VINTAGE MODELS OF SPATIAL STRUCTURAL CHANGE

by

Lars Westin

VINTAGE MODELS OF SPATIAL STRUCTURAL CHANGE

by

Lars Westin

ABSTRACT

In the study a class of multisector network models, suitable for simulation of the interaction between production, demand, trade, and infrastructure, is presented. A characteristic feature of the class is a vintage model of the production system. Hence, the rigidities in existing capacities and the temporary monopolies obtainable from investments in new capacity at favourable locations are emphasized.

As special cases, the class contains models in the modelling traditions of "interregional computable general equilibrium", "spatial price equilibrium", "interregional input-output" and transportation networks.

On the demand side, a multihousehold spatial linear expenditure system is introduced. This allows for an endogenous representation of income effects of skill-differentiated labour.

The models are represented by a set of complementarity problems. This facilitates a comparison of model properties and the choice of an appropriate solution algorithm.

The study is mainly devoted to single period models. Such equilibrium models are interpreted as adiabatic approximations of processes in continuous time. A separation by the time scale of the processes and an application of the slaving principle should thus govern the choice of endogenous variables in the equilibrium formulation.

Keywords: adiabatic approximation, dynamics, input-output, interregional computable general equilibrium, slaving principle, spatial models, spatial linear expenditure system, spatial price equilibrium, transportation networks, vintage function.

VINTAGE MODELS OF SPATIAL STRUCTURAL CHANGE

av

Lars Westin
Fil. kand.

Author and title

Lars Westin
Vintage Models of Spatial Structural Change. Umeå Economic Studies No. 227,
1990. 232 pages.

Abstract

In the study a class of multisector network models, suitable for simulation of the
interaction between production, demand, trade, and infrastructure, is presented. A
characteristic feature of the class is a vintage model of the production system. Hence,
the rigidities in existing capacities and the temporary monopolies obtainable from
investments in new capacity at favourable locations are emphasized.

As special cases, the class contains models in the modelling traditions of "inter-
regional computable general equilibrium", "spatial price equilibrium", "interregional
input-output" and transportation networks.

On the demand side, a multihousehold spatial linear expenditure system is introduced.
This allows for an endogenous representation of income effects of skill-differentiated
labour.

The models are represented by a set of complementarity problems. This facilitates a
comparison of model properties and the choice of an appropriate solution algorithm.

The study is mainly devoted to single period models. Such equilibrium models are
interpreted as adiabatic approximations of processes in continuous time. A separation
by the time scale of the processes and an application of the slaving principle should
thus govern the choice of endogenous variables in the equilibrium formulation.

Keywords: adiabatic approximation, dynamics, input-output, interregional com-
putable general equilibrium, slaving principle, spatial models, spatial linear expen-
diture system, spatial price equilibrium, transportation networks, vintage function.

"Time present and time past
Are both perhaps present in time future
And time future contained in time past"

T.S. Eliot "Four Quartets"

ACKNOWLEDGEMENTS

This dissertation has emerged from mainly three fields of research which I have been involved in during my time at the Department of Economics at Umeå University.

I started with work in the field of interregional input-output modelling with an application to the municipality Vilhelmina in the inland of northern Sweden. Later on, this lead me to applied equilibrium models and a project together with Håkan Persson. In the project, a model of the Swedish economy with a spatial vintage formulation was developed. The third source is my work together with Börje Johansson on the dynamics of structural change, labour skill and transportation networks. In this work, applied studies have been made related to regions in Northern Sweden, the Swedish transportation network and the interaction on networks between Sweden and the rest of Europe.

This study may be seen as an attempt to summarize and put together some of my experiences from those fields, which all deal with the general problem of structural change in spatial economies.

During the time, several people have inspired me and supported my work. In particular, I wish to thank Åke E. Andersson, David Batten, Erik Bylund, Börje Johansson, Anders Karlqvist, Håkan Persson, Tönu Puu and Folke Snickars.

Åke E. Andersson was my supervisor during the initial part of my PhD studies. His personal network and fluent ideas has been an important source of inspiration.

I owe a special debt of gratitude to Börje Johansson who has been my supervisor during the last years, and an engaged partner in an immense number of discussions of problems related to the study. He has also read through and made comments on successive versions of the manuscript and persistently encouraged me to finalize the work. I would also like to thank Thomas Aronsson, Mats Bergman, Runar Brännlund, Bengt Kriström, Karl-Gustaf Löfgren, Henry Ohlsson, Jørn Stage and

Magnus Vikström at the Department of Economics for their critical comments and suggestions on an early version of the manuscript. The "visiting Venetian", Roberto Roson, and Gunnar Lindberg at the Swedish Road Administration have both been valuable discussion partners during the completion of the thesis.

Furthermore, I wish to thank all colleagues and friends at the Department of Economics, in the network connected to CERUM, and at many other disciplines at Umeå University for many inspiring discussions through the years. Equally important have been all discussions and activities in various student organisations and other more informal networks. Those structures have offered a variety of ideas and unexpected meetings, at an extent which is not possible within the formal part of the academic world.

The financial support from the Faculty of Social Science at Umeå University and the Gösta Skoglund International Fund is hereby gratefully acknowledged.

Allan Sugg has corrected my "Swedish-English" language, Solweig Brand and Roger Jacobsson have given invaluable guidings in my attempts to understand the world of word processing, while Ing-Marie Nilsson excellently has transformed my manuscript into a form ready for printing.

Last, but not least, I wish to express my gratitude to the support and trust from my family, Christin and Kim, and my parents Paul and Margit Westin.

Umeå, May 1990

Lars Westin

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

1.1 INTRODUCTION AND OBJECTIVES OF THE STUDY

The aim of this study is to investigate a class of models suited for analysis of structural change and spatial interaction. Our objective is to give a systematization of models focusing on changes in, and the interaction between, commodity and passenger flows, industrial structures, employment, and expenditure patterns.

We are dealing with models in the traditional Spatial Price Equilibrium (SPE) class and relate those to the more recently developed Interregional Computable General Equilibrium (ICGE) models. So far, those two model classes generally have been treated as separate. We relate them to each other, and investigate their similarities and differences. It is suggested that the two classes should be treated as a single class of spatial equilibrium models.

This class of models has an important role in consequence analyses where the problems are of system character, have economy-wide implications, and where inherent consistency is important. The explicit consideration of space and spatial interaction in the models emphasize the mutual interdependence, as well as the game-like nature of, the development at different locations in space. Transportation systems, other means of infrastructure, and interaction costs are essential parts of this development and are, to a varying extent, represented in the models. Since spatial interaction takes time, space also contributes to the complexity, the fundamental uncertainty, and the rigidities involved in the dynamics of an economy.

Models which represent this dynamic interaction between economy-wide spatial structural change and infrastructure are rare. In the sequel we suggest an extension of the above spatial equilibrium models into multisector network equilibrium models. Those contain an integrated multimodal network representation of the

infrastructure, together with a multisector, income-sensitive formulation of commodity markets.

Analysis of structural change deals with the competition and connection between new and old, creation and destruction, growth and decline. Both between industries and commodities, as well as production techniques, design concepts, ideas and system solutions. Multisector and multiactivity models reflect an ambition to understand and to represent this steadily ongoing process. The vintage production function utilized in the sequel gives rich possibilities to investigate rigidities and changes as a result of an introduction of a new technique on employment, commodity flows and production in a spatial system.

Moreover, by treating labour, where possible, as a nonhomogeneous production factor, differentiated by skill, preferences and location, the problem of structural change is not merely reduced to choice of technique and output level, but the models also touch upon the interaction among, and initiatives of, human beings.

"Structural change", or "structural adjustment" as it also may be labelled, is a concept with several interpretations. In the following the interpretation is related to changes in the industrial and trade patterns that take place within a medium-term horizon. Hence, the interest is concentrated on changes in the absolute growth, and the changing relative shares of different industries, measured for example as value added, sales value or the number of employees engaged. The elasticities of the distributions over production techniques of profits, productivity, utilized intermediate inputs, and labour with different skills in relation to price and demand changes are other important aspects of our interpretation. Changes in demand at different locations in the economy and the related changes in trade and communication patterns give a further dimension to the analysis of structural change.

The above interpretation may be clarified by a short discussion of the relation between our own and other definitions of the term. More narrow definitions of structural stability, instability and change are used in the theory of mathematical topology and dynamic systems [Hirsh and Smale (1974), Poston and Stewart (1978)]. In this context, structural change denotes the process by which a structurally stable system, via a phase of instability, moves into a new stable structure. Hence, struc-

tural change denotes the period of transition between stable patterns. Such points of transitions are generally unstable singularities and may thus only represent a system during relatively short time intervals. However, in our context, structural change is the generic outcome of a dynamic spatial multisector economy.

One attempt to relate those interpretations with each other may be by use of the slaving principle as it has been formulated by e.g. Haken (1983a, 1983b). The slaving principle implies separation and ordering of different processes and sub-systems in accordance with their adjustment time.[1]

Consider the medium-term models that we will present in the sequel; they explicitly depict the adjustment in the economic structure and commodity flows. The slow change in infrastructure during this time horizon makes it possible to treat the infrastructure as fixed. The commodity flows on the network may then be structurally stable from a topological point of view. In spite of this, changes in production and demand may occur. However, nonmarginal changes in the infrastructure may be initiated by either new "critical" links or new transportation modes. Those may then cause a transition of demand, supply and trade which force the commodity flows into a new and, from topological view, different pattern. The slow change of infrastructure thus slaves the faster processes in transportation and production. Our interpretation of structural change in this study, comprehends changes in both infrastructure and economic structure, while the topological interpretation only would refer to such changes in the slow processes which generate changes between stable structures among the faster ones.

The above discussion about the slaving principle has implications also for applied modelling. If variables are clustered into groups with similar speed of adjustment, for example into slow and fast variables, the dynamics of a fast cluster may follow different trajectories dependent on the initial state and the parameters given by the slow cluster. Each such trajectory may be characterized as a member of an archetype system within which the behaviour of the model, e.g. viability, existence of attractors, or other long-run properties, are similar. If a small change in initial state or parameters of a model does not change the system archetype, the model is structurally stable. Such stable archetype systems are delimited by unstable singu-

---

1   See also Karlqvist (1987) for an introduction.

larities. A model of an economy near such a singularity is difficult to interpret, since small, measuring or modelling, errors may generate solutions far from each other. The models presented in the following are constrained to analyses of stable structures and parameter sets where equilibria, i.e point attractors exist. However, instability properties and qualitative system shifts may be analyzed by exogenous parameter changes.

Another field in which the notions of structure and structural change are used is related to market structure. Recent contributions in this field have emphasized the relation between spatial trade and market structure, especially the effects of scale economies, differentiated products, oligopoly, and other forms of imperfect competition.[2] Our interpretation of structural change does not exclude such analysis in the thesis, but we are not dealing with models of scale economies and price-setting. However, an analysis of structural change cannot neglect economic agents' search for monopoly profits, either by means of price or product competition, as an essential driving force in the economy. Models of structural change thus have to introduce Schumpeterian entrepreneurs and "creative destruction", which cause entry and exit of productive capacities and network links.

The vintage model of the production system, based on information from the establishment level, has properties of this type. It emphasizes the costs associated with changes in an industry structure, without which the problem of structural change would be nonexistent. It also emphasizes the empirically observed rigidities in the structure of production, interaction and localized resources. The vintage approach introduces an intuitive and attractive element of rigidity in equilibrium analysis of structural change.

Vintage analysis represents a "structuralistic" tradition with strong roots in Scandinavian research. It has an origin in the "small open economy" character of the Nordic countries, where adjustment to changes in the rest of the world is an important problem. Issues related to monopoly and other strong forms of imperfect competition, often studied in North American market structure analyses, some of which are mentioned above, do not play the same role in small countries facing inter-

---

2   Harris (1984), Helpman and Krugman (1985).

national competition. This gives a further motivation to the surpressed treatment of those topics in this study.

The policy problems, appropriate to analyze by means of the models outlined in the following, may be exemplified by questions regarding the broader effects of educational programs, network investments and introduction of new techniques among interdependent locations. Numerical modelling is of special interest under such circumstances. It gives an opportunity to study simultaneous effects of the interaction between for example investments, labour market adjustment, and household demand in a spatial framework. That is, problems characterized by the difficulties to grasp the system effects from different actions with analytical methods solely.

All models in the study are for this reason designed for numerical application. Moreover, instead of limiting the study to "the model" of structural change, a class of models is described, within which a choice may be made when an empirical implementation is at hand. The reason is that the choice of an appropriate model always has to be related to the amount and type of information available, suitable behavioral assumptions for the actual region and time period, as well as the accessible solution algorithms.

The focus of the thesis is directed towards spatial multisector models. When dealing with such models, there is a huge body of existing theoretical and applied literature including the initial works by e.g. Ricardo, von Thünen, Walras, and Leontief. It is also easy to observe the time gap between theoretical development and numerical applications of multisector models. However, the number of applications have, as a result of swift computer development, increased since the middle of the seventies and the gap between theory and at least simple applications has decreased. Hence, the use of the input-output model as the basic tool in applied multisector analysis has been replaced by price equilibrium models, although still with an input-output representation as the core.

In spatial economics, the interregional input-output model and the classical transportation model were replaced by more complex allocation and price equilibrium models of optimization type. Spatial equilibrium analysis has for some time been represented by the quadratic programming formulations of Spatial Price Equi-

librium (SPE) models and various gravity, entropy and logit formulations. The SPE model has now also been extended into nonlinear and network oriented directions. Especially freight and traffic problems have been addressed. The eighties has also brought about an introduction of Interregional Computable General Equilibrium (ICGE) models. Generally, SPE and ICGE models have been treated separately. But an integration of the models is possible. In the following we suggest that such an integration may be accomplished within the common format of complementarity problems. This mathematical representation of a large set of economic models imply simplified comparison of model properties and choice of appropriate solution algorithms.

In the ICGE models, which are developed from the Walrasian tradition, the implication of the notion "general", is that the connections between factor payments, factor incomes, commodity prices, and demand are taken care of in a complete multisector price endogenous model of the economy. This introduction of induced demand is an important contribution of ICGE models compared to the earlier SPE models. However, in spite of the theoretical elegance of the Walrasian "general" equilibrium model, it is easy to observe, even in a long-term framework, rigidities and sustainable unbalances in many markets. This reveals itself in the form of a slow reshaping of resources, networks, and human knowledge. It is also possible to identify time periods and regions where those patterns have reached a deadlock and made the process of change slower than usual as well as periods and regions where such structures have changed very fast and where unforeseen patterns have emerged. Hence, the Walrasian assumption that all markets clear within the same time period is in most cases too strong. In the following, we instead suggest that one should apply the slaving principle to a problem and limit the equilibrium analysis to markets and processes with a common time scale. Because of this, reliable applied equilibrium models should be partial models, but when appropriate, with an endogenous treatment of income effects.

Although common, price sensitive equilibrium analysis may not always be relevant in analysis of structural change. Other approaches such as fixed price, input-output, or other allocation models of optimization type, as well as disequilibrium models may therefore, both from an empirical and theoretical point, be satisfactory alternatives to equilibrium analysis. However, those may often be seen as special cases of equili-

brium models. This further motivates studies on the relation between, and the possibility to combine, different model approaches in the analyses of structural change. In this case, the recent development in the fields of numerical algorithms and complementarity problems should be utilized. Together they give possibilities to clarify model properties and to chose appropriate algorithms.

An equilibrium model in discrete time should be considered as an adiabatic approximation of a dynamic model in continuous time. The adiabatic approximation makes it possible to exploit the relations between the discrete time and continuous time models. It is our belief that the next step in applied spatial multisector modelling has to go beyond the discrete time formulations and to develop applied nonlinear models in continuous time while the appropriate use of equilibrium approximations are investigated further.

1.2   PLAN OF THE STUDY

With the previous introduction as a background, the objectives of the dissertation may be specified as follows:

*   Give a systematic presentation of the vintage model of production and technical change and evaluate the properties of the model in relation to empirical information.

*   Introduce this vintage approach into SPE and ICGE models.

*   Integrate the above two frameworks into a class of spatial equilibrium models, by demonstration of the conditions under which they generate equivalent solutions. Demonstrate that a number of existing models are special cases of this class.

*   Show how the transport sector may be introduced in this class through a detailed network representation with a connection between network activities and direct, indirect and income effects in the producing and demanding nodes.

* Formulate the models as complementarity problems and discuss how different solution algorithms constrain the possible behavioural formulations.

* Relate equilibrium analysis in discrete time to modeling in continuous time by use of the slaving principle and the adiabatic approximation.

