due to the time lag, $t_c = L_c/v$, between the two sets of moving loads the wheel load function in Eq. (2.1) is modified and represented as:

$$F(t) = \sum_{i=1}^{N} p \cdot [U_i(t, v, L) + U_i(t - t_c, v, L)]$$  \hspace{1cm} (2.3)$$

The expressions given in Eq. (2.1) and Eq. (2.3) consider only the effect of moving forces according to the vehicle model in Fig. 2.1(a). They neglect the effect of inertia of the moving masses and the interaction between the train cars and supporting beam. If to consider those effects for the moving mass case, the load term $p$ can be replaced by the function (Bolotin, 1964)

$$F(p, M, v) = p - M(\ddot{u} + 2v\dot{u}' + v^2u'')$$  \hspace{1cm} (2.4)$$

where $M$ denotes the vehicle mass ($m_1 + m_2$) lumped at each load or wheel position, $u$ is the vertical deflection of the beam and dots (') and primes (') represent differentiation with respect to time $t$ and coordinate $x$, respectively. Most important is the physical meaning of the added terms in Eq. (2.4). The term $M\ddot{u}$ represents the inertial force acting along the direction of deflection $u$ of the beam, the term $2M\dot{u}'$ is the Coriolis force relating to the rate of inclination of the beam and the term $Mv^2u''$ is the centrifugal force associated with the curvature of the beam,
induced by the mass with speed $v$ at the position of action. Consequently, based on Eq. (2.4), the equation of motion for the beam can be written as

$$m \dddot{u} + c_e \dot{u} + c_i \dddot{u} + E I \dddot{u} = \sum_{i=1}^{N} F(p, M, v) \cdot [U_i(t, v, L) + U_i(t - t_c, v, L)]$$  \hspace{1cm} (2.5)$$

where $m$ denotes the bridge mass per unit length, $c_e$ the external damping coefficient, $c_i$ the internal damping coefficient, $E$ the modulus of elasticity of the beam and $I$ the moment of inertia of the beam. Most commonly, the second and third term on the left hand side is replaced with $c \dot{u}$ only.

Beam bridges for high speed trains are, however, often so stiff that the effects of both the Coriolis force and the centrifugal force are very much smaller than that of the added masses. They can therefore generally be excluded without losing accuracy in the solutions.

Olsson (1986) and Samani and Pellicano (2009) also showed that the behaviour of beams under moving forces or moving masses is very similar when the moving mass is assumed to be small in comparison to the beam mass. Karoumi (1998) concluded that too, assuming a rail surface with no roughness and a normal vehicle speed. Then, the moving force model is fully adequate and there is no need for using complicated vehicle models. However, the moving force model has been found not suitable for the study of short span bridges ($L \ll 20 - 25$ m) since the result they produce (displacements and accelerations) are much larger than those obtained from more sophisticated models with distribution of the loads due to the presence of the sleepers and a ballast layer, together with the train-bridge interaction (Museros et al., 2002). Thus, the track model in many cases also becomes important. How the beneficial effect of the track (i.e. the rails, sleepers and ballast) should be considered in an enhanced assessment is given emphasis to in SB-LRA (2007), which also refers to the investigations performed in ERRI D214 (1999). Thus, for enhanced assessments of short span bridges, the favourable effect of the track distributing the load should be considered in a track model, resulting in reduced acceleration levels.

Consideration of load distribution is not the only factor that complicates the moving load assessments on railway bridges. Fig. 2.3 includes potential factors to eventually consider in more enhanced designs and assessments.

Powerful numerical methods, based on finite element methods, are most often employed to analyse the dynamic behaviour of bridges and moving vehicles. Virtually, the limit of complexity in the subsystem models is then nonexisting. Obviously, the use of simplified vehicle and bridge models is helpful and requires less preparation and computation efforts, still allowing the identification of key parameters dominating the dynamics in the structural system (Yang et al., 2004). However, the required time step $\Delta t$ and the number of modes to include in a mode superposition solution must be investigated in a study of convergence in results. Various
Bridge damping
Irregularities (wheel, rail)
Dynamic modulus of elasticity
The transition zone between embankment and bridge
The stiffness and damping of the track
The mass of the train
Boundary conditions
The transition zone between embankment and bridge
Bridge/soil interaction
Cracked/uncracked concrete
Load distribution ballast/sleepers
Train damping
Bridge/train interaction

