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Enhancement in the electromagnetic beam-plasma instability due to ion streaming

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We investigate the Weibel instability in counterpropagating electron-ion plasmas with the focus on the ion contribution, considering a realistic mass ratio. A generalized dispersion relation is derived from relativistic theory by assuming an initially anisotropic temperature, which is represented by a waterbag distribution in momentum space which shows an enhanced growth rate due to the ion response. Two-dimensional particle-in-cell simulations support the theoretical analysis, showing a further amplification of the magnetic field on the ion time scale. The effect of an initially anisotropic temperature is investigated showing that the growth rate is monotonously decreased if the transverse spread is increased. Nevertheless, the presence of ions generates that the instability can develop for significantly higher electron temperatures. Suppression of the oblique mode is also explored by introducing a parallel velocity spread.

PACS codes:

1. Introduction

The importance of plasma instabilities for explaining the generation of magnetic fields has been recognized in the context of various astrophysical phenomena, such as gamma-ray bursts (GRBs) (Piran 2004), active galactic nuclei (AGNi) (Birk *et al.* 2001; Lyubarsky & Kirk 2001), pulsar wind outflows (Kirk & Skjæraasen 2003), shock formation and the associated acceleration of cosmic rays (Schlickeiser 2002). When particles are ejected by these objects, e.g. by an exploding GRB, they interact with the surrounding medium, causing several types of plasma instabilities, which lead to the generation of magnetic fields. The detected radiation allows to measure the field strength that can be compared with theoretical models and simulations. On large scales, the interaction of particle jets with a pre-existing plasma can lead to shock formation, where the magnetic field was observed to be amplified substantially at the shock front (Drenkhahn & Spruit 2002). Often, an electromagnetic wave propagating ahead of the shock is present, the so-called precursor (Panaitescu & Kumar 2002). The large size of the shock provides enough time for plasma instabilities to occur between the precursor and the surrounding medium (Frederiksen *et al.* 2004). Here, we do not provide a full consistent theory of the shock model, but focus on the precursor regime and the generation of plasma instabilities and the self-consistently generated magnetic fields.

Magnetic fields can also be generated by the interaction of an intense laser pulse with a

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solid density plasma target (Shukla *et al.* 2009). The non-stationary ponderomotive force of a large-amplitude electromagnetic wave pushes the electrons in propagation direction, thus generating a temperature anisotropy. This is of interest in fast ignition scenarios, where spontaneously generated magnetic fields are an undesirable effect. One of the key issues in the FIS is the deposition of the beam energy in the plasma core, which is required to ignite fusion (Honda *et al.* 2000). It was found that a short laser pulse with an intensity of the order $10^{21} - 10^{22}$ W/cm² accelerates electrons in an overdense plasma in forward direction (Norreys *et al.* 2009). For such high intense laser pulses, the electron velocity becomes relativistic and a return electron current is formed in the opposite direction to maintain the global charge neutrality. The interaction between the currents drives several plasma instabilities similarly to the astrophysical scenario including the longitudinal electrostatic (Mendonça *et al.* 2005) and transverse electromagnetic modes (Kong *et al.* 2009). Both, analytical results and simulations, reveal that these instabilities can play an important role in stopping the hot electrons in the core (Mason 2006).

Many theoretical and numerical studies were performed to identify and understand the fastest growing instability. An efficient process of magnetic field generation is provided by the Weibel instability (Weibel 1959). Medvedev and Loeb suggested that the Weibel instability is capable of generating a strong small-scale magnetic field in an initially unmagnetized plasma (Medvedev & Loeb 1999) arising from a plasma anisotropy. The energy, which is stored in the temperature anisotropy, is transferred to magnetic field energy and causes an initially exponentially growing magnetic field. After this linear stage, the instability saturates and the magnetic field energy reaches a quasi-steady level. In the case of a pure electron-positron plasma, computer simulations have demonstrated that the generated magnetic field grows to about 15% of the initial kinetic energy density (Fonseca *et al.* 2002a) which is in agreement with the values inferred from GRB afterglows (Panaitescu & Kumar 2002).