To achieve the above objectives, we will proceed as follows. In **Chapter 2**, what we consider to be the central features of structural change in spatial economies are presented. Hence, features of appropriate models may be identified. This is followed by a compressed survey of some existing models which have been used in analyses of structural change. In the chapter an introduction to complementarity problems and the relation between complementarity problems, linear and nonlinear programming, variational inequalities, and equilibrium problems is also provided. Chapter 2 is finalized by a discussion of how behavioral assumptions, solution algorithms, and empirical constraints together impose a trade-off pattern as regards feasible model formulations.

Chapter 3 penetrates the vintage model in a systematic way. From the vintage assumption we derive and relate production, cost, and profit functions at the industry level, as well as the inverse and ordinary supply functions, to each other. The imputed values under different exogenous constraints on prices, factors and demand are studied and interpreted. The estimation of the model, given the available empirical information is discussed. The relevance of the exit and entry assumptions in the model are evaluated in relation to empirical observations and possible extensions are suggested. Other extensions of the vintage model, such as introduction of heterogeneous labour and endogenous investments, are also considered in the chapter. The discussion is confined to nonspatial single sector formulations throughout the chapter.

Chapter 4 contains extensions of the nonspatial single sector vintage model into Isard's and Koopmans arcetype spatial multisector formulations with exogenously given demand or prices. In this chapter the models are, as the models in Chapter 3, supply side oriented. Conditions for consistent treatment of the transportation sector in spatial multisector models are also discussed in the chapter.

Price elastic formulations of the demand side are introduced in **Chapter 5**. The vintage formulation is here introduced into SPE models of both Marshallian and Walrasian type. Hence, a first step towards an integrated representation of SPE and ICGE models is taken.

The next chapter, **Chapter 6**, contains vintage formulations of ICGE models. First a combined national-regional model is introduced. Subsequently, a model with a spatial multihousehold linear expenditure system is presented. This model contains link-related transport costs. The model is then extended to include endogenous capacity increase and the pertinent demand for investment commodities. Finally, explicit budget constraints and a detailed network representation are introduced. By this, a model is suggested which introduces the ICGE model into an important area of combined structural change and transport network modelling.

The models discussed so far, are integrated into a single class of network equilibrium models in **Chapter 7**. The chapter also contains a discussion of nested CES functions as a way to allow for combinations of the substitution possibilities found in SPE and ICGE models.

The suggested models are single-period models. In **Chapter 8**, the approximation of structural change dynamics in continuous time by discrete time models is addressed. The slaving principle and the adiabatic approximation are discussed in a more formal way. An outcome of this is a theoretical tool, related to the time scale of processes, for appropriate formulations of various versions of models within the multi-sector network model class. Pros and cons of multiperiod models are also discussed in this context.

Finally, in **Chapter 9** some summarizing conclusions are drawn and further research is suggested.

2 MODELLING SPATIAL STRUCTURAL CHANGE

2.1 INTRODUCTION

An initial purpose of this chapter is to give a broad introduction to the problem of spatial structural change and to survey different existing models suitable for applied, i.e. numerical, analyses of this problem. The ambition is not to give a complete picture of "the state of the art" in spatial modelling. Instead the ambition is to identify desirable features of models of structural change and to facilitate an understanding of how the models we suggest in the sequel are related to, and extend, existing models.

Another purpose of the chapter is to give an introduction to the representation of mathematical models as complementarity problems. A complementarity representation contains both optimization and equilibrium concepts. We argue that it is advantageous to reformulate economic models into such a unifying form, since comparisons of properties and the choice of appropriate solution algorithm becomes facilitated. The progress in the field of algorithms for complementarity problems has been rapid during the eighties, and some of the constraints on the solution of applied models have been reduced by this. For this reason, a broad overview of different solution algorithms for complementarity problems is also given.

In section 2.2, some observations regarding fundamental features of spatial structural change are made. By this, we identify features which should be included in a model of the involved dynamics. Section 2.3 contains an overview of existing models. The complementarity problem is treated in section 2.4, while algorithms are discussed in section 2.5. The concluding section contains a discussion of how the modeler has to consider trade-offs between introduction of behavioral assumptions, collection of empirical information and developing solution algorithms, while constructing applied models.

2.2   ON THE DYNAMICS OF SPATIAL STRUCTURAL CHANGE

Modelling of spatial structural change has to start from an identification of the important properties of the dynamics of this process. In this section, our aim is to characterize the processes for which we develop our class of models and the appropriate utilization of those models.

The most fundamental observation is the differences in speed of change and adjustment among various processes in an economic network. The dynamics of localized resources, such as production capacities, natural resources and infrastructure, are characterized by inertia and a low average speed of change. The link structure transforms at a lower speed than the production capacities located in the nodes. On the other hand changes in the amount, speed, direction, and attributes of commodities, traffic, assets and information transferred in this rigid structure are generally much faster. Hence, our first conclusion is:

*   A fundamental feature of structural change and economic dynamics is the differences in the adjustment time between processes.

The observation calls attention to a careful examination of the form, length, and timing in the change of variables. Differences in time scales also allow for an ordering of the dynamic processes in the economy and an application of "the slaving principle". This principle implies that slow processes are treated as more fundamental than the faster processes. The former give a structure within which the latter may develop. In applied modelling, the pertinent interpretation is that the slow processes determine the parameters in the models of the faster processes. The main advantage gained from this is that subsystems of a larger system may be analyzed separately, with a reduction of the complexity of a problem.

Differences in time scales may be further illustrated by the relatively fast change of output, possible within the existing capacity in an industry, compared with the slower and more costly adjustment of the production capacity to meet increases in demand or changes in technology and factor prices. The rigidities in the capacity structure have, as will be illustrated in Chapter 3, empirically been established from Swedish

data[1]. The overall structure in a sector remains, due to those observations, fairly unchanged in a medium term perspective although the processes of exit and entry of capacities are active. The slow adjustment of capacities is due to the fact that only a marginal part of the aggregate capacity is replaced each year. This inertia may be given two interdependent explanations:

* Investment costs and indivisibilities of new structures bring about thresholds for capacity augmentation.

* Sunk costs in the existing structure of machines, buildings and networks give options to price competition in relation to new techniques and introduce friction in the removal and exit processes.

This slow transformation of localized resources provides a constraint on the economic development, although the reshaping of the economy is steadily ongoing. The existing capacity structure of old and recently introduced "vintages" represents at each point in time both the history, and a hint on the future development, of an industry or region. This duality is further exemplified by the fact that while a specific pattern of infrastructure, labour knowledge, productive capacities, and trade connections at one instant facilitate growth, this pattern may also, by locking economic resources, conceal a limit on the further speed of change and growth.

Especially decisions related to the slow processes, such as costly infrastructure, thus have important long-run effects. A limit may only be released at a high cost of abolition. Two implications of this are:

* Programs for exit of obsolete structures are as important as programs for the promotion of new structures.

* The slower the time scale of a process is, the more widespread is the influence on the shape of an economy.

The normally unbalanced exit of scrapped capacities and entry of productive capacities in each location generate a steady reshaping of the economy and a spatial relo-

---

1   Johansson and Strömqvist (1980), Johansson and Holmberg (1982), Westin (1987).

cation of activities within the boundaries set by the slower processes. The result is unbalanced growth, further strengthened by a uneven initiation of technological innovations and diffusion of technical knowledge. Other signs of this relocation are the spatial flows of investment funds to profitable locations and migration from stagnating regions to sites with a variety of opportunities, education facilities and possibilities, for instance to obtain "on the job training".

The resulting specialization of activities in space adds cost of communication and transportation to the costs of transformation. It reinforces disparities in the accessibility to information and competence in the economic network and promotes a diversification of preferences and taste among households. A never ending need for network maintenance and initiation of new links and communication systems arise, which together with "link catastrophes" are expressions of change in the infrastructure. It is an outcome of attempts among actors to adapt to changes in the environment and to encourage the growth of welfare. Increased interaction as a result of increased accessibility may then also counteract spatially unbalanced growth paths. These observations may be summarized as follows:

*   Interaction costs generate diversity among commodities, production techniques, and communication systems. Differences in accessibility and in resource endowments generate unbalanced growth in sectors and locations.

Product cycle theory has been developed to explain how the production of, and the demand for, a commodity evolve over time and how change by this is transmitted between regions. It provides, with an assumption of global profit maximization among firms, a dynamic stage theory for the spatial reallocation of production. Modern product cycle theory and studies of firm location have furthermore emphasized the importance on spatial development of labour skill, accessibility and the information transfer inherent in commodity flows [e.g. Johansson and Westin (1987)]. Jacobs (1984) has in this context stressed the importance of import substitution in all types of regional economies as a means to generate growth.

Investments in education, attractive environments, culture, and communication systems as well as attempts to "create creativity" have in this way become new political instruments to support development. Those initiatives may be seen as responses to

the part of the product cycle theory which indicate where new products may be "born" and where the imitation is quickest and most successful. Other parts of the theory indicate how the character of investments changes from product development to cost reducing investments in the production process and increased market penetration over the cycle. This transformation is often connected with changes in location and the composition of labour demand. Attempts among public decision makers to create economic advantages and attractive locations by reduction of taxes, investments in and subsidies to production facilities, stimulations to introduce new techniques etc. may be seen as a response to this part of the theory. The policy problems here is to find a mix of activities and education efforts which both satisfies the existing and the future structure on the supply and demand for labour. The choice of a given policy is thus generally determined by objectives which combine overall growth and income distribution. These observations may be transferred to the following conclusion about model features:

*   The growth and distributional effects caused by differences in labour skill, initial resources and taste heterogeneity emphasize the need to treat labour as a heterogeneous factor and to utilize spatial multihousehold approaches in the analysis of structural change.

Product cycle models are generally constrained to analysis of a single commodity or an industry. The models are intuitively attractive but in applications criticism may be directed toward the delimitation of the commodity or industry and the length of each stage in the cycle. Although the market mechanism is not usually explicit in the models, price movements are often implicitly assumed to work as equalizing signals between demand, supply and choice of location. Furthermore, the existence of intermediate and competitive commodities generate, together with the limits set by the infrastructure, a need for an explicit treatment of the mutual interdependences in a multicommodity economy. Hence:

*   An understanding of growth and change may be supported by spatial multisector models of the dynamic interaction between demand, supply, trade, and location.

Moreover, it is our opinion that applied models are needed to obtain a deeper understanding of the process of structural change. However, there is a trade-off

between the representation of dynamics and the detailed refinement in economic models. Applied multisector models with an elaborate representation of the dynamics still remain to be developed.

The specification and estimation difficulties involved in the application of multisector models in continuous time generate a need for simplifications. Equilibrium analysis becomes relevant as such an approximation of a continuous model, since the dimension of the problem may be decreased.

This gives a motivation for a development of multisector equilibrium models. These models emphasize in an explicit way the interaction between different markets by assessing and comparing different equilibria. To establish equilibrium, gap closing in markets, maximization among actors, and decreasing marginal returns are assumed. Although those not are the sole forces generating change, they represent observable and useful abstractions of economic behaviour. An approximation of continuous dynamics by comparison of equilibrium states is possible when the involved processes work on a common time scale and when the slower processes may be assumed to be almost constant during the time period.[2] This gives a criterion for the appropriate utilization of equilibrium models:

* Equilibrium analysis is appropriate as an approximation of a continuous dynamic system when the endogenous processes operate on a common time scale and the relative adjustment time between the slower and the endogenous processes are large.

Most product cycle and multisector approaches are used in a context of initially existing products or industries and are as such dependent on imitation and "habitation" as means of dynamic adaptation among the actors. Schumpeter (1934) and others, e.g. Day (1987), have stressed experimentation, creativity and search for "monopolistic profits" both by financial/producing and intellectual/cultural agents as alternative adaptive strategies. Such strategies give an almost continuous development of new commodities and ideas. However, creativity requires scarce resources and the amount of novelties emerging at each instant of time is always limited. This fact contributes to the explanation of economic rigidity. Equilibrium analysis may

---

2   A more formal discussion of this is found in Chapter 8.

explain the nature of such rigidites, but in explanation of forces behind differen-
tiation by quality and design as well as creation of novelties, the possible contri-
butions are less evident.

Many features recognized in structural change have obviously not, or may not, be
captured by endogenous formulations in multisector equilibrium models. Other less
developed fields are the game situation generated by the intertemporal character of
structural change, with pertinent irreversibilities, formation of expectations and
learning. Further weaknesses, especially of applied Walrasian equilibrium models,
are simplified network descriptions and a crude treatment of accessibility and
interaction costs. The system effects on choice of location and flow patterns of in-
vestments, which change the accessibility in the economy, have not yet been
adequately analyzed in equilibrium models.

We have indicated above areas where further research may take place. Some of the
addressed problems may be impossible to find a solution for within an equilibrium
framework. Despite those limitations we emphasize in the following discussion that;

* Spatial flows are constrained by interaction costs, reflect interdependencies and
  reinforce couplings.

* Taste for variety and diversity of preferences, necessitate the use of heteroge-
  neous commodity and multihousehold approaches.

* Labour heterogeneity constrains the development of productivity, growth, and
  change.

* The speed of growth and change over space is influenced by the interactions
  between rigidities in the existing structure and the gap-closing forces between
  existing and warranted states.

Our choice of a vintage approach, illustrates the last point. Spatially distributed pro-
duction rigidities slow down growth and change, but prospects to obtain temporary
monopolies generate novelties which both create new and close existing gaps.

2.3 MODELS OF STRUCTURAL CHANGE AND GROWTH

A comparison of models focusing on the problem of spatial structural change and growth, may be accomplished along several different lines. One aspect may be assumptions about the mobility of production factors, and the determination of prices under different time horizons. Formulations of temporal interdependences is another aspect. The determination of trade and the treatment of infrastructure and the transport sector are other interesting fields, since they generally bring about complex problems.

In this section some existing models are compared with respect to how they treat prices and time. Later on, we will return to other aspects. The aim is to make it possible to relate our own models to existing models. The overview also contains some nonspatial models and aggregated models, but the survey is limited to models of real flows. Financial markets are only introduced as exogenously given rates of return criteria. Pure time-series models are also excluded in an attempt to only focus on models with explicit causal relations. In addition, there is a bias towards models which have been important for the Swedish modelling of structural change.

As regards the formulation of dynamics, a model classification may be made into three traditional groups,

* Single-period models
* Multiperiod models in discrete time
* Models in continuous time

The first group consists of models which generate a solution for the terminal point without an explicit treatment of the process which takes the system from the initial point to the terminal point in time. The second group includes models with at least two periods and explicit intertemporal connections. Both those groups may be further classified according to the assumed period lengths. Finally, the third group contains continuous time models on different time scales based on differential equations.

A crude division in relation to how the models treat prices and quantities may be made in the following way:

* Quantity-oriented models, with implicit prices.
* Models with exogenously fixed prices and endogenously determined quantities.
* Models with endogenously determined prices and quantities.

Such a division always includes vague parts. For instance, a sequence of single-period fixed-price models becomes a price endogenous model if prices are adjusted towards equilibrium between each period. Models may also be formulated as combinations of two of the above categories. For example, a two-period model where the first period contains a short-term fixed-price model and the second a long-run equilibrium model has been used by Ohlsson (1988). Furthermore there is a class of disequilibrium models with flexible but bounded prices. Those belong to the price-endogenous type, but have many properties in common with the fixed-price models. In "quantity-oriented" models, formulated as programming problems, the dual variables may, as is well known, often also be interpreted as prices. Models in the above groups may thus be treated as special cases of a broad class of equilibrium models where elasticities, the time horizon and the speed of adjustment determine the model category. This is also one of the themes in the following discussion. However, the classification will help to delineate models discussed in the subsequent parts of the study into broad groups.

The classification may also be seen as a division in relation to refinement. When moving from sole quantity, via fixed-price models with rationing schemes to models with endogenous prices and quantities, the behavioral assumptions normally have to be more developed, while the statistical estimation problems also increase.

Although theoretical work has been devoted to almost all models which can be obtained by combining the above six categories, the applied use of price equilibrium models has mainly been constrained to single-period models. Multiperiod and continuous-time models are often quantity-oriented. Models with both price and quantity interaction in continuous time are infrequent in applied modelling. However, such a model is not an aim in itself. A "good" model is a model which provides insights into the problem under study, with the help of as few variables and relations

as possible. The time horizon of the problem then becomes important. For example, in long-run models, prices may be omitted because they are, at best, short-to medium-term signals of scarcity. The underlying quantity changes then provides the system with all necessary information. Traditional gravity models are, because of this, interesting mainly in long-run analysis.

