The Stick or the Carrot?
Modeling Reference Price Dependence and Loss Aversion in an Environmental Policy setting

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June 13, 2012

Abstract

This paper concerns how loss averse consumers react to positive and negative price changes. A growing empirical literature suggest that price increases relative to a reference price, defined as what the consumer is used to pay for the good, have more effect on demand than same-sized price decreases, implying an asymmetry in price elasticities. However, little has yet been done in terms of theoretical modeling. Using standard economic framework but incorporating loss aversion, we develop a model where the consumer is loss averse for price changes relative to the reference price. We then use numerical simulations to illustrate the implications for the policy maker trying to change the consumption pattern. Our results suggest that the policy maker’s choice between imposing taxes or same-sized subsidies should depend on the position of the pre-policy prices relative to the consumer’s reference price.

Keywords: Reference price, loss aversion, taxes.

Introduction

Policy makers are trying to change consumption patterns towards a more sustainable one, mixing taxes on externality-generating activities and subsidies on the environmental friendly alternatives. Take car driving as an example. Some policy makers propose taxes on gasoline, whereas others suggest subsidizing sustainable fuels such as bio-fuel or electricity. However, there seems to be little concern on how consumers actually react to these different policies from a behavioral perspective. Does the increase in price due to a tax only affect the consumer via his budget restriction, or does the higher price also imply less utility from the consumption of the good? How does this affect the demand for the good? And, does the change in demand from a tax differ in magnitude

∗The author would like to thank supervisor Tomas Sjögren for great inputs. Also thanks to participants at thesis seminar for valuable comments.
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from the change in demand from a subsidy of equal size? If there is a difference between the two, should we then use the stick or the carrot in order to make consumers switch consumption pattern?

Interest in behavioral economics has grown in recent years, stimulated largely by the accumulating evidence that standard models of consumer decision making is sometimes inadequate in describing human behavior. Numerous empirical studies over the last four decades reveal that rational choice might, in some circumstances, be a poor guide for modelling human behavior. By departing from this assumption, proponents of behavioral economics argue that we can obtain a deeper understand how the consumer acts, and the practical and theoretical relevance of a behavioral perspective on consumption is growing in influence.

In behavioral economics, many empirical and experimental studies have found an asymmetry in individuals reaction to gains and losses. This behavior is most often explained by the concept of loss aversion, and refers to peoples’ tendency to prefer avoiding losses than acquiring same-sized gains. As formulated by Kahneman and Tversky (1979) in their often-cited Prospect Theory\(^1\), the basic idea is that utility is defined for changes; gains and losses relative to a reference point, and that losses relative to this reference point loom larger than gains. They define the value function as

\[
v(x) = \begin{cases} 
(x - r)\beta & \text{if } x \geq r \\
-\lambda(r - x)\beta & \text{if } x < r
\end{cases}
\]

where \(r\) is the reference point, \(x\) is final wealth and \(\lambda\) and \(\beta\) are the loss aversion parameters. By defining \(\beta\) to be less than one, the changes in utility are characterized by diminishing sensitivity which means that the marginal utility of gains and losses decreases with their size relative to the reference point. This gives the value function its concave shape above the reference point and a convex shape below this reference point. Thus, the Prospect Theory value function is s-shaped with a steeper slope for losses than for gains.

\(^1\)Kahneman and Tversky (1992) is an extension of their original theory. See also Schoemaker (1982) for a review of the differences between Prospect Theory and other theories on risky choices.
Figure 1: The Prospect theory value function. The steeper slope for losses than for gains imply loss aversion.

Although Prospect Theory originally was formulated as a theory on risky choices\(^2\), the idea has been applied to certain outcomes as well. For example, Kahneman and Tversky (1991) model consumption behavior given reference dependence and its implications. Also relevant is Thaler (1980) who proposes a positive theory of consumer choice, based in part on Prospect Theory.

In marketing science, there is a growing literature that tries to capture psychological aspects of price perception. Relevant to this paper, there are some empirical support that the consumer, at least in part, compares the observed price of a frequently purchased good\(^3\) to what is called his reference price, which often is assumed to be what the consumer is used to pay for the good\(^4\). See for example Winer (1986), Briesch et. al. (1997) and Mazumdar et. al. (2005). Deviations of the observed price from this reference price is then incorporated into the consumers decision problem. If the observed price is above the reference price, empirical findings suggest that the consumer purchases less quantity of the good than if the same observed price equals the reference price. Failure to incorporate these reference price effects can then result in a bias of price elasticity and thus lead to non-optimal pricing.

Further, some of the empirical support for reference price effects also suggest that the asymmetry caused by loss aversion also holds for negative and positive price changes, where a price increase is a loss to the consumer and a price de-


\(^3\)It is often argued that these effects are strongest for frequently purchased goods where the consumer has a clear picture of what the good usually cost.

