Genuine Saving and the Social Cost of Taxation*

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Abstract

Following the 1987 report by The World Commission on Environment and Development, the genuine saving has come to play a key role in the context of sustainable development, and the World Bank regularly publishes numbers for genuine saving on a national basis. However, these numbers are typically calculated as if the tax system is non-distortionary. This paper presents an analogue to genuine saving in a second best economy, where the government raises revenue by means of distortionary taxation. We show how the social cost of public debt, which depends on the marginal excess burden, ought to be reflected in the genuine saving. By presenting calculations for Greece, Japan, Portugal, U.K., U.S. and OECD-average, we also show that the numbers published by the World Bank are likely to be biased and may even give incorrect information as to whether the economy is locally sustainable.

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1. Introduction

Since the 1970s, a theory of welfare accounting has gradually evolved. One of the basic ideas behind the research in welfare accounting has been to provide a coherent framework for measuring changes in welfare in a dynamic economy, as well as understanding how the current procedures for national accounting ought to be modified with this particular objective in mind.[1] In this paper, we revisit the relationship between capital formation and the subsequent welfare change by presenting a measure of "genuine saving" for a second best economy where the public revenue spent on environmental policy is raised by distortionary taxes. We argue below that such a measure is not only interesting from a theoretical point of view; it has also bearing on statistics of relevance for environmental policy frequently published by the World Bank.

The genuine saving is an indicator of comprehensive net investment, i.e. the value of the net investment in all capital stocks of relevance for society. As such, genuine saving does not only reflect the social value of net investment in physical capital (the measure of net investment used in conventional national accounting); it also reflects the social value of changes in other capital stocks, such as natural and human capital. The remarkable feature with genuine saving is that it constitutes an exact measure of welfare change over a short time-interval.[2] Following the 1987 report by The World Commission on Environment and Development, it has also come to play an interesting role as an indicator of sustainable development. The World Commission wrote that development is sustainable if it meets "the needs of the present without compromising the ability of future generations to meet their own needs" (Our Common Future, page 54). One possible interpretation is that sustainable development requires welfare to be non-declining.[3] This suggests, in turn, that the genuine saving is a local indicator of sustainable development, where the emphasis on the word "local" is due to that we are measuring the welfare change over a short time-interval.[4]

[1] The seminal contribution to the theory of welfare accounting is Weitzman (1976), showing how a welfare-equivalent measure of net national product ought to be defined if the resource allocation is first best. Aronsson (1998, 2008) analyzes the corresponding welfare measurement problem in second best economies, where the public revenue is raised by distortionary taxes. See also the literature reviews by Weitzman (2003) and Aronsson, Löfgren and Backlund (2004).

[2] Although Weitzman (1976) did not attempt to analyze genuine saving, it shows up in the proof of his main result, i.e. we need Weitzman's welfare measure to relate the indicator of welfare change to the genuine saving. Standard references for genuine saving are Pearce and Atkinson (1993) and Hamilton (1994, 1996).

[3] This definition is used in Arrow et al. (2003).

[4] See also Asheim (1994) and Pezzey (1993), who show that a positive value of genuine saving does not give any information as to whether the current level of utility or consumption is sustainable forever.

Another interpretation of sustainable development is that the current (instantaneous) utility level must not fall short of the maximum level that can be sustained forever, in which case non-positive genuine saving indicates that the current utility level faced by the representative consumer exceeds the maximum sustainable level (Pezzey and Toman 2002, Pezzey 2004). As such, the genuine saving has become an important statistic underlying the environmental policy debate, and the World Bank regularly publishes numbers for genuine saving on a national basis for a large number of countries.[5]

However, the appropriate procedures for calculating the genuine saving have not received sufficient attention. In fact, the calculations that we have seen either assume that the resource allocation is first best, or that the resource allocation is suboptimal in the sense that society has not reached the best possible outcome given its objective and constraints (due to uninternalized market failures).[6] To our knowledge, there are no studies dealing with the measurement of genuine saving (or an analogue thereof) in economies where the resource allocation is second best optimal; a scenario that will arise if restrictions faced by policy makers prevent them from implementing the first best resource allocation. This gap in the literature is somewhat surprising considering that the revenue raised by the public sector in real world economies typically necessitates distortionary taxes, which are associated with an excess burden that may affect both the sign and magnitude of the welfare change that the economy experiences during a short time-interval. Arguably, the principles for measuring genuine saving ought to be modified accordingly; at least if the welfare economic foundation is to be taken seriously. Therefore, the purpose of this paper is to present an analogue to genuine saving in a second best economy, where the government raises revenue through a distortionary tax (instead of a lump-sum tax).