It is easy to find representatives for the quantity-oriented models where prices are absent or implicitly obtained as shadow prices. The most obvious members of the single-period group are quadratic and rectangular interregional input-output models developed from the path-breaking model by Leontief (1951).[3] The classical transport assignment model and various extensions of it also belong to this class of models.[4] Other members of the group are the multiregional, multisector investment planning and forecasting model developed in Sweden by Snickars and Granholm (1981) and the vintage allocation model by Karlqvist and Strömqvist (1982). In summary, this category comprises the most commonly used tools in applied spatial multisector modelling. A simplifying advantage of the models is that price and income processes not are modelled endogenously, while internal consistence is obtained.

The second group of the quantity type is the multiperiod models in discrete time. A well known representative for the category is the discrete time, multisector multi-plier-accelerator model by Leontief (1970). Dynamic programming models also belong to this class. Examples of models of regional development are Snickars and Lundqvist (1978), Lundqvist (1980), and Karlqvist et al. (1978). Models of technological change and industrial development with recursive dynamics may be represented by Nelson (1971) and Day (1970).

Among the quantity models in continuous time, one finds differential equation versions of the dynamic Leontief model, [e.g. Medio (1987)], and the optimal control model by Brody (1970). An example of an aggregate, biregional multiplier-accelerator model in continuous space is Puu (1987).

Models with exogenously fixed prices may be represented by the single-period multisector, multiregional optimization models in Johansson and Strömqvist (1980,

---

3   See for example Isard (1951), Chenery (1953), Moses (1955, 1960), Oosterhaven (1984).
4   Hitchcock (1941) and Koopmans (1949). See also Takayama and Judge (1971).

1981)' and in Westin (1987). Those are all utilizing the vintage production function discussed further in Chapter 3 below.

A first attempt to extend this fixed-price multisector vintage model from a single-period to a recursive multiperiod model was made in a single region model by Johansson (1986). The model makes it possible to study the adjustment process in a vintage structure in more detail, but also introduces problems with the estimation of adjustment times.

The third model category consists of price and quantity endogenous models. The theoretical work by Ricardo (1821) and von Thünen (1826) are classical points of departure here.[5] Ricardo focused both on the spatial and vintage aspects of an economy when formulating the theory of diminishing marginal return and land rent. In the theory of comparative advantage he also emphasized the benefits of exchange between spatially separated economies and formulated determinants for the direction of commodity flows on a simple network. Von Thünen formulated an integrated theory of production, location and trade. His simple system has, as noticed by Samuelson (1983), elements of Ricardian trade and rent theory, the Heckscher-Ohlin theory of factors and goods pricing, the Leontief input-output system, as well as general equilibrium analysis.

One may although observe that, despite this early contribution by von Thünen, the theory of trade and the theory of location, [e.g. Weber (1909), Hotelling (1929), and Christaller (1935)] developed side by side for a long time. Ohlin (1933) made an attempt to integrate the fields. He reintroduced, more than one hundred years after von Thünen, "transfer costs" into international and interregional trade theory. However, these elements are although suppressed in the "Heckscher-Ohlin" model.

Walras' (1874) nonspatial general equilibrium theory emphasizes the mutual inter-dependencies between actors in the economy. The important "general" part of the theory is the connection between factor costs, incomes and household demand. Proofs of the existence of a Walrasian equilibrium was given in the fifties by Arrow and Debreu (1954) and Debreu (1959). Palander (1935) took a step towards a

---

5   Elaborated historical notes on spatial and nonspatial equilibrium models may be found in
     Takayama and Judge (1971), Scarf (1973), Dervis et al. (1982), Ponsard (1983), and Shoven and
     Whalley (1984).

theory of general equilibrium in an economy with a spatial separation between the location of production and consumption, but it was not until Lösch (1940) that spatial theory and general economic equilibrium analyses were integrated in an albeit constrained system of spatial equilibrium.

Inspired by the work of Cournot (1838) and Enke (1951), the spatial price equilibrium (SPE) model was formulated as a mathematical programming problem by Samuelson (1952). A quadratic programming solution of the SPE model was obtained by Takayama and Judge (1964). The model was initially formulated in a "Marshallian" tradition with inverse demand and supply functions but has also been given a "Walrasian" formulation with ordinary functions [Takayama and Woodland (1970), Friesz et al. (1983)]. The SPE model has, besides the analysis of commodity production and trade,[6] frequently been used in freight and passenger transport models.[7] The latter are also named freight-network equilibrium models, because of the considerable increase in the network complexity compared with the traditional SPE model. Disequilibrium formulations of the SPE model have also been suggested by Thore (1986) and Nagurney and Zhao (1988). The SPE model has primarily been connected with partial studies of a set of markets, without endogenous income effects.

Isard and Ostroff (1958), starting from the work by Arrow and Debreu (1954) formulated conditions for a general spatial equilibrium with an explicit spatial price condition, a transport sector and endogenous income effects. Harris and Nadij (1985) argue that the Arrow-Debreu system already contains a spatial economy and it is thus a generalization of the Isard and Ostroff model. Among the differences between the two one may mention the allowance for spatially heterogeneous commodities in the Arrow-Debreu system, while Isard and Ostroff assumed commodities to be spatially homogeneous as in the SPE model. Cross-hauling then is excluded. However, and as will be discussed further in Chapter 7, the existence or not of cross-hauling is generally a question of an appropriate notation and choice of elasticities. Another assumption made by Isard and Ostroff, which is often necessary in applied work, is an introduction of particular exporting and importing firms. Arrow-Debreu instead gave firms a possibility either to sell to a trading firm or to

---

6   Kennedy (1974), Uri (1978), Takayama and Labys (1986), Kjellman (1988).
7   Friesz et al. (1983), Harker (1985), Friesz and Harker (1985), Nagurney (1987).

trade the commodities themselves. Even if the Arrow-Debreu model is a general system, the contribution by Isard and Ostroff should thus be seen as a way towards an operationalization of a spatial general equilibrium model.

Although the existence of a general equilibrium was proved during the fifties, the development of algorithms for the computation of an equilibrium is more recent. Such applied numerical models in the Walras-Arrow-Debreu tradition are often named Computable General Equilibrium (CGE) models. An early application within this class is the multisector growth (MSG) model by Johansen (1960). However, it is first during the seventies, and the modern computer development, that the CGE model became available for a larger group of researchers. The algorithms for computation of nontrivial general equilibria have been improved during the last fifteen years. As a consequence, the number of reports on applied equilibrium models also have increased. Comparison between different model types and their properties have also been presented.[8]

In Sweden, one finds early applications of the CGE model in energy studies [Bergman (1978), Lundgren (1985)]. The vintage approach has been used in CGE models of structural change at the national level by Johansson and Persson (1983, 1987) and Persson (1983). A putty-clay approach displaying some similarities with the vintage formulation may also be found in a model by Bergman (1982, 1986). In Norway, CGE models have been formulated by e.g. Haaland et al. (1987) and Haaland (1988).

The spatial extensions of the applied Walrasian equilibrium model have mainly been accomplished in international trade.[9] It is only recently, during the second half of the eighties, that the approach has been extended to interregional computable general equilibrium (ICGE) models. Kimbell and Harrison (1984), Liew (1984), Persson and Westin (1985), Buckley (1987), Madden (1987), Suknam and Hewings (1987), Higgs et al. (1988), and Westin (1988), are examples of contributions in this direction.

---

8   Scarf (1973), Ginsburgh and Waelbroeck (1981), Dervis et al. (1982), Scarf and Shoven (1984), Shoven and Whalley (1984).

9   Examples are found in Shoven and Whalley (1974), Deardorf and Stern (1986), Whalley (1985), and Srinivasan and Whalley (1986).

Two traditions of applied spatial equilibrium models have, as is evident from the above survey, developed during the seventies and eighties. Some of the contributions in the fields are given in Figure 2.1.

A comparison of the SPE and ICGE frameworks reveals that besides the Marshallian formulation traditionally found in the former and the exclusion of income effects, other differences also exist. The SPE framework is commodity-oriented and, in accordance with classical and many modern international trade and locational theories, treats commodities as spatially homogeneous. The ICGE models have commodities aggregated into sectors at the "meso" level, as input-output models. They treat sector outputs as spatially heterogeneous and make use of the "Armington" assumption [Armington (1969)]. As a consequence, goods from a sector may be treated as imperfect substitutes over space. The approach allows for cross-hauling and implies that extreme specialization may be avoided. This gives an empirically tractable property to the ICGE model.

The conflict between empirically observed cross-hauling and the assumptions of homogeneous commodities, perfect competition and complete information in the SPE model has also raised an interest for integration of gravity [Harker (1988), Bröcker (1988a, 1988b)], entropy [Johansson and Batten (1983)] or logit [Echenique et al. (1988)] approaches with the SPE model. The gravity extended models have by Harker (1988) been called Dispersed Spatial Price Equilibrium models (DSPE). The close relation between gravity, logit, and entropy models makes this name appropriate also for this larger group of models. DSPE models represents an alternative to the Armington assumption when heterogeneity is introduced in order to decrease the tendencies in the SPE models towards over-specialization in trade. So far no comparative evaluation of Armington and DSPE models have been made.

A homogeneous commodity assumption may be appropriate in models of raw materials or when a very detailed level of aggregation is possible. However, in models of multisector economies aggregation generally becomes necessary, due to empirical and computational limitations.

Figure 2.1   Contributions to two traditions of spatial equilibrium modelling.

SPATIAL PRICE EQUILIBRIUM AND FREIGHT-NETWORK EQUILIBRIUM MODELS	INTERREGIONAL AND INTER-NATIONAL COMPUTABLE GENERAL EQUILIBRIUM MODELS

Cournot (1838)	Cournot (1838)
Walras (1874)	Walras (1874)
Koopmans(1949)	
Enke (1951)	Isard (1951)
Samuelson (1952)	
Arrow and Debreu (1954)	Arrow and Debreu (1954)
Beckman, McGuire and Winston (1956)	
Isard (1956)	
Isard and Ostroff (1958)	Isard and Ostroff (1958)
	Debreu (1959)
	Moses (1960)
	Johansen (1960)
Takayama and Judge (1964)	
Hardley and Kemp (1966)	Hardley and Kemp (1966)
Woodland (1969)	Armington (1969)
Takayama and Woodland (1970)	
Takayama and Judge (1971)	
	Shoven and Whalley (1974)
Dafermos (1980)	
Florian and Los (1982)	
Friesz et al. (1983)	
	Kimbell and Harrison (1984)
	Liew (1984)
	Whalley (1985)
	Whalley and Trela (1986)
Nagurney (1987)	Buckley (1987)
Harker (1988)	Westin (1988)
Bröcker (1988b)	Westin (1990)

The attempts to formulate multiperiod formulations of SPE, CGE and ICGE models fall into the two classes; recursive models with myoptic or static expectations and intertemporal equilibrium models. The first type is represented in Takayama and Judge (1971)[10], de Melo and Dervis (1977), Johansson and Persson (1983), Persson and Johansson (1982) and Bergman (1986), while the intertemporal equilibrium models are represented by Manne and Perckel (1985), Takayama et al. (1984) and Nagurney and Aronson (1988).

The problems inherent in discrete time models, which we discuss in Chapter 8, have also lead to an interest in continuous time models with integrated price and quantity processes. An early model in this category is the dynamic input-output system by Brody (1970), a more recent model is Andersson and Zhang (1988), where the possibility to stabilize a Leontief system is studied.

The class of spatial multisector models suggested in the following contains formulations of quantity, fixed-price, and price equilibrium models discussed above. Both SPE and ICGE models, as well as input-output and transport models are contained in the class. One of our main topics in the following chapters is to examine the relations between those traditions. In Chapter 4 quantity and fixed-price models are discussed. Those models are supply-side oriented. Price endogenous SPE models with price elastic demand are introduced in Chapter 5. In Chapter 6 are various ICGE models discussed, while in Chapter 7 the integrated class is formulated. In Chapter 8 some attempts to formulate multiperiod equilibrium models and the relation between continuous-time and equilibrium models are considered.

---

10   An intertemporal formulation may also be found here.

2.4   FORMULATION OF ECONOMIC MODELS AS COMPLEMENTARITY PROBLEMS

A number of economic models may be represented as complementarity problems. This representation is appropriate when one is dealing with equilibrium as well as linear and nonlinear programming models. As was discussed in the previous section, an extensive part of the spatial multisector models are of those types. In the sequel, our models are formulated in a unifying complementarity framework. By such a reformulation of different models into a common format, comparisons of the model properties become simplified. In this way, equivalence between different models may be demonstrated. Furthermore, the choice of an efficient solution algorithm is facilitated once a model has been shown to belong to a specific subclass of complementarity problems. This separation between model formulation and model solution is an important step in order to clarify model properties and development of wide range algorithms.

In this section, the complementarity problem is introduced. The relation between complementarity problems, variational inequalities, linear and quadratic programming problems, and equilibrium systems is then discussed. In the next section, some algorithms for the solution of complementarity problems are presented briefly.

Assume that z is a finite vector in $\mathbf{R^n}$ and that a mapping $\mathbf{F}: \mathbf{R^n} \to \mathbf{R^n}$ exists. Then the general complementarity problem may be defined as [Cottle and Dantzig (1968)],[11]

(CP):

Find $z^* \in \mathbf{R^n}$ that solves

$$\mathbf{F}(z^*) \geq 0, \quad z \geq 0, \quad z^{*T}\mathbf{F}(z^*) = 0. \tag{2.1}$$

When z is a price vector and $\mathbf{F}$ contains price dependent excess supply functions, this formulation is familiar as the Walrasian equilibrium condition [Varian (1984)]. The formulation may also represent the Kuhn-Tucker conditions in nonlinear programming [Intriligator (1981)]. A complementarity problem is thus characterized by

---

11   A T used as a superscript denotes a transposed vector or matrix.

weak inequalities and complementarity slackness. Members of this family are problems in which a subset of alternative techniques [e.g. vintages] shall be chosen and problems with bounds on prices and quantities, where it is not a priori clear which inequalities hold strictly in the solution.

Furthermore, given a convex and compact, i.e. closed and bounded, set $K \in R^n$ and the mapping $F: K \rightarrow R^n$, a variational inequality problem $(VIP[K,F])$ may be defined as [Pang and Chan (1982)],

$(VIP[K,F])$:

Find a vector $z^* \in K$ such that

$$F(z^*)^T[z - z^*] \geq 0, \qquad \text{all } z \in K. \qquad (2.2)$$

The complementarity problem (CP) corresponds to a problem $(VIP[R^n+, F])$, i.e. a variational inequality problem where K is constrained to the nonnegative orthant [Karamardian (1971), Lemke (1980)]. Hence, complementarity problems are a subset of variational inequality problems.[12]

Linear complementarity problems (LCP), where the mapping F is linear in z, are formulated as a subset of (CP) [Mathiesen (1985a)],

(LCP):

Find z^* and w such that,

$$w - f + Mz^*, \quad w \geq 0, \quad z^* \geq 0, \quad z^{*T}w - 0. \qquad (2.3)$$

Above, f is a vector and M is a matrix of constants. A linear programming problem is an (LCP). Furthermore, a quadratic programming problems also may be interpreted

---

12   It may be proved that when $F(z)$ is continuous, at least one solution of (2.2) exists, if $F(z)$ also is strictly monotone then (2.2) has at most one, globally unique, solution. Furthermore, if $F(z)$ is strongly monotone then a unique computable solution exists [Kinderlehrer and Stampacchia (1980), Nagurney (1987), Zhao (1989)].

as an (LCP).[13] To be a linear or quadratic optimization problem, the matrix M in (2.3) has to have the form,

$$M = \begin{bmatrix} 0 & -A^T \\ A & Q \end{bmatrix} = \begin{bmatrix} 0 & -A^T \\ A & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \qquad (2.4)$$

The matrix M is here decomposed into two matrices, the first is skew-symmetric and the second contains the symmetric submatrix Q. When Q is positive semidefinite, M is also positive semidefinite, and (LCP) may be reformulated and solved by quadratic programming methods. When Q is a zero matrix, (LCP) may be solved as a linear programming problem. Alternatively, any algorithm that solves (LCP) also solves a (LP) or (QP).

The connection between optimization, complementarity, and variational inequality problems may also be obtained in the following way. Let $G(z)$ be a continuously differentiable function defined on a open neighborhood of K and denote its gradient by $\nabla G(z)$. If there exists a $z* \in K$ such that,[14]

$$G(z*) = \underset{z \in K}{\text{Min}} \ G(z), \qquad (2.5)$$

then $z*$ also solves the following variational inequality problem,

Find $z* \in K$ such that
$$\nabla G(z*)^T[z - z*] \geq 0, \qquad \text{for all } z \in K. \qquad (2.6)$$

Hence, if $\nabla G(z) = F(z)$ or if appropriate integrability, i.e. symmetry, conditions hold, so that $G(z) = \int F(z) \, dz$, the variational inequality problem $(VIP[K, F])$ may be solved as an optimization problem. That is, if $z*$ solves (2.6), it also mini-

---

13   Cottle and Dantzig (1968), Mathiesen (1985a), Takayama and Labys (1986).
14   Friesz et al. (1985), Nagurney (1987).

mizes $G(z)$ over K. If $G(z)$ is strictly convex, the solution is global. Hence, monotonicity in variational inequality problems plays an analogous role to convexity in optimization. One may also note that a gradient mapping is characterized by a symmetric Jacobian matrix, that is $\delta F_i / \delta z_j = \delta F_j / \delta z_i$, all i and j. The differentiated matrix $\nabla F(z)$ must thus be symmetric if $F(z)$ should be the gradient of $G(z)$. This integrability condition gives a connection with the symmetric matrix in (2.4) and is an essential point of separation between equilibrium problems which are convertible into extremal problems and those which not are convertible.

