This is the accepted version of a paper presented at 14th International Conference on Advanced Robotics (ICAR 2009), 22-26 June 2009, Munich.

Citation for the original published paper:


N.B. When citing this work, cite the original published paper.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:umu:diva-30079
Trajectory Planning and Time-Independent Motion Control for a Kinematically Redundant Hydraulic Manipulator

Uwe Mettin, Pedro X. La Hera, Daniel Ortíz Morales, Anton S. Shiriaev, Leonid B. Freidovich, Simon Westerberg

Abstract—In this paper we consider the problem of motion planning and control of a kinematically redundant manipulator, which is used on forestry machines for logging. Once a desired path is specified in the 3D world frame, a trajectory can be planned and executed such that all joints are synchronized and constrained to the Cartesian path. We introduce an optimization procedure that takes advantage of the kinematic redundancy so that time-efficient joint and velocity profiles along the path can be obtained. Differential constraints imposed by the manipulator dynamics are accounted for by employing a phase-plane technique for admissible path timings. In hydraulic manipulators, such as considered here, the velocity constraints of the individual joints are particularly restrictive. We suggest a time-independent control scheme for the planned trajectory which is built upon the standard reference tracking controllers. Experimental tests underline the benefits and efficiency of the model-based trajectory planning and show success of the proposed control strategy.

Index Terms—Trajectory Planning, Motion Control, Robotics in Agriculture and Forestry, Kinematically Redundant Manipulator

I. INTRODUCTION

Nowadays most of the harvesting and logging in forestry is performed by human-operated machines. There is a clear trend towards autonomous processes due to the overall benefits from robotics applications in industry [1]. Full automation of forestry machines, however, can be considered as a long-term goal, since serious challenges are addressed regarding motion planning and control of the manipulation tasks, maneuvering through the rough forest environment, machine perception, localization and mapping. Semi-autonomous schemes, still involving a human operator but with less complex tasks, on the other hand, are to be expected in the nearest future. For instance, automated motions from initial to target points are expected to provide great assistance and stress relaxation to the operator. Teleoperation can further increase safety and efficiency, since several machines could be remotely operated by one person [1].

In this paper we consider the problem of trajectory planning and feedback control design along Cartesian paths for a forwarder crane. Forwarders are used to collect logs from the ground to the tray for transportation out of the harvesting site. The human-operated crane is a kinematically redundant manipulator whose end-effector position is controlled in a 4D joint space; its orientation is not controlled. Thus, the movement of the end effector from a start point to an end point is described in a 3D Cartesian space, its coordination, however, is subject to a higher dimensional configuration space.

Our investigation aims for an intelligent strategy of trajectory planning and control design for a hydraulic manipulator, such as depicted in Fig. 1. Once a desired path is specified in the 3D-world frame, a motion can be planned and executed such that all joints are synchronized and constrained to the Cartesian path. We suggest an optimization procedure that takes advantage of the redundancy from task space to configuration space. Those joint profiles that yield the maximum speed performance along the path are found by employing a phase-plane technique for admissible path timings subject to certain constraints of the mechanical construction and dynamics. Eventually, a time-efficient trajectory along a specified Cartesian path is obtained.

Planning a path at first and then computing a timing function along the path subject to differential constraints is known as decoupled approach [4]. We should note that such methods were already introduced in the 1980’s by [3], [2], [8] proposing a time scaling of trajectories in order to accommodate actuator torque limitations. The resulting trajectories are time-optimal and require a bang-bang control for switching between accelerating and decelerating at full speed.
speed. For hydraulic actuators, however, the apparent velocity constraints are dominant and will therefore be treated carefully in our study.

The rest of the paper is organized as follows. A kinematic model of the crane together with constraints in configuration and velocities is introduced in Section II. In Section III we specify a particular Cartesian path of the boom tip in order to illustrate the strategy of path-constrained trajectory planning without its explicit dependence on time. An optimization procedure for time-efficient joint and velocity profiles is suggested. Moreover, two trajectories along the predefined path are given as examples. The time-independent control scheme is discussed in Section IV. Experimental results are presented in Section V. The paper ends with concluding remarks.