Our study is based on a model developed by Chamley (1985), which is an extension of the Ramsey model in the sense of adding a public sector and assuming that the public revenue is raised by using a linear, yet time-varying, labor income tax. We show that the marginal excess burden of taxation affects the second best analogue to genuine saving via the accumulation of public assets. Finally, we exemplify by adjusting the World Bank numbers for genuine saving and show that neglecting the social costs of taxation (as the World Bank does) may give rise

[5] See also Hamilton (2010) for an overview of research on genuine saving.
[6] See, e.g., Aronsson and Löfgren (1998) and Löfgren and Li (forthcoming).

to biased estimates of genuine saving and, in some cases, alter our conclusions as to whether the economy is locally sustainable.

2. The Model

The model presented below contains consumers, firms and a government. We start by describing the decision problems faced by agents in the private sector and then continue with the policy problem facing the government. Following much earlier literature, the second best problem will be described as a Stackelberg game, where the government is acting leader and the private agents are followers.

Consumers and Firms

The model developed in this section largely resembles the Ramsey-type models used in earlier literature on welfare accounting with the modification that the public revenue is raised by a labor income tax.[7, 8] Following the convention in earlier literature, we assume that the economy is populated by a fixed number of identical consumers normalized to one. The preferences are described by a time-separable utility function. The objective function facing the consumer is represented by the present value of future utility,

$$U(0) = \int_0^\infty u(c(t), z(t), q(t)) e^{-\theta t} dt \,, \tag{1}$$

where c is the consumption of a private good, z leisure and q the quantity of a public good decided upon by the government, while the parameter θ denotes the utility discount rate (i.e. the marginal rate of time preference). The public good is a state-variable and may be thought of as public capital that leads to higher environmental quality (e.g., environment-friendly infrastructure, public parks, publicly provided carbon-sinks, etc.). This is clearly a somewhat naïve description of environmental quality; by focusing solely on the public sector contribution to such quality, it leaves out a number of vital relationships between production, consumption and damages to the environment. Yet, this simplification is analytically

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[7] Adding another distortionary tax will not affect the principal findings below. See Chamley (1986) for a dynamic representative agent model with linear taxes on labor income and capital income.

[8] Aronsson (2008) uses a similar model to derive a second best analogue to Weitzman's (1976) welfare measure (i.e. a second best analogue to the comprehensive net national product) when public revenue is collected through distortionary taxes, as well as analyzes the role of public goods in welfare accounting.

convenient and is of no practical importance for the qualitative relationship between genuine saving and tax distortions, which is the main focus in this paper. As a consequence, we abstract from other aspects of environmental quality. The determination of the public good is discussed below. Leisure is defined as a fixed time endowment, $\bar{l}$, less the hours of work, l. The instantaneous utility function, $u(\cdot)$, is increasing in each argument and strictly concave.

The consumer holds two assets; capital, k, and government bonds, b, which are assumed to be perfect substitutes. If we define $a = k + b$, the asset accumulation equation can be written as

$$\dot{a}(t) = r(t)a(t) + w_n(t)l(t) - c(t) \tag{2}$$

with $a(0) = a_0$, where w_n is the marginal wage rate defined as $w_n(t) = w(t)[1-\tau(t)]$, in which w is the gross wage rate and τ the tax rate. The variable r is the interest rate. The price of the private consumption good has been normalized to one.