The behavioral assumptions thus determine the type of (CP), the character of the solution and the computation difficulties different economic models give rise to. A list of economic assumptions and their conseqences on the computation of a solution include the following items:

* Separability, monotonicity, and continuity of demand, supply and transport cost functions imply existence of an equilibrium and simplify computation [Compare footnote 12].

* Replacement of monotone transport cost functions by non-monotone functions gives multiple equilibria. When transport costs are obtained endogenously from the behaviour of actors working on the network, monotonicity may be retained but computation difficulties will increase.

* Demand functions with income as an endogenous variable, instead of exogenously given, tends to cause indefiniteness of $F(z)$ and thus violate the integrability condition [Mathiesen (1985b)]. This also concerns multihousehold formulations with explicit budget constraints and multiregional models with balance of payment conditions, instead of overall conservation of flow conditions for the whole economy.

To illustrate how some of those features make an algorithm incapable of handling a specific problem, it may be appropriate to start with linear programming methods. Utilization of those implies with necessity that equilibria with two households and two commodities or more may not be solved by (LP) if each household has to fulfil a

budget constraint [Ginsburgh and Waelbroeck (1981)]. Production and demand functions also have to be modelled as activity analysis or step functions.

Another well known example of how an algorithm constrains the model behaviour may be obtained from linear programming trade models, where extreme specialization easily is obtained. If smooth price-sensitive demand and supply functions are introduced, the problem extends beyond the linear programming format. However, if those functions are given a linear form with symmetric substitution matrices, the problem is solvable as a quadratic programming problem [Takayama and Judge (1964)]. But, as soon as the substitution matrices of such demand or supply functions are nonsymmetric, it is no longer possible to formulate an optimization problem. Lack of symmetry is albeit often the result of unconstrained empirical estimations of multicommodity models. However, if the linear formulation is retained, such a model still is a (LCP).

When nonlinear demand, supply and transport cost functions or multihousehold formulations with endogenous budget constraints are introduced as endogenous elements of a multicommodity problem, it converts to a nonlinear complementarity problem (NCP). Hence, nonlinear Walrasian equilibrium models of the ICGE type and nonlinear SPE and Freight Network models belong to this class of problems.

Since algorithms for and properties of linear and quadratic problems are well studied, it pays off to reformulate a problem into a quadratic format whenever this is possible. However, recently there has been fast progress in the development of algorithms for general nonlinear formulations, some of which are referred in the following section.

2.5   ON ALGORITHMS FOR COMPLEMENTARITY PROBLEMS

The step from theoretical to applied modelling has always been constrained by the efficiency and availability of algorithms. Because of this, it is appropriate to make some comments on the algorithms for the solution of complementarity and variational inequality problems.[15] It has, due to the development of computers during the seventies and eighties, been a rapid increase in the dimension and complexity of the problems which are possible to solve. A tendency towards a unified framework for different types of algorithms may also be seen. However, there is still a need for development of special algorithms which, for example by exploitation of sparsity in large matrices, may be advantageous compared to standard tools.

One may distinguish between four broad groups of algorithms for solutions of complementarity and variational inequality problems; log-linearization methods, optimization methods, fixed-point methods and iterative methods. Among each of these, different variants exist and still other methods utilize, at different stages of computation, methods from more than one of the groups. In general, the problems associated with algorithms are related to existence of feasible solutions, global or local uniqueness of an obtained solution, and convergence properties.

Johansen (1960), used a log-linearization approach to solve the MSG model. In this approach, the model is first reformulated into a linear system in growth rates of the endogenous variables. This system is solved by matrix inversion. Hence, the algorithm does not deal with the complementarity problem directly but approximates a solution. The approach has been further developed, by use of a second order Taylor expansion, and is utilized by Dixon et al. (1982) in the Australian ORANI model and by Liew (1984) in an interregional model. Among the advantages of this approach is the ease by which variables may be moved between the exogenous and endogenous sets. The disadvantage is that only an approximate solution is obtained.

A second type of solution methods is optimization algorithms. They have played a central role in applied work since the last world war. The properties of linear pro-

---

15   Further details may be found in Lemke (1980), Dervis and de Melo (1982), Pang (1984), and
      Talman and van der Laan (1987).

gramming algorithms such as the simplex algorithm and some nonlinear programming algorithms are nowadays well known. Because of the equality between a single household Negishi welfare optimum and a general equilibrium solution, the equilibrium may be found by optimization. However, as was discussed above, optimization is not generally a sufficient tool for computation of general equilibrium models. On the other hand their applicability may, if used together with iterative algorithms, increase considerably.[16]

Among the algorithms based on fixed-point theorems, the one by Scarf (1973) is the most well known. The advantage of Scarf's method is that if a solution exists it may be found at a chosen level of accuracy. A disadvantage of the method, and the reason for the ongoing search for other algorithms, is that the search may be very costly and time consuming.

Programming algorithms and fixed-point algorithms are based on pivoting. They find a solution or show that a feasible solution doesn't exist. Convergence may in those cases be proved analytically. Iterative methods do normally not give such an exact solution. The simplest and most well known iterative algorithm is the tatonnement process. Given a sign preserving mapping $\phi: \mathbf{R}^n \to \mathbf{R}^n$, usually simplified to an adjustment constant, this iterative process, where k denotes the iteration number, may be written:

$$\mathbf{z}^{k+1} = \mathbf{z}^k - \phi[\mathbf{F}(\mathbf{z})]. \qquad (2.7)$$

If $\mathbf{z}$ is a vector of prices and $\mathbf{F}(\mathbf{z})$ a vector of excess supply functions, the process is the Walrasian price process. Gross substitutability between all goods is a sufficient condition for stability of the process. This is a strong assumption which is a theoretical disadvantage of the tatonnement method. However, in practical work, where the modeler has insights into the structure of the problem, this method and other iterative methods have shown to converge fast. [Ginsburgh and Waelbroeck, (1981), Dervis et al. (1982), Persson (1983)]. The tatonnement algorithm (2.7) is a special case of a larger class of iterative methods for nonlinear systems. Other members of this class are Jacobi, Gauss-Seidel and Newton methods. The theoretical con-

---

16   Ginsburgh and Waelbroeck (1981) give a comprehensive treatment of the relation between equilibrium and programming solutions.

vergence properties of iterative methods are also generally unclear [Mathiesen (1987)]. Convergence problems may either be a result of an incorrect step size adjustment function, an inappropriate decomposition of the problem, or non-existence of a feasible solution.

The complementarity formulation has also been used directly to develop algorithms based on the solution of sequences of linear complementarity problems (SLCP). Mathiesen (1985a) has suggested an iterative approach with an outer part of Newton type where the (LCP) linearization is made by a first-order Taylor approximation. The resulting problem is solved in the inner part by an algorithm due to Lemke (1965), which has properties similar to a pivoting fixed-point algorithm. Existence and uniqueness of equilibrium and the computational properties of complementarity algorithms, especially for equilibria with endogenous incomes, are also unclear. However, a lot of research is going on in the field. Alternative algorithms are analyzed in Mathiesen (1985a), and Talman and van der Laan (1987). In applied work, the algorithms often have shown to be well behaved. Complementarity algorithms for freight-network models have also been explored by Friesz et al. (1983).

Variational inequality formulations have been used to develop decomposition algorithms for SPE models [Dafermos (1980), Nagurney (1987, 1988)]. Such a decomposition may for example be made over demand or supply markets into iterative solution of sequences of programming problems. Those may then be solved as nonlinear problems or after linearization by quadratic programming. Recently, Zhao (1989) claims to have developed convergent variational inequality algorithms for Walrasian equilibrium problems in productive economies i.e. for CGE and ICGE models. Reports on the speed of the algorithms for different sizes of problems has so far not been presented.

The rapid progress in the field of algorithms, where nonintegrable models are approximated by sequences of integrable models and where fixed point methods are accelerated by iterative methods indicate an integration of different algorithms for optimization, fixed-point and nonlinear equations [Mathiesen (1985a)].

2.6   TRADE-OFF'S IN SPATIAL MULTISECTOR MODELLING

An aim of this work is, as was discussed in Chapter 1, to present a class of spatial multisector models for the analysis of structural change. The vintage formulation of the sector production function constitute a common characteristic of this model class. We have furthermore emphasized that the models should be possible to apply to real problems. This is motivated by the need to test theories and different formulations against empirical observations. It is a necessary condition if the theory of spatial structural change shall develop in productive directions.

For each given situation, the choice of a particular model from this class has to be made with regard to:

*   the modelling purpose and context,
*   the behavioral assumptions,
*   the empirical information,
*   the available algorithms.

Previously it was discussed how available algorithms may constrain the possible behavioral assumptions. The behavioral assumptions are in applied modelling further constrained by the available empirical information. An appropriate use of empirical information is in turn related to the purpose and context of the model formulation. The more nonlinearities we want to introduce, the more empirical information is generally needed.

In cases where good estimates are nonexistent, one may always argue that guesses are possible. Such guesses may be used to show the sensitivity of, and the restrictions inherent in a model. Such an analysis is especially important with regard to parameters which are assumed to be fixed at some arbitrary or conventional value. The obvious problem in relation to such exercises is that uncertain guesses give unreliable results.

We have in this chapter given a characterization of important and desirable features in models of structural change. We have also reviewed some existing models and algorithms for this category of models. Generally, the available algorithms do not

any longer constrain the model development in the same strict way as before. These type of constraints will probably be reduced even more in the future. Hence, more efforts may be directed towards model formulation and empirical estimation.

3 VINTAGE STRUCTURES, INVESTMENTS AND TECHNICAL CHANGE

3.1 INTRODUCTION

Hotelling (1932) argued that the industry supply function should be determined from the unit cost distribution of the units producing a commodity. This distribution would more or less follow the normal distribution and the supply function would thus be a sigmoid function.

In this chapter we introduce a vintage model of a single industry with the properties suggested by Hotelling. A fundamental feature of the model is that the industry is represented by a distribution of techniques and not, as is usual in traditional neoclassical models, by an average production function. The approach may be traced back to the above mentioned work by Hotelling, but also to earlier work in this direction by Heckscher (1918) and Marshall (1920), as well as the contributions by Svennilson (1938), Houthakker (1955), Salter (1960), Johansen (1965, 1972), and Hildenbrand (1981).

This vintage formulation has also strong relations to models with putty-clay production functions in continuous time[1] and models which are built on activity analysis and multiplant formulations.[2] The model can be estimated with empirical information at the establishment level. A sequence of such empirical studies have also confirmed the suggestion made by Hotelling.[3] The establishment orientation gives a

---

1 See e.g. Solow (1962), Phelps (1963), Bardhan (1966), and Bliss (1968).
2 See e.g. Takayama and Judge (1971), Kennedy (1974), Takayama (1978), Uri (1978) and Ginsburgh and Waelbroeck (1981).
3 Examples of empirical and theoretical contributions with close connections to this vintage formulation may be found in Johansson and Strömqvist (1980, 1981), Johansson and Persson (1983), Karlqvist and Strömqvist (1982), Wibe (1980, 1982), Førsund and Hjalmarsson (1987), Strömqvist (1983), Førsund (1984), Seierstad (1985), Johansson (1986), Westin (1987) and Westin (1988).

further relation to the engineering production functions developed by for instance Chenery (1949).

The vintage model is formulated as a set of linear programming problems. In this chapter, the industry production, cost, and profit functions as well as the ordinary and inverse supply functions are derived. The relations between the above functions, the interpretation of their dual problems, and the implicit prices obtained under different exogenously imposed price and quantity conditions are analyzed and assessed. In this way models already established are brought together and arranged in a systematic way. Linear complementarity formulations of the functions are derived as a prelude to the introduction of the vintage supply-side in multisector and spatial models.

In section 3.2, the presentation is limited to a formulation without investments. The model may thus be interpreted as a short-run model [Johansen (1972)] since the capacity is not allowed to adjust within the time horizon. The adjustment process is instead concentrated on the output adjustment in relation to given constraints on factor supply, demand and prices. A model, which may be interpreted as a medium-term model, and where new capacities are introduced endogenously, is discussed in section 3.3. The exit and entry properties of the model are compared with empirical observations in section 3.4. Introduction of heterogeneous labour is discussed in section 3.5 while concluding comments are given in section 3.6.

The chapter is only devoted to single-period models of nonspatial economies. Extensions into spatial multisector models and a discussion of multiperiod formulations may be found in the subsequent chapters.

3.2 A VINTAGE MODEL OF THE PRODUCTION SYSTEM

3.2.1 CHARACTERIZATION OF AN INDIVIDUAL VINTAGE

A sector or industry which produces a single homogeneous commodity, q_i, is at a given point in time described by a set of vintages, denoted by $\tau = 1, \ldots,$ NV. Each vintage is characterized by a capacity constraint, $\bar{q}_i(\tau)$ and an ex post constant returns to scale (CRTS) technique of the following Leontief type,[4]

$$q_i(\tau) = \min(q_{1i}(\tau)/a_{1i}(\tau), \ldots, q_{NSi}(\tau)/a_{NSi}(\tau),$$
$$L_i(\tau)/l_i(\tau)), \tag{3.2.1}$$

$$q_i(\tau) \in [0, \bar{q}_i(\tau)], \tag{3.2.2}$$

Deliveries of intermediate inputs from sector j to vintage τ in sector i are denoted $q_{ji}(\tau)$; $i, j = 1, \ldots,$ NS, with $a_{ji}(\tau)$ as the associated input coefficients. The latter determine intermediate commodity demand, $q_{ji}(\tau)$, in the same way as in an input-output model,

$$q_{ji}(\tau) = a_{ji}(\tau)q_i(\tau). \tag{3.2.3}$$

Labour demand in each vintage is denoted by $L_i(\tau)$ and the unit labour coefficient is denoted $l_i(\tau)$. Labour is thus for the moment assumed to be homogenous and the following labour demand function is obtained,

$$L_i(\tau) = l_i(\tau)q_i(\tau). \tag{3.2.4}$$

The Leontief production function assumes away substitution possibilities within each vintage, and reflects the rigidities in a production process ex post of the installation

---

4   All definitions range, if not otherwise indicated, over the complete set of a variable.

of production equipment. An implemented technique may be seen as chosen from an ex ante "frontier" production function at the micro level with such substitution possibilities.[5]

In Figure 3.1, a graphical description of a discrete "vintage capacity distribution" in a two dimensional input space is given.
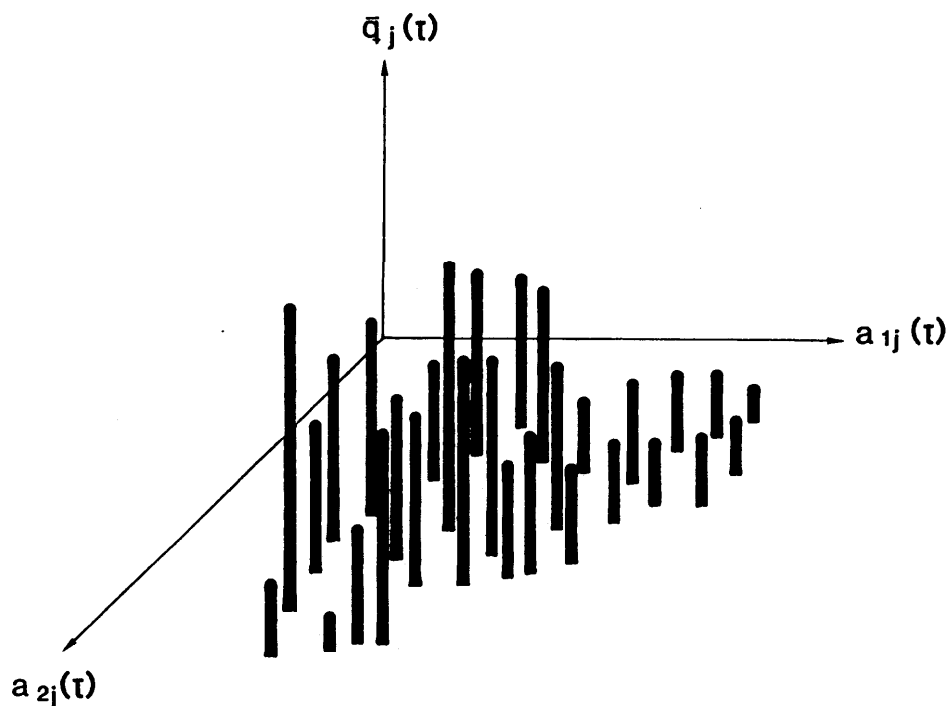
Figure 3.1   A discrete capacity distribution in a two-dimensional input space. [Adapted from Johansson and Strömqvist (1980)]

---

5   This was suggested by Johansen (1972). However, restricted ex post substitution between groups of inputs may be realistic even in short-term and may be taken care of by CES or nested CES functions in each vintage. The nonnested Leontief production function in (3.2.1) may be seen as a special case of such a general formulation.