Figure 2.3: Example of modelling aspects that may be necessary to consider in more enhanced moving load designs and assessments of railway bridges.

types of bridges has been studied in analyses of the moving load problem. The simply-supported beam is adopted most often but other more complex bridge models also include the related effects of e.g. rail irregularities (Paultre et al., 1992) and torsional vibration (Hsu, 1996). The objective with more realistic simulations with design loads or actual traffic loads, based on measurements, is to predict actual dynamic load effects instead of using the traditional dynamic amplification factors, described and evaluated in e.g. Chaallal and Shahawy (1998) and James (2003). A typical example of this is the study of Karoumi and Wiberg (2006) where actual dynamic load effects on bridges along the Bothnia Line in Sweden were determined based on simulations.

Typical guidelines for the necessary kind of measurements and calculations has also been produced, see e.g. UIC (2006). In addressing dynamic effects and actual traffic loads, an integrated international research project, Sustainable Bridges - Assessment for Future Traffic Demands and Longer Lives, recently considered this subject in a background document on assessment of actual traffic loads (SB4.3, 2007). This background document was followed by the guideline for assessing the actual load carrying capacity of existing railway bridges (SB-LRA, 2007).

The requirements in the Eurocodes, see CEN (2002), is reported on by Bucknall (2003), relating to high speed railway bridge design. The code includes requirements on design checks, acceptance criteria, structural analysis and structural properties to be adopted. Three principle behaviours that contribute to the total dynamic response of a railway bridge are identified: inertial response, resonance effects and additional dynamic effects due to track irregularities, wheel defects and suspension defects. In addition, the importance of taking into account resonance effects and bridge deck acceleration, along with other deformations and load effects is focused on. The code states explicitly that compliance with vertical deflection limits is not sufficient to guarantee satisfactory behaviour where resonance effects occur. These requirements were from 2005 also implemented in the Swedish railway bridge design code, BV Bro (Banverket, 2008).
CHAPTER 2. TRAIN-BRIDGE MODELLING ASPECTS

In the following, some typical aspects to consider are presented, mostly based on the Eurocode requirements for the design of high speed railway bridges. When determining whether a structure will be prone to resonance effects, the relationship between frequency of loading (and associated traffic speed) and the natural frequencies of a structure is paramount. The natural frequency of a structure depends upon its element length or span, mass, stiffness, boundary conditions and vibration mode shape. Of these parameters, stiffness is the most difficult to predict accurately.

**Stiffness**

The relationship between frequency of loading and the natural frequencies of the bridge is particularly important when any resonant peaks occur just above the loading frequency and hence the speed range being considered. It is therefore necessary to make a lower bound assessment of the natural frequency (i.e. lower bound value of stiffness) of the structure to obtain a lower bound (and safe) estimate of maximum permitted speed.

**Damping and other dynamic characteristics**

Damping in railway bridges is a complex phenomenon and its accurate description and representation in a numerical model is difficult (Majka and Hartnett, 2008). Generally, it depends on the material, the state of the bridge (presence of cracks, ballast, support conditions) and the amplitude and frequency of vibrations (Fryba, 1996). As the overall dynamic behaviour is more sensitive to the value of damping rather than the mathematical model used to represent damping, viscous damping assumptions are often made as they also result in easier computational methods. The assumption of viscous damping is sufficiently accurate for design, as bridge damping values are highly variable. Highly importantly, the maximum acceleration or load effect is dependant upon the rate at which previous dynamic motions are dissipating through damping. The less the damping, the larger are of course the maximum dynamic effects. Therefore, lower bound values of damping should be used in design calculations to ensure that safe estimates of peak dynamic effects at resonance are obtained. However, as the dynamic vehicle/bridge interaction effects tend to reduce the peak response at resonance, especially for short span bridges, account of this may according to the codes be taken by increasing the value of assumed damping with the term $\Delta \zeta$, see CEN (2002).

In assessment of existing bridges, a way to identify the dynamic characteristics of a bridge (i.e. natural frequencies and mode shapes in addition to modal damping ratios) is to use powerful operational modal analysis, based on output-only analyses in ambient vibration, see e.g. Brincker et al. (2003). An assessment of damping identification methods is presented in Prandina et al. (2009).

