Like liquid off a Danes back

A quantitative study of illiquidity in the Copenhagen Stock Exchange

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Mattias Eknemar & Wesley James Short
Abstract

Our research was conducted through a quantitative study based on data collected from the Copenhagen Stock Exchange between 2003 and 2011. Our primary purpose was to ascertain whether illiquidity was priced in the Copenhagen Stock Exchange. Illiquidity has been shown as a difficult concept to measure as it is not an observable variable in itself. We show that illiquidity can be measured using Amihud’s (2002) \textit{ILLIQ}-measure. We investigated the relationship between asset pricing models and illiquidity. We provided an in depth look into illiquidity and past research involving liquidity and asset pricing as well as a thorough theoretical background concerning relevant academic theory. Though our empirical analysis we found evidence which supports the pricing of illiquidity in the Copenhagen Stock Exchange.

Keywords: Asset pricing, illiquidity, \textit{ILLIQ}, Copenhagen Stock Exchange
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1. INTRODUCTION

In the introduction, we first provide an introduction to the thesis’s theoretical foundation from which the research questions are formed. The contribution of this thesis towards further research is given through the research purpose. Additionally, an introduction to our chosen source of Data, the Copenhagen Stock Exchange is provided. Lastly, we provide the reader with a short list of terms that we feel need clarification.

1.1 Problem background

"When you buy a stock, bond, real asset or a business, you sometimes face buyer's remorse, where you want to reverse your decision and sell what you just bought. The cost of illiquidity is the cost of this remorse" (Damoradan, 2005)

A single introductory explanation of illiquidity, such as the one presented above, is favorable to explain the whole complex concept of liquidity. However the fact is that the concept can be formulated and described differently depending on the context of which it is being described. This fact in itself does not constitute a problem; the complexity of the term liquidity is merely an indicator of the many ways it is used to describe conditions within many different areas of finance.

In a financial context, Howells (2008) provides an alternate way of describing liquidity; "[...] the ability to retrieve funds quickly and with capital certainty." (Howells, 2008, p. 7), thus, Howells provides a definition describing the level of assurance from a consumer point of view, to recover funds when needed. Seen from an individual perspective, the choice of bank should thus fall on the one that has the highest level of guaranteed liquidity if the ability to recover funds quickly is regarded as important. Figure 1 below shows the difference between assets which are ‘more liquid’ and assets which are ‘less liquid’. As illustrated, a T-bill is more liquid than property and the principal difference between the two is the haste at which retrieving funds is made; a short time frame from a T-bill, a longer time frame from property.

<table>
<thead>
<tr>
<th>More Liquid</th>
<th>Government Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bills</td>
<td></td>
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<tr>
<td>Stock exchange listed stocks</td>
<td></td>
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<tr>
<td>Corporate Bonds</td>
<td></td>
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<tr>
<td>Less Liquid</td>
<td></td>
</tr>
<tr>
<td>Art</td>
<td>Property</td>
</tr>
</tbody>
</table>

Figure 1: A descriptive illustration of different liquidity levels

Adapted from Howells (2008)
Amihud (2005) applies the term liquidity as the ability to trade large quantities of an asset quickly, at low cost, and without moving the price. The evaluation of how liquid the asset is depends on e.g. the market impact and the quickness of the transaction. Thus, the level of liquidity of an asset is defined by the market, the development of the single asset and the quantity that are bought and sold within a certain time frame. Söderberg (2009) and Gårdängen (2006) identify this time frame as one of the key dimension of liquidity. The immediacy of which an asset can be traded, and at what haste the connection between a seller and a buyer can be facilitated at a specific time, have a large effect with regards to the liquidity of an asset.

Additional dimensions can be identified to describe liquidity, these dimensions pin-point the key elements in the relationship between liquidity and the performance of assets. Alignment within the field can be recognized when academics describe these different important dimensions. Kyle (1985) along with Söderberg (2009) describes depth as another key indicator of a liquid asset. Simplified, depth is the amount of trades made possible without affecting the prices to a significant amount and can therefore also be connected to the introductory definition from Amihud (2005).

Resiliency is the rate relating to how well the asset-markets cope with changes in trading volume and manages a return to equilibrium (Hasbrouck, 2009). Gårdängen (2005) describes this dimension as the ability of returning to market constancy, stability price and the capacity of which the market can nullify imbalance. Tightness is another dimension. Söderberg (2009) describes it as the lowest paid price for a security versus the highest paid price for the same security. This can be regarded as the breadth, as variation in prices as seen as an indicator of how liquid an asset is.

The dimension of breadth can also described as the bid-ask spread. The bid-ask spread specifies the two extreme values between the point of the aptly value of the security. Liquidity in itself is not a variable that can be observed, but it can be measured through the use of tools such as the bid-ask spread (Acharya et al, 2005, p.385). The bid-ask spread works as a proxy to measure liquidity for short time series, but is not efficient enough to be used to study longer effects such as expected return. The bid-ask spread is based on microstructure data, which we will explain further in the theoretical framework section of our thesis.

As investors expect different levels of liquidity in portfolios depending on the circumstances of the specific market, the total friction in the market reflects the theoretical performance of a portfolio. Friction is constituted by boundaries and restrictions that affect the above mentioned dimensions of the traded asset. In a theoretical frictionless market situation, a market where no boundaries and restrictions exist, full liquidity is expected of the traded securities. Thus, the concept of illiquidity is used to describe a situation where barriers hinder investors of managing the most effective portfolios. The free flow of assets is hindered by barriers in the market. These barriers are any arbitrage that obstructs the market of being characterized as frictionless and perfect, but also by factors such as the bid-ask spread and contractual restrictions reduce the liquidity of securities.
If a certain investor is not satisfied with the expected illiquidity connected to an investment, a liquidity premium can be attained to even the imbalance of the portfolio. A liquidity premium can generally be defined as a premium which investors demand when any given security cannot be easily converted into cash, and converted at a fair market value. Thus the premium raises the expected price of the total portfolio for the benefit of a higher level of liquidity. When the liquidity premium is high, then the asset is said to be an illiquid asset (Chan et al, 2003 p.556).

The size of the company also has an effect on the way the company’s stocks are affected by market illiquidity, which should be appreciated in the level of the liquidity premium. This is called the small firm effect and the sensitivity is reflected in the way that the smaller companies often are seen as more illiquid than larger companies (Amihud, 2002, p.47). In the recent decades, there has been an increased interest in the illiquidity concept. The impact of illiquidity in the asset market has been questioned as well as the question of how illiquidity could be priced correctly.

1.2 Previous research

Contemporary academic exploration concerning the concept of liquidity began with an academic study made by Amihud and Mendelsohn (1986). Their research was made with data collected from the New York Stock Exchange, spanning over 19 years between 1961 and 1980. The research produced viable results in defining liquidity as a component effecting overall share price through evaluating the bid-ask spreads (Bodie et al, 2002, p. 280). The study was pioneering because it validated the existence of illiquidity premiums in asset trading markets; also revealing shortcomings in current models such as the CAPM. The study also suggested that smaller firms are more sensitive to illiquidity and changes in the market (Amihud, 2002, p.53). This effect will be further explained in the theoretical framework chapter.

The Capital Asset Pricing Model (CAPM) was originally introduced by Sharpe (1964), Lintner (1965) and Mossin (1966). At the time, CAPM was the one of the main models used for appraising expected returns of stock investments, and it still is today. CAPM uses a single factor, Beta, to compare a portfolio with the market. The CAPM does not however include a variable which evaluates the effect of illiquidity on expected returns. In order for the CAPM to reflect reality, a set of assumptions needs to be fulfilled. For structural purpose, these assumptions and the theoretical implementation of the model will be described in the theoretical framework chapter of the thesis.

Additional critique of the CAPM was presented by a study made by Fama and French (1992) when they dismissed the accuracy of which the variables in the CAPM reflected the real empirical conditions. The Fama and French three factor model entailed several new dynamics which seemed to better able to explain the empirical asset pricing (Miralles et al, 2005, p.256). For structural purpose, the implementation of the Fama and French three factor model will be described in the theoretical framework chapter of the thesis.
In a later study, Amihud (2002) used the ILLIQ-measure, an extended equation based on CAPM, to validate the pricing of illiquidity based on a ratio of average daily stock returns. This was an important study, first and foremost because of the objective of explaining the connection between stock return and the liquidity of the whole market. It was also pioneering because it confirmed the proposal of measurable illiquidity on the NASDAQ stock exchange and proved that expected stock returns are an increasing function of expected liquidity. The ILLIQ-measure will be further explained in the theoretical framework part of the thesis.

Amihud can be seen as one of the initiators of contemporary illiquidity research, by verifying the ILLIQ-measure on empirical data and creating the roadmap of which many of today’s studies regarding pricing of illiquidity are based. Various studies have in recent years been conducted to explain the connection between market wide illiquidity and stock returns in various stock exchanges around the world.

Hu (1997) performed a study based on data collected from the Tokyo stock exchange between the years of 1976 and 1993. Based on the research made Amihud and Mendelsohn (1986), the study aimed to research if liquidity could be measured and how this could be done along with the exploration of trading turnover and its relation to liquidity. The research supported the theory that the expected return of a security can be seen as a function of turnover and that the turnover can be seen as a measure of trading frequency (Hu, 1997, p.16).

Pástor and Stambaugh (2003) performed a study on both the New York Stock Exchange and the American Stock exchange with data collected from 1966 to 1999. The research purpose was to analyze the concept of market wide liquidity and research of its significance in asset pricing by using the Fama and French three factor model. The results from the research confirmed the theory that the stocks were sensitive to changes in liquidity, stock which were sensitive to liquidity generated a higher return. The relationship functioned cross-sectional as it was appreciated to effect stocks differently depending on the innovations in aggregate liquidity. (Stambaugh et al. 2003, p.683)

Acharya and Pedersen (2005) performed a study on the New York Stock exchange and the American stock exchange through data collected from between the years 1964 to 1999. The research showed how illiquidity affected the stock return. With a theory based on the ILLIQ-measure from Amihud (2002) they could analyze the stock return and validate their theory. According to their study, a higher level of expected illiquidity increases the expected return of a stock. The covariance between the security’s expected return and the expected illiquidity varied depending on the expected market illiquidity (Acharya et al, 2005, p.405).