II. MODELING THE MANIPULATOR

The manipulator used for our study (see Fig. 1) is a slightly downsized version of a typical forwarder crane, very similar to it in configuration and dynamics. It is hydraulically powered and consists of a series of links. We are concerned with the manipulation task of moving the end effector from a start point to an end point in the world frame, i.e. grasping tasks performed by the end effector are not considered here. Thus, the robot geometry to be described is an open kinematic chain of four links from the base to the joint where the end effector is attached. The joints are structured as follows:

(0) Base of the robot manipulator.
(1) Revolute joint for slewing, associated with \( q_1 \).
(2) Revolute joint for the inner boom, associated with \( q_2 \).
(3) Revolute joint for the outer boom, associated with \( q_3 \).
(4) Prismatic joint for telescopic extension of the outer boom, associated with \( q_4 \).
(5) Joint where end effector is attached (boom tip).

The joint variables form the vector of generalized coordinates \( q = [q_1, q_2, q_3, q_4]^T \) for this 4 degree-of-freedom system. They are measured by high-resolution encoders.

The forward kinematics can be conveniently expressed using the Denavit-Hartenberg (DH) convention [10], where each link configuration is represented by the homogeneous transformation

\[
A_i = \text{Rot}_{z, \theta_i} \text{Trans}_{x, d_i} \text{Trans}_{x, a_i} \text{Rot}_{x, \alpha_i},
\]

parameterized by joint angle \( \theta_i \), link offset \( d_i \), link length \( a_i \), and link twist \( \alpha_i \). In Table I the parameters are provided that describe the configuration of the forwarder crane used in this study. Eventually, the Cartesian position of the boom tip with respect to the base frame of the robot is defined by

\[
p^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 1} \end{bmatrix} T_4^0 \begin{bmatrix} 0_{3 \times 1} \\ 1 \end{bmatrix}
\]

where \( T_4^0 = A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4) \).

Inverse kinematics from a configuration of the boom tip to the joint variables can be found as a solution of a set of nonlinear trigonometric equations given by \( T_4^0 \) in (2). In our case only the boom-tip position shall be specified along some motion such that corresponding joint variables are computed in closed form by a function

\[
q = F(p^0, q_4),
\]

where \( q_4 \) is the chosen redundant joint variable. Here non-uniqueness issue is resolved so that the function is continuous along the target path.

The configuration space of the manipulator is spanned by the joint variables and it is restricted by the mechanical construction of the robot. Differential constraints, on the other hand, are imposed by the system dynamics. The system dynamics can be described by the following Euler-Lagrange equation [10]

\[
dt \left( \frac{\partial L(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial L(q, \dot{q})}{\partial q} = B(q) u
\]

with the Lagrangian given by \( L(q, \dot{q}) = K(q, \dot{q}) - V(q) \), where \( K(q, \dot{q}) \) represents the kinetic energy of the system, \( V(q) \) the potential energy, and \( B(q) u \) forms the vector of external and controlled forces and torques. For a hydraulically powered crane, velocity constraints show up naturally due to a maximum flow rate through the hydraulic system, while acceleration constraints are given through maximum producible forces and torques of the hydraulic actuators. Differential constraints are typically configuration dependent.

In this study we will only use configuration and velocity constraints as given in Table II. Procedures for computing reliable estimates of dynamical parameters as well as obtaining accurate quantitative description of external forces in (4) are currently under study. The range of allowable velocities has been obtained from experiments with the individual joints as follows. We have applied the minimum and maximum control input\(^1\) over some time intervals and have recorded the associated evolution of the joint angles, which are used to estimate angular velocities. Eventually, the velocity constraints are chosen as the most conservative value

\[
\text{TABLE II}
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>( q_i \text{ min} )</th>
<th>( q_i \text{ max} )</th>
<th>( q_i \text{ min} )</th>
<th>( q_i \text{ max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1 rad</td>
<td>0.5 rad</td>
<td>-0.4 rad/s</td>
<td>0.4 rad/s</td>
</tr>
<tr>
<td>2</td>
<td>-0.45 rad</td>
<td>1.37 rad</td>
<td>-0.16 rad/s</td>
<td>0.21 rad/s</td>
</tr>
<tr>
<td>3</td>
<td>-2.7 rad</td>
<td>-0.1 rad</td>
<td>-0.43 rad/s</td>
<td>0.39 rad/s</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.55 m</td>
<td>-0.67 m/s</td>
<td>0.5 m/s</td>
</tr>
</tbody>
</table>

\(^1\)The electro-hydraulic valve is controlled by direct current, which allows for actuation back and forth depending on the polarity.
not to be violated. It is important to note that in hydraulic actuators the velocity constraints are way more restrictive compared to standard electric motors, since they directly depend on the maximum flow of hydraulic oil.