\(^4\)Kőszegi and Rabin (2006) define the reference point as being the consumers expectations about future outcomes. Using their definition, the reference price can then be seen as the expected price. There are other definitions of reference prices as well. For example, Xia et. al. (2004) define this to be what the consumer perceive to be a fair price.
crease is a gain. For example, Putler (1992) finds asymmetric price elasticities in the purchase of eggs and shows that price increases have almost two and a half times the effect of demand compared to a price decrease. Hardie et. al. (1993) performed a similar study on the purchase of orange juice and found almost identical results. Also, Kalwani et. al. (1990) found empirical support for asymmetric price elasticities for coffee. Uhl and Brown (1971) conducted a survey in which customers were asked to indicate how they would respond to price increases and decreases of 5, 10, and 15 percent for food. The authors report that customers were considerably more sensitive to price increases than to decreases. Further, Dawes (2004) found that consumers had a higher propensity to change car insurance if the price of their current insurance increased, compared to when the price of the alternative decreased. Reference dependence in pricing context has also been suggested by Thaler (1985) and Camerer (2000).

If the consumers' behavior exhibits both reference price effects and loss aversion, this will affect how different policies are perceived by the consumer, and hence affect the outcome of the policy. Although there are some empirical work on reference price effects and loss aversion, little if any work has been done on how to theoretically model this behavior. Further, to our knowledge there exist no previous literature on reference price effects and loss aversion in an environmental policy context.

In this thesis we ask the question whether the consumer is more prone to change consumption pattern depending on whether the policy maker taxes the bad good or impose a same-sized subsidy on the environmental friendly good. By incorporating reference price effects and loss aversion into consumer analysis, we try to answer this question from a behavioral point of view. If taxes have more impact on the demand compared to a same-sized subsidy, it seems likely that it is more effective to impose a tax on the externality-generating good than to subsidize the sustainable alternative. Further, we test how this result depends on the position of the reference price relative to the pre-policy observed price. To answer these questions, we first develop a theoretical model including gains and loss effects. We then use numerical simulations to test how the demand for two goods are affected by taxes and subsidies, and how the change in demand depend on the reference price. Thus, the purpose of this paper is two-fold; we both construct a general theoretical model of consumption behavior with reference dependence in prices and loss aversion, and also analyze how this behavior affects the policy makers choice between subsidies and taxes.

The outline of this paper is as follows. In section two we present the basic model describing the consumers decision problem including reference price effects and loss aversion. In section three, we analyze this model using numerical simulations to see what parameter values are most reasonable. In this section, we also discuss the policy maker's problem. The paper is concluded with a
discussion in section four.

The Model

The consumer receives utility from an activity $R$, which is exemplified by car driving. Fuel is used as input for the activity $R$, and the consumer can choose between an environmentally-friendly input factor $x_e$; for example electricity, and an input factor that is bad for the environment $x_g$; for example gasoline,

\[ u = u(R(x_e, x_g)) \]  

Further, the consumer is loss averse relative to a reference price $p_i$, where $i = e, g$. The reference price $p_i$, defined as the latest paid price for the good, is exogenously given by the time of the choice, and deviations from this reference price are perceived as gains and losses by the consumer. These gains and losses enter the consumer’s utility function but not the budget constraint. $p_i$ is in this paper denoted as the observed price, since this is the price the consumer observes when purchasing the two goods. Further, the observed price is assumed to be the same for all sellers. Hence, in case of price increases the consumer does not bother to find a better price somewhere else.

We define the gain term in utility due to price decreases as

\[ G_i = \theta \mu_1 ((p_i - p_i) x_i)^{\beta_1} \]  

and loss in utility due to price increases enter the utility as

\[ L_i = (1 - \theta) \mu_2 ((p_i - p_i) x_i)^{\beta_2} \]

where $0 < \beta_1, \beta_2 < 1$ and

\[ \theta = \begin{cases} 1 & \text{if } p_i < p_i \\ 0 & \text{otherwise} \end{cases} \]

$\beta_1$ and $\beta_2$ being less than one imply diminishing sensitivity to gains and losses in line with previous literature, for example Kahneman and Tversky (1979). Further, $\mu_1$ and $\mu_2$ define how much gains and losses matters to the consumer, relative to the utility of consumption. For loss aversion to hold, we assume that $\mu_2 > 1$. More attention to these gains and loss parameters are given later in this paper when simulating the model. If the price is below the reference price, $\theta$ takes a value of one and hence only the gain term (3) is relevant. On the other hand, if the price is above the reference price then $\theta$ is equal to zero and only the loss term (4) matters. It is assumed that it is net gains and losses

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7For example, we can assume that the consumer uses a hybrid car.