The consumer chooses his/her consumption of the private good, c, and hours of work, l, at each instant to maximize the present value of future utility subject to equation (2), the initial condition, and a No Ponzi Game condition (which is a restriction on the present value of the terminal asset). The consumer also treats the factor prices and policy variables at each point in time as exogenous. By using the first order conditions, one can write the demand for the private good and labor supply as functions of the net-of-tax wage rate, the marginal utility of wealth and the public good, respectively,[9]

$$c(t) = c(w_n(t), \phi(t), q(t)) \tag{3}$$

$$l(t) = l(w_n(t), \phi(t), q(t)) . \tag{4}$$

The marginal utility of wealth obeys, in turn, the differential equation

[9] Note that the current value Hamiltonian implied by the consumer's decision problem can be written as (if the time-indicator is suppressed)

$$J = u(c, z, q) + \phi\dot{a}$$

where the marginal utility of wealth in current value terms appears as the costate variable attached to wealth. Equations (3) and (4) are derived from the first order conditions $u_c(c, z, q) - \phi = 0$ and $-u_z(c, z, q) + \phi w_n = 0$.

$$\dot{\phi}(t) - \theta\phi(t) = -\phi(t)r(t)\ . \tag{5}$$

Finally, by substituting equations (3) and (4) into the instantaneous direct utility function, we obtain the instantaneous indirect utility function defined conditional on the marginal utility of wealth

$$v(t) = v(w_n(t), \phi(t), q(t)) = u\left(c(w_n(t), \phi(t), q(t)), \bar{l} - l(w_n(t), \phi(t), q(t)), q(t)\right). \tag{6}$$

Turning to the production side, we assume that identical competitive firms use labor and capital to produce a homogenous good under constant returns to scale and normalize the number of firms to one. The production function is given by $f(l(t), k(t))$, and the firm obeys the first order conditions $f_l(l,k) - w = 0$ and $f_k(l,k) - r = 0$.

The Government

The social welfare function coincides with the objective faced by the representative consumer. By using the conditional indirect utility function presented in equation (6), the social welfare function at time 0 can be written as

$$V(0) = \int_0^\infty v(w_n(t), \phi(t), q(t))e^{-\theta t}dt\ . \tag{7}$$

Turning to the state-variables faced by the government, the accumulation equation for the public good is assumed to take the following form:

$$\dot{q}(t) = g(t) - \delta q(t) \tag{8}$$

where $g(t)$ is the contribution to the public good at time t and δ the rate of depreciation. We can think of g as representing public expenditure on investment in environment-friendly infrastructure or abatement of the natural environment. The government uses the income tax to finance the contributions to the public good, although it does not necessarily balance the budget at each instant. If we write the unit tax on labor as $\tau w = w - w_n$, the equation of motion for government bonds is written as

$$\dot{b}(t) = r(t)b(t) + g(t) - [w(t) - w_n(t)]l(t)\,. \tag{9}$$

Finally, by combining equations (2), (9) and the zero profit condition, $f(l,k) - wl - rk = 0$, we can derive the resource constraint

$$\dot{k}(t) = f(l(t),k(t)) - c(t) - g(t)\,. \tag{10}$$

To simplify the notation, we assume that $f(\cdot)$ measures output net of capital depreciation, which means that the left hand side of equation (10) represents the net investment in physical capital. Equation (10) means that output is used for private consumption as well as private and public investment.

The decision problem facing the government will be to choose the tax rate (or net wage rate) and contribution to the public good at each instant to maximize the social welfare function presented in equation (7) subject to the state-differential equations (5), (8), (9) and (10), as well as subject to the first order conditions for the private control variables given by equations (3) and (4), and the first order conditions of the firm (which define the gross wage rate and interest rate by the marginal product of labor and capital, respectively). The reason as to why equation (5) appears as a state-differential equation in the government's decision-problem is that the equation of motion for the private marginal utility of wealth is part of the necessary conditions faced by the consumer and, therefore, a constraint that the optimal tax and expenditure policy must fulfill.[10]

The current value Hamiltonian associated with the public decision problem can be written as (suppressing the time-indicator for notational convenience)

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[10] The resource allocation must also obey initial conditions for k and b as well as a No Ponzi Game condition for b. As pointed out by Chamley (1985), the government does not face any explicit constraint on the initial private marginal utility of wealth, $\phi(0)$.

$$H = v(w_n, \phi, q) + \lambda \dot{k} + \psi \dot{q} + \mu \dot{b} + \varsigma \dot{\phi} \tag{11}$$

where λ, ψ, μ and ς are the costate variables (measured in current value utility) attached to the state variables in the decision-problem faced by the government, i.e. the stock of physical capital, the environmental public good, the stock of government bonds and the private marginal utility of wealth, respectively. The first order conditions are presented in the Appendix. Here, we use these conditions to derive a measure of welfare change.

