COMMUNICATING MATHEMATICS OR MATHEMATICAL COMMUNICATION? AN ANALYSIS OF COMPETENCE FRAMEWORKS

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In this study we analyse the communication competence included in two different frameworks of mathematical knowledge. The main purpose is to find out if mathematical communication is primarily described as communication of or about mathematics or if it is (also) described as a special type of communication. The results show that aspects of mathematics are mostly included as the content of communication in the frameworks but the use of different forms of representation is highlighted both in the frameworks and also in prior research as a potential cause for characterising mathematical communication differently than "ordinary" communication.

INTRODUCTION

It is often stated that reading mathematics demands a specific type of reading ability, separate from an "ordinary" reading ability, that needs to be taught at all educational levels (e.g. Burton & Morgan, 2000; Shanahan & Shanahan, 2008). Research has also indicated that it might be the presence of symbols in mathematical texts, and not the mathematics in itself, that primarily creates such a demand of a specific type of reading ability (Österholm, 2006). This discussion about aspects of reading in mathematics can be expanded to aspects of communication, and it is relevant to examine how mathematical communication is described within frameworks that describe (school) mathematics (e.g. NCTM, 2000; Palm, Bergqvist, Eriksson, Hellström, & Häggström, 2004) to determine the relation between mathematical communication and communication in general, as well as between mathematical communication and other aspects of mathematics. The following overarching question is focused on in this paper: Is mathematical communication described simply as communication of mathematics (i.e. ordinary communication but regarding a specific topic) or as a special type of communication?

BACKGROUND

At a general level, two "extreme" examples of different theoretical perspectives can be given regarding relationships between communication and mathematics. Sfard (2008) does not describe communication and cognition as separated, but sees thinking as the individualised form of interpersonal communication and mathematics as a form of discourse. From this perspective, a mathematical communication competence is the same as mathematical knowledge in general, and whether there is something special about mathematical communication is the same as asking if there is something special about mathematics. Another perspective is to see a separation between mathematics and mathematical knowledge on the one hand and the communication of mathematical

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content and the ability to communicate on the other hand. From this perspective, a certain use of language "indicates" whether individuals "in fact" conceive of something a certain way (Tall, Thomas, Davis, Gray, & Simpson, 2000, p. 230).

Other researchers focus more specifically on potentially special properties of mathematical communication (or communication in any other content area), when focus tend to be on literacy, and primarily reading. For example, McKenna and Robinson (1990) define the concept of content literacy as consisting of three components; general literacy skills, content-specific literacy skills, and prior knowledge of content. Similarly, Behrman and Street (2005, p. 8) suggest that "the ability to read with understanding would not be constant across disciplines, since learning depends upon domain-based declarative knowledge [prior knowledge of content] and domain-related strategies [content-specific literacy skills], in addition to more generalized strategies [general literacy skills]". These three components frame our discussion and analysis in this paper, and we focus on content-specific literacy skills.

A question addressed in some research studies is whether there are such things as content-specific literacy skills, examined by comparing reading in different domains. Results from such empirical studies tend to highlight similarities between domains. In particular, several studies show strong or moderate correlations between different tests of reading comprehension; between social studies and general reading comprehension ($r = 0.79$) (Artley, 1943), between reading comprehension in an anatomy course and general reading ability ($r = 0.72$) (Behrman & Street, 2005), and also between reading comprehension for a mathematical text and a historical text ($r = 0.47$) (Österholm, 2006). These results are seen as evidence of general literacy skills.

Another type of comparison between domains shows that experts from different domains read texts within their domain in different ways (Shanahan & Shanahan, 2008). However, a limitation in this study is that it is based on the reading of singular texts from each domain, but there is a great variety of texts within a domain (Burton & Morgan, 2000), making it difficult to draw conclusions about domains in general.