III. MOTION PLANNING

A. Path Planning

At first we have to specify a path that the boom tip should describe over time. In case of a forwarder crane, one is generally interested to move from a given point $A$ to a target point $B$. There are numerous ways for connecting two points in the Cartesian space that yield feasible inverse kinematics (3) in the configuration space. However, in this study we are not concerned with the path-planning problem. Our goal is to compute a timing function along a predefined path subject to differential constraints.

Let us specify a desired path that we are going to work with from here on. This path describes a 2D circle in the $x$-$z$-plane of the base frame as depicted in Fig. 2. For simplicity we skip one dimension of the boom-tip movement by restricting the slewing angle to be zero, i.e. $q_1(t) = 0$. In particular, we define

$$p^0(\theta) = \begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix} = R \begin{bmatrix} \cos(\theta) \\ 0 \end{bmatrix} + \begin{bmatrix} x_C \\ 0 \end{bmatrix},$$

where $R = 0.7 \text{m}$, $[x_C, z_C] = [3, 2.5] \text{m}$ and $\theta \in [-\pi, \pi]$.

Even though, this path might be unusual in practice, it is very illustrative for the following analysis. Note that any other path could have been chosen connecting two points in the workspace and involving all or less degrees of freedom, if desired. Note that for the 2D circle we still have kinematic redundancy of the manipulator, since the motion lives in a 2D task space and the coordination is realized in a 3D configuration space.

![Fig. 2. Path of the boom tip describing a 2D circle in the x-z-plane of the base frame.](image)

B. Path-Constrained Trajectory Planning

In this section we illustrate the parameterization of motions along predefined paths without explicit dependence on time. The procedure is exemplified for the circular path (5).

At first, we define a new variable that describes the path as a function of the generalized coordinates. For instance, the arc length along the path would be one choice that naturally yields a monotonic evolution. In the case of the circular path (5), such a path coordinate is already introduced by the angular position

$$\theta = \arctan2(z - z_C, x - x_C)$$

associated with a point $p^0$ on the circle (see Fig. 2).

As the second step, all joint variables must be parameterized as functions of the new variable $\theta$ instead of time:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} := \Phi(\theta) = \begin{bmatrix} \phi_1(\theta) \\ \phi_2(\theta) \\ \phi_3(\theta) \\ \phi_4(\theta) \end{bmatrix},$$

where $\phi_1(\theta) = 0$ due to the choice of (5) and the other functions are obtained from (3). It can be interpreted as synchronization of all joints along the path clocked to an independent configuration variable. With such representation the explicit dependence of time disappears and, thus, $\dot{\theta}(t)$ can be viewed as a motion generator. Once a velocity profile for $\theta$ is chosen, all joint velocities are directly assigned by

$$\ddot{q} = \Phi'(\theta)\dot{\theta}.$$

It means that the nominal evolution of the full state space vector $[q, \dot{q}]^T$ is parameterized along the path, without even using the system dynamics (4). This approach is known as path-constrained trajectory planning [6], which is subject to velocity and acceleration constraints. In the context of control theory, the geometric function (7) is called a virtual holonomic constraint [9], if it is preserved by some control action along solutions of the closed-loop system.

Finally, we have to assign a velocity profile along the path taking into account that there are configuration-dependent differential constraints. In this study only velocity constraints (see Table II) shall be considered due to their dominating relevance in hydraulic actuators. It is straightforward to also account for acceleration constraints linked to the system dynamics (4) as soon as a reliable quantification of them is obtained.

Considering each link $i$ of the manipulator, we get the individual constraints for allowable $\dot{\theta}$ along the path using the equation for constrained joint velocities (8). It means that the phase-space of $\theta$-dynamics can be directly shaped such that the individual joint velocity constraints are not violated. Hence, a velocity profile along the path can be obtained as a solution of the differential equation relating $\dot{\theta}$ and $\ddot{\theta}$. The evolution of $\dot{\theta}$ with respect to time and thus the evolution of all joints in (7), is derived by integrating this differential equation. As a result, the whole motion of the manipulator along a specified path can be generated by following the simple numerical steps introduced above.
C. Path-Constrained Time-Efficient Trajectory

The relation between \( \theta \) and \( \dot{\theta} \) is instrumental and helps with assignments of velocity profiles along the same path such that we can optimize trajectories for execution time or other performance indices.