3. Measuring Genuine Saving

The conventional approach to measuring genuine saving is to add the value of changes in environmental and/or natural capital stocks to the net investment in physical capital, as well as adding the value of net investment in other capital goods such as human capital. In our simple model, which abstracts from human capital, this suggests that we should define genuine saving by adding the value of net investment in the environmental public good to the value of net investment in physical capital, i.e. $\lambda \dot{k} + \psi \dot{q}$.[11] We show below that this procedure gives a correct measure of welfare change if the resource allocation is first best, while it does not give a correct measure of welfare change in the second best framework addressed here.

Define the optimal value function at time *t* as follows:

$$V^0(t) = \int_t^{\infty} u(c^0(s), z^0(s), q^0(s)) e^{-\theta(s-t)} ds \tag{12}$$

where $c^0 = c(w_n^0, \phi^0, q^0)$ and $z^0 = \bar{l} - l(w_n^0, \phi^0, q^0)$ are defined by equations (3) and (4). We use the superindex "0" to denote "second best optimal resource allocation". By totally differentiating the optimal value function represented by equation (12) with respect to time, we obtain a measure of welfare change over a short time-interval

[11] Although investment in human capital would affect the exact form of the genuine saving measure, adding human capital to the model would not affect the qualitative results presented below for how the principles of measuring genuine saving ought to be modified in a second best economy by comparison with the corresponding principles in the first best. The welfare measurement problem associated with human capital is addressed by Aronsson and Löfgren (1996).

$$\frac{dV^0(t)}{dt} = -u(c^0(t), z^0(t), q^0(t)) + \theta V^0(t) \,. \tag{13}$$

To explore the relationship between the right hand side of equation (13) and the measure of genuine saving suggested above, i.e. $\lambda\dot{k} + \psi\dot{q}$, and to be able to relate our study to earlier comparable literature (see the introduction), we begin by evaluating the welfare change measure in a first best resource allocation. We will then continue with the second best analogue to genuine saving.

Special Case: Genuine Saving in the First Best

In terms of our model, the first best resource allocation constitutes a special case where $\mu(t) = \varsigma(t) = 0$ and $\phi(t) = \lambda(t)$ for all *t*. Such an allocation would follow if the labor income tax were replaced by a lump-sum tax to finance the contribution to the environmental public good at each instant. By using that $\theta V(t) = H(t)$ at the optimal resource allocation,[12] and if we use the superindex "*" to denote the first best (to distinguish it from the second best), we obtain the familiar result

$$\frac{dV^*(t)}{dt} = \phi^*(t)\dot{k}^*(t) + \psi^*(t)\dot{q}^*(t) \,. \tag{14}$$

If applied to the model set out above with a two-dimensional capital concept, the right hand side of equation (14) is the conventional genuine saving measure. In our model, the genuine saving is given by the sum of the value of net investment in physical and environmental capital. This approach to measure the genuine saving is also consistent with the approach taken by the World Bank; let be that they use a broader capital concept than we do (that also includes human capital).

Genuine Saving in the Second Best Model

Let us now return to the more general second best model set out above. As we show in the Appendix, by applying the same procedure as above, we can derive the following result:

[12] The optimal control problem is time-autonomous, except for the time-dependence of the utility discount factor.