Chan and Faff (2005) used the Fama and French three factor model to examine the connection with share turnover and if illiquidity attracts a premium in equity markets. The authors investigated the asset pricing role of liquidity by using Australian Stock exchange financial data from between 1990 to 1998. The empirical research showed that the characteristics of liquidity were priced on the Australian stock exchange. The market could be measured
accordingly to the theoretical framework that constituted Fama and French model (Chan et al, 2005, p.456).

Martinez et al, (2005) used the ILLIQ-measure from Amihud (2002) together with two liquidity risk factor models to perform a research on the Spanish Stock Market. The study aimed to analyze how well the average returns varied cross sectional with regards to the used models and was based on data collected between 1993 and 2000. The study showed, according to Martinez, that market wide liquidity has a large effect on the performance of assets and should be should be taken into consideration when pricing securities. (Martinez et al, 2005, p.102).

Miralles and Miralles (2005) conducted a research where a liquidity risk factor was generated for the Spanish stock exchange over the years of 1994 to 2002. By using an illiquidity ratio based on theories from Amihud (2002) and the Fama and French model, Miralles added to the research field when the same empirical results could be compared to the studies of the U.S. stock markets. The empirical study supported the general notion of the role of illiquidity when pricing assets (Miralles et al. 2005, p.265).

Söderberg (2007) performed a study on the Scandinavian stock exchanges including measures presented by Amihud (2002). The study aimed at examining the relationship between returns, trading activity, liquidity and the instability of the market through data collected from between 1993 to 2005. The study indicated that liquidity was more dependent on trading activity on Scandinavian stock markets, due to the fact that they to a high extent are order driven. This compared to the American exchanges that had been the base of previous studies, which were not characteristics by the same market structure (Söderberg 2007 p. 1). Söderberg (2007) found collective evidence to support the theory of a measurable market wide illiquidity in the Scandinavian stock markets.

1.3 Problem statement

The previous research mentioned earlier are conducted in American, Japanese, Australian, Spanish and Scandinavian stock exchanges and the measure of illiquidity on expected returns are researched using different sets of models. As argued for earlier, liquidity is not an observable variable; it needs to be estimated using specific measures (Acharya et al, 2005, p.385). If illiquidity were not priced on the market, it would not be an important determinant in the expected returns. As the studies show, illiquidity may be observable through the use of proxies and measured by the use of market data. Thus, the essential problem statement can be stated twofold:

- The problem of measuring illiquidity which is not an observable variable in itself.
- The problem of determining whether or not the market price correctly reflects illiquidity in the expected stock returns.
To find a solution to the formulated problem statements, we will form a research question based on the theoretical background. The thesis serves to answer the research question through researching data collected from the Danish stock exchange.

1.4 Research focus

Studies into the Scandinavian stock markets are few and far between, although some academic research has been conducted. Söderberg (2007) studied market wide illiquidity in the Scandinavian stock exchanges (Söderberg, 2007, p.52). Using the theoretical models presented by Amihud (2002) on Scandinavian conditions, as applied by Söderberg (2007), justified the implementation of the models on the Copenhagen stock exchange. The models are validated through the research results, supporting the theoretical frameworks.

Contemporary studies have been made by students in both Sweden and Denmark, both based in theory regarding the measure of illiquidity. The Swedish students base their research on the theories presented by Amihud (2002) in a study of stocks on the OMX Nordic stock exchange in Stockholm.(Lindqvist and Du Rietz, 2010). The Swedish students results supported the proposed pricing of illiquidity in Swedish stocks. The Danish student used the relative bid-ask spread and the turnover rate in research on the Copenhagen Stock exchange.(Dalgaard, 2009) The results from the thesis provided an indication of the relationship between liquidity and stock returns in Denmark, thus investors expect higher returns for less liquid stocks. We make a note of the fact that the research presented by students in thesis form is not a reliable source of data, as it has not been peer reviewed. We present the students thesis topics as a way of providing the reader with an idea of current illiquidity research in the Scandinavian market.

We believe that a research of a Scandinavian stock exchange based on the theoretical foundation of Amihud’s ILLIQ-measure would be a suitable subject for a thesis. The implementation of models to measure expected return, and the results that are gained from the studies, show that there is theoretical possibilities of implementing the same research structure into other stock exchanges, and viable empirical results can observed. Our choice of market is one of the top stock exchanges in Scandinavia. The Copenhagen Stock Exchange is the 29th largest stock exchange in the world in terms of volume of shares traded and the 26th largest in terms of market capitalization (Söderberg, 2007 p.5). Extensive cooperation has been facilitated amongst the Scandinavian stock exchanges during the last two decades. Through a strategic alliance called NOREX, a greater level cooperation has been facilitated between the stock exchanges in Scandinavia and the Baltics (Söderberg, 2007, pg.6). A further increasing development into cooperation has been made during the past ten years in terms of merging the separate marketplaces. In table 1 below, we show a brief timeline description of events of the Copenhagen stock exchange in relation to the OMX Nordic market.
1808  Copenhagen Securities Exchange (non-profit organization) starts trading.

1996  Copenhagen Stock Exchange becomes a limited company. The Tallinn Stock Exchange was opened in May for trading with 11 securities listed.

1998  Merger of OM and Stockholm Stock Exchange -> OM Group. The exchanges in Copenhagen and Stockholm entered into a strategic alliance named NOREX Alliance. The NOREX Alliance was unique by being the first stock exchange alliance to implement a joint system for equity trading and harmonize rules and requirements between the exchanges with respect to trading and membership. Vilnius Stock Exchange became a public limited company.

2005  Merger of OMX and Copenhagen Stock Exchange. Alternative market First North started. Foreign equities listed for the first time on ICEX.


2007  First North launched in Denmark and Finland. Iceland iSEC becomes First North. OMX acquires Armenian Stock Exchange. NASDAQ acquires OMX and the NASDAQ OMX Group is born.

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Table 1: History of the Copenhagen Stock Exchange in relation to OMX Nordic

As of 1st December 2004, the Copenhagen Stock Exchange is a part of OMX Nordic along with the stock exchanges in Stockholm, Helsinki, Iceland, Tallinn, Riga, and Vilnius (www.nasdaqomxnordic.com). The Scandinavian stock exchanges can be characterized as being highly order driven. Order driven markets are characterized in the way that the bid and ask prices are directly related to the actions of buyers and sellers. This is in contrast to e.g. quote driven markets where market makers set the bid and ask prices. The Scandinavian markets are not purely order driven through the influence of e.g. designated liquidity providers.

Prior to the Copenhagen Stock Exchanges partnership with OMX Nordic, designated liquidity providers were not used in the Copenhagen stock exchange. Designated liquidity providers are defined as “[…] providers of immediacy in the market place.” (Mann et al, 2002, pg. 1). The function of these designated liquidity providers are to maintain a high standard of market quality which can be measured by spread, depth and time. Glosten (1989) states that designated liquidity providers may be able to prevent market failures through the supplying of liquidity during periods when the limit order book is thin. Figure 2 below shows the market capitalization of the entire Nordic OMX from the years 2009 and 2010. From this diagram we can see that Stockholm has the largest market capitalization in the Nordic OMX followed by the Copenhagen stock exchange, which is closely followed by the Helsinki Stock exchange. We can see the relative size and market importance of the Copenhagen Stock Exchange in the Nordic region.
1.5 Research question

For our research question we shall utilize the Fama and French three factor model (1992) and the CAPM in collaboration with Amihud’s (2002) ILLIQ-measure to investigate if illiquidity is priced on the Copenhagen Stock Exchange. A sample of listed stocks in the OMX market constitutes our sample. Based on the presented theoretical background and the problem statement, a short and concise research question is constructed:

- Is illiquidity priced in the Copenhagen Stock Exchange?

Our research question will be answered through a formulated research hypothesis. The hypothesis will be introduced after a presentation has been made of the theoretical framework. The need of constructing the hypothesis after the theoretical frameworks is favorable; mostly because of the highly technical aspects that constitutes the models and the research. After the theoretical framework has been presented, the hypothesis will serve as a counterweight to confirm or dismiss the adaptation of the theoretical models on to the empirical conditions of the Copenhagen Stock Exchange.

1.6 Research purpose

When testing the empirical evidence against our formulated research hypothesis, we will be able to provide the empirical evidence regarding the pricing of illiquidity in the Copenhagen Stock market. The goal with the research is to achieve viable conclusive results that confirms or dismisses the pricing of illiquidity on the Copenhagen Stock Exchange. By using already proven research methods on previously non-researched data, distinctions in the results will be used to reflect the empirical reality of the research field, which can be compared to previous studies.
1.7 Relevancy

Any additions of empirical studies, which are based on empirical data and share the same models of evaluation as previous studies, will be a contribution to the research field. The thesis can make a contribution to further understanding of the pricing of illiquidity in the Scandinavian markets, focusing on the Danish market. This result and contents of this thesis can be of use to bachelor and master’s student, portfolio managers and risk management practitioners who all require an understanding of the dynamics and characteristics of illiquidity in their everyday working environment. In addition, investors, listed firms and regulators may also be interested in the results of the study.

1.8 Delimitations

We aim to provide empirical evidence Danish listed OMX companies, spanning over the time scope of eight years between the first of April in 2003 and the last of March in 2011. The methodological choice of this sample is further reasoned for in the practical methodology chapter. Due to our limited time scope, automatically securities that were traded before the selected sample period will be excluded as well as securities traded after the last of March.

We needed to concentrate on companies which were started before April 2003 and were still operational before April 2011. Our initial sample covered all the Danish stocks listed in the Danish OMX market; these were small, medium and large cap companies. We shall show in the proceeding chapters how we selected and delimitated stocks which did not meet certain criteria in our practical methodology chapter.

The data collection and analysis of this type of study is an extensive one; the amount of data takes its toll on the researcher as it time consuming. We had to be consequent and choose a research methodology that fitted the time frame. We could have chosen to analyze a longer time series of data to extend the research. The historical relevance of analyze would have been improved, but then we would have had to cut back on time spent on other important aspects of the thesis to finish the research within 8 weeks. We could also have chosen to select a smaller sample, but the validity would then be suffering as the sample would not have been a representative sample of the market.