Another way to address the issue of content literacy is to think about what could be seen as content-specific literacy skills. At a general level, to be familiar with a certain genre or linguistic register (i.e. that mathematical texts might have a certain style or form, and that they might use words and formulations for purposes different than in other domains) could be seen as part of content-specific literacy skills. However, it is difficult to find a common description of all kinds of mathematical texts, since even when limiting the selection to mathematical research articles, Burton and Morgan (2000) notice a large variety of writing styles.

Empirical studies of students reading comprehension of mathematical texts have highlighted the use of symbols in mathematical texts as the most important potential cause for a need of content-specific literacy skills (Österholm, 2006). The use of different forms of representation is often also noted as a critical property of

mathematics, for example in Sfard's (2008) commognitive theory and in the cognitively oriented framework by Duval (2006).

In summary, we have not found any studies focusing more broadly on characterizing mathematical communication, except at more general theoretical level. When comparing different domains, focus tend to be on aspects of reading, where we have not found any clear empirical evidence for separating reading in different domains in general, instead the variation within a domain seems equally important. For mathematics, some empirical and theoretical evidence exist that different forms of representations can create a potential need for content-specific literacy skills.

PURPOSE

As a way of expanding our knowledge of a potential need for content-specific literacy skills in mathematics, in this paper we examine if and how content-specific literacy skills are described as part of a mathematical communication competence within frameworks of mathematical knowledge. Our research questions are:

1. What aspects of communication are included in frameworks describing mathematical competence?
2. How is mathematics described as the content of communication in frameworks of mathematical competence?
3. How is communication described as having special character due to aspects of mathematics in frameworks of mathematical competence?
4. Is mathematics described mainly as the content of communication or as part of other aspects of communication, in frameworks of mathematical competence?

METHOD

We acknowledge that many different types of analyses could be used to fulfil the described purpose, but in this paper we focus on one type of linguistic analysis, and do not include several different types of analyses, partly due to space limitations. However, we aim to expand our analyses in future publications, since different types of analyses might give different types of information.

Our method for analysing competence frameworks consists of two main steps. In step 1 we read each framework and highlight parts that specify some aspect of communication. In step 2 we analyse the highlighted parts from step 1 regarding how aspects of mathematics are related to the noted aspects of communication, in particular if mathematics is described as the content of communication or related to other aspects of communication. In both these steps, both authors perform the analysis separately and we then compare our results. Before performing the second step, we compare our results from the first step and agree on how to interpret the text and code the data, and we use our common agreement as a basis for the second step.

In this study we analyse two different frameworks of mathematical competence; a framework from NCTM (2000) and a framework created based on an analysis of the Swedish national curriculum (Palm et al., 2004). We shortly refer to these frameworks

as the NCTM framework and the Swedish framework respectively. These frameworks are chosen since they include a communication competence, and we only analyse the parts of the frameworks that explicitly address the communication competence. Aspects of communication could be included also in other frameworks of mathematical competence, which do not include a communication competence, and also in other parts of the analysed frameworks (e.g. when representations are discussed in a separate competence), but we limit our analysis to the communication competence. The main reason for this limitation is that another type of method of analysis could be needed to handle more implicit descriptions of aspects of communication.

The main analytical tool used in this study consists of a description of different aspects of communication. Based on definitions of *communication* we create a description of these aspects. We use definitions from dictionaries; from Merriam-Webster Online and the Swedish National Encyclopaedia (NE) for a standard type of definition and from Wikipedia (in English and Swedish) for a more colloquial type of definition, and also the definition from Sfard (2008) for a more non-standard perspective. We use different types of definitions in order to not exclude potential references to communication in the analysed frameworks. Based on these definitions, the following aspects of communication are identified; agent, technique, quality, content, and unspecified (first column in Table 1, in the results section). Common for all definitions is a focus on some *type of exchange of "information" between agents*. Deliberately, we do not define notions used here, but instead focus on words or phrases that in some way signal or specify some aspect of this "exchange" (third column in Table 1). The components within each aspect (second column in Table 1) are added in order to distinguish between words and phrases that specify a certain aspect of communication differently.