8It seems reasonable that most consumers have a clear picture of what they normally pay for fuel. Hence, that reference price effects influence decisions is in this case a reasonable assumption. Further, we argue that the same reasoning would hold for other types of frequently purchased goods, such as electricity.
that matters to the consumer. Hence, the deviations in prices are multiplied by the consumed quantity. This is in line with other reference dependent models of consumer behavior, for example Putler (1992) and Kahneman and Tversky (1991a).

One should note that Prospect Theory define the reference point as the status quo, whereas the previous mentioned literature on reference prices and also this paper defines the reference point as being some previous outcome, for example prices paid earlier by the consumer. Further, Prospect Theory is defined only for gains and losses, whereas we in this paper follow Putler (1992), Köszegi and Rabin (2006) and Sugden (2003) in defining the utility both for absolute levels of consumption as well as gains and losses. For the time being assuming a concave functional form for the utility of consumption $u$, the consumer’s utility function can then be written as

$$U = u(R(x_g, x_e)) + G_i - L_i$$

where it follows that the consumer’s utility is concave in price decreases and convex in price increases. The consumers budget restriction is defined as

$$m = p_g x_g + p_e x_e$$

where $p_g$ and $p_e$ is the price of $x_g$ and $x_e$ and $m$ is the income spent on the activity $R$.

The consumer chooses $x_g$ and $x_e$ to maximize (6) in every time period, subject to (7). The Lagrangian is then defined as

$$L = u(R(x_g, x_e)) + G_i - L_i + \lambda (m - p_g x_g - p_e x_e)$$

with the first order conditions

$$\frac{\partial u}{\partial R} \frac{\partial R}{\partial x_g} + \beta \theta \mu_1 (p_g - \bar{p}_g)^{\beta_1} x_g^{\beta_1 - 1} + \beta (1 - \theta) \mu_2 (p_g - \bar{p}_g)^{\beta_2} x_g^{\beta_2 - 1} - \lambda p_g = 0$$

$$\frac{\partial u}{\partial R} \frac{\partial R}{\partial x_e} + \beta \theta \mu_1 (p_e - \bar{p}_e)^{\beta_1} x_e^{\beta_1 - 1} + \beta (1 - \theta) \mu_2 (p_e - \bar{p}_e)^{\beta_2} x_e^{\beta_2 - 1} - \lambda p_e = 0$$

$$m - p_g x_g - p_e x_e = 0$$

As can be seen in (9) and (10) the first order conditions are made up of four terms. If $\bar{p}_i = p_i$, the first order conditions reduces to only include the the first and last terms. If the price is less than the reference price, $\theta$ is equal to one and the third term vanish. On the other hand, if the price is above the reference price, then $\theta$ is equal to zero and the fourth term vanishes. It then follows that in case of deviations in price relative to the reference price, the marginal utility of consumption does not only depend on quantity of $x_i$ consumed, but also on whether the purchase is perceived as a gain or as a loss. That is, marginal utility

\footnote{see Köszegi and Rabin (2006) for a further discussion on reference points and status quo.}
of consumption consist of the first and the second or the third term, depending on whether the price has increased or decreased.

Given the non-linearity in utility, this implies an interior solution. The first order conditions (9), (10) and (11) then implicitly define the demand functions $x^*_e$ and $x^*_g$ such that

$$ x^*_e = x_e(p_e, p_g, \theta (p_e - p_e), (1 - \theta) (p_e - \bar{p}_e), m) $$  \hspace{1cm} (12) 

and

$$ x^*_g = x_g(p_g, p_e, \theta (p_g - p_g), (1 - \theta) (p_g - \bar{p}_g), m) $$  \hspace{1cm} (13) 

Throughout this paper, the demand function without gains and loss terms will be referred to as the standard demand function, whereas the demand function including these effects, as in (12) and (13) are referred to as the modified demand functions. At the price $p_i = \bar{p}_i$, the demand function reduces to the standard demand function since gains and losses only enters for deviations in price relative to the reference price. At the price equal to the reference price, the demand function is kinked. However, above and below this reference price, the slope of the demand function will differ. Previous empirical studies suggest that since price increases have more impact on utility than a same-sized price decrease, the effect on demand should be larger for price changes above the reference price. That is, we expect the derivatives of the demand functions with respect to the price to be

$$ \frac{\partial x^*_i}{\partial p_i} |_{p_i < \bar{p}_i} < \frac{\partial x^*_i}{\partial p_i} |_{p_i > \bar{p}_i} $$  \hspace{1cm} (14) 

where the term on the left is for price decreases and the right term is for price increases. This implies that the slope of the modified demand function is steeper for prices above the reference price than for prices below the reference price. Since we have assumed that $\mu_2 > \frac{\mu_1}{\mu_2} > 1$, we expect our model to exhibit the same kind of behavior.

However, since evaluating these derivatives involve two different Hessian determinants, it is rather difficult to analytically determine the difference between these two derivatives\textsuperscript{10}. When we later in this paper simulate the model we will be able to confirm whether this is in fact the case.