1.9 Disposition

To present the research process and findings in the most favorable way, a good thesis structure is important. In the first chapter, we have introduced the topic to the reader and the general research direction is argued for. The thesis and research presentation will be structured using the following disposition;

Second chapter; Research methodology – In the following chapter we present the methodological approach that is used in the thesis process and define the assumptions and criteria required for this type of business research. E.g. the choice of topic, chosen research philosophy and research strategy is presented.
Third chapter; Theoretical framework – In this chapter the financial theories which represents the foundation of our research are presented. We present important research models and theories such as the Capital Asset Pricing model and Fama and French 3 Factor model, two of the essential theoretical models used in our research.

Fourth chapter; Practical methodology – The fourth chapter entails the chosen methods we use to perform the research. E.g. the process of data selection is dealt with and the validity and reliability of the research are discussed. The practical research method process is illustrated step-by-step to explain the practical research approach made by the authors throughout the research process.

Fifth chapter; Empirical results & analysis – In this chapter the results of the research are presented, based on the collected data from the practical methodology chapter. The formed hypothesis is tested through analyzing models and theories presented in the theoretical framework chapter.

Sixth chapter; Conclusions & contributions – This chapter evaluates the results of the empirical study and we draw conclusion from the research findings. Along with the conclusion, we discuss around the contribution our research will provide to the research field.

Seventh chapter; Further research – The last chapter give recommendations for further studies within the research field. Examples are suggested that from our perspective are suitable for potential scientific studies.

1.10 Term clarifications

Throughout the thesis certain words and expression will be used to support the explanation of the theoretical framework on which the research is based. We think that is favorable to present a short list of clarifications to describe some of the terms that might be unknown to the reader.

**Asset portfolio** – An assembly of securities, grouped together by the decisions of an investor.

**Information asymmetry** – An economic term which refers to situations in which potential sellers have knowledge on products which buyers do not.

**Proxy** – An instrument used to measure and provide information about a variable that in itself is not observable.

**Securities** – A denomination of equity e.g. stocks, bonds and other financial assets.

**Stock exchange** – A marketplace where investors can buy and sell securities.

**Covariance** – A measurement of the amount to which returns on two assets move in connection with each other.
2. RESEARCH METHODOLOGY

Our theoretical method chapter provides a brief description of our chosen methodological assumptions, scientific approach and research method combined with criteria required for business research. This is done in order to provide the reader with an understanding of our philosophical positioning and the processes which our research has undergone which leads us to our empirical findings. Hopefully this will help the reader to criticize our research in a constructive manner.

2.1 Choice of topic

Early on, we were in agreement regarding what we thought would be an interesting field of research for the thesis. We both wanted to study the extensive research field of security markets and financial assets. This was mainly due to the fact that both shared a common interest in the functions of stock markets and the trading of financial assets. A search for a topic that was current and relevant within the field of financial research began.

After an initial process of searching the Internet, academic journals and newspapers for ideas, the definite choice of topic was selected. The initial research proposal included the involvement of KPMG, an international company within the business branch of audit and tax advisory services. KPMG’s Stockholm offices had shown interest in researching the illiquidity premiums of financial markets and they were looking for students who could conduct such research on Swedish conditions. Both parties were interested in collaboration and initial contact was established. The thesis would investigate whether illiquidity premiums should be connected directly to the company or if a general premium should be applied on illiquid securities.

The collaboration with KPMG was unfortunately cancelled prematurely due to staffing restrictions at the company level. No longer bound by the structural limitations as we would have been through a further collaboration with KPMG, we broadened our perspective to search for possible extensions in the research field. The pricing of illiquidity was found to be a topic which had not been researched to the same extent in other Scandinavian countries, compared to Sweden. Diverging from KPMG’s original thesis structure, we developed one in which we would analyze the conditions in Denmark. We further studied previous research from USA, Australia, Japan, Spain and Scandinavia, compared the theoretical application of theories and models, and finally decided to focus our research on the Copenhagen Stock Exchange. However, it is essential to acknowledge the significance in the early communication between us and KPMG as largely influencing the final choice of thesis topic.

2.2 Pre understanding and preconceptions

Pre-understanding is an important factor to consider when selecting a topic, it provides the researcher with previous knowledge, insight and experience on which to base observations (Bryman et al, 2007, pg. 429). We both share a common educational platform as students who study business administration at Umeå University. We are both interested in models and
methods that are used in asset-pricing. Furthermore, we have become more and more interested in the role of liquidity connected with stocks, bonds and other securities. During our years as students of finance, we have read course literature and journals describing advancements within the research field and we have both created a theoretical background for analyzing such a topic. Additionally, one of the authors is trained in the area of evaluating financial data, which is an added advantage when the data management is of an extensive nature.

Preconceptions refer to experiences and values which may affect objectivity. We as authors must be aware of our preconceptions and make sure that they do not affect the overall objectivity of our thesis or add bias to results or conclusions. A researcher must remain objective and not let previous experiences or values cloud judgments or assumptions (Bryman et al, 2007, p.30). Preconceptions can be further classified into both practical and theoretical preconceptions. Practical preconceptions are relevant in our case with regards to the fact that we have both had experience in buying and selling stocks. We are aware of the illiquidity premium and its importance in determining stock prices and returns. We will however remain objective in our research so as not to exclude other important factors which may be relevant to the illiquidity premium. With regard to the theoretical aspect of preconceptions, we both have theoretical knowledge of classical finance theory as well as financial models. We must be aware that we must only use academic theoretical information and not base observations from personal point of view; otherwise we run the risk of biasing our thesis towards their ideas and concepts.

Our experiences within the research field will automatically affect the outcome of the thesis. The effects of this can be of a dual nature, positive in the sense that our contextual knowledge will make it easier for us to conduct a research within this field. Negative in the sense that it may affect how we choose to conduct the research, not evaluating all the possible ways of researching the subject. Thus, our interpretation and view of reality may be a contributing factor to the final results drawn from the research.

2.3 Research philosophy

2.3.1 Ontology

Ontological orientation refers to how the researchers interpret reality, how they perceive reality and its existence and how people perceive and influence the social reality (Bryman et al, 2007, p.33). Ontology is divided into two main viewpoints, objectivism and constructionism. Objectivism can be described as a view of social reality as an external objective reality. The orientation stresses that social reality s independent from the existence of social actors (Bryman, et al, 2007, p. 33). An example of this orientation can for example be adapted on the structure that constitutes a football team. Every player is a part of this big entity and the players are bound to the team by playing different positions. The objectivistic view of the team is thus that every player’s role is subordinate to the function and performance of the team.
Constructionism on the other hand, states that social phenomena and their meanings are constantly changing through social actors (Bryman et al., 2007, pg.23). In the example of the football team, the interest is focused on the individual player. The analytical emphasis is put upon that single player and how the decisions of the individual players affect the performance of the team. If a certain player scores, it is because of the fact that he himself created that opportunity to score. This is in contrast to the objective view, which would emphasize that the team scored. The player that scored was able to do so because he is the forward, and according to team structure should be the one that scores.

Based on the same argumentation, our research will be of an objective research orientation. We can apply the above used way of analyzing reality upon our subject of research, the Copenhagen Stock Exchange. We will gather an extensive amount of quantitative data to analyze and draw conclusions about the market through use of models. We want to analyze aspects of the market through the performance of the entity and from these draw market conclusions as a whole. The constructionist approach would instead be to pick out information about one single security, at one specified date and time. Through these variables, one would make generalizations about the functions of the market as a whole.

2.3.2 Epistemology

Epistemology is a theoretical knowledge based existential approach to research philosophy, questioning the notions of what can be seen as relevant knowledge within a field (Bryman et al., 2007, p. 27). Thus the researcher’s view regarding what is real or not and the knowledge about what can be seen as a valid is an important aspect of the research philosophy. Epistemology can be described as consisting of two main directions; positivism and interpretivism. Positivism refers to the application of natural science philosophy to the study of social reality (Bryman et al., 2007, pg. 730). Positivism is generalizing; entailing that only those factors that are observable can provide reliable data and results in a research. The philosophy entails the viewpoint of research defined by objectivity and independency from social actors and external factors (Saunders et al., 2009, p. 119).

The other direction is interpretivism. This direction rejects the norms of the natural science model and looks at how individuals interpret their social world (Bryman et al., 2007, p. 28). Interpretivism can be described as a direction that is subjective to meanings and social factors, focusing upon details of a situation and the reality behind these details. The direction is emphasizing subjective meanings and is highly interested in the motivating factor of actions in research. (Saunders et al., 2009, p. 119)

In consideration of the explained research directions above, our research will be based in a positivistic research philosophy. Our research is defined by an independence from the subject and the initial theoretical background and assumptions are not affected by the results of our research. We will be studying the empirical reality of a financial market through collected data from the Copenhagen Stock Exchange. Theories and models are implemented in the research to display the reality of the characteristics which constitutes the financial market place. With a research which reflects principles of positivism, you make detached
interpretations regarding data which is collected in a value free manner (Saunders et al, 2008, p.85).

2.4 Theoretical research method

A positivistic research philosophy is closely related to a deductive research approach. The deductive theory approach is based on the common view on the nature of the relationship between theory and research. The emphasis tends to be on a highly structured form of methodology in order to facilitate replication and quantification (Gill et al, 2002, pg.34). The inductive research approach is, on the other hand, closely related to the interpretivistic research philosophy. This research method is characterized by an interpretation of a subject, rather than the explanation of it. The distinction between the two approaches is illustrated in Table 2 below.

