Mathematics in the Swedish Upper Secondary School Electricity Program: A study of teacher knowledge

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Abstract

Mathematical knowledge is often a prerequisite for students at Swedish upper secondary vocational programs to be able to study vocational courses, for example electricity courses in the Electricity Program. Electricity Program students study mathematics in their electricity courses as well as in their mathematics course. The mathematics in those two settings has a different character. A goal of this thesis is to investigate what constitutes that character. In this study three mathematics and five electricity teachers have been interviewed about how they would explain three mathematical electricity tasks to students on the Electricity Program. Teacher knowledge in both electricity and mathematics has been used in the analyses and has been compared between the different teacher groups. In addition to providing an overview analysis of all the teachers’ explanations, detailed analyses have been carried out, comparing pairs of teachers’ explanations. The teachers’ choices of explanations and their use of specific and general mathematical knowledge have been studied.

Mathematics contains a wide range of subject areas but also a wide range of representations and methods that highlight different aspects of mathematics. This study shows that different teachers emphasize different aspects of mathematics in their explanations of the same tasks, even though intended to the same students, both in their choices of explanation and in their use of mathematics. The electricity teachers drew upon their practical electrical knowledge when they connected their explanations of mathematics to vocational work. The electrical knowledge they used not only grounded the tasks in a, for them, well-known real-world environment. The electrical knowledge actually helped them to solve the tasks, albeit in a more concrete/specific way than the mathematics teachers. The electricity teachers drew upon more specific mathematical knowledge in their explanations of the interview tasks, whereas the mathematics teachers drew upon more general mathematical knowledge in their explanations. The different explanations of mathematics from the two kinds of teachers are markedly different, depending on whether they have a more practical/vocational or a more general/algebraic approach. The solutions to the interview tasks turned out to be the same but the character of the solutions paths are substantially different. This raises questions regarding the students’ abilities to reconcile the different approaches.
Sammanfattning


1 Introduction

Mathematics is taught in many vocational courses in Swedish upper secondary vocational education. For students in upper secondary education mathematical knowledge is often a prerequisite to study some of the vocational courses and learn an occupation. At least one mathematics course is mandatory for students enrolled in Swedish upper secondary education. In their vocational education, students study mathematics in some of their vocational courses as well as in their mathematics course, but mathematics is used differently in the different settings and alternative aspects of mathematics are highlighted in the various contexts. The Swedish national curriculum document states that the teaching of the first mathematics course should develop the students’ competence in using mathematics in different contexts, and especially in the chosen study program. For example, the teaching of the mathematics course should include algebraic expressions and formulas that are relevant and required in the vocational courses. It has been shown that the practical use of mathematics could be visible to students by integrating the teaching of mathematics and vocational subjects (L. Lindberg & Grevholm, 2011), but there have been problems in trying to accomplish this integration, and the collaboration between mathematics and vocational subjects needs to be developed by schools in Sweden (Sverige. Gymnasieutredningen, 2008). It is not clear how mathematics is used and taught in the vocational courses or if there are differences from how this is used and taught in the students’ mathematics course. That is what this study will start to investigate and hopefully the results can be used to develop the collaboration of mathematics and other subjects.

In the Electricity Program\(^1\) in Sweden, the national curriculum document states that mathematical calculations are an important part in all electricity subjects (Sverige, 2000a). So, mathematics is needed to do electricity work, but what does that mathematics look like at school level? The students on the Electricity program use mathematics in two different courses, their mathematics course and their electricity course, which are taught by two different teachers - mathematics teachers and electricity teachers, with different educational background. The hypothesis of this study is that the use of mathematics and the teaching of mathematics by mathematics teachers and the electricity teachers does differ in some ways. How is mathematics presented by mathematics teachers and electricity teachers, what aspects of

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\(^1\) Since autumn 2011 the program has been called the Electricity and Energy Program, before it was called the Electricity Program. This study was conducted during autumn 2010, so from here the program will be referred to as the Electricity Program.
mathematics are treated and what is this difference? This study will start to systematically compare how mathematics teachers present and explain mathematics when working with mathematical electricity tasks and this is compared with electricity teachers’ presentations and explanations.

This study focuses on the analysis of teacher knowledge. Teacher knowledge has been widely researched since the 1980s and has been showed to be one aspect influencing the quality of mathematics educations (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Tchoshanov, 2011). By looking at teacher knowledge the researchers started to study teachers’ knowledge and thinking and focus on looking at teachers’ knowledge of specific subjects (Shulman, 1986), which is studied in this dissertation. The mathematics teachers and the electricity teachers have different backgrounds, mathematics teachers are educated in mathematics and teaching and electricity teachers have usually been working in vocational positions and have then taken teacher education. In this study, mathematics teachers and electricity teachers have been interviewed about how they would have explained mathematical electricity tasks to potential students. With different education and experience they are explaining the interview tasks, and this study will explore what teacher knowledge the teachers are drawing upon in their explanations. In this study the teacher knowledge of mathematics and electricity is studied as the interview tasks involve both subjects.

1.1 Aim and research questions

The aim of this study is to explore the similarities and differences in mathematics teachers’ and electricity teachers’ explanations of mathematical electricity tasks, using a framework of teacher knowledge.

Specifically the research questions are:

What teacher knowledge in mathematics and electricity do mathematics teachers and electricity teachers in the Electricity Program draw upon to explain mathematical electricity tasks?

Are there characteristic similarities and differences in the knowledge that the teachers draw upon to explain these mathematical electricity tasks?

The rest of this chapter will describe the research that is related to this study. This research includes mathematics in workplaces, in vocational educations and in electricity education. Also included is a description of Swedish upper secondary education and research related to Swedish vocational education.
1.2 Background

This study deals with mathematics education for students who have chosen a vocational and non-theoretical education - students at the Electricity Program at upper secondary schools in Sweden. These students are using mathematics in their electricity courses as well as in their mathematics courses, but these take place in different settings and with different teachers. Mathematics is used and taught differently in the electricity and mathematics courses and this study will highlight some of these similarities and differences. Niss writes about the overall, long-term goals of mathematical didactical research, where the first described goal is to be able to answer the question of whom in society needs what mathematical insights and competences, and for what purpose. Another goal is about how teaching and learning of mathematics is related to other subjects (Niss, 2007), which is also of interest in this study, as it compares mathematics teaching in mathematics and electricity courses. Niss describes what is needed to reach these goals of mathematical didactical research and what that would involve, among other things:

"In addition, we would know and understand how the teaching and learning of mathematics can be viewed in relation to the teaching and learning of other subjects. What significant similarities (and differences) are there between mathematics and other educational subjects in these respects? This would also imply that we knew to what extent, and how, mathematics teaching and learning can gain from interaction with other subjects, both in the terms of content and in the terms of organized cooperation between subjects." (Niss, 2007, p.1294)

This study will focus on mathematics within electricity education. It will start to investigate how mathematics in the mathematics subject is related to mathematics in the electricity subject in the Electricity Program by studying mathematics teachers’ and electricity teachers’ explanations of the same tasks. Electricity education aims to educate students so that they can use what they have learned in their working life. Researchers have studied mathematics at workplaces and this background section starts with a description of that work.

1.2.1 Research in mathematics at work

Mathematics has many faces and is used differently in different contexts. Mathematics can be seen as totally different in various contexts. The classic example of the problem of recognizing mathematics as the same activity is Nunes, Schliemann and Carraher’s study of Brazil street vendors who were calculating prices in the street but were unable to make the same calculations in a school setting (1993). Educational research discusses the situated
component of knowledge, arguing that knowledge is situated and dependent on the activity, context and culture in which it is developed and used (Brown, Collins, & Duguid, 1989). This study focuses on mathematics in electricity education, which aims to teach the students skills that can be used in their future workplaces.

Research studies in the area of workplace mathematics have shown that mathematics in workplaces is used differently than in school mathematics. Workplace mathematics could be said to be situated in the context of the workplace (Fitzsimons, 2002; Triantafillou & Potari, 2010; Williams & Wake, 2007). Noss et al. have studied mathematics in various workplaces, speaking to investment bank employees, nurses and pilots and this gave the researchers “the opportunity of making connections between what are largely two disparate worlds – the world of mathematics learning and the world of mathematics in work” (Noss, Hoyles, & Pozzi, 2000, p. 19). Williams, Wake and Boreham (2001) studied a student who interpreted a graph from an industrial chemistry laboratory using college mathematics knowledge. They observed that the mathematical practice in the workplace was significantly different from that of school mathematics, and pointed out that “one feels like the anthropologists who visit exotic cultures” (p. 81) when they studied workplace mathematics. Mathematics in workplaces is, in addition to school mathematics knowledge, the competence to relate mathematics to its meaning and interpret it in the workplace context. The meaning and interpretation of mathematics has been studied, for example, in the study of technicians’ use of mathematics in a computer-aided design production company (Magajna & Monaghan, 2003), and in the study of nurses use of mathematics (Pozzi, Noss, & Hoyles, 1998). Mathematical knowledge used in technological workplaces is called techno-mathematical literacies. It concerns individuals’ understanding and use of mathematics in workplaces through technology, e.g. the knowledge of how to interpret computer output (Kent, Noss, Guile, Hoyles, & Bakker, 2007). To understand techno-mathematical literacies and to design mathematical learning materials for workplaces, case studies of employees’ use of mathematics at manufacturing and financial service workplaces was carried out (Hoyles, 2010). Hoyles et al. discuss the use of mathematics in workplaces and also state that the use of workplace mathematics differs from school mathematics:

“most adults use mathematics to make sense of situations in ways that differ quite radically from those of the formal mathematics of school, college and professional training. Rather than striving for consistency and generality, which is stressed by formal mathematics, problem-solving at work is characterized by pragmatic goals to solve particular types of problems, using techniques that are quick and efficient for these problems.” (Hoyles, 2010, p. 7)
1.2.2 Mathematics in vocational education

Given that workplace mathematics differs from school mathematics, it is not surprising that there are differences between vocational mathematics and theoretical mathematics education. Students in a vocational education are studying to get an occupation; they are not directly heading towards further education. But most technical vocational education includes some mathematics education, “in most, especially industrialised countries, some sort of mathematical knowledge is part of the vocational training” (Straesser, 2007, p. 167). In the description of the Electricity Program in the Swedish national curriculum document it states that mathematics is an important part of the electricity subject, that mathematical calculations are a prerequisite to exercise the students’ future profession and that this education should therefore develop the students’ mathematical knowledge (Sverige, 2000a, 2011). Below research findings concerning mathematical knowledge in vocational education are described.

Mathematical knowledge could be a prerequisite for students to be able to study a vocational education. In electricity education, mathematics knowledge helps students to understand the vocational situation, and could even be a prerequisite: “understanding electro technical devices is simply impossible without mathematics” (Straesser, 2007, p. 168). Hill points out the need for algebra knowledge in electricity education: “Electricians need to know concepts that just cannot be understood and worked with unless they have some knowledge of algebra” (R. O. Hill, 2002, p. 452)

Mathematics in vocational education is not studied for the sake of mathematics; it is studied to reach the vocational education’s goals. Mathematics in vocational education has a purpose only if it adds value to the vocational course (Gillespie, 2000). The vocational field offers a vast array of applications for mathematics, which is often used as a tool to model the workplace situation, but mathematics is not needed as such (Straesser, 2007). The priorities of mathematics teaching could also be different in mathematics and vocational courses, “the vocational theory is not interested in a deep understanding of mathematical concepts or procedures, but in an effective and fast prediction of the vocational situation.” (Straesser, 2007, p. 168).

Vocational teachers are not usually educated to teach mathematics. They have usually been working for several years and then studied education and pedagogy to become vocational teachers. The mathematical education they have is from their own studies to become a professional in their former vocation. Straesser and Bromme report from a study where they interviewed
vocational teachers about what they think about the relation between mathematical and vocational knowledge. When they analyzed the professional knowledge of these vocational teachers they found that the vocational teachers’ curricular knowledge could be divided into mathematics knowledge and vocational knowledge, and they state that these domains of knowledge differ widely and could even be taken for different cultures. In their study, half of the vocational teachers found that mathematics was a helpful tool in vocational contexts (and nothing else): ”Mathematics has its fundamental purpose in helping with vocational problems, serving as an operative tool” (Strasser & Bromme, 1989, p. 194). A third of the vocational teachers said that mathematics could be a help to understand vocational situations.

In Swedish upper secondary vocational education, one mathematics course is compulsory for all students. Swedish upper secondary mathematics education is described below.

1.2.3 Swedish upper secondary mathematics education

Swedish upper secondary schools, called gymnasiums, are divided into theoretical and vocational programs. Students choose either a theoretical or a vocational program after compulsory school and, since 1994, they study this program over a three-year period. In all of the programs there are some core subjects that all students study, including the first mathematics course and, for example, Swedish and English. For the students on the Electricity Program it is compulsory to study at least the first mathematics course. The subject of mathematics in Swedish gymnasium builds on the mathematics knowledge the students have learned at compulsory school by broadening and deepening the subject. The national curriculum document stated, in the year 2000, that:

“The power of mathematics as a tool for understanding and modelling reality becomes evident when the subject is applied to areas that are familiar to pupils. Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course. Knowledge of mathematics is a prerequisite for achieving many of the goals of the programme specific subjects.” (Sverige, 2000b)

So the mathematics course should be related to the program that the students are studying and mathematics knowledge could be a prerequisite

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2 When this study was conducted, during the autumn of 2010, this mathematics course was named Mathematics A. After the reform of the Swedish gymnasium, autumn 2011, this first mathematics course for the vocational education is called Mathematics 1a.
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for students to study other subjects, e.g. electricity. In 2011, the Swedish gymnasium was reformed, which lead to the creation of a new national curriculum document, which emphasize the integration of mathematics and other subjects. For example, in the new curriculum document it states that mathematics is a tool within science and for different vocations, and that mathematics furthermore deals with the discovery of patterns and the formulation of general relations (Sverige, 2011). Also, the curriculum document states that the mathematics courses should relate mathematics to its meaning and use in other subjects, professions, society and historical contexts. Since 2011 the first mathematics course has been divided into three versions, one for vocational programs, one for social science programs and one for nature science programs, with some differences in the content of the courses. The teachers in each course decide how to plan, teach and evaluate the course according to the national curriculum documents. The national curriculum document consists of descriptions of general and process goals of the subject and, for each course, highlights the content goals and assessment criteria. How the structure of the course is modified and which problems are chosen, according to the students’ study program, is up to the teachers.

How mathematics teachers relate mathematics to vocational situations and use in professions is up to the teachers. There are different ways of adapting the mathematics course to the students chosen study orientation, e.g. problem-based learning, projects and themes, or infused subjects (Rudhe, 1996). It is called an infused subject when the theoretical subject is used as a tool in the vocational subject, or when the vocational subject is used as material in the theoretical subject. For example, mathematics teachers can design their teaching using examples from the vocational courses and often in collaboration with vocational teachers (Berglund, 2009). The aim of infused subjects is to make a functional whole of the vocational subjects and the more theoretical subject, like mathematics, and this could motivate the students to study subjects which are more theoretical (Rudhe, 1996). Adapting the mathematics course to working life often leads to cooperation between mathematics teachers and vocational teachers, so they work together to plan of their courses. This could lead to shared understandings of each other’s viewpoints (Gillespie, 2000; L. Lindberg & Grevholm, 2011).

In mathematics education research, there is a discussion about the goal of school mathematics. School mathematics should prepare students not only for academic studies but also for work and life. The American high school program Functional Mathematics is developed to prepare students for life and work and state: “focusing on useful mathematics increases total learning” (p. 128) and they emphasize: “that the goal of mathematics education is not just mathematical theory and word problems, but authentic
Introduction

mathematical practice” (Forman & Steen, 2000, p. 133). Tasks from workplace and everyday contexts can also motivate students and stimulate students’ thinking, and workplace context could also be a way of concentrating more on concepts and less on procedures (Taylor, 1998).

At the beginning of 2000, a Swedish research team developed a research project to test if it was possible to integrate mathematics teaching with vocational courses. The mathematics and vocational teachers in the project cooperated around the mathematical content, models and teaching methods in their courses. The results show that it is possible to integrate mathematics with vocational courses and both the teachers and the students considered this teaching to be more meaningful and the students became more motivated (L. Lindberg & Grevholm, 2011). When Lindberg and Grevholm discussed the arguments for and against integrating the mathematics course with vocational courses, they wrote: “pupils in vocational education need a different kind of mathematics course that is directly related and seen by the students to be directly related, to their vocational studies” (p. 41).

1.2.4 Research in electricity education

In Sweden, electricity education is included in the science curricula in primary education and in the physics curricula in theoretical secondary education. In vocational secondary education physics is not included, but in the Electricity Program there are, of course, several courses in electricity.

Research in electricity education has shown that the concepts of electricity are problematic in education as they are abstract, not visible and the teaching of electricity is therefore dependent on models and analogies (Mulhall, McKittrick, & Gunstone, 2001). Research also shows that secondary students have different conceptions of electricity circuits and they used different mental models or explanation models, with different conceptions of current and energy and alternative views of how the current circulates in the circuit (Borges & Gilbert, 1999; Kärrqvist, 1985). Kärrqvist (1985) found six explanation models that secondary students used to make sense of an electrical circuit (ordered from least to most correct description of the electrical circuit):

- Unipolar model: the current flows from the positive terminal of the battery in one conductor to the base of a bulb where it is consumed and lights the bulb.
- Two-component model: ‘plus’ and ‘minus’ currents travel from the battery terminal in one conductor each to a bulb, where the currents meet and produce energy that lights the bulb.
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- Closed circuit model: all components in the circuit have two poles that are connected in the closed circuit so the current can circulate. Current and energy have the same meaning.
- Current consumption model: current circulates the circuit and is consumed as it goes through a resistive component.
- Constant current source model: the current circulates the circuit but the current from the battery is always the same, regardless of the circuit. The current is not consumed in the circuit but can vary in the different parts of the circuit.
- Ohm’s model: current flows around the circuit transmitting energy. The circuit is seen as a whole interacting system, a change at one point in the circuit affects the entire system. The amperage depends not only on what battery is used, but also of the components in the circuit. This corresponds to the ‘scientific view’.

Electricity is a complex and sometimes difficult area both to teach and learn. In one study, experienced teachers were interviewed about their teaching of DC electricity, their understanding of the concepts of DC electricity, their use of models and analogies in this teaching and their views of students’ difficulties with this subject (Gunstone, Mulhall, & McKittrick, 2009). In their results they describe how teachers who had a better understanding of the area were also more informed about the difficulties in teaching it, and teachers who saw the electricity area as not as difficult to teach had more simple views of learning and understanding the concepts. Also, electricity textbooks have been studied and their authors have been interviewed about electricity concepts and inadequacies in both the authors’ conceptions and their textbooks were found (Gunstone, McKittrick, & Mulhall, 2005).

1.2.5 Research in Swedish vocational education

Some research studies of upper secondary vocational education in Sweden have been carried out, but not specifically focusing on mathematics education. In the Building and Construction Program, students’ perspectives of the mandatory academic subjects, called core subjects (e.g. mathematics, Swedish and English) have been studied by Högberg (2009), who followed two classes during one year. Most of the students stated that they thought there was no use in studying these core subjects as they did not think that they would be beneficial to them in their occupations, but mathematics was an exception:

“However, in line with their orientation towards their future work, they considered mathematics to be important because of its relevance to work in construction. Some pupils also said that the core subjects could be acceptable, but only if they were relevant to their future work.” (Högberg, 2009, p. 167).
Vocational education aims to teach students the practice and the social practice of the students’ future occupation. Lindberg (2003) studied the classroom tasks that vocational teachers’ gave students to work with. She concludes that vocational education seems to employ tasks that bridge the practice of school and the practice of work. Also, in school, the focus is on learning the social practice of the vocation while the focus in work is on production. Johansson (2009) describes the differences between pure school subjects and vocational subjects in the vocational education. He highlights that vocational teachers teach vocational theories, practical skills and also mediate the culture of the profession. Vocational teachers are introducing the students to the socialization of their future profession. Johansson points out that in vocational education there is a strong connection between learning and usefulness in vocational didactics. Things that are not useful are not counted as worth learning.

1.2.6 Summary

Mathematics is used differently in different contexts, and school mathematics and workplace mathematics could even be seen as two separate worlds. Also, in vocational education, mathematics is often used differently than in the mathematics courses, as mathematics is usually used as a tool in vocational education.

In the Electricity Program in upper secondary vocational education in Sweden, mathematics is taught in both the mathematics and electricity courses. Mathematics is presented by both mathematics teachers and electricity teachers and these teachers have different types of mathematical education and experience. This study will explore what teacher knowledge mathematics teaches and electricity teachers at the Electricity Program draw upon in their explanations of some mathematical electricity tasks. Furthermore, this study suggests a way to characterize the similarities and differences in the teachers’ explanations.
2 Theoretical constructs

What teachers do in their classrooms is influenced by a number of things. Schoenfeld (2011) presents goals, resources and orientations as important parts in his model of teachers’ decision-making in the classrooms. The teacher’s orientations determine what they see as relevant and what resources they will use to achieve their goals. Here, resources here mean knowledge and include different kinds of knowledge: facts, procedures, conceptual knowledge and knowledge of problem-solving strategies. The focus of this study was chosen to be teacher knowledge, as the two teacher groups have totally different educational backgrounds and, therefore, also different knowledge of their subjects, mathematics and electricity. However, they are teaching the same students the same kind of tasks with the same goal of educating the students for their future occupation. In this study, where the differences of the teachers’ presentations were analyzed, teacher knowledge was chosen, as it was expected to be one of the main features that differentiate the mathematics and the electricity teachers. The content of that knowledge is complex enough to study in its own right.