For the time being assuming that (14) holds, we can depict a hypothetical modified demand function as below

\textsuperscript{10}Comparing two derivatives with different Hessian determinants is somewhat complicated, but will be easier when numerically simulating the model.
Figure 2: Modified demand function with kink where the reference price equals the observed price.

which can be compared to the standard demand function below

Figure 3: Standard demand function

To the left and right of the kink in Figure 2, we expect an asymmetry in
price elasticities. Specifically, we expect that the demand is more elastic for price increases than for decreases. Thus, as long as we keep the reference price fixed, the change in demand due to a price increase is greater than the change in demand due to a price decrease.

Further, we expect an increase in the reference price will shift the demand function outward,

\[ \frac{\partial x^*_i}{\partial p_i} > 0 \]  

That is, if the reference price is higher, then the demand is higher for all observed prices. A common sales strategy is to have a discounted price displayed together with the higher suggested retail price. The idea is that the discounted price is supposed to seem even lower compared to the suggested retail price. Explained in terms of reference price effects, the purpose of this strategy is to increase the reference price and thereby shift the whole demand curve to the right, where the demand is higher for all observed prices.\(^{11}\)

**Simulations**

We now turn to the simulations part of this paper, where we numerically simulate the model derived in the previous section\(^ {12}\). The purpose of these simulations is as follows; first, we calculate simulated price elasticities for different parameter values to find reasonable values of these parameters. Secondly, we use simulations to test whether it is more efficient for the policy maker trying to decrease the ratio \( x_g / x_e \) to impose a tax on \( x_g \) than to impose a same-sized subsidy on \( x_e \). Thirdly, we test what happens when the reference price differs from the pre-policy price, and how the results are affected depending on whether the reference price is high or low relative to the pre-policy price. Finally, we also discuss what would happen if the policy maker could change the reference price instead of the observed price.

First, we need to make assumptions of the functional forms. Assuming a Cobb-Douglas functional form for the utility of consumption of \( x_g \) and \( x_e \), we have that

\[ u (R (x_g, x_e)) = x^\alpha_g x^{1-\alpha}_e \]  

where \( 0 < \alpha < 1 \). The consumers utility function, including gains and loss terms, can then be written as

\[ U = x^\alpha_e x_g^{1-\alpha} + \theta \mu_1 ((p_i - p_i) x_i)^\beta - (1 - \theta) \mu_2 ((p_i - p_i) x_i)^\beta \]  

where the budget constraint is as in (7). The first order conditions with respect to \( x_g \) and \( x_e \) are then

\[ (1 - \alpha) x^\alpha_e x_g^{1-\alpha} + \theta \beta \mu_1 (p_i - p_g)^\beta x_g^{\beta-1} \]
\[ + (1 - \theta) \beta \mu_2 (p_g - p_g)^\beta x_g^{\beta-1} - \lambda p_g = 0 \]  

\(^{11}\)See Thaler (1985) and Putler (1992).

\(^{12}\)The model is simulated using Maplesoft Maple software.
\[ \alpha x_e^{\alpha-1} x_g^{1-\alpha} + \theta \beta \mu_1 (p_e - p_c)^\beta x_e^{\beta-1} \\
+ (1 - \theta) \beta \mu_2 (p_c - \bar{p}_e)^\beta x_e^{\beta-1} - \lambda p_c = 0 \]  
\[ m - p_g x_g - p_e x_e = 0 \]

which implicitly define the demand functions for \( x_e \) and \( x_g \).

We will use the demand functions without any gains and loss effects as our benchmark, defined as

\[ x_e^S = \frac{\alpha m}{p_e} \]  
\[ x_g^S = \frac{(1 - \alpha) m}{p_g} \]

where the superscript \( S \) refers to the standard demand function. The demand including gains and loss terms is then referred to as the modified demand.

Now consider the exogenous variables and parameters. Since we are mainly interested in changes in demand and not in absolute levels of consumption, we choose arbitrary reference prices and income (initially \( p_g = p_e = 6 \) and \( m = 1000 \)) but follow the previous literature on the loss aversion parameters\(^\text{13}\). Further, we set \( \alpha = 0.5 \).

In previous empirical literature on loss aversion, there are some estimates of the magnitude of loss aversion. Most empirical studies have measured loss aversion as the ratio of price elasticities for negative and positive price changes. For example, Putler (1992) finds this ratio to be 2.4 for coffee, whereas Hardie et. al. (1993) find that the demand for orange juice is 1.5 times more elastic for price increases than for decreases. Kahneman and Tversky (1992) do not use elasticities to measure loss aversion, but instead a model somewhat similar to this paper. They estimate that for money gambles, monetary losses have 2.25 times more effect on utility than gains. These types of goods differ from the ones exemplified in this paper, gasoline and electricity, but since there exist no empirical estimates of the magnitude of loss aversion for the types of goods studied in this paper, we have to assume that the loss aversion for them are of similar magnitude. Of course, this implies that the results from the simulations might differ from reality and should be interpreted with care.