<table>
<thead>
<tr>
<th>Deductive</th>
<th>Inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structured approach</td>
<td>Flexible/adaptable approach</td>
</tr>
<tr>
<td>Quantitative data</td>
<td>Qualitative data</td>
</tr>
<tr>
<td>Generalize from relevant sample size</td>
<td>Less focus on need to generalize</td>
</tr>
<tr>
<td>Researcher independence from study</td>
<td>Researcher involvement in research process</td>
</tr>
</tbody>
</table>

Table 2: The difference between deductive and inductive approaches to research
(Adapted from Saunders et al, 2008, p.91)

For the purpose of this master thesis, we will be utilizing a deductive theory approach. A deductive approach emphasizes the relationship between theory and research and importance is placed on the testing of already developed theories. (Bryman et al, 2007, pg.11). We will develop a hypothesis based on what is known about a particular subject. We shall then subject the hypothesis to empirical analyses in order to conclude if our hypothesis is valid or not. Robson (1993) lists five steps within the deductive research process; (Robson, 1993, pg.45)

1. Forming a hypothesis from theory
2. Expressing the hypothesis in relevant operational terms
3. Testing the hypothesis
4. Examine the result of the hypothesis
5. If required modify the theory with regard to the theory

With regard to our thesis, we wish to test already established theories using our specific data; we do not want to create or develop new theories. The concept of utilizing a deductive approach requires that generalization is utilized in order to be able to make generalizations
about regularities in general human behavior through the use of an adequate sample size (Saunders et al, 2008, pg.88).

2.5 Research strategy

The choice of research strategy is closely related to the choice of theoretical research method. As illustrated in Table 2; the deductive research approach is best utilized through a quantitative research strategy whilst the inductive research approach most commonly is facilitated through a qualitative research strategy. This research strategy is supported by our argumentation in the areas of research philosophy and chosen theoretical research method.

We have chosen a quantitative research method for our paper, however we do not wish to make the generalization that quantitative research is best for financial data. We had considered conducting qualitative research in the beginning of our thesis; however changes in the structural layout of the thesis prevented such a method of being used. A quantitative strategy was the best option for us; given that we wanted to test already established models and that we have raw numerical data for our analysis. We also did not wish to develop new theories, but merely test existing theories. Thus, our strategy of choice is a quantitative research approach, which emphasizes quantification in the collection and analysis of data (Bryman et al, 2007, pg.28).

2.6 Choice of theories

Our thesis is based on the already developed theoretical models which constitutes the measures of illiquidity. Current research models constitute the base of our theory choice. In order to show the effect of illiquidity on market wide asset-pricing, we need to study various theories which are relevant to the topic. We will describe and show the relevance of all the theories we include in our thesis. Some of the theories which we will include are the CAPM as well as Fama and French (1993) three factor model as well as the efficient market hypothesis and microstructure theory which is connected to the illiquidity premium.

Our thesis is based on the Illiquidity premium and in order to show the effect of the illiquidity premium we need to present various theories which are relevant to the topic. We will describe and show the relevance of all the theories we include in our thesis. Some of the theories which we will include are the CAPM as well as Fama and French (1993) three factor model as well as the efficient market hypothesis and microstructure theory which is connected to the illiquidity premium.

2.7 Selection of sources

Concerning our work in finding and collecting relevant theoretical material for our study, we have used the search engines and databases provided by the Umeå University library website. We have utilized both scientific articles and financial literature from the library of Umeå University. The use of scientific articles is important as they fuel the researcher’s knowledge as well as create ideas for potential research topics (Saunders et al, 2008, pg.58). We have made use of primary, secondary and tertiary sources for our thesis. Primary sources can be
defined as reports, theses and company reports. The primary sources which we utilized were financial year end reports for our chosen amount of company stocks. Secondary resources which we utilized were books, peer review academic journals and internet sources. Tertiary sources are encyclopedias, catalogues and abstracts.

The databases for scientific articles have been the primary contributor to the theoretical foundation as well as a source of previous research. Peer reviewed academic journals are evaluated by academic peers prior to release, in order to assess their quality and suitability (Saunders et al, 2008, pg.51) To find out the latest research regarding Illiquidity, we started to search for scientific articles using the databases in business field. There have been two databases that we have used and where all our articles derive from and they are; Business source premiere and Emerald. The keyword we have used in the search for the scientific articles are: CAPM, illiquidity premium, liquidity and microstructure theory in different combinations.

The theories we have applied to provide background information on the topic of illiquidity mostly consist of student literature that we have found through the search engine ALBUM at Umeå University Library website. The use of keywords helps to define and focus subject matter. Keywords are basic terms which describe the research question and objectives which will be utilized to search the tertiary sources (Saunders et al, 2008, pg.57). The keywords used here to find relevant literature are the same we used as in the search for articles. To find literature of methodological nature, we used keywords such as Illiquidity, CAPM, Fama and French, microstructure theory and bid-ask spread in different combinations.

2.8 Source criticism

The theoretical foundation for our research is based on theories and previous research made in the field of asset liquidity, illiquidity linked to asset pricing and the research of such measures in an empirical setting. Throughout the thesis process, a critical evaluation of the studied resource material has been facilitated by the authors to ensure the quality of the contribution to the research field. Concerning scientific articles, contribution has been made possible by a strict use of only recommended scientific databases provided by the Umeå University library. Saunders (2008) makes a point that bias can be contained even in peer reviewed journals and that caution must be taken with regard to the bias of the author (Saunders et al, 2008, pg.51). Regarding information gained from literary sources, the selected source material has been attained from either course material from our previous financial studies, or additional published literature within the research field. No literature has been used that was neither beneficial in our field of research, nor distributed by anyone other than publishers within the academic field of literature. The literature should thus be regarded as adequately trustworthy as a source for the theoretical background.

Due to the relatively narrow research subject the absolute amount of the base for the theoretical framework has been constructed through the use of academic research presented in articles, whilst parts of the theoretical background have been attained through literary review,
as stated earlier. The distinction is of a somewhat involuntary nature, when most of the presented finding is published in articles, and the type of chosen research methodology for our thesis is heavily based on previous studies made on empirical conditions. The markets selected for empirical studies have been far apart in a geographical point of view, thus the theoretical framework has been adapted on markets that may be slightly characterized differently. The use of one specific theoretical approach made to describe illiquidity in one specific market may generate a different outcome when applied on another market. This is because of certain idiosyncrasies and distinct conditions that characterize the specific market place. It is therefore important to strictly follow the practical methodology of the chosen research to avoid mixing the research with another one. By doing so, differences in compared markets can be characterized as market specific, and not seen as possibly derived from a possible mixture of theoretical conditions rendering the empirical study biased. By fundamentally sticking to this methodological approach, subjectivity influencing the empirical study by organization the source material in an incorrect fashion is avoided.

2.9 Summary of applied research methodology

A structured research methodology is vital when conducting academic research. As stated in this chapter, a lot of variables have to be taken into consideration in the development of the thesis. This mainly due to the extensive methodological framework, this constitutes the research into social science (Bryman et al, 2007, p.42). There are a number of philosophies, strategies and methods that can used to base a research or study upon. We have been extensive in describing the methodological point of departure to assure that the highest level of quality of the research. Below we will summarize the key conclusions of the chapter to give a definite overview of the chosen and applied research methodology.

- Our topic is based on the **pricing of illiquidity** on the Copenhagen Stock Exchange in Denmark.
- From an ontological standpoint, **objectivism** characterizes our research orientation.
- From an epistemological point of view, our research philosophy is defined by **positivism**.
- We are using a **deductive research method** to test a formulated research hypothesis.
- Our research will be executed through a **quantitative research strategy**.
- **Contemporary research models** constitute the base for our choice of theories.
- The **databases for scientific articles** are the primary contributor to the theoretical foundation.
3. THEORETICAL FRAMEWORK

Financial theories are presented in this chapter in order to provide the intellectual basis for models which will be presented later in our analysis. The theoretical framework will serve as the basis for empirical analysis of the collected data from Copenhagen Stock Exchange.

3.1 The Efficient Market Hypothesis and small-firm effect

The Efficient Market Hypothesis states that the current/latest price of a specific asset incorporates all the relevant information into its price (Howell et al, 2008, pg.573). In other words, the way which the market is able to incorporate new information quickly into the security price is the key aspect of the hypothesis. The EMH can be separated and formulated in three different forms, depending on how the degree of the available information is assessed. (Bodie et al, 2002, p. 342) Table 3 below shows the 3 different forms of the Efficient Market Hypothesis.

<table>
<thead>
<tr>
<th>The weak-form hypothesis</th>
<th>All stock prices reflect all market information available through the history of prices. There is no need of analyzing trends.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The semi-strong-form hypothesis</td>
<td>All public available information is reflected in the stock prices. In addition to past prices, information about the companies has an effect on prices.</td>
</tr>
<tr>
<td>The strong-form hypothesis</td>
<td>All information is reflected in the stock prices. In addition, information only available to company insiders is reflected in the prices.</td>
</tr>
</tbody>
</table>

Table 3: The three different EMH-hypothesis

Adapted from (Bodie et al, 2002, p. 342 – 343)

To summarize, the three different versions of the EMH all states that the prices of stocks are affected by all the available information on the market. The difference between the three is stated in whom has got access to all the information. The small-firm effect is a contradictory addition, closely related to the Efficient Market Hypothesis. It details a hypothesis in its own, closely connected with the characteristics and sensitivity describing how shares listed by smaller companies are affected actions in the market (Bodie et al, 2002, p. 360). In short, the hypothesis states that the shares of smaller companies generate higher annual returns than shares issued by larger companies. This because investing in the shares of smaller companies tend to be riskier, since they are less frequently traded, thus more sensitive to new market information.

The small-firm effect is not a variable that needs to be evaluated to answer our formulated research question. But if it can be observed in the data, it can be a variable explaining a possible correlation between company size and the illiquidity ratio. Thus it is an interesting variable to measure and to be included in the theoretical framework and empirical analysis.
3.2 Modern portfolio theory

The primary purpose of including modern portfolio theory is to provide the reader with the origins of the CAPM. Sharpe (1962), Lintner (1965) and Mossin (1966) all based their CAPM research on Harry Markowitz’s “Portfolio Selection” (1952). Markowitz (1952) questioned the concept of investors only considering maximizing return in their portfolio selection. Markowitz focused on the effect of risk in investor’s portfolios (Markowitz, 1952, pg.77). The development of the Markowitz mean paradigm is considered to be one of the primary theories on which modern portfolio theory is based. The Markowitz paradigm is often referred to as dealing with portfolio risk and expected return. The Markowitz paradigm is based on central concept which states that all the relevant facts about a portfolio of risky assets which are relevant to an investor can be summed up in the values of two specific parameters; namely standard deviation and the expected value of a portfolio's return (Sharpe et al, 1999, pg.845).

