Vibro-acoustic analysis of a satellite reflector antenna using FEM

Author: Johannes Sikström*

Supervisors:
Per SJÖVALL†
Magnus BAUNGE‡

Examiner:
Martin BERGGREN§

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*E-mail: johannes8506@gmail.com
†E-mail: per.sjovall@afconsult.com
‡E-mail: magnus.baunge@ruag.com
§E-mail: martin.berggren@cs.umu.se
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I would also like to thank RUAG Space and ÅF Ingemansson, both located in Göteborg, for the resources I have been given access to during my thesis work. They have provided me with good workspaces and superior computational resources. RUAG Space was accommodating when an upgraded version of the software was required.

Apart from the technical goals of the thesis my personal aim include to gain experience from working with the main responsibility for a large project and to be introduced to the working environment of engineers.
Abstract
The acoustic environment generated during launch is the most demanding structural load case for large, lightweight satellite reflector antennas. The reflector is exposed to extremely high sound pressure levels originating from the structural excitation of the rocket engines and exterior air flow turbulence. This thesis aims to predict the structural responses in the reflector due to the acoustic pressure load with a model based on Finite Element Modelling (FEM). The FE-model is validated against a previously performed Boundary Element Method (BEM) analysis. An approach called Split Loading together with a combination of BEM and FEM will be utilized to handle the surrounding air mass and the applied sound pressures. The idea of Split Loading is to divide the structure into several patches and apply a unit pressure load to each patch separately. In the last step the unit pressure is scaled and correlated by a power spectral density calculated from the acoustic pressures. Split Loading will be implemented in software packages MSC.NASTRAN/PATRAN. The model developed in this thesis handles both the added mass of the surrounding air and the sound pressure applied to the reflector. The model can qualitatively well reproduce the results of the BEM-analysis and the test data. However, the model tends to overestimate responses at low frequencies and underestimate them at high frequencies. The end result is that the model becomes too conservative at low frequencies to be used without further development.

Sammanfattning
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1 Introduction

1.1 Background

RUAG Space is the largest independent supplier of space technology in Europe. The office in Göteborg, Sweden, specializes in highly reliable on-board satellite equipment that among other things include light-weight satellite reflector antennas. These antennas have state-of-the-art RF design combined with very low mass. The vibro-acoustic environment generated during launch, displayed in Figure 1, is usually the most demanding structural load case. The acoustic pressure load originates from the sound produced by the rocket engines and the exterior air flow turbulence. The antennas are stowed on a satellite, which in turn is stowed in the rocket, as shown in Figure 2. The dynamic interaction between a fluid and a structure is often of great importance when dealing with engineering problems. This concerns everything from offshore structures to aircrafts or as considered in this thesis, a satellite reflector antenna. For such a light-weight structure the fluid interaction significantly changes the dynamic behaviour of the structure. It is required to model these systems accurately while taking into consideration the fluid-structure interaction in a coupled structure-acoustic analysis.

![Launch of ESA rocket ARIANE 5, potentially loaded with satellite equipment provided by RUAG Space.](image)

The traditional method for performing vibro-acoustic analysis is by means of the Boundary Element Method (BEM). RUAG Space does currently not have access to special software packages required to use this method. This is making the cost of performing these analyses relatively high. However
recent development of Finite Element Modelling (FEM) in vibro-acoustic analyses has increased the possibilities of using FE softwares and combining FEM with other methods.

This thesis aims to study and develop an alternative method to perform the vibro-acoustic analysis based on Finite Element Modelling. The goal for the model is to be able to reproduce the data produced by the BEM-analysis [4].

Figure 2: Schematic view of ESA rocket ARIANE 5 at the launch pad. Notice the satellite stowed inside the payload fairing in the top.
1.2 Reflector antenna

The reflector antenna considered in this thesis is shown in Figure 3. The reflector is an Engineering Model (EM) which has been developed within an European Space Agency (ESA) financed study which is called "In-Flight Thermo-Elastic Stability Improvement of Carbon Reflectors". The reflector is made with the Simplified Technology for Advanced Adaptable Reflector (STAAR) concept.

A physical test has been performed on the reflector in the reverberation chamber of IABG’s Acoustic Facility in Germany. During the test the reflector was subdued to sound pressure levels equal to those created at launch. The sound field created at launch is called a diffuse sound field due to its random properties. The diffuse sound field is best described with statistical methods and it is further explained in section 2.2. The structural responses were measured with 34 accelerometers, in 16 different positions. The sound pressures levels where measured with 6 microphones placed around the reflector as shown in Figure 4. The reflector antenna FE-model used in NASTRAN throughout this thesis is presented in section 3.2.1 Model.
1.3 Problem specification

The problem considered in this thesis is how to create a finite element model that takes into account the vibro-acoustic (fluid-structure interaction) environment. The physical phenomena that need to be accounted for are:

- Surrounding air increasing the effective mass of the reflector hence increasing the damping.
- The acoustic pressures applied to the reflector in the form of a diffuse sound field.

The acoustic effects on structures can often be neglected for heavy and small structures. However, in the case considered in this thesis the structure has a large area ($\approx 5 \text{ m}^2$ on each side) and is very light-weight ($\approx 12 \text{ kg}$). Because of these dimensions acoustic pressures significantly affect the behaviour of the reflector.

The acoustic effects may be difficult to include in the FE-model, presented in section 3.2.1, since the model is large already. Extending it further to include the fluid-structure interaction could make it computationally heavy. Hence, it is vital that the computational time required for the calculation is kept as short as possible. The analysis will be performed with use of the software package MSC.NASTRAN/PATRAN.

The following questions will be a starting point for this thesis:

- Is it possible to create a computationally efficient model that take into account vibro-acoustic effects and the loading in terms of a diffuse acoustic field?
1.4 Aim

- What are the advantages and disadvantages of this model?
- Can this model be used to reproduce the results produced by RUAG Space’s BEM-analysis?
- Can this model reproduce the physical test data provided by RUAG Space?

1.5 Limitations

This thesis does not consider the effects of structure borne vibrations transferred via the attachment of the reflector. The reflector will be in the free-free configuration\(^1\), as opposed to the stowed configuration, in all calculations. Development of damping devices for this kind of reflectors has already been investigated by Löfgren and Simon-Bálint [15] on RUAG Space’s request.

This thesis does not consider the creation of the FE-model of the reflector. The reflector FE-structure is already developed and provided by RUAG Space.

1.6 Scope

This project is split into two parts. These parts will be presented respectively in two separate MSc Theses.

The first part consists of literature studies concerning vibro-acoustic modelling, the difference between BEM and FEM and the definition of a diffuse and direct acoustical field. It also aims to develop methods to achieve a diffuse field in a acoustic FE-model and simplified modelling of the antenna and the surrounding fluid. The first part was concluded by Hansson and is presented in [13].

The second part consist of literature studies of vibro-acoustic modelling, random analysis and its implementation. The work will be focused on creating a FE-model that can calculate the effects of an diffuse acoustic field on the reflector. The final step is to validate the model with reference to earlier

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\(^1\) See section 3.2.3 for more information on different boundary conditions.
simulations and physical test performed on the satellite reflector antenna. The work from the second part is presented in this thesis.

1.7 Outline

The outline of this thesis is as follows. In section 2, a comparison of the numerical methods available together with the theory utilized during this thesis work will be presented. In section 2, the diffuse sound field and split loading approach, which is the essential method used in the thesis, is also explained. Section 3 is dedicated to describing the practical construction of the model and the input data used. The last two sections 4 and 5 describe and discuss the results produced by the model and compare it to the already existing data.
2 Theory

In this section the fundamental theory utilized in this thesis will be presented. The method is based on random vibration theory and combines FEM and BEM to include the acoustic excitation caused by the diffuse sound field. Theory on random vibrations is outlined in Appendix A.

2.1 Available numerical methods - Overview

Differential equations are used to model all physical phenomenon encountered in engineering. Problems that are simple enough to be calculated analytically are rare and most problems need to be calculated with the aid of computers. The differential equations that describe the problem are assumed to hold over a certain region that may be one-, two- or three-dimensional. Numerical methods can help to give an approximate solution to general differential equations. In this section the two most interesting computational methods for vibro-acoustic calculations will be presented briefly.

2.1.1 The Boundary Element Method - BEM

Bies and Hansen [6] explains that the boundary element method (BEM) only involves discretizing the boundary of an enclosed space or a noise radiating structure. This method requires shorter computational time than for example FEM. This is because in FEM the entire enclosed volume in interior noise problems and a large space around the structure in noise radiating problems has to be discretized.

There are two main methods that can be used to evaluate an acoustic field: the direct method and the indirect method. The direct method is best suited for problems where interior or exterior domains are considered. If the domain of interest contains both interior and exterior domains, then the indirect method is a better choice. The indirect method is the least sophisticated method which in its simplest form starts with a solution that satisfies the governing equations in the domain but which has some unknown coefficients [8]. These coefficients are then determined by enforcing the boundary condition at a number of points or sub-regions. The direct method on the other hand is more versatile and general than indirect methods. The indirect method can be presented as a special case of the direct method (for further reading on BEM see for example [8]). It is possible to combine this method with other methods such as FEM. This is further discussed in section 2.1.3 Summary.
2.1 Available numerical methods - Overview

2.1.2 The Finite Element Method - FEM

Ottosen and Petersson [21] explains that a characteristic feature of the finite element method is that it does not seek approximations that hold over the entire region of interest directly. Instead the region is divided into smaller parts, finite elements, and the approximation is carried out over each element. The collection of all the element in the model is referred to as the finite element mesh, or simply the mesh [14].

The procedure of dividing the structure into elements makes it possible to go from, in principle, infinitely many unknowns (degrees of freedom - DOF) to a finite number of unknowns, i.e. the unknown values at the finite number of nodes. This system or mesh is called a discrete system in comparison with the original system that is continuous.

Some of the limitations for this model is that when the problem is large (i.e. contains many elements) the matrix calculations become very heavy. When considering FEM for use in acoustics it is important to take into consideration the ability to perform in different frequency ranges. A rule of thumb for the element size in acoustics is that the model should contain six elements per wavelength. It is easy to understand that if the frequency is high the number of elements needed to get an accurate solution is far too large to be computationally effective. For more details on basics of FEM and its theory, further reading of references [14], [21] and [26] is recommended.
2.1.3 Summary

If the frequency range of interest is high, with respect to the structure, there is a third method available called statistical energy analysis (SEA). SEA will not be of interest in this thesis since the frequency range of the analysis is too low for SEA. SEA is described in Appendix B.

FEM and BEM both aim to solve the same type of problem, however they have different strengths and weaknesses. A summary of BEM and FEM is presented in Table 1.

Table 1: Comparison of BEM and FEM.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>BEM</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknown</td>
<td>Displacement</td>
<td>Displacement</td>
</tr>
<tr>
<td>Frequency</td>
<td>Discrete, low</td>
<td>Discrete, low</td>
</tr>
<tr>
<td>Spatial detail</td>
<td>Discrete</td>
<td>Discrete</td>
</tr>
<tr>
<td>Excitation</td>
<td>Discrete</td>
<td>Discrete</td>
</tr>
<tr>
<td>Procedure</td>
<td>Intermediate, mostly for surfaces</td>
<td>Complicated, established</td>
</tr>
<tr>
<td>Model</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>Advantage</td>
<td>-Fast</td>
<td>-Accurate</td>
</tr>
<tr>
<td></td>
<td>-Simple pre-/post processing</td>
<td>-Non-linear problems easily solved</td>
</tr>
<tr>
<td></td>
<td>-Open regions not a problem</td>
<td>Complex systems allow various problems to be solved</td>
</tr>
<tr>
<td>Disadvantage</td>
<td>-Mostly applicable to surface/shell</td>
<td>-Computationally heavy</td>
</tr>
<tr>
<td></td>
<td>-Difficulties with non-linearity</td>
<td></td>
</tr>
</tbody>
</table>

Both FEM and BEM are interesting since they both have advantages suitable for this thesis. FEM can describe the complex system while BEM offers a computationally efficient way of including the surrounding fluid to the structure.