2.1 Overview of research on teacher knowledge

Mathematics teachers’ and electricity teachers’ explanations of mathematical electricity tasks are the focus of this study. In this study, the similarities and differences in the two teacher groups’ explanations are studied. The analysis uses teacher knowledge to discern similarities and differences in the teachers’ explanations. This chapter describes the theoretical background for teacher knowledge and presents examples of how teacher knowledge has been used in research studies. The theory of teacher knowledge provides a framework for analyzing the knowledge the teachers draw upon in their teaching. The literature on teacher knowledge, in general, tries to categorize different kinds of knowledge that teachers have. This work has been helpful in recognizing that teaching involves many kinds of competencies other than that of subject matter and identifying different kinds of expertise. However, this research is still in development, with some of the main terms undefined, and a number of different, still un-synthesized, attempts to characterize teacher knowledge in a variety of different settings. In this chapter, an overview of how teacher knowledge is used in mathematics and science education is presented. The theoretical framework that is used to analyze the data in this study is a simpler one derived from Shulman’s earlier work (Shulman, 1986), and this is described in this chapter. At the end of the chapter three popular models of teacher knowledge are discussed in relation
Theoretical constructs

to this study to give a sense of this research field and to contrast the chosen definitions.

2.1.1 Teacher knowledge in general

Teacher knowledge is a widely researched area. The area grew out of work by Shulman (1986) and others who wanted to systematically study the knowledge teachers draw upon in their everyday practice. The approach to systematically studying teachers’ knowledge led to a shift in understanding and a new valuation of the teachers’ practical work rather than previous approaches that focused on evaluation and labeling of teachers and teaching behavior (Feiman-Nemser & Floden, 1986).

The importance of content knowledge in teaching was highlighted by Shulman (1986). Shulman described three categories of content knowledge in teaching - subject matter content knowledge, pedagogical content knowledge and curricular knowledge. Shulman stated that teachers need content knowledge and general pedagogical knowledge, but teachers must also be able to explain their subject - “Mere content knowledge is likely to be as useless pedagogically as content-free skill.” (Shulman, 1986, p.8). Shulman highlighted teachers’ need and use of subject-specific pedagogical knowledge in addition to content knowledge of their subjects. He introduced pedagogical content knowledge as an important part of teacher knowledge, “pedagogical knowledge, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching.” (Shulman, 1986, italicizing in original, p.9). Pedagogical Content Knowledge (PCK) highlights the importance of specific pedagogical knowledge for specific Subject Matter Knowledge (SMK).

What is included in teacher knowledge and the definitions of teacher knowledge is not clear. Here, the main parts of teacher knowledge are discussed and problems with the definitions are highlighted. In 1987, Shulman expanded his model of teacher knowledge in which he included seven categories of teachers’ knowledge base (Shulman, 1987). Shulman’s categories of teacher knowledge consisted of:

- Content knowledge,
- General pedagogical knowledge,
- Curriculum knowledge,
- Pedagogical content knowledge,
- Knowledge of learners and their characteristics,
- Knowledge of educational contexts
- Knowledge of educational ends.
He highlighted the importance of PCK:

“Among those categories, pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction.” (Shulman, 1987, p.8)

Teacher knowledge and especially PCK has since then been used in a wide range of research studies.

Shulman’s model of teacher knowledge was intended for different school subjects. His research group has been studying secondary teachers in English, biology, mathematics and social studies, and the group has followed them during their teacher preparation and their first year of teaching (Shulman, 1986). The research group found that teachers of different subjects have different pedagogical strategies to help the students learn the subject. This has inspired the educational research community to continue to study different subject’s specific pedagogic or PCK in a variety of fields, as there is an agreement that this contributes to effective teaching and student learning. This study aims to compare mathematics teachers’ and electricity teachers’ explanations of the same interview tasks, and teacher knowledge was chosen as the first analyzing instrument. The teachers in this study mixed both mathematical and electrical knowledge in their explanations, so this study deals with both teacher knowledge in mathematics and in electricity. This study identifies and compares teacher knowledge of both mathematics and electricity that the two teacher groups draw upon to explain the same interview tasks.

Below, examples of research studies using teacher knowledge in mathematics and in science education are given. Teacher knowledge is rarely used in vocational education research, but electricity is included as a topic in the science curriculum both in primary and secondary schools and teacher knowledge is widely investigated in science education research. Therefore examples of teacher knowledge in science education will also be given.

2.1.2 Studies of teacher knowledge in mathematics

Since Shulman’s introduction of the PCK concept, there has been a lot of interest in identifying what effective teachers know or should know when teaching mathematics (Baumert et al., 2010; Hill et al., 2008; Hill et al., 2005). Researchers conclude that the quality of teaching is dependent on the subject-related knowledge the teachers are able to bring to their teaching, but there is no agreement regarding what kind of mathematical knowledge
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This is and how it could be studied (Rowland & Ruthven, 2011). Investigations of the nature of teachers' mathematical knowledge in teaching have been carried out and researchers have started to identify “mathematical knowledge for teaching” (Ball, Thames, & Phelps, 2008) or “mathematical knowledge in teaching” (Rowland & Ruthven, 2011).

The main fields in studies of teacher knowledge in mathematics contain studies of the relation between teacher knowledge and the effect of students’ outcomes and the quality of teaching. Teacher knowledge has also been used in comparative studies of different teachers.

Hill et al. (2005) studied teachers' mathematical knowledge for teaching, including both PCK and SMK, and found that teacher's mathematical knowledge was significantly related to students’ achievements in both the first and third grades. Tchoshanov (2011) studied how the cognitive types of teachers’ content knowledge affected both student achievement and teaching practice. The cognitive types of content knowledge were knowledge of facts and procedures, knowledge of concepts and connections and/or knowledge of models and generalizations. Tchoshanov concludes: “teacher content knowledge of concepts and connections is significantly associated with student achievement and lesson quality in middle grades mathematics” (Tchoshanov, 2011, p.162). Whether PCK and SMK each make a contribution to explaining differences in quality of instruction and student progress was investigated in the German COACTIV study (Baumert et al., 2010; Krauss et al., 2008). In this study, tests were constructed to assess secondary mathematics teachers’ PCK and SMK. They assumed that PCK is inconceivable without sufficient SMK, but that SMK cannot substitute PCK, and they found that teachers’ PCK was distinguishable from their SMK in grade 10 classes. They also concluded that students’ learning gains were positively affected by the teachers’ PCK in mathematics.

“PCK- the area of knowledge relating specifically to the main activity of teachers, namely, communicating subject matter to students – makes the greatest contribution to explaining student progress and that SMK has lower predictive power for student progress (Baumert, 2010, p.168).

Krauss et al. report from the same COACTIV study that “the degree of cognitive connectedness between PCK and SMK in secondary mathematics teachers is a function of the degree of mathematical expertise” (Krauss et al., 2008). Hill et al. (2008) studied mathematical knowledge for teaching, including both PCK and SMK, and found a strong and positive association between the teachers’ mathematical knowledge for teaching and the mathematical quality of the teachers’ instruction.
Teacher knowledge has been used in comparative studies of different teachers. One group of studies looks at differences of expert and novice teachers. Leinhardt and Smith (1985) studied expert and novice mathematics teachers’ knowledge of fractions and made an in-depth analysis of the teachers’ explanations. They concluded that experts had deeper and more elaborated knowledge than the novice teachers, but also that among the experts there were differences in levels of SMK. Krauss et al. (2008) report from a study where they tested secondary-level mathematics teachers’ content knowledge and PCK and found that mathematics teachers with an in-depth mathematical training outscored teachers from other school types on both content knowledge and PCK, and that they had a higher level of connectedness between the two knowledge categories. Mathematical teacher knowledge has also been studied in international comparative studies. Ma (1999) studied elementary school mathematics teachers in China and the United States (US) and documented the differences between the Chinese and US teachers’ knowledge of mathematics-for-teaching. Ma made a suggestion of how Chinese teachers’ understanding of mathematics and of their teaching contributed to their students’ success. She suggested that the most skilled Chinese teachers had a “profound understanding of fundamental mathematics” (PUFM) or a deep understanding of the domain and knew how to help the students to understand it, but she did not find that the US teachers had this knowledge. This PUFM contains both SMK and PCK, as it aims at meaning-making and deep understanding. The first property of PUFM is connectedness and deals with the teachers’ intentions to make connections among mathematical concepts and procedures, which will help the students to learn a unified body of knowledge.

2.1.3 Studies of teacher knowledge in science

Research of teacher knowledge in science education usually involves the use of PCK (Gess-Newsome, 1999), whereas in mathematics several models of teacher knowledge exist within different categories. PCK is highly valued for its potential to define important dimensions of expertise in science teaching, both for teacher education and in-service teachers.

“Many science teachers and science teacher educators have a wealth of knowledge about how to help particular students understand ideas such as force, photosynthesis, or heat energy; they know the best analogies to use, the best demonstrations to include, and the best activities in which to involve students. Our identification of this knowledge as pedagogical content knowledge recognizes its importance as distinguished from subject matter or pedagogical knowledge.” (Magnusson, Krajcik, & Borko, 1999) (p.116)
Theoretical constructs

Magnuson et al. argue that PCK lies outside the expert knowledge of a content specialist and the general educator, and that PCK is an important construct in the development of effective teachers of science. PCK has been used to capture, document and portray science teachers’ expert knowledge of teaching (Loughran, Mulhall, & Berry, 2004). They state:

“The foundation of (science) PCK is thought to be the amalgam of a teacher’s pedagogy and understanding of (science) content such that it influences their teaching in ways that will best engender students’ (science) learning for understanding” (Loughran, Mulhall, & Berry, 2004, p. 371).

Loughran et al developed a method for studying science teachers’ PCK that consisted of at first looking at particular science content and for this they studied teachers’ different professional and pedagogical experience repertoires of this particular content. Nilsson (2008) studied student-teachers’ development of PCK, where PCK was seen as a teacher’s integration of SMK and pedagogy in ways which intended to enhance students’ learning. The student-teachers’ reflections on their own teaching were analyzed in the terms of three knowledge bases (subject matter knowledge, pedagogical knowledge and curricular knowledge) and Nilsson suggested that experience and reflection helped the student-teachers to transform their knowledge bases into PCK and develop their PCK.

2.2 Theoretical issues

There is no strict definition of teacher knowledge and no common understanding of what is included. Ball et al. point out that this field is in need of analytic and theoretical development, and argue:

“the field has made little progress on Shulman’s initial charge: to develop a coherent theoretical framework for content knowledge for teaching. The ideas remain theoretically scattered, lacking clear definition.” (Ball et al., 2008, p.394)

The SMK and PCK categories are frequently used in research studies of teacher knowledge, but the definitions for these terms are not exact. There are also researchers who argue that there is no distinction between SMK and PCK, and say that, for teachers, all knowledge is pedagogic (McEwan & Bull, 1991). However, in a large number of studies of teacher knowledge the distinction is used.

“We have already pointed out the ambiguous boundary between SMK and PCK, but feel that the two categories are useful organizing devices in describing teacher knowledge for research purposes” (Petrou & Goulding, 2011, p.22)

Below, this study’s focus on teacher knowledge is described and defined.
**2.2.1 Focus on PCK/SMK**

The teachers in this study explain mathematical electricity tasks in the electricity context, and the teachers use mathematical knowledge and electrical knowledge in their explanations. The frameworks for teacher knowledge described above only look at one subject at a time, e.g. mathematics or physics. In Shulman’s model, he also looks at one subject at a time, and Ball’s framework ‘Mathematical Knowledge for Teaching’ characterizes mathematics knowledge. But the teachers in this study draw upon both mathematical knowledge and electrical knowledge when they explain the interview tasks, as these involve both mathematics and electricity. In this study teacher knowledge in both mathematics and in electricity is studied.

In this study, the goal is to study similarities and differences in the teachers’ explanations and to explore and compare the mathematical and electrical knowledge the teachers draw upon in their explanations. The two teacher groups teach different courses, mathematics and electricity courses, with different curriculums, but mathematics is covered in both courses. Therefore, no questions about the curriculum were asked in the interviews in this study. In this study Shulman’s (1986) original categories; Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK) are used, and the third category curricular knowledge is left out. Of course, the teachers’ explanations could be an indirectly expression of their curricular knowledge, but no teacher in this study explicitly talked about the curriculum as a reason for their explanation. Furthermore, both Ball and Grossman proposed to include curricular knowledge into the PCK category (Ball et al., 2008; Grossman, 1990).

In the interviews, one teacher’s explanation of a task was often a mix of several mathematical and electrical explanations. First, a distinction between mathematical teacher knowledge and electricity teacher knowledge was made, then, all explanations were further divided into SMK or PCK.

Below, different definitions of the terms PCK and SMK are described, and then the definitions for this study are presented.

**2.2.2 Definitions of PCK in research studies**

Pedagogical Content Knowledge (PCK) deals with content knowledge for teaching, and has been used in a variety of research studies with different or vague definitions, and there have been attempts to analyze its contents. It is
difficult to theoretically define PCK, but, practically, PCK represents the knowledge teachers are using in the classrooms:

“In a practical sense, however, it represents a class of knowledge that is central to teachers' work and that would not typically be held by nonteaching subject matter experts or by teachers who know little of that subject.” (Marks, 1990, p.9)

In Shulman’s original article, PCK consists of two parts: first, knowledge of representations of the subject and second, knowledge of students and the subject. The first part of PCK, knowledge of different representations of the subject, was described as:

“Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one’s subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others.” (Shulman, 1986, p. 9)

In the second part of PCK, Shulman included knowledge of what makes the subject difficult to learn, preconceptions students usually have and how to help students overcome these issues (Shulman, 1986). Schoenfeld also draws attention to the importance of including knowledge of students’ conceptions and preconceptions in PCK.

“Knowing to anticipate specific student understandings and misunderstandings in specific instructional context, and having strategies ready to employ when students demonstrate those (mis)understandings – is an example of pedagogical content knowledge (PCK).” (Schoenfeld, 2006, p.480)

PCK is usually described as containing these two parts: knowledge of how to explain the subject and knowledge of students and the subject. Schoenfeld points out how PCK differs from SMK: one can understand how to do something (SMK) without knowing how to explain it to students and anticipate students’ problems with it (PCK) (Schoenfeld, 2006).

Different subcategories of PCK have been proposed based on research studies. In the rest of this section a summary of some research studies that have been described in PCK subcategories is given. In all these studies the subcategories of PCK include both the knowledge of how to explain the subject and the knowledge of students and the subject. Ball et al. have elaborated on the construct of PCK in their theory of Mathematical Knowledge for Teaching (2008). They have divided PCK into two subcategories;

- Knowledge of content and students
- Knowledge of content and teaching.
Knowledge of Content and Students (KCS) is knowledge about students and mathematics: knowledge of what students are likely to think and what they will find confusing, what the students will find interesting, motivating, easy and hard. KCS also includes knowledge of common students’ conceptions and misconceptions about particular mathematical content. The category Knowledge of Content and Teaching (KCT) combines knowledge about teaching and knowledge about mathematics. For example when designing instructions, the teacher will decide in what sequence a particular content will be taught and what examples to use. Marks (1990) writes that the common view of PCK is that it is an adaption of SMK for pedagogical purposes, but he also found the reverse process: the application of general pedagogical principles to particular subject matter contexts. For his study of fifth-grade teachers teaching fractions, Marks suggests that PCK could be portrayed composed of the following four highly integrated parts:

- Subject matter for instructional purposes.
- Students’ understanding of the subject matter.
- Media for instruction in the subject matter.
- Instructional processes for the subject matter.

Krauss et al. (2008) used PCK in a study that assessed secondary-school teachers’ mathematical knowledge. They characterized PCK as:

- Knowledge of mathematical tasks.
- Knowledge of student misconceptions and difficulties.
- Knowledge of mathematics-specific instructional strategies.

In their study they used questionnaires and test items which tested highly and lowly educated mathematics teachers’ PCK and content knowledge. Their results indicate that the highly educated mathematics teacher had a greater level of connection between PCK and content knowledge.

PCK has been used in science education and Magnusson, Krajcik and Borko (1999) conceive PCK as a transformation of the knowledge of subject matter, pedagogy and context.

“Pedagogical content knowledge is a teacher’s understanding of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented, and adapted to the diverse interests and abilities of learners, and then presented for instruction” (Magnusson, Krajcik and Borko, 1999, p. 96).

They include the following five parts in PCK of science teaching:

- Orientations towards science teaching.
- Knowledge and beliefs about the science curriculum.
- Knowledge and beliefs about students’ understanding of specific science topics.
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- Knowledge and beliefs about assessment in science.
- Knowledge and beliefs about instructional strategies for teaching science.

In all these research studies PCK contains both knowledge of how the subject is taught and knowledge of students and the subject. That will also be used in this study.

### 2.2.3 Definitions of SMK in research studies

Subject Matter Knowledge (SMK) involves knowledge of the subject that the teacher is teaching. Shulman (1986) pointed out the importance of content knowledge in teaching and that it is important for teachers to know the concepts and facts of the subject as well as to understand the structures of the subject.

> “The teacher need not only understand that something is so; the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied.” (Shulman, 1986, p. 9)

In Ball et al’s (2008) theory of Mathematical Knowledge for Teaching, SMK has been divided into two sub-categories;

- Common content knowledge.
- Specialized content knowledge.

Common Content Knowledge (CCK) is mathematical knowledge and skills that are not unique to teaching and are used in settings other than teaching, e.g. correctly calculating or solving mathematical problems. Specialized Content Knowledge (SCK) is mathematical knowledge and skill that is unique to teaching, and it is not needed for anything else but teaching and it is knowledge beyond what is taught to the students, e.g. an explanation of why you ‘add a zero’ when you multiply by 10.

Krauss et al. (2008) and Baumert et al. (2010) have conceptualized SMK as in-depth background knowledge of the content of the secondary-level mathematics curriculum, with both conceptual and procedural skills. When they used questionnaires to assess teachers’ content knowledge in mathematics, they used items that required complex mathematical argumentation or proof.
Teacher knowledge is widely used in educational research, and since Shulman’s introduction in 1986, different researchers have used teacher knowledge and worked on developing definitions for the terms. Teacher knowledge was originally not intended for a specific subject and has been used both in mathematics and science educational research (the science curriculum usually includes electricity as a topic).

In this study, teacher knowledge was chosen to analyze the teachers’ explanations, and as the teachers used both mathematical knowledge and electrical knowledge, teacher knowledge of both mathematics and electricity was studied. Teacher knowledge is in this study limited to Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), and the teachers’ explanations are categorized in SMK and PCK in mathematics and SMK and PCK in electricity.

In research studies, PCK usually includes both knowledge of how to explain the subject and knowledge of the subject and students. Knowledge of how to explain the subject includes representations, how the subject is taught and useful analogies to help students understand the subject. Knowledge of the subject and students includes knowledge of students’ common preconceptions and problems/difficulties with the subject.

Below, the definitions of teacher knowledge used in this study are presented, with a small example from this study. Thereafter three frameworks of teacher knowledge in mathematics are presented to contrast the definitions chosen in this study.
Theoretical constructs

2.3 Theoretical framework for this study

2.3.1 Definitions of constructs

In this study, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) in both mathematics and electricity are used in the analyses of the teachers’ explanations of the interview tasks. The categories in this study build on Shulman’s original definitions of teacher knowledge (Shulman, 1986).

In this study, SMK (in mathematics and in electricity) is defined as the knowledge of the subject, which includes both conceptual and procedural knowledge. PCK (in mathematics and in electricity) is defined in this study to include two parts:

Part a) knowledge of useful representations, examples and illustrations to make the content accessible to the students.

Part b) knowledge of common students’ conceptions, misconceptions and difficulties with the content.

In this study, the following definitions will be used:

**Subject Matter Knowledge (SMK) in mathematics:**

The ability to calculate and reason directly related to mathematics, including knowledge of facts and concepts and an understanding of how they are related and why they are valid. Included is knowledge of methods and procedures and why they are valid. Examples include making calculations, manipulating formulas and reasoning about formulas.

**Pedagogical Content Knowledge (PCK) in mathematics:**

a) Knowledge of how to teach the subject and knowledge that a teacher could draw upon to help a student move forward with their mathematical reasoning. This includes explanations, representations, analogies, illustrations and examples to make the mathematical content accessible to students. For instance, using an easier example to explain something.

b) Knowledge of students and the subject, including the teachers’ view of students’ common conceptions and mistakes and what students are likely to find difficult. Also included is knowledge of how to help students with this. Included here is teachers’ experience with students’ pre-knowledge of mathematics and students’ difficulties with mathematics in tasks like this. This involves everything that the teachers say about their experience of students and the subject of mathematics.
Theoretical constructs

**Subject Matter Knowledge (SMK) in electricity:**
The ability to reason about electricity, including knowledge of electricity facts and concepts, and an understanding of how they are related and why they are valid. This includes common content knowledge needed by someone working with electricity and content knowledge about electricity, usually included in the physics course. Examples include applying Ohm’s law and reasoning about the circuit diagram.

**Pedagogical Content Knowledge (PCK) in electricity:**
a) Knowledge of how to teach the subject and knowledge that a teacher could draw upon to help a student move forward with their electrical reasoning. This includes explanations, representations, analogies, illustrations and examples to make the electrical content accessible to students. For instance, using an illustration or a model to explain the electrical circuit.

b) Knowledge of students and the subject, including the teachers’ view of students’ common conceptions and mistakes and what students are likely to find difficult. Also included is knowledge of how to help students with this. This includes teachers’ experiences with students’ pre-knowledge of electricity and students’ difficulties with electricity in tasks like this. This involves everything that the teachers say about their experience of students and the subject electricity.