In many previous works, the linear and nonlinear stages of the Weibel instability have been investigated theoretically and numerically by assuming immobile ions, which form a neutralizing background, because their impact is visible on larger time scales only. Thus, the role of ions is relatively unexplored yet, but necessary for the treatment of realistic scenarios and the large time scales of the shock formation process (Wiersma & Achterberg 2004). The question arises whether the inclusion of the ion dynamics still drives the electromagnetic or Weibel-type instability on the ion time scale after the electron Weibel instability has saturated. At this stage, the electron temperature is quite high but the evolution of ions is slow due to their inertia and thus the ion temperature of the ions T_i is still different from T_e .

2D and 3D numerical simulations have been performed by colliding two electron-positron or electron-ion plasmas with or without temperature anisotropy (Silva *et al.* 2003; Ren *et al.* 2004) but the ion dynamics for realistic mass ratios have not been investigated with simulations yet because of limited computational resources. We have performed two-dimensional kinetic simulations with the particle-in-cell (PIC) code OSIRIS (Fonseca *et al.* 2002b) to probe the features of the Weibel instability of two interpenetrating, initially unmagnetized electron-ion plasma clouds with zero net charge and initial temperature anisotropy. We demonstrate the possibility of driving the electromagnetic instability on the ion time scale and compare the simulations with analytical predictions. In the case of an electron-ion plasma, the Weibel instability is a two stage process. Initially, only the electrons respond to the instability since they are much lighter than the ions. This process shuts down due to a strong electron heating and the quasi-isotropization of the electron distribution function. At this stage, the streaming ions start to respond by generating micro-currents and associated magnetic fields that grow to the expense of the

energy stored in the ion flow. As a consequence the instability is further amplified on ion time scale.

The manuscript is organized as follows. In Section 2, the dispersion relation of the Weibel instability in an initially unmagnetized temperature anisotropic electron-ion plasma is derived analytically. The theoretical results are supported by our simulations, which are discussed in Section 3. Summary and conclusions of our work are presented in Section 4.

2. Derivation of the dispersion relation

We consider two identical counterpropagating warm electron-ion plasmas. In order to obtain the dispersion relation, we use the relativistic Vlasov and Maxwell's equations. The obtained dispersion relation is based on the model described by Yoon and Davidson (Yoon & Davidson 1987) and Silva et al. (Silva *et al.* 2002) and considers electromagnetic waves propagating with the wave vector $\mathbf{k} = k \hat{\mathbf{z}}$.

At equilibrium, we have equal densities in each particle component

$$n_{e0} := n_{e10} = n_{e20} = n_{i10} = n_{i20} =: n_{i0},$$

where n_{j10} and n_{j20} denote the densities of species $j = e, i$ in beam 1 and 2, respectively. We choose a waterbag distribution function for each species with opposite bulk momenta along the x -direction, p_{x0} , and with a thermal spread along the z -direction, p_{z0} , while the plasma species are cold in the x - and y -directions, obtaining

$$F_{j0} = \frac{1}{2p_{zj0}} \{ \delta(p_x - p_{xj10}) \delta(p_y) \{ \Theta(p_z - p_{zj10}) - \Theta(p_z + p_{zj10}) \} \\ + \delta(p_x - p_{xj20}) \delta(p_y) \{ \Theta(p_z - p_{zj20}) - \Theta(p_z + p_{zj20}) \} \} \quad (2.1)$$

with the Heaviside step function Θ . The initial bulk motions of the species are $p_{ex10} = -p_{ex20} =: p_{xe0}$, $p_{xi10} = -p_{xi20} =: p_{xi0}$. We consider the electron bulk momenta and transverse temperature to initially dominate over the ions, $p_{xe0} \gg p_{xi0}$ and $T_e \gg T_i$, respectively. The normalized dispersion relation for a purely transverse mode ($\mathbf{k} \cdot \mathbf{E} = 0$) is thus given by

$$\tilde{\omega}^2 - (\tilde{k}^2 + \mathcal{F}) - \frac{\mathcal{G}_I \tilde{k}^2 \beta_{zi0}^2}{\tilde{\omega}^2 - \tilde{k}^2 \beta_{zi0}^2} - \frac{\mathcal{G}_E \tilde{k}^2 \beta_{ze0}^2}{\tilde{\omega}^2 - \tilde{k}^2 \beta_{ze0}^2} = 0, \quad (2.2)$$

where frequency is normalized to the electron plasma frequency ($\tilde{\omega} = \omega / \omega_{pe}$) and the wave number to the inverse electron skin depth ($\tilde{k} = k \omega_{pe} / c$). With the definitions