2.2 Diffuse sound field

The diffuse sound field is of great importance to this thesis since the sound field generated at launch is such a field. Due to the random properties of the sound field, with origin in the rocket engines and the exterior air flow...
2.2 Diffuse sound field

An ideal diffuse sound field is a sound field in which the time average of the mean-square pressure is the same everywhere and the flow of acoustic energy in all directions is equally probable [7]. As explained by Pierce [23], the diffuse sound field results from the idealization of the sound field as a superposition of a number of unique and freely propagating plane waves as exemplified in Figure 5.

![Figure 5: Superposition of travelling plane waves make up the diffuse sound field.](image)

The complex amplitude of the pressure, \( \hat{p} \), in the diffuse field in the constant frequency case is written as

\[
\hat{p} = \sum_q \hat{p}_q e^{i\bar{n}_q \cdot \bar{x}} \tag{1}
\]

where \( \bar{n} \) is the randomly distributed normal vector, subscript \( q \) is the index of plane waves and \( k = \frac{2\pi}{\lambda} \) is the wave number. The summation is over all plane waves considered to make up the diffuse field.

The random properties of the diffuse sound field is simulated in NASTRAN by utilizing the random analysis tool.
2.3 Acoustic FE-modelling in NASTRAN

In this section some of the theoretical aspects of the computer implementation will be presented. In sections 2.3.2 and 2.3.3 theory used by NASTRAN is presented.

2.3.1 Approach to fluid-structure interactions

As mentioned in section 1.3 there are two physical phenomenon to include in the model. The surrounding fluid (air) mass and the acoustic pressures applied to the structure. The fluid-structure problem at hand will be solved using MSC.NASTRAN and the approach is a mixture of FEM and BEM. The FE-model will not contain any fluid volume elements. Instead the effect of air mass will be added in NASTRAN through a BEM approach where fluid volume properties are added to the surface elements of the structure in order to generate a virtual mass matrix. In addition to the added effect of air mass the structure will be exposed to acoustic excitation in the form of a diffuse sound field. Diffuse sound fields are not deterministic, hence they can only be described with statistical methods. Due to the properties of diffuse sound fields the acoustic pressure excitation is handled through random analysis. The acoustic excitation will be included through an approach called split loading which utilizes the random analysis tool in PATRAN. Split loading is further explained in section 2.4 Split loading.

2.3.2 Virtual fluid mass

To add effective mass from the surrounding fluid to the reflector, the method of virtual fluid mass will be used since it is simple to implement in MSC.NASTRAN and well suited for this type of problem.

In Nastran 2005 Reference Manual [18], it is explained that a virtual fluid volume produces a mass matrix which represent the fluid coupled to a boundary of structural elements. The method used by NASTRAN solves Laplace’s equation by distributing a set of sources over the outer boundary. Each of these sources produces a simple solution to the differential equation. Then by matching the motion, assumed to be known, of the boundary to the effective motion caused by the sources, it is possible to solve a linear matrix equation for the magnitude of the sources. The values of the sources in turn determine the effective pressures and hence also the forces on the grid points. How to combine this into a matrix equation is presented below.

Let \( \sigma_j \) be the value of a point source of the fluid (in units of volume flow rate per area) located at \( r_j \). Assume that the source is acting over an area \( A_j \) and that \( e_{ij} \) is the unit vector in the direction from point \( j \) to point \( i \). The reference manual [18] shows that using these assumptions the vector
velocity at any other point can be written as

\[ \dot{u}_i = \sum_j \int_{A_j} \frac{\sigma_j e_{ij}}{|r_i - r_j|^2} dA_j \] (2)

The pressures, \( p_i \), at any point, \( r_i \), is another set of necessary equations. They are described by density, \( \rho \), sources and geometry and becomes

\[ p_i = \sum_j \int_{A_j} \frac{\rho \dot{\sigma}_{ij} e_{ij}}{|r_i - r_j|} dA_j \] (3)

Collecting the results obtained when integrating equations (2) and (3) over the finite element surfaces yields two matrices, \([\chi]\) and \([\Sigma]\) where

\[ \ddot{\bar{u}} = [\chi] [\dot{\sigma}] \] (4)

and

\[ \bar{F} = [\Sigma] \ddot{\sigma} \] (5)

where \( \bar{F} \) are the forces at the grid points in vector form. The NASTRAN Manual [18] goes on to explain that \([\Sigma]\) is obtained by integrating equation (3) and one additional integration is necessary to convert pressures to forces. By using equation (4) and (5) a mass matrix can be defined as

\[ \bar{F} = [M_f] \ddot{\bar{u}} \] (6)

where the virtual fluid mass matrix is

\[ [M_f] = [\Sigma] [\chi]^{-1} \] (7)

This is implemented in NASTRAN through use of the MFLUID card. See references [16], [17] and [18] and their references for more information and detailed theory.

### 2.3.3 Frequency response analysis

Frequency response analysis is a method used to compute structural response to steady-state oscillatory excitation [18]. In this type of analysis the excitation is explicitly defined in the frequency domain and all of the applied forces are known at each forcing frequency. Forces in this case can be applied forces or enforced motions such as displacement, velocities or accelerations. There are two primary methods available for solving this type of problems, the direct method and the modal method. The modal method is a simplification of the direct method which solves the coupled equations of motion in terms of forcing frequency (for more theory on the direct method see [18]).
The modal method, which will be used in this thesis, utilizes mode shapes of the structure considered in order to reduce and uncouple the equations of motion. The solution, for a specific forcing frequency, of the modal method is created through summation of the individual modal responses. The formulation of this method is presented below.

If the damping is ignored, temporarily, we have the undamped equation for harmonic motion [18]

\[-\omega^2 \begin{bmatrix} M \\ K \end{bmatrix} \bar{x} + \begin{bmatrix} K \end{bmatrix} \bar{x} = \bar{P} (\omega)\]  

(8)

at a forcing frequency \( \omega \). \([M]\) and \([K]\) are the mass and stiffness matrices respectively. In order to utilize the modal method the variables are transformed from physical coordinates \( \bar{x} \) to modal coordinates \( \bar{\xi}(\omega) \) by assuming

\[ \bar{x} = [\phi] \bar{\xi}(\omega) \]  

(9)

where \([\phi]\) are the mode shapes which are used to transform the problem from terms of grid point behaviour to mode shape behaviour. Inserting Equation (9) into equation (8) gives

\[-\omega^2 \begin{bmatrix} M \\ K \end{bmatrix} [\phi] \bar{\xi}(\omega) + \begin{bmatrix} K \end{bmatrix} [\phi] \bar{\xi}(\omega) = \bar{P} (\omega)\]  

(10)

Equation (10) is the equation of motion in terms of the modal coordinates. In order to uncouple these equations premultiply by \([\phi]^T\) to obtain


(11)

where now \([\phi]^T [M] [\phi]\) is the modal generalized mass matrix, \([\phi]^T [K] [\phi]\) is the modal generalized stiffness matrix and \([\phi]^T \bar{P} (\omega)\) is the modal force vector. Using the orthogonality property, described by

\[ [\phi_i]^T [M] [\phi_j] = 0 \text{ if } i \neq j \]
\[ [\phi_i]^T [K] [\phi_j] = 0 \text{ if } i \neq j, \]  

(12)

of the mode shapes to formulate the equation of motion in terms of the generalized mass and stiffness matrices is the final step. The generalized matrices are diagonal matrices which means that the equations of motions are uncoupled. In this uncoupled form the equation of motion can be written as

\[-\omega^2 m_i \xi_i(\omega) + k_i \xi(\omega) = p_i(\omega)\]  

(13)

where \(m_i\) is the \(i\):th modal mass, \(k_i\) is the \(i\):th modal stiffness and \(p_i\) is the \(i\):th modal force. Since the equations of motions in this case is uncoupled this method is much faster than the alternative direct method. To recover the
physical responses a summation of the individual modal responses, $\xi_j(\omega)$, is used together with equation (9). The key feature of this analysis type is that it makes it possible to discard modes from frequency ranges of no interest. Equation (13) is yet without damping. The Nastran Reference Manual [18] states that when using modal damping each mode has the damping $b_i = 2m_i\omega_i\zeta_i$. The equation can then, still in the uncoupled form, be written as

$$-\omega^2 m_i \xi_i(\omega) + i\omega b_i \xi_i(\omega) + k_i \xi_i(\omega) = p_i(\omega)$$

for each mode. The modal responses are then computed using

$$\xi_i(\omega) = \frac{p_i(\omega)}{-\omega^2 m_i + i\omega b_i + k_i}$$

The frequency response analysis will be performed in the frequency range 20-300 Hz, however the modal analysis will be performed in the frequency range 0-400 Hz since mode shapes outside the frequency range of interest also affect the results.

2.3.4 Random analysis in NASTRAN

The approach used to simulate the effects of the diffuse acoustic field involves using NASTRAN to apply a random load to the antenna. This is possible since the diffuse acoustic field may be applied in the form of random loading through a PSD. It is done as a post-processing step in a frequency response analysis.

In the frequency response analysis a transfer function is generated. The transfer function represent the ratio between input and output. In order to form a response PSD the transfer function is multiplied with the input PSD. This procedure is explained in section 3 Implementation.
2.4 Split loading

In this section the main idea and theory of a numerical method that handles vibro-acoustics will be presented. The method is called split loading and it is based on FEM.

2.4.1 Introduction

Some factors that influence the structural responses to acoustic loading are fluid mass, damping, diffraction and spatial correlation. Note that when considering acoustic loads these effects can be neglected for relatively heavy structures with small exposed surfaces. However, in the case considered in this thesis the structure is a large, light-weight reflector antenna in which case the effects of acoustic pressures are significant. The approach of split loading handles the factors that can influence the vibro-acoustic behaviour without becoming computationally heavy.

Barrett [1] describes that the split loading method splits the loading on the structure to account for the excitation of the non-symmetric structural modes. In order to represent an acoustic excitation the simplest way is to apply a frequency dependent, but spatially uniform random pressure to the structure surface.

2.4.2 Split loading - Overview

The method used in this thesis is here broken down into six principal steps to increase the readers understanding.

1. Perform a modal analysis, including added air mass, of the entire reflector.
2. Split the reflector into smaller sections, called patches.
3. Apply a uniform unit pressure to the patches separately.
4. Calculate the transfer functions for each patch.
5. Scale the unit pressures applied to the patches with a pressure power spectral density\(^2\) matrix corresponding to the sound pressure levels\(^3\) applied to the reflector. In this step the correlations between the patches are also included. Finally multiply the result with the transfer function.
6. Extract the responses, at desired nodes and directions, from the analysis.

\(^2\)The power spectral density correspond to the power carried by a signal per Hz. For further details see appendix A.3.

\(^3\)Sound pressure levels are explained in section 2.4.3
2.4.3 Conversion of SPL to PSD

The acoustic load studied in this thesis is specified as sound pressure levels (SPL) for full octave bands. Bies and Hansen [6] states that an octave band is a frequency range where the upper limit is approximately twice the lower frequency limit. Octave bands are identified by their respective geometric mean called the band center frequency, $f_c$. If the full octave band should offer to low resolution the one-third octave band (1/3 octave band) analysis may be used instead. Frequency bands often used for acoustic measurements and calculations are shown in Table 2.