The main advantage gained from a utilization of vintage distributions compared with average sector functions, is that the latter does not describe the structure in the sector. Structural change is reflected by the rigid adjustment of vintage distributions in the input space. Such adjustment cannot be captured by CES sector production functions, for instance. Actually, Hildenbrand (1981) argues that the ex post hypothesis implies that constant returns to scale never prevail, that the production function never is homotetic, and that the elasticity of substitution never is constant. A CES function would then impose a hard constraint on the behaviour in the industry.

The above capacity distribution is a technical description of an industry. However, technical information is clearly not enough to understand the decision to operate, renew, or close down a plant. Prices have to be introduced in order to add economic considerations to the problem.

Assume that the economy is competitive in the sense that the sector consists of a large number of vintages [production units or firms] and that each decision maker acts as a price-taker. Moreover, let p_j, for the time being, denote an exogenously given market price, referring to the supply from sector j, and let w_i denote an exogenous wage level in sector i. The unit cost function related to the Leontief production function of vintage r, is then specified by the following linear, scale-independent function:

$$v_i(r) - \sum_j p_j a_{ji}(r) + w_i l_i(r). \qquad (3.2.5)$$

With a Leontief technique, the factor demand remains unchanged when factor prices change. The isocost curve, dual to the Leontief production function, is thus a linear function.

Each vintage has factor demand functions which are linear in output up to full capacity. As long as a change does not cause a close down of a vintage, its demand functions are independent of relative factor prices.

The linear unit gross profit function for each vintage, dependent on output and factor prices is, by an extension of the unit cost function, given as:

$$\pi_i(\tau) = p_i - v_i(\tau). \tag{3.2.6}$$

Due to the linear properties of this function, gross profits are maximized at a production level constrained either by the available capacity or by demand. When the gross profit covers an ordinary rate of return on the capital in the vintage, it may generate a net profit. This net profit would in such cases represent control of other scarce resources, such as technical, management, marketing or localizational advantages.

3.2.2   THE VINTAGE PRODUCTION FUNCTION

Given the above definitions of the production, cost, profit and factor demand functions which characterize the price-taking "Leontief vintage", the corresponding industry [or sector] functions may be derived. The vintage production function is built by the set of production functions described in (3.2.1) - (3.2.4).[6] For sector i, the function is determined by solution of the linear programming problem (A:I) below, as the set of constraints varies,[7]

---

[6]   This function was called the restricted or short-run macro production function by Johansen (1972).

[7]   If not otherwise indicated all summations range over the complete set of a variable.

(A:I):

$$\text{Max}_{\{q(\tau)\}} \quad \sum_{\tau} q_i(\tau) \tag{3.2.7}$$

s.t. $\sum_{\tau j} a_{ji}(\tau) q_i(\tau) \leq q_{ji},$ all j   (3.2.8)

$\quad\quad\quad \sum_{\tau} l_i(\tau) q_i(\tau) \leq \overline{L}_i$ (3.2.9)

$\quad\quad\quad\quad\quad q_i(\tau) \leq \overline{q}_i(\tau),$ all τ (3.2.10)

$\quad\quad\quad\quad\quad q_i(\tau) \geq 0,$ all τ (3.2.11)

We have, although the constraints may be formulated in a more general way, used a fixed set of intermediate resources, q_{ji}, and a fixed labour supply, $\overline{L}_i$. The problem determines, for any convex set of input constraints, a concave sector production function, $q_i(q_{ij}; \text{ all } j, \overline{L}_i)$, bounded from above by the aggregate sector capacity, $\overline{q}_i$. Hence,

$$0 \leq q_i - \sum_{\tau} q_i(\tau) \leq \sum_{\tau} \overline{q}_i(\tau) - \overline{q}_i. \tag{3.2.12}$$

The problem (A:I) is based purely on a technical description of the sector, comparable with the information in Figure 3.1. However, and as is well known, as soon as the programming problem has been formulated, economic interpretations may be derived from the dual problem. Let nonnegative dual variables σ_j; all j, μ_i and $\overline{\sigma}_i(\tau)$; all τ, be associated with conditions (3.2.8) - (3.2.10) respectively. Those may be interpreted as the marginal productivities of each input and the quasi-rent obtained from each capacity, i.e. their shadow prices in terms of units output.[8] The dual problem to (A:I) is,[9]

---

8   See Førsund (1984) for a discussion regarding the concept of "quasi-rent" and the origins from Marshall (1920).

9   We will use the following notation in relation to the optimization problems. A maximization is denoted (A) while (B) denotes a minimization problem. Irrespective of which of those is the primal, a primal-dual pair will have the same number, i.e. (A:I)-(B:I).

(B:I):

$$\underset{\{\sigma_j, \ \mu_i, \ \overline{\sigma}_i(\tau)\}}{\text{Min}} \quad \underset{j}{\Sigma} \ \sigma_j q_{ji} + \mu_i \overline{L}_i + \underset{\tau}{\Sigma} \ \overline{\sigma}_i(\tau) \overline{q}_i(\tau) \qquad (3.2.13)$$

$$\text{s.t.} \ \underset{j}{\Sigma} \ \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) + \overline{\sigma}_i(\tau) \geq 1, \quad \text{all} \ \tau \qquad (3.2.14)$$

$$\sigma_j, \ \mu_i, \ \overline{\sigma}_i(\tau) \geq 0, \quad \text{all} \ \tau, \ j \qquad (3.2.15)$$

Problem (B:I) may be interpreted as minimization of the imputed cost for utilization of given factors and capacities under a nonnegativity condition on the quasi-rent per unit of output in each vintage. The duality theorem of linear programming states that if feasible solutions exist, the solution of (A:I) equals the solution of (B:I).[10]

A partial analysis of an individual vintage production function implies, as discussed previously, that the choice of production level is constrained only by the demand or the production capacity. However, in this formulation at the industry level, zero output from a vintage is also obtained when the opportunity cost of production in a vintage, under a given set of input constraints, is unfavorable in comparison with other vintages in the industry.

This efficiency condition may also be illustrated by an inspection of the Kuhn-Tucker complementary slackness conditions, associated with each vintage output level in (A:I)-(B:I). The conditions are,

$$[1 - \underset{j}{\Sigma} \ \sigma_j a_{ji}(\tau) - \mu_i l_i(\tau) - \overline{\sigma}_i(\tau)] q_i(\tau) = 0,$$

$$q_i(\tau) \geq 0, \text{all} \ \tau. \quad (3.2.16)$$

It is obvious from (3.2.16) and (3.2.10) that a positive output less than full capacity, requires that the following equality is satisfied,

---

10 Dorfman, Samuelson and Solow (1958).

$$\sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) - 1. \tag{3.2.17}$$

Consequently, close down and zero output is obtained when,

$$\sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) > 1. \tag{3.2.18}$$

Full capacity utilization implies that, $\bar{\sigma}_i(\tau) > 0$, so the following equality holds,

$$\sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) + \bar{\sigma}_i(\tau) - 1. \tag{3.2.19}$$

This condition also gives a definition of the quasi-rent per unit of output in vintage τ in terms of the imputed prices of the input factors measured in units of output. An "active vintage" is defined as a vintage with $q_i(\tau) > 0$. Such a vintage satisifies (3.2.19) with $\bar{\sigma}_i(\tau) \geq 0$. The condition further suggests the interpretation of $\bar{\sigma}_i(\tau)$ as a measure of vintage efficiency, in spite of the fact that market prices have not been introduced.

It is not possible to give a simple analytical formulation of this vintage production function, as is possible with the CES and translog functions, for example. The shape of a specific function can only be obtained after repeated solutions of problem (A:1 - B:1), under variation of the input constraints.[11]

Capital does not, as in many neoclassical production functions, enter explicitly in the vintage function. Although capital is implicitly reflected by the capacity constraint, only variable inputs are considered. In addition, observe that we do not consider the costs associated with the utilization of the fixed factors. Hence, the model is useful mainly in short- and medium-term analyses.

---

11 However, solution of the industry cost function [See the following section 3.2.3] corresponding to this production function is often a faster way. This method was also used by Johansen (1972). Moreover, in Førsund et al. (1980) an algortihm is presented which is not based on repeated solutions of programming problems, but on a search procedure among input coefficients and capacity limits.

3.2.3 THE VINTAGE COST FUNCTION

So far, market prices have not been introduced at the industry level. If factor prices are introduced, it becomes possible to formulate the vintage cost function. In this subsection some properties of the cost function are derived and the linear programming formulation of the function is recast into a linear complementarity problem.

Consider a situation with a given level of demand d_i, less of aggregate capacity, $d_i \leq \overline{q}_i$.[12] For this level of demand, the vintage cost function, $\tilde{c}_i (p_j;$ all $j, w_i, q_i)$, can be obtained from solutions of the following problem under variation of prices and demand,

(B:II):

$$\underset{\{q_i(\tau)\}}{\text{Min}} \; \sum_\tau v_i(\tau)q_i(\tau) \qquad\qquad (3.2.20)$$

$$\text{s.t.} \qquad \sum_\tau q_i(\tau) \geq d_i \qquad\qquad (3.2.21)$$

$$q_i(\tau) \leq \overline{q}_i(\tau), \qquad \text{all } \tau \qquad (3.2.22)$$

$$q_i(\tau) \geq 0, \qquad \text{all } \tau \qquad (3.2.23)$$

With this formulation, quantitative factor resource constraints are assumed away. Information about the scarcity of a factor is instead given exogenously from the market factor prices facing the industry. If, in (B:II), d_i is continuously increased from zero to $\overline{q}_i$, while the vintages are numbered in order of activation, a monotone sequence of increasing cost levels is obtained. This sequence may be written as,

$$0 \leq v_i(1) \leq v_i(2) \leq \ldots \leq v_i(\tau) \leq \ldots \leq v_i(NV). \qquad (3.2.24)$$

The marginal cost level, $\tilde{v}_i$, is thus given as a function of the sector production level and the unit cost in the marginal technique $\tilde{\tau}$,

---

12 This condition is necessary to guarantee a nonempty solution space.

$$\tilde{v}_i(q_i) - \{\min v_i(\bar{\tau}): \sum_{\tau=1}^{\bar{\tau}} \bar{q}_i(\tau) \geq q_i\}. \tag{3.2.25}$$

This multivalued step function is also utilized in the inverse vintage supply function, which determines the supply price $\tilde{p}_i(q_i)$ as a function of the level of industry output.[13] The multivalued character of the function is revealed by the price indeterminacy which is obtained at the full capacity limit of each vintage, i.e. at the end of each step in the function. The inverse supply function is written as,

$$\tilde{p}_i(q_i) - \left[\begin{array}{ll} \tilde{p}_i - v_i(\bar{\tau}) & : q_i(\bar{\tau}) < \bar{q}_i(\bar{\tau}) \\[2mm] v_i(\bar{\tau}) \leq \tilde{p}_i \leq v_i(\bar{\tau}+1): & q_i(\bar{\tau}) - \bar{q}_i(\bar{\tau}), \ \bar{\tau} < NV \\[2mm] v_i(\bar{\tau}) \leq \tilde{p}_i \leq \infty & : q_i(\bar{\tau}) - \bar{q}_i(\bar{\tau}), \ \bar{\tau} - NV \end{array} \right.$$

$$\tag{3.2.26}$$

The shape of the function is obviously determined by the input prices and the technical and capacity constraints given by the vintage production function. Examples of such inverse vintage supply functions, also called "Heckscher - functions", for the manufacturing industry in the Swedish county of Norrbotten, are given in Figure 3.2.

The name of the "Heckscher - function" originates from the Swedish economist E. Heckscher, who used a similar illustration while discussing the effecs of decreases in the customs duty on a sector [Heckscher (1918)]. The supply price for each year is normalized by the unit cost which gives a zero quasi-rent. The figure reveals some of the structural problems faced by the industry in the county in the second half of the seventies. Between 1973 and 1978, total supply decreased while the cost level increased, the supply function thus shifted backwards and upwards. After changes in the relative prices, due to devaluations, as well as scrapping of old and investments in new techniques, the industry could recover in the beginning of the eighties and shift the supply function back to and beyond the 1973 position.

---

13   The supply function may also be written with input prices and wages as further arguments.

SUPPLY PRICE
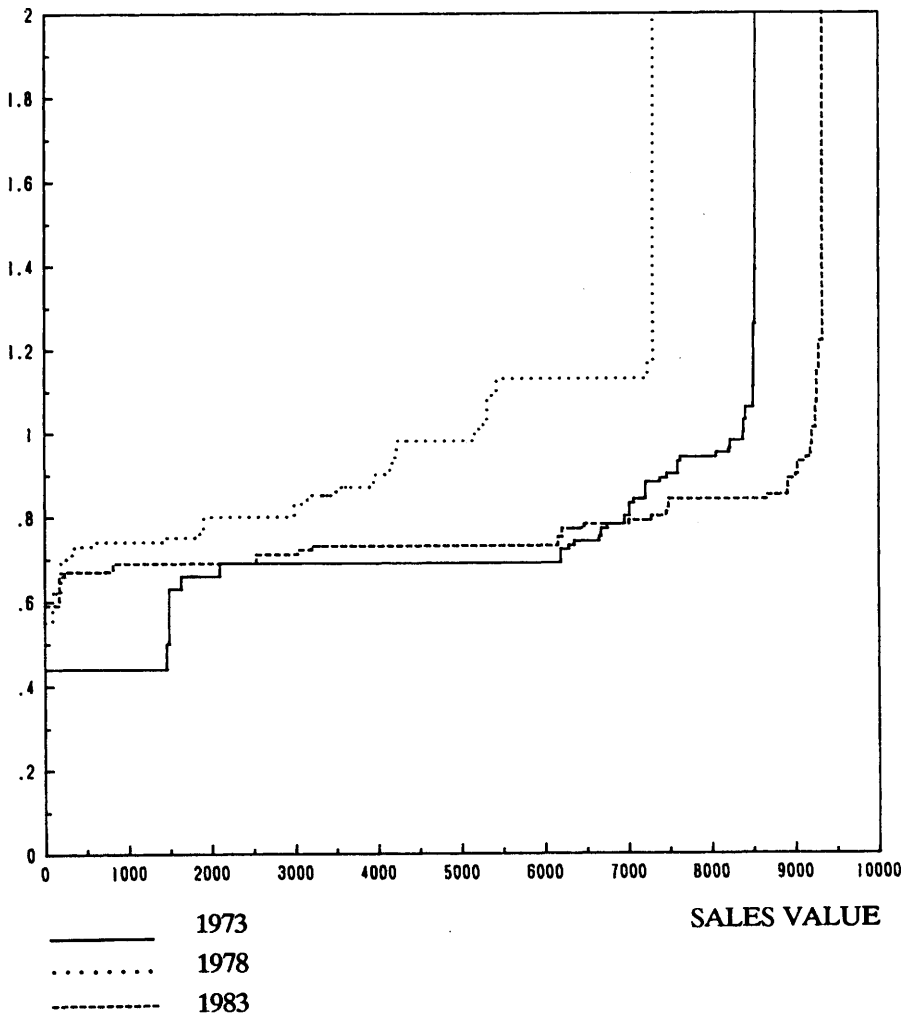
Figure 3.2   Inverse supply functions of "Heckscher" type for the manufacturing
                  industry in Norrbotten, 1973, 1978, and 1983. [Data from SIND,
                  1980 prices, thousands SEK].

The marginal cost level was used in the formulation of the supply function. It is possible to show that this level always is equal to or exceeds the average cost level.[14] It thus reflects the mix of constant and decreasing returns to scale in a vintage production function of "ex post" type.

The vintage cost function, $\tilde{c}_i(\cdot)$, is nondecreasing, homogeneous of degree one and concave in factor prices.[15] Hence, the factor demand functions are homogeneous of degree zero, i.e. a uniform change of all factor prices has no effects on the amount and relative shares of demanded factors.