$$\mathcal{F} = \frac{2}{\gamma_e} - \frac{2}{\gamma_{e0}} \frac{u_{xe0}^2}{1 + u_{xe0}^2} + \frac{2}{M\gamma_i} - \frac{2}{M\gamma_{i0}} \frac{u_{xi0}^2}{1 + u_{xi0}^2} \quad (2.3)$$

with

$$\frac{1}{\gamma_j} = \frac{1}{2u_{zj0}} \ln \left(\frac{1 + \beta_{zj0}}{1 - \beta_{zj0}} \right) \quad (2.4)$$

and

$$\mathcal{G}_E = \frac{2}{\gamma_{e0}} \frac{\beta_{xe0}^2}{\beta_{ze0}^2}, \quad (2.5)$$

$$\mathcal{G}_I = \frac{2}{M\gamma_{i0}} \frac{\beta_{xi0}^2}{\beta_{zi0}^2}, \quad (2.6)$$

where $M = m_i / m_e$ is the ratio of the ion to electron masses. We introduced the standard

FIGURE 1. (Color online) Approximated growth rate given by Eq. (2.12) versus wave number. The blue, red, green, orange, pink and cyan colors refer to different transverse temperatures $T_p = 10, 20, 30, 40, 50, 60$ keV, respectively. (a) Pure electron Weibel instability ($m_i \rightarrow \infty$) and (b) inclusion of ion motion.

definitions $\gamma_{j0} = (1 + p_{xj0}^2/m_j^2 c^2 + p_{zj0}^2/m_j^2 c^2)^{1/2}$, $u_{xj0} = \gamma_{j0} \beta_{xj0} = p_{xj0}/m_j c$, $u_{zj0} = \gamma_{j0} \beta_{zj0} = p_{zj0}/m_j c$ for $j = e, i$. The reduced dispersion relation can then be written as

$$\tilde{\omega}^6 - \tilde{\omega}^4 \mathcal{A} + \tilde{\omega}^2 \mathcal{B} - \mathcal{C} = 0 \quad (2.7)$$

with the definitions

$$\mathcal{A} = \tilde{k}^2 (\beta_{zi0}^2 + \beta_{ze0}^2 + 1) + \mathcal{F}, \quad (2.8)$$

$$\mathcal{B} = \tilde{k}^4 \beta_{zi0}^2 \beta_{ze0}^2 + \tilde{k}^2 (\tilde{k}^2 + \mathcal{F}) (\beta_{zi0}^2 + \beta_{ze0}^2) - \tilde{k}^2 (\mathcal{G}_I \beta_{zi0}^2 + \mathcal{G}_E \beta_{ze0}^2), \quad (2.9)$$

$$\mathcal{C} = \tilde{k}^4 [\tilde{k}^2 \beta_{zi0}^2 \beta_{ze0}^2 + \mathcal{F} \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_I \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_E \beta_{ze0}^2 \beta_{zi0}^2]. \quad (2.10)$$

The exact analytical expressions for the solutions can be found, but due to their complexity we focus on the limit $\text{Im } \tilde{\omega} \ll \tilde{k}$ and introduce the real growth rate $\tilde{\Gamma} = -i\tilde{\omega}$. We assume that $\tilde{\Gamma}^6 \ll \tilde{\Gamma}^4 \ll \tilde{\Gamma}^2$ and obtain the dispersion relation

$$\tilde{\Gamma}^4 \mathcal{A} + \tilde{\Gamma}^2 \mathcal{B} + \mathcal{C} = 0, \quad (2.11)$$

which is bi-quadratic and analytically easily solvable. The roots are given by

$$\begin{aligned} \tilde{\Gamma}^2 = & \frac{1}{2[\tilde{k}^2(\beta_{zi0}^2 + \beta_{ze0}^2) + (\tilde{k}^2 + \mathcal{F})]} \\ & \times \left\{ -[\tilde{k}^4 \beta_{zi0}^2 \beta_{ze0}^2 + \tilde{k}^2(\tilde{k}^2 + \mathcal{F})(\beta_{zi0}^2 + \beta_{ze0}^2) - \tilde{k}^2(\mathcal{G}_I \beta_{zi0}^2 + \mathcal{G}_E \beta_{ze0}^2)] \right. \\ & \pm \left\{ [\tilde{k}^4 \beta_{zi0}^2 \beta_{ze0}^2 + \tilde{k}^2(\tilde{k}^2 + \mathcal{F})(\beta_{zi0}^2 + \beta_{ze0}^2) - \tilde{k}^2(\mathcal{G}_I \beta_{zi0}^2 + \mathcal{G}_E \beta_{ze0}^2)]^2 \right. \\ & \left. \left. - 4\tilde{k}^4 [\tilde{k}^2 \beta_{zi0}^2 \beta_{ze0}^2 + \mathcal{F} \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_I \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_E \beta_{ze0}^2 \beta_{zi0}^2] [\tilde{k}^2(\beta_{zi0}^2 + \beta_{ze0}^2 + 1) + \mathcal{F}] \right\}^{1/2} \right\} \end{aligned} \quad (2.12)$$