Table 2: Preferred frequency bands.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Octave bands, $f_c$ [Hz]</th>
<th>Band limits [Hz]</th>
<th>Lower</th>
<th>Upper</th>
</tr>
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<tr>
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<td></td>
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<td>80</td>
<td></td>
<td>71</td>
<td>88</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td></td>
<td>88</td>
<td>113</td>
</tr>
<tr>
<td>21</td>
<td>125</td>
<td>125</td>
<td>113</td>
<td>141</td>
</tr>
<tr>
<td>22</td>
<td>160</td>
<td></td>
<td>141</td>
<td>176</td>
</tr>
<tr>
<td>23</td>
<td>200</td>
<td></td>
<td>176</td>
<td>225</td>
</tr>
<tr>
<td>24</td>
<td>250</td>
<td>250</td>
<td>225</td>
<td>283</td>
</tr>
<tr>
<td>25</td>
<td>315</td>
<td></td>
<td>283</td>
<td>353</td>
</tr>
<tr>
<td>26</td>
<td>400</td>
<td></td>
<td>353</td>
<td>440</td>
</tr>
<tr>
<td>27</td>
<td>500</td>
<td>500</td>
<td>440</td>
<td>565</td>
</tr>
<tr>
<td>28</td>
<td>630</td>
<td></td>
<td>565</td>
<td>707</td>
</tr>
<tr>
<td>29</td>
<td>800</td>
<td></td>
<td>707</td>
<td>880</td>
</tr>
<tr>
<td>30</td>
<td>1000</td>
<td>1000</td>
<td>880</td>
<td>1130</td>
</tr>
</tbody>
</table>

The reflector is exposed to sound pressure levels in the six first full octave bands 31.5-1000 Hz. These octave bands have center frequencies equal to 31.5, 63, 125, 250, 500 and 1000 Hz. Bies and Hansen [6] states that the level of sound pressure, $p$, is $L_p$ decibels (dB) greater than or less than a reference sound pressure, $p_{ref}$, according to

$$L_p = 10 \log_{10} \frac{p^2}{p_{ref}^2} = 10 \log_{10}(p^2) - \log_{10} p_{ref}^2 \text{ (dB)}.$$  \hspace{1cm} (16)

This is useful when comparing sound pressures, however for the purpose of absolute level determination, the sound pressure is determined in comparison...
to the lowest sound pressure which a young ear can hear. This corresponds to 20 µPa i.e. $p_{\text{ref}} = 2 \times 10^{-5}$ N/m$^2$. If this value is substituted into equation (16) the result is called the sound pressure level (SPL) and takes the form

$$L_p = 10 \log_{10}(p^2) + 94 \text{ (dB)}.$$  

The source of the SPL is the diffuse acoustic field. In order to compare the SPL in the acoustic field with the structural responses which is typically specified as a Power Spectral Density (PSD) it is necessary to convert the SPL into a pressure PSD. Barrett [1] states that by assuming that the PSD level is constant over each octave band the PSD is given by

$$\text{PSD} = \frac{p_{\text{ref}}^2 10^{\frac{\text{SPL}}{10}}}{\Delta f}$$

where PSD and SPL is given for each octave band, $p_{\text{ref}} = 2 \times 10^{-5}$ [Pa] is a reference pressure and $\Delta f$ is the bandwidth of the corresponding octave band.

### 2.4.4 Spatial correlations

Loading of one patch will affect the other patches as well, hence the spatial correlations of the acoustic pressure field is essential to accurately couple the acoustic pressures with the structural modes. This is done by choosing uncorrelated patches which makes different normal modes excited based on the size and location of the specific patch. The structure is divided into patches with sizes based on the wavelength of the acoustic pressure at the frequency ranges of interest to the analysis. Basically the loading of the structure is simulated by separately applying multiple different load cases. The results from these cases are correlated when the random responses are calculated.

Correlation can influence the applied acoustic loading by as much as a factor of 2. The correlations applied in this project depends on the frequency of the sound pressure and the distance between the patches. The spatial correlation corresponds to

$$f_s(r_{ij}, \omega) = \frac{\sin |k||r_{ij}|}{|k||r_{ij}|}$$

where $r_{ij}$ is the distance between the center of patch $i$ and patch $j$ and $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$. Since both $r_{ij} \geq 0$ and $k \geq 0$ the absolute signs may be omitted. The spatial correlation function is displayed in Figure 6 and in Appendix A.7 the spatial correlation equation is derived.
2.4 Split loading  

To get the actual correlation PSD the spatial correlation function needs to be multiplied with a frequency dependent reference PSD which gives

\[ S(r_{ij}, \omega) = S_{ref}(\omega) f_s(r_{ij}, \omega) = S_{ref}(\omega) \frac{\sin kr}{kr} \]

(20)

where \( S(r_{ij}, \omega) \) is the correlation and \( S_{ref}(\omega) \) is a frequency dependent PSD i.e. the input PSD calculated from the sound pressure levels. The complete PSD input matrix then becomes

\[
[S_r(\omega)] = S_{ref}(\omega) \begin{bmatrix}
1.0 & f_s(r_{1j}, \omega) & \cdots & \cdots \\
& 1.0 & f_s(r_{ij}, \omega) & \vdots \\
& & \ddots & \vdots \\
& & & 1.0 \\
\end{bmatrix}
\]

(21)

where the ones in the diagonal correspond to the input PSD and the off-diagonal terms correspond to the spatial correlations calculated with equation (20). The matrix in equation (21) is symmetric. Wirsching, Paez and Ortiz [25] state that in order to extract the responses this matrix is multiplied with the transfer function matrix, where each term correspond to a transfer function, according to

\[
[S_X(\omega)] = [H(\omega)] [S_F(\omega)] [H^*(\omega)]^T
\]

(22)

Equation 22 is derived in Appendix A.6. The implementation in NASTRAN is described in section 3 Methodology.

It is important to note that the diffuse field specification considers an acoustic field that is undisturbed by the presence of any structure. Placing a struc-
ture in the field causes the sound waves to diffract around that structure making the pressure applied to the structure increase with approximately 3 dB [1]. This is simulated in the model by increasing the input PSD with a factor of 2 (see equation (18)). It is assumed that the diffraction factor is the same at all frequencies. Possible effects of this assumption are discussed in section 5.

2.4.5 Random analysis in Split loading

The random analysis of split loading can be summarized into two main steps:

1. Transfer function calculation - This step defines a fundamental behavior of the FE-model. It is calculated through a frequency response solution with a unit load applied.

2. Random response calculation - This step applies a random loading to the fundamental behavior of the FE-model. The random input is in the form of a power spectral density.

In the case of split loading each patch is utilized in an individual load case and have a weighted transfer function calculated. These transfer functions are then combined and correlated in the random portion of the analysis. Both of the described steps are performed by NASTRAN.

2.4.6 Model considerations

As mention in section 2.1.2 a good rule of thumb is to have six elements per wavelength at the maximum frequency considered in frequency response analysis. At 300 Hz this would make the maximum element size

\[
\text{Element Size} \leq \frac{\lambda}{6} = \frac{c}{6f} = \frac{343}{6 \times 300} \approx 0.2 \text{ m}
\]

(23)

But since the element size of the model considered in the thesis is smaller than 5 cm this will cause no problem.

The size of the patches are based on the half wavelength of the acoustic pressure waves at a certain frequency. This gives

\[
\text{Patch Size} \approx \frac{\lambda}{2} = \frac{c}{2f}
\]

(24)

With this as a starting point it is possible to create appropriate patches to the structure of interest. This is described in section 3.3.1 Patches.
3  Methodology

3.1  Computational aspects

In this section some of the main computational aspects of the implementation will be discussed.

3.1.1  Software versions

The reason for this section is that when using computer aided engineering (CAE) software the version have great impact on the results produced. The software versions used4 for the computations in the thesis is displayed in Table 3.

<table>
<thead>
<tr>
<th>Software</th>
<th>Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSC.PATRAN</td>
<td>v2007r2</td>
</tr>
<tr>
<td>MSC.NASTRAN</td>
<td>v2008r1a</td>
</tr>
<tr>
<td>MATLAB</td>
<td>R2010a</td>
</tr>
</tbody>
</table>

When dealing with dynamic problems it is essential to be aware that different versions of the same software can give the studied model specific behaviour. This is the case for NASTRAN when using the MFLUID card to include the added air mass effect. When MFLUID is utilized the peak in structural responses (acceleration, displacement etc.) of a frequency response analysis is supposed to coincide with the eigenfrequency for the reflector with added air mass. NASTRAN v2005r2 was initially used in this thesis and even when using MFLUID NASTRAN v2005r2 produces peaks in the structural response at the eigenfrequencies without added air mass. This is of course a big problem since it gives non-physical behaviour when adding the air mass, which is a important part of the analysis performed in this thesis. The solution to this problem was simply to upgrade to NASTRAN v2008r1a which handles MFLUID exactly the way it is supposed to.

3.1.2  Computational time

The computational time is always an important factor when dealing with computer aided engineering. There is often a compromise between speed and accuracy. The FE-model of the reflector was already created, by RUAG Space, at the start of the thesis so the concern has been with the actual method. The time used for the frequency response analysis and to retrieve the PSD responses is comparatively small. The most time consuming part of the analysis is the eigenvalue extraction.

4Other versions may have been used for simple calculations, training and basic plotting,
3.2 Vibro-acoustic analysis model

In the analyses performed in this thesis the MFLUID card is present. This makes the eigenvalue extraction take significantly more time since the air mass is included in the mass matrix as a virtual mass.

In this thesis there are several different frequency response analyses (32 patches, 42 patches, different resolutions etc.) that needs to be performed. It would be very ineffective if the time consuming eigenvalue extraction had to be run prior to every frequency response analysis. In NASTRAN there is a restart-command which can be used to avoid this problem. The restart command enables reuse of the solution to the computationally heavy modal analysis in a simple way. This means that the modal analysis only needs to be run once and that the different frequency response analyses all can reuse the modal solution.

3.2 Vibro-acoustic analysis model

In this section the specific implementation of the split loading approach in NASTRAN will be described.

3.2.1 Model

The NASTRAN FE-model of the reflector is provided by RUAG Space and summarized in Tables 4 and 5.

Table 4: FE-model summary, see [18] for information on the different elements.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Points</td>
<td>21552</td>
</tr>
<tr>
<td>CBUSH Elements</td>
<td>4</td>
</tr>
<tr>
<td>CHEXA Elements</td>
<td>11202</td>
</tr>
<tr>
<td>CONM2 Elements</td>
<td>7</td>
</tr>
<tr>
<td>CQUAD4 Elements</td>
<td>15196</td>
</tr>
<tr>
<td>RBE2 Elements</td>
<td>2420</td>
</tr>
</tbody>
</table>

Table 5: FE-model element types.

<table>
<thead>
<tr>
<th>Element name</th>
<th>Element type</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBUSH</td>
<td>Spring</td>
</tr>
<tr>
<td>CHEXA</td>
<td>Solid</td>
</tr>
<tr>
<td>CONM2</td>
<td>Point mass</td>
</tr>
<tr>
<td>CQUAD4</td>
<td>Shell</td>
</tr>
<tr>
<td>RBE2</td>
<td>Rigid body</td>
</tr>
</tbody>
</table>
3.2 Vibro-acoustic analysis model

The reflector FE-model is shown from the front in Figure 7 and from the back in Figure 8. The total mass of the antenna is 11.72 kilograms and it has a diameter of 2.4 meters. In [3] the reflector is described to consist of a carbon fiber reinforced polymer (CFRP) sandwich reflector supported by an all CFRP support structure (ribs on the backside of the reflector). The honeycomb core height is 6 mm and the support structure has a total height of 60 mm. The reflector contains up to 7 layered laminates.