Proposition 1. *The welfare change measure for the second best economy is given by*

$$\frac{dV^0(t)}{dt} = \lambda^0(t)\dot{k}^0(t) + \psi^0(t)\dot{q}^0(t) + \mu^0(t)\dot{b}^0(t) + \varsigma^0(t)\dot{\phi}^0(t)\,. \tag{15}$$

If we follow convention and define genuine saving as $\lambda(t)\dot{k}(t) + \psi(t)\dot{q}(t)$, then the right hand side of equation (15) is interpretable as a generalized measure of genuine saving. The generalization follows because the social planner faces two additional state variables here (in addition to *k* and *q*); namely, the stock of public debt, b, and the private marginal utility of wealth, ϕ.

The costate variable $\mu(t)$ attached to the government debt at time *t* is interpretable as the negative of the marginal excess burden at time *t*; it reflects that increased government debt at present necessitates higher distortionary taxes in the future. If $\mu(t) < 0$, as one would normally expect, the intuition is that public debt (asset) accumulation gives rise to a social cost (benefit) due to the distortions generated by the tax system. Therefore, public debt or asset accumulation affects the genuine saving in the second best (which it does not in the first best where $\mu = 0$). As pointed out by Chamley (1985), the marginal excess burden measured in real consumption units, $MEB = -\mu(t)/\phi(t) = -\mu(0)/\phi(0)$, is constant over time along the optimal path.[13] Otherwise, it would be possible for the government to reduce the overall welfare cost of taxation by changing its debt policy. We will return to the marginal excess burden below.

The welfare effect of changes in the private marginal utility of wealth, i.e. the fourth term in equation (15), is also due to the appearance of distortionary taxation, although for another reason. The tax system distorts the labor supply and private consumption and, therefore, also the path for the private marginal utility of wealth, causing it to differ from the path for the shadow price of physical capital, $\lambda(t)$. The associated welfare cost of this discrepancy is captured by the variable $\varsigma(t)$.[14] To understand why changes in the private marginal utility of wealth affect the welfare change measure, recall that the public decision-problem is

[13] This is seen by solving the equations of motion for $\phi(t)$ and $\mu(t)$ subject to transversality conditions.

formulated in terms of demand functions and an indirect instantaneous utility function, which are defined conditional on the private marginal utility of wealth. If evaluated in the first best, $\partial H / \partial \phi = 0$ (in which case $\varsigma = 0$) because the private cost benefit rule for c and l, respectively, would in that case coincide with the corresponding social cost benefit rule, whereas $\partial H / \partial \phi$ is generally nonzero in the second best due to discrepancies between the private and social cost benefit rules.

If we measure the conventional genuine saving in consumption units, $GS = [\lambda \dot{k} + \psi \dot{q}] / \phi$, we can rewrite equation (15) as follows:

$$\frac{dV^0(t)}{dt} = \phi^0(t)\left[GS^0(t) - MEB^0\,\dot{b}^0(t) + \frac{\varsigma^0(t)}{\phi^0(t)}\dot{\phi}^0(t)\right]. \tag{16}$$

The expression within square brackets of equation (16) is the generalized measure of genuine saving expressed in consumption units. Since $\phi^0(t) > 0$, it follows that welfare increases over the short time-interval $(t, t+dt)$ if, and only if, the expression within square brackets is positive. Equation (16) constitutes the basis for the application below.

4. Application

In this section, we exemplify by calculating how the marginal value of public debt will modify the numbers for genuine saving published by the World Bank. Our starting point is the World Bank measure of genuine saving, which is defined by subtracting natural resource depletion and damages from carbon dioxide emissions from the net investment in physical capital and then adding education expenditures (which serve as a proxy for investment in human capital; let be that the proxy is somewhat misleading, as the connection between such expenditure and the future earnings of the investors is unclear).