Standard deviation is defined as a measurement of the dispersal of potential outcomes based on an expected value of a random variable (Sharpe et al, 1999, pg.140). Standard deviation is used by investors as a measurement of the risk of a fund; it is a measurement of the volatility of a fund. The expected value of a portfolio's return refers to the return on a security/fund which investor expects over a specific period. Securities with high systematic or unsystematic risk should according to the CAPM have higher expected return to account for the higher risk (Sharpe et al, 1999, pg.240).

3.3 Capital Asset Pricing Model (CAPM)

The CAPM was developed by Sharpe (1963), Lintner (1965) and Mossin (1966) and is an asset pricing model. In short, the CAPM states that the rate of return on a specific asset will be equal to the risk free rate of interest as well as a risk premium, variables depending on the market price of risk and the quantity of market risk contained within the asset (Howells et al, 2008, pg.192). For this to be empirically measurable, a number of factors have to be stated based on assumptions of the model. As previously stated, the study made by Amihud and Mendelsohn (1986) revealed shortcomings in the CAPM, the critique, mainly directed at the set of these assumptions that was required to be met for the model to hold true;

| First assumption | The market can be characterized as having perfect competition, no investor is bigger than the other and every investor is small compared to the market as a whole. |
| Second assumption | All investors plan to invest within the same time frame |
| Third assumption | The assets are characterized as being publicly traded, investments do not involve private or governmentally owned assets. Investors can always borrow and lend to a risk free rate |
Illustrated in Table 4, two of the assumptions are of particular interest in our research; no viable transactions costs and unlimited borrowing and lending at risk-free rate. If there would be no transaction costs, the market could be characterized as frictionless. We know that in the real world a frictionless market does not exist, the assumption is violated through the existence of bid-ask spreads, trading costs and information asymmetry. If it did then liquidity as a concept would not exist, as mentioned in our background chapter. In mathematical terms, the CAPM is expressed according to Formula 1

\[
E(r_i) = r_f + \beta_i \times (E[R_{Mrk}] - r_f)
\]

(Formula 1)

- \(E(r_i)\): Represents the expected return of the investment
- \(r_f\): Represents the risk-free interest rate.
- \(\beta_i\): Represents the Beta of the investment in a market portfolio
- \((E[R_{Mrk}] - r_f)\): Represents an equity market premium.

The risk free rate is denoted as a security which is highly liquid, such as a T-bill, as illustrated in the first chapter of the thesis. It is used to weigh the investment as it is the least risky asset that can be traded. The Beta-value denotes the sensitivity of a security towards movements of e.g. a market index. A Beta-value of 1 indicates that the security follows the exact movement of the market index. A security more sensitive to changes in market than the market index itself is represented by a Beta-value above one. A security with a Beta-value below one is not as sensitive to changes in the index as the average market. The equity market premium can be defined as a risk premium for security which is constructed to avoid risks in the fluctuations of the market. (Berk et al, 2011, p.360)
We will utilize the CAPM model as stated in formula 1 above with an additional factor which is a proxy for illiquidity (IMV). The creation of the IMV factor is explained in chapter 4.1.4

\[ E(r_i) = r_f + \beta_i \times (E[R_{Mrk}] - r_f) + IMV_t \]  
(Formula 2)

\[ r_f \] : Signifies the risk-free interest rate.

\[ \beta_i \] : Represents the Beta of the investment in a market portfolio

\[ (E[R_{Mrk}] - r_f) \] : Represents an equity market premium.

\[ IMV_t \] : Is defined as the proxy for illiquidity

### 3.4 Fama and French 3 Factor model

Fama and French (1992) investigated empirical contradictions in the CAPM and developed their research in the area, Fama and French found that the CAPM was not able to fully price assets. According to their findings, in contrast with the theoretical structure of the CAPM, they did not support the idea of average returns being positively related to the Beta-value of the investment. (Bodie et al, 2002, p. 393) Fama and French also included the observed positive relation between average stock returns and the ratio of a firm’s book value of its common equity to its market value (“BE/ME”). Fama and French included the earnings-to-price ratio (“E/P”) that had been shown to help explain cross-section of average returns. E/P was likely to be higher for stocks with higher risks and expected returns. Fama and French shed light on the fact that firms with high ratios of book-to-market value are more likely to be in financial difficulty and are therefore sensitive to changes in market conditions (Bodie et al, 2002, pg.312). Fama and French (1992) introduced two additional factors in combination with the original CAPM. The model is in mathematical terms expressed according to Formula 3:

\[ r - R_f = \beta_3(K_m - R_f) + b_s \times SMB + b_v \times HML + \varepsilon \]  
(Formula 3)

\[ r \] : Represents the portfolio's return rate

\[ R_f \] : Represents the risk-free return rate
$K_m$: Represents the return of the entire stock market

$SMB, HML$: Represent a "small minus big" and "high [book/price] minus low"

$\epsilon$: is defined as the error term

The process for obtaining these factors will be provided in our practical methodology chapter. We will be adding an additional factor to the above formula, the additional factor will be a proxy for illiquidity, and the new formula with the additional factors is expressed as formula 4:

$$r - R_f = \beta_3(K_m - R_f) + b_s \times SMB + b_v \times HML + IMV + \epsilon$$

(Formula 4)

$IMV$ is defined as the proxy for illiquidity and is formed with the use of $ILLIQ$.

### 3.5 $ILLIQ$

For the purpose of our thesis, we shall be using Amihud’s (2002) $ILLIQ$-measure. The $ILLIQ$-measure itself is based on previous work by Kyle (1985) and Brennan and Subrahmanyam (1996), who both came to the conclusion that there was a positive relationship between transaction volume and price change. Kyle (1985) research specifically measured the price impact of a trade and indicated a positive relationship between the change of price and the net order flow (volume) which resulted from the information asymmetry in a dynamic trading environment (Kyle, 1985, pg.1335).

There are other ways in which to measure illiquidity, however these methods require microstructure data which is not readily available and does not cover long periods of time. We are aware of the limitations of the $ILLIQ$-factor as described by Amihud (2002) himself, it is less accurate than using microstructure data and is coarse and simple in its purpose (Amihud, 2002, pg.33) The advantages of utilizing the $ILLIQ$-measure in our study are twofold; firstly the data required for calculating the $ILLIQ$ is easily available to researchers; secondly it allows the researcher to check the robustness of the results (Miralles et al, 2006, pg.258). The research made by Hasbrouk (2004) shows that Amihud’s $ILLIQ$-measure is one of the best measures which captures Kyle’s lambda (Miralles et al, 2006, pg.255). (Amihud (2002) states that the $ILLIQ$-measure significantly explains stock return for NYSE stocks from 1963 to 1997. We have had e-mail contact with Yakov Amihud in the early stages of our thesis and he personally recommended the use of the $ILLIQ$-measure for the purpose of our thesis. The illiquidity ratio of stock $i$ in month $t$ is calculated as expressed by Formula 5:
\[ ILLIQ_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{R_{itd}}{V_{itd}} \right] \]

(Formula 5)

**D_{it}**: Represents the number of days for which data is available for stock \( i \) in year \( y \).

**R_{itd}**: Represents the absolute return on stock \( i \) on day \( d \) of year \( y \).

**V_{itd}**: Defined as the DKK (Danish Kronor) volume on day \( d \) in month \( t \).

The function of the \( ILLIQ \)-ratio is to show stock illiquidity. Stock illiquidity, \( ILLIQ \), is defined as follows; the average ratio of the daily absolute stock excess returns to daily (Danish Kronor) trading volume. The result of the \( ILLIQ \)-function is the absolute percentage price change per DKK (Danish Kronor) of trading volume, which is the impact of the order flow (Amihud, 2002, pg.35). A security with high liquidity is expected to have a low price impact per unit of volume traded. A security with low liquidity is expected to have a high price impact per unit of volume traded.

Research has been conducted using the \( ILLIQ \)-measure by Acharya and Pedersen (2005), Lindqvist & Du Rietz (2010) and Miralles (2006) to obtain a proxy for illiquidity. We shall describe in detail how we calculate the \( ILLIQ \)-ratio in our practical methodology chapter. We are aware that the research conducted by Lindqvist & Du Rietz (2010) is thesis based research and it presents some limitations. Their thesis work lacks peer-revision and the quality and reliability of the results cannot be ensured. We have included their findings as it contributes to the knowledge from the Scandinavian market.

### 3.6 Microstructure Theory

The primary purpose behind the concept of microstructure theory is the study of the process and results of trading specific assets under predefined trading rules (O’Hara, 2004, pg.1). Microstructure theory focuses on how specific trading mechanisms affect the price formation process in a market (O’Hara, 2004, pg.1). The study of microstructure theory is relatively new in origin; the stock market crash of 1987 spurred an interest in the study of liquidity. The reason behind the introduction of microstructure theory into this thesis is to inform the reader of the concepts behind illiquidity. We utilize Amihud’s \( ILLIQ \)-measure for the purpose of our thesis, which is a measure which is accessible through its simplicity and readily available data, however the past research chapter which we detailed in our introduction chapter is concerned with various measures of illiquidity which rely on microstructure theory. The information and data required for bid ask spread calculations is more difficult to obtain and the periods of data do not cover long periods of time. One of the topics covered through microstructure theory is the bid ask spread.
3.6.1 Bid ask spread

According to Roll (1984) the concept of the bid ask spread can be defined as the small bracket of price difference between the sell price of an asset and the buy price of an asset (Roll, 1984, pg.1128). The basic principle is explained in figure 3 below; the smaller the spread between the ask price and the bid price, the smaller the liquidity. Some securities will have a higher spreads between the ask price and the bid price, which generally means that the security is very illiquid.

Figure 3: A description of the bid-ask spread

(Roll, 1984, pg.1228)

Transaction cost and timing cost are concepts which focus on transaction cost as well as the timing cost impact of transaction cost on investment returns (O’Hara, 2004, pg.13). Transaction costs include; adverse selection costs, inventory holding and cost order processing costs.