![Figure 7: The front of the antenna structure used in NASTRAN.](image)

3.2.2 Interesting nodes

The nodes of interest for the analysis are taken from [2]. The nodes used in this thesis will be the same as the nodes used in the BEM-analysis i.e. the sensor positions used in the physical test [9]. Thereby, it is possible to correlate the model with available test and analysis data. The nodes are presented in Figure 8 and their numbers are 1-14, 41 and 61 giving a total of 16 nodes to consider in the analysis.

In the physical test the highest acceleration was found in the sensor position corresponding to node 1. This makes node 1 of high interest to the analysis. Node 1 will represent nodes close to the edge while node 5 and 8 will be used for representing results closer to the center of the reflector.

3.2.3 Boundary conditions

There are two different boundary conditions for the reflector:

- Free-free
- Stowed configuration
3.3 Implementation of Split loading

3.3.1 Patches

The number and sizes of the patches are based on the frequencies of interest to the vibro-acoustic analysis of the reflector. The frequency range of interest is 20 - 300 Hz. The test described in [9] is performed in the frequency range 20 Hz - 2.5 kHz. However the FE-model constructed in this thesis will not consider frequencies above 300 Hz since it is mainly in low frequency modes that the reflector is affected, and potentially damaged, by the acoustic pressure load. Two different patch choices will be studied.

Case 1 32 patches of the same size is used for all frequencies. (Typical dimension 0.6 m).

Case 2 42 patches of different sizes are used in different frequency ranges. (Typical dimensions 0.6, 1.2 and 2.4 m).

The free-free condition is, as the name indicates, the case when the reflector is without fixation. In the case when the antenna is stowed on the satellite the boundary condition is called stowed configuration. In this case the reflector model is fixed with the spring elements (CBUSH).

In this thesis only the free-free boundary condition will be studied further.
With this in mind the structure will initially be split into 42 different patches. These 42 patches can be sub categorized into 3 sets according to

**Subset 1** Obtained through splitting each side (front and back) into 16 patches each. Typical dimension of the patches is 0.6 m.

**Subset 2** Obtained through splitting each side into 4 patches each. Typical dimension is 1.2 m.

**Subset 3** Simply the entire front and the entire back. Typical dimension is 2.4 m.

The first and second sets of patches are displayed in Figure 9. The jagged edges of the patches is due to the fact that the patches are made up of the QUAD4 elements which do not always align at the edge of the patches.

![Figure 9: Example of different patches used in the analysis.](image)

In **Case 1** the patches of **Subset 1** will be used and in **Case 2** all three subsets will be used.

The patches on the front of the reflector are made up by elements on the front of the reflector, obviously. But for the patches on the back side of the reflector there are two different ways to create the patches due to the support structure. The elements either under or on the back of the support structure can be chosen. In this thesis the elements on the back of the support structure have been chosen. The loading utilizing this choice of elements is displayed in Figure 10.

Referencing equation (24) and the fact that the reflector has a diameter of 2.4 meters gives a way to calculate the frequency range for the different
3.3 Implementation of Split loading

Figure 10: Unit loading of part of a patch on the back of the reflector.

patches. An example of calculating the maximum frequency for patches 33 and 34 (see Table 6) using equation (24) is

\[ f_{\text{max}} \approx \frac{c}{2 \times \text{Typical patch dimension}} = \frac{343}{2 \times 2.4} \approx 70 \text{ [Hz]} \]  

(25)

Repeating this calculation for the other patches as well gives the result presented in Table 6.

Table 6: Patches and corresponding frequency ranges.

<table>
<thead>
<tr>
<th>Patch number</th>
<th>Typical patch dimension [m]</th>
<th>Front (F) or Back (B)</th>
<th>Frequency range [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>2.4</td>
<td>F</td>
<td>20 - 70</td>
</tr>
<tr>
<td>34</td>
<td>2.4</td>
<td>B</td>
<td>20 - 70</td>
</tr>
<tr>
<td>35 - 38</td>
<td>1.2</td>
<td>B</td>
<td>70 - 140</td>
</tr>
<tr>
<td>39 - 42</td>
<td>1.2</td>
<td>F</td>
<td>70 - 140</td>
</tr>
<tr>
<td>1 - 16</td>
<td>0.6</td>
<td>B</td>
<td>140 - 300</td>
</tr>
<tr>
<td>17 - 32</td>
<td>0.6</td>
<td>F</td>
<td>140 - 300</td>
</tr>
</tbody>
</table>

The data in Table 6 will be utilized for Case 2 i.e. for 42 patches. The power spectral density multiplied with the transfer function will be scaled to zero outside of the patch specific frequency range. For example this means that the PSD applied to patch 35-42 will be zero except for when the frequency, \(f\), is in the range \(70 < f < 140 \text{ Hz}\). This is explained in further detail in section 3.4 Input data.

The goal when creating the patches should always be to make them equal in size. This is simple if the structure is rectangular but if it is circular, like the
reflector considered in this thesis, it is difficult. This leads to different sizes of the patches exemplified in Figure 11 which shows that patch 7 consist of more than twice as many elements as patch 1.

![Figure 11: Elements in patches 1 and 7.](image)

(a) Patch 1 consist of 165 elements.  
(b) Patch 7 consist of 424 elements.

However, the difference in size should not strongly effect the results.

### 3.3.2 Graphical user interface - PATRAN

Since the ordering of the elements of the structure is complex and not always intuitive it is hard to make the process of creating patches automated. In this thesis the pre-/post processing software MSC.PATRAN is used to create the patches and apply the loads. This is a graphical approach where the user manually selects elements for each patch. Even though it is a manual method the guide lines presented in this section makes it reasonably effective. The method is described step by step below. The first part is to create the patches:

1. Decide the size and location of the patches according to section 3.3.1.

2. Create a group containing all elements on the front of the reflector and one containing all elements on the back of the reflector i.e. the elements that the uniform unit pressure should be applied to.

3. Create geometric lines next to the structure to help when picking elements. It is not recommended to create the lines to close to the structure. Since the plane view is used it does not matter that the lines are not close. See Figure 12.

4. Post the groups, one at the time, containing all elements on either side of the structure.
5. Start with the smallest patches of the model. This makes it easy to create larger patches later on by combining several smaller patches.

6. Use the plane view to see the structure from the side. (See Figure 12 (b).)

7. Use the geometric lines to mark the elements to include in each patch. Make these elements a group with the desired patch name.

8. To create larger patches: Post the smaller patches (groups of elements) to include in the larger patch and then create a new patch with the posted elements.

9. When all elements are grouped into patches: Check your patches so that no elements exist in more than one patch.

![Figures](a) ISO view of the lines created on both sides to help create patches of desired size. (b) View from the side of the lines created on both sides to help create patches of desired size.

**Figure 12:** Visualization of the geometric lines created in PATRAN to help create the patches.

After the patches have been created the unit loads, considered during the frequency response analysis, should be applied as separate load cases. This is done for each patch according to:

1. Post the current patch (group of elements).

2. Use the ‘Load/BC’ menu to apply an element uniform unit pressure (PLOAD card) to each element in the patch. This is easiest done by simply marking the entire patch. Check orientation of the elements! The load applied, for example, to patch number 22 is displayed in Figure 13.
3. Connect this load to a load case under the 'Load Case' menu.

Now the model is ready to be used. Simply allow PATRAN to create the first draft for the NASTRAN input file. When the file is created the user can manually add things to the input file. This needs to be done to add the MFLUID (virtual fluid mass) card.

For more information on using PATRAN see the Combined Documentation 2005 [19].

3.4 Input data

3.4.1 Applied PSD

Sound pressure levels for each octave band are presented in Table 7. The SPL needs to be converted to a PSD before it can be applied to the structure. The PSD obtained through use of equation (18) is also presented in Table 7.

Table 7: SPL applied to the structure in the form of a PSD. PSD values are here multiplied by 2 to account for acoustic diffraction.

<table>
<thead>
<tr>
<th>Frequency band, $f_c$ [Hz]</th>
<th>Analysis SPL [dB]</th>
<th>PSD $\frac{\text{Pa}^2}{\text{Hz}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.5</td>
<td>132.4</td>
<td>623.4</td>
</tr>
<tr>
<td>63</td>
<td>137.9</td>
<td>1110.9</td>
</tr>
<tr>
<td>125</td>
<td>140.6</td>
<td>1033.2</td>
</tr>
<tr>
<td>250</td>
<td>142.3</td>
<td>767.6</td>
</tr>
<tr>
<td>500</td>
<td>138.8</td>
<td>171.9</td>
</tr>
<tr>
<td>1000</td>
<td>136.6</td>
<td>52.1</td>
</tr>
</tbody>
</table>

The PSD in Table 7 is the frequency dependent reference PSD that is multiplied with the correlation matrix presented in equation (21).
When 32 patches of approximately equal size are used in the analysis the PSD is applied directly over the entire frequency range (20-300 Hz). When 42 patches of 3 different sizes are used the PSD can not be applied without thinking about the different frequency ranges for the different patch sizes. This means that the PSD that is applied to a specific patch needs to be scaled to zero outside of the frequency range of that patch size. Reference Table 6 on page 30 for information on patches and corresponding frequency ranges.

The PSD is applied in NASTRAN via the TABRND1 and RANDPS cards. In Appendix C a simple example of how to implement RANDPS and TABRND1 is presented. The reasoning above gives us two different sets of TABRND1 entries to implement in NASTRAN. When 32 patches of the same typical dimension are utilized the TABRND1 entry in the NASTRAN input file becomes

\begin{verbatim}
$Input PSD (diagonal) 20-300 Hz
TABRND,100022,LOG,LINEAR, , , ,
+ ,31.5,623.42,63.1110.98,125.,1033.21,250.,767.57,
+ ,500.,171.91,1000.,52.08,ENDT
\end{verbatim}

where 100022 is the tableID. The strings LOG and LINEAR specify logarithmic or linear interpolation for the X- and Y-axis. The reason for using linear interpolation on the Y axis is that some of the spatial correlations are negative which creates problem if logarithmic interpolation is used. When 42 patches are used the TABRND1 entries becomes

\begin{verbatim}
$PSD in frequency range 20-60 Hz
TABRND,100020,LOG,LINEAR, , , ,
+ ,31.5,623.42,63.1110.98,125.,1.e-20,250.,1.e-20,
+ ,500.,1.e-20,1000.,1.e-20,ENDT

$PSD in frequency range 60-140 Hz
TABRND,100021,LOG,LINEAR, , , ,
+ ,31.5,1.e-20,63.1110.98,125.,1033.21,250.,1.e-20,
+ ,500.,1.e-20,1000.,1.e-20,ENDT

$PSD in frequency range 140-300 Hz
TABRND,100022,LOG,LINEAR, , , ,
+ ,31.5,1.e-20,63.1.e-20,125.,1033.21,250.,767.57,
+ ,500.,171.91,1000.,52.08,ENDT
\end{verbatim}

The NASTRAN PLOAD card, used to apply the unit pressure loads in the frequency response analysis, overestimates the responses of the reflector. Through breakdown of the problem into a smaller model a correction factor have been developed. The factor is 2.19 and it is applied at the same time as the responses are scaled to be presented in terms of the gravitational acceleration \( g = 9.81 \text{ m/s}^2 \).