To be able to adjust the current numbers for genuine saving, we make two simplifying assumptions: (i) the resource allocation is second best optimal in the sense discussed above,

[14] In a simplified version of this model without the public good, Chamley (1985) shows that the variable ς is equal to zero at time 0 (due to that there is no initial condition on the private marginal utility of wealth in the second best problem), while it is negative in a steady state.

and (ii) the interest rate is constant and equal to the rate of time preference, so $\dot{\phi}(t) = 0$ for all t, in which case the fourth term on the right hand side of equation (15) vanishes. The second assumption is needed because there is no way to estimate the value of changes in the private marginal utility of wealth, given the data to which we have access.[15] Therefore, we augment the numbers for genuine saving published by the World Bank by the value of the change in government debt defined as the public deficit times the marginal excess burden measured in consumption units, where the latter is based on estimates in empirical literature.[16]

To be more specific, we will subtract

$$MEB \frac{db(t)}{dt}$$

from the number for genuine saving published by the World Bank. We consider three different numbers for marginal excess burden; 0.1, 0.3 and 0.5, respectively, which are well in line with – although in the lower range of – the estimates summarized by Jacobs (2009).[17] The results are presented in Figure 1, which contains the numbers published by the World Bank as well as the numbers following the adjustment mentioned above.

Figure 1 HERE

Figure 1 presents the numbers for genuine saving (*GS*) published by the World Bank, as well as our generalized measure of genuine saving (*GGS*), for Greece, Japan, Portugal, U.K., U.S. and OECD-average, for the period 1991-2009.[18] All numbers on which the curves are based are given in hundreds of U.S. dollars in 2009 prices and measured per capita.[19] We make three

[15] This is, of course, a restrictive assumption. It may serve as reasonable approximation if the economy is relatively close to a stationary equilibrium.

[16] See Jacobs (2009) for a recent comprehensive literature review.

[17] We realize, of course, that the studies surveyed by Jacobs are typically based on models different from ours. The mean value of marginal excess burden among the studies in his survey is 0.5.

[18] Data for the genuine saving originates from the World Bank and were obtained from the World Development Indicators (collected in the spring of 2011) at http://databank.worldbank.org/ddp/home.do, whereas data for budget surpluses and deficits were collected from the OECD Economic Outlook 87 data base. We used the GNP deflator (UN statistics) to convert the nominal numbers for genuine saving and budget surpluses/deficits into real numbers.

[19] We assumed away population growth in Section 2, because such growth is not important for our qualitative understanding of how the principles for measuring genuine saving in a second best economy differ from the corresponding principles that apply in a first best resource allocation. In practice, population growth adds complications to welfare measurement, since changes in the population affect the welfare change between two subsequent periods (depending on the form of the objective function). We abstract from these complications here.

broad observations from the figure. *First*, neglecting the accumulation of public debt may lead to the wrong conclusion as to whether the economy is locally sustainable. This is the case for the U.S. in 2008, where the conventional genuine saving is positive, whereas the generalized genuine saving is negative if based on the numbers 0.3 and 0.5, respectively, for the marginal excess burden. Similar findings apply for the U.K. in 2009, Greece in 2004-2005 and Portugal in 2006. *Second*, with the highest number for marginal excess burden, i.e. 0.5, which is in line with the empirical evidence referred to above, public debt accumulation may have a considerable effect on the generalized genuine saving. *Third*, and perhaps even more important, since the conventional genuine saving and the budget deficit move together to some extent (e.g., the net investments in physical capital typically fall and budget deficits typically increase during recessions), the conventional genuine saving measure may be a poor indicator as to when the economy is at the risk of becoming locally unsustainable, i.e. the signal that this statistic is designed to give may come several years after which the generalized genuine saving has turned negative.

It is necessary to exercise caution in the interpretation of the results in Figure 1. One reason is, of course, that the World Bank numbers are uncertain, and it is not always clear that the measure used by the World Bank covers all important aspects of the conventional genuine savings measure - as it ought to be defined in a world without tax distortions - or that all components are measured in the best way possible. The estimates of marginal excess burden are also subject to uncertainty, and the appropriate value to be used may also differ between countries. However, to arrive at an accurate picture of the savings behavior of society, our results suggest, nevertheless, that the savings by the public sector may be of practical relevance when determining whether or not the economy is locally sustainable.