### 2.3.2 An example from this study

To clarify these definitions, I will use an example from one interview task from this study to exemplify how the definitions of SMK and PCK in mathematics and SMK and PCK in electricity were used in this study. In this interview task, the teachers were asked to help a student calculate the total resistance in a parallel circuit with two given parallel resistances (120 Ω and 220 Ω). The formula for calculating the total resistance in a parallel circuit was given: \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \), where \( R_1 \) and \( R_2 \) are the parallel resistances and \( R \) is the total or equivalent resistance in the circuit.

The teachers explained this in different ways; both using different mathematical approaches and different electrical explanations that they thought were relevant to this task. A teacher’s explanations of this task were categorized as follows:
Theoretical constructs

**SMK in mathematics:**
Calculating the total resistance in the parallel circuit was categorized as SMK in mathematics, as this involves knowledge of calculations and mathematical methods. Also included here was the statement that rounding in the part calculations would affect the accuracy of the final answer, as this involves understanding that taking away accuracy within the calculation will affect the final answer.

**PCK in mathematics:**
Knowledge of how to teach the subject included explanations that helped the students to make calculations in the task, e.g. the use of an easier example in the beginning. Knowledge of students and the subjects were included in all teachers’ statements concerning their experience of students working with tasks like this, e.g. when a teacher said that it was too difficult for students to solve the fraction task or that the task was easier to solve if the students used a specific structure.

**SMK in electricity:**
This category includes knowledge of the parallel circuit, e.g. a teacher’s statement that the total resistance in a parallel loop is always less than the smallest resistance in the loop. This category also includes teachers’ statements about the parallel circuit, which is useful in an electricity workplace, e.g. knowledge of a simpler formula if the resistances are equal.

**PCK in electricity:**
Knowledge of how to teach the subject included explanations of how to help the students understand the parallel circuit. For instance the use of an analogy to explain the parallel loop, e.g. to compare the loop with bridges over a river, or explanations with electrons moving in the conductors. Knowledge of students and the subjects were included in all teachers’ statements concerning their experience of students working with tasks like this, e.g. that students can mix the concepts of serial and parallel circuits.

**2.4 Frameworks for mathematical teacher knowledge**

There are different ways of talking about teacher knowledge in mathematics education research and different ways of studying the complicated relation between teachers’ knowledge and their teaching practices. The choice of analyzing the data in this study with categories developed from Shulman’s original framework for teacher knowledge will now here be compared with other frameworks. The rest of this chapter will present other ways of
studying teacher knowledge in mathematics education research and discuss these frameworks in relation to this study. These studies point out the complexity of teacher knowledge and attempt to deal with this complexity by categorizing knowledge in different ways. In this section three prominent frameworks are presented and discussed. These frameworks were discussed at the 33rd PME conference (Ball, Charalambous, Thames, & Lewis, 2009). The three perspectives are ‘Mathematics-for-Teaching’ (Davis & Simmt, 2006), the ‘Knowledge Quartet’ (Rowland, Huckstep, & Thwaites, 2005) and ‘Mathematical Knowledge for Teaching’ (Ball et al., 2008), and they are all studying teachers’ mathematical knowledge used in teaching mathematics.

In the PME Research forum, two lesson segments were analyzed by each of the three frameworks, with the purpose of contributing to the understanding of addressing the mathematics that matters for teaching by the educational research field. In this section, three parts are given in the presentation of each of the three frameworks: first, an overview of the framework, second, a summary of the framework’s analysis of the PME lesson segments; and finally, a short description of how an analysis of an example from this study could have looked like with the presented framework.

An example from this study

In this study, one of the interview tasks given to the teachers was how they would help a student to calculate the total resistance in a parallel DC-electrical circuit with two parallel resistances (120 Ω and 220 Ω). The formula for calculating the total resistance in a parallel circuit was given and the teachers explained this in different ways - both using different mathematical approaches and different electrical explanations that they considered relevant to this task. In this chapter, this example is used to exemplify how these three presented frameworks could have been used to analyze data from this study.

2.4.1 Mathematics-for-Teaching

In Davis and Simmt’s (2006) theoretical discussion of teachers’ ‘Mathematics-for-Teaching’, they distinguish between four intertwining categories. They argue that, for teachers, the knowledge of established mathematics is inseparable from the knowledge of how mathematics is established. So they are not separating the teachers’ formal mathematics knowledge from their knowledge of how this might be taught. They say that teachers usually have sufficient knowledge to teach the subject well, but that the teachers may have a limited conscious awareness of their own mathematical representations and relations among them. The ‘Mathematics-
Theoretical constructs

for-Teaching’ model contains of two categories of knowledge and two categories of knowing (see Figure 1). The categories of knowledge are: mathematical objects and curriculum structures, which are usually treated as stable. The categories of knowing are: classroom collectivity and subjective understanding, which are usually treated as dynamic.

![Figure 1. Perceived relationships among some aspects of teachers’ ‘Mathematics-for-Teaching’, from Davis and Simmt, 2006, p. 298](image)

Davis and Simmt conjecture that a particular fluency with these four aspects is important for mathematics teaching and that “a key (and perhaps the key) competence of mathematics teachers is the ability to move among underlying images and metaphors” (Davis & Simmt, 2006, p.303). Connections seem to be one important issue in this framework, both interconnections of mathematical concepts and representations and also cross-grade interconnections.

In the analysis of the two lessons in the PME research forum, Davis and Simmt point out that although the teachers have a rich mathematical understanding, many teachers have a limited awareness of their own mathematical understandings. They argue that in many mathematical classrooms the use of rules and predetermined procedures limits the mathematics by closing off interpretive possibilities. One of the teachers, Chloe, used several representations for subtraction but she did not connect these representations, so this was confusing for the students, and she did not understand their confusion as she was taking the connections for granted. The other teacher, Karen, limited her students by teaching them a limited set
Theoretical constructs

of mathematical concepts, probably chosen to be best practice, but taking away the depth of mathematical interactions with her students and also closing off mathematical thinking and imagination.

Analyzing the parallel resistance example with the ‘Mathematics-for-Teaching’ framework

In my interpretation, one category of the ‘Mathematics-for-Teaching’ framework is relevant for this study. The mathematical objects category would, in my interpretation, consist of the knowledge of how to calculate the total resistance in the parallel circuit and also all different explanations of this calculation, such as using fractions, decimal numbers or how to manage the calculator. Also included in this category would be the teachers’ competence to connect these concepts or images. The rest of the categories in the ‘Mathematics-for-Teaching’ framework are not useful for this study. The curriculum category would consist of knowledge about how concepts are connected with the curriculum used in previous and in future classes. In this study, the mathematics teachers and the electricity teachers do not have the same curriculum as they teach different courses, so comparing the teachers’ curricular knowledge would not be of interest here. The classroom collectivity category refers to how knowledge is produced by the collective, e.g. the students in the classroom or the teachers in the concept study group. This knowledge production involves the creation of means to represent what is already known. The research group’s focus is on the learner’s production of new interpretive possibilities because the focus of the collective supports the development of robust, flexible individual understanding.

“The principal pedagogical concern is thus more than the provision of a good explanation or foundational experience (although these may be important concerns), but the creation of means to represent what is already known (albeit often tacit) in a manner that allows it to be knitted together into more broadly available conclusions.” (Davis & Simmt, 2006, p. 309)

In this study, the classroom collectivity is not an explicit issue, as it investigates teachers’ diverse explanations in clinical interviews and not how knowledge representation is treated in the classrooms. The last category, subjective understanding, involves how human learning is both biological and cultural, e.g. through language. Experiences, images and interpretations included in an individual’s understanding of a concept, for example multiplication, are not simply illustrations, rather embodiments of the concept for the knower. However, this is beyond the scope of this study.

This framework has been used in concept studies with practicing teachers, where the teachers have been participating in the elaboration of
mathematical knowledge (Davis, 2008). The aim of these studies has been to improve and reconfigure the teachers’ learning, teaching and knowledge of mathematics. In this study, the goal is to compare two teacher groups mathematical knowledge, so this framework is not really useful.

2.4.2 Knowledge Quartet

The ‘Knowledge Quartet’ framework is practice-based and developed from teacher educators’ lesson observations of pre-service teacher’s practice and the framework is used to discuss and reflect teaching and mathematics teaching development (Rowland et al., 2005). Elementary trainee teachers’ mathematics content knowledge, both SMK and PCK, was studied during practical school-based training, and Rowland et al. described ways in which the teacher students drew on their knowledge of mathematics and mathematics pedagogy in their teaching. They observed four dimensions of mathematics related knowledge, called the ‘Knowledge Quartet’:

- Foundation.
- Transformation.
- Connection.
- Contingency.

Foundation contains theoretical knowledge and understanding of mathematics and beliefs about mathematics, e.g. why and how it is learned. The other three dimensions refer to ways and contexts in which knowledge is applied in the preparation of, and in actual, teaching. Transformation contains behavior that is directed towards students to enable learning, e.g. the representation of ideas to learners in form of examples and explanations. Connection deals with the coherence of planning and teaching a lesson or series of lessons, e.g. the sequencing of topics, ordering of tasks and exercises, including both knowledge of connections within mathematics and an awareness of the relative cognitive demands of topics. The last category, contingency, concerns classroom events that are impossible to plan for, e.g. the teachers’ readiness to respond to students’ ideas and change their lesson plans.

In the analysis of the two lessons in the PME research forum, Rowland et al. highlighted moments from the two lessons and categorized them in their four categories, they also noted elements that they felt would be interesting to discuss with the teachers. In the ‘Knowledge Quartet’ analysis, the lesson with Chloe, who was not connecting her representations in the ‘Mathematics-for-Teaching’ analysis, was divided into episodes for the ‘Knowledge Quartet’ categories. In all of the categories they found moments where the students did not understand the teacher’s strategies or
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representations and that the teacher dismissed the students’ ideas. The other lesson, with Karen, the teacher that in the ‘Mathematics-for-Teaching’ analysis was categorized as using a limited set of mathematical concepts, was, in the ‘Knowledge Quartet’, analyzed as using a relaxed and confident management of the mathematics classroom.

Analyzing the parallel resistance example with the ‘Knowledge Quartet’ framework

In the ‘Knowledge Quartet’ framework, the first two categories might have been useful for this study, but the remaining two categories are not relevant.

- The ‘Foundation’ would hold mathematical knowledge of how to calculate the total resistance in the parallel circuit and also knowledge of why certain mathematical methods and concepts are valid, e.g. when adding two fractions the same denominator should be used. Also included here are mathematical vocabulary and beliefs about mathematics. Beliefs about mathematics would include the nature and purposes of mathematics and how students will best learn mathematics and no questions about this were asked in the interviews in this study.
- The ‘Transformation’ category would consist of the teachers’ choice and use of examples, e.g. the use of an easier fraction example in the parallel resistance interview task.

The remaining categories are not relevant to this study:

- ‘Connection’ category would contain the planning of a lesson or a sequence of lessons and no questions about this were asked the teachers in this study.
- ‘Contingency’ category would contain how to deal with on-the-spot classroom events are not dealt with in this interview study, as this study only analyzes teachers’ explanations in interviews.

The ‘Knowledge Quartet’ has been used as a framework for teacher educators in lesson observations of student-teachers practicing teaching in elementary school. The framework is meant to be used to reflect on and develop teaching and teacher knowledge. In this study, the goal is to compare two teacher groups’ mathematical knowledge when explaining interview tasks. In this study, no questions of lesson planning were asked and no lesson observations were conducted so the ‘Knowledge Quartet’ framework is not really useful in this study.
2.4.3 Mathematical Knowledge for Teaching

The ‘Mathematical Knowledge for Teaching’ framework has been developed by studying teaching practices and seeking to identify knowledge needed by teachers in their work of teaching mathematics (Ball et al., 2008). This practice-based theory of mathematical content knowledge for teaching was built on Shulman’s model of teacher knowledge, with the categories: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), see Figure 2.

![Diagram of Domains of Mathematical Knowledge for Teaching](image)

**Figure 2 From Ball et al. 2008 p. 403.**

Ball et al., studied mathematics teaching and identified mathematical knowledge for teaching containing of two subdomains of PCK:

- Knowledge of Content and Students (KCS).
- Knowledge of Content and Teaching (KCT).

KCS is knowledge about students and mathematics: knowledge of what students are likely to think and what they will find confusing; and what the students will find interesting, motivating, easy and hard. KCS also involves knowledge of common students’ conceptions and misconceptions about particular mathematical content. The KCT category combines knowledge about teaching and knowledge about mathematics. For example, when designing instructions, the teacher will decide in what sequence a particular content will be taught and what examples to use.
Theoretical constructs

This framework also contains two subdomains of SMK:
- Common Content Knowledge (CCK).
- Specialized Content Knowledge (SCK).

CCK is mathematical knowledge and skill that is not unique to teaching and used in settings other than teaching, e.g. correctly calculating or solving mathematical problems. SCK is mathematical knowledge and skill that is unique to teaching, not needed for anything else but teaching and it is knowledge beyond what is taught to the students, e.g. an explanation of why you ‘add a zero’ when you multiply by 10.

Curricular knowledge has been placed in its own category within PCK, and they state that curricular knowledge might be a part of KCT. A third category, horizon knowledge, has been placed within SMK, which concerns knowledge of how mathematical topics are related in the curriculum and seeing connections to future mathematical ideas.

In the ‘Mathematical Knowledge for Teaching’ analysis of the two lessons in the PME research forum, Ball et al. were trying to understand the mathematical issues and demands that arose without evaluating the teachers. They studied the teachers’ use of representations of mathematical ideas, recorded the mathematical work, selection and sequencing of examples and use of mathematical language. For example, they discussed affordances and limitations with the teachers’ examples and representations in the lesson and how the examples may have clarified or obscured students’ mathematical ideas and processes. With episodes from these PME lessons, they gave examples of their categories of mathematical knowledge for teaching.

Analyzing the parallel resistance example with the ‘Mathematical Knowledge for Teaching’ framework

Most of the categories in this framework may have been useful in this study. The explanations the teachers gave in the parallel resistance example would have been categorized with the ‘Mathematical Knowledge for Teaching’ framework as followed:
- ‘Common content knowledge’, CCK, knowledge of how to make the calculations.
- ‘Specialized content knowledge’, SCK, - no example in this study.
- ‘Knowledge of content and students’, KCS, teachers’ experience with common students’ problems and pre-knowledge.
Theoretical constructs

- ‘Knowledge of content and teaching’, KCT, teachers’ different examples of how to calculate the sum of two fractions and also the teachers’ comments of which examples they should use to start and which examples they can use next.

This framework includes the mathematical knowledge teachers need to reply and validate students’ solutions and suggestions on the spot in the classrooms. This could often be included in the SCK category in their framework, but in this study this is not an issue, as this study is an interview study. Ball et al. also state that there are problems with this framework, e.g. the boundary problem: how to discern one category from another, especially in the case with the CCK and SCK categories (Ball et al., 2008). In this study, a framework built on Shulman’s original categories is used (see section 2.3.1 in this chapter).

2.4.4 Summary of three frameworks used to describe mathematical knowledge of teaching

These three research teams are, in different ways, characterizing teacher knowledge for teaching mathematics, and they are all acknowledging the complexities of knowing mathematics for teaching by dealing with both the mathematics and the work of teaching. The ‘Knowledge Quartet’ (Rowland et al., 2005) framework and ‘Mathematical Knowledge for Teaching’ (D. L. Ball et al., 2008) framework both depart from Shulman’s model of teacher knowledge and from their own empirical studies of teachers, where they come up with suggestions of how to further categorize teacher knowledge in mathematics in additional categories to the categories Shulman suggested. On the other hand, ‘Mathematics-for-Teaching’ framework (Davis & Simmt, 2006) intends, in their concept studies, to improve teachers’ mathematical understanding by involving the teachers in deeper mathematical insights, e.g. by identifying connections between different interpretations. This framework differs from the ‘Knowledge Quartet’ and ‘Mathematical Knowledge for Teaching’ frameworks, which originates from empirical studies. The ‘Mathematic-for-Teaching’ framework originates from Davis and Simmt’s theoretical discussions of mathematical knowledge (mathematical objects and curriculum structures) and knowing (classroom collectivity and subjective understanding).

The analysis of the two lesson segments by these three frameworks in the PME research forum showed some similarities and some differences. For example, the teacher Karen was analyzed as using singular meanings of mathematical concepts, resulting in diminishing the depth of mathematical interactions with her students in the ‘Mathematics-for-Teaching’ framework.
Theoretical constructs

(Davis & Simmt, 2006), but in the ‘Knowledge Quartet’ framework (Rowland et al., 2005), she was analyzed as using a relaxed and confident management of the mathematical classroom. In the ‘Mathematical Knowledge for Teaching’ framework (Ball et al., 2008), affordances and limitations of Karen’s selection and sequencing of examples were discussed, and also her use of mathematical language was discussed. It has been shown that different theoretical perspectives can lead to different interpretations and understandings of the same lesson (Even & Schwarz, 2003). To be able to understand the practice of teaching and learning mathematics may require the use of different perspectives (Even, 2009). In the comments of the PME research forum, Neubrand writes that the ‘Mathematical Knowledge for Teaching’ (Ball et al., 2008) and ‘Mathematics-for-Teaching’ (Davis & Simmt, 2006) seems to be complementary, where the former focuses on the mathematics itself and the latter deals with the horizon of mutual understanding (Neubrand, 2009).

These three frameworks all focus on different aspects of teaching practice. This study focuses on the knowledge different teachers (mathematics and electricity) draw upon when explaining the same interview tasks. This study compares teachers’ mathematical explanations of the same interview tasks, not developing or evaluating teachers, so the ‘Mathematical Knowledge for Teaching’ framework (Ball et al., 2008) is, among the frameworks presented here, the one that is the most useful for this study. The ‘Mathematical Knowledge for Teaching’ framework builds on Shulman’s notations of teacher knowledge and could serve as a specific example of Shulman’s work in the area of mathematics teaching. But the ‘Mathematical Knowledge for Teaching’ framework is including teacher knowledge used by teachers in the classrooms in their meetings with students, e.g. when evaluating students’ suggestions. In this study, this is not an issue, as this study is a clinical interview study. Furthermore, this study also deals with electrical knowledge, so therefore the categories in this study are based on Shulman’s original categories (Shulman, 1986).
3 Methods

In this section the methods both for collecting and analyzing data in this study will be described. The study is built on task-based interviews with teachers. The aim of the study is to study the teacher knowledge mathematics and electricity teachers draw upon in explaining mathematical electricity tasks. In this section I will describe how the interview tasks were constructed, who the participants were and how the data was analyzed and presented.

3.1 Interviews

There are many ways of studying teacher knowledge, for example studying teachers and teaching in different kinds of case studies. Teacher knowledge have successfully been studied in lesson observations, interviews with mathematical tasks and pencil-and-paper tests (Hill, Sleep, Lewis, & Ball, 2007). In this study, interviews with mathematical tasks have been chosen, as I am interested in closely study mathematics and electricity teachers’ explanations and the mathematics in their explanations. The method, interviews with tasks, is used to understand the nature and extent of teacher knowledge (Hill et al., 2007). Pencil-and-paper tasks are used in big scale studies whereas this study focuses on detailed information of teachers’ explanations. Lesson observations would have required very many observations to be able to find similar tasks in the lessons to compare, and it is not sure that the mathematics teachers are using tasks from the students chosen study program. So, to be able to compare the two teacher groups this method was chosen with interviews with the same interview tasks to both teacher groups. Furthermore, this method gives the opportunity for the teachers to explain the interview tasks in all possible ways without having to consider the on the spot classroom context. In a classroom the teachers may give different explanations in classroom lectures and to individual students, but in these interviews they were asked to explain in all kinds of ways they could come up with.

3.1.1 Interview tasks

The interview tasks used in this study were mathematical electricity tasks, task with mathematical calculations in the electricity context. The interview tasks are tasks of a type that are commonly used in the first electricity course at the Electricity Program. The tasks may also be used in the first mathematics course at the Electricity Program by mathematics teachers who are adapting the mathematics course to the students’ study program. In this
Methods

In this study, both mathematics and electricity teachers have been interviewed, and the teachers have been asked about how they would explain and help students on the Electricity Program with the interview tasks. The interview tasks are modified versions of the electricity education examples in the Danish Competence project (Niss & Højgaard Jensen, 2002). The tasks in the Danish Competence project are electricity tasks used in electricity education. In this study some of the electricity context is removed from these Competence project tasks so that also a mathematics teacher, not educated in electricity, can understand and consider using the interview tasks in their own teaching (for example see interview task 1 in figure 3).

1. Student task: Ohm’s law

Your students have started with Ohm’s law in their electricity course. Suppose that you have a student working with the following task:

\[ U = R \cdot I \]

\[ I = 0.1 \text{ A} \]

\[ R = 68 \text{ ohm} \]

\[ U = 10 \text{ V} \]

Task:
Calculate the current, I, in this circuit.

Ohm’s law:
\[ U = R \cdot I \]

Calculations of the effect:

Ohm’s law:
\[ U = R \cdot I \]

The effect law:
\[ P = U \cdot I \]

Figure 3 Interview task 1 given to mathematics and electricity teachers.