which exhibits unstable solutions when $\tilde{\Gamma} > 0$. Hence, the Weibel instability will grow if the condition

$$\tilde{k}^4 [\tilde{k}^2 \beta_{zi0}^2 \beta_{ze0}^2 + \mathcal{F} \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_I \beta_{zi0}^2 \beta_{ze0}^2 - \mathcal{G}_E \beta_{ze0}^2 \beta_{zi0}^2] [\tilde{k}^2(\beta_{zi0}^2 + \beta_{ze0}^2 + 1) + \mathcal{F}] > 0 \quad (2.13)$$

is fulfilled. Moreover, Eq. (2.13) is satisfied if

$$0 < \tilde{k}^2 < \mathcal{G}_I + \mathcal{G}_E - F, \quad (2.14)$$

which is the condition for the existence of unstable modes in the presence of ion streaming. If $T_e = T_i$, then $\mathcal{G}_I \simeq \mathcal{G}_E$, and the contribution of the ions just give rise to a slight change

on the threshold. In this case, the Weibel instability shuts down when $\mathcal{G}_\mathcal{E} < \mathcal{F}$. On the other hand, if $T_i \ll T_e$ which is the case right after the shutdown of the electron Weibel instability stage, $\mathcal{G}_\mathcal{I}$ will dominate over $\mathcal{G}_\mathcal{E}$ and it will push the instability threshold condition to larger wave numbers, as illustrated in Fig. 1.

The growth rate of the instability is analyzed numerically by plotting the growth rate $\tilde{\Gamma}$ against the wave number $\tilde{\mathbf{k}}$ (Fig. 1). If the ions are not included, the analysis shows that the electron Weibel instability ($m_i \rightarrow \infty$) will shut down at $T_\perp \simeq 70$ keV. Fig 1 (a) shows that the maximum growth rate strongly depends on the perpendicular temperature of the electrons. Our analysis predicts that the ion streaming enhances the growth rate of the electromagnetic beam-plasma instability for larger wave vectors and prevents the instability to cease at the threshold condition, which is due to the fact that it involves two different time scales. The theoretical results presented herein will be verified by PIC simulations in the next section.

3. Simulations

To confirm our theoretical findings, we perform two-dimensional simulations of two initially unmagnetized electron-ion plasma beams, propagating in opposite directions. The simulation box contains the counterpropagating beams, allowing for transverse modes as well as for parallel modes. We also investigated the transverse modes in a set up with the beams propagating perpendicularly to the simulation plane and achieved similar results.

A temperature anisotropy is described by a waterbag distribution function in momentum space. The periodic box contains 1024×1024 cells with a total size of $(102.4 \times 102.4)\lambda_e^2$, normalized to the electron skin depth $\lambda_e = c/\omega_{pe}$, and initially 4 particles per cell and species. The total time of the simulation is $1200\omega_{pe}^{-1}$ with a temporal resolution $\Delta t = 0.07\omega_{pe}^{-1}$. The density ratio between the electrons and ions, $n_{e0}/n_{i0} = 1$, is considered in all simulations and the ion to electron mass ratio is $m_i/m_e = 1836$. The relativistic electron bulk velocity is initially $u_{xe0} = 10$. We perform several simulations for different thermal spreads $u_{ze0} = \gamma_{e0}\beta_{ze0} = 1, 5, 25, 50, 80, 120$, transverse to the bulk motion of the electrons. We fix the parameters for the relativistic ions to $u_{xi0} = 0.9$ for the bulk velocity and the thermal spread $u_{zi0} = 0.001$. In order to be able to distinguish between the different contributing effects, the ion temperature is chosen to be significantly smaller than the electron temperature, $T_{\perp e} \gg T_{\perp i}$ with the definition of the rest frame temperature

$$\frac{k_B T_\perp}{mc^2} = \int d^3p p_\perp v_\perp F_{b0}, \quad (3.1)$$

where F_{b0} is the distribution function of the particular species. This choice for $T_{\perp e} \gg T_{\perp i}$ can reproduce the conditions close to the saturation of the electron Weibel instability.