We test for different parameter values to obtain similar loss aversion effects as in the previous literature while at the same time having reasonable price elasticities, and also test how sensitive the model is to changes in these parameter values. The price elasticities are approximated by

\[ \frac{\partial x(p)}{\partial p} \frac{p}{x(p)} = \lim_{\Delta p \to 0} \frac{x(p + \Delta p) - x(p)}{\Delta p} \frac{p}{x(p)} \]

For the standard demand function, the concavity of the utility function imply a convex demand function. For infinite-small price changes, this demand

\(^{13}\)However, most empirical studies only estimates elasticities and not explicit parameter values.
function has a unitary price elasticity of $-1$. Further, as we increase the change in price to discrete changes, the standard demand function will be more elastic for price decreases than for increases. That is, even with no addition to the utility function in the form of gains and loss terms, price decreases have a higher effect on demand than price increases\textsuperscript{14}. Hence, with the standard demand function and for large price changes we have the opposite of loss aversion. As can be seen in Table 1, this changes when we introduce gains and loss terms \textsuperscript{15}.

Kahneman and Tversky (1992) estimate the loss aversion parameters in equation (1) to be $\beta = 0.88$ and $\lambda = 2.25$ for monetary gambles which provides a starting point when choosing loss aversion parameter values. However, one should bear in mind that their model differ from our model. For example, Kahneman and Tversky define their model in terms of utility of monetary gains and losses, whereas we define our model in terms utility of consumption. Further, they only include changes relative to the reference point, whereas we include both changes but also consumption levels. The simulated elasticities given these parameters are unfortunately somewhat unrealistic, rendering price elasticities of $-25$ for price increases and $-12$ for price decreases. That is, for relatively large parameter values, the slope of the modified demand function will be concave and rather steep to the immediate left of the kink, rendering unrealistic high elasticities.

Thus, we need to choose rather low values for the parameter values in order to have reasonable\textsuperscript{16} price elasticities for price decreases relative to the reference price. Unfortunately, this implies that the convexity of the standard demand function dominates the concavity of price decreases, rendering increasing sensitivity to price decreases for very large price decreases. Of course this could be remedied by choosing different parameter values, but again this results in unrealistic price elasticities. Hence, the concave utility function is somewhat limiting when modeling reference price effects, at least for gains relative the reference point. This has not been discussed in previous literature. However, the model is still able to provide some insights concerning the implications of these effects.

For prices above the reference price, the concavity of the utility function is less of a problem. We want the demand function to the right of the kink to be convex and below the standard demand function. This imply that the slope will be flatter as the magnitude of the price change increase and in turn yield a less elastic demand. Here, the convexity of the demand function imply diminishing

\textsuperscript{14}One can then interpret our model as what happens to a commonly assumed utility function when we include reference price effects and loss aversion. Hence, the concavity of the utility function does not necessarily limit the scope of loss aversion.

\textsuperscript{15}The often-assumed concavity of utility has not been discussed in previous loss aversion literature. First of all, most reference dependent models are only defined for gains and losses, whereas we specify the decision problem both as a function of absolute consumption levels and price changes. Secondly, since previous literature that do include both changes and absolute quantities (Putler 1992) have not used specific functional forms, this has not been more than a hypothetical problem.

\textsuperscript{16}For example, previous literature on demand for gasoline suggest price elasticities around -1 (see Dahl and Starmer (1991) for a review).
sensitivity to price increases, which is in line with previous reference dependent literature.

Below we summarize the simulated own-price elasticities for different parameter values and both infinite-small as well as larger price changes. As long as $\alpha = 0.5$, we only need to calculate the price elasticity for one good. All price elasticities are evaluated for price changes relative to the reference price $p_0 = 6$.

<table>
<thead>
<tr>
<th>$\Delta p_g$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>elasticity for price increases</th>
<th>elasticity for price decreases</th>
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<td>-</td>
<td>-</td>
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<td>-1</td>
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<tr>
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<td>-1.200</td>
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</tbody>
</table>

Table 1: Price elasticities to the left and right of the kink, and for different values of $\Delta p$. * indicates values used by Kahneman and Tversky, albeit for a completely different specification of the model.

In Table 1, we see that by setting $\beta_1 = 0.04$, $\beta_2 = 0.9$, $\mu_1 = 0.05$ and $\mu_2 = 0.15$ we have a price elasticity of $-1.8$ for price increases and $-1.012$ for price decreases. These results are perhaps not precise replications of empirical estimates of gasoline elasticities but it is close enough\(^{17}\) while at the same time being able to illustrate the reference price effects and loss aversion. Given these parameter values, the demand is more elastic for price increases than for decreases, as expected. This holds both for infinite-small price changes as well as larger price changes. We also test for other parameter values than those displayed in Table 1. For larger parameter values, the elasticities becomes unrealistically large, while smaller parameter values makes the loss aversion effect vanish.