In this study three interview tasks were used, all of them written as textbook tasks and the teachers were asked about how they would have explained the task to a student who does not understand it. This format of interview tasks has been used in studies comparing different mathematics teacher groups (Ma, 1999). In the three interview tasks the teachers were asked about how they would have explained the task to a student. The teachers were asked of all kinds of explanations they thought was important to teach to students. The explanations include alternative ways the teachers could explain the tasks in. An explanation could be a mathematical method of solving the task or an electrical property of the task. Explanations in this study include concepts that the teachers think are important to consider when helping a student with this type of task. The teachers did for example talk about their
experience of students working with tasks like this and of common problems students use to have with them and that is also included in the analyses. The teachers gave several different kinds of explanations for each interview task and in the analysis the knowledge used in these different kinds of explanations are analyzed.

3.1.2 Teachers

In this study eight mathematics and electricity teachers have been interviewed. The teachers are three mathematics teachers and five electricity teachers, teaching in the first year at the Electricity Program at four different schools in northern Sweden (see Table 1). The mathematics teacher on the third school did not want to be filmed so he is not included in the analysis (M3 is missing in table 1).

<table>
<thead>
<tr>
<th>Mathematics teachers</th>
<th>Teacher abbreviation</th>
<th>Subject</th>
<th>Additional subject</th>
<th>Teaching experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Mathematics</td>
<td>Physics</td>
<td>&gt; 10 years</td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>Mathematics</td>
<td>Biology</td>
<td>&lt; 5 years</td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>Mathematics</td>
<td>Physics</td>
<td>&gt; 5 years</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electricity teachers</th>
<th>Teacher abbreviation</th>
<th>Subject</th>
<th>Electricity working experience</th>
<th>Teaching experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Electricity</td>
<td>&gt; 10 years</td>
<td>&gt; 5 years</td>
<td></td>
</tr>
<tr>
<td>E2a</td>
<td>Electricity</td>
<td>&gt; 10 years</td>
<td>&lt; 10 years</td>
<td></td>
</tr>
<tr>
<td>E2b</td>
<td>Electricity</td>
<td>&gt; 10 years</td>
<td>&gt; 10 years</td>
<td></td>
</tr>
<tr>
<td>E3</td>
<td>Electricity</td>
<td>&gt; 10 years</td>
<td>&lt; 5 years</td>
<td></td>
</tr>
<tr>
<td>E4</td>
<td>Electricity</td>
<td>&gt; 10 years</td>
<td>&gt; 10 years</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. A presentation of the interviewed teachers.

At one of the schools two electricity teachers were interviewed as they were both teaching in the first year (called E2a and E2b in Table 2). The mathematics teacher M1 and the electricity teacher E1 are teaching at the same school, and so on for the rest of the teachers. Two of the three interviewed mathematics teachers also teach physics, which includes electricity as a topic. These teachers teach some electricity content in the physics course to other students, not at the Electricity Program. One of the mathematics teachers teaches in mathematics and in biology, and he does not have knowledge in the electricity content or context, but he can use examples like these tasks for his classes in mathematics at the Electricity
Program. The mathematics teachers in this study are all educated mathematics teachers and have been working at least three years full time teaching and at least partly at the Electricity Program. All of the electricity teachers in this study have been working as electricians for many years, before they studied to become vocational teachers in electricity and all of them have been teaching for at least three years. They have no formal education in mathematics or in teaching mathematics, but they use mathematics when they teach some of the electricity courses.

Each teacher was interviewed alone at his own school and was told to use as long time as he liked on each of the three interview tasks. The interviews lasted one to one and a half hour each. The questions in the tasks were on the form of how to explain this task to a student at the Electricity Program who does not understand this type of task. The teachers were told to give as many different explanations as they thought could be useful for students to understand this type of task. The teachers were given a paper with the tasks, formulas and circuit diagrams and an extra paper to make calculations on. The teachers’ written calculations were videotaped and their oral explanations recorded. The importance of videotaping the teachers’ calculations, actions and statements while explaining has been highlighted for example by the Learning Mathematics for Teaching Project where they point out that videotaping is needed as:

“the mathematically substantive elements of classroom instruction occur in quick sessions and thus cannot be reliably disaggregated and analyzed in the moment. Similarly, it is hard to judge the mathematical nature of class time spent exploring and making conjectures without knowing the outcome of these activities.” (Project, 2011, p.31)

In this study the camera was directed downwards and captured the paper where the teachers wrote and pointed at the electricity circuits in the tasks. The interviews, that lasted approximately one hour each, were transcribed.

3.2 Analyses of data

In general it is difficult, maybe even impossible, to study knowledge, as teachers’ knowledge is not directly accessible. In this study indications of knowledge have been studied, and indications of knowledge is similar to what Speer refers to as “attributed beliefs” (Speer, 2005). Speer’s distinguish between “attributed” and “professed” beliefs, where “attributed” are those that researchers infer based on data and “professed” are those stated by teachers. What the teachers say and do (calculate, point to, draw pictures and so on) is in this study attributed to the teachers as indications of knowledge that the teachers have and draw upon to explain these tasks. The
teachers’ statements and actions is used as indications of their teacher knowledge and “something will count as knowledge, and be modeled as knowledge, if it appears to be used as such by the individual being modeled” (Schoenfeld, 2011, p.53).

A summary of all teacher’s statements and actions of each interview task was made. This summary consists of identified portions of explanations, where each portion of explanation expressed one idea that the teacher explained. This is similar to Marks’ study where “contiguous portions of the text that expressed a single coherent idea were identified” (Marks, 1990). These portions of explanations included statements and actions like written calculations and sometimes pointing in the circuit diagram. Each portion of explanation represents a single idea that the teacher choses to use as an explanation for this task. When a teacher repeated an explanation, these explanations were grouped together to one portion of explanation and analyzed once. A summary of one teacher’s explanations of a task consists of a list of several different portions of explanations. Each portion of explanation was categorized in this study’s teacher knowledge categories: SMK and PCK in both mathematics and electricity.

3.2.1 An example of a teacher’s explanations

To illustrate the categorization of the teacher interviews, one of the teacher’s explanations of one task will be presented in this section. It is an example of how one of the mathematics teachers explained the second interview task. Interview task 2 is presented below (see figure 4). In this task the teachers are asked to help a student to calculate the total resistance in a parallel circuit with two given parallel resistances. One mathematics teacher’s explanations of this interview task are presented to show how the interviews were categorized (see Table 2). First the teacher’s statements and actions are written down in chronological order. The statements are descriptions of what the teacher said and the actions are descriptions of what the teacher calculated or showed in the circuit diagram, for example pointed to. After that, ideas of explanations are identified and statements and actions belonging to a specific idea of explanation are grouped together. One teacher’s explanations of one task were most often a list of several different ideas of explanations, both mathematical and electrical. The different ideas of explanations were categorized in SMK or PCK in both mathematics and electricity.
2. Student task: Parallel resistance

Your students have started to work with electrical circuits with parallel resistance. Many students have difficulties in using the formula to calculate the total resistance in a parallel circuit.

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}
\]

R1 is 220 ohm and R2 is 120 ohm

Task:
Calculate the total resistance in the circuit.

How would you explain how to solve this task to a student?

Figure 4 Interview task 2

In Table 2, a mathematics teacher’s explanations of task 2 are presented. The first column in table 2 shows a summary of the actions and statements that the teacher gave in chronological order. In the second column, the actions and statements are grouped together into ideas of explanations. One idea of an explanation is for example a calculation, an explanation of a fact, or an analogy to make the subject more understandable. Most often, one idea of explanation consists of one or some consecutive statements and actions. But in some cases a teacher started with one idea of explanation and after a while left that idea and started another idea of explanation and at last finished the first explanation. Sometimes teachers did repeat some ideas of explanations, in this study they are also grouped into one idea of explanation, so in the analysis the explanations are only counted and categorized once.

Each idea of explanation in the summaries (column 2 in table 2) was coded as an indication of mathematical knowledge or electrical knowledge, depending on the main idea of the statement. After that the ideas of explanations was coded as SMK or PCK in mathematics or SMK or PCK in electricity. In the third column in table 2 the teachers’ explanations are reworded into a sentence used for all teachers’ similar ideas of this explanation. All teachers’ ideas of explanations are presented in an overview picture for each task, where all teachers using the same ideas of explanations are grouped into the same box.
### Mathematics teacher M1, statements and actions explaining task 2

<table>
<thead>
<tr>
<th>Chronological order</th>
<th>Grouping and Categorization</th>
<th>Presentation in the overview picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Inserted the given value into the formula</td>
<td>Grouped with 3</td>
<td></td>
</tr>
<tr>
<td>2. Said that this task is too difficult for students to solve with fractions</td>
<td>Categorized as PCK in mathematics</td>
<td>Said that the task is too difficult for students to solve with fractions</td>
</tr>
<tr>
<td>3. Calculated the total resistance in decimal numbers with the calculator</td>
<td>Categorized as SMK in mathematics, along with 1, 4</td>
<td>Calculated the total resistance in the parallel circuit</td>
</tr>
<tr>
<td>4. Explained that when $1/R=0.0129$ we have to invert to get $R$</td>
<td>Grouped with 3</td>
<td></td>
</tr>
<tr>
<td>5. Said that he would have given the students a simpler example in the beginning, e.g. 4Ω and 5Ω</td>
<td>Categorized as PCK in mathematics, along with 6, 7, 8, 9</td>
<td>Showed an example with easier numbers</td>
</tr>
<tr>
<td>6. Calculated the simpler example with decimal numbers</td>
<td>Grouped with 5</td>
<td></td>
</tr>
<tr>
<td>7. Got $1/R=0.45$, compared this with an equation and solved it</td>
<td>Grouped with 5</td>
<td></td>
</tr>
<tr>
<td>8. Calculated the simpler example with fractions</td>
<td>Grouped with 5</td>
<td></td>
</tr>
<tr>
<td>9. and explained that $1/R=9/20 \Leftrightarrow R/1=20/9$</td>
<td>Grouped with 5</td>
<td></td>
</tr>
<tr>
<td>10. Said that it is important to understand the currents in this circuit, explained by bridges over a river</td>
<td>Categorized as PCK in electricity</td>
<td>Used an analogy to explain the circuit, e.g. of bridges over a river or traffic on roads</td>
</tr>
<tr>
<td>11. Reasoned that it will be faster to pass the river with two bridges, therefore the total resistance less in parallel circuits</td>
<td>Categorized as SMK in electricity</td>
<td>Reasoned about the total resistance in a parallel circuit</td>
</tr>
<tr>
<td>12. Said that the students should know that the total resistance in a parallel circuit always is less than the smallest resistance in the circuit</td>
<td>Categorized as SMK in electricity</td>
<td>Said that the total resistance in a parallel circuit is always less than the smallest included resistance</td>
</tr>
</tbody>
</table>

**Table 2** A mathematics teacher’s explanation of task 2.

**Column 1:** all statements and actions in chronological order, **column 2:** statements and actions are grouped together into ideas of explanation and categorized, **column 3:** wording of the idea of explanation in the overview.
3.2.2 Analysis of the teacher example

In this study a revised version of Shulman’s definitions of teacher knowledge is used (Shulman, 1986) in the analyses. An idea of explanations is categorized as an indication of mathematics or electrical knowledge and both mathematics and electrical knowledge is further divided in PCK or SMK. These definitions are described in chapter 2, and here is a short version as a reminder:

- **Subject matter knowledge (SMK),** in both mathematics and in electricity, is defined as knowledge of the content. Teachers’ statements of facts, concepts and reasoning about the subject and also teachers’ actions as calculation procedures and methods are categorized as SMK.

- **Pedagogical content knowledge (PCK),** in both mathematics and in electricity, is defined as knowledge of useful representations, examples and analogies to make the content accessible to the students. Also included in PCK is knowledge of common students’ conceptions, misconceptions and difficulties with the content. Teachers’ statements and actions aiming at helping the students to understand the subject are categorized as PCK. Also categorized as PCK is teachers’ statements of common students’ preconceptions and common difficulties.

For example, the teacher’s explanation of the formula for the total resistance in a parallel electrical circuit, with a simpler number example (see line 5, table 2), was categorized as indications of mathematical knowledge. A teacher’s explanation of the currents in the same parallel circuit task, by talking about bridges over a river (see line 10, table 2), was categorized as indications of electrical knowledge. After that all the indications of knowledge in the mathematical category were divided into indications of SMK or PCK and the same was done with the ideas in the electricity category. For example the teacher’s calculation of the total resistance in a parallel circuit (see line 3, table 2) was categorized as SMK in mathematics, as this involves facts and procedural knowledge of mathematics. The teacher’s explanation with a simpler number example (see line 5, table 2) to help a student understand the formula was categorized as PCK in mathematics, as this involves knowledge of an example that can make the mathematical content accessible for the students. The teacher’s statement that the total resistance in a parallel circuit is always less than the smallest resistance in the circuit (see line 12, table 2) was categorized as SMK in electricity, as this is factual knowledge of the parallel circuit. The teacher’s explanation of the parallel circuit with an analogy, here two bridges over a
river (see line 10, table 2), was categorized as PCK in electricity, as this involves knowledge of this analogy aiming at making the electricity content understandable for students.

3.2.3 Remarks concerning the analysis

The categories SMK and PCK are related and could be difficult to separate (e.g. Marks, 1990). In this study, when the teachers explicitly say that this explanation is given to help the students understand a specific area, it is categorized as PCK. However, to have this knowledge, to be able to explain this is dependent of the teachers’ SMK, but not categorized as SMK here. Pedagogical content knowledge is dependent of content knowledge, for a teacher to be able explain something, (s)he must know the content. Teachers’ knowledge of mathematics influence their pedagogical content-specific decisions (Even, 1993; Marks, 1990):

“Theyr pedagogical decisions – questions they ask, activities they design, students’ suggestions they follow – are based on, in part, their subject-matter knowledge.”
(Even, 1993, p.113)

For example when a teacher said that to help a student understand how to calculate the total (equivalent) resistance in a parallel circuit, he used an example with simpler numbers to explain this and he calculated this example with both decimal numbers and fractions. “To use an easier example with simpler (smaller) numbers” was categorized as PCK in this study, but the teacher must have SMK in mathematics about different representations of numbers to be able to calculate this task. Here, “to use an easier example” is categorized as PCK in mathematics.

Teachers’ knowledge of representations of science concepts to help students understand them seems necessarily dependent upon having subject matter knowledge of the concepts, but having subject matter knowledge does not guarantee representations that will help students understand or that teachers will use particular representations when it is pedagogically best (Magnusson et al., 1999). In this study, when a teacher said that he uses to use an analogy to explain the electrical circuit, that is categorized as PCK in electricity. Here the teacher’s use of this analogy is assumed to be used to help students understand the electricity content, and therefore it is PCK.

In the cases when a teacher reasoned about a formula or a fact, that was in this study categorized as SMK as this is interpreted as knowledge of how concept are related. The teacher may have reasoned about this to help a student understand this concept and in that case the reasoning could have
been categorized as PCK, but in this study, teachers’ reasoning about a concept is categorized as SMK, as this have been interpreted as knowledge of how concepts are related. In this study, the teachers who are reasoning about the formulas are most often both reasoning about the mathematical properties (for example how the involved variables are related, categorized as SMK in mathematics) and the electrical properties (how electrons are moving in the conductor, categorized as PCK in electricity).

3.3 Presentation of data

The data in this study are presented in two parts. First, an overview of all data for each interview task is presented in an overview picture, with all teachers’ explanations of that task. After that detailed descriptions of explanations of a specific topic by some teachers are presented for each task.

3.3.1 Overview analyses

The first analysis involved all indications of teacher knowledge from all the teachers, and in the analysis the similarities and differences between the mathematics and the electricity teachers groups were studied.

For each task all categorizes of teacher knowledge from all teachers’ interviews were put together in an overview picture and the similarities and differences between the two teacher groups were studied. The teachers’ explanations are categorized in the four categories; subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in mathematics, and SMK and PCK in electricity. In the overview picture, all teachers’ explanations of one task is presented in four rows representing SMK and PCK in mathematics and SMK and PCK in electricity, and similar explanations are put together in boxes. The boxes in this overview picture are made yellow if the explanation in the box is given exclusively by mathematics teachers, blue if the explanation is given exclusively by electricity teachers and green if the explanation is used by both mathematics and electricity teachers. The abbreviation for each teacher represented in the box is written at the bottom row in each box, the mathematics teachers are abbreviated M1, M2 and M4 and the electricity teachers E1, E2a, E2b, E3 and E4.

Below is the overview picture for task 2 is presented in Figure 5 where all the teachers’ explanations are included and grouped for SMK and PCK in mathematics and SMK and PCK in electricity. Mathematics teacher M1 is made bold in figure 5 to show how his explanations, described in Table 2, are presented in the overview picture.
Methods

In the definitions of PCK for this study, PCK contains two types of knowledge (see chapter 2):

- knowledge of representations of the subject and
- knowledge of students and the subject.

In this overview picture, boxes with indications of pedagogical content knowledge of the second type, knowledge of students and the subject, are made with rounded corners. For example, a teacher’s mentioning of common student difficulties or common student preconceptions of the subject, was categorized as PCK, knowledge of students and the subject, and inserted in a box with rounded corners.

**Indications of Teacher knowledge, task 2**

**Subject matter knowledge in mathematics**

- Said that rounding too early in the calculations gives a wrong answer  
  M4
- Calculated the total resistance in the parallel circuit  
  M1, M2, M4, E1-4

**Pedagogical content knowledge in mathematics**

- Said that the task is too difficult for students to solve with fractions  
  M1
- Showed an example with easier numbers  
  M1, M4, E2a, E2b
- Said that some students use to forget to invert at the end of their calculations  
  E2a, E2b, E4
- Said that students may have a bad self-confidence in mathematics  
  E1

**Subject matter knowledge in electricity**

- Reasoned about the total resistance in a parallel circuit  
  M1, M4, E1, E3, E4
- Said that the total resistance in a parallel circuit is always less than the smallest included resistance  
  M1, M4, E2a, E2b, E3, E4
- Electrician knowledge: knowledge of an alternative formula  
  E2b, E3, E4
- Calculated an estimation to be able to check the answer  
  E4

**Pedagogical content knowledge in electricity**

- Used an analogy to explain the circuit e.g. bridges over a river or traffic on roads  
  M1, M4, E1, E2b, E4
- Explained the origin of the formula:  
  \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \)  
  M4, E3
- Said that the two new concepts, series and parallel circuit, may be difficult for the students  
  E1, E2a
- Said that using a structure when solving a task makes it easier for the students  
  E2a, E2b

Figure 5. Overview of explanations for task 2. Yellow boxes show explanations given only by mathematics teachers; blue boxes show explanations given only by electricity teachers and green boxes show explanations given by both mathematics and electricity teachers. M1-M3, represents mathematics teachers, E1-E5 represents electricity teachers. The M1 teacher’s explanations in the example in Table 2 are bold.

Boxes in the category PCK (both in mathematics and electricity) with rounded corners are explanations that are based on the teachers’ experience of students’ pre-knowledge or common mistakes.
In chapter 4 all the teachers’ explanations of each task is presented in overview pictures. In these overview pictures it is possible to discover some similarities and differences among the two teacher groups’ explanations. But this overview did not reveal all differences that were found during the interviews and transcriptions, therefore a new analyzing session began with detailed studies of some of the teachers’ explanations.

3.3.2 Detailed analyses

Detailed studies have been used in teacher knowledge research in mathematics (Leinhardt & Smith, 1985; Ma, 1999). Leinhardt and Smith chose three of the eight teachers in their study and made close examinations of them, to study differences among them even though their performance was similar in the first analysis. Ma (1999) was studying the coherence of the teachers’ mathematical knowledge. Across four topics, subtraction, multiple digit multiplication, division with fraction and geometry, she found that the American teachers had fragmented knowledge and the Chinese teachers had coherent knowledge, both in their algorithmic knowledge as well as in conceptual knowledge.

In the detailed analyses in this study, explanations from mathematics and electricity teachers were selected that in the overview were categorized in the same category, but in a more detailed analysis these explanations had differences in the teachers’ use of mathematics. Most often was the explanations for the detailed analyses chosen from the category SMK in mathematics, but one explanation from PCK in mathematics and one from PCK in electricity was also included, as the teachers’ use of mathematics was different. Detailed analysis of mathematics and electricity teachers’ specific explanations of the same topic have been done for some explanations of each task. Marks (1990) highlights the fact that the level of analysis is important:

“The difference among these interpretations of the same example is a matter of focus. If this statement is taken in its entirety, it represents pedagogical content knowledge, but if it is analyzed into components it represents other forms of teacher knowledge. Each level of interpretation is valid; the main difference is that the general pedagogical and subject matter components derive less directly from the teacher’s assertion and must be inferred.” (Marks, 1990, p.8).

The detailed analyses of mathematics and electricity teachers’ specific explanations of mathematics indicate differences in the teacher knowledge of mathematics that the teachers drew upon in their explanation. To distinguish between the teachers different use of mathematics, the mathematical knowledge the teachers used in their explanations was in this study categorized as either specific or general. In the detailed analysis, an
Methods

Explanation based on specific mathematical knowledge consisted of a solution of this specific task most often a calculation of the requested quantity with the given values. Also included were explanations with examples where the teachers chose values in their calculations to make the mathematics in the example easier. An explanation based on general mathematical knowledge consisted of a general solution, not specific to this task instead useful for a wide range of tasks, most often including algebra. Explanations categorized as general mathematical knowledge was not only calculating the given task, these explanations were also explaining general mathematical properties. Also generic examples using numerical values were categorized as general mathematical knowledge, when the numerical values were used by the teacher to explain general mathematics. For example when a teacher in this study used numerical values to reason about a mathematical formula, saying that when a conductor’s area is doubled then the conductor’s resistance is halved, that was categorized as using general mathematical knowledge.