Appendix

For the analysis to be carried out below, it will be more convenient to use the present value Hamiltonian than the current value Hamiltonian. We will, therefore, reformulate equation (11) in present value terms, through multiplying by $e^{-\theta t}$, and also write out the constraint functions explicitly. We have

$$\begin{aligned} H_p(t) &= H_p(w_n(t), g(t), k(t), q(t), b(t), \phi(t), \lambda_p(t), \psi_p(t), \mu_p(t), \varsigma_p(t), t) \\ &= v(w_n(t), \phi(t), q(t)) e^{-\theta t} + \lambda_p(t)[f(l(t), k(t)) - c(t) - g(t)] \\ &+ \psi_p(t)[g(t) - \delta q(t)] + \mu_p(t)[r(t)b(t) + g(t) - (w(t) - w_n(t))l(t)] \\ &+ \varsigma_p(t)\phi(t)[\theta - r(t)] \end{aligned} \tag{A1}$$

where the subindex p attached to the Hamiltonian and costate variables denotes "present value", the factor prices $w(t)$ and $r(t)$ are defined by the marginal products of labor and capital, respectively, while $c(t) = c(w_n(t), \phi(t), q(t))$ and $l(t) = l(w_n(t), \phi(t), q(t))$ according to equations (3) and (4).

The first order conditions for the control variables are (suppressing the time indicator)

$$\frac{\partial H_p}{\partial w_n} = 0 \text{ and } \frac{\partial H_p}{\partial g} = 0 \tag{A2}$$

while the equations of motion for the costate variables become

$$\dot{\lambda}_p = -\frac{\partial H_p}{\partial k}, \ \dot{\psi}_p = -\frac{\partial H_p}{\partial q}, \ \dot{\mu}_p = -\frac{\partial H_p}{\partial b}, \text{ and } \dot{\varsigma}_p = -\frac{\partial H_p}{\partial \phi}. \tag{A3}$$

Derivation of Equation (15)

By totally differentiating equation (A1) with respect to time, we obtain

$$\begin{aligned} \frac{dH_p}{dt} &= \frac{\partial H_p}{\partial w_n}\frac{dw_n}{dt} + \frac{\partial H_p}{\partial g}\frac{dg}{dt} + \frac{\partial H_p}{\partial k}\frac{dk}{dt} + \frac{\partial H_p}{\partial q}\frac{dq}{dt} + \frac{\partial H_p}{\partial b}\frac{db}{dt} + \frac{\partial H_p}{\partial \phi}\frac{d\phi}{dt} \\ &+ \frac{\partial H_p}{\partial \lambda_p}\frac{d\lambda_p}{dt} + \frac{\partial H_p}{\partial \psi_p}\frac{d\psi_p}{dt} + \frac{\partial H_p}{\partial \mu_p}\frac{d\mu_p}{dt} + \frac{\partial H_p}{\partial \varsigma_p}\frac{d\varsigma_p}{dt} + \frac{\partial H_p}{\partial t} \end{aligned} \tag{A4}$$

where $\partial H_p / \partial t = -\theta v(w_n, \phi, q)\exp(-\theta t)$, since the direct effect of time is due to the utility discount factor. Now, $\partial H_p / \partial \lambda_p = dk/dt$, $\partial H_p / \partial \psi_p = dq/dt$, $\partial H_p / \partial \mu_p = db/dt$ and

$\partial H_p / \partial \varsigma_p = d\phi / dt$ according to the definition of the Hamiltonian. Therefore, by using the first order conditions in equations (A1) and (A2), equation (A4) reduces to read

$$\frac{dH_p}{dt} = -\theta v(w_n, \phi, q)e^{-\theta t} . \tag{A5}$$

For notational convenience, we have suppressed the superindex *0* for "second best" in equation (A5). Solving equation (A5) for $H_p(T)$ gives

$$H_p(T) = H_p(t) - \theta \int_t^T v(w_n(s), \phi(s), q(s))e^{-\theta s} ds . \tag{A6}$$

If T approaches infinity, and by using the transversality condition $\lim_{T\to\infty} H_p(T) = 0$,[20] we obtain

$$H_p(t) = \theta \int_t^\infty v(w_n(s), \phi(s), q(s))e^{-\theta s} ds . \tag{A7}$$

Finally, transforming equation (A7) to current value, through multiplying both sides by $e^{\theta t}$, we obtain $\theta V(t) = H(t)$, where $H(t) = H_p(t)e^{\theta t}$ is the current value Hamiltonian and $V(t) = \int_t^\infty v(w_n(s), \phi(s), q(s))e^{-\theta(s-t)} ds$ the social welfare function.[21] Substitution of $\theta V(t) = H(t)$ into equation (13) gives equation (15).