The list of words and phrases is created according to the following procedure: First we include words used in the definitions of communication in the dictionaries and also add words from a brainstorming activity around the different aspects and components. Then we look up the included words in dictionaries and include more words from the given definitions, and repeat this procedure for all new words. The purpose with the list of words and phrases is not only to search for those specific words included in the list, but also to more easily find relevant *types* of words when analysing the frameworks. That is, new words and phrases are also added to the list during the process of analysis.

In the first step of the process of analysis, each framework is read from start to end and all relevant words and phrases are highlighted in the text. The context is taken into account in the process of analysis to decide if a certain word should be highlighted. For example, "understand" could refer to the process of understanding a written text, an aspect of communication, but could also refer to a cognitive state that does not fit our (broad) type of characterisation of communication. All highlighted words are then included in a table as shown by Table 1, which is used for answering research question 1, regarding what aspects of communication are included in the frameworks.

In the next step of analysis, focus is on relationships between aspects of mathematics and aspects of communication, and each framework is read from start to end again. For

each occasion when some word has been highlighted in a framework, it is decided if and how any aspect of mathematics is included in the highlighted aspect of communication, based on the following six types of how an aspect of mathematics is specified:

1. Some form of the word "mathematics" or "mathematical" is used.
2. Some mathematical form of representation is referred to (e.g. through words like table, graph, or symbol).
3. Some mathematical concept or object is referred to (e.g. through words like triangle, number, or function).
4. Some mathematical activity is referred to, by referring to any other type of mathematical competence (e.g. problem solving) or to any procedure or operation that can be linked to a mathematical concept (e.g. derive or multiply).
5. Something mathematical is referred to, other than what is included in types 1-4.
6. Nothing mathematical is referred to.

For each occasion when one of types 1-5 has been noted, it is also noted what aspect of communication the mathematics is related to (i.e. agent, technique, quality, content, or unspecified). All occasions when some aspect of mathematics is specified in relation to some aspect of communication are then used when answering research questions 2-4, regarding how mathematics is included in different aspects of communication.

The following is an example of the process of analysis. In the excerpt below from the NCTM framework, the relevant words and phrases are highlighted:

> Students in the lower grades need help from teachers in order to share mathematical ideas with one another in ways that are clear enough for other students to understand.

Three aspects of communication are here noted; "share" refers to a creative agent, "ideas" refers to content, and "clear enough..." refers to quality regarding the exchange. One occasion is noted where an aspect of mathematics is specified; type 1 (using "mathematical") and related to the aspect of content in the communication.

In this paper, focus is not on quantifying occurrences of different aspects in a detailed manner, but rather on the existence of different aspects and general tendencies. Although the two authors' separate analyses resulted in several discrepancies, the main results and conclusions reported in this study are representative of each of our separate analyses and our common agreement regarding interpretation and analysis of data, which shows good reliability of the procedure in order to produce answers to the specific research questions of the present study.

RESULTS

Table 1 shows the words and phrases found in the NCTM framework. Due to space restrictions, the table for the Swedish framework is not presented, but the result is summarised. In both frameworks of mathematical competence, all aspects of communication are described through the use of corresponding words or phrases. The NCTM framework describes all components (i.e. specifications of aspects) while the Swedish framework does not describe *bodily* as technique or *breadth of information* as quality.