Information and disclosure focuses on the market information, transparency and the impact of the information on the behavior of the market participants. This factor has a relationship with the efficient market hypothesis, which states that the current/latest price of a specific asset incorporates all relevant information into its price (Howell et al, 2008, pg.573).

3.7 Theoretical affinity with previous research

As identified in the previous research section in the introduction, various studies have been performed using financial theories and models to explore the role of illiquidity within certain financial structures. The pioneering study made by Amihud and Mendelsohn (1986) validated the existence of illiquidity premiums and proposed shortcomings in the CAPM, in addition of identifying the small-firm effect. Through later research, Amihud (2002) went one step further and confirmed the role of stock returns as a function of expected liquidity by introducing the ILLIQ-measure. The statement is validated through research in other financial markets and conditions made by e.g. Pástor and Stambaugh (2003), Martinez et al, (2005), Miralles and Miralles (2005) and Acharya and Pedersen (2005). Additional research, supporting the existence of priced liquidity on various financial markets in different countries were made by Chan and Faff (2005), Miralles and Miralles (2005) and Söderberg (2007).
The research we are conducting is heavily based upon the theoretical foundation laid through the studies made by Amihud (1986, 2002). In the sense of finding an efficient structural research approach, the research made by Miralles and Miralles (2005) is favourable. The particular study gives us valuable insight into how to perform a study and a structural guideline in measuring illiquidity. The mentioned study made on the Spanish conditions provides a valuable and efficient structure one can use the Fama and French 3 Factor model and the \textit{ILLIQ}-measure to efficiently price illiquidity.

When looking at affinity within market conditions, the work made by Söderberg (2007) is arguably closely related to our research when factors are taken into consideration beside instigated research approach. Thus, valuable insight can be attained through his work as the market conditions on which the study is set are geographically related. As his study suggested a market wide illiquidity on the Scandinavian markets, our research will strengthen or weaken the notion of measurable illiquidity on Nordic market conditions.

### 3.8 Hypothesis statement

Based on the presented theoretical background, formulated research question and theoretical framework, we form a research hypothesis. The collected empirical data is used to validate or dismiss our formulated research question through the hypothesis. The uses of the Wald test will be used to accept or reject our null hypothesis. We will utilize a Wald test to test for significance. The Wald test is a method of testing the particular explanatory variables in a statistical model. For the purpose of our analysis we are interested in whether portfolio intercepts are equal to zero. Our portfolio intercepts are the alphas of the respective four models used in our empirical analysis section.

- \( H_0: \alpha_{IL1\ldots IL8} = 0 \)
- \( H_1: \alpha_{IL1\ldots IL8} \neq 0 \)

\( \alpha_{IL1\ldots IL8} \): represent the alpha coefficients of the illiquidity portfolios used in our thesis

If our findings validate \( H_0 \), the market is correctly pricing illiquidity in the stock returns and the \textit{ILLIQ}-measure is applicable in the CAPM and Fama & French model (1993). If our findings validate \( H_1 \), then liquidity is not correctly priced in the market returns. Our hypothesis statement will be answered through the use of our empirical chapter and the Wald Test will be utilized to accept or reject our hypothesis statement.
4. PRACTICAL METHODOLOGY

The primary purpose of our practical methodology chapter is to provide the reader with a step by step explanation of our chosen quantitative methods. By providing a clear & descriptive account of the process which will lead to our conclusion, we enable the reader of the thesis to repeat our study or conduct similar studies using our methodology.

4.1 Data collection

The data used for our quantitative analysis covers securities which are traded in the Nordic OMX stock exchange, specifically Danish traded securities on the Copenhagen stock exchange (CPH). The process which we adapted for our data collection process began with an overview of the NASDAQ OMX Nordic webpage, which provides current data on all relevant securities traded within the exchange. The website is the official page of the Nordic OMX exchange therefore we considered the home page as an accurate and up to date source of data. The purpose of this step was to examine which stocks are traded in the Copenhagen stock exchange.

The sample period spanned from sample period of 8 years between the first of April 2003 and the last of March 2011. The sample period based on research conducted by Miralles et al (2007), who utilized a time period of 8 years to conduct research on illiquidity in the Spanish stock market. As we will be utilizing the Miralles (2007) research method for our empirical section, we wanted to make sure that the time variables used were the same. This to ensure that the practical research methodology was adequate in aspect of reliability. We utilized Thomson Reuters DataStream to assess the periodic validity of Danish securities in the NASDAQ OMX Copenhagen market according to our chosen time period.

4.2 Selection of variables

Our next step requires use of two data sources namely Thomson Reuters DataStream and NASDAQomxnordic.com. We first utilized Thomson Reuters DataStream to download the following variables for our selected data;

- Date
- Adjusted closing price

We then compared the adjusted closing prices in NASDAQomxnordic.com and downloaded the following variables;

- Trading volume (given in Danish Kronor)

The purpose of utilizing two sources is to obtain accurate and up to date data. For the purpose of our research question we need to collect additional data. The sources of all security data was taken from the relevant company websites, the relevant variables are listed as follows; Equity, amount of shares outstanding and company size. Additional variables required are a relevant market index. For the purpose of our thesis we have selected the following market index, OMX Copenhagen All Share (OMXC). The OMX Copenhagen Stock Exchange's All-
Share Index includes all the shares listed on the Copenhagen Stock Exchange. The aim of the index is to reflect the current status and changes in the market. The base date for the Copenhagen Stock Exchange's All-Share Index is December 31, 1995. We deem this as a relevant index to utilize in order to calculate Beta (see chapter 3.CAPM for definition).

We also require a risk free rate which is the interest rate that can be obtained from investing in the default-free financial instruments, which are short-term liquid government papers. We came across an issue regarding the Danish market namely the fact that the T bill was discontinued in 2008 (Danish National Bank, 2009). We contacted the Danish national bank through email and were told that the best alternative T bill to utilize would be a Danish Central-government bond (Bullet issues) with 2 years to maturity. This data was downloaded from the Danish national bank website (http://www.nationalbanken.dk/).

4.3 Sample selection

Our next step was to assess the total amount of securities traded in the OMX Nordic Copenhagen stock exchange. The total amount of securities traded on the Copenhagen OMX Nordic market was 192 securities within the small, medium and large cap sectors. The securities are presented in table 5 below;

<table>
<thead>
<tr>
<th>Small Cap</th>
<th>Medium Cap</th>
<th>Large Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>38</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 5: Numbers of securities traded on Copenhagen OMX Nordic within each cap

Data adapted from NASDAQomxnordic.com

The purpose of the previous step was to determine our sample size, we chose to use a sample of securities and not a population, the primary reason behind this decision was time restrictions. Keller (2005) states that by using statistical inference, the researcher is able to draw conclusions about a population perimeter (Keller, 2005, pg145). The basic principle behind sample size is that more is better, in that a higher sample size allows the researchers to more accurately make assumptions (Keller, 2005, pg.151). We initially chose the entire large cap and medium cap companies as well as a larger proportion of small cap companies. The reason for selecting a larger amount of small cap securities was that small cap companies primarily tend to be newly formed and hence they have less historical data. We were aware that potentially a large proportion of the small cap stocks would be excluded by the Amihud (2002) selection criteria, which shall be discussed shortly. Our next consideration was to look at which securities met our chosen period, which was April 2003-April 2011. We required the securities to be active during the entire period. We utilized Thomson Reuters DataStream to assess the time period validity of our chosen sample. We were able to check each individual securities historical data and assess whether the security fitted our timeline criteria. We utilized convenience sampling for the purpose of our thesis. We selected the Copenhagen Stock exchange as our sample and from that we chose the companies which best suited our required criteria as presented by Amihud.
When selecting our security data we needed to consider the selection criteria as recommended by Amihud (Amihud, 2002, pg.36)

1. The security must have at least 200 days of daily stock price and volume date.
2. The security must be listed at the end of the year (y-1)
3. The third step as stated by Amihud looks at minimum tick effect and is relevant to the New York Stock Exchange., therefore for the purpose of our thesis we will not utilize this criteria point.

We utilized the above criteria for the selection of our thesis data, we have included a list of the included securities in appendix no.1

The results of the above mentioned procedures are presented in table 6 below;

<table>
<thead>
<tr>
<th>Small cap</th>
<th>Medium cap</th>
<th>Large cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>21</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 6: Summary of financial data on securities selected for analysis

The data above consists of an approximately equal amount of small, medium and large cap companies. Small, medium and large cap companies are defined as follows by the NASDAQ Nordic OMX stock exchange; (www.nasdaqomxnordic.com)

- Small Cap: Companies with a market value below 150 000 000 €
- Medium Cap: companies which have a value between 150 000 000 € and 1000 000 000 €
- Large Cap: Companies which have a market value over 1000 000 000 €

4.4 Practical research method

After acquiring and sorting the data from chapter 4 in our thesis, we are able to move on with the procedures which will enable us to answer our research question. We shall be utilizing a procedure based on the research of Miralles and Miralles (2005) who we mention in the introductory chapter of our thesis. For the purpose of clarity we will divide the entire procedure into steps, this will improve the replication aspect of our research.

4.4.1 First step – A calculation of the \textit{ILIQ}-measure

The first step in this procedure is the calculation of the illiquidity ratio as calculated utilizing formula 3 below

\[ \text{Formula 3} \]

\[ 1115636482.06 \text{ DKK} \text{ € converted as of 04/05/2011 through www.eur.kurs24.com} \]

\[ 7437576547.05 \text{ DKK converted as of 04/05/2011 through www.eur.kurs24.com} \]
\[
\text{ILLIQ}_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{R_{itd}}{V_{itd}} \right|
\]

(Formula 6 (5))

We divided the absolute returns of our securities by the volume of trades in DKK (Danish Kronor). We then needed to take note of the days each security was traded in a month, as not all securities were traded an equal number of days.

Once individual monthly ILLIQ-ratios had been calculated on all the securities, we constructed the 8 illiquidity portfolios according to smallest to largest illiquidity ratios. This was made to be able to calculate the IMV-factor, which will be discussed in Portfolio monthly returns were calculated for each of the IL portfolios, given equal weight to each security. We conducted a descriptive statistical analysis of these ILLIQ-measures. The primary purpose in creating these illiquidity portfolios is that the most liquid stocks and the most illiquid stocks will be utilized to create an illiquidity factor, which will be utilized in the CAPM and Fama and French three factor model as described in chapter 3. Descriptive statistics on all selected securities and their ILLIQ values are provided in appendix no.3.

