Adaptive Finite Element Methods for Multi-physics Problems

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Akademisk avhandling

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Abstract
In this thesis we develop and evaluate the performance of adaptive finite element methods for multiphysics problems. In particular, we propose a methodology for deriving computable error estimates when solving unidirectionally coupled multiphysics problems using segregated finite element solvers. The error estimates are of a posteriori type and are derived using the standard framework of dual weighted residual estimates. The main feature of the methodology is its capability of automatically estimating the propagation of error between the involved solvers with respect to an overall computational goal. The a posteriori estimates are used to drive local mesh refinement, which concentrates the computational power to where it is needed the most. We have applied the methodology to several common multiphysics problems using various types of finite elements in both two and three spatial dimensions.

Multiphysics problems often involve convection-diffusion equations for which standard finite elements are known to be unstable. For such equations we formulate a robust discontinuous Galerkin method of optimal order with piecewise constant approximation. Sharp a priori and a posteriori error estimates are proved and verified numerically.

Fractional step methods are popular for simulating incompressible fluid flow. However, since these methods are based on operator splitting, rather than Galerkin projection, they do not fit into the standard framework for a posteriori error analysis. Here, we formally derive an a posteriori error estimate for a prototype fractional step method by separating the error in a quantity of interest into a finite element discretization residual, a time stepping residual, and an algebraic residual.

Key words
finite element methods, multiphysics, a posteriori error estimation, duality, adaptivity, discontinuous Galerkin, fractional step methods

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