\footnote{NASTRAN free field input format is used for simplicity.}
3.4 Input data

3.4.2 Applied correlations

The correlations (off-diagonal elements of the PSD matrix) are calculated using equation (20). Since equation (20) is a function of both frequency and the distance, $r$, between the patches this is done outside of NASTRAN and manually inserted to a NASTRAN *.rnd file containing all RANDPS and TABRND1 entries.

The correlation needs to be calculated for a total of 16 different distances in order to include correlation between all 42 patches. The distances between the patches is calculated from center to center. Two of the distances are between patches 35 - 38 and 39 - 42 (see Figure 14(b)). The other 14 distances all concern the smallest patches (see Figure 14(a)). Some distances are obviously the same due to symmetries and these can of course be used for all pairs of patches which have that specific distance between them. For example the distance between patch 1 - 6 is the same distance as between the pairs 4 - 7, 10 - 13 and 11 - 16. Similarly all distances are used for every pair of patches, where it applies, in order to calculate the correlations. Figure 14 and Table 8 together show more information about the different distances used for correlation calculation.

In Figure 14(a) all patches except patch number 1, 4, 13 and 16 are assumed to have there center coincide with the center of the corresponding square in the grid. Patches 1, 4, 13 and 16 however are assumed to have there respective centres 14.1 cm closer to the center of the reflector than the grid would suggest. The same reasoning of course applies to both sides. The 16 different distances gives a total of 226 correlation terms to be implemented.

---

In Figure 14(a) all patches except patch number 1, 4, 13 and 16 are assumed to have their centers coincide with the center of the corresponding square in the grid. Patches 1, 4, 13 and 16 however are assumed to have their respective centers 14.1 cm closer to the center of the reflector than the grid would suggest. The same reasoning of course applies to both sides. The 16 different distances give a total of 226 correlation terms to be implemented.

---

6 The distances between patches 35 - 38 and 39 - 42 are 10 cm in x-direction and 10 cm in y-direction.

---

Figure 14: Patches with their respective number.

---

6 The distances between patches 35 - 38 and 39 - 42 are 10 cm in x-direction and 10 cm in y-direction.
Table 8: Distances for correlation calculation (see Figure 14 for patch numbering).

<table>
<thead>
<tr>
<th>Distance name</th>
<th>Distance between patches</th>
<th>Distance [m]</th>
<th>Number of times used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>1 - 2</td>
<td>0.51</td>
<td>16</td>
</tr>
<tr>
<td>$r_2$</td>
<td>1 - 3</td>
<td>1.10</td>
<td>16</td>
</tr>
<tr>
<td>$r_3$</td>
<td>1 - 4</td>
<td>1.60</td>
<td>8</td>
</tr>
<tr>
<td>$r_4$</td>
<td>1 - 6</td>
<td>0.71</td>
<td>8</td>
</tr>
<tr>
<td>$r_5$</td>
<td>1 - 11</td>
<td>1.56</td>
<td>8</td>
</tr>
<tr>
<td>$r_6$</td>
<td>1 - 16</td>
<td>2.26</td>
<td>4</td>
</tr>
<tr>
<td>$r_7$</td>
<td>1 - 7</td>
<td>1.21</td>
<td>16</td>
</tr>
<tr>
<td>$r_8$</td>
<td>1 - 8</td>
<td>1.77</td>
<td>16</td>
</tr>
<tr>
<td>$r_9$</td>
<td>6 - 7</td>
<td>0.60</td>
<td>30</td>
</tr>
<tr>
<td>$r_{10}$</td>
<td>6 - 8</td>
<td>1.20</td>
<td>16</td>
</tr>
<tr>
<td>$r_{11}$</td>
<td>6 - 11</td>
<td>0.85</td>
<td>28</td>
</tr>
<tr>
<td>$r_{12}$</td>
<td>3 - 10</td>
<td>1.34</td>
<td>32</td>
</tr>
<tr>
<td>$r_{13}$</td>
<td>5 - 15</td>
<td>1.70</td>
<td>8</td>
</tr>
<tr>
<td>$r_{14}$</td>
<td>5 - 12</td>
<td>1.90</td>
<td>8</td>
</tr>
<tr>
<td>$r_{15}$</td>
<td>35 - 36</td>
<td>1.00</td>
<td>8</td>
</tr>
<tr>
<td>$r_{16}$</td>
<td>35 - 38</td>
<td>1.41</td>
<td>4</td>
</tr>
</tbody>
</table>

In addition to the correlations between the patches on the same side of the reflector, discussed above, there is of course correlation between patches on opposite sides of the reflector as well. Even though this is possible to include it is a bit tricky. The only correlations between front and back on the reflector that are considered are between patches in the vicinity of the edges. Through testing the best choice of correlations between the front and back side of the reflector have been found. For 32 patches the method utilizes patches that are in connection with the edge of the reflector for correlation between front and back. The correlation between two patches in contact with the edge, lying directly behind each other, is put equal to -1.0 in the entire frequency range. When 42 patches are used only patches on the same side of the reflector are correlated. For the full correlation to be present it would be desirable to use every correlation between every patch on both sides. This would however make the method extremely ineffective to work with. Full correlation between front and back would drastically increase the number of correlation terms and it is not straight forward to derive an expression for the correlation of patches on opposite sides that are not directly behind each other.

The frequencies at which the correlations are calculated depends partly on the input PSD frequencies but also on the frequency range of the patches.
3.4 Input data

This means that the correlations will be calculated at the same frequencies as the input PSD is given. The correlation values between these frequencies will be found from interpolation.

In the case where 32 patches are used in the analysis the correlations are applied directly over the entire frequency range (20 - 300 Hz). However in the case of 42 patches the correlations need to be scaled for the patches respective frequency range. This scaling is done in the same way as for the PSD shown in section 3.4.1 Applied PSD.

There is a large number of combinations of patch settings and correlations that could be studied. The settings that produce the best results will be further compared in this thesis and they are presented in Table 9.

Table 9: Correlation settings.

<table>
<thead>
<tr>
<th>Number</th>
<th>Patches</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>No correlations.</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>Correlation both between patches on the same side of the reflector and between the patches on opposing sides that is in connection with the edge.</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>No correlations.</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>Correlations between patches on the same side of the reflector.</td>
</tr>
</tbody>
</table>

Notice that it is not physically justifiable to not use any correlations at all. It is obvious that the patches behaviour are in fact correlated. The non-correlation options are however of interest for increased understanding of the effects of correlations.

3.4.3 Damping

The modal damping used has been taken from the modal test results (post thermal vacuum), according to the EM test report [5]. This damping is defined in Table 10.

Table 10: Modal damping given in % of critical damping.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Modal viscous damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 45</td>
<td>0.4%</td>
</tr>
<tr>
<td>45 - 100</td>
<td>0.6%</td>
</tr>
<tr>
<td>100 - 130</td>
<td>0.3%</td>
</tr>
<tr>
<td>130 - 300</td>
<td>1.0%</td>
</tr>
</tbody>
</table>
The damping described in Table 10 can be applied in either intervals or as points in which case NASTRAN interpolates between the points. These two cases are displayed between 0 - 140 Hz in Figure 15.

Figure 15: Damping applied in intervals and as interpolation points.

Integrating the surfaces beneath the two curves gives an idea of how big the difference in the total damping is between the two cases. The result is that the interpolation method gives an area that is 0.18 % smaller than the interval case. Since this difference is small enough to be neglected and the fact that the interpolation method is simpler to implement, the interpolation point method of applying the damping will be used in the model created in this thesis.
4 Results

In this section the results obtained during the vibro-acoustic analysis will be presented and explained. Main interest is in the acceleration power spectral densities of the nodes mentioned in section 3.2.2. In sections 4.1 and 4.2 the results from the first two steps of the analysis, the modal analysis and the frequency response analysis, are presented. In section 4.3 the results including the final step, random analysis, are presented and compared to the results from the physical test and the previously performed BEM-analysis. In section 4.4 results regarding the computational time of the model is presented. Patch numbers in this entire section all refer to the numbering presented in Figure 14 on page 35.

4.1 Modal analysis

The eigenfrequencies or natural frequencies of the reflector is presented in Table 11, where modes both with and without added air mass are included. Table 11 shows that the addition of air mass decreases the eigenfrequencies of the reflector by approximately 9%.

Table 11: Eigenfrequencies below 300 Hz for the reflector with and without air effect.

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Frequency [Hz]</th>
<th>Without added air effect</th>
<th>With added air effect</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43.1</td>
<td>38.8</td>
<td>9.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>55.6</td>
<td>50.2</td>
<td>9.8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>104.1</td>
<td>96.1</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>120.8</td>
<td>109.8</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>171.6</td>
<td>157.9</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>187.1</td>
<td>168.5</td>
<td>9.9</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>207.1</td>
<td>189.7</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>228.6</td>
<td>209.4</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>240.4</td>
<td>215.2</td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>273.8</td>
<td>253.4</td>
<td>7.4</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>289.0</td>
<td>268.2</td>
<td>7.2</td>
<td></td>
</tr>
</tbody>
</table>

In Figure 16 the modes at the four first eigenfrequencies (rigid body modes excluded) of the reflector, is displayed with added air mass.
4.2 Frequency response analysis

The results from the frequency response analysis is no end result, but it gives an idea about the differences in loading of different patches. Hence the results in this section is to be considered an orientation.

In Figure 17 an example of displacements obtained during the uniform unit pressure loading of patch 1 and patch 7, using 32 patches (see Figure 14(a)), at approximately 30 and 95 Hz is displayed.

Comparing the results in deformation, in Figure 17, for the loading of Patch 1 to the loading of Patch 7 it is easy to see that the deformations are larger for Patch 1. This is expected since the loading of the patches close to the edge of the reflector will give larger effect on the reflector behaviour. Remember that all patches are loaded equally with uniform unit pressure. Patch 7 consist of more than twice as many elements as patch 1, still the effect is greater when loading patch 1 due to the location close to the edge. Patch 7 is close to the center of the reflector and hence the loading of this patch has smaller influence on the deformation of the reflector.

---

Easiest to detect the difference between the loadings at 30 Hz(Figure 17(a) and (c)).
4.2 Frequency response analysis

RESULTS

(a) Displacement of the reflector during the loading of patch 1 at $\approx 30$ Hz.

(b) Displacement of the reflector during the loading of patch 1 at $\approx 95$ Hz.

(c) Displacement of the reflector during the loading of patch 7 at $\approx 30$ Hz.

(d) Displacement of the reflector during the loading of patch 7 at $\approx 95$ Hz.

Figure 17: Displacements for loading of patch 1 and 7 at 30 and 95 Hz.

The frequency response analysis is just a sub step in the analysis which is why this section is only an orientation and no further studies of these differences will be performed.
4.3 Split loading

In this section the main results of this thesis is presented. The results from
the model created with split loading. The results are discussed and compared
to previously performed BEM-analysis as well as the physical test data.

4.3.1 Patch choices and correlation

Focus will be on the acceleration power spectral density of the response.
In addition to this the root mean square values, from here on out referred
to as RMS values, will be compared as a qualitative measure. RMS values
are presented in terms of the gravitational acceleration $g = 9.81 \text{ m/s}^2$. The
nodes that will be used for the comparison are nodes 1, 5 and 8. Node 1 is
located at the very edge of the reflector, node 5 is close to the center of the
reflector and node 8 is close to one of the attachments (see Figure 21). The
correlations settings number refer to Table 9 on page 37. The only direction
considered is the x-direction since this direction is normal to the surface of
the reflector. The response values will be highest in the normal direction
and these responses will also contribute most to the stresses.