One teacher’s explanation of a task could consist of several explanations based on both general and specific types of mathematical and also based on electrical knowledge, as some teachers connected their mathematical explanations to electricity.

After the overview analysis, detailed analyses are presented with groups of mathematics and electricity teachers’ specific explanations of the same topic. For each interview task some explanations of the same topic from both mathematics and electricity teachers are described and analyzed in detail. These explanations are chosen to highlight contrasts between the mathematics and electricity teachers’ use of mathematical knowledge. As this study is too small to generate any general results, the goal is to illustrate some characteristic patterns and rich examples of teachers’ use of mathematics in their explanations. The detailed analyses most often presents one mathematics and one electricity teacher giving an explanation of the same topic, in two cases one mathematics teacher’s explanation is compared with two or three electricity teacher’s explanations. The specific explanation is represented in the overview analysis picture in a green box, an explanation given by both mathematics and electricity teachers.

These specific explanations of the same topic came mostly from the category SMK in mathematics in the overview analyses, as it is the mathematical teacher knowledge that is in focus in this study. But there is also one detailed study from the category PCK in mathematics and one from the category PCK in electricity. Sometimes a teacher connected his explanations to electricity and to show that some of the detailed studies of SMK in mathematics also
include electrical knowledge. The detailed analyses are presented after the overview picture for each task in chapter 4.

### 3.4 Reliability of the method

This method of clinical interviews with teachers does have some implications for the results of the study. The teachers had no chance to plan their explanations of the task. In ordinary teaching, the teachers can use teaching material in their planning of their teaching, so the teachers may be able to give richer explanations in ordinary teaching. The teaching of these concepts in ordinary teaching is a part of a whole sequence of lessons and that is not considered here. Furthermore, the teachers knew that the interviewer is a mathematics teacher and therefore they might have concentrated on the mathematical parts of their explanations. In addition to this, it is not known in what degree the mathematics teachers are using tasks like this in their teaching at the Electricity Program.

To make sure that the categorization was reliable, two external researchers have been involved. One researcher in mathematics education has independently of the author categorized one section of two interviews. This categorization was compared to the authors’ categorization and an agreement of the results was made. An experienced science researcher was involved in discussions with the author about the categorization of all the teachers’ explanations for one task and an agreement was made. Furthermore was a part of one of the video filmed interviews presented at a research seminar where the participants discussed the categorization of this teacher’s statements and actions.

### 3.5 Ethical considerations

The teachers in this study participated voluntarily, they were informed of the goal of this research and they had the right to interrupt their participation if they wanted to, according to informed consent (Codex, 2011). All the materials from the interviews are kept in secret and the teachers are anonymous, in this report the mathematics teachers are named M1, M2 and M4 and the electricity teachers are named E1, E2a, E2b, E3 and E4. In this report all the teachers are written as male, even though there are female teachers in this study and this because there are not very many female upper secondary electricity educators in the northern parts of Sweden.
4 Results

4.1 Analysis task 1

In this section the results for task 1 (figure 6) will be presented, first with an overview of all teachers’ explanations and later with detailed analyses of three specific explanations. The specific explanations that are analyzed in detail are explanations of the same topic given by both mathematics and electricity teachers.

Figure 6, Interview task 1

In interview task 1 the teachers are first asked to help a student to calculate the current in a simple direct current circuit with one resistance where the voltage and the resistance in the circuit are given. In the interview task it is written that the student has difficulties with resolving a variable from Ohm’s law, and the teachers are asked to give explanations to this student. In the second part of this interview task the teachers are asked how they would have explained how to calculate the effect in the resistance in the circuit.

Figure 7 shows an overview of all the teachers’ explanations of task 1 in the categories Subject matter knowledge (SMK) and Pedagogical content knowledge (PCK) in mathematics, and SMK and PCK in electricity. Yellow boxes show explanations given only by mathematics teachers, green boxes
Results

explanations given by both teacher groups and blue boxes shows explanations given solely by electricity teachers.

**Indications of Teacher knowledge, task 1**

**Subject matter knowledge in mathematics**

- Reasoned about proportionality: M4
- Calculated the current in the circuit: M1, M2, M4, E1-4
- Calculated the effect in the resistance: M1, M2, M4, E1-4
- Merged Ohm’s law and the Effect law to get a new formula for the effect: M1, M2, M4, E2a, E3
- Explained rounding of the answer: M4, E4
- Reasoned about the Ohm’s law formula: M1, E3

**Pedagogical content knowledge in mathematics**

- Said that for some students it is difficult to see: $R_1$ / $R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$: M4
- Used a triangle to rearrange the formula: M1, M2, E1, E2a, E2b, E3
- Said that using a structure when solving a task makes it easier for the students: M1, E2a, E2b
- Said that it is easier for some students to rearrange a formula with numbers than with variables: M1, M4, E2a

**Subject matter knowledge in electricity**

- Said that units and prefix is important when working with electricity: M4, E2b, E3, E4
- Electrician knowledge: transferred effect and loss effect in high tension lines, or loss effect can be noticed by heat: E1, E3

**Pedagogical content knowledge in electricity**

- Used a analogy to explain the electrical circuit, e.g., water buckets on different heights, or electrons in the conductor: M2, E1, E3

Figure 7 Yellow boxes show explanations given only by mathematics teachers; blue boxes show explanations given only by electricity teachers and green boxes shows explanations given by both mathematics and electricity teachers.

M1, M2, M4, represents mathematics teachers; E1-E4 represents electricity teachers.

Boxes in the category PCK (both in mathematics and electricity) with rounded corners are explanations that are based on the teachers’ experience of students’ pre-knowledge or common mistakes.

At this overview level most explanations are given by both mathematics and electricity teachers. The two teacher groups use mostly the same explanations, with only a few exceptions, the mathematics teachers have one explanation categorized as SMK in mathematics and one explanation in PCK in mathematics that the electricity teachers do not mention. Two electricity teachers use SMK in electricity that no mathematics teachers use. These electricity teachers connect their explanation to practical electricity work. These explanations that only one of the teacher groups did use are discussed further below.
In SMK in mathematics, one mathematics teacher referred to proportional reasoning when he reasoned about the effect law. The teacher said that it is good if the students understand how the effect changes according to the current, saying that the current and voltage are proportional to each other. He said that if the current increases, because in the effect formula the current is multiplied with the voltage, the effect will also increase and vice versa³.

“It is good that they (the students) know how the effect changes if the current changes, that effect and current are proportional to each other, when the current increases here, as it is multiplication with the voltage, so the effect must increase and vice versa.” (M4 12:40)

In PCK in mathematics, one mathematics teacher, M4, discussed students’ difficulties with rearranging and simplifying formulas, here the Ohm’s law. The teacher (M4) said that some students can have difficulties in rearranging and simplifying the Ohm’s law formula, for example: \( \frac{R \cdot I}{R} = 1 \cdot R = R \), and he said that the priority rules for calculations are important and not obvious for all students.

Two electricity teachers on the other hand had one explanation exclusively used by them in the category SMK in electricity. These teachers used knowledge from practical work with electricity in the reasoning and argumentations for their explanations of the interview task. One of the teachers said that during electrical laboratory work the students can feel the effect in a resistance when the resistance is getting warm. The other electricity teacher did use the rearranged effect formula to explain why energy is transferred in high voltage lines (see detailed study in section 4.1.3).

In the next section detailed studies of specific explanations in the category SMK in mathematics and SMK in electricity will be presented. These specific explanations, given by both mathematics and electricity teachers, were chosen to highlight differences in the argumentation and reasoning in the teachers’ explanations that did not emerge in the overview analysis.

### 4.1.1 Detailed analysis 1 of task 1

In the overview analysis, the approach of calculating the current in this direct current circuit with one resistance on 68Ω and the voltage 10V was

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³ All the citations are translated from Swedish to English by the author.
categorized as SMK in mathematics. All teachers gave explanations of how to calculate this task. Here a presentation will be given of one mathematics and one electricity teacher’s explanations of this task and their different use of mathematics in their explanations will be analyzed. These teachers are chosen to highlight how different teachers’ use of mathematics could be in explaining the same task. The mathematics teacher used both general and specific mathematics knowledge in his solution and the electricity teacher used specific mathematical knowledge.

**Mathematics teacher M1**

This teacher first discussed the importance of the student’s understanding of the Ohm’s law formula and he said that the formula is abstract. Then he explained how to calculate the task in two ways:

<table>
<thead>
<tr>
<th>Teacher wrote:</th>
<th>Teacher did:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U = I \cdot R )</td>
<td>The teacher said that he uses to write the formula and then rearranges it, to get ( I ). He divided with ( R ) on both sides of the equal sign and cancelled ( R/R ) on the right hand side and got a formula for ( I ).</td>
</tr>
<tr>
<td>( \frac{U}{R} = \frac{I \cdot R}{R} )</td>
<td>The teacher inserted the given values and calculated the current ( I ).</td>
</tr>
<tr>
<td>( \frac{U}{R} = I )</td>
<td>Then the teacher said that there is also another way, when you first insert the given values and then solve that equation. The teacher said that to solve the equation, we want ( I ) alone on one side of the equal sign, so we divide on both sides with ( 68 ).</td>
</tr>
<tr>
<td>( I = \frac{10}{68} )</td>
<td>The teacher cancelled ( 68/68 ) and got the same expression for the current as in the first explanation.</td>
</tr>
</tbody>
</table>

This mathematics teacher showed two ways of solving the task, first rearranging the formula with variables and inserting the given values into the rearranged formula and calculating the answer. Secondly, he inserted the given values into the formula and solved the equation he got. The teacher said that he usually starts with the first alternative, but if the students do not understand this he tends to show the other way of solving the task, where he first inserted the given values.
Analysis

This mathematical teacher used two different mathematical ways of solving this task. In doing so the teacher not only solved this specific task, the teacher also explained to the hypothetical student that both formulas and equations is descriptions of equality and how to rearrange them. To rearrange the formula with variables is categorized as general mathematical knowledge in this detailed analysis, as it is an algebraic solution method. The teacher’s second explanation, first inserting the given values into Ohm’s law and then solving the equation, is categorized as specific mathematical knowledge, as this explanation uses specific numbers for this task.

Electricity teacher E1

This electricity teacher solved this task with a help-triangle. For students the use of this help-triangle may be new, but some students may have used a similar help-triangle in tasks with velocity, distance and time in compulsory school.

![Teacher wrote:](image)

**Teacher wrote:**

\[
\frac{U}{I \cdot R}
\]

\[
I = \frac{U}{R} = \frac{10}{68} = 0.147 A
\]

**Teacher did:**

The teacher said that most of the students have difficulties with rearranging formulas, so therefore he uses the help-triangle for Ohm’s law.

The teacher said that with this help-triangle it is easy to solve this task by covering the variable that you are searching for. He said that in this task we want to calculate the current, so we cover the I and get the formula U/R.

The teacher wrote the formula for I and inserted the given values and calculated the answer.

The teacher said that to rearrange formulas is difficult for many students and that some students have a bad self-confidence in mathematics, so it is important to make this as simple as possible for the students and that is why this teacher used the help-triangle to explain how to solve a task like this.

Analysis

This electricity teacher taught a method to solve tasks like this, a mechanical method that avoided mathematical explanations. To use the help-triangle to rearrange the Ohm’s law formula is in the detailed analysis categorized as specific mathematical knowledge, as it works with this specific formula and
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not every formula. This teacher might not know any other ways of solving this task and to be able to solve the task this is enough.

Comparison of the teachers’ explanations

In this detailed analysis both teachers are explaining the same thing; how to calculate the current in the circuit. But there are differences in both what they explained and in their use of mathematics. Both teachers explained how to solve the task and calculated the requested value of the current. The mathematics teacher explained the task in two mathematical ways, first solving the task algebraically and then inserting the given values into the formula and solving the equation. The mathematics teacher started with an algebraic, general, mathematical way to explain this task. If the students did not follow that explanation he said that he would have used a more concrete or specific explanation, where he first inserted the given values into the formula and then solved the equation that he got. In the mathematics teachers explanations he also explained equality in formulas and equations. The electricity teacher, on the other hand, used one explanation that he considered suitable and easy as he pointed out that students could have and often have bad experiences with mathematics. In his explanation of this task he used specific mathematical knowledge, the help-triangle - a specific method for the Ohm’s law formula. With this method he avoided as much as possible the use of mathematics and solved the task simply and effectively.

4.1.2 Detailed analysis 2 of task 1

One mathematics teacher and one electricity teacher talked about how to round the answer in this interview task. In this detailed study these two teachers’ explanations of how to round the answer will be presented and analyzed. Knowledge about rounding is in the overview analysis categorized as SMK in mathematics, but the electricity teacher did not refer to mathematics when he argued for his explanation, instead he used SMK in electricity in his explanation. The two teachers’ explanations of rounding were different and that will be described here.

Mathematics teacher M4

This mathematics teacher talked about rounding of the answers in this task. He said that a lot of students think that they should have as many decimals as possible to get the answer as exact as possible. So when the students have calculated an answer with the calculator they would give an answer with all the decimals displayed on the calculator. The teacher said that often students do not realize that the given values are not exact values, and that they should
Results

look at the number of significant digits in the given values and round the answer according to that.

In this interview task the current should be calculated with the given values of the resistance, 68 Ω, and the voltage, 10V. The mathematics teacher highlighted the fact that it is not certain in the task how many significant numbers there are in the value of the voltage of 10V.

“In this task it would have been super if a student writes something about that the ‘10’ have two significant numbers and that you can answer with two significant numbers. If the given numbers have two decimals then you can answer with two decimals, so the student must be able to round their answers.” M4 (15:30)

Analysis

The teacher used general mathematical knowledge of the rules for rounding an answer, according to how many significant numbers the given values in the task have and that is categorized as general mathematical knowledge.

Electricity teacher E4

This electricity teacher also said that it is important that the students round their answers. When he talked about rounding the answers in this task, he referred to how rounding is done in an electricity workplace. He said that one or at most two decimals are used practically in electricity workplaces, depending on the magnitude of the number. So in tasks like this he said that he wanted the students to round the answer to at most 2 decimals.

“I am not interested in more than maybe one decimal, because practically is it most often rough calculations you do at work.” (E4 9:10)

“One or at most two decimals is what we use practically, depending on the magnitude, of course, on the number. In 9 cases of 10 in practice, if you are constructing something, and I have been working as a constructor, you are not out for exact values; you are out for estimations to know if you are within the limit of what the components tolerate and what the voltage source can give and to be within rules and laws.” (E4 16:00)

Analysis

The teacher pointed out how rounding is used in electricity workplaces and in constructions. In his arguments of how to round the answer, he did not refer to mathematics; instead he used electrical knowledge of how this is done in electricity workplaces, categorized as electrical knowledge.
Comparison of the teachers’ explanations

These two teachers used different knowledge when they explained the same thing; rounding of an answer. Both teachers talked about the importance of rounding the answers, but they used different knowledge in their explanations of how the students should round their answers. The mathematics teacher used general mathematics knowledge when he talked about significant numbers in calculations. The electricity teacher did not use mathematical knowledge in his arguments instead he used electrical knowledge when he talked about how rounding is done in electricity workplaces.

4.1.3 Detailed analysis 3 of task 1

The second part of task 1, Calculations of the effect, urged the teachers to help a student to calculate the effect in the resistance in this circuit. The calculation of the effect in the resistance in task 1 is categorized in the overview analysis as SMK in mathematics.

In this detailed description of one mathematics teacher’s explanation of the calculation of the effect in the resistance in task 1 will be compared with two electricity teacher’s explanations. The teachers’ explanations are different compared to each other, and the example will highlight a number of interesting details of the teachers’ explanations.

The effect in the resistance could either be calculated with the value of the current that was calculated in the first part of this interview task, or the effect formula could be rearranged so that it contains the given values in the task by merging the effect formula with Ohm’s law. The mathematics teacher merged the formulas of Ohm’s law and the effect law in two ways, to get a new formula for the effect. The mathematics teacher used these two new formulas to calculate the effect in the resistance. The first electricity teacher did not merge Ohm’s law and the effect law; he calculated the effect with a calculated value for the current. The second electricity teacher did merge Ohm’s law and the effect law to get a new formula for the effect. With this new effect formula he explained why high voltage is used in transferring energy high tension lines.

Mathematics teacher M4

This mathematics teacher said that if he would have used this task, then he would not have planned for students to calculate the current first. He said that when we have the effect formula, where the effect is equal the voltage
times the current, and we have voltage and current in Ohm’s law too, we can rewrite and merge the two formulas. Calculating the effect and merging the formulas is categorized as SMK in mathematics in the overview analysis. This mathematics teacher also said that it is important to teach the student to use correct units and that is categorized as SMK in electricity in the overview analysis.

The teacher also said that the students need to realize that the variable U is the same in the different formulas so they can exchange them when merging the two formulas. He also said that simplifying expressions with variables is not easy or obvious for all students.

**Analysis**

The mathematics teacher did not only explain how to calculate the effect in the resistance in this task. Furthermore he explained general mathematics of how to merge formulas; here the effect formula and the Ohm’s law formula, and he merged the formulas in two ways. Merging the two formulas is categorized as general mathematical knowledge in this detailed analysis, as it involves algebraic reasoning. He also pointed out the importance of using correct units and that is categorized as electrical knowledge.
**Electricity teacher E4**

This electricity teacher calculated the effect in the resistance in this task in a direct way. He also talked about the importance of using correct units for the values and that is categorized as SMK in electricity in the overview analysis.

He said that it is important to use the correct units, a lot of students write only numbers, and we want them to realize the difference between the parameters, so that the students do not use un-technical talk. He said that when working in electricity workplaces it is important to use the correct technical terms for the magnitudes that are used.

**Analysis**

The teacher showed a direct way of calculating this task. He used the calculated value for the current to calculate the effect in this resistance. He did not say anything about merging the two formulas, and probably he did not see any use of doing that. This method is straight-forward and solves the task simply and quickly. In this detailed analysis it is categorized as specific mathematical knowledge, as it solves this specific task with specific values. He also connected the task to electricity workplaces when he pointed out how important it is in the electricity workplaces to use the correct units and that is categorized as electrical knowledge.

**Electricity teacher E3**

This electricity teacher did also merge Ohm’s law and the effect law, as the mathematics teacher did, but only in one way. In contrast to the mathematics teachers in this detailed analysis this electricity teacher merged the formulas to be able to explain that when high voltage is used in high tension lines the loss effect in the lines is reduced.

This electricity teacher said that he tends to start calculating a task like this with the values for the current and the voltage and use the effect formula
direct. Then he said that when he has done that a couple of times, he would explain the loss of effect in distribution lines, in the overview analysis categorized as SMK in electricity. In this explanation of effect loss in distribution lines, he said that he tends to merge the effect formula with Ohm’s law. The teacher’s merging of the effect law and Ohm’s law is described here:

\[
\begin{align*}
  P &= I \cdot U \\
  U &= I \cdot R \\
  P &= I \cdot I \cdot R
\end{align*}
\]

He then exemplified the use of this new effect formula, \( P = I \cdot I \cdot R \) with a concrete example of high tension lines. He said that if this \( R \) is the resistance in the distribution line to a customer and he pointed out that distribution lines have a resistance, all cables have that and distributions lines are long, so they have a substantially resistance. The effect that is generated in this resistance is pure loss effect, so he wrote: \( P_{\text{loss}} = I \cdot I \cdot R_{\text{cable}} \). This loss effect becomes heat, the cable gets warm, and we have no use of that effect. After that the teacher went back to the original effect formula; \( P = U \cdot I \) and he explained that to distribute a certain effect to a customer, the best is to have as little loss effect in the distribution lines as possible. The distribution of effect to a customer can be done using different voltages, a higher voltage and gives a lower current but anyway the same effect to the customer, according to the effect formula: \( P = U \cdot I \). Then he said that he tends to show a calculation example where he chooses to distribute an effect, for example 10kW, to a customer. With this effect he said he uses to prepare two voltages, and he calculates the current in each case, with the \( P = U \cdot I \) effect formula. After that he calculates the resistance for a certain cable and uses the new effect formula, \( P = I \cdot I \cdot R \), to calculate the loss effect in the distribution line for the two prepared voltages with different currents. He said that this example will show that the case with a higher voltage and smaller current will give a lower loss effect in the distribution line. He exemplified this further by reasoning and said that if we for example double the voltage used in the distribution line, we do not get half the loss effect, instead we get a fourth of the loss effect (according to \( P = I \cdot I \cdot R \)). Because we use the current times the current in the new effect formula, so a doubled voltage will give halved the current and then we have halved the effect double, the loss effect will be a quarter. In the end he explained that this is why we have 400 kV in the distribution lines in Sweden, when effect is transmitted; we use
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high voltage that gives low current and a low loss effect in the transmission lines.

Analysis

The teacher’s reasoning about how the loss effect is reduced in high tension lines is categorized in the overview analysis as SMK in electricity. The teacher has mathematical knowledge of how to merge the effect law with Ohm’s law, in the overview categorized as SMK of mathematics, and he used it when he explained how the loss effect is reduced in high tension lines.

The teacher first used specific mathematical knowledge when he calculated the effect in the resistance. Then he used general mathematical knowledge when he merged Ohm’s law and the effect formula, as it involved algebraic reasoning not needed to solve this task. The teacher used electrical knowledge in his explanations of high tension lines, when he both explained transferred and loss effect and why the loss effect is reduced when high voltage is used.