**Mass**

The natural frequency of a structure decreases as the mass of the structure increases (providing other parameters such as stiffness do not change). Any underestimation
of mass will overestimate the natural frequency of the structure. As a result the loading frequency and hence speed at which resonance occurs will be overestimated. Therefore safe upper bound estimates of bridge mass are required to ensure a safe prediction of resonant speeds. In addition, the maximum acceleration of a structure at resonance is inversely proportional to the distributed mass of the structure. Therefore, it is important that a second case is also taken into account with lower bound values of mass used in calculations to ensure that a safe estimate of peak dynamic acceleration effects at resonance is obtained.
Chapter 3

The research work

The requirement of more accurate dynamic analyses of railway bridges calls for reliable but rather simplified system models to make moving load simulations practical for large and complex bridges. In the present research, the New Årsta Railway Bridge was instrumented and subjected to static and dynamic load testing for verification and updating of a simplified Bernoulli-Euler beam FE model, intended for more accurate and time efficient analyses of passing trains on bridges.

Fig. 3.1 has the intention of clarifying the performed research work for the reader. The appended papers which are located in the middle suggest a methodology that can be adopted for analysis of the effect of passing trains on bridges. Two possible directions of the developed and implemented analysis concept are indicated. The right hand side of Fig. 3.1, valid for assessment of existing bridges, briefly explains the contribution in each paper and the total process work flow in this thesis, preparing the simplified model for dynamic load effect predictions based on system identification and measurements. The left hand side of Fig. 3.1, valid for design of new bridges, emphasises an alternative use in optimised bridge design, based on the implemented statistical identification routine and the optimisation algorithm. The order of Paper C and Paper D is not strict in the context of the performed work. Paper D could as well be placed before Paper C, which is indicated with the arrows in two directions between those two papers.

The right hand side in Fig. 3.1 uses static and dynamic analyses to find an optimal FE model based on real measured load effects. This optimised FE model is then adopted in new simulations for future trains having higher axle loads and/or speeds. The left hand side in Fig. 3.1 instead uses simulations to find an optimal bridge design in the bridge design phase, based on design code requirements. As an example, a typical problems that a bridge designer encounters is to find acceptable combinations of span lengths, stiffness and mass distribution along the bridge, and so forth, resulting in a maximum vertical bridge deck acceleration lower than the code limit of $3.5 \text{ m/s}^2$. Using this maximum acceleration limit value as “measured”
in the optimisation process, the optimal intended bridge characteristics are calculated. This could be valuable and time-saving for bridge designers even though the optimisation routines on their own can be time consuming in complicated cases.

The following are brief summaries of the appended papers:

In Paper A, *Monitoring traffic loads and dynamic effects using an instrumented railway bridge*, an example of vehicle and bridge system identification is presented. This paper is very much of a more general kind, demonstrating possibilities with railway bridge measurements. The bridge in question is not the New Årsta Railway Bridge, analysed in the following papers, but an adjacent integral-type railway bridge. A complete Bridge Weigh-in-Motion (B-WIM) system, with axle detection and accurate axle-load evaluation was implemented, together with general evaluation of eigenfrequencies, prediction of possible wheel/rail defects and identification of acceleration levels. Some very early but representative results are presented, and the efficiency of the algorithms and usefulness of the monitoring program highlighted. This paper was included in a special issue of Engineering Structures, based on a presentation of the author at the International Conference on Structural Engineering, Mechanics and Computation (Karoumi et al., 2004). The B-WIM algorithm was later refined and presented as a Matlab toolbox in Liljencrantz and Karoumi (2009).

In Paper B, *Monitoring dynamic behaviour of a long-span railway bridge*, the bridge in question, the New Årsta Railway Bridge, is introduced. The bridge and its instrumentation is described, together with the simplified FE model and its concept of an equivalent modulus of elasticity. The paper focuses on investigating the dynamic characteristics of the bridge using operational modal analysis, but with fixed monitoring sensors. In addition, extreme bridge acceleration values from different train passages are collected and compared with the recommended limit value in bridge design codes. However, the theory of operational modal analysis relies on movable accelerometers. Such a measurement was performed as a complement to this paper and presented by the author at the 2nd International Conference on Experimental Vibration Analysis for Civil Engineering Structures (Wiberg, 2007). Not much new information was received and results from these measurements are included in Paper E.

In Paper C, *An equivalent modulus of elasticity approach for simplified modelling and analysis of a complex prestressed railway bridge*, the possibility of using an equivalent modulus of elasticity in simplifying the modelling is investigated, based on measured and predicted axial strain, including the Vlasov portion of the torsional moment due to constrained warping. The study involves several static load tests with a fully loaded macadam train and Swedish Rc6 locomotives. The definitions of modulus of elasticity values are discussed, followed by a description of the field measurements and the analysis techniques.