In this section the four different correlation settings presented in Table 9 will
be compared. In Figures 18 and 19 the acceleration power spectral density
response at node 1 and 5 are displayed for the four correlation settings
described in Table 9.

Looking at Figure 18 it is clear that for 32 patches the correlations do not
have a large impact on the results. The 32 patch method gives slightly
better RMS values when no correlation are used (setting 1). For 42 patches
the correlations have more effect on the results. The first two peaks are
significantly lower when correlations (setting 4) are used for 42 patches.

Figure 18: Comparison of different correlation settings for Node 1 Tx.
4.3 Split loading

RESULTS

(a) 32 patches.

(b) 42 patches.

Figure 19: Comparison of different correlation settings for Node 5 Tx.

Table 12: Correlation settings RMS results. RMS values closest to the BEM-analysis are marked in boldface and setting numbers refer to Table 9.

<table>
<thead>
<tr>
<th>Analysis/Setting number</th>
<th>Node 1 rms [g]</th>
<th>Node 5 rms [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>157</td>
<td>39.7</td>
</tr>
<tr>
<td>1</td>
<td>185</td>
<td>29.8</td>
</tr>
<tr>
<td>2</td>
<td>211</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>213</td>
<td>37.6</td>
</tr>
<tr>
<td>4</td>
<td>142</td>
<td>29.7</td>
</tr>
</tbody>
</table>

Table 12 shows that 42 patches produces the RMS values closest to the BEM analysis. Utilizing 42 patches with correlations according to correlation setting 4, gives the best overall result. It does not however produce a good value at node 5 which is in the center of the reflector. However it is more important that the model produce results close to the BEM-analysis in the vicinity of the edges since this is where the highest stresses are created.

It is, as mentioned before, not physically justifiable to _not_ use any correlations. It is however much simpler and faster to create the model without correlations. So the question if one of the correlation settings is better than the others does not have a simple yes or no answer. The different models are further discussed in section 5 Conclusions. In the rest of the result section correlation setting 2 and 4 (see Table 9) will be used since it is difficult to physically justify not using correlations.
4.3 Split loading

4.3.2 Validation against known data

In Figure 20 (a) and 20 (b) an overview of the power spectral density accelerations in the interesting nodes is presented. In Figure 20 (c) the corresponding data from the previously performed BEM analysis is presented. Notice that the RMS values in Figure 20 (c) are calculated up to 500 Hz while the RMS-values in Figures 20 (a) and (b) only include frequencies up to 300 Hz. In Table 13 the acceleration response RMS is presented for the different patch choices and for the physical test data as well as for the BEM analysis.

<table>
<thead>
<tr>
<th>Node</th>
<th>RMS-acceleration [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEM analysis</td>
</tr>
<tr>
<td>1</td>
<td>157.8</td>
</tr>
<tr>
<td>2</td>
<td>73.9</td>
</tr>
<tr>
<td>3</td>
<td>45.4</td>
</tr>
<tr>
<td>4</td>
<td>21.4</td>
</tr>
<tr>
<td>5</td>
<td>39.5</td>
</tr>
<tr>
<td>6</td>
<td>62.4</td>
</tr>
<tr>
<td>7</td>
<td>84.8</td>
</tr>
<tr>
<td>8</td>
<td>24.9</td>
</tr>
<tr>
<td>9</td>
<td>86.9</td>
</tr>
<tr>
<td>10</td>
<td>61.0</td>
</tr>
<tr>
<td>11</td>
<td>83.7</td>
</tr>
<tr>
<td>12</td>
<td>83.7</td>
</tr>
<tr>
<td>13</td>
<td>59.0</td>
</tr>
<tr>
<td>14</td>
<td>59.0</td>
</tr>
<tr>
<td>41</td>
<td>21.4</td>
</tr>
<tr>
<td>61</td>
<td>62.5</td>
</tr>
</tbody>
</table>

The data in Table 13 further stress that the method with 42 patches produces acceleration power spectral densities that are closer to the available reference data, especially the BEM-analysis data. This is much due to the first two peaks that are significantly lower for 42 patches with correlations than for the other patch and correlation choices. The mean difference in RMS value between split loading with 42 patches and BEM-analysis is approximately 10%.

To study the differences between the methods and patch choices a comparison of all methods at the same node and in the normal direction will be presented.

---

4 For 32 and 42 patches with correlations according to correlation settings 2 and 4.
4.3 Split loading

RESULTS

(a) 32 patches.

(b) 42 patches.

(c) BEM-analysis.

Figure 20: Acceleration PSD response - Node 1-14, 41 and 61.
In Figure 21 the power spectral density response at node 1 is presented for both 32 and 42 patches compared to the physical test data and the BEM-analysis. The main interest is to compare the Split loading approach with the BEM-analysis data since the same damping has been used for these analyses.

Figure 21: Acceleration PSD response - Node 1 Tx.

Figure 21 shows that split loading with 32 patches produces responses that are overestimated compared to both the BEM analysis and the physical test. Split loading with 42 patches produce responses significantly closer to the BEM analysis. This is due to the fact that the two initial peaks are heavily overestimated by split loading with 32 patches. In general, the shape of the graphs produced by the split loading approach are well correlated with the shapes of the physical test data and the BEM-analysis.

Figure 22: Acceleration PSD response - Node 5 Tx.

In Figure 22 the acceleration response at node 5 is displayed. It is obvious that the split loading approach overestimates the responses at low frequencies and underestimates the responses at high frequencies compared to the
BEM-analysis. However, the Split loading approach displays behaviour similar to that of the physical test between 40-95 Hz. Notice that this behaviour is not seen in the BEM-analysis.

In Figure 23 the acceleration response at node 8 is displayed. Figure 23 further confirms that split loading underestimates responses at low frequencies and overestimates them at higher frequencies.

![Figure 23: Acceleration PSD response - Node 8 Tx.](image)

The displacement responses are presented next, the displacement could be considered a better representation of what creates the stresses. In Figure 24 the cumulative displacement RMS is presented for node 1 and 5 respectively.

![Figure 24: Cumulative RMS Displacement.](image)

Figure 24 shows that the two first peaks, corresponding to the first and second mode of the reflector, are the peaks that mainly contribute to the overestimation of the responses. The differences are larger close to the edge of the reflector where the responses are higher. It is obvious that the two first peaks are responsible for the big differences in the total RMS values.
It is clear that 42 patches produces values more correlated with the BEM-analysis than the 32 patches method.

### 4.4 Computational time

The analysis consist of 3 parts which require computer time:

- Modal analysis.
- Frequency response analysis.
- Random analysis.

The modal analysis is by far the most time consuming process while the frequency response analysis is rather fast due to the possibility to reuse the solution obtained from the modal analysis by using the RESTART commando in NASTRAN. The random analysis only uses a couple of seconds and hence it will not be further discussed here. The hardware used in all calculations is presented in Table 14 and the computational times for the modal and the frequency response analysis are presented in Tables 15 and 16.

#### Table 14: Computer hardware.

<table>
<thead>
<tr>
<th>Processor</th>
<th>Intel Core 2 Duo, 2.66 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAM</td>
<td>7.87 GB</td>
</tr>
<tr>
<td>System</td>
<td>Windows XP Pro 2003, x64 Edition</td>
</tr>
</tbody>
</table>

#### Table 15: Computational time for modal analysis.

<table>
<thead>
<tr>
<th>Option</th>
<th>Time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without MFLUID</td>
<td>91</td>
</tr>
<tr>
<td>With MFLUID</td>
<td>2249</td>
</tr>
<tr>
<td>Difference</td>
<td>2158</td>
</tr>
</tbody>
</table>

#### Table 16: Computational time for frequency response analysis.

<table>
<thead>
<tr>
<th>Option</th>
<th>Time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 patches</td>
<td>239</td>
</tr>
<tr>
<td>42 patches</td>
<td>280</td>
</tr>
<tr>
<td>Difference</td>
<td>41</td>
</tr>
</tbody>
</table>

Both the times and the time differences for frequency response analyses is comparatively small and they should not be a major factor when choosing number of patches.
5 Discussion

In this section the results presented in section 4 will be discussed and conclusions regarding the developed model will be drawn.

5.1 Results discussion

In this section the results, presented in section 4, will be further discussed along with possible reasons for the deviations from the physical test and the BEM-analysis.

Split loading with 42 patches and correlation between patches on the same side produces the results closest to the BEM-analysis. Qualitatively (RMS-values) the results are close to the BEM-analysis, but the model overestimates the responses at low frequencies and underestimates them at high frequencies. Split loading displayed behaviour at node 5, seen in the physical test data, that the BEM-analysis could not reproduce (see Figure 22). This suggests that FEM have some advantages compared to BEM.

The difference between 42 patch analysis and the one with 32 patches is of course that several different patch sizes are used in different frequency ranges for the former. The different patch sizes in different frequency ranges probably contribute to the 42 patch models ability to correctly calculate structural responses.

As mentioned above 42 patches with correlations is the setting for the split loading analysis that gives the best results overall. The acceleration power spectral density RMS values are close to the values of the BEM-analysis but still there are some differences. The main reasons that might effect the results are here presented and discussed.

- Air mass - MFLUID (low frequencies).
- Difference in damping - FEM/BEM.
- Correlations.
- Diffraction factor (low frequencies).
- Number of modes included in the modal analysis.

Air mass - MFLUID

The peaks in the responses produced by split loading are sharper than the ones produced by the BEM-analysis. The difference in damping on the peaks is easily seen in Figure 19 where the two peaks at approximately 170 Hz are much sharper for split loading than for the BEM-analysis. MFLUID adds
the air mass to the model through virtual mass and it is possible that it does not increase the damping as much as would be desired.

**Difference in damping**

The damping specified is the same for all split loading analyses and it is the same damping that is used in the BEM-analysis. Still, as mentioned above, looking at the peaks of the PSD-responses it seems the peaks are significantly sharper for the split loading approach than for the BEM-analysis. The reason for this might be that the same damping has different impacts in FEM and BEM. The height of the peaks, specially the first two peaks, is the main difference between the split loading model and the BEM-analysis. The peaks contribute much to the RMS values and hence part of the differences can be deduced to the applied damping.

**Correlation**

Next factor that influence the results are, of course, the correlations between patches. The best result was found for 42 patches with correlation between patches on the same side of the reflector. It is, as mentioned before, not physically justifiable not to use any correlations, so correlations between patches on the same side of the reflector can be considered a minimum. The most physically correct method is to include correlations between every patch, on both sides. It is not certain that this would give better results though. For 32 patches the best result (including correlations) was found when correlations was used between patches on the same side as well as between patches on different sides of the reflector that are directly behind each other and is connected with the edge of the reflector. This correlations probably gives good results since the response is highest at the edges of the reflector and the correlations between front and back can help to decrease the responses there.

**Diffraction factor**

The diffraction factor (=2) applied to the input power spectral density due to the acoustic diffraction could be another reason for the differences. The control microphones used during the acoustic test are placed approximately 2 m away from the reflector. The distance where the diffraction is present depends on the wave length of the sound pressure. This means that at low frequencies the wave lengths are larger and hence the microphones are, in terms of wavelength, closer to the reflector. At 40 Hz the wavelength is \( \approx 8.5 \text{ m} \), this means that the microphones are placed a small fraction of the wavelength away from the reflector. In this case the microphones could incept the diffraction phenomenon, in which case the applied diffraction factor would be excessive at low frequencies. Utilizing this thought in the
model would result in no, or a smaller, diffraction factor for low frequencies. Decreasing the diffraction factor at low frequencies, or making it frequency dependent, could decrease the overestimation at low frequencies and improve the behaviour of the split loading model.