For our circular path (5) we are interested to achieve a rather small execution time of the motion. Given the fact that the manipulator is redundant allows for optimization of the virtual holonomic constraint (7). The following procedure is suggested:

- Parametrize the redundant joint variable \( q_4 := \phi_4(\theta) \) by some function, chosen to be a trigonometric polynomial of order \( M \)
  \[ \phi_4(\theta) = \phi_{40} + \sum_{i=1}^{M} (\phi_{4a,i} \cos(i\theta) + \phi_{4b,i} \sin(i\theta)) \]  

- Apply inverse kinematics (3) to compute the full vector function \( \Phi(\theta) \) along the given path.
- The optimal joint profile (9) is found for the polynomial coefficients \( x = [\phi_{40}, \phi_{4a,1}, \phi_{4b,1}] \) that maximize the area under the envelope function formed by the individual joint velocity constraints along the path using (8). A time-efficient trajectory is finally obtained by constructing a smooth curve in the \((\theta, \dot{\theta})\)-phase-plane close enough to the velocity constraints without violating them.

D. Example of Two Trajectories

Here, we show two examples of possible trajectories along the circular path (5) in order to illustrate the framework discussed in Sections III-B and III-C. The trigonometric function (9) is used for the redundant joint variable \( q_4 \) such that the joint profiles (7) along the path are well defined. In particular, the following periodic trajectories are chosen:

**A:** Trajectory with a constant angular velocity and disadvantageous joint profiles given by

\[ \phi_4(\theta) = 0.5 + 0.15 \cos(\theta). \]

**B:** Trajectory with time-efficient joint profiles obtained from the optimization procedure in the previous section as

\[ \phi_4(\theta) = 0.806 + 0.513 \cos(\theta) - 0.106 \sin(\theta). \]

The corresponding joint profiles—the individual virtual holonomic constraints \( \Phi(\theta) \)—are depicted in Fig. 3 versus each other. As indicated by Table II, it is advantageous to use the telescopic boom as much as possible for fast motions since the allowable velocities are much higher as compared to the inner boom. That is why, we can assign a high velocity profile for Trajectory B. The phase portraits for \( \theta \)-dynamics of both trajectories are shown in Fig. 4a indicating the significant difference of the individual velocity profiles along the path. For Trajectory A we wanted a constant angular velocity, which is found to be 0.3 rad/s at maximum due to velocity constraints of mainly the first link. Trajectory B, on the other hand, is shaped such that the potential of the individual joints gets exploited.

Given the velocity profiles along the path, we can also get the respective time evolution by integrating \( \theta \)-dynamics. In Fig. 4b the solutions are depicted; the execution time for Trajectory B is 3 times smaller than the one for Trajectory A. An illustrative summary of Fig. 3–4 is given in Fig. 5 showing the crane kinematics along the path for the two trajectories.

IV. TIME-INDEPENDENT CONTROL STRATEGY

Motion planning should be complimented with a feedback control design. We are interested here to derive a time-independent control strategy that achieves invariance of the preplanned trajectory \([\theta_s, \dot{\theta}_s]\) and contraction to it if the system is moved away by external forces. In particular, the virtual holonomic constraint, which is specified in the trajectory planning step, must be satisfied such that the vectorial error signal

\[ y(t) = \Phi(\theta(t)) - q(t) \]  

is shown in Fig. 3 versus each other.
goes to zero. Additionally, the configuration variable must evolve over time as preplanned, i.e. we need
\[
|\theta(t) - \theta_\star(t)| \to 0 \quad \text{as} \quad t \to \infty. \tag{11}
\]

It is not an option to generate a desired time reference for the individual joints by simply applying \(q_\star(t) = \Phi(\theta(t))\) to standard tracking controllers, since we want a strategy that is configuration dependent. It means that the reference shall be generated based on the current position of the boom tip projected on the path by the function (6). The equation to generate the reference \(\theta_\star(t + \Delta_s)\) for the next time-sampling step is then derived from an approximation of the desired velocity profile. In this way, the reference signal for the evolution along the path is not explicitly depending on time, however, it can be used with a standard tracking controller. It is also possible to include an estimated time delay into the algorithm.

In the next section we finally show experimental tests of the proposed control strategy implementing such an algorithm for a preplanned trajectory.