In Paper D, *Statistical screening of individual and joint effect of several modelling factors on the dynamic FE response of a railway bridge*, the potentials of statistical
ANALYSIS OF PASSING TRAINS ON BRIDGES

NEW BRIDGE TO BE DESIGNED

- Design based on optimisation
- Design code requirements
  - “Measurements”
- Statistical identification of influencing modelling parameters
  - FE modelling
- Optimised bridge design
  - “Measurements” + FE modelling

EXISTING BRIDGE

- Dynamic responses based on system identification and measurements
- Manual FE model tuning
  - Measurements + FE modelling
- Statistical identification of influencing modelling parameters
  - FE modelling
- Optimised FE model
  - Measurements + FE modelling
- Simulations
  - (predicted load effects)
- Check code requirements

Figure 3.1: Schematic thesis structure. The right hand side presents the subjects included in the appended papers, intended for dynamic responses based on system identification and measurements. The left hand side emphasises the possible implementation in the design phase, resulting in an optimal bridge design.
methods are highlighted. Factorial experimentation in simulating railway bridge
dynamics is exemplified. Unlike the usual one factor at a time parameter studies,
factorial experimentations also identify the effect of significant modelling parameter
interactions. The statistical effect estimations are based on main-effect and inter-
action sum of squares in the theory of design and analysis of experiments. For large
factorial experiments these calculations become tedious and are simplified consider-
ably by using Yates’s algorithm. The algorithm is fairly simple but its mathematical
proof is seldom given. It is therefore included here in Appendix A. Additionally, the
paper includes the necessary code for implementation of the statistical procedure
in the mathematical software package Matlab®.

In Paper E, Optimized model updating of a railway bridge for increased accuracy
in moving load simulations, an optimised updating method based on load tests
and statistically identified influencing updating parameters is used for more time
efficient and accurate load effect predictions. A benchmark test is presented to
demonstrate the high potential of the adopted Nelder-Mead simplex optimisation
algorithm. This algorithm is also described in detail here in Appendix B. High
speed train model simulations are performed with the optimised FE model of the
New Årsta Railway Bridge and more accurately predicted load effects are exem-
plified. In addition, the Matlab® syntax for implementation of the optimisation
routines is given. The high potential FE model updating procedure is used tradi-
tionally, based on measurements, but the relevant area of introducing it in the early
bridge design phase is outlined.
Chapter 4

Conclusions

The New Årsta Railway Bridge in Stockholm was successfully instrumented during construction. A simplified 3D Bernoulli-Euler beam element FE model of the bridge was prepared. The FE model was first manually tuned based on static railway load testing. The most extensive work was performed in a statistical identification of significantly influencing modelling parameters. Those were finally used in FE model optimisation based on both static and dynamic load testing, resulting in increased accuracy in predicted load effects from moving load simulations.

The following was concluded and noted:

- The study confirmed that, for a complex bridge structure, measurements are necessary to obtain a reliable FE model to be used for dynamic analyses of passing trains.

- The complex bridge could be simplified by means of beam theory and an equivalent modulus of elasticity, still producing approximate but reliable results for simplified global analyses. The typical value of an equivalent modulus of elasticity was in this case approximately 25% larger than the specified mean value for the concrete grade in question.

- With statistical identification of significantly effecting modelling parameters the amount of parameters included in the optimisation were kept at an optimally low level.

- The implemented statistical and optimisation algorithms operated efficiently and the accuracy in static and dynamic load effect predictions was considerably improved.

- The first pure bending and torsional frequencies for the New Årsta Railway Bridge were found at 1.30 Hz and 3.55 Hz, respectively.
CHAPTER 4. CONCLUSIONS

- The optimisation technique gave a modal damping ratio of between 0.92\% and 2.10\%. Results from peak picking methods and modern stochastic subspace identification techniques in operational modal analysis varied considerably and generally gave lower damping ratios. This proved the difficulties in damping estimation. Also, the lower bound value of 1\%, given in the design codes for prestressed concrete bridges, is not as much on the safe side as was earlier thought.

- The New Årsta Railway Bridge did not suffer from high bridge deck accelerations. This may indicate that this and other multispan continuous concrete bridges are suitable for high speed train traffic.