The small-firm effect will also be measured, used as variable in the evaluation of the structure of the Copenhagen Stock Exchange. The market data is categorized in small, medium and large caps depending on the size of the company as listed by OMX Nordic. The relationship between company size and ILLIQ-ratio will be analyzed to evaluate if the small-firm effect can be observed on the Copenhagen Stock Exchange. If companies listed in the small cap are observed to have higher ILLIQ-ratios, the small firm effect may be an explanatory variable.

4.4.2 Second step – Procedure for calculations with Fama and French Model (1993)

The next step is to calculate the market return factor (MKT) for each security, the variables required for this step are as follows; monthly security returns, risk free rate, and monthly market index returns. Regressions were run to calculate the market return using the variables mentioned. We utilized SPSS to conduct the regressions.

4.4.3 Third step – Creation of size, book-to-market and VMI factors for the Fama and French model (1993)

We first created the size portfolios, by ranking the securities according to size at the end of December y-1, we divided them into small (s) and big (b) companies. Next we needed to rank companies according to their book to market value; we calculated the book to market value by taking total equity at y-1 divided by shares outstanding at y-1. The companies were then sorted according to largest to smallest book to market values and were placed into the following categories; high (H), medium (M) and low (L). Lastly, we utilized the liquidity ratio to rank securities into very liquid (V), moderately liquid (M) and illiquid (I) securities.
Through the above process we construct 18 portfolios from the intersection of the above created factors (two size, three book to market and three illiquidity groups) which leads to the following:

\[(S/L/V), (S/L/N), (S/L/I), (S/M/V), (S/M/N), (S/M/I), (S/H/V), (S/H/N), (S/H/I)\]

\[(B/L/V), (B/L/N), (B/L/I), (B/M/V), (B/M/N), (B/M/I), (B/H/V), (B/H/N), (B/H/I)\]

The purpose of creating these 18 portfolios will become clear in the next step.

4.4.4 Fourth step – Creation of the size factor “SMB” (small minus big), book to market factor “HML” (high minus low) and the illiquidity factor “IMV” (illiquid minus very liquid)

The size factor SMB is calculated each month by taking the difference between the average returns on the nine small security portfolios \((S/L/V), (S/L/N), (S/L/I), (S/M/V), (S/M/N), (S/M/I), (S/H/V), (S/H/N), (S/H/I)\) and the average of return from the nine big security portfolios \((B/L/V), (B/L/N), (B/L/I), (B/M/V), (B/M/N), (B/M/I), (B/H/V), (B/H/N), (B/H/I)\)

The book to market factor HML is calculated each month through the process of taking the difference between the average returns on the six high book to market security portfolios \((S/H/V), (S/H/N), (S/H/I)\) \((B/H/V), (B/H/N), (B/H/I)\) and the average returns of the six low book to market security portfolios \((S/L/V), (S/L/N), (S/L/I)\) \((B/L/V), (B/L/N), (B/L/I)\)

Finally, we create the illiquidity factor for each month by taking the difference between the average returns on the six illiquid security portfolios \((S/L/I), (S/M/I), (S/H/I)\) \((B/L/I), (B/M/I), (B/H/I)\) and the average of the returns on the six very liquid securities portfolios \((S/L/V), (S/M/V), (S/H/V)\) \((B/L/V), (B/H/V)\).

4.4.5 Fifth Step – The role of illiquidity factor in asset pricing

In this step we will investigate the role of illiquidity factor in asset pricing. We will utilize our eight illiquidity portfolios (IL1-IL8) which were created in the first step of our practical research chapter. We will analyze these eight illiquidity portfolios with the standard CAPM, CAPM + IMV, Fama and French 3 factor and lastly the Fama and French 3 factor model with additional IMV factor. We will look at the individual alphas (risk adjusted average return) of each portfolio using the different models specified. The relevant formulas for this analysis are listed below

\[E(r_i) = r_f + \beta_i \times (E[R_{Mr,k}] - r_f)\]

(Formula 7 (1))

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We will compare the above four models and look at their respective alpha values. We will analyze these alpha values in our empirical result chapter. We will then utilize the Wald test as a method of testing the particular explanatory variables in a statistical model. For the purpose of our analysis we are interested in whether portfolio intercepts (alphas) are equal to zero. The Wald test analyzes whether the alpha coefficients are jointly equal to zero. By utilizing the Wald test we will be able to answer whether our models can capture returns when used as asset pricing models, which provides us with the ability to accept or reject our hypothesis statement which was presented in chapter 3.7.

From this step we will be able to ascertain which model(s) perform best in capturing returns when used as asset pricing models. We will utilize our monthly data from 2003-2011, for each illiquidity portfolio. We will utilize a 5% significance level for the Wald test. The Wald test is directly related to our hypothesis statement.

### 4.5 Reliability, replication, generalization and validity of the practical methodology

We need to look at four key factors when conducting business research namely; reliability, replication, generalization and validity. The reliability criteria are related to how repeatable the study is and whether the results are stable between various researchers (Bryman et al, 2007, p.40). We will achieve reliability in our thesis through the data capturing procedure as well as through the implementation of the models. The practical methodology chapter provides step by step descriptions of how we acquired, processed and analyzed our collected data.
data. We are aware that there are four primary threats to reliability according to Robson (1993), these threats are; subject error, subject bias, observer error and observer bias (Robson et al, 1993, pg.63). We have reviewed our metrology and feel that these threats to reliability have been addressed. Our sources of stock data (www.nasdaqomxnordic.com) and Thomson Reuters DataStream are both well-established source of financial data. We utilize the Danske National Banken (Danish national bank) to acquire the relevant risk free rates. We are confident that if another researcher is to follow our same method then the results will match ours if conducted with the same data. We are aware that if other data is used then the results may differ.

The replication criterion evaluates how well the researchers have presented the steps and procedures in order to get their final results (Bryman et al, 2007, p.41) Our practical methodology chapter is detailed in describing the chain of applied models, any other researcher would get the same results if they were to conduct the same research given they use the same data.

The concept of generalization refers to the authors/researchers ability to conclude that the results of a study are applicable to situations outside of the proposed research context (Bryman et al, 2007, pg.76). We as authors are aware of the restrictions of our study and do not intend to generalize regarding our results to other markets. Our findings are only applicable to the Copenhagen Stock Exchange.

The validity criterion concerns the integrity of conclusions which are concluded from research (Bryman et al, 2007, pg.41). We believe that our study fulfills this criterion as we are repeating the core procedures that have been put forward by other respected researchers in previous study and thus should be valid also for our study. The models used in our research are well known and highly regarded within the field of financial theory (Bryman et al, 2007, pg.63). According to characteristics of validity, the finding in our research should be possible to generalize through the previous research (Bryman et al, 2007, pg.42). This criterion is also reinforced by the way that we are supporting previous research. It is important that the results of one study are able to be used in another research setting (Saunders et al, 2008, pg.102).

4.5.1 Statistical Tests

We utilized both Microsoft excel and SPSS 17 to conduct our statistical tests. Our primary statistical test is the Wald test which was conducted in SPSS. The Wald test is an F-test for the significance of all variables in the four different models. The Wald test analyzes whether the alpha coefficients are jointly equal to zero. The reason why we utilize the alpha coefficients is that it can be shown that in an efficient market, the expected value of the alpha coefficient is zero. By utilizing the Wald test we will be able to answer whether our models can capture returns when used as asset pricing models. We are able to accept or reject our hypothesis through the use of this test. Other statistical tests which we utilized were variance, mean and standard deviation.
5. EMPIRICAL RESULTS & ANALYSIS

In this chapter we present the results gathered from our practical research method section. We also present the analysis of the empirical data and discuss the relevancy of the finding in connection with the formulated research question and stated research hypothesis. The results presented in this chapter will validate or reject our proposed hypothesis.

5.1 Testing the hypothesis statement

The previous five steps in chapter 4.4 are developed in order to enable us to answer our research question by utilizing our hypothesis statement:

- \( H_0: \alpha_{\text{IL1...IL8}} = 0 \)
- \( H_1: \alpha_{\text{IL1...IL8}} \neq 0 \)

If our findings validate \( H_0 \), the market is correctly pricing illiquidity in the stock returns and the \( \text{ILLIQ} \)-measure is applicable in the CAPM and Fama & French model (1993). If our findings validate \( H_1 \); then liquidity is not correctly priced in the market returns. Our hypothesis statement will be answered through the use of our empirical chapter and the Wald Test will be utilized to accept or reject our hypothesis statement. The steps which follow in our empirical analysis will enable us to accept or reject the above hypothesis statement, which will provide us with the ability to answer our research question as well as purpose.
5.2 Analysis of the ILLIQ-measure

The figure 4 above represents a graphical summary of the eight illiquidity portfolios over an eight year period of time (2003-2011). We can see from the figure that the first four illiquidity portfolios (IL1, IL2, IL3 and IL4) are liquid and this is illustrated by the small yearly fluctuations and values of the ILLIQ-measure. The next four illiquidity portfolios (IL5, IL6, IL7 and IL8) show signs of a higher average ILLIQ-measure, which indicates they are portfolios which are less liquid.

We calculated the ILLIQ-measure as stipulated in the first step of the practical methodology. We had a total of 57 different securities, which we divided into 8 portfolios, all but one of those portfolios contained 7 securities each, and one contained 8 securities. We have attached a complete list of the securities together with their individual ILLIQ ratios in the appendix no.2
From the above descriptive statistics presented in Table 7, we can see that the portfolio IL1 contains the most liquid securities; these have the lowest mean return per portfolio. The portfolio IL8 has the least liquid stocks and the highest mean return per portfolio; we can also see that the standard deviation (volatility) is higher with the more illiquid stocks (IL6-IL8) as compared with the more liquid portfolios (IL1-IL3). The result of the \textit{ILLIQ}-measure is the absolute percentage price change per DKK (Danish Kronor) of trading volume. A security with high liquidity is expected to have a low price impact per unit of volume traded. A security with low liquidity is expected to have a high price impact per unit of volume traded. The portfolio IL8 contains only small cap securities and it contains the least liquid stocks.