This teacher chose to explain why the loss effect in high tension lines is reduced when high voltage is used. To explain this electricity fact, he used general mathematical knowledge of merging formulas.

Comparison of the teachers’ explanations

In this detailed study there are several things that are different between the teachers, see table 3. It is not only the teachers’ use of specific and general mathematical knowledge that are different. The teachers chose to calculate the requested effect in two different ways, both with specific mathematical and general mathematical knowledge. Also different in this task is what the teachers explain.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Knowledge</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E4, E3</td>
<td>Spec math</td>
<td>Inserted values in the effect formula to calculate the requested effect.</td>
</tr>
<tr>
<td>M4</td>
<td>Gene math</td>
<td>Merged the formulas and calculated the requested effect.</td>
</tr>
<tr>
<td>E3</td>
<td>Gene math</td>
<td>Merged formulas and explained why high voltage is used in high tension lines.</td>
</tr>
<tr>
<td>E4, M4</td>
<td>Electrical</td>
<td>The importance of using correct units</td>
</tr>
<tr>
<td>E3</td>
<td>Electrical</td>
<td>Transferred effect and loss effect</td>
</tr>
</tbody>
</table>

Table 3 Comparisons of the teachers’ explanations in detailed analysis 3 of task 1.
Two teachers (M4, E3) did merge Ohm’s law and the effect law into a new effect law by the use of general mathematical knowledge. These two teachers were merging the two formulas to be able to explain different things, the mathematics teacher explained mathematics and the electricity teacher explained electricity. The mathematics teacher explained how to merge formulas in two ways and the electricity teacher explained how the loss effect in high tension lines is reduced with using high voltage.

The mathematics teacher used this task to explain how to merge two formulas to a new effect formula and he did this in two ways. With these new effect formulas he calculated the requested effect. The explanations contained algebraic knowledge that was not needed to solve the task, but in the second explanation could the given values of the resistance and the voltage in the interview task be used in the calculation instead of the calculated approximation of the current and that may increase the accuracy of the answer. This mathematics teacher used this task to teach how to merge formulas. The second electricity teacher (E3) also used general mathematics to merge the two formulas into a new formula, but not to be able to calculate the effect requested in the task. Instead he used this new effect formula to be able to explain why high voltage is used in high tension lines.

The teachers used different kinds of mathematical knowledge, specific and general, in explaining this task. Furthermore, the mathematics teacher and one electricity teacher both used general mathematical knowledge to explain different things. The mathematics teacher used this electricity example to teach the students more about general algebra, merging formulas. The second electricity teacher (E3) used general mathematics, merging formulas, to be able to explain electricity, here why high voltage is used in high tension lines.
4.2 Analysis task 2

In this task the students are asked to calculate the total resistance, the equivalent resistance, in a direct current circuit with two parallel resistances (see figure 8). The teachers in the interviews are asked how they would explain how to solve a task like this to a student on the Electricity Program and what is important for the students to understand to be able to solve tasks like this.

2. Student task: Parallel resistance

Your students have started to work with electrical circuits with parallel resistance. Many students have difficulties in using the formula to calculate the total resistance in a parallel circuit.

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]

R1 is 220 ohm and R2 is 120 ohm

Task:
Calculate the total resistance in the circuit.

How would you explain how to solve this task to a student?

Figure 8 Interview task 1

Figure 9 gives an overview of all the teachers’ explanations. This overview indicates that there are both similarities and differences between the two teacher groups’ use of mathematical and electrical knowledge. In this task the both teacher groups use explanations that are similar in all the knowledge categories, but there are explanations given by one of the teacher groups and not the other teacher group.

The overview shows that in SMK in mathematics, one mathematics teacher gave an explanation that no electricity teacher mentioned. This explanation is knowledge about that rounding of the part answers in calculations during the solving of a task could affect the precision of the final answer. In PCK in mathematics there are one explanation given only by one mathematics teacher. He said that with the given values of the resistances in this task it is far too difficult for students to solve the task with fractions. Both these mathematics teachers base their explanations on mathematical knowledge that no electricity teacher mention.
**Indications of Teacher knowledge, task 2**

**Subject matter knowledge in mathematics**

- Said that rounding too early in the calculations gives a wrong answer
- Calculated the total resistance in the parallel circuit
- Said that some students use to forget to invert at the end of their calculations
- Said that students may have a bad self-confidence in mathematics

**Pedagogical content knowledge in mathematics**

- Said that the task is too difficult for students to solve with fractions
- Showed an example with easier numbers
- Calculated an estimation to be able to check the answer

**Subject matter knowledge in electricity**

- Reasoned about the total resistance in a parallel circuit
- Calculated an estimation to be able to check the answer
- Explained the origin of the formula
- Said that the two new concepts, series and parallel circuit, may be difficult for the students

**Pedagogical content knowledge in electricity**

- Used an analogy to explain the circuit e.g bridges over a river or traffic on roads
- Said that using a structure when solving a task makes it easier for the students

Figure 9 Yellow boxes show explanations given only by mathematics teachers; blue boxes show explanations given only by electricity teachers and green boxes show explanations given by both mathematics and electricity teachers.

M1, M2, M4, represents mathematics teachers; E1-E4 represents electricity teachers.

Boxes in the category PCK (both in mathematics and electricity) with rounded corners are explanations that are based on the teachers’ experience of students’ pre-knowledge or common mistakes.

In SMK in electricity the electricity teachers had some explanations exclusively used by them. One of the explanations is knowledge of an alternative formula useful in parallel circuits to calculate the total resistance when all the parallel resistances have the same value, independent of how many they are. These electricity teachers said that if the resistances in a parallel circuit have the same value, then the total resistance could be calculated by dividing the value of one of the resistances with the number of resistances in the circuit. One electricity teacher used the alternative formula to test the value he calculated to see if it was reasonable.

The electricity teachers had some indications of knowledge about students and the subject in the category PCK in mathematics and PCK in electricity that the mathematics teachers do not mention in this study. This is knowledge about students working with tasks like this that the electricity
Results

teachers probably have experience of, as this is a commonly used task in the electricity course.

One mathematics teacher (M2), educated in mathematics and biology, is only represented once in this overview picture. He had not seen or used this task before, so he had no experience with it. He solved it correctly but he had no experience in helping students with it. His solution will be shown in one of the detailed analysis below.

In the next section three examples of detailed analyses of specific explanations given by one mathematics and one electricity teacher are described. The first detailed analysis is an example of an explanation of the calculation of the total resistance in the parallel circuit, SMK in mathematics. The second detailed analysis is a description of two teachers who explain a simpler example, categorized as PCK in mathematics and the last detailed analysis is two teachers who explain the origin of the formula for the total resistance, categorized as PCK in electricity.

**4.2.1 Detailed analysis 1 of task 2**

In this task the teachers were asked how they would explain to a student on the Electricity Program how to calculate the total resistance in a parallel circuit, using the given formula for the total resistance. Calculating the total resistance in a parallel circuit is categorized as SMK in mathematics in the overview analysis. Here two explanations of how to calculate the total resistance in this task given by one mathematics and one electricity teacher will be presented. The mathematics teacher had not seen or used this formula before, as he is educated in mathematics and biology and the electricity teacher had a lot of experience using this formula. The electricity teacher did in his explanation also explain electrical properties of this circuit and that is categorized as SMK in electricity in the overview analysis. This example is chosen to show how different the reasoning and argumentation for a specific explanation of this task could be. This mathematics teacher explained this task in a solely mathematical way, which is different from how the experienced electricity teacher explained it.

**Mathematics teacher M2**

This mathematics teacher, teaching mathematics and biology, has taught electricity students for some years and he had not used this example before in his teaching, so he had no experience of what problems a student might have with it. The teacher solved the task in a correct way, but afterward he said that this solution is probably a little complicated for the students to
follow and it may also be complicated for the students to use this rearranged formula.

Results

The teacher rearranged the formula with the variables and he got a big parenthesis that he dealt with correctly. The teacher wrote down every step in the calculations carefully. But he said afterwards that it could be difficult for the students to rearrange the formula in this way. The teacher also said that it could be difficult for the students to insert the given values into this rearranged formula and calculate the answer on their calculators, because this rearranged formula is complicated with a parenthesis as denominator.

Analysis

The teacher used a common mathematical way of rearranging a formula with the unknown as a denominator. First he rearranged the formula to get a new formula for the unknown variable and then he inserted the given values to calculate an answer. In the detailed analysis rearranging the formula this algebraic way is categorized as general mathematical knowledge, as it includes rearranging a formula with the unknown as a denominator. The teacher did not provide any electricity explanation.
Results

Electricity teacher E2b

In contrast to the mathematics teacher, this electricity teacher started his explanation with some electricity explanations, in the overview analysis categorized as SMK in electricity. After his electricity explanations he calculated the total resistance in the interview task and discussed his calculations. He started his explanation saying that the total, or the equivalent, resistance in a parallel circuit is always less than the smallest resistance in the circuit. He also said that this given formula is always usable for two resistances, and he meant that it is useful for all different resistances (both equal and different). He also said that the given formula is a simpler version for two resistances of the common formula, valid for any number of resistances:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

He then said that if the resistances in the parallel circuit have the same resistance then you can simply calculate the total resistance in the circuit by dividing one of the resistance’s value with the number of resistances in the parallel circuit. This could be described:

$$R_{tot} = \frac{R_n}{n}$$

where $R_n$ is one of the resistances with the same value and $n$ is the number of resistances in the parallel circuit.

He gave the example: if you have two 220Ω resistances connected in parallel, the total resistance will be $220/2=110Ω$. In cases with equal resistances in parallel circuits, you do not have to use the common formula; you can instead use this simpler formula.

Then the teacher calculated the total resistance in this task, described below:

**Teacher wrote:**

$$R_1 = 220Ω$$

$$R_2 = 120Ω$$

$$\frac{1}{R_{era}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{era} = 77.69Ω$$

**Teacher did:**

The teacher wrote the given values and the formula.

The teacher inserted the given values into the formula and said that it is incorrect to add these fractions as they are written now with different denominators. He said that the simple method using the calculator is useful, and this method is presented in the textbook.

The teacher used the calculator and the button $[x^{-1}]$ to calculate the correct answer.
Results

The teacher said that he in class tends to show some simpler examples in the beginning and calculate them with fractions. In solving those examples he reasons with the students about common denominators in addition with fractions. The teacher said that maybe you do not need to show the fraction calculations in this task, you can use the easy way with the calculator that is explained in the textbook.

“The question is how careful you should be, should you do the full fraction calculation, or should you do the simpler way with the calculator. [...] Actually it is good if they can do this (fraction calculations) but for the electricity subject it is enough for us to get the value, the total resistance.” (E2b 19:55)

Analysis

This teacher used electrical knowledge when he explained that the total resistance in a parallel circuit is always smaller than the smallest resistance in the circuit. He did not give any reasons for why this is true, but this can be used to check if a calculated value is reasonable. Some interviewed teachers in this study did reason within electricity about why the total resistance in a parallel circuit is always less than the smallest resistance in the circuit (see section 4.2 Analysis task 2, figure 9). No teacher in this study used mathematics in reasoning why the total resistance in a parallel circuit is always less than the smallest included resistance. Also categorized as electrical knowledge, is the teacher’s explanation of an alternative formula when the resistances in a parallel circuit are equal. In that case, the total resistance in the circuit is one of the resistances divided by the number of resistances in the parallel circuit. The teacher used electrical knowledge when he said that the given formula is a version for two resistances of the common formula, \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \). To calculate the total resistance in this task, the teacher inserted the given values for the resistances and made the calculations with the calculator, categorized as specific mathematical knowledge, as it was a method to solve this specific task.

This electricity teacher chose to start his explanations of this task with explanations of electricity properties. After that he calculated the total resistance in the circuit that was asked for in the task.

Comparison of the teachers’ explanations

There are several differences in these two teachers’ explanations of the calculation of the total resistance in a parallel circuit, see table 4. They are both solving the task, calculating the total resistance in the parallel circuit, but they are using different approaches. The two teachers are using different
Results

mathematical approaches and they chose to explain different things in this task.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Knowledge</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>Gene math</td>
<td>Calculate the total resistance by rearranging the formula algebraically.</td>
</tr>
<tr>
<td>E2b</td>
<td>Spec math</td>
<td>Calculate the total resistance by inserting values into the given formula.</td>
</tr>
<tr>
<td>E2b</td>
<td>Electrical</td>
<td>Electrical properties of the circuit.</td>
</tr>
</tbody>
</table>

Table 4 Comparisons of the teachers’ explanations in detailed analysis 1 of task 2.

This mathematics teacher explained this task in a solely mathematical way using an algebraic method. Here we have to remember that the mathematics teachers are not only teaching this first mathematics course, they are also teaching higher mathematics courses with, among other topics, more algebra. The electricity students do most often not study any higher mathematics courses. The mathematics teacher used a general mathematical method to solve the task, not referring to electricity at all in his solution. The mathematics teacher’s explanation was based on general mathematical knowledge; he used an algebraic method that could be used for all different formulas.

The electricity teacher on the other hand, first explained electrical properties of this specific circuit and then used a practical mathematical method to calculate the requested total resistance in the circuit. The electricity teacher even highlighted that it is only specific mathematical knowledge that is needed for the electricity subject, when he said that for the students it is enough to be able to calculate the answer.

4.2.2 Detailed analysis 2 of task 2

In this detailed analysis one mathematics teacher and one electricity teacher are described, both choosing to explain this kind of task with a simpler example in the beginning, and calculating this simpler example with fractions. Both teachers said that they use to start explaining tasks like this with a simpler numerical example to show the students how to calculate it with fractions. To use a simpler mathematics example to help the students understand a task is categorized as PCK in mathematics.

Mathematics teacher M1

This mathematics teacher said that he would have used an easier example in the beginning to explain a task like this and that this easier example would
help the students to follow the reasoning in the calculations. The teacher showed how to calculate the simpler example with both fractions and decimal numbers. He said that the easier example would have had easier numbers, for example the resistances 4Ω and 5Ω, and with this easier example he calculated the total resistance in the parallel loop, first with decimal numbers.

Teacher wrote:
\[
\frac{1}{R} = \frac{1}{5} + \frac{1}{4}
\]

Teacher did:
The teacher chose the easier resistances 5Ω and 4Ω and inserted them into the given formula.

\[
\frac{1}{R} = 0.2 + 0.25
\]

He calculated this example with decimal numbers.

\[
\frac{1}{R} = 0.45
\]

He got 1/R equals 0.45.

\[
R \cdot \frac{1}{R} = 0.45 \cdot R
\]

To be able to calculate R he compared this with a small equation and extended both sides of the equal sign with R.

\[
\frac{1}{0.45} = \frac{0.45}{0.45} \cdot R
\]

The teacher divided with 0.45 on both sides of the equal sign and cancelled 0.45/0.45.

\[
2.2 = R
\]

He got the answer 2.2Ω, for this easier example.

He also explained how to solve the same easier example using fractions, by extending the denominators to get a common denominator and adding the fractions.

Teacher wrote:
\[
\frac{1}{R} = \frac{1}{5} + \frac{1}{4}
\]

Teacher did:
The teacher inserted 5Ω and 4Ω into the formula and calculated this example with fractions calculations.

\[
\frac{1}{R} = \frac{1 \cdot 4}{5 \cdot 4} + \frac{1 \cdot 5}{4 \cdot 5}
\]

He extended the terms to the common denominator 20.

\[
\frac{1}{R} = \frac{5}{20} + \frac{5}{20}
\]

He calculated the sum of the terms.

\[
\frac{1}{R} = \frac{9}{20}
\]

The teacher explained that to calculate R, you can invert both sides of the equal sign.

\[
R = \frac{20}{9}
\]

He switched both sides of the equal sign, and calculated the answer on this easier example again.

The teacher also explained how to calculate R when the result from the formula was given as 1/R. He did this in two different ways, first comparing it to a small equation and then explaining how to invert both sides of the equal sign.
Results

Analysis

The teacher showed how to calculate this easier example with both decimal numbers and with fraction numbers and this is categorized as specific mathematical knowledge in this detailed analysis, as this is an example of using concrete numbers in the given formula to solve the task. The teacher also showed two ways of calculating R, when you have 1/R, in this easier example. First he compared 1/R=0.45 with a small equation that he solved by multiplying by R on both sides of the equal sign and cancelling R/R. Secondly, he showed how to invert to get R, by switching both sides of the fractions (1/R=9/20 ⇔ R=20/9). Both ways are categorized as specific mathematical knowledge in this detailed analysis, as it involves two concrete explanations of calculating R when you have 1/R.

Electricity teacher E2a

The teacher started solving the original task trying to find a common denominator, but after a little while he said that he could not remember how to do this. Then he said that he in class tends to start explaining how to solve a task like this by using a simpler example, an example with easier numbers, for example the resistances 2Ω and 4Ω and calculating it with fractions. His calculation of this easier example is described below:

\[
\begin{align*}
\text{Teacher wrote:} & & \text{Teacher did:} \\
\frac{1}{R} &= \frac{1}{2} + \frac{1}{4} & \text{The teacher inserted the easier values of the resistances into the given formula.} \\
\frac{1}{R} &= \frac{1 \cdot 4}{2 \cdot 4} + \frac{1 \cdot 2}{4 \cdot 2} & \text{He extended the terms to get a common denominator.} \\
\frac{1}{R} &= \frac{4}{8} + \frac{2}{8} & \\
\frac{1}{R} &= \frac{6}{8} & \text{He added the fractions, and got a sum that equals } 1/R. \\
R &= \frac{8}{6} & \text{The teacher said that to get } R \text{ we have to switch nominator and denominator on each side of the equal sign.} \\
R &= 1.3Ω & \text{He calculated an answer for his easier example.}
\end{align*}
\]

The teacher showed how to invert 1/R to get R in the end of the explanation.
Results

Analysis

This electricity teacher said that he tends to start showing tasks like this with easier numbers, smaller resistances and solving this easier example with fraction calculations. He solved the easier example in one way, with fractions, and that is categorized as specific mathematical knowledge in this detailed analysis. In his easier example he showed how to invert to get R, by switching both sides of the fractions ($1/R=6/8 \Leftrightarrow R=8/6$), also categorized as specific mathematical knowledge in this detailed analysis.

Comparison of the teachers’ explanations

To use an easier example to help a student to understand a task is categorized as PCK in mathematics, as it is an explanation of how to help the students move forward in their understanding of the formula. Both teachers said that when they explain tasks like this with an easier example, they tend to use an example with smaller numbers, so that they can show how to calculate the task with fractions that are not too difficult for the students. The teachers said that it will be easier for the students to understand this if they use smaller/simpler numbers in the example, numbers that the students do not have problems with and/or recognize. To calculate this easier example is in this detailed analysis categorized as using specific mathematical knowledge, as it is a solution of this specific task.

In this detailed analysis, both teacher are explaining an easier example and they are both doing this in similar ways, except that the mathematics teachers did it in two ways, both with decimal numbers and fractions, whereas the electricity teacher did it in one way, with fractions. This analysis indicates that this mathematics teacher had knowledge of explaining the easier example in more ways than this electricity teacher had. In the mathematics teacher’s explanations, he is also indirectly explaining the relation of decimal numbers and fractions, besides helping the student to understand how to use the formula for parallel resistances. The mathematics teacher also explained how to get R when you have $1/R$ in two different ways, while the electricity teacher showed one way. The mathematics teacher said that you can explain this as a small equation and solve this equation to get R or you can invert both sides of the equal sign, which the electricity teacher also did. The mathematics teacher showed how to note an inversion in the written calculations with raised in -1. In this example the mathematics teacher wrote: \( \frac{1}{R} = 0.45 \Leftrightarrow R = 0.45^{-1} \). In addition to teach how to calculate this task, the teacher also explained how to solve similar equations, with the unknown as a denominator.
Results

The mathematics teacher said that it is too difficult for these students to find the least common denominator in this task, while the electricity teacher started to try to find a common denominator for a little while, until he said that the numbers were too complicated and he did not remember how to do it. The reason for this may be that the mathematics teacher has more experience with fraction calculations than the electricity teacher.

4.2.3 Detailed analysis 3 of task 2

This section presents one mathematics teacher’s and one electricity teacher’s explanation of the origin of the formula for the total resistance in a parallel circuit. All teachers solved this task, but it is only one mathematics teacher and one electricity teacher that explained the origin of the formula for the total resistance in a parallel circuit: $1/R = 1/R_1 + 1/R_2$. These two teachers’ explanations of the origin of the formula are analyzed in detail here. Both teachers say that they tend to start explaining the parallel formula's origin to their students to help the students understand and use the formula; therefore this is categorized as PCK in electricity. This example is chosen because the detailed analyses show that the two teachers’ explanations differ in several aspects and they use different kinds of mathematical knowledge in their reasoning of the formulas origin.

These two teachers explained the origin of the formula for the total resistance in a parallel circuit. Both teachers started with Kirchhoff’s first law stating that the sum of the part currents in the parallel branches equals the head-current ("Kirchhoff's laws," 2009). After that the mathematics teacher used an algebraic way to show how the total resistance formula of a parallel circuit origin from Kirchhoff's law. The electricity teacher on the other hand, showed the origin of the formula by reasoning and calculating in the electricity circuit.