Aspect	Component	Words and phrases
Agent	Sender/ creator	Write; Draw; Speak; Talk; Describe; Explain; Convey; Express (oneself); Articulate; State; Build; Use (e.g. certain technique); Present; Reason; Claim; Justify; Give account of; Describe; Clarify; Formulate; Share; Convince; Act out; Think out loud; Pose a question; Question; Complete; Make public; Work out (in public); Provide; Critique.
	Receiver/ interpreter	Read; Listen; Interpret; Analyze; Evaluate; Examine; Consider; Probe; Explore.
	Both of the above	Converse; Discuss; Dialogue; Respond; Paraphrase; Participate (in conversation).
Technique	Oral	Talk; Speak; Listen; Oral; Discuss; Dialogue; Converse; Think out loud.
	Written	Write; Draw; Read; Symbol; Diagram; Picture; Mathematical expression; Sketch.
	Bodily	Act out; Use object.
	Unspecified	Tools; Ways (of communicating); Informal means; Verbal; Word; Vocabulary; Terminology; Terms; Representation; (Some type of) language; Genre.
Quality	Depth of info	Precise; Coherent; Clear; Thoughtful; Rigorous.
	Breadth of info	Complete; Rich; Elaborate.
	The exchange	Understandable; (Sufficiently) convincing; Audience; Purpose; Communicative power.
	Unspecified	Mathematically; Sophisticated; Well-constructed; Exemplary; Problematic; Informally; Formally; Standards (of dialogue/argument); Carefully.
Content	-	Understanding; Viewpoint; Argument; Idea; Situation; (Result of) thinking; Strategy; Explanation; Solution; Mathematics; Reasoning; Method; Task; Problem; Question; Answer; Evidence; Example; Procedure; Result; Insight; Claim.
Unspecified	-	Communicate; Discourse.

Table 1: Aspects of communication found in the NCTM framework.

Regarding how aspects of mathematics are included in aspects of communication, the analysis of the NCTM framework shows that most often mathematics is part of content (approximately 60 % of all occasions) and otherwise part of technique, except on one

occasion when it is part of quality and one occasion when it is unspecified, using the following words and phrases:

- *Specifying content*: mathematical thinking, strategy, mathematical idea, solution, mathematics, mathematical/procedural task, method, reasoning, mathematical argument, proof, procedure, result, mathematical property, mathematical understanding.
- *Specifying technique*: language of mathematics, (mathematical/algebraic) symbol, diagram, communicate in mathematical ways, mathematical terminology/term, mathematical writing, write mathematically, mathematical language, mathematical style, mathematical expression.
- *Specifying quality*: mathematically rigorous.
- *Unspecified*: communicate mathematically.

The same type of analysis of the Swedish framework shows that most often mathematics is part of content (approximately 70 % of all occasions) and otherwise part of technique, using the following words and phrases (translated from Swedish):

- *Specifying content*: mathematics, information/question with mathematical content, mathematical idea, mathematical line of thought, (mathematical) concept, the concept of pie chart, law, method, reasoning.
- *Specifying technique*: language of mathematics, mathematical language, symbols of mathematics, mathematical terminology, pie chart.

CONCLUSIONS AND DISCUSSION

Communication in general is well represented in the frameworks of mathematical competence through many specifications of different aspects of communication, although all specifications focused on in this study are not included in both frameworks. Besides the general aspects of communication, for both frameworks, aspects of mathematics are mostly included as the content of communication and otherwise as technique, except one occasion when an aspect of quality is specified. Mathematics is often specified through labelling something as "mathematical" in some way (e.g. by referring to the language of mathematics or mathematical ideas/thinking), thereby tending to keep descriptions at a general level, since it is not clear in itself what the notion of "mathematical" refers to.

In prior research no clear evidence of the need for content-specific literacy skills have been found, and similar can be said about the analysis of competence frameworks since aspects of mathematics are mainly included as content of communication and aspects of mathematics are often referred to only by labelling something as "mathematical", and it is not clear if or how this could be seen as creating a need for content-specific literacy skills. This conclusion is valid at least for communication using natural language, but the use of different forms of representation is highlighted both in prior research (empirical and theoretical) and in the frameworks (through certain mathematical techniques) as a potential cause for the need for content-specific literacy skills.

Is there a need to teach a specific kind of communication ability in mathematics? There exist much literature about content literacy that discuss benefits of teaching reading also in content areas (Hall, 2005), but perhaps it is not about learning a special kind of reading ability but an effect of a good way of teaching the content that focuses on processes of interpretation and comprehension (Draper, 2002). This perspective can perhaps also be applied on the NCTM framework, since there is much focus in this framework on effects and benefits of using communication in teaching and learning, and guidance on how to create communication-rich mathematics classrooms.

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