Below are plots of the demand function for $x_g$. The first plot is the standard demand function, defined as in (22). The second one is the modified demand function for the chosen parameter values and in the third plot we increase the loss aversion effect to better illustrate the kink.

\(^{17}\)We do not try to replicate real world data but only try to find reasonable approximation.
Figure 4: Standard demand function defined as in equation (20). We have $p_g$ on the horizontal axis and the demand for the two goods on the vertical axis. The solid line is the demand for $x_g$ for different prices. The dashed curve is the demand for $x_e$ for different $p_g$ (but with $p_e$ fixed).

Figure 5: Modified demand function for $x_g$ when $\beta_1 = 0.04$, $\beta_2 = 0.9$, $\mu_1 = 0.05$ and $\mu_2 = 0.15$. We have prices on the horizontal axis and the demand for the two goods on the vertical axis. The solid line is the demand for $x_g$ for different prices, including reference price effects and loss aversion. The dashed curve is the demand for $x_e$ for different $p_g$ (but with $p_e = \overline{p}_e$), including reference price effects and loss aversion. The dotted line indicates the reference price.
As can be seen in the Figure 5 and most visible in Figure 6, there is a kink in the demand function where the observed price \( p_g \) is equal to the value of the reference price \( \bar{p}_g \). This kink exist for all values of \( \beta \) and \( \mu \) and becomes more distinct as we increase these parameters. Further, we indeed have the expected asymmetry to the right and left of the kink with the demand function being more steep for prices above the reference price than below. In other words, our model is consistent with previous empirical literature on loss aversion.

As also can be seen in the figures, there is a kink in the demand for \( x_e \) where \( p_g = \bar{p}_g \). This is the case even in the absence of any such effect on the own price since we have that \( p_e = \bar{p}_e \). To compare with the standard demand function, in the case with no reference price effects the demand for \( x_e \) will be independent from \( p_g \) and thus be illustrated by a horizontal line. Analogue to the graphs above, if we instead let the prices for \( x_e \) vary and keep the price of \( x_g \) fixed, the demand function for \( x_e \) exhibits the same kind of price effects. Since we have set \( \alpha = 0.5 \), the demand functions for the two goods are symmetric.

**The Policy Maker’s Problem**

We now turn to the policy maker trying to decrease the ratio \( x_g/x_e \) using either a tax on \( x_g \) or a same-sized subsidy on \( x_e \). First we need to consider the reference price a bit more. In the model above we assume that the reference price is exogenously given. However, since the slope of the demand function and hence much of the analysis is depending on this reference price relative to the observed prices, we need to consider what determines this and also how different reference prices affect the analysis. Most previous literature defines the reference price as the price paid in previous time periods. For example, Putler (1992) assumes that
the reference price is what the consumer payed for the good in the previous time period. This implies that without any exogenous price changes, the observed price always equals the reference price. Hence, reference price effects only enter for the time period where the price change occur, and in the following time periods the demand function reduces to the standard demand function without these effects. This case is also partly in line with what Winer (1986) describes as extrapolative expectations hypothesis, where the reference price is formed by the most recent observed price and a general price trend. We can think of the reference price as being defined as

\[ p_i^t = (\delta)p_i^{t-1} + (1-\delta)p_i^{t-1} \text{ where } \delta \leq 1 \]

which is in line with previous literature on reference price formation, see for example Tridib Mazumdar et. al. (2005). The case above, with the reference price being equal to last periods observed price, could then be thought of as \( \delta \) being equal to zero, so that the reference price always equals previous time periods observed price.

Consider now the policy maker trying to decrease the ratio \( x_g/x_e \) by either imposing a tax on \( x_g \) or subsidize \( x_e \). We simulate this scenario setting the reference equal to 6 and start off with both \( p_g \) and \( p_e \) being equal to 6. We then analyze the effects of taxes on \( x_g \) and subsidies on \( x_e \). Given the parameter values used above we have that the ratio initially, with no price change, is 1 and the consumer demand the same amount of both goods. The question is now, is it more effective to increase the price than to decrease the price of electricity? Since the demand is more elastic for price increases than for price decreases so that price increases have a bigger impact on quantity consumed, we expect it to be more effective to impose a tax on \( x_g \) than to use a same-sized subsidy on \( x_e \). We also make the same experiment with the standard demand function, ignoring reference price effects and loss aversion. The table below summarizes the simulated results. \( t_i \) indicates what policy is used. Positive \( t \) is a tax (price increase), whereas negative \( t \) indicates a subsidy (price decrease).