Mathematics teacher M4

The teacher explained the origin of the parallel formula, starting with Kirchhoff’s currents law.
Results

Teacher wrote:  
\[ I = I_1 + I_2 \]

Teacher did:  
The teacher started with writing Kirchhoff’s law, and explained it while pointing to the part currents and head current in the circuit diagram.

\[ U = I \cdot R \]
\[ I = \frac{U}{R} \]

He said that the voltage is the same over the two resistances and that the currents could be rewritten by Ohm’s law.

\[ \frac{U}{R_{\text{pars}}} = \frac{U}{R_1} + \frac{U}{R_2} \]

The teacher substituted the currents in Kirchhoff’s law with expressions for the currents derived from Ohm’s law.

\[ \frac{1}{R_{\text{pars}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

The teacher then factorized the expressions on both sides of the equal sign, and he divided by \( U \) on both sides of the equal sign.

\[ \frac{1}{R_{\text{pars}}} \cdot \frac{U}{U} = U \cdot \left( \frac{1}{R_1} + \frac{1}{R_2} \right) / U \]

He cancelled \( U/U \) and got the formula for the total resistance in a parallel circuit.

By using algebra in this way, the teacher verified the origin of the formula for the total resistance in a parallel circuit.

Analysis

The teacher used electrical knowledge in the beginning when he introduced Kirchhoff’s law and when he explained that the voltage is the same over the two resistances and the same as the voltage source. Thereafter he used mathematical knowledge. He rewrote the two part currents, \( I_1 = U/R_1 \) and \( I_2 = U/R_2 \), derived from Ohm’s law and substituted the currents in Kirchhoff’s law with these expressions, categorized as general mathematical knowledge as merging formulas is not specific to this task. After that he used algebra to factorize the expressions on each side of the equal sign and then cancelled \( U \) divided by \( U \) and got the original formula for the total resistance in a parallel circuit.

Electricity teacher E3

This electricity teacher also used Kirchhoff’s law to explain the origin of the parallel formula, but he did it in a different way compared to the mathematics teacher. The electricity teacher said that as there is no voltage chosen in this task, he usually let the students in class choose a voltage to use when calculating the part and head currents in the circuit and the total resistance in the circuit by Ohm’s law.
This electricity teacher showed with a couple of examples that independent of which voltage you use in this parallel circuit you will have the same total resistance. He did this in a practical way, he said that he uses to let the students chose which voltage they are going to calculate the currents in the circuit with. For the chosen voltage, he calculated the two part currents, added them to a head current and said that when you know the head current and the voltage in the circuit, you can calculate the total resistance in the loop. After he had calculated the total resistance for one voltage, he said that he tends to calculate the total resistance for some other voltages and with these examples he concluded that independent of which voltage is used, the total resistance will be the same. Then he said that, as the total resistance is independent of which voltage you use, in the formula they have used the voltage 1 V because it is easy and you have the special button on the calculator that you can use (1/x).
Results

Analysis

The teacher started the explanation using electrical knowledge when he introduced Kirchhoff’s law and explained that the voltage is the same over the two resistances and the same as the voltage source. Then this electricity teacher used mathematical knowledge when he calculated the different electrical magnitudes, but the idea for his explanation of the origin of the parallel formula is electrical and the reasoning is based on electricity. The teacher used electrical knowledge when he chose different voltages to be able to calculate the currents and the total resistance in this circuit. The mathematical knowledge this teacher used is categorized as specific mathematical knowledge, since he is calculating the electrical magnitudes (currents and resistance) in this particular circuit. With the result of these calculations of the total resistance with different voltages, the teacher concluded that the total resistance in the circuit is independent of what voltage there is, categorized as specific mathematical knowledge, as this conclusion is based on concrete examples.

Comparison of the teachers’ explanations

These two teachers chose to explain the origin of the formula for the total resistance in a parallel circuit. But their reasoning and argumentations were in many ways different, see table 5. The mathematics teacher derived the formula algebraically and the electricity teacher verified the formula with some numerical examples.

Both teachers started their explanations in the same way with Kirchhoff’s current law, stating that the sum of the part currents equals the head current in the parallel circuit. After that the mathematics teacher continued basing his explanation only on mathematics, whereas the electricity teacher continued with electricity reasoning in the electrical circuit. The electricity teacher made calculations for some voltages in the circuit and based on these calculations he concluded that the formula is valid for all different voltages.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Knowledge</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E3</td>
<td>Spec math</td>
<td>Explained the origin of the formula by verifying the formula.</td>
</tr>
<tr>
<td>M4</td>
<td>Gene math</td>
<td>Explained the origin of the formula by deriving the formula.</td>
</tr>
<tr>
<td>M4, E3</td>
<td>Electrical</td>
<td>Kirchhoff’s law, voltage in the circuit</td>
</tr>
</tbody>
</table>

Table 5 Comparisons of the teachers’ explanations in detailed analysis 3 of task 2.
Both teachers’ explanations were based on their knowledge of Kirchhoff’s current law in parallel circuits, electrical knowledge of electrical circuits. They also used electrical knowledge when they reasoned about the voltage in the circuit, as they did when they said that it is the same voltage over the two resistances and over the voltage source. After that the teachers used different argumentation and reasoning in their explanations. The mathematics teacher used the task to explain how the formula for the total resistance in a parallel circuit could be derived with general mathematics. The electricity teacher used specific mathematical knowledge to verify the formula for the total resistance in a parallel circuit.
4.3 Analysis task 3

In this interview task, see figure 10, the teachers are presented with a student’s incorrect solution of the length of an electrical cord with a given inner resistance and the teachers are asked to explain how to help the student solve this task.

3. Student task: The resistance of a conductor (resistivity)

You have given your students tasks to calculate the resistance in a conductor.

The formula for a conductor’s resistance:

\[ R = \frac{\rho \cdot L}{A} \]

(\( \rho \) is the conductor’s resistivity (\( \rho = 0.0175 \)), \( L \) is the length of the conductor (m) and \( A \) is the conductor’s cross-sectional area (mm\(^2\)).

In one of the tasks the students should calculate how long a copper conductor with a cross-sectional area of 1.5 mm\(^2\) should be to have a resistance of a 1.2 ohm.

Your student solves it like this:

\[
\begin{align*}
1,2 &= 0,0175 \cdot L / 1,5 \\
1,2 - 0,0175 \cdot 1,5 &= L \\
L &= 1,77 \text{ m}
\end{align*}
\]

How would you help the student? How would you calculate the task?

Figure 10 Interview task 3

All the teachers’ explanations of this task are presented in the overview picture in figure 11. The overview picture indicates that there are some differences between the mathematics and the electricity teachers and that the electricity teachers in this study had some more explanations exclusively used by them and not used by mathematics teachers. In SMK in electricity three electricity teachers said that in tasks like this you have to be careful with the meaning of the task, if it is the cable’s or the conductor’s length, as most often a cable contains at least two conductors. This is called Electrician knowledge as it is practical electrical knowledge. No mathematics teacher mentioned this.
**Results**

*Indications of Teacher knowledge, task 3*

**Subject matter knowledge in mathematics**

- Calculated the length of the conductor in the task
  - M1, M2, M4, E1-E4
- Reasoned about the formula, what will happen if the length or the area increase
  - M1, E3
- Explained rounding of the answer
  - E2b

**Pedagogical content knowledge in mathematics**

- Said that it is easier to rearrange the formula for students if it is written: \( R = \frac{\rho \cdot L}{A} \)
  - M1, M2, M4
- Said that it is easier for students to use several steps in the calculations
  - M3, E1
- Said that it is easier for some students to rearrange a formula with numbers than with variables
  - M2, M4, E2a
- Explained the formula by testing it with an invented length
  - E1, E2a
- Used an example with easier numbers
  - E1

**Subject matter knowledge in electricity**

- Said that units and prefix is important when working with electricity
  - M2
- Said that this is a practical formula that uses both meter and square-millimeter
  - M1, E4
- Electrician knowledge: a cable contain at least two conductors, so the cable length is not the same as the conductors length
  - E2b, E4
- Explained what the variables stand for
  - M1, E2a, E3, E4
- Used an analogy to explain resistivity eg electrons in a conductor or an electrical cable connected to a trailer in the winter
  - M1, E1, E3

**Pedagogical content knowledge in electricity**

Figure 11 Yellow boxes show explanations given only by mathematics teachers; blue boxes show explanations given only by electricity teachers and green boxes shows explanations given by both mathematics and electricity teachers.

M1, M2, M4, represents mathematics teachers; E1-E4 represents electricity teachers.

Boxes in the category PCK (both in mathematics and electricity) with rounded corners are explanations that are based on the teachers’ experience of students’ pre-knowledge or common mistakes.

In PCK in mathematics the two teacher groups differ in their preferred writing of the formula for a conductor’s inner resistance. The mathematics teachers preferred to write the formula with one common division sign and some of the electricity teachers preferred to write the formula without any division signs. The electricity teachers had some explanations in PCK in mathematics that the mathematics teachers did not mention. The electricity teachers used a practical way to explain how to solve this task, two of the teachers tested the formula with an invented length to get an inkling of the answer, and one electricity teacher tested the formula with easier numbers to be able to remember how to rearrange the formula.

One mathematics teacher (M4) said that these interview tasks are similar, mostly concerned with rearranging formulas. From the mathematical view
this is true, but from the electricity view this is a new content area of resistivity of conductors.

In the next section detailed analyses of two specific explanations of the same topic given by mathematics and electricity teachers will be presented and analyzed. The first detailed analysis describes one mathematics and one electricity teacher’s reasoning about the electrical resistivity formula, categorized as SMK in mathematics in the overview analysis. These both teachers did in their reasoning of the formula also explain the formula in electricity terms, here categorized as PCK in electricity. The second detailed analysis describes one mathematics and three electricity teachers’ calculations of the length of the conductor in this task, also categorized as SMK in mathematics in the overview analysis. In this detailed study, the two electricity teachers included electricity explanations, which were categorized as indications of SMK and PCK in electricity in the overview analysis.

4.3.1 Detailed analysis 1 of task 3

Here one mathematics and one electricity teacher’s explanations by reasoning about the resistivity formula for calculating a conductor’s resistance will be analyzed in detail. Reasoning about the resistivity formula is categorized as SMK in mathematics in the overview analysis as this is knowledge of how the parameters in the formula are related. Both teachers in this detailed analysis did combine their reasoning of the formula with an electricity explanation talking about the electrons moving in the conductor, categorized as PCK in electricity in the overview analysis.

Mathematics teacher M1

The mathematics teacher said that he would explain the formula for the students. While pointing to the variables in the formula he reasoned about what will happen if one variable increased. He said that if the variable L in the formula increased then the variable R must also increase and if the variable A in the formula increased then must the variable R in the formula decrease.

“I would explain the formula in itself to the students, reason about the length, when the length increase the resistance will increase, and the cross sectional area, when it becomes bigger there is room for more electrons.” (M1 30:10)

He also made a picture of two conductors with different width and with two electrons moving in each conductor. He continued to explain in physical
terms, saying that the electrons have more space in the thicker conductor and then the resistance or opposition will be less.

“We have a certain amount of electrons that are going to pass, from one place to another. And then you understand that the opposition will be less if the conductor is thicker, if the cross-sectional area increases.” M1 (31:05)

Analysis

The teacher reasoned about the formula in two different ways, first studying the variables' position in the formula and then by talking about electrons moving in the conductor, with the picture he made. This teacher first explained the mathematical formula and then exemplified this with electrons moving in the conductor. Reasoning about the variables in the formula is categorized as general mathematical knowledge in this detailed analysis, as this involves general knowledge of formulas. Reasoning with the illustration of two conductors with electrons moving in them is categorized as electrical knowledge.

Electricity teacher E3

The teacher reasoned about the formula by talking about electrons moving in a cable.

“The electrons are going to rush forward in this cable and they will hit into other atoms, and the more atoms they will hit into the more will the resistance be, because then they will have more opposition. If we take a longer cable then there will be more that they will hit into and the resistance will be bigger. If I take a broader cable there will be more area and less resistance.” (E3 36:35)

The teacher explained the formula by reasoning about electrons moving in two conductors of different width, and he gave an explanation of how the electrons will hit into atoms.
Results

Then he made an explanation based on calculations in the formula. He pointed to the variable $L$ in the formula and reasoned about what will happen if we increase the length. He said that if the length increased ten times, then there will be ten times more things for the electrons to hit into, and the resistance will increase. While pointing to the $A$ in the formula he said that as it is division by the $A$ in the formula, so if the area increased then the resistance will decrease, so if we double the area, the resistance will be halved.

Analysis

The teacher reasoned about the formula in two different ways, first by talking about electrons moving in the conductor, comparing conductors with different length and width, and then by reasoning about the formula with numerical examples. The teacher started to explain the formula in the electricity context and the exemplified this with numerical examples of the formula. The teacher’s reasoning about electrons in the cable is categorized as electrical knowledge. This teacher reasoned about general mathematical properties of the resistivity formula, even if he used numbers to exemplify this with (ten times and double), this was categorized as general mathematical knowledge.

Comparison of the teachers’ explanations

These two teachers’ reasoning about the resistivity formula is similar, with only slight differences. Both teachers did reason about general properties of formulas, the mathematics teacher reasoned about the variables position in the formula, whereas the electricity teacher exemplified his reasoning with simple numeric values, as a generic example.

Both teachers reasoned about the resistivity formula both with mathematics and electrical knowledge. The difference of the two teachers reasoning of the resistivity formula was that the mathematics teacher started his reasoning with reasoning of the mathematical formula and exemplified his reasoning with an illustration of an electrical conductor and reasoning of electrons moving in the conductor. The electricity teacher started his reasoning of the resistivity formula with reasoning about electrons moving in a conductor and exemplified this with reasoning of the mathematical formula by using numerical examples. This is one example of how a mathematics teacher highlights the mathematics whereas the electricity teacher highlights the electricity in those interview tasks.
### 4.3.2 Detailed analysis 2 of task 3

To calculate the conductor’s length in this task was categorized as SMK in mathematics in the overview analysis. In this detailed analysis one mathematics teacher’s explanation of this calculation will be compared with two electricity teachers’ explanations of the same task. The electricity teachers did in their explanations also include electrical explanations that are also described. In several ways these explanations differ from each other.

**Mathematics teacher M4**

The mathematics teacher started to point out how similar the interview tasks 1-3 are and he said that in all of them you are going to rearrange the formulas to be able to calculate one unknown. He said that he uses to rearrange the formula one variable at a time, because otherwise students could easily make mistakes. The teacher started with writing the formula and explaining that it is easier for the students to rewrite it when you have one common sign of division.

\[
\begin{align*}
\text{Teacher wrote:} & \quad R &= \rho \cdot \frac{L}{A} \\
R &= \frac{\rho \cdot L}{1 \cdot A} = \frac{\rho \cdot L}{A}
\end{align*}
\]

He rewrote \( \rho \) to \( \rho/1 \) and then multiplied the fractions. He said that this formula looks easier, to start rearrange, when you see that \( A \) is a common denominator for the whole right hand side.

\[
\begin{align*}
R \cdot A &= \frac{\rho \cdot L \cdot A}{A} \\
R \cdot A &= \rho \cdot L \\
R \cdot A &= \frac{\rho \cdot L}{\rho} \\
\frac{R \cdot A}{\rho} &= L
\end{align*}
\]

The teacher said that these interview tasks are similar; you should know how to rewrite a formula, and the students do not have to do it fast, it is important that they know what they do.

“I always use to try to get them to take one letter (variable) at a time, so that they see what happens and that it would not be wrong.” (M4 46:25)
Results

When the teacher looked at the student’s solution in the task, he said that he rather wants the students to rewrite the formula with variables instead of values, as it can be many numbers to write and part answers if you use the values. He also said that students most often have difficulties with rewriting formulas and that rewriting formulas is often more difficult than solving equations with specific values. He said that most often the students have difficulties in seeing that the variables stand for some values/numbers when they are rewriting a formula.

In this interview task the teacher said that it is easier if the formula is written with one common division sign, and he also said that to rewrite the formula to one common division sign is not obvious for all students. This mathematics teacher showed carefully how to rewrite the formula to one common sign of division. In this task he rewrote $\rho$ to $\rho/1$, to show how to multiply a constant to a fraction and then he multiplied $\rho/1$ with $L/A$. He said that the formula is easier with one common division sign, because it is easier for the students to see how to rearrange the formula and move the variable $A$. After he finished the task he said that the new letter $\rho$ also may be a difficulty for some students, and he compared it with exchanging the letter $x$ in an equation to another letter, that in his experience, some students may think is more difficult.

Analysis

The teacher used algebra in rearranging the formula to be able to calculate the length of the conductor, and that is categorized as general mathematical knowledge in this detailed analysis. The teacher also explained how to rearrange a multiplication of a constant to a fraction, categorized as general mathematics. He did not refer to any electrical knowledge in this explanation.

Electricity teacher E1

This teacher started his explanation with a real world story about a trailer in the snow with an electrical cable connected to it. He said that of course there is a resistance in a copper cable, and that he uses to illustrate this with a real world story of a trailer on a winter camping with an extension cord. He said that after some days the cable has melted down two to three centimeters in the ice and that is because we have used 2000W through 0.75’s cable and it will be warm. He continued his story reasoning that if there has been heat generated there has to be a resistance in this cable to generate this effect, which is this heating. After this he calculated the task, described below:
This teacher started his explanation of the task situating it in a real world context. In solving the task he also connected the calculations to the real world context, when he tested the formula with a length to see what kind of magnitude the answer can have.

"Test with for example 10 meters, which is very little, to get an inkling, think practically, what seems to be likely." (Ét 20:00)

The teacher did not seem to be totally sure of how to solve this task, but he had a solid understanding of the electricity in the task and he had methods to overcome his weakness in the calculation. He first tested the formula with an invented value for L and then he studied an easier example to remember how to do. When he tested the formula with the invented length 10m, he did not calculate the resistance for this length; he only said that this resistance is far too small, it is less than 1 ohm, compared to 1.2 ohm as it should be in the task. When he rearranged the formula to get a formula for the length, he said he uses to discuss with the students how to do it.
Analysis

The teacher solved the task by inserting the given values and solving the equation that he got, this is categorized as specific mathematical knowledge in this detailed analysis, as this is a calculation of the requested length. Also his testing of the formula with an invented value for the cable’s length is categorized as specific mathematical knowledge, as it is a concrete way of testing the magnitude of the expected answer. The teacher used electrical knowledge when he introduced the task with the trailer example to explain that a cable has an inner resistance.

Electricity teacher E2a

This electricity teacher said that in tasks like this the students should be able to rearrange the formula to be able to calculate any of the involved variables. The teacher showed a way of rewriting the formula so that it is easy for the students to rearrange it to get one unknown variable. He rewrote the formula so it had two variables on each side of the equal sign and no division sign and said that when the formula is written like this, it is easier for the students to rearrange it.

\[
\begin{align*}
\text{Teacher wrote:} & \quad R_L = \frac{\rho \cdot L}{A} \\
& \quad A \cdot R_L = \frac{\rho \cdot L}{A} \\
& \quad \frac{A \cdot R_L}{\rho} = \frac{\rho \cdot L}{\rho} \\
& \quad \frac{A \cdot R_L}{\rho} = L \\
& \quad 1.5 \cdot 1.2 = L \\
\end{align*}
\]

\[
\begin{align*}
\text{Teacher did:} & \quad \text{The teacher said that in tasks like this he usually rewrite the formula so it has no division sign. He multiplied with } A \text{ on both sides of the equal sign and cancelled } A/A. \\
& \quad \text{He said that in this task it is the length that is missing, so he divided with } \rho \text{ on both sides of the equal sign and cancelled } \rho/\rho. \\
& \quad \text{He inserted the given values and calculated the length of the conductor.}
\end{align*}
\]

This teacher said that it is easier for the students to resolve one of the variables in this formula if the formula is written: \( A \cdot R_L = \rho \cdot L \). This formula could now be used as a starting-formula that is easy to rearrange to get any of the involved parameters. He said that with this formula it is easy to resolve one unknown variable; you divide with the other variable on that side of the equal sign, and you get the unknown variable left alone on that side of the equal sign. For this specific formula it works well to do like this. He also
said another way of explaining this to students could be to insert the given values into the formula and then solve this equation. He said that students, who say that they do not understand this with variables, may understand if you insert the given values into the formula before you rearrange it.

**Analysis**

The teacher rearranged the formula algebraically, categorized as general mathematical knowledge. But here the teacher rearranged the formula in a specific way with two variables on each side of the equal sign, this specific solution method is only valid for this specific formula and it is not a general solution, so this may also be categorized as specific mathematical knowledge. The teacher did not use any electrical knowledge in this explanation.

**Electricity teacher 2b**

This electricity teacher started to rearrange the given formula algebraically, but before he calculated the in the task requested length of the conductor, he discussed practical electricity issues connected to this task. He discussed the difference between the length of a cable and a conductor.

The rearrangement of the formula was done like this:

\[
\begin{align*}
\text{Teacher wrote:} & \quad R &= \rho \cdot \frac{L}{A} \\
& \quad A \cdot R &= \rho \cdot L \\
& \quad \frac{A \cdot R}{\rho} &= L \\
\text{Teacher did:} & \quad \text{He wrote the formula for calculating the length of a conductor.}
\end{align*}
\]

Then the teacher pointed out how important it is to distinguish between the length of a cable and the length of a conductor in tasks like this. The resistivity formula is used in calculations of the length of a conductor and a cable usually includes at least two conductors so the length of the cable and the conductor could sometimes be incorrectly mixed. The teacher said that in tasks like this, it is important to know if it is the length of the cable or the length of the conductor that is asked for, as this is not the same thing. To explain this he made a picture of an electrical circuit with a load, a resistance denoted \( R \). To and from the resistance \( R \) he made conductors and illustrated the conductors’ inner resistance with long, thin resistances, denoted \( R_{L1} \) and \( R_{L2} \).
The teacher explained that the task is to calculate the resistance in one conductor, and he pointed out that a cable usually contains two conductors, with inner resistances both of them. So one conductor in a cable could be used to transport the current to the load in the circuit and another conductor in the same cable could be used to transport the current from the load. The length of the cable is not the same as the length of the conductors; the length of the conductors in his picture is twice as long as the cable length.