**Number of modes**

When performing the frequency response analysis the modal solution is used. It is important to include mode shapes outside of the studied frequency range, since these mode shapes contribute to the behaviour of the reflector during the frequency response analysis. Mode shapes between 0-400 Hz was included even though 20-300 Hz was the frequency range of interest. This allows both the rigid body modes and several modes above 300 Hz to contribute to the results. Further increasing the number of included mode shapes should not significantly effect the results since mode shapes far outside of the frequency range of interest already is included.
5.2 Conclusions

When discussing the results the questions asked at the start of this thesis should be kept in mind:

- Is it possible to create a computationally efficient model that take into account vibro-acoustic effects and the loading in terms of a diffuse acoustic field?

- What are the advantages and disadvantages of this model?

- Can this model be used to reproduce the results produced by RUAG Space’s BEM-analysis?

- Can this model reproduce the physical test data provided by RUAG Space?

This thesis shows that it is possible to create computationally efficient model that take into account vibro-acoustic effects. However the model can not (yet) accurately reproduce the data of the BEM-analysis and the physical test. The model produces results that qualitatively well correspond to the BEM-analysis but there are differences particularly at low frequencies. The advantages with the split loading is that it is computationally efficient since there are no volume fluid elements present. The disadvantage is that it is non-trivial to implement the correlations and that patches are created through the graphical interface and not as a script.

The conclusion regarding the results is that since the model is conservative at low frequencies, where the highest stresses are created, it can not be used practically without further development. The model would break reflectors that actually are strong enough for the acoustic qualification test.

5.3 Further research

To obtain a model that could be used practically some further research would have to be performed. The factors mentioned in section 5.1 could be a starting point to further develop the split loading approach.

Another option could be to abandon split loading and instead use volume fluid elements to engulf the reflector and include fluid-structure interaction at the surface. This approach would of course become significantly heavier computationally since the reflector model is large and the surrounding fluid would have to be really large to produce the proper physical environment. This would increase the degrees of freedom drastically.
References


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A Random vibration theory

In this section basic theory of random analysis will be presented. The acoustic load applied to the reflector in the form of a diffuse acoustic field will be applied as a random load in NASTRAN.

A.1 Introduction

Petyt [22] states that harmonic, periodic and transient forces are termed deterministic since they can be described by explicit mathematical relationships. With random forces, which are caused by for example gales and diffuse acoustic fields, there is no way of predicting an exact value at a future instant in time. Such forces can only be described with statistical methods.

A.2 Autocorrelation function

Consider two random variables varying in time. Petyt [22] explains that in general random processes are time dependent. In many practical situations however the probability density functions, $p(f_1)$ (one random force) and $p(f_1, f_2)$ (two random forces) are independent of time. Random processes with probability density functions that are independent of the time origin are said to be weakly stationary. When considering weakly stationary processes the mean and the variance is constant i.e.

$$\mu_f = \langle f_k(t) \rangle = \text{Constant}, \quad (A.1)$$
$$\sigma^2_f = \langle \{f_k(t) - \mu_f(t)\}^2 \rangle = \text{Constant}. \quad (A.2)$$

In this case the covariance is a function of $\tau = (t_2 - t_1)$ only and it becomes

$$\sigma_{f_1,f_2} = \langle \{f_k(t_1) - \mu_f\}{f_k(t_1 + \tau)} \rangle - \mu_f. \quad (A.3)$$

The covariance defined in equation (A.3) can be rewritten as

$$\sigma_{f_1,f_2} = R_f(\tau) - \mu_f^2 \quad (A.4)$$

where the autocorrelation function is introduced as

$$R_f(\tau) = \langle f_k(t_1)f_k(t_1 + \tau) \rangle \quad (A.5)$$

Notice that if the mean is zero the covariance and the autocorrelation function are identical. Petyt [22] states that a stationary random process is said to be ergodic if the time averages are equal to the equivalent ensemble averages. Ergodic processes are assumed by NASTRAN when performing the calculations.
A.3 Power Spectral Density

Physics handbook [20] defines the Fourier Transform as

\[ f(t) = \int_{-\infty}^{+\infty} F(i\omega)e^{i\omega t} d\omega \quad (A.6) \]

\[ F(i\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t)e^{-i\omega t} dt. \quad (A.7) \]

To determine the Fourier transform of the signal \( f(t) \) the new signal \( f_T(t) \) is introduced. This signal \( f_T(t) \) is the same as \( f(t) \) in the interval \(-T \leq t \leq T\) and zero everywhere else since it is a condition for the Fourier transform that the signal span all times. Using this signal together with the definition of the Fourier transform repeatedly Petyt [22] shows that the mean square of \( f_T(t) \) can be written as

\[ f^2_T(t) = \frac{1}{2T} \int_{-\infty}^{+\infty} f^2_T(t) dt \]

\[ = \int_{-\infty}^{+\infty} \frac{\pi}{T} |F_T(i\omega)|^2 d\omega \quad (A.8) \]

Using equation (A.8) the mean square of the original signal, \( f(t) \), is

\[ \overline{f^2(t)} = \lim_{T \to \infty} f^2_T(t) \]

\[ = \int_{-\infty}^{+\infty} \lim_{T \to \infty} \frac{\pi}{T} |F_T(i\omega)|^2 d\omega \]

\[ = \int_{-\infty}^{+\infty} S_f(\omega) d\omega \quad (A.9) \]

where \( S_f(\omega) \) is introduced as

\[ S_f(\omega) = \lim_{T \to \infty} \frac{\pi}{T} |F_T(i\omega)|^2 d\omega \quad (A.10) \]

and is called the power spectral density. The power spectral density is an even function over all real frequencies, both positive and negative. However it is more convenient to deal with only positive frequencies which is why the one-sided spectral density function is defined as

\[ G_f(\omega) = 2S_f(\omega) \quad \text{for } \omega > 0. \quad (A.11) \]

Using equation (A.11) gives the mean square of \( f(t) \) as
A.4 Cross-correlation function

\[ f^2(t) = \int_0^{+\infty} G_f(\omega) d\omega. \quad (A.12) \]

Petyt [22] shows that the power spectral density is in fact the Fourier transform of the autocorrelation function. This implies that

\[ R_f(\tau) = \int_{-\infty}^{+\infty} S_f(\omega) e^{i\omega\tau} d\omega. \quad (A.13) \]

A.4 Cross-correlation function

Petyt [22] goes on to explain that if a structure is subjected to two (or more) random forces there is a possibility that they are related in some way. The autocorrelation function can be used to describe the main features of the process \((f_1(t) + f_2(t))\) (where \(f_1(t)\) and \(f_2(t)\) are two stationary, ergodic processes with zero mean) and it becomes

\[
R_c(\tau) = \{f_1(t) + f_2(t)\} \{f_1(t + \tau) + f_2(t + \tau)\} = f_1(t)f_1(t + \tau) + f_1(t)f_2(t + \tau) + f_2(t)f_1(t + \tau) + f_2(t)f_2(t + \tau)
\]

where \(R_{f_1f_2}(\tau)\) and \(R_{f_2f_1}(\tau)\) are called cross correlation functions. If \(f_1(t)\) and \(f_2(t)\) are completely unrelated the cross-correlation functions will be zero. Since the two signals \(f_1(t)\) and \(f_2(t)\) are stationary it is easy to show that

\[
R_{f_1f_2}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{f_1f_2}(\tau) e^{-i\omega\tau} d\tau
\]

Using the same reasoning, for the Fourier transform, as in the previous section it is possible to show that the Fourier transform of the cross-correlation function is a function called the cross-spectral density defined by

\[
S_{f_1f_2}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{f_1f_2}(\tau) e^{-i\omega\tau} d\tau
\]

Petyt [22] goes on to show that if the structure is subjected to distributed forces the random process describing the forcing will be a function of both position and time. If the process is stationary, ergodic and has a zero mean the essential features of the probability density are described by the correlation between the forces at two positions \(\vec{r}_1 = (x_1, y_1, z_1)\) and \(\vec{r}_2 = (x_2, y_2, z_2)\) as

\[
R_f(\vec{r}_1, \vec{r}_2, \tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{f_1f_2}(\tau) f(\vec{r}_1, t) f(\vec{r}_2, t + \tau) d\tau
\]
Equation (A.17) is known as the space-time correlation function. Now the cross-spectral density of the forces at $\vec{r}_1$ and $\vec{r}_2$ is the Fourier transform of the cross-correlation function which gives

$$S_f(\vec{r}_1, \vec{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_f(\vec{r}_1, \vec{r}_2, \tau)e^{-i\omega\tau}d\tau$$  \hspace{1cm} (A.18)

A.5 Spatial correlations

In the case of a diffuse sound field, like the one present during launch of a satellite, it is important to include the spatial correlations. Theory describing the spatial correlations is presented in this section.

Pierce [23] derives the cross-correlation function of the constant-frequency pressure field to be the average over the volume of the product between $p(\vec{x}, t)$ and $p(\vec{x} + \Delta\vec{x}, t + \Delta t)$ for a fixed $\Delta x$ and $\Delta t$. Pierce [23] shows that this gives

$$\langle p(\vec{x}, t)p(\vec{x} + \Delta\vec{x}, t + \Delta t) \rangle = \frac{1}{2} \sum_q |\hat{p}_q|^2 \cos \omega(\Delta t - \vec{n}_q \cdot \frac{\Delta\vec{x}}{c})$$  \hspace{1cm} (A.19)

where $|\hat{p}_q|^2$ is given by equation (1), $q$ is the plane wave index, $\vec{n}_q$ is the normal vector of plane wave $q$ and $c$ is the speed of sound. However in the diffuse field idealization the cosine is replaced by its average over propagation direction, the sum of $|\hat{p}_q|^2$ is replaced by $2\langle p^2 \rangle$ and the average of solid angle of $\cos \omega(\Delta t - \vec{n}_q \cdot \frac{\Delta\vec{x}}{c})$ can be performed in spherical coordinates taking $\Delta x$ in the z-direction. Together this reduces\(^9\) equation (A.19) to\(^10\)

$$\langle p(\vec{x}, t)p(\vec{x} + \Delta\vec{x}, t + \Delta t) \rangle = f(\vec{x}, \vec{x} + \Delta\vec{x}, \tau)$$

$$= \langle p^2 \rangle \frac{1}{2} \int_0^\pi \cos \omega(\Delta t - \frac{\Delta x}{c} \cos \theta) \sin \theta d\theta$$

$$= \langle p^2 \rangle \cos \omega \Delta t \frac{\sin k|\Delta x|}{k|\Delta x|}$$  \hspace{1cm} (A.20)

where $k$ is the wave number defined as $k = \frac{2\pi}{\lambda}$. In this thesis $\cos \omega \Delta t = 1$ since we are only interested in the frequency domain i.e. $\Delta t = 0$.