References

Aronsson, T. (1998) Welfare Measurement, Green Accounting and Distortionary Taxes. *Journal of Public Economics* **70**, 273-295.

Aronsson, T. (2008) Social Accounting and the Public Sector. *International Economic Review* **49**, 349-375.

[20] See Seierstad and Sydsaeter (1987, Chapter 4, page 245).

[21] A result analogous to equation (A7) was also derived by Aronsson (1998), although his study focuses on Hamiltonian-based welfare measures and does not address genuine saving.

Aronsson, T. and Löfgren, K-G. (1996) Social Accounting and Welfare Measurement in a Growth Model with Human Capital. *Scandinavian Journal of Economics* **98**, 185-201.

Aronsson, T. and Löfgren, K-G. (1998) Green Accounting: What Do We Know and What Do We Need To Know? In Tietenberg, T. and Folmer, H. (eds) *International Yearbook of Environmental and Resource Economics 1998/1999*. Edward Elgar Publishing Limited.

Aronsson, T., Löfgren, K-G. and Backlund, K. (2004) *Welfare Measurement in Imperfect Markets: A Growth Theoretical Approach*. Edward Elgar Publishing Limited.

Arrow, K. J., Dasgupta, P. and Mäler, K-G. (2003) Evaluating Projects and Assessing Sustainable Development in Imperfect Economies. *Environmental and Resource Economics* **26**, 647-685.

Asheim, G.B. (1994) Net National Product as an Indicator of Sustainability. *Scandinavian Journal of Economics* **96**, 257-265.

Chamley, C. (1985) Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium. *International Economic Review* **26**, 451-468.

Chamley, C. (1986) Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica* **54**, 607-622.

Hamilton, K. (1994) Green Adjustments to GDP. *Resources Policy* **20**, 155-168.

Hamilton, K. (1996) Pollution and Pollution Abatement in the National Accounts. *Review of Income and Wealth* **42**, 13-33.

Hamilton, K. (2010) Genuine Saving, Social Welfare, and Rules for Sustainability. In Aronsson, T. and Löfgren, K-G. (eds) *Handbook of Environmental Accounting*. Edward Elgar Publishing Limited.

Jacobs, B. (2009) The Marginal Cost of Public Funds and Optimal Second-Best Policy Rules. Erasmus University. Mimeo.

Löfgren, K-G. and Li, C-Z. (forthcoming) Introduction. In Löfgren, K-G. and Li, C-Z. (eds) *Green National Accounting and Sustainability*. Edward Elgar Publishing Limited.

Pearce, D.W. and Atkinson, G. (1993) Capital Theory and the Measurement of Sustainable Development: An Indicator of Weak Sustainability. *Ecological Economics* **8**, 103-108.

Pezzey, J. (1993) The Optimal Sustainable Depletion of Nonrenewable Resources. University College, London.

Pezzey, J. (2004) One-Sided Sustainability Tests with Amenities, and Changes in Technology, Trade and Population. *Journal of Environmental Economics and Management* **48**, 613-631.

Pezzey, J. and Toman, M. (2002) Progress and Problems in the Economics of Sustainability." In Tietenberg, T. and Folmer, H. (eds) *International Yearbook of Environmental and Resource Economics 2002/2003*, 165-232. Cheltenham: Edward Elgar Publishing Limited.

Seierstad, A. and Sydsaeter, K. (1987) *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.

Weitzman, M.L. (1976) On the Welfare Significance of National product in a Dynamic Economy. *Quarterly Journal of Economics* **90**, 156-162.

Weitzman, M.L. (2003) *Income, Capital, and the Maximum Principle*. Harvard University Press.

World Commission on Environment and Development (1987) *Our Common Future*. Oxford: Oxford University Press.

Figure 1 Genuine Saving and Generalized Genuine Saving 1991-2009