After that he calculated the length of the conductor in the interview task, by inserting the given values into the rearranged formula.

Teacher wrote: \[ L = \frac{1.5 \cdot 1.2}{0.0175} \]

Teacher did: The teacher inserted the given values into the rearranged formula, and calculated the length of the conductor.

\[ L = 102.86m \]

Analysis

This electricity teacher used general mathematical knowledge when he rearranged the formula algebraically. The teacher used electrical knowledge when he pointed out that a cable usually includes two conductors, and that you have to have that in mind when you ask and answer a question like this. This is a kind of vocational electrical knowledge or practical electrical knowledge of how electricity works in a laboratory or real world setting. The teacher used general mathematical knowledge to solve this task and electrical knowledge when he highlighted practical electrical issues relevant for the task.
Comparison of the teachers’ explanations

In this detailed analysis, there are differences both between the teachers’ use of mathematical knowledge in their calculations and differences in the teachers’ choices of explanations, see table 6. The mathematical knowledge used in the calculations was both of specific and general type. Two electricity teachers chose to, in addition to calculate the task, give explanations of electrical properties of conductors in their explanations.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Knowledge</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Spec math</td>
<td>Inserted the given values into the formula to calculate the conductor’s length.</td>
</tr>
<tr>
<td>E1</td>
<td>Spec math</td>
<td>Tested the formula with an invented value for the length.</td>
</tr>
<tr>
<td>M4, E2a, E2b</td>
<td>Gene math</td>
<td>Rearranged the formula algebraically to calculate the conductor’s length.</td>
</tr>
<tr>
<td>E2a</td>
<td>Spec math</td>
<td>Rearranged the formula: ( A \cdot R_L = \rho \cdot L )</td>
</tr>
<tr>
<td>M4</td>
<td>Gene math</td>
<td>Multiplication of a constant and a fraction</td>
</tr>
<tr>
<td>E1, E2b</td>
<td>Electrical</td>
<td>Real word illustration of loss effect, conductor versus cable</td>
</tr>
</tbody>
</table>

Table 6 Comparisons of the teachers’ explanations in detailed analysis 2 of task 3.

The first electricity teacher (E1) used a practical way of calculating this task. He tested the formula with an invented length, before he inserted the given values and solved the equation. The mathematics teacher (M4) and the other two electricity teachers (E2a, E2b) used algebra to rearrange the formula before they inserted the given values and calculated the length of the conductor. The mathematics teacher rearranged the resistivity formula to get a new formula that described the relationship between the length and the rest of the variables. The electricity teacher E2a did also rearrange the resistivity formula, but he did not rearrange it to a formula for determining the length but a formula that described the relation of pairs of the involved parameters. This way is not a common way to present mathematical relations in mathematics education. The electricity teacher believed that this way of presenting the formula could help students find a formula to calculate any of the involved parameters.

Two electricity teachers (E1, E2b) also chose to explain electrical properties of the task. The first electricity teacher explained resistivity in conductors with a real world example of a trailer in snow connected with a wire. The last electricity teacher explained the difference between a conductor and a cable.
### 4.4 Summary of results

Eight detailed analyses of explanations of the same topic given by mathematics and electricity teachers were described. These detailed analyses were three explanations from task 1, three explanations from task 2 and two explanations from task 3. Table 7 gives an overview of all the detailed analyses. Most of them, all but two, were explanations categorized as SMK in mathematics in the overview analysis.

<table>
<thead>
<tr>
<th>Task 1 Description of the explanation</th>
<th>Over-view cat</th>
<th>Teacher</th>
<th>Gene math</th>
<th>Spec math</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation of I in the circuit</td>
<td>SMK ma</td>
<td>M1</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rounding of the answer</td>
<td>SMK ma</td>
<td>M4</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect calculation</td>
<td>SMK ma</td>
<td>M4</td>
<td>x x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E4</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E3</td>
<td>x</td>
<td>x</td>
<td>x x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task 2 Description of the explanation</th>
<th>Over-view cat</th>
<th>Teacher</th>
<th>Gene math</th>
<th>Spec math</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation of the total resistance</td>
<td>SMK ma</td>
<td>M2</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E2b</td>
<td>x</td>
<td>x</td>
<td>x x x</td>
</tr>
<tr>
<td>Easier example</td>
<td>PCK ma</td>
<td>M1</td>
<td></td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E2a</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Origin of formula</td>
<td>PCK el</td>
<td>M4</td>
<td>x x x</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E3</td>
<td>x x</td>
<td>x x</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task 3 Description of the explanation</th>
<th>Over-view cat</th>
<th>Teacher</th>
<th>Gene math</th>
<th>Spec math</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning resistivity</td>
<td>SMK ma</td>
<td>M1</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E3</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculation of the conductors length</td>
<td>SMK ma</td>
<td>M4</td>
<td>x x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E1</td>
<td>x x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E2a</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E2b</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

Table 7 Summary of the detailed analyses presented in the Result chapter.
Results

The reason for choosing most SMK mathematics explanations is that the starting point in this research was differences in the use of mathematics in the teachers’ explanations, so it was also here that the most interesting differences were found. Besides the six detailed analyses of explanations in SMK in mathematics was one explanation each analyzed in the categories: PCK in mathematics and PCK in electricity.

The teachers’ use of specific and general mathematical knowledge and also electrical knowledge in their explanations is reported in the last three columns in table 7, with an “x” for each different explanation. Some of the teachers chose to explain electricity properties in addition to their mathematical explanations and in these cases that is noted in the last column in table 7. The details of all these explanations are found in the result chapter for each task (4.1, 4.2 and 4.3).

Table 7 shows that the mathematics teachers used general mathematics in all their explanations except in the explanation in PCK in mathematics with an easier example. The electricity teachers mainly used specific mathematics in their explanations and in some cases electrical knowledge. Table 7 also shows that it is mainly the electricity teachers that connect their mathematical explanations to electricity when they in these explanations of SMK in mathematics also explain electricity.

How the teachers are represented in the detailed analyses is reported in table 8. Mathematics teacher M2, teaching in mathematics and biology, have no formal education in electricity or physics and had no experience in teaching tasks like this and is therefore not represented more than once in these detailed analyses, as his explanations were not so varied and rich.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>M1</th>
<th>M2</th>
<th>M4</th>
<th>E1</th>
<th>E2a</th>
<th>E2b</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of detailed analyses</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Total per teacher group</td>
<td>Mathematics teachers: 8 detailed analyses</td>
<td>Electricity teachers: 11 detailed analyses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8 The number of detailed analyses for each teacher in this study.
Discussion

5 Discussion

In this chapter I will return to the research questions and I will summarize and discuss the themes discovered in the previous chapter. After that some connections to previous research will be made and implications for future research will be discussed.

5.1 Back to the research questions

5.1.1 Research question 1

**RQ1:** What teacher knowledge in mathematics and electricity do mathematics teachers and electricity teachers in the Electricity Program draw upon to explain mathematical electricity tasks?

To be able to answer the first research question, teacher knowledge, in this study limited to subject matter knowledge (SMK) and pedagogical content knowledge (PCK), in both mathematics and in electricity, was used for a general overview of the main ideas in the teachers’ explanations. The teachers chose different explanations, electricity or mathematical explanations, as both knowledge bases are involved in these tasks. This study reveals that teachers from both teacher groups drew upon both mathematical and electrical knowledge when they explained these interview tasks. Both mathematics and electricity teachers, used explanations from all the categories; SMK and PCK in mathematics and SMK and PCK in electricity. The mathematics teachers that also teach physics are using electrical knowledge, but the mathematics teacher who is also teaching biology is not using electrical knowledge. The categorization in SMK and PCK in both mathematics and electricity in the overview analyses proved to be one way of organizing and studying the different teacher groups’ knowledge.

To summarize the three interview task, there are several similarities between the two teacher groups, as there are examples of both mathematics and electricity teachers who do mention most of the explanations. However, there are some kinds of explanations that one of the teacher groups mentions and not the other teacher group and these explanations will be described below. There are also differences between the individual teachers that are not analyzed in this study.

The overview analyses of the three interview tasks indicate that the mathematics teachers used some instances of mathematical knowledge in
Discussion

their explanations that the electricity teachers did not mention in this study. In SMK in mathematics there are explanations given only by one mathematics teacher (M4) in task 1 and 2. This mathematics teacher used proportional reasoning in task 1 and said that rounding too early in task 2 may affect the precision of the answer. No electricity teacher mentioned proportions at all and when the electricity teachers did talk about rounding, they referred to how rounding is done in electricity work.

In PCK in mathematics the mathematics teachers had some knowledge in the subcategory knowledge of mathematics and the students that no electricity teacher mentioned. In task 1, one mathematics teacher (M4) said that some students can have difficulties in rearranging and simplifying a formula; \( \frac{R_1}{R} = \frac{R}{R} \cdot l = 1 \cdot R = R \), and he compared this with fraction calculations, that in his experience students may have difficulties with. In task 2, one mathematics teacher, M1, said that solving this task with the given resistances is far too difficult for students using fractions. These mathematics teachers had experience of mathematics and students that no electricity teacher mentioned.

These overview analyses also indicate that the electricity teachers connected their explanations to practical electricity work, and no mathematics teacher did that in this study. The electricity teachers used knowledge in SMK in electricity that the mathematics teachers did not mention. In the overview pictures, this knowledge is called Electrician knowledge, as it has to do with practical electricity work. In task 1, two electricity teachers talked about the loss effect in conductors. One of the teachers (E1) talked about how you can feel conductors getting warm in a laboratory setting and the other electricity teacher (E3) reasoned and calculated an example of loss effects in high tension lines. This teacher compared the use of high or low voltage in a conductor and showed how the loss effect in the conductor was reduced by using higher voltage. In task 2, three electricity teachers mentioned an alternative formula for the total resistance in a parallel circuit, an alternative formula when the parallel resistances are equal. In the last interview task, two electricity teachers highlighted that in tasks like this, with electrical resistivity, it is important to know if it is a cable or a conductor’s length that is asked for, as most often a cable hold two or more conductors.

The electricity teachers also used knowledge in PCK in both mathematics and in electricity that the mathematics teachers did not mention. The electricity teachers showed different experience with students common mistakes on these tasks (e.g. that students use to forget to invert in the end of their calculations of the total resistance in a parallel circuit), maybe because
the electricity teachers have used tasks like this in their teaching several years, while the mathematics teachers might not have used these tasks very much in their teaching. The electricity teachers also used more practical way of explaining the task, for example when they inserted an invented value into a formula to test the formulas.

In answering research question 2 detailed analyses were done, and they are discussed below.

5.1.2 Research question 2

RQ2: Are there characteristic similarities and differences in the knowledge that the teachers draw upon to explain these mathematical electricity tasks?

At the overview level, with the teachers’ explanations categorized in subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in both mathematics and in electricity, it appears that the two teacher groups’ explanations are mostly similar. But the overview did not display all details in the teachers’ explanations and therefore detailed analyses of some explanations given by both mathematics and electricity teachers were done. The explanations that were chosen came mostly from the category SMK in mathematics, but some of the teachers connected their explanations with electrical knowledge, so that is also included in these detailed analyses. The detailed analyses of explanations of a specific topic were done by studying what explanations and what kind of mathematical knowledge the teachers drew upon. The teachers’ use of mathematics in their explanations was categorized as general mathematical knowledge when they involved explanations for more general mathematics, whereas specific mathematical knowledge involved explanations specific for this task. In the detailed analyses was also the electrical knowledge the teachers connected to their explanations of the tasks studied.

There are some things that stand out as different between the mathematics and the electricity teachers’ explanations in these detailed analyses. The first thing is that the teachers’ drew upon different kinds of mathematics. The mathematics teachers drew upon general mathematical knowledge in their explanations and the electricity teachers mostly drew upon specific mathematical knowledge in their explanations. The summary of the detailed analyses reveals that the mathematics teachers drew upon general mathematics in all their explanations that were categorized as SMK in mathematics in the overview. It is only in the detailed analysis of PCK in mathematics, with an easier fraction example to explain how to calculate the total resistance in a parallel circuit, where the mathematics teacher did not
draw upon general mathematics knowledge. That seems to be natural as this explanation is given to concretize a general formula. In all the other detailed analyses the mathematics teachers drew upon general mathematical knowledge, i.e. algebra, although some of the mathematics teachers said that if the students do not understand their explanation they might have explained with concrete numbers. These mathematics teachers said that they would have started with the more general way. The electricity teachers did not draw upon general mathematics as frequent as the mathematics teachers did. In the detailed analyses of the three interview tasks there are some examples of electricity teachers using general mathematics.

5.2 Connections to research

The pattern of mathematics teachers using more general mathematics and electricity teachers using more specific mathematics is not very surprising and can be derived from the different curriculums that guide the mathematics and the electricity courses. The curriculum for the electricity subject and the electricity courses states that to be able to make correct calculations is a prerequisite to exercise the profession, so the electricity education should therefore develop the students’ mathematical knowledge (Sverige, 2011). The mathematics subject’s curriculum states that mathematics is a tool within science and for different vocations, and that mathematics furthermore deals with the discovery of patterns and formulation of general relations (Sverige, 2011).

The mathematics teachers explained the mathematics in not only these tasks but the general mathematics, so that the students get a chance to learn mathematics and may be able to use mathematics in similar tasks in other contexts. The electricity teachers used mathematics in the tasks to explain the electricity subject to the students. For example in the explanation where one mathematics and one electricity teacher both used general mathematics knowledge, merging Ohm’s law and the effect law in task 1, they did that to be able to explain different things, mathematics or electricity. The mathematics teacher explained how to merge formulas and the electricity teacher explained why high voltage reduces the loss effect in high tension lines. It seems that the mathematics teachers choose to explain the mathematics in the tasks and used the electricity context to show how mathematics works and the electricity teachers used mathematics to explain electricity. The two teacher groups used mathematics in different ways, for the electricity teachers mathematics is only needed if it adds value to and can predict the vocational situation (Gillespie, 2000; Straesser, 2007). In vocational didactics, mathematics is only needed if it is useful in the vocational courses (Johansson, 2009).
The mathematics teachers explained general mathematics and the electricity teachers explained electricity. In the explanations of the same task the mathematics teacher had the mathematics in the foreground and electricity in the background and the vice versa for the electricity teachers, who had electricity in the foreground and mathematics in the background. Although all the teachers in this study did solve the interview tasks, they do send different message to the students about how to reason in tasks like this, which of course may be an obstacle for the students in learning mathematics.

Other research studies have shown that general mathematics is not part of workplace mathematics which may also be a reason in this study. For example, mathematical generalizations are discussed in workplace mathematics:

“From a mathematical point of view, efficiency is usually associated with a general method that can then be flexibly applied to a wide variety of problems [...] The crucial point is that orientations such as generalizability and abstraction away from the workplace are not part of the mathematics with which practitioners work.” (Noss, Hoyles, & Pozzi, 2000). p.32

The electricity teachers in this study used specific mathematical and practical electrical knowledge, sufficient for their explanations and also this is discussed in workplace mathematics research:

“For the practitioners in our studies, the computational and estimation methods of routine activity – in all there many forms – were more than adequate for their purposes.” (Noss et al., 2000) p.32

This study contributes to the question of the role mathematics could play in vocational education. Lindberg and Grevholm (L. Lindberg & Grevholm, 2011) showed that collaboration between mathematics and vocational subject could have positive effect on students’ learning of mathematics, and this study further highlights aspects that needs to be taken into consideration in collaborations between mathematics and other subjects. Even when the mathematics content is the same, as in this study, it could be presented differently by different teachers, and this could very well be an obstacle for the students in their efforts to understand the mathematics in the Electricity Program. This raises questions if it is reasonable to assume that students should be able to reconcile these two different mathematical approaches.

5.3 Implications

So what can be learned from this study? Mathematics contains a wide range of subject areas but also a wide range of representations and methods that
highlight different aspects of mathematics. This study shows that different teachers emphasize different mathematics in their explanations of the same tasks even though intended to the same students, both in the teachers’ choices of explanation and in the teachers’ use of mathematics. Contrary to expectation, electricity teachers had a lot of knowledge to draw upon to explain mathematical tasks, even though their mathematics was not always very strong. The electrical knowledge they used not only grounded the tasks in a, for them well-known real world environment, but the electrical knowledge actually helped them solve the tasks (albeit in a more concrete/specific way than for the mathematics teachers.) The differences between electricity and mathematics teachers do not show one approach to be necessarily better than the other, but the subject matter has a distinctly different character when done by these different types of teachers. This thesis provides specific data which could help teachers understand how their students experience the subject matter in the different courses.

Mathematics was presented in different ways by the different teachers, even when used in the same tasks. In this study the differences in the teachers’ explanations have been analyzed in the light of teacher knowledge. But the differences could also depend on other things, like for example the teachers’ goals and orientations. The tasks in this study were used differently by the different teachers. The mathematics teachers used the electricity context to explain mathematics and the electricity teachers used mathematics to explain electricity. The electricity context was used to explain the mathematics of formulas and equations, for example when reasoning about a formula. Mathematics was used to explain electricity practice, for example why energy is transferred in high voltage in high tension lines.

5.3.1 Lessons for research

There are many different ways of teaching mathematics and mathematics is used differently by different kinds of teachers. In this study, electricity teachers, not educated in mathematics, turned out to have mathematical knowledge both similar and different compared to mathematics teachers. The tasks in this study involve different subject matter knowledge, both mathematics and electrical knowledge. In explaining the same interview tasks different teachers are choosing different kinds of explanations and using different kinds of knowledge, both mathematical and electrical knowledge and both SMK and PCK. An overview picture with all teachers’ explanations divided in SMK and PCK in mathematics and in electricity proved to be one way of studying all teachers’ explanations in this study. But this overview picture could not discern all differences in the details of the teachers’ explanations. Detailed studies of some teachers’ specific
mathematical explanation of the same topic indicate that the teachers’ use of mathematics could be substantially different, although categorized in the same teacher knowledge category. This could probably be valid for all kinds of frameworks for mathematical knowledge for teaching. To be able to discern all aspects of this knowledge it is not enough to use broad categories of teacher knowledge, detailed analyses of the teachers’ use of mathematics are also needed (Ma, 1999). Probably there are different levels of understanding the content knowledge in the different categories and different ways of analyzing that.

The teachers’ use of mathematics in their explanations was in this study analyzed in their use of specific or general mathematical knowledge. Specific mathematical knowledge involved calculations of the specific task, and general mathematical knowledge involved a general explanation of the type of task in addition to calculate the task. The mathematics teacher used mostly general mathematical knowledge in their explanations and the electricity teachers used mostly specific mathematical knowledge in their explanations. This is interpreted as the two teacher groups are using the same tasks but with different purposes. The mathematics teachers are using the tasks to explain mathematics and the electricity teachers are explaining electricity with the help of mathematics.

5.3.2 Lessons for teachers

Mathematics is explained differently in the mathematics and the electricity classroom; both the teachers’ choices of what to explain and the teachers’ use of mathematics showed to be different in the two classrooms. The distinction between SMK and PCK may be a way of motivate and inspire teachers in their in-service development as teachers. With this distinction it could be easier for teachers with different educational backgrounds to reflect and discuss their teaching practices with each other. Teacher knowledge is not only dependent on the teachers’ educational backgrounds; the teachers in this study have a lot of knowledge that could be useful for the involved teachers. Electricity teachers without university studies in mathematics may have teacher knowledge in mathematics that mathematics teachers could learn from. And electricity teachers could probably learn from mathematics teachers about different mathematical ways of solving electricity task that could help them in explaining the mathematics in the electricity courses in more varied ways. Teachers from different backgrounds could probably expand their teacher knowledge by collaboration with each other, both within the teacher group and also between different teacher groups, as in this study mathematics teachers and vocational teachers. Niss (2007)
discussed teacher collaboration as an important source of teacher competencies and teacher development:

“And we know that although teachers’ initial preparation does matter a lot for the ways in which they are initiated to their professional career, continual professional development focused on the teaching and learning of mathematics as an integrated entity, and the building of a professional identity in collaboration with different sorts of colleagues – peers, mentors, researchers – are even more critical to the way and extent to which they can develop as teachers who can help students learn mathematics to the best of their capacities.”(Niss, 2007, p.1305)

5.3.3 Further research

This study has only started to investigate teachers’ different mathematics teaching practices and several new interesting research areas could continue from this study. Continuing studies could include teachers, students and new subjects. In this study the difference in teacher knowledge was studied, but there are certainly other reasons for differences in teachers’ explanations. Teachers have different goals and orientations and there are different teaching cultures in different educations and these aspects could also be explored. More studies of reasons of why teachers have different explanations would be highly interesting. This study also opens up for the fact that also other kinds of teachers than mathematics teachers use mathematics in their teaching. Furthermore, different kinds of teaching experiments could be developed from this study, aiming at develop teaching practices. Possible continuing research could include students and students’ experience of different explanations and students understanding of different explanations. There are certainly other ways, than the distinction of specific and general mathematical knowledge that was used in this study, which could be used in analyses of different levels of content knowledge and probably could these be used in the electricity subject as well.
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