To the increasing sequence of unit cost levels in (3.2.24), corresponds a monotone decreasing sequence of dual variables associated with the capacity constraint (3.2.22) in problem (B:II)[16],

$$\bar{\sigma}_i(1) \geq \bar{\sigma}_i(2) \geq \ldots \geq \bar{\sigma}_i(NV) \geq 0. \qquad (3.2.27)$$

We may therefore also interpret $\bar{\sigma}_i(\tau)$ as an efficiency measure related to the unit cost advantage of vintage τ, i.e. quasi-rents.[17]

Let σ_i denote the shadow value of the demand condition (3.2.21) in problem (B:II), interpreted as the opportunity cost to produce the marginally demanded unit. The dual to problem (B:II) is then,

---

14 Førsund and Hjalmarsson (1987).

15 Proofs may be obtained from Varian (1984). Homogeneity implies that a change of all factor prices by a positive scalar, does not affect the ordering of the vintages. Hence, the costs increases with a factor given by the scalar. Concavity is a result of the substitution process away from factors with increasing prices.

16 With an inelastic demand function, as in (B:II), the last inequality holds as an equality.

17 Observe that a similar notation on a dual variable to a specific condition is utilized independent of the objective function. The dimension in which the dual is measured therefore varies with the objective function.

(A:II):

$$\underset{\{\overline{\sigma}_i(\tau),\ \sigma_i\}}{\text{Max}} \quad \sigma_i d_i\ -\ \underset{\tau}{\Sigma}\ \overline{\sigma}_i(\tau)\overline{q}_i(\tau) \qquad\qquad (3.2.28)$$

$$\text{s.t.} \qquad\qquad \sigma_i\ -\ \overline{\sigma}_i(\tau)\ \leq\ v_i(\tau), \quad \text{all } \tau \qquad (3.2.29)$$

$$\overline{\sigma}_i(\tau),\ \sigma_i\ \geq\ 0, \qquad\qquad \text{all } \tau \qquad (3.2.30)$$

This problem may be interpreted as maximization of the imputed sales value less the "imputed profits", under an efficiency condition on each vintage. From condition (3.2.29) and the fact that $q_i(\tau)$ is the dual variable of this condition, we obtain the following rules for the activity of each vintage,

$$\overline{q}_i(\tau)\ -\ q_i(\tau)\ >\ 0, \quad \text{if}\quad \sigma_i\ >\ v_i(\tau)\ ;\ \overline{\sigma}_i(\tau)\ >\ 0$$

$$\overline{q}_i(\tau)\ \geq\ q_i(\tau)\ \geq\ 0, \quad \text{if}\quad \sigma_i\ =\ v_i(\tau)\ ;\ \overline{\sigma}_i(\tau)\ =\ 0 \qquad (3.2.31)$$

$$q_i(\tau)\ =\ 0, \quad \text{if}\quad \sigma_i\ <\ v_i(\tau)\ ;\ \overline{\sigma}_i(\tau)\ =\ 0$$

It is clear from above that the imputed price of demand corresponds to the sum of the variable cost of production and the quasi-rent in the marginal vintage. By that, conditions similar to conditions (3.2.17) - (3.2.19) given by problem (A:I)-(B:I) have been received. Moreover, if p_j, w_i, and d_i in (A:II)-(B:II) are set to the optimum values of $\overline{\sigma}_j$, μ_i, and q_i in a solution of (A:I), the solutions of (A:I) and (A:II) are equal. This illustrates a duality between cost and production functions. The cost function reveals the technological properties given by the sector production function. As mentioned previously, this may be used to explore the isoquant map and the region of substitution of the vintage production function.

The problem (A:II)-(B:II) may be reformulated as an (LCP). Hence, the formulation is easily introduced in more general problems and solved by algorithms other than (LP). To simplify, we change to a matrix notation while developing the (LCP) formulation.

Let $q_1(\tau)$ denote a vector of vintage production levels and $\bar{q}_1(\tau)$ a vector of vintage capacities in a sector,

$$q_1(\tau) - \begin{bmatrix} q_i(1) \\ \cdot \\ \cdot \\ q_i(NV) \end{bmatrix}, \quad \bar{q}_1(\tau) - \begin{bmatrix} \bar{q}_i(1) \\ \cdot \\ \cdot \\ \bar{q}_i(NV) \end{bmatrix}. \quad (3.2.32)$$

$[NV \times 1]$ $[NV \times 1]$

Moreover, let $v_1(\tau)$ be a vector of fixed vintage unit costs as defined in (3.2.5), and S_1 a summation vector of ones,

$$v_1(\tau) - \begin{bmatrix} v_i(1) \\ \cdot \\ \cdot \\ v_i(NV) \end{bmatrix}, \quad (3.2.33)$$

$[NV \times 1]$

$$S_1 - \begin{bmatrix} 1 & \cdots\cdots\cdots & 1 \end{bmatrix}. \quad (3.2.34)$$

$[1 \times NV]$

The capacity distribution of active vintages at a given demand, d_i, is obtained from a reshapening of problem (B:II) into,

(B:II'):

$$\underset{\{q_i(\tau)\}}{\text{Min}} \quad v_i(\tau)^T q_i(\tau) \tag{3.2.35}$$

$$\text{s.t.} \quad S_i q_i(\tau) \geq d_i \tag{3.2.36}$$

$$q_i(\tau) \leq \bar{q}_i(\tau) \tag{3.2.37}$$

$$q_i(\tau) \geq 0 \tag{3.2.38}$$

Let σ_i as previously be the dual variable to the demand constraint (3.2.36) and let $\bar{\sigma}_i$ be a [NV x 1] vector of dual variables to constraint (3.2.37). Then, the dual problem to (B:II') is,

(A:II'):

$$\underset{\{\sigma_i, \bar{\sigma}_i(\tau)\}}{\text{Max}} \quad d_i \sigma_i - \bar{q}_i(\tau)^T \bar{\sigma}_i(\tau) \tag{3.2.39}$$

$$\text{s.t.} \quad S_i^T \sigma_i - \bar{\sigma}_i(\tau) \leq v_i(\tau) \tag{3.2.40}$$

$$\sigma_i, \bar{\sigma}_i(\tau) \geq 0 \tag{3.2.41}$$

From the Lagrangean function associated with (A:II')-(B:II') one may derive the following Kuhn-Tucker conditions,

$$\delta L/\delta q_i(\tau) - v_i(\tau)^T - \sigma_i S_i + \bar{\sigma}_i(\tau)^T \geq 0,$$

$$q_i(\tau)^T [\delta L/\delta q_i(\tau)] = 0, \quad q_i(\tau) \geq 0, \tag{3.2.42}$$

$$\delta L/\delta \sigma_i - d_i - S_i q_i(\tau) \leq 0,$$

$$\sigma_i [\delta L/\delta \sigma_i] = 0, \quad \sigma_i \geq 0, \tag{3.2.43}$$

$$\delta L/\delta \overline{\sigma}_1(\tau) - \overline{q}_1(\tau) + q_1(\tau) \leq 0,$$

$$\overline{\sigma}_1(\tau)^T[\delta L/\delta \overline{\sigma}_1(\tau)] - 0, \quad \overline{\sigma}_1(\tau) \geq 0. \qquad (3.2.44)$$

Those conditions may be rewritten as the nonnegative system,

$$\begin{bmatrix} \delta L/\delta q_1(\tau) \\ -\delta L/\delta \sigma_i \\ -\delta L/\delta \overline{\sigma}_1(\tau) \end{bmatrix} - \begin{bmatrix} v_1(\tau) \\ -d_i \\ \overline{q}_1(\tau) \end{bmatrix} + \begin{bmatrix} 0 & -S_1^T & I \\ S_1 & 0 & 0 \\ -I & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1(\tau) \\ \sigma_i \\ \overline{\sigma}_1(\tau) \end{bmatrix}$$

$$(3.2.45)$$

with the complementarity and nonnegativity conditions,

$$q_1(\tau)^T[\delta L/\delta q_1(\tau)] + \sigma_i[\delta L/\delta \sigma_i] + \overline{\sigma}_1(\tau)^T[\delta L/\delta \overline{\sigma}_1(\tau)] - 0,$$

$$q_1(\tau), \quad \sigma_i, \quad \overline{\sigma}_1(\tau) \geq 0.$$
$$(3.2.46)$$

Together, (3.2.45) and (3.2.46) gives the definition of an (LCP). The skew symmetric shape of, and the zero's in, the coefficient matrix verify the linear programming character of the problem. The complementarity condition gives the following equilibrium condition between sales value and factor costs,

$$d_i \sigma_i - v_i(\tau)^T q_i(\tau) + \overline{q}_i(\tau)^T \overline{\sigma}_i(\tau) \qquad (3.2.47)$$

The complementarity problem may thus be interpreted as a problem of finding the output price, the activity levels and quasi-rents which satisfy this equilibrium condition. Since (3.2.45) is a monotone system such an equilibrium exists, although it is not neccessarily unique.

We have in this subsection formulated the vintage cost function and the inverse supply function. Both are increasing monotonously in output. The shapes of the functions are obtained by formulation and solution of a set of linear complementarity problems. Hence, any algorithm that solves a (CP) or a (VIE) may be used to obtain the functions.

3.2.4   THE VINTAGE PROFIT FUNCTION

The vintage profit function is determined by the vintage production function and the prices of output and factors.[18] In this section, properties of the profit function and the duality between the production and profit functions are discussed. In addition, it is also shown how the ordinary vintage supply function is obtained from profit maximization. Finally, the (LCP) representation of the vintage profit function is derived.

The vintage profit maximization problem is stated as,

(A:III):

$$\underset{\{q_i(r)\}}{\text{Max}} \quad \sum_r \pi_i(r) q_i(r) \qquad\qquad (3.2.48)$$

$$\text{s.t.} \qquad q_i(r) \leq \overline{q}_i(r), \qquad \text{all } r \qquad (3.2.49)$$

$$\qquad\qquad q_i(r) \geq 0, \qquad \text{all } r \qquad (3.2.50)$$

The problem has a special character in the sense that there are no bounds on available factor quantities or on the level of demand. The constraints are obtained from the technological limitations on each vintage, which together give the industry production possibility set. Compare this with the vintage production function. In that case a pure quantity constrained problem was solved although it may be scaled to a sales value maximization problem after multiplication of output by it's price. The

---

18   The profit function has also been formulated as a vintage value added function [Johansson and Strömqvist (1980)]. The value added function has the same properties as the profit function but shape and optimal solution differ.

vintage cost function was a function of input prices and the demand level. Hence it was a mixed price-quantity constrained problem.

The vintage profit function is obtained as the solution to (A:III) under the sole variation of prices. The function is continuous, nondecreasing in the output price, and nonincreasing in input prices. At each fixed set of prices there exists an optimal, although not always unique, solution due to the concavity in output of the objective function. Hence, without quantity restrictions on inputs or aggregate output, an optimal solution, also bounded from above by the existing aggregate capacity, is obtained. This further illustrates that the underlying vintage production function has a mix of constant and decreasing returns to scale although each vintage has CRTS. The duality between the profit and production functions makes it also possible to derive the isoquants in the production function by varying the prices in the profit function instead of using the cost function [Seierstad (1985)].

Further insights regarding the profit function may be obtained by considering the following dual to the above maximization problem,

(B:III):

$$\underset{\{\overline{\sigma}_i(\tau)\}}{\text{Min}} \quad \sum_\tau \overline{q}_i(\tau)\overline{\sigma}_i(\tau) \qquad\qquad (3.2.51)$$

$$\overline{\sigma}_i(\tau) \geq \pi_i(\tau), \qquad \text{all } \tau \qquad (3.2.52)$$

$$\overline{\sigma}_i(\tau) \geq 0, \qquad \text{all } \tau \qquad (3.2.53)$$

In problem (B:III), the imputed nonnegative quasi-rents are minimized under the condition that the quasi-rent always is equal to or exceeds the unit gross-profit in a vintage. Together (A:III)-(B:III) yield a new set of vintage activity conditions,

$$\bar{q}_i(\tau) - q_i(\tau) > 0, \quad \text{if} \quad \pi_i(\tau) > 0 \; ; \; \bar{\sigma}_i(\tau) > 0,$$

$$\bar{q}_i(\tau) \geq q_i(\tau) \geq 0, \quad \text{if} \quad \pi_i(\tau) = 0 \; ; \; \bar{\sigma}_i(\tau) = 0, \qquad (3.2.54)$$

$$q_i(\tau) = 0, \quad \text{if} \quad \pi_i(\tau) < 0 \; ; \; \bar{\sigma}_i(\tau) = 0.$$

The fact that gross profits may be negative while quasi-rents always are nonnegative is an important distinction which, in the sequel, is utilized in (CP) formulations of the problem. The conditions above may be compared with the conditions in (3.2.31). It is obvious that the conditions are similar if p_i in (3.2.54) is set equal to σ_i in (3.2.31). Hence, if the sector output price in the profit problem equals the shadow value of demand in the cost minimization problem, the solutions are equal.

When factor prices are kept fixed and the output price is varied, a monotone supply function is obtained where q_i is a multivalued function of the market price. The form of the function is conditional on the quasi-rent in the marginal vintage $\tilde{\tau}$,

$$q_i(p_i) = \begin{cases} \text{if } \bar{\sigma}_i(\tilde{\tau}) > 0 \; : \; \sum_{\tau=1}^{\tilde{\tau}} \bar{q}_i(\tau) \\[2em] \text{if } \bar{\sigma}_i(\tilde{\tau}) = 0 \; : \; \sum_{\tau=1}^{\tilde{\tau}-1} \bar{q}_i(\tilde{\tau}-1) \leq q_i \leq \sum_{\tau=1}^{\tilde{\tau}} \bar{q}_i(\tilde{\tau}) \end{cases} \qquad (3.2.55)$$

Such ordinary "Hotelling" supply functions are, for the metal, machinery and equipment industry in Norrbotten, depicted in Figure 3.3. The supply price for each year is normalized by the unit cost in the zero gross-profit vintage.

The sigmoid form of the functions verifies the previously discussed observations made by Hotelling (1932). The development in the metal, machinery and equipment industry in Norrbotten shows a different pattern then what was given in Figure 3.2 for the total manufacturing industry in the county. In 1973 the industry was small

SALES VALUE
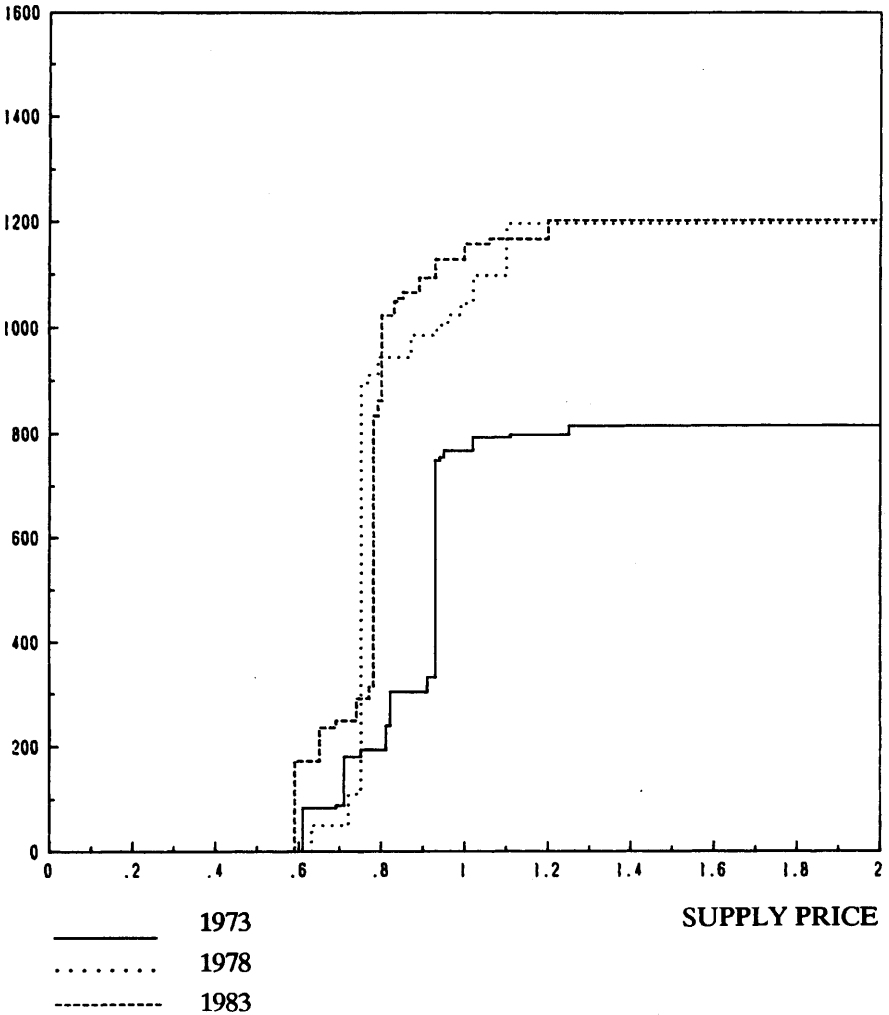
Figure 3.3   "Hotelling" vintage supply functions for the metal, machinery and
                  equipment industry in Norrbotten 1973, 1978 and 1983. [Data from
                  SIND, 1980 prices, thousands SEK]

and not especially robust, minor changes in the relative prices may in such situations result in major close downs. During the middle of the seventies, the industry increases both its aggregate supply and the gross profits. However, between 1978 and 1983 the supply function is almost unchanged. This may reflect the problems in the rest of the industry during the second half of the seventies.