<table>
<thead>
<tr>
<th></th>
<th>standard demand function</th>
<th>modified demand function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_g ) ( p_e ) ( x_g/x_e ) ( x_g/x_e )</td>
<td>( x_g/x_e )</td>
<td>( x_g/x_e )</td>
</tr>
<tr>
<td>no policy</td>
<td>6 6 6 1</td>
<td>1</td>
</tr>
<tr>
<td>( t_g = 1 )</td>
<td>6 7 6 0.857</td>
<td>0.727</td>
</tr>
<tr>
<td>( t_g = 2 )</td>
<td>6 8 6 0.750</td>
<td>0.56</td>
</tr>
<tr>
<td>( t_e = -1 )</td>
<td>6 6 5 0.81</td>
<td>0.833</td>
</tr>
<tr>
<td>( t_e = -2 )</td>
<td>6 6 4 0.64</td>
<td>0.666</td>
</tr>
</tbody>
</table>

Table 2: effects of different policies when the reference price equals the pre-policy price.

From Table 2, we first have that with no price effects the ratio is almost the same irrespectively of whether the policy maker imposes a tax on \( x_g \) or subsidizes
the subsidy being slightly more effective. This is due to the convexity of the demand function. However, as soon as we introduce price effects, the outcomes of the policies differ. We now have that a tax on $x_g$ is more effective than a subsidy on $x_e$ on decreasing the ratio $x_g/x_e$. That is, if the observed price initially is equal to the reference price, so that we initially stand at the kink, a price increase have more effect on demand than a price decrease. This result is in line with Putler (1992), Hardie et. al. (1993) and Winer (1998) although they do not formulate price changes as origin from any policy maker but instead only consider exogenous price shocks.

We could also have the case where there is no or extremely slow updating of reference prices. This could be thought of as $\delta$ in (24) being equal or close to one. For example, assume that the reference price is the first observed price, and that it has not changed over time. Further, assume that there over time has been some changes in the observed price (for any reason, for example inflation) but that the consumer never adapts to these changes. Hence, all observed prices are perceived as gains or losses as long as a price change does not bring the price back to the first observed level. For example, consider the case where the first observed price for any reason is very low, and that the observed price since then has increased\(^{18}\). This imply that we now stand to the right of the kink where the observed price is higher than the reference price. Now, consider the policy maker trying to decrease the ratio $x_g/x_e$. In this case, we could have that even if the policy maker subsidizes $x_e$ the price is still perceived as a loss if the price after the subsidy still is above the reference price. A tax on $x_g$ is naturally perceived as a loss. Here, it is not necessarily the case that a tax on $x_g$ always is more effective. We simulate this scenario setting the reference price equal to 4 for both goods and the observed price initially equal to 6. We then impose taxes on $x_g$ and same-sized subsidies on $x_e$. The results are summarized in Table 3. As in previous table, $t_i$ indicates what policy is used. A positive $t$ is a tax (price increase), whereas negative $t$ indicates a subsidy (price decrease).

<table>
<thead>
<tr>
<th>$t_i$</th>
<th>$p_i$</th>
<th>$p_g$</th>
<th>$p_e$</th>
<th>$x_g/x_e$</th>
<th>$x_g/x_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no policy</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$t_g = 1$</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>0.857</td>
<td>0.78</td>
</tr>
<tr>
<td>$t_g = 2$</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>0.750</td>
<td>0.63</td>
</tr>
<tr>
<td>$t_e = -1$</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>0.81</td>
<td>0.74</td>
</tr>
<tr>
<td>$t_e = -2$</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>0.64</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 3: effects of different policies when reference prices are relative low.

\(^{18}\)For example, due to inflation
Indeed, some interesting results emerge. Clearly, the ratio is smaller when subsidizing $x_e$ compared to when taxing $x_g$. At first sight these results might seem counter intuitive. However, since both cases are perceived as losses irrespectively of policy it actually make sense to have subsidies as the more effective policy. Remember that gains and losses are defined as diminishing functions of the deviations in price. Since both cases are perceived as losses, the marginal utility of a subsidy is larger than the marginal disutility from the tax. Hence, in cases where the price of the goods already have increased and the consumers haven’t had the time to adapt to these price levels, it makes sense for the policy maker to subsidize the environmental-friendly good.

Finally, we now assume that the reference price initially is above the observed price for any reason. In this case, all prices are perceived as gains. Unfortunately, and as discussed above, the demand function is not very well behaved to the left of the kink. Here, we will have increasing sensitivity to price decreases which makes analyzing different magnitudes of gains problematic. Previous loss aversion literature assume that as the magnitude of the gains increases, the marginal effect on utility decreases. This would imply that small deviations in price from the reference price has a higher marginal effect than if the already lowered price decreased even more. Hence, in a situation where the observed prices initially are to the left of the kink, an increase in the price of $x_g$ would have more effect on consumption than a decrease in the price of $x_e$. However, with the convexity of our demand function to the left of the kink, our model suggest the opposite. In our case, the convexity of the demand function dominates the concavity of the gain effect for all reasonable parameter values.