---


\(^10\)Compare with result of the expression for the cross-correlation function in equation (A.18) in section A.4 Cross-correlation function
A.6  Spectral density of response

Wirsching, Paez and Ortiz [25] states that in order to determine the cross-spectral density function of the response the best approach is to employ the relationships between the PSD of a random function \( X(t) \) and the finite Fourier transform of \( X(t) \). Let the vector of Fourier transforms be defined according to [25] as

\[
\bar{X}(\omega) = \begin{bmatrix} X_1(\omega) \\ X_2(\omega) \\ \vdots \\ X_n(\omega) \end{bmatrix}
\]

(A.21)

and

\[
\bar{F}(\omega) = \begin{bmatrix} F_1(\omega) \\ F_2(\omega) \\ \vdots \\ F_n(\omega) \end{bmatrix}
\]

(A.22)

The cross-spectral density of the force, which is assumed to be given, is

\[
[S_F(\omega)] = 
\begin{bmatrix}
S_{F_1F_1} & \cdots & S_{F_1F_n} \\
S_{F_2F_1} & \cdots & S_{F_2F_n} \\
\vdots & \ddots & \vdots \\
S_{F_nF_1} & \cdots & S_{F_nF_n}
\end{bmatrix}
\]

(A.23)

Further the cross-spectral density of \( X_j(t) \), \( X_k(t) \), \( F_j(t) \) and \( F_k(t) \) can be written as

\[
S_{X_jX_k}(\omega) = CE [X_j(\omega)X_k^*(\omega)]
\]

(A.24)

\[
S_{F_jF_k}(\omega) = CE [F_j(\omega)F_k^*(\omega)]
\]

(A.25)

where \( C \) is a constant. The Fourier transform of the response is related to the force by

\[
\tilde{X}(\omega) = [H(\omega)] \bar{F}(\omega)
\]

(A.26)

where \([H(\omega)]\) is the transfer function matrix. Now take the complex conjugate and then the transpose of equation (A.26) and then post multiply to get

\[
\tilde{X}(\omega)\tilde{X}^*(\omega)^T = [H(\omega)] \bar{F}(\omega)\bar{F}^*(\omega)^T [H^*(\omega)]^T
\]

(A.27)

To continue take the expected value of both sides and multiply by a constant \( C \) to get

\[
CE \left[ \tilde{X}(\omega)\tilde{X}^*(\omega)^T \right] = [H(\omega)] CE \left[ \bar{F}(\omega)\bar{F}^*(\omega)^T \right] [H^*(\omega)]^T
\]

(A.28)

By substituting the expressions in equations (A.24) and (A.25) into equation (A.28) the spectral density function response matrix can be written as

\[
[S_X(\omega)] = [H(\omega)] [S_F(\omega)] [H^*(\omega)]^T
\]

(A.29)
In equation (A.29) the diagonal elements are the spectral density functions of each coordinate and the off-diagonal terms are the cross-spectral density functions.

A.7 Coupling spatial correlation

As presented by Thiébaut [24] a useful formulation when performing calculations in vibro-acoustics is the "u-p" formulation which is

\[
\begin{bmatrix}
K_S - \omega^2 M_S & C \\
\omega^2 C^T & K_F - \omega^2 M_F \\
\end{bmatrix}
\begin{bmatrix}
U \\
P \\
\end{bmatrix}
= 
\begin{bmatrix}
F_S \\
F_F \\
\end{bmatrix}
\]

(A.30)

where the indices S and F stand for structure and fluid respectively, K and M are the stiffness and mass matrices and C is the camping matrix. U is the nodal displacement vector and P is the nodal pressure vector.\(^{11}\)

The input to output relation used in NASTRAN is the one of equation (A.29) in section A.6 Spectral density of response. The before mentioned cross-spectral density is actually the product of the power spectrum (PSD at a reference point), \(S_{\text{ref}}(\omega)\), and the spatial correlation function. This is defined in equation (A.20) in section A.5 Spatial correlations if \(\langle \bar{p}^2 \rangle = S_{\text{ref}}(\omega)\). This results in the equation for cross-power spectral density and it becomes

\[
S(r_{ij}, \omega) = S_{\text{ref}}(\omega) f_s(r_{ij}, \omega) = S_{\text{ref}}(\omega) \frac{\sin |k||r|}{|k||r|}
\]

(A.31)

where

\[
f_s(r_{ij}, \omega) = \frac{\sin |k||r_{ij}|}{|k||r_{ij}|}
\]

(A.32)

is the spatial correlation, \(S(r_{ij}, \omega)\) is the cross-correlation and \(S_{\text{ref}}(\omega)\) is a frequency dependent PSD i.e. the input PSD calculated from the sound pressure levels. The complete PSD input matrix containing both auto- and cross-correlations then becomes

\(^{11}\)As explained by Carlsson [10] the structure part in fluid structure interaction problems is by convention discretized by a displacement-based finite element method. The acoustic formulation of the fluid can be discretized as either vector field fluid-particle displacements (or velocities) or a scalar field as the pressure or displacement-potential. The use of a displacement field discretization for the fluid gives a symmetric vibration problem and no coupling terms are needed. The number of degrees of freedom is however significantly higher (3 DOF/node compared to 1 DOF/node) than when using the pressure. This fact may give a large number of spurious modes that correspond to the rotational fluid motions at zero frequency. Using the scalar field automatically enforces the irrotationality of fluid motions which minimizes the existence of spurious modes. In this thesis the scalar field pressure will be used as primary variable mainly because this is how the method is defined in NASTRAN.
where the diagonal consist the autocorrelations and the off-diagonal consist of the cross-correlations. In the case of a diffuse sound field $S_r$ is real symmetric and positive definite[24].
B Statistical Energy Analysis - SEA

Statistical energy analysis (SEA) is an engineering method used for vibro-acoustic computations. The basic idea of SEA is to decompose the structure of interest into different subsystems. These subsystems should have similar characteristic properties. Once the structure is decomposed into subsystems the method estimates the flow of resonant vibration energy between the subsystems. The commonly used variable of interest is the surface mean velocity that occurs in a certain wave type within the structure.

Fahy [12] talks about two main approaches within the SEA - The modal approach and the travelling wave approach (see [12] for more information on travelling wave approach). For systems with fluid-structure interaction the modal approach is best suited due to the fact that the power transfer takes place over the surface of the shared boundary. The modal approach works well with vibro-acoustic problems because of the normally weak coupling, however the model performs badly when coupling of structures is present. The modal approach is based on two main concepts in linear acoustics. The orthogonality of modes that allow describing the storage of energy in different subsystems and the statistical theory that makes it possible to unify the external and the internal influences on the subsystems.

The main problem with SEA for the application in this thesis is that it is not valid for low frequencies (< 100Hz depending on problem). The frequency range of interest for this thesis is mainly 20-300 Hz. If higher frequencies would have been of interest SEA in combination with for example FEM could be of interest. SEA has good performance in the high frequency range while FEM is limited in capturing high frequency modes. In Table 17 the main features of SEA is presented.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>SEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknown</td>
<td>Subsystem energy</td>
</tr>
<tr>
<td>Frequency</td>
<td>Band average, High</td>
</tr>
<tr>
<td>Spatial detail</td>
<td>Average</td>
</tr>
<tr>
<td>Excitation</td>
<td>Average/random</td>
</tr>
<tr>
<td>Procedure</td>
<td>Quick, demanding at first</td>
</tr>
<tr>
<td>Model</td>
<td>Small</td>
</tr>
<tr>
<td>Advantage</td>
<td>-High frequency</td>
</tr>
<tr>
<td></td>
<td>-Fast</td>
</tr>
<tr>
<td>Disadvantage</td>
<td>-Only works in high frequency domain</td>
</tr>
<tr>
<td></td>
<td>-Limited knowledge of structural definition</td>
</tr>
<tr>
<td></td>
<td>-No point recovery</td>
</tr>
</tbody>
</table>
C Using RANDPS ans TABRND1

In NASTRAN the PSD is applied through use of the RANDPS and TABRND1 cards. This is written in a .rnd file which is applied as a post-processing step to the frequency response analysis. The RANDPS card is a power spectral density specification and is used according to

\[ \text{RANDPS, SID, J, K, X, Y, TID} \]

where the option terms are explained in Table 18.

<table>
<thead>
<tr>
<th>SID</th>
<th>Random analysis set identification(ID) number.</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Excited subcase ID number.</td>
</tr>
<tr>
<td>K</td>
<td>Applied subcase ID number. J≤K is required.</td>
</tr>
<tr>
<td>X and Y</td>
<td>Components of the complex number.</td>
</tr>
<tr>
<td>TID</td>
<td>Referenced TABRND1 number.</td>
</tr>
</tbody>
</table>

The TABRND1 card is a power spectral density table and is used according to

\[ \text{TABRND1, TID, XAXIS, YAXIS, f1, g1, f2, g2, ... , ENDT} \]

where the option terms are explained in Table 19.

<table>
<thead>
<tr>
<th>TID</th>
<th>Table identification(ID) number.</th>
</tr>
</thead>
<tbody>
<tr>
<td>XAXIS and YAXIS</td>
<td>Interpolation for respective axis.</td>
</tr>
<tr>
<td>fi and g1</td>
<td>Frequency - PSD pairs.</td>
</tr>
</tbody>
</table>

To demonstrate this, assume a frequency response analysis performed on a thin plate that is divided into the 4 different patches shown in Figure 25. Let \( r_1 = 1 \) be the distance between the patches 1-2, 1-3, 2-4 and 3-4 and let \( r_2 = 1.41 \) be the distance between patch 1-4 and 2-3. It is important to remember that the input matrix is symmetric and real which makes all Y on the RANDPS card equal to zero. Since the matrix is symmetric only the upper half needs to be used in the matrix. If this is the case the .rnd file implementing the random analysis, using made up values, becomes

\[ \text{Diagonal Terms} \]

\[ \begin{array}{c}
\text{RANDPS, 101, 1, 1, 1, 0, 202} \\
\text{RANDPS, 101, 2, 2, 1, 0, 202} 
\end{array} \]

\[ \text{Copyright} \]

\[ \text{The NASTRAN format called free field is used in these examples in order to make overview easy. For more information on different formats see [16], [17] and [18].} \]
C USING RANDPS ANS TABRND1

Figure 25: Simple example of plate with 4 patches.

| RNDPS, 1, 0, 1, 3, 1, ., 0, ., 202 |
| RNDPS, 1, 0, 1, 4, 1, ., 0, ., 202 |

$OFF-diagonal terms – Cross-correlations$

$Distance r1$
- RNDPS, 1, 0, 1, 1, ., 2, 1, ., 0, ., 203
- RNDPS, 1, 0, 1, 1, ., 3, 1, ., 0, ., 203
- RNDPS, 1, 0, 1, 2, 4, 1, ., 0, ., 203
- RNDPS, 1, 0, 1, 3, 4, 1, ., 0, ., 203

$Distance r2$
- RNDPS, 1, 0, 1, 1, 4, 1, ., 0, ., 204
- RNDPS, 1, 0, 1, 2, 3, 1, ., 0, ., 204

$Table of PSD input$-------
$Autocorrelation$
- TABRND, 202, log, linear, , , , , +
  +, 31.5, 300, 63, 400, 125, 200, 250, 350, +
  +, ENDT

$Cross-correlations$
- TABRND, 203, log, linear, , , , , +
  +, 31.5, 270, 63, 320, 125, 140, 250, 17.5, +
  +, ENDT
- TABRND, 204, log, linear, , , , , +
  +, 31.5, 240, 63, 280, 125, 80, 250, 3.5, +
  +, ENDT

The values in TABRND1 203 and 204 are found by multiplying the correlations ($0 < \text{Correlation} < 1$) with the PSD value in the diagonal at the corresponding frequency. This .rnd file corresponds to the symmetric input correlation matrix in equation (A.33) that for this simple example at 31.5 Hz becomes...
\[ [S_r(\omega)] = 300 \begin{bmatrix}
1.0 & 0.9 & 0.9 & 0.8 \\
1.0 & 0.8 & 0.9 \\
1.0 & 0.9 \\
sym. & 1.0
\end{bmatrix} \]  \quad (C.1)

For more detailed information on how the RANDPS and TABRND1 cards are used see references [16], [17] and [18].