We clearly observe the collective dynamics of electrons and ions on two different time scales. In the beginning of the simulations, only the electrons respond to the unstable plasma waves. The random charge separation generates micro-currents which reinforce the initial perturbations causing the growth of the plasma instability. Then, any inhomogeneity in the current will amplify the corresponding magnetic field. The transverse magnetic field $B_\perp$ dominates over the growth of the longitudinal $B_\parallel$. Fig. 2 shows the temporal evolution of the transverse magnetic field component for a thermal velocity $u_{ze0} = 0.44$. At about $t\omega_{pe} = 14$, the electron Weibel instability generates a magnetic field, which starts to merge into larger filaments later on. At $t\omega_{pe} = 56$, the electron Weibel instability is slowed down significantly and finally saturates, due to the fact that the thermal pressure of the electrons prevails over the magnetic pressure (Silva *et al.*

FIGURE 2. (Color online) Temporal evolution of the transverse magnetic field B_3 for $u_{ze0} = 0.44$. (a) Weibel instability on electron time scale at $t\omega_{pe} = 14$, (b) nonlinear stage of electron Weibel instability at $t\omega_{pe} = 56$, (c)-(e) Weibel instability on ion time scale at $t\omega_{pe} = 119$ -350, (f) nonlinear stage of Weibel instability on ion time scale at $t\omega_{pe} = 700$.

2002). After the saturation of the electron Weibel instability at $t\omega_{pe} = 119$, the ions begin to participate in the instability, due to which we observe a further enhancement of the magnetic field, and the field growth is eventually saturated at $t\omega_{pe} = 350$. At this stage, the filament size is $10 c/\omega_{pe} \approx c/4\omega_{pi}$.

The temporal evolution of the energy of the transverse magnetic field component B_3 , shown in Fig. 3, demonstrates the different stages. After the initial linear phase of the electron Weibel instability until $t\omega_{pe} = 50$, a local maximum is observed according to the saturation of the electron Weibel instability, followed by a decrease of the energy. The second increase in the energy is due to the ions, which is demonstrated in Fig. 3 (a). The comparison of two simulations with immobile and mobile ions clearly shows the contribution of the ions, yielding a further increase in the magnetic field energy. Fig. 3 (b) shows the fit of the theoretical growth rate, obtained from Eq. (2.13). The theoretical predictions $\Gamma = 0.43 \omega_{pe}$ for the electron Weibel instability and $0.019 \omega_{pe}$ for the ions matches well with the simulation result, where we have calculated the analytical growth rate for the ion Weibel instability assuming $T_e \gg T_i$ which implies the condition $\mathcal{G}_I > \mathcal{G}_E$. At this point $t\omega_{pe} \sim 50$ the electron Weibel instability shuts down due to strong electron heating. Later on, the ions start to participate to the Weibel instability and grow at the expense of the free energy stored in the ion flow. As a consequence, the ion contribution amplifies the Weibel instability. Because of the geometry of our two-dimensional simulations, the unstable modes are not restricted to purely transverse modes i.e. the oblique modes predicted in (Silva *et al.* 2002) are captured in the simulations. This is responsible for the slight deviation between the predicted growth rate and the simulation results.

We estimate the saturation magnetic field based on the magnetic trapping mechanism (Davidson *et al.* 1972; Yang *et al.* 1994; Silva *et al.* 2002). When the magnetic field

FIGURE 3. (Color online) Transverse magnetic field energy for perpendicular electron thermal velocity $u_{ze0} = 0.44$ (a) without (blue) and with ions (red) and determination of the growth rate on ion (b) and electron time scale (c).

FIGURE 4. (Color online) Temporal evolution of the transverse magnetic energy for thermal velocities $u_{ze0} = 0.44$ (red) and 2.0 (blue).

reaches the value

$$B_{sat} = \frac{\gamma_i^2 \Gamma_{i0}^2 M}{k_i u_{i0} e} \quad (3.2)$$

the particles cannot overcome the magnetic potential and are trapped. We find that Eq. (3.2) matches the saturation magnetic field for the electron Weibel instability, as well as the additional increase of $0.57 m_e c \omega_{pe} / e$ in the field due to the ion dynamics.