\section*{V. Experimental Results}

The experimental test are carried out with a real-time prototyping platform of the type dSPACE 1401 at a sampling time of \(\Delta_s = 0.001\) s. The utilized joint controllers consist of a two-loop MFC (Model-Following Control) structure \cite{7}, which is based on an identified nonlinear second order model of each link presented in \cite{5}. The nonlinear friction is compensated by a model-based addition to the control signal, whereas the plant controller is formed by a discretized PID structure with encoder measurements used for computation of the error signals.

Here, we show results for Trajectory B planned along the circular path illustrated in Section III-D. The reference signals to the individual joint controllers are generated as described in Section IV, fed through look-up tables containing the desired joint profiles \(\Phi(\theta)\). The measurement delay is identified to be \(\Delta_d = 0.26\) s.

In Fig. 6 the desired circular path is shown together with experimental results for the periodic trajectory, where the crane’s boom tip was initialized about 0.5 m away from the path. We see convergence to the path with acceptable error to it along the motion keeping in mind that the trajectory has a close to maximum velocity profile. In Fig. 7 the time evolution is shown corresponding to Fig. 6. Here, \(\theta\) is the projection of the boom tip onto the circular path using the function (6). Several turns are depicted, where the period of the first one is slightly longer due to the initialization away from the path. However, the real trajectories for each turn closely fit the planned one once the boom tip converged to the path. Let us also check that the specified joint profiles

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6}
\caption{Desired circular path for the boom tip and real experiment of the periodic trajectory shown for several turns.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7}
\caption{Time evolution of the boom tip along the path for several turns. The crane was initialized away from the path.}
\end{figure}

\(\Phi(\theta)\) are invariant along the motion. Fig. 8 shows that the individual joint controllers work sufficiently well such that equation (10) is indeed satisfied with a reasonable accuracy.

In Fig. 9 we finally show a comparison of a time-reference tracking scheme to the time-independent virtual-constraints-based control strategy. The 3D plot depicts the evolution along the path for the individual controllers, where both trajectories are initialized at the same point but the time reference is lagging behind at the start. It is clearly seen that the time-independent control strategy is advantageous because the distance to the path is diminished and at the
same time the desired evolution along the path is obtained. Since the time-reference tracking controller diminishes the error to a point evolving along the path, which is initially different to the boom-tip position, the execution time for one turn is much bigger compared to the desired one.

![Fig. 8](image1.png)  
**Fig. 8.** Invariance of joint profiles along the motion illustrated by reference \( \Phi (\theta(t)) \) (dashed line) versus real joint evolution \( q(t) \) (solid line).

![Fig. 9](image2.png)  
**Fig. 9.** Evolution along the path for a time-reference tracking scheme (dashed line) compared to the time-independent virtual constraints based control strategy (solid line). Here, the time reference is lagging behind at initialization.

## VI. Conclusions

In this paper we have considered the problem of trajectory planning and control redesign for a kinematically redundant manipulator, which is used on forestry machines for logging. We introduce an optimization procedure that takes advantage of the kinematic redundancy so that time-efficient joint and velocity profiles along the path can be obtained. Differential constraints imposed by the manipulator dynamics are accounted for by employing a phase-plane technique for admissible path timings. In this study we concentrated on the dominant velocity constraints which are apparent in hydraulic actuators. It is straightforward to also account for acceleration constraints.

We have illustrated the case of a circular Cartesian path and two trajectories are chosen as examples. The angular position of a point on the path is used as an independent configuration variable for parameterization of the whole motion in terms of a virtual holonomic constraint and a velocity profile of the new variable along the motion. Such a parameterization can be instrumental and helps with assignments of velocity profiles along the same path so that we can optimize trajectories for execution time or other performance indices. The explicit dependence on time disappeared, which allows for implementation of a time-independent control strategy, built upon standard tracking controllers, that achieves invariance of a preplanned trajectory and contraction to it if the system is moved away by external forces. Experimental tests demonstrate the feasibility of the proposed strategy.

Our study has been motivated by current needs of forestry industry. The following semi-autonomous scenario seems realizable in the nearest future: Fill the working space of the crane with numerous paths required for the process (say 20) and have an operator select a target point. An off-line/on-line motion planning strategy together with a designed feedback controller would allow moving the end effector in an optimal way (say as fast as possible) to the target point. Whenever its vicinity is reached, the driver can take over for the local grasping task.

## References