4.1 Further research

Examples of possible subjects and recommendations for further research are:

- Detailed 3D modelling of the instrumented part of the bridge to include the in-plane stresses and a deformed cross section for comparison with the simplified model included here, especially in torsional vibration.

- Study the unique (in Sweden) slab track system on The New Årsta Railway Bridge in detail.

- Identification of changes in bridge behaviour, i.e. damage detection based on the installed monitoring equipment as alarm system. With a more refined bridge model it will be possible to detect local damages based on monitoring data and the already implemented optimisation routine. In this way it should also be possible to indicate eventual changes in track condition.

- Develop and implement general computer code for statistical experimental design and FE model optimisation in the process of FE model updating. With such routines included in a toolbox the user will be able to choose from different types of statistical and optimisation algorithms to use in the specific project.

- The implemented routines for statistical identification and FE model optimisation are possible to use also in the bridge design phase. If this is further developed and made user-friendly it may be a timesaving action for bridge designers.

- Generally, perform more measurements on this and other railway bridges to verify the recommended lower bound damping ratios in design codes and to test the performance of the operational modal analysis concept on railway bridges.
4.1. FURTHER RESEARCH

- Implement B-WIM for measurements and real-time analyses of train traffic which can be used for assessment of bridges and for calibration of load models in design codes.

- Study the effect or influence of additional factors such as for example braking and acceleration of trains, the crossing of two vehicles moving in opposite directions, the mass ratio of the vehicles to the bridge, elastic bearings and supporting columns.

In addition, the author actually put a great deal of effort into developing a vehicle-bridge interaction model that, based on the bridge deck curvature in both the horizontal and vertical direction, was shown to operate successfully. This model is possible to utilise for more detailed studies in the future.
Bibliography


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Appendix A

Yates’s algorithm

Yates’s algorithm calculates sums of squares of all the contrasts simultaneously from a $2^k$ factorial design. It achieves a saving in multiplying one form of matrix by a vector. Usually, multiplying a $2^p \times 2^p$ matrix by a $2^p$ vector requires $2^{2p}$ multiplications. A matrix of the form

\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}^p
\]

can however be multiplied by a $2^p$ vector in $2p^{2p+1}$ operations by taking advantage of its particular structure.

The algorithm is simple to implement and is mathematically represented in the following:

1. Define the tensor product of the matrices $A = (a_{ij})_{m \times m}$ and $B = (b_{kl})_{n \times n}$:

\[
(a_{ij})_{m \times m} \otimes (b_{kl})_{n \times n} = (a_{ij}b_{kl})_{mn \times mn}
\]

where $a_{ij}b_{kl}$ is at position $(i-1)n + k, (j-1)n + l$. 

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2. Let $\tilde{A}_{m \times m}$ be the matrix $A$ expanded as:

$$
\begin{bmatrix}
    a_{11} \cdots a_{1m} & 0 & \cdots & 0 \\
    0 & a_{11} \cdots a_{1m} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & a_{11} \cdots a_{1m} \\
    a_{21} \cdots a_{2m} & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & a_{m1} \cdots a_{mm}
\end{bmatrix}
$$

3. Let $\tilde{A}$ and $\tilde{B}$ be the expansions of $A$ and $B$ according to (2). Then:

$$
A \otimes B = \tilde{A} \tilde{B}
$$

4. Generalise the result to powers, i.e.

$$
A^{\otimes p} = \tilde{A}^p
$$

Example

With $m = n = 2$:

$$
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}
$$

$$
A \otimes B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \otimes \begin{bmatrix} e & f \\ g & h \end{bmatrix}
$$

$$
= \begin{bmatrix}
    ae & af & be & bf \\
    ag & ah & bg & bh \\
    ce & cf & de & df \\
    cg & ch & dg & dh
\end{bmatrix}
$$

$$
= \begin{bmatrix}
    a & b & 0 & 0 \\
    0 & 0 & a & b \\
    c & d & 0 & 0 \\
    0 & 0 & c & d
\end{bmatrix} \begin{bmatrix}
    e & f & 0 & 0 \\
    0 & 0 & e & f \\
    g & h & 0 & 0 \\
    0 & 0 & g & h
\end{bmatrix}
$$

Multiplying $A \otimes B$ by the vector $v$, takes the sparsity advantage of the matrices $A$ and $B$ into account. Each row of $B$ has only two non-zero elements, so
$Bv$ is computed in $2 \cdot 4$ multiplications. Also $\tilde{A}(Bv)$ is computed in $2 \cdot 4$ operations. Thus, the number of multiplications is in general $mn^2 + m^2n = mn(m + n)$. Therefore, this example does not save anything since the total number of multiplications (16) is equal to the ordinary matrix multiplication. However, for $m, n > 2$, $mn(m + n) < (mn)^2$.

