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Magnetization of an anisotropic plasma by electromagnetic waves

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Abstract

It is shown that magnetic fields can be generated in a warm plasma by the nonstationary ponderomotive force of a large-amplitude electromagnetic wave. In the present Brief Report, we derive simple and explicit results that can be useful for understanding the origin of the magnetic fields that are produced in intense laser-plasma interaction experiments.

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Several mechanisms have previously been proposed for the generation of magnetic fields in plasmas, e.g. non-parallel density and temperature gradients [3], the electron temperature anisotropy (known as the Weibel instability [4]), counterstreaming electron beams [5], and the ponderomotive forces of laser beams [6–10]. Spontaneously generated magnetic fields are of great importance in laser produced plasmas [11–14], in our Universe [15, 16], in many cosmic environments [5], as well as in galactic and intergalactic spaces [17–20].

In laser produced plasma experiments [12] and in cosmic plasmas [19] the electrons are heated by lasers and electron beams, respectively. Consequently, there is an electron-temperature anisotropy. Our objective here is to consider the nonlinear interaction between a large amplitude electromagnetic wave and a hot plasma with electron temperature anisotropy. We then find that the nonstationary ponderomotive force of the electromagnetic wave creates slowly varying electric fields and vector potentials, which generate magnetic fields.

Let us consider the propagation of an electromagnetic wave, with the electric field $\mathbf{E}(\mathbf{r}, t) = (1/2)\hat{x}E_0(x,t)\exp(-i\omega t + ikz) + c.c.$, in an unmagnetized non-relativistic plasma with an electron temperature anisotropy $T_\perp/T_\parallel \neq 1$, where $T_\perp$ and $T_\parallel$ are the electron temperature perpendicular and parallel to $\hat{z}$, where $\hat{z}(\hat{x})$ is the unit vector along the $z(x)$ axis in a Cartesian coordinate system. The ions are immobile. Here, $\hat{x}E_0(x,t)$ is the envelope of the electromagnetic field at the position $x$ and time $t$, and $c.c.$ stands for complex conjugate. The frequency $\omega$ and the wave vector $\mathbf{k} = k\hat{z}$, are related by [21]

$$\frac{k^2c^2}{\omega^2} = N = 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \frac{T_\perp}{T_\parallel}W(\xi)\right],$$

where $c$ is the speed of light in vacuum, $N$ is the index of refraction, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, $n_0$ is the electron number density, $e$ is the magnitude of the electron charge, $m_e$ is the electron mass, $W(\xi) = -1 - \xi Z(\xi)$, $\xi = \omega/\sqrt{2kV_{T\parallel}}$, $V_{T\parallel} = (T_\parallel/m_e)^{1/2}$, and $Z(\xi)$ is the plasma dispersion function [22]. Equation (1) is simply obtained from the Maxwell equations and the linearized Vlasov equation, with an equilibrium anisotropic electron distribution function [21].

For $(T_\perp - T_\parallel)/T_\parallel \gg 1$ and $\xi \gg 1$, Eq. (1) gives

$$N \approx 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{k^2V_{T\perp}^2\omega_{pe}^2}{\omega^4},$$

We refer to the supplementary material for further details and calculations.
where \( V_{T\perp} = (T_{\perp}/m_e)^{1/2} \) is the electron thermal speed defined in terms of the perpendicular electron temperature.

The electromagnetic wave exerts a ponderomotive force \( \mathbf{F}_p = \mathbf{F}_{ps} + \mathbf{F}_{pt} \) on the plasma electrons, where the stationary and non-stationary ponderomotive forces \([6]\) are, respectively,

\[
\mathbf{F}_{ps} = \frac{(N - 1)}{16\pi} \nabla |E_0|^2, \tag{3}
\]

and

\[
\mathbf{F}_{pt} = \frac{1}{16\pi} \frac{k}{\omega^2} \frac{\partial [\omega^2 (N - 1)]}{\partial \omega} \frac{\partial |E_0|^2}{\partial t}. \tag{4}
\]

The ponderomotive force pushes the electrons locally, and creates the slowly varying electric field

\[
\mathbf{E}_s = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = \frac{1}{n_0 e} \mathbf{F}_p, \tag{5}
\]

where the scalar and vector potentials are, respectively,

\[
\phi = -\frac{(N - 1)}{16\pi n_0 e} |E_0|^2, \tag{6}
\]

and

\[
\mathbf{A} = -\frac{c}{16\pi n_0 e} \frac{k}{\omega^2} \frac{\partial [\omega^2 (N - 1)]}{\partial \omega} |E_0|^2. \tag{7}
\]

The induced slowly varying magnetic field \( \mathbf{B}_s \) is then \( \mathbf{B}_s = \nabla \times \mathbf{A} \). Noting that

\[
\frac{\partial [\omega^2 (N - 1)]}{\partial \omega} = \frac{2k^2 V_{T\perp}^2 \omega_{pe}}{\omega^3}, \tag{8}
\]

where \( N \) has been replaced by \((1b)\), we can express the magnitude of the magnetic field as

\[
|\mathbf{B}_s| = \frac{eck^3 V_{T\perp}^2 |E_0|^2}{2m_e L \omega^5}, \tag{9}
\]

where \( L \) is scale length of the envelope \( |E_0|^2 \). We note from (8) that the magnetic field strength is proportional to \( T_{\perp} \). The electron gyrofrequency \( \Omega_e \) is

\[
\Omega_e = \frac{e|\mathbf{B}_s|}{m_e c} = \frac{k^3 V_{T\perp}^2 V_0^2}{2L \omega^3}, \tag{10}
\]
where \( V_0 = e|E_0|/m_e \omega \) is the electron quiver velocity in the electromagnetic field. Equation (9) depicts an interesting scaling of \( \Gamma_c \) against the perpendicular electron temperature \( T_\perp \), the wave electric field squared \( |E_0|^2 \), the laser frequency \( \omega \), and the wave number \( k \), as well as the length scale of the laser envelope \( |E_0|^2 \).

We take some typical parameters that are representative of laser-plasma interaction experiments: \( n_0 \approx 10^{20} \text{cm}^{-3} \), \( T_\perp \approx 10 \text{ keV} \), the laser intensity \( I = 10^{20} \text{W/cm}^2 \), and the laser wavelength \( \lambda = 0.25 \mu \text{m} \). For these values, we have \( \omega = 8 \times 10^{15} \text{s}^{-1} \), \( V_\perp = 4.19 \times 10^9 \text{cm/s} \), and \( V_0 = 6c \times 10 \times 10^{-10} \lambda \sqrt{I} = 0.15c \). Hence, over the laser wave envelope length of 0.25\( \mu \text{m} \), the magnetic field strength, estimated from Eq. (9), turns out to be of the order of 60 MG.

To summarize, we have shown that magnetic fields in a plasma with an electron temperature anisotropy can be generated by the ponderomotive force of a large-amplitude electromagnetic wave. Specifically, the non-stationary ponderomotive force of the electromagnetic wave pushes the electrons locally and creates slowly varying electric fields and vector potentials. The latter, in turn, produce magnetic fields in an anisotropic electron plasma. The present results are useful for understanding the origin of magnetic fields in intense laser-solid-density plasma interaction experiments [10] and in cosmic plasmas [19].

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               (Addition Wesley, Reading, 1994).
[22] B. D. Fried and S. D. Conte, The Plasma Dispersion Function  