\textbf{5.3 Analysis of the Fama and French 3 factors and the IMV factor}

\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
Portfolio & Mean Return & Standard Deviation & Average \textit{ILLIQ} & Average no. of stocks \\
\hline
IL1 & 0.014490483 & 0.000207036 & 0.004958988 & 7 \\
IL2 & 0.015842488 & 0.000203171 & 0.011928069 & 7 \\
IL3 & 0.017275299 & 0.000252934 & 0.07587976 & 7 \\
IL4 & 0.018451474 & 0.000275896 & 0.389781021 & 7 \\
IL5 & 0.100025556 & 0.000298745 & 0.905049193 & 8 \\
IL6 & 0.125545779 & 0.000325416 & 2.26262561 & 7 \\
IL7 & 0.154789546 & 0.000365874 & 5.839343391 & 7 \\
IL8 & 0.195477889 & 0.000558748 & 7.4265136 & 7 \\
\hline
\end{tabular}
\end{center}
\caption{Summary of average monthly \textit{ILLIQ} and Mean return of 8 portfolios}
\end{table}

\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|}
\hline
Factor & Mean & Std. Deviation \\
\hline
MKT & 0.6212 & 5.1276 \\
HML & 0.1239 & 2.9292 \\
IMV & 0.0996 & 0.8795 \\
SMB & 0.0721 & 1.6521 \\
\hline
\end{tabular}
\end{center}
\caption{Descriptive statistics of Fama and French factors and the IMV illiquidity factor}
\end{table}
Table 8 above provides a short descriptive summary of the factors utilized in the Fama and French 3 factor model plus the additional illiquidity factor IMV and the market return factor. These factors were created through steps 2-4 in our practical research method section. We look at the mean of the market return factor (0.6212), this positive figure is consistent with the assumption of risk aversion. The mean return for the SMB factor is positive (0.0721), this supports the small firm effect theory which states that shares of smaller companies generate higher annual returns than shares issued by larger companies. This because investing in the shares of smaller companies tend to be riskier, since they are less frequently traded, thus more sensitive to new market information. If our SMB factor was negative, then we would have evidence against the small firm effect or possible evidence that the small firm effect may not be stable over time. Our results differ from that of Miralles et al (2007) in that they had a negative SMB factor, which they attributed to small firm effect may not be stable over time (Miralles et al, 2007, pg.261).

5.4 Analyzing the role of illiquidity in asset pricing

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>CAPM alpha</th>
<th>FAMA FRENCH 3 alpha</th>
<th>FAMA FRENCH 3+IMV alpha</th>
<th>CAPM + IMV alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL1</td>
<td>0.456</td>
<td>0.854</td>
<td>0.241</td>
<td>0.328</td>
</tr>
<tr>
<td>IL2</td>
<td>0.325</td>
<td>0.513</td>
<td>0.278</td>
<td>0.521</td>
</tr>
<tr>
<td>IL3</td>
<td>0.589</td>
<td>-0.258</td>
<td>0.235</td>
<td>-0.854</td>
</tr>
<tr>
<td>IL4</td>
<td>-0.158</td>
<td>0.123</td>
<td>0.412</td>
<td>-0.025</td>
</tr>
<tr>
<td>IL5</td>
<td>-0.258</td>
<td>-0.587</td>
<td>-0.129</td>
<td>-0.102</td>
</tr>
<tr>
<td>IL6</td>
<td>-0.365</td>
<td>-1.003</td>
<td>-0.126</td>
<td>0.547</td>
</tr>
<tr>
<td>IL7</td>
<td>0.125</td>
<td>0.258</td>
<td>-0.025</td>
<td>0.787</td>
</tr>
<tr>
<td>IL8</td>
<td>0.987</td>
<td>0.186</td>
<td>0.776</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

Table 9: Alpha values for individual illiquidity portfolios using four different asset pricing models

Table 9 represents the individual Alpha’s for each portfolio using 4 different asset pricing models namely: CAPM, Fama and French 3 Factor model, CAPM+IMV and Fama and French 3 factor model + IMV (IMV represents an illiquidity risk factor, which was created in 4.4.3).
We can see the results from the above analysis that with regard to the CAPM, CAPM+IMV Fama and French 3 factor model and the Fama and French 3 factor model + IMV, the least liquid IL8 portfolio has a higher alpha value than the more liquid portfolios of IL1, IL2, and IL3. This is not however the case if we look at the Fama and French 3 factor model, Portfolio IL8 has a lower alpha value than portfolio IL1. Using our data, we can summarize that generally stocks with high liquidity have higher alpha values and therefore higher returns when used with certain asset pricing models.

If we compare our results to that of Miralles (2006), we have certain similarities with regards to the least liquid portfolios having higher alpha (risk adjusted average returns) values than the liquid portfolios.

The Wald test is a method of testing the particular explanatory variables in a statistical model. For the purpose of our analysis we are interested in whether portfolio intercepts are equal to zero. By utilizing the Wald test we will be able to answer whether our models can capture returns when used as asset pricing models. When we look at the P Values of the above table we can see that we reject H0 at the 5% significance level for the CAPM, Fama and French 3 factor model and the CAPM + IMV.

However with regard to the Fama and French 3 factor model + IMV, we accept the null Hypothesis at the 5% significance level. We can therefore conclude that there is evidence of illiquidity being priced in the Copenhagen Stock Exchange using the Fama and French 3 factor model +IMV asset pricing model.

If we once again compare our results in this section to Miralles (2007) we have one primary difference in that with their Wald test analysis they reject all models presented above except the CAPM + IMV model, which they stated was the model which provided the best risk specification (Miralles et al, 2006, pg.263).
6. CONCLUSIONS & CONTRIBUTIONS

In this chapter we discuss the conclusions of the empirical findings. We will also discuss the contributions that we through this study have made to the research field through the implementation of the measure of illiquidity in asset pricing

6.1 Conclusions

Based on our research statement we formulated a research question which we would answer through analysis of empirical data. The research question is;

- Is illiquidity priced in the Copenhagen Stock Exchange?

Based in a positivistic research philosophy and a deductive research method, we performed a quantitative study on 8 years of gathered empirical data. The application of theoretical models and an analysis of the results will prove or dismiss our formulated research hypothesis. If our findings validate H0, the market is correctly pricing illiquidity in the stock returns. If our findings validate H1; then liquidity is not correctly priced in the market returns.

We can draw several conclusions after analyzing our empirical data. Through the use of the ILLIQ-measure, we have shown that a portfolio of securities can be assigned an illiquidity value based on the variables; absolute return and turnover. This is in accordance with the first point of our problem statement, as we have shown that illiquidity can be measured through a theoretical model. Through the analysis of the empirical results we find evidence supporting the existence of liquidity on the Copenhagen Stock Exchange. Furthermore we have found evidence supporting the fact that illiquidity is priced in the Copenhagen Stock Exchange, this has been proven through the use of the practical research method and the use of the Fama and French 3 factor model with the inclusion of an illiquidity factor (IMV), this conclusion supports our second problem statement as well as providing support to our hypothesis statement H0.

Finally, to answer our research question; we have found evidence through our empirical analysis to support the fact that illiquidity is priced in the Copenhagen Stock Exchange. We hope that this thesis and its findings can be of use to scholars, financial institutions and private investors.

6.2 Contributions

In our study of the Copenhagen Stock Exchange, we have implemented a theoretical model for appraising expected returns of stock investments, represented by the CAPM. The model is then used as the base of an inclusion of an earnings-to-price ratio, represented by the Fama and French 3 factor model. These models are measures together with a measure of the price impact of trades, represented by the ILLIQ-measure. Through data collection and an extensive
data processing method, we have been able to evaluate the validity if the implementation of theoretical theories on the Copenhagen stock exchange.

The results of the thesis add valuable contributions to the research field, first and foremost since our research confirms that illiquidity is priced on the Copenhagen stock exchange. Our results provide evidence to support the belief that illiquidity should be an important factor to consider when utilizing asset pricing models. We can establish empirical confirmation in the connection of increasing illiquidity and the increase in stock returns. Additionally, our research further adds contributions to a research field which generally is in need of exploration. The results may be used by e.g. investors, regulators and portfolio managers who are interested in the effect of illiquidity on asset pricing. Perhaps our research will add contribution, be a little step on the way in a future forming of a generally accepted method of accurately estimating the impact of illiquidity in the pricing of assets.
7. FURTHER RESEARCH

In this chapter we present ideas for further research, these ideas were formed through the process of writing our thesis and may be of use for researchers in the field of asset pricing and illiquidity as well as provide students with potential research topics for their theses.

7.1 Recommendations for further research

Based on the conclusions and contributions of our study on Copenhagen Stock Exchange, we have identified interesting aspects that might be of interest for further research. The effect of illiquidity on asset pricing on the Scandinavian stock markets, is in many aspects unexplored and could be the focus of many further research.

OMX Nordic has grown considerably during the past decade, through the merger of several stock exchanges around the Baltic Sea. The effects of the increasing size of OMX Nordic would be an interesting area of research, as differences in illiquidity in the market could be identified and measured within a long time frame. The research would study if the merger between the markets in some way effects the changes in market wide liquidity on expected returns. One interesting aspect would be to use longer time series of data, and see if the results would vary compared to our time data sample. Also, spillover-effects between the different countries could be an interesting research focus and give valuable insights.

Additionally, further analyzing the effect of the implementation of designated liquidity drivers on the Copenhagen Stock exchange would be an interesting base of study. A research could be made to identify differences in the characteristics of the market based on the merger with OMX Nordic and the implementation of designated liquidity providers. It would be interesting to see and compare the conditions, prior to and after the introduction of these market makers. Furthermore, such a research could give valuable insight into how these market makers affect the market wide liquidity in a longer time frame.
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APPENDIX

Appendix no. 1: Table of Selected data from the Copenhagen Stock Exchange

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Appendix no. 2: Table of listed companies within each portfolio

The table below shows portfolios of which they were placed according to their respective average monthly *ILLIQ*-measure.

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