We can also see what would happen if the policy maker somehow could shift the reference price of one of the two goods. This is similar to the salesman trying to shift the reference price with high suggested retail prices relative to the observed price to increase the demand for the good. For example, assume that the policy maker somehow were able to increase the reference price for $x_e$. Note here that the policy maker does not want to increase the price level which would decrease the demand of $x_e$, but rather make consumers perceive the observed prices of $x_e$ as low relative to the now higher reference price. In other words, the policy maker want to do what the salesman does with high suggested retail prices. If the reference price for $x_e$ increase without affecting the price level, this shifts the demand curve for $x_e$ to the right, where demand is higher for all prices. The shift is illustrated in the figure below.
Figure 7: Shift in reference price of $x_e$ from 6 to 7. The dashed curve illustrates the new demand function.

As can be seen in Figure 7, the increase in the reference price shifts the demand function upwards, as we expected and consistent with previous literature. The demand for $x_e$ is now higher for all prices. Of course, compared to the seller it might be hard or even impossible for the policy maker to actually shift the reference price. This results is nevertheless interesting since it provides the policy maker with an additional tool for changing consumption patterns, at least in theory. If the policy maker were able to change consumers perception of the reference price, it would be a costless way of changing consumption behavior. For example, if the policy maker were able to increase the reference price of the environmental-friendly good, $x_e$, the model suggest that this would increase the demand for $x_e$ without the policy maker having to spend any money on subsidies.

We could also have the opposite case where the policy maker tries to decrease the reference price of $x_g$ to make observed prices $p_g$ look relatively high. If the reference price for $x_g$ decrease without affecting the price level, all observed prices are now perceived as higher relative to the reference point, which shifts the demand curve for $x_g$ to the left implying a lower demand for all prices. The effects of this shift in reference prices is illustrated in Figure 8.
Figure 8: Shift in reference price of $x_g$ from 6 to 4. The dashed curve illustrates the new demand function.

Conclusions

This paper addresses the question on how consumers perceive different policies, and how this affect the outcome of different policies. These topics are analyzed within a microeconomic framework with the addition of reference price effects and loss aversion. This paper rests heavily on these assumptions. So, are they reasonable? We think so. The assumption on how reference prices affect decisions is rather intuitively appealing. When we go out to buy a good, we often compare the observed price to something like the reference price. Both the reference price effect and loss aversion have been found empirically valid as well. However, as defined in this paper, these effects can be argued to be relevant mostly for frequently purchased goods. In other words, one must be careful when applying these ideas to different goods.

Further, we see that it is not trivial to theoretically model these effects. When assuming a concave utility function, which in turn implies a convex demand function, the standard demand function exhibits an increasing sensitivity to gains relative to any reference price. This is the opposite of loss aversion, since price decreases have bigger impact on utility than price increases for larger price changes. This is something not discussed in previous literature in this field, and needs to be considered for us to have a reasonable theoretical description of these effects. Although somewhat limiting when it comes to modeling loss aversion in prices, our model is still able to provide some interesting results and implications of the studied behavior.

The main conclusion to be drawn is that in the case of reference price effects
and loss aversion, the asymmetry in price elasticities will imply different outcomes in terms of consumption behavior depending on whether a tax or subsidy is chosen as policy instrument. Further, and as the simulation shows, the choice between a tax and a subsidy depends heavily on the reference price. For a very low reference price, subsidies are more effective due to the diminishing marginal disutility from taxes. When the reference price equals the pre-policy observed price and when the reference price is high, taxes are more effective on changing demand patterns. Hence, it is of great importance for policy maker to have an idea of the consumers reference price when choosing which policy instrument to use. This result is not depending on the specific functional forms used to model this behavior, but rather due to the diminishing sensitivity to gains and losses.

We also illustrate the effects of a shift in the reference price similar to how the salesmen have a high suggested retail price relative to the discounted observed price. This is related to the discussion on framing effects, which suggest that the framing of the decision problem affects whether the consumer perceive the outcome as a gain or as a loss. We only mention these effects briefly, and it is clearly more to be done on this issue.

For future research, an obvious challenge would be to test these hypotheses empirically. Although there have been some empirical studies mentioned earlier, it would from the policy makers point of view be crucial to test for these effects for goods relevant to policy making. In this paper we exemplify the model with car fuel. Other goods that could be relevant are for example energy and electricity.

To fully exploit the scope of these effects it would be necessary to know how consumers update their reference prices. For example, consider the price of gasoline that has increased continuously over time. Still, the demand has increased. To explain this behavior in terms of reference price effects, we would have to consider how the consumers update their reference price. If the consumer updates his reference price rather fast, the reference price effects would only persist for a relative short period of time.

In general, there seems to be a scope for reference dependence in many more applications. Much more work remain, both empirically and theoretically, but hopefully this paper provides a step forward.

References