Whether the vintage supply functions should be illustrated as Heckscher or Hotelling functions is a question of convenience, since they are inverses of each other. The condition for positive supply in a vintage is in the cost minimization problem,

$$\bar{\sigma}_i(r) + v_i(r) - \sigma_i = 0, \qquad (3.2.56)$$

and in the profit maximization problem,

$$\bar{\sigma}_i(r) + v_i(r) - p_i = 0. \qquad (3.2.57)$$

Since σ_i is a nondecreasing monotone function of d_i, vintages are activated in the same order when p_i increases from zero to infinity, as when d_i increases from zero to the capacity limit of the industry. This gives the inverse relation between the "Heckscher" and the "Hotelling" supply functions.

The vintage profit function is linearly homogeneous and convex in prices with factor demand functions homogeneous of degree zero.[19] The vintage production, cost, and profit functions give factor demand functions which are more complex and general than what normally may be obtained from smooth standard production functions. The factor demand functions are thus not possible to derive in closed form by, for instance, means of Hotelling's or Sheppard's lemma [Compare Hildenbrand (1981)]. Except for very trivial cases, only numerical simulations may give the exact shape of the functions. Usually, it also implies that one cannot make quantitative assumptions regarding the form, e.g. symmetry, of the Jacobian matrix of derivatives in advance of a simulation. However, the own-price effect on output is always nonnegative.

---

19   Apply the proofs in Varian(1984).

Formulated as a problem of optimization, the industry profit function may be represented as an (LCP). Define the vector, $\pi_1(\tau)$, of unit gross profit coefficients as:

$$\pi_1(\tau) = \begin{bmatrix} \pi_1(1) \\ \cdot \\ \cdot \\ \pi_1(NV) \end{bmatrix}. \qquad (3.2.58)$$

$$[NV \times 1]$$

The problem (A:III)-(B:III) may then be formulated as,

(A:III'):

Max $\pi_1(\tau)^T q_1(\tau)$ s.t. $q_1(\tau) \leq \bar{q}_1(\tau)$, $q_1(\tau) \geq 0$ (3.2.59)

(B:III'):

Min $\bar{q}_1(\tau)^T \bar{\sigma}_1(\tau)$ s.t. $\bar{\sigma}_1(\tau) \geq \pi_1(\tau)$, $\bar{\sigma}_1(\tau) \geq 0$ (3.2.60)

From the Lagrangean functions associated with (A:III') and (B:III'), one may derive the following Kuhn - Tucker conditions,

$$\delta L/\delta q_1(\tau) = - \pi_1(\tau) + \bar{\sigma}_1(\tau) \geq 0,$$

$$q_1(\tau)^T[\delta L/\delta q_1(\tau)] = 0, \; q_1(\tau) \geq 0, \qquad (3.2.61)$$

$$- \delta L/\delta \bar{\sigma}_1(\tau) = \bar{q}_1(\tau) - q_1(\tau) \geq 0,$$

$$\bar{\sigma}_1(\tau)^T[\delta L/\delta \bar{\sigma}_1(\tau)] = 0, \; \bar{\sigma}_1(\tau) \geq 0. \qquad (3.2.62)$$

Those conditions may be rewritten as the system,

$$
\begin{bmatrix} \delta L/\delta q_1(r) \\ -\delta L/\delta \bar{\sigma}_1(r) \end{bmatrix} - \begin{bmatrix} -\pi_1(r) \\ \bar{q}_1(r) \end{bmatrix} + \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \begin{bmatrix} q_1(r) \\ \bar{\sigma}_1(r) \end{bmatrix}
$$

$$(3.2.63)$$

and the following complementarity and nonnegativity conditions,

$$q_1(r)^T[\delta L/\delta q_1(r)] + \bar{\sigma}_1(r)^T[\delta L/\delta \bar{\sigma}_1(r)] - 0,$$

$$q_1(r), \ \bar{\sigma}_1(r) \geq 0. \qquad (3.2.64)$$

Together, conditions (3.2.63)-(3.2.64) give the (LCP). The monotonicity of (3.2.63) guaranties that at least one solution exists as long as the solution space not is empty. The multivalued character of the supply function implies that the solution may not be unique. The system (3.2.63) once again empazises the distinction between gross profits and quasi-rents which was observed in (3.2.54).

3.2.5   SUMMARY OF THE VINTAGE FUNCTIONS

We have in this section discussed some properties of the vintage production, cost, and profit functions. Independent of which of the functions we are chosing, the vintage activity rules contain a measure of the quasi-rent in each vintage. In Table 3.1 below, those rules are summarized in order to clarify the relation between different prices and quasi-rents.

Table 3.1	Summary of the activity rules for each vintage.

THE VINTAGE PRODUCTION FUNCTION:

$$\overline{\sigma}_i(\tau) + \sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) - 1 \geq 0, \quad q_i(\tau) \geq 0,$$

$$q_i(\tau)[\overline{\sigma}_i(\tau) + \sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau) - 1] = 0.$$

THE VINTAGE COST FUNCTION:

$$\overline{\sigma}_i(\tau) + v_i(\tau) - \sigma_i \geq 0, \quad q_i(\tau) \geq 0,$$

$$q_i(\tau)[\overline{\sigma}_i(\tau) + v_i(\tau) - \sigma_i] = 0.$$

THE VINTAGE PROFIT FUNCTION:

$$\overline{\sigma}_i(\tau) - \pi_i(\tau) \geq 0, \quad q_i(\tau) \geq 0,$$

$$q_i(\tau)[\overline{\sigma}_i(\tau) - \pi_i(\tau)] = 0.$$

With the interpretation of the dual variables as shadow prices and quasi-rents, the similarity between the conditions in Table 3.1 is evident. After normalization of unit profits, as defined in (3.2.4) - (3.2.5), by the output price, the relation between the vintage profit and production functions may be studied further,

$$\pi_i(\tau)/p_i = 1 - \sum_j (p_j/p_i)a_{ji}(\tau) - (w_i/p_i)l_i(\tau). \tag{3.2.65}$$

If the exogenous unit gross profits in the profit maximization problem (A:III) are given by,

$$\pi_i(\tau)/p_i = \bar{\sigma}_i(\tau), \tag{3.2.66}$$

where $\bar{\sigma}_i(\tau)$ now is the dual variable to the vintage capacity constraint in the vintage production function (A:I)-(B:I), measured in output quantities, then the solutions obtained from the production and profit functions would be equivalent.

If quantity constraints on input and demand are imposed on the profit function, the interpretation of the dual variables is modified. Consider problem (A:IV),

$(A:IV)$:

$$\underset{\{q_i(\tau)\}}{\text{Max}} \ \sum_\tau \pi_i(\tau)q_i(\tau) \tag{3.2.67}$$

$$\text{s.t.} \quad \sum_\tau q_i(\tau) \geq d_i \tag{3.2.68}$$

$$\sum_\tau a_{ji}(\tau)q_i(\tau) \leq q_{ji}, \qquad \text{all } j \tag{3.2.69}$$

$$\sum_\tau l_i(\tau)q_i(\tau) \leq \bar{L} \tag{3.2.70}$$

$$q_i(\tau) \leq \bar{q}_i(\tau), \qquad \text{all } \tau \tag{3.2.71}$$

$$q_i(\tau) \geq 0, \qquad \text{all } \tau \tag{3.2.72}$$

In this profit maximization problem, the constraints both from the industry production and cost functions have been introduced. Differentiation of the associated Lagrangean, gives the following activity conditions,

$$\Phi = - \pi_i(\tau) + \sum_j \sigma_j a_{ji}(\tau) + \mu_i l_i(\tau)$$

$$+ \bar{\sigma}_i(\tau) - \sigma_i \geq 0,$$

$$q_i(\tau)\Phi = 0, \quad q_i(\tau) \geq 0. \qquad (3.2.73)$$

From the definition of unit gross profits in (3.2.6), we may rearrange the inequality into,

$$\bar{\sigma}_i(\tau) + \sum_j (p_j + \sigma_j) a_{ji}(\tau) + (w_i + \mu_i) l_i(\tau)$$

$$- (p_i + \sigma_i) \geq 0. \qquad (3.2.74)$$

The dual variables σ_i, σ_j and μ_i are, in this problem, interpreted as the difference between the exogenously given prices and the imputed prices, corresponding to the alternative cost of each utilized resource. Such differences between exogenous prices and dual variables may be used to obtain prices based on opportunity costs. An iterative algorithm around an (LP) with a master program which adjusts the previous prices towards the shadow values could be used to solve this type of problem.

3. 2. 6   EMPIRICAL INFORMATION OF VINTAGE STRUCTURES

In this subsection some further empirical illustrations of vintage distributions are given. The limitations of the empirical source for the models presented previously are also indicated. The data is derived from the industrial statistics obtained from Statistics Sweden (SCB). Annually, SCB collects information from all establishments with more than five employees within the mining, manufacturing and building industry. From this source, the Swedish National Industrial Board (SIND) then produces the following vintage information,

*   Number of employees, $L_i(\tau)$.
*   Labour costs, $w_i(\tau)L_i(\tau)$.

* Sales value, $p_i(\tau)q_i(\tau)$.

* Costs of intermediate goods, $v_i(\tau) = \sum_j p_j(\tau)q_{ji}(\tau)$.

Above, each variable is given an appropriate notation related to what has previously been used. Prices have an extended notation to elucidate the fact that establishments obtain differentiated prices even in sectors producing commodities traditionally regarded as homogeneous, as shown by e.g. Wibe (1985). This reflects variations in commodity composition and other heterogeneities and gives a remainder of the limitation of the homogeneity assumption.

When the vintage model is estimated with this empirical material, two problems especially arise. The effects of aggregation of heterogeneous commodities into sectors which are assumed to produce a homogeneous output, and the confidentiality rules which prohibit utilization of all details in the material.

The need for aggregation implies a utilization of variables in terms of values. In the model, the following accounting relation may be determined for each vintage,

$$p_i q_i(\tau) = \pi_i(\tau)q_i(\tau) + w_i(\tau)L_i(\tau) + \sum_j p_j q_{ji}(\tau). \qquad (3.2.75)$$

It is thus assumed to be possible to separate price and quantity variables from each other. The database does not allow for such a separation in all cases. For example, the unit cost, which was introduced in equation (3.2.5) often at best may be estimated as,

$$v_i(\tau) = [\sum_j p_j(\tau)q_{ji}(\tau) + w_i(\tau)L_i(\tau)]/p_i(\tau)q_i(\tau). \qquad (3.2.76)$$

Hence, when vintages are compared, one cannot differentiate between price and technical causes to cost advantages and quasi-rents. The simple solution of this problem, which is best in sectors with homogeneous commodities, is to set the bench mark prices equal to one in all sectors. Possible price advantages, for example as a

result of location, financing or marketing efforts, are then interpreted as technical advantages. An entirely satisfactory estimation of the production function in problem (A:I) which relies on technical information only, is generally cumbersome.

The second problem, the confidentiality problem, is by SIND solved by aggregation of workplaces into groups of three or more. The aggregation is made in order to obtain as homogeneous units as possible.[20] A "vintage" does because of this primarily reflect the economic age of a group of, in some sense equal, establishments measured, for example, by their profitability. Technically, this vintage is composed of equipment with a variation in the time of installation. The assumption is that the economic age summarizes the combination of technical, management, and other features into a single measure. Although information is lost with the aggregation, this loss has to be related to the fact that the model is oriented towards the meso level.

The vintage formulation introduces structural rigidities at this meso level. Processes observable at other aggregation levels, some of which also work on different time scales, then have to be simplified or neglected. In the model, this approximation of the micro level is reflected by the aggregation and Leontief representation of each establishment.

We close this subsection by a short discussion of two "snapshot pictures" obtainable from the database.[21] After calculation of the value added per employee in each vintage, the productivity distributions in Figure 3.4 may be defined.

Those distributions give indications about the sensitivity of the industrial structure, especially the labour demand, in relation to changes in prices and wages. The labour demand elasticities at different points may be calculated from the functions, which may be approximated by inverted logistic functions.[22] The flatter the function becomes nearby the average wage level (the horizontal lines in the figure) the more sensitive the industry is in relation to disturbances. The figure illustrates the large problems the industry in Norrbotten met in the end of the seventies. The situation in

---

20 Johansson and Marksjö (1984).
21 Compare also with Figures 3.2 and 3.3 which are created with information from the same database.
22 Johansson and Marksjö (1984).

WAGE
VALUEADDED/EMPLOYED
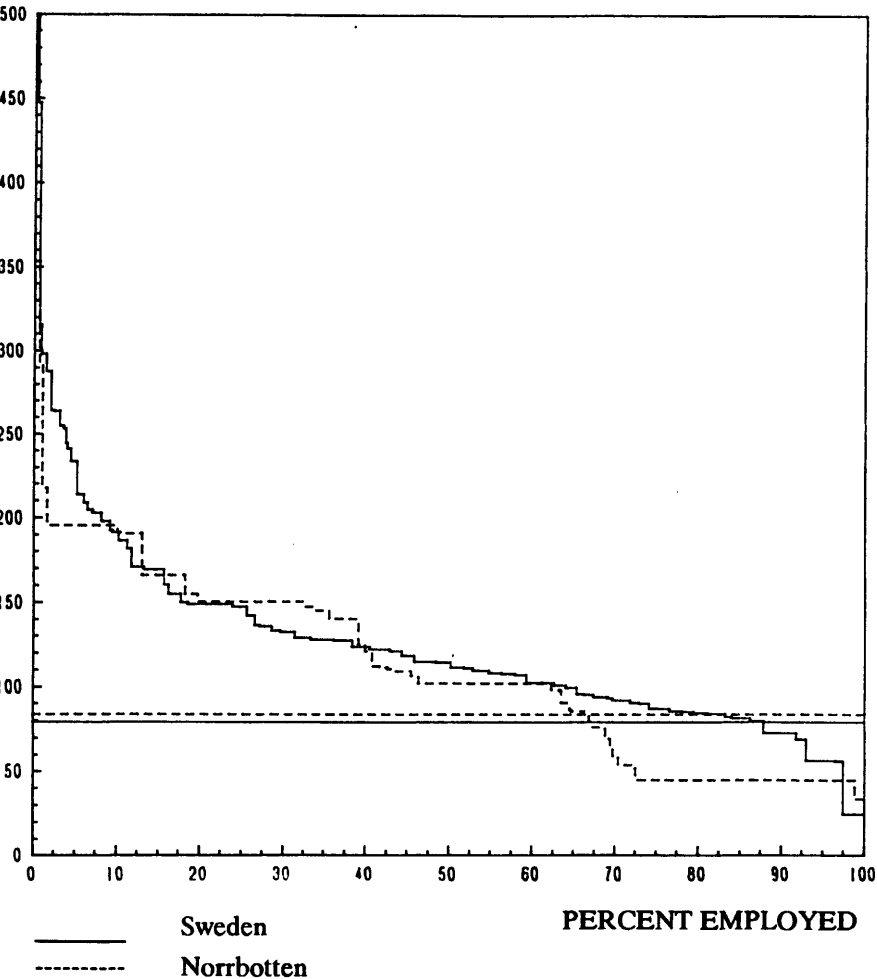
	Sweden
--------	Norrbotten

Figure 3.4   Productivity distributions and average wages (the horizontal lines) in the manufacturing industry in Norrbotten and Sweden 1978. [Data from SIND, thousands SEK].

the total Swedish industry was also problematic, but the figure makes it clear that large regional differences occurred. Almost 25 percent of the labour force in Norrbotten was employed in establishments with a labour productivity less then the average wage level. It is also interesting to observe that the average wage level was higher in Norrbotten compared with the average of Sweden.

The next two figures give related information but from another angle. The share of labour cost out of value added in a vintage is defined as,

$$\theta_i(\tau) = w_i(\tau)L_i(\tau)/[w_i(\tau)L_i(\tau) + \pi_i(\tau)q_i(\tau)]]. \qquad (3.2.77)$$

In Figure 3.5 on the next page, the labour cost share is depicted in percent for the Swedish manufacturing industry 1973, 1978 and 1983, while in Figure 3.6 the same measure is given for the industry in Norrbotten.