Multiplying $A^{\otimes p}$ by a $m^p$ vector $v$, the $m^p \times m^p$ matrix $\tilde{A}$ has only $m$ non-zero elements on each row. The product $\tilde{A}v$ involves $m \cdot m^p$ multiplications. To get $\tilde{A}^p v$, the product is repeated $p$ times, resulting in a total of $pm^{p+1}$ multiplications. Consequently, the saving is huge even for a moderately huge $p$, compared to the $m^{2p}$ multiplications of an ordinary matrix by vector product. If for example $m = 2$ and $p = 20$, $m^{2p} \approx 10^{12}$ but $pm^{p+1} \approx 4 \cdot 10^6$. 
Appendix B

The Nelder-Mead simplex algorithm

The Nelder-Mead simplex algorithm is implemented in the optimisation toolbox of Matlab as the `fminsearch` solver. The algorithm uses a simplex of \( n + 1 \) points for \( n \)-dimensional vectors \( x \). Firstly, the algorithm makes a simplex around the initial guess \( x_0 \) by adding 5\% of each component \( x_0(i) \) to \( x_0 \) and uses these \( n \) vectors as elements of the simplex in addition to \( x_0 \). Secondly, the algorithm modifies the simplex repeatedly according to the following procedure:

1. Let \( x(i) \) denote the list of points in the current simplex, \( i = 1, \ldots, n + 1 \).

2. Order the points in the simplex from smallest function value \( f(x(1)) \) to largest \( f(x(n+1)) \). At each step in the iteration, the current worst point \( x(n+1) \) is discarded and another point is accepted into the simplex (or, in the case of step 7 below, all \( n \) points with values above \( f(x(1)) \) are changed).

3. Generate the reflected point

\[
r = 2m - x(n+1)
\]

where

\[
m = \sum \frac{x(i)}{n}, \quad i = 1, \ldots, n
\]

and calculate \( f(r) \).

4. If \( f(x(1)) \leq f(r) < f(x(n)) \), accept \( r \) and terminate this iteration.

5. If \( f(r) < f(x(1)) \), calculate the expansion point \( s \)

\[
s = m + 2(m - x(n+1))
\]

and calculate \( f(s) \).

a) If \( f(s) < f(r) \), accept \( s \) and terminate the iteration.
b) Otherwise, accept $r$ and terminate the iteration.

6. If $f(r) \geq f(x(n))$, perform contraction between $m$ and the better of $x(n+1)$ and $r$.
   
   a) If $f(r) < f(x(n+1))$, i.e. $r$ is better than $x(n+1)$, calculate
   
   $$c = m + \frac{r - m}{2}$$
   
   and calculate $f(c)$. If $f(c) < f(r)$, accept $c$ and terminate the iteration. Otherwise, continue with Step 7.
   
   b) If $f(r) \geq f(x(n+1))$, calculate
   
   $$cc = m + \frac{x(n+1) - m}{2}$$
   
   and calculate $f(cc)$. If $f(cc) < f(x(n+1))$, accept $cc$ and terminate the iteration. Otherwise, continue with Step 7.

7. Calculate the $n$ points
   
   $$v(i) = x(1) + \frac{x(i) - x(1)}{2}$$
   
   and calculate $f(v(i)), i = 2, \ldots, n + 1$. The simplex at the next iteration is $x(1), v(2), \ldots, v(n + 1)$.

The iterations proceed until a stopping criterion is met.

Fig. B.1 below illustrates the points that may be calculated in the algorithm procedure, along with each possible new simplex. The original simplex is represented in the bold outline.
Part II

APPENDED PAPERS
Paper A

Monitoring traffic loads and dynamic effects using an instrumented railway bridge

Paper B

Monitoring dynamic behaviour of a long-span railway bridge

Paper C

An equivalent modulus of elasticity approach for simplified modelling and analysis of a complex prestressed railway bridge

Paper D

Statistical screening of individual and joint effect of several modelling factors on the dynamic FE response of a railway bridge

Paper E

Optimized model updating of a railway bridge for increased accuracy in moving load simulations

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