To investigate the predicted decrease of the electron Weibel growth rate for an increase of the transverse electron temperature, shown in Fig. 1, we have performed simulations for different velocity spreads transverse to the bulk motion of the plasma species. In agreement with the theory, Fig. 4 shows the growth of the magnetic energy in the simulations for $u_{ze0} = 0.44, 2.0$, revealing a reduction in the magnetic field energy of one order of magnitude for the higher temperature.

Interestingly, Fig. 4 shows the reduction in the magnetic field growth at higher transverse electron temperature, but it does not show the full suppression of the B field growth, even at a very high temperature. This is due to the fact that oblique modes are excited.

FIGURE 5. (Color online) Transverse magnetic energy for $u_{ze0} = 0.44$ and parallel spreads $u_{xe0} = 0$ (red), 0.44 (cyan), 10 (green), 50 (blue).

In Fig. 2 (b) for $t\omega_{pe} = 56$, we observe that the magnetic field has started to tilt, which confirms the presence and excitation of the oblique modes.

The oblique modes investigated in previous studies include an electrostatic character (Bludman *et al.* 1960). Bret *et al.* have studied the effect of temperature on oblique modes (Bret *et al.* 2010), and found that the growth rate of oblique modes is strongly decreased by the temperature ($\Gamma \propto T_b^{-1}$) and saturates above a critical temperature. These modes are stabilized due to a relatively small phase velocity and saturate through particle trapping, followed by an efficient heating of the plasma (Bret *et al.* 2007). To test this saturation and to suppress the oblique modes, we have carried out further simulations by introducing a parallel temperature, keeping the transverse temperature fixed as $u_{ze0} = 0.44$. Fig. 5 shows the dependence of the magnetic field energy on such a parallel spread. The initial bump, which is due to the electron Weibel instability and therefore sensitive to the geometry of the velocity spread, is decreased as predicted by (Bludman *et al.* 1960), but it requires significant increase of the parallel temperature to achieve a strong reduction. These simulation results illustrate that the saturation of the oblique modes regimes a contribution of parallel and perpendicular thermal spread, and this will be explored elsewhere.

4. Summary and conclusions

We have derived the linear dispersion relation for the Weibel instability of two counter-propagating electron-ion beams by using a waterbag distribution in the momenta. The threshold condition for the occurrence of the Weibel instability has been derived and an analytical expression for the growth rate has been found. Our analytical results show a strong dependence of a perpendicular electron momentum spread on the growth rate, which can significantly lower the electron Weibel growth rate. The inclusion of ions enhances the Weibel instability at large wave numbers, generating the occurrence of the (ion) Weibel instability even for high T_e (with low) T_i .

To confirm our theoretical findings, we have performed 2D PIC simulations of two counter-propagating electron-ion beams with a realistic mass ratio, considering different

perpendicular electron momentum spreads. We have observed that the obtained level of magnetic field energy is eventually the same and independent of the initial temperature. To identify the role of the ions, we have also simulated the same setup with immobile ions. After the first linear stage, the instability saturates and the energy in the magnetic field decays rapidly. If the ions are mobile, their streaming enhances the instability and the fields are amplified showing a second linear stage. The comparison of the temporal evolution of the total magnetic field energy for both cases showed a difference of more than an order of magnitude in the final energy. Our simulation results have confirmed the theoretical predictions for the exponential growth in the magnetic field energy, showing a good agreement for purely growing modes on electron and ion time scales. The magnetic field energy reaches up to 0.01% of the initial total kinetic energy ϵ_p .

Our simulation results also show the presence of oblique modes which can be suppressed by increasing the momentum spread in longitudinal direction. We observe that a very high temperature is necessary to achieve a significant reduction in the oblique mode suppression but still even high T_e are not able to shutdown the growth of these modes completely. This shows that suppression of these modes is more complex and further work is required to understand the mechanism for the suppression.

In conclusion, we found that the energy stored in the ion flow can enhance the electromagnetic instability at high wave numbers and prevent an immediate shutdown of the Weibel instability. Hence, the present work emphasizes the importance of including the ion dynamics when studying the generation of sub-equipartition quasi-static magnetic fields.

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