Figure 3.5 shows how the labour cost share increased almost uniformly between 1973 and 1978, and how it decreased again between 1978 and 1983, so that the industry became more profitable compared with the situation in 1973. In Norrbotten, the shifts were larger especially in 1978, which clearly was a year with large losses.

Inspection of Figures 3.4 to 3.6 reveal two circumstances of special interest. The first observation is that the "activity condition" derived previously, i.e. zero production in vintages with negative unit gross profits, is not completely in accordance with empirical facts. Vintages with negative profits may be active. In the theoretical world which is described by our model, this would represent a misallocation of resources.

A second feature is a rather stable overall structure along a sequence of years, especially for larger regions and similar stages in the business cycle. One may argue that this is an expected result of a normally distributed material and that the movements of individual establishments have not been followed. However, such studies of individual establishments and their transition patterns indicate that movements of establishments between classes of gross profit shares under consecutive years show a high degree of stability and a tendency towards diagonal dominance in the annual

LABOUR COST SHARE
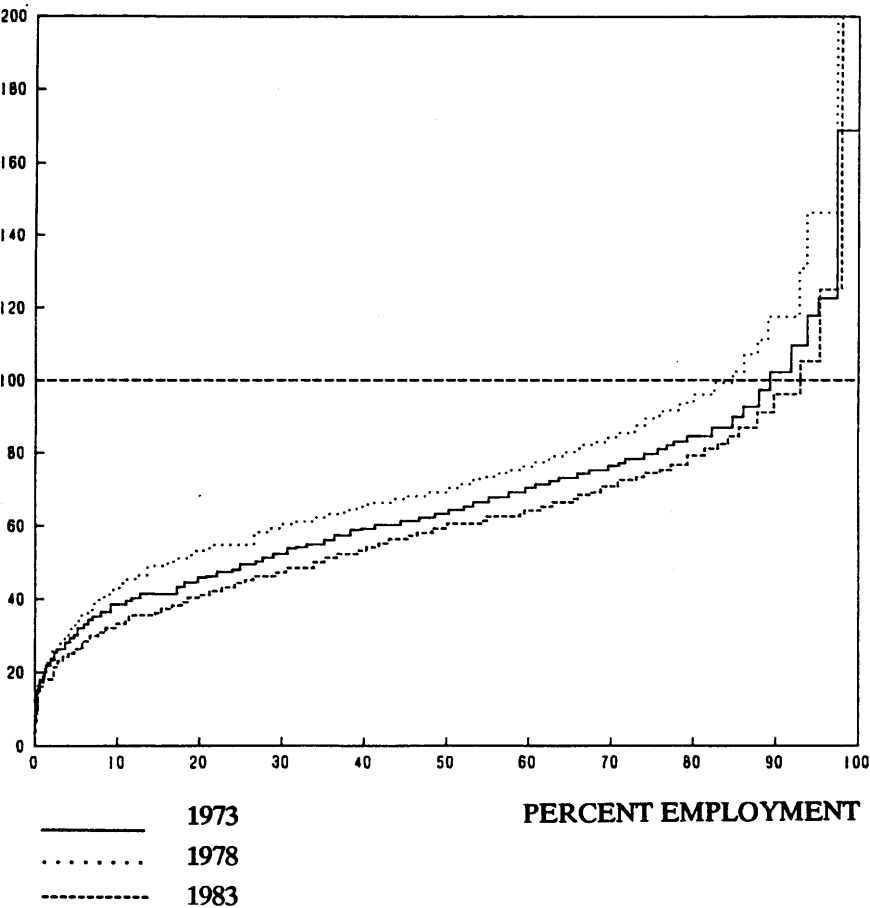
Figure 3.5   Labour cost shares for the Swedish manufacturing industry 1973, 1978 and 1983. Percent. [Data from SIND.]

LABOUR COST SHARE
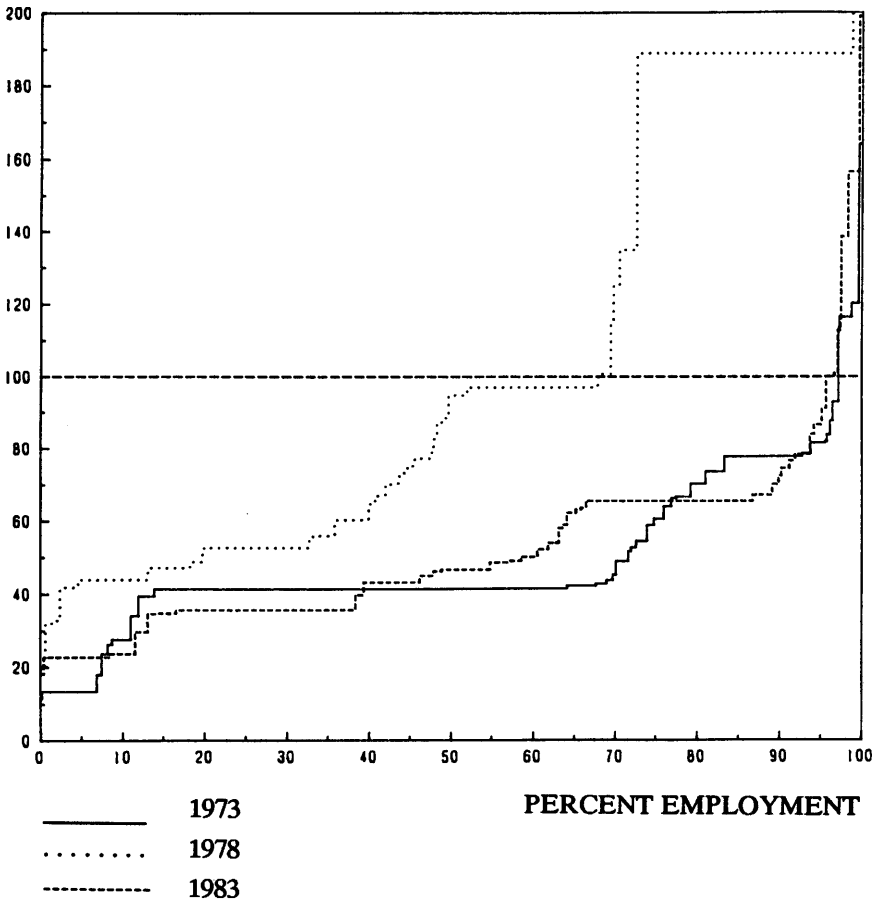
Figure 3.6   Labour cost shares for the manufacturing industry in Norrbotten 1973, 1978 and 1983. Percent. [Data from SIND.]

transition matrices.[23] Such rigidities may be explained by the durability of existing capacities caused by temporary monopolies as well as by the ongoing investments in new, and scrapping of old, machinery and routines to maintain a satisfying profit level. Nevertheless, the structures are not completely robust, and effects of business cycle movements, exit, entry and changing international competition may be observed in the figures. This also becomes more evident the smaller the region is.

With those observations in relation to the empirical material, both the advantages and problems with the estimation of the vintage model has been illustrated. The vintage approach gives an interesting alternative to conventional production functions and turns the interest towards the dynamics of vintage structures, i.e. the process of structural change. Therefore, the next section is devoted to inclusion of investments in the vintage model. With investments, the model offers a deeper understanding of the interaction between rigidities and change.

3.3 INVESTMENT IN NEW VINTAGES, TECHNICAL CHANGE, AND TEMPORARY MONOPOLIES

Although restricted, the vintage formulations presented so far gives a tool for analysis of structural change. They represent an existing structure and may be used in sensitivity analysis of this structure. However, demand for investment commodities were exogenously given and unconnected with an entry of new vintages. The entry of new capacity was neither modelled as a competition with sunk costs in existing capacity. Hence, fundamental aspects of production dynamics were excluded.

Entry of new capacity may be treated by introduction of a "best practice" technique as a new vintage [Salter (1960)]. The best practice technique is defined as the most efficient (cost minimizing) existing technique for the current, or expected, set of prices at a point in time. The concept is often also related to the ex ante macro production frontier from which the appropriate equipment is chosen [Johansen (1972)].

---

23   Johansson and Holmberg (1982), Strömqvist (1983).

Estimation of the "best practice" technique in a sector and location at a point in time, allows for a variety of approaches.[24] Estimation of a traditional smooth ex ante frontier is, beacuse of the problem to separate price and quantitative information, cumbersome with the Swedish establishment database but may be obtained from time series of data at the industry level. An alternative approach is to utilize vintage functions based on sales value, value added or gross profits. Efficiency may then be measured as the labour productivity or the gross profit share out of value added. The technical description of the best practice technique is obtained from an average of the input coefficients in the 10 or 25 percent most efficient vintages at the expected relative prices.

In Figure 3.7 on the following page, this approach is shown in the upper figure and compared with an ex ante frontier approach in the lower figure at two different price relations. The areas marked with lines separate the ten most efficient percentages of all vintages under two different relative prices. The best practice technique is the weighted average of those ten percent values. Instead, the "frontier approach" implies that a smooth function is estimated from which the new technique is derived at a given price relation. The frontier approach thus generally gives a more efficient "best practice" technique.

Extensions of the vintage approach involve estimation of the trend in the growth of efficiency in the best practice, by which forecasts of "the future best parctice set" may be done. This is necessary in order to introduce technological progress besides the substitution effects discussed in relation to Figure 3.6. In both those approaches, the technique embodied in new capacity does not correspond to an exisiting production technique, as is the case in e.g. engineering analysis. Combinations of the engineering approach, the ex ante production function, and the vintage approach are possible as long as consistency is secured.

In a model with endogeneous entry, the vintage function is extended to include conditions for introduction of the new capacity. New capacity competes with sunk costs in existing vintages and the latter have by this a situation of temporary monopolies. Existence of such advantages is on the other hand also a force to make investments in new advanced techniques. Entry of a new technique characterized by high invest

---

24 Compare Førsund and Hjalmarsson (1987).
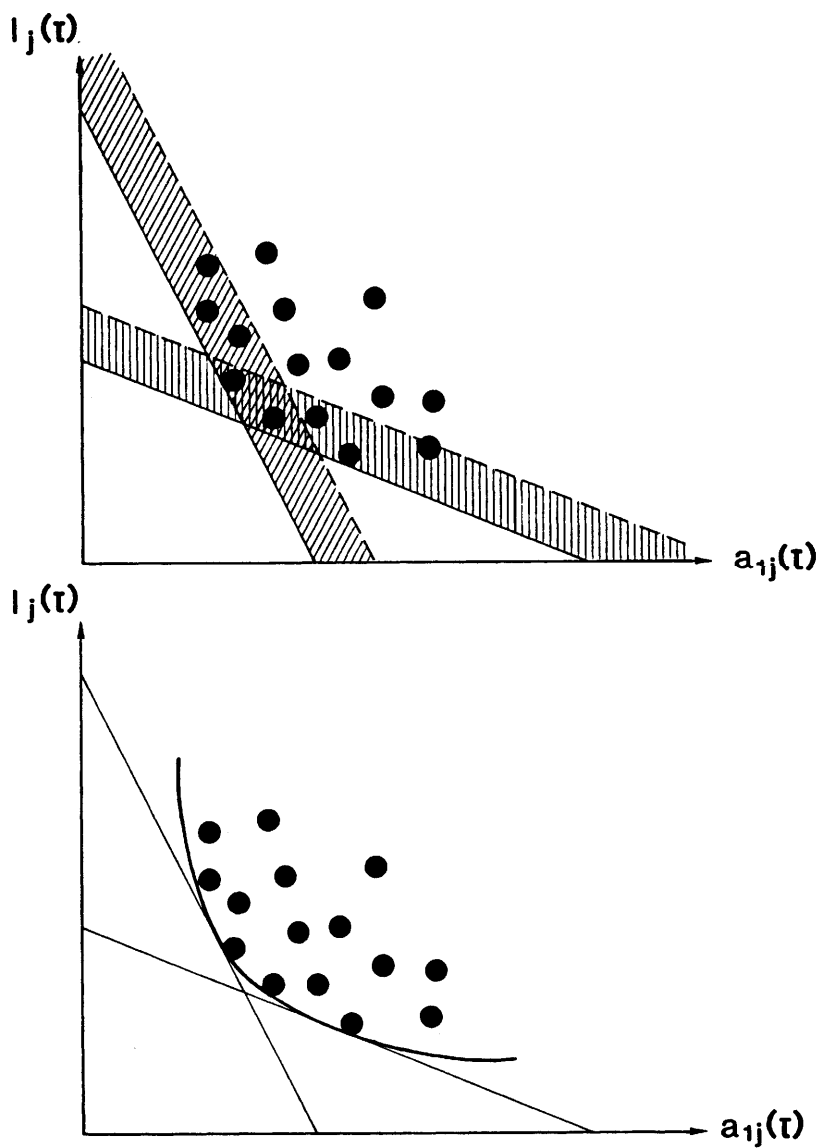
Figure 3.7   Best practice sets [the upper figure] and an ex ante frontier with two relative price lines.

ment costs and long durability may, if the technical advantage is not large enough, be facilitated by an increase in demand. Investments in marketing and design may help the entering technique to obtain a better price compared to the sector average. In the model the latter situation will be represented by an increased advantage for the technique.

Below, such a single commodity model with endogenous entry of new capacity is introduced. A similar model with a vintage profit function has previously been suggested by Johansson and Westin (1987).[25]

Let the production in new capacity be denoted by $q_i(\tau*)$ and let the unit gross profit in this technique be,

$$\pi_i(\tau*) - p_i - v_i(\tau*). \tag{3.3.1}$$

Moreover, let the annual [periodical] capital cost per unit new capacity be given by $\delta_i k_i$. Here δ_i is an annuity factor [a function of the rate of interest and the expected lifetime of, and risk involved in, the investment] and k_i is the cost of one unit capacity in sector i. Introduction of those features into the profit maximization problem (A:III) results in the following objective function,

$$\sum_\tau [p_i - v_i(\tau)]q_i(\tau) + [p_i - v_i(\tau*) - \delta_i k_i]q_i(\tau*). \tag{3.3.2}$$

A number of comments may be made in relation to this objective. The new technique will be introduced if the gross profit minus the periodical cost of capital is positive. The exlcusion of capital costs in existing vintages represents the sunk costs, which new capacity has to compete with in the introduction phase.

Entry of the new capacity involves a press on the output price in the sector which may make old capacities obsolete. However, if this is the case and no further constraints are imposed, the problem may be unconstrained and contradict observed

---

25   Models based on maximization of value added may be found in Johansson and Strömqvist (1980).

behaviour.[26] Three different constraints may be suggested to bound the problem. One is to limit available funds of investment capital.[27] Let $\bar{K}_1$, denote the available investment capital, e.g. loanable funds. The constraint is formulated as,

$$k_1 q_1(r^*) \leq \bar{K}_1. \tag{3.3.3}$$

Implicitly, this constraint assumes existence of imperfections in the financial markets, which may be difficult to motivate in the medium term. A less binding but still rigid alternative is a step function for the annuity factor. The rate of interest would then increase with the demand for funds. Another possible constraint may be a limit on the demand at the given exogenous market price. Such a constraint has been introduced earlier in the text, for example constraint (3.2.20), and reflects exogenous knowledge regarding the demand function. This approach would correspond to an accelerator approach where the investment in new capacity is constrained by the gap between perceived demand, at the price possible to obtain after the new technique has been introduced, and the output in the existing capacity,

$$q_1(r^*) = d_1 - \sum_r q_1(r) \tag{3.3.4}$$

A third type of constraint may be a limit on available input resources i.e. labour supply and intermediate goods. This was previously exemplified in problem (A:IV). With an introduction of investments, this group may be further extended to include constraints on the availability of investment goods, of land, and of floor space for the installation of the new capacity.[28]

The value of δ_1 is important for the speed of structural change in the model. It reflects the expected durability of the new capacity from an economic point of view. A long expected life-time may reflect an expected slow rate of structural change, which gives a low expected yearly cost and a low δ_1. Entry is then facilitated. How-

---

26 Observe that there is no capacity constraint on the new technique. This is also reflected by the linearity in the capacity cost. Ex post of investment is the capacity set to the production level at the time of investment.

27 Karlqvist and Strömqvist (1982).

28 Johansson and Snickars (1988).

ever, in the model there is no penalty if a faster structural change than expected involves bankruptcy for the owners of a vintage. In a period of fast structural change and short expected life times, δ_i will increase and large scale projects become costly, which will have a dampening effect on change. When δ_i becomes a function of the speed of structural change, an interesting nonlinear stability problem is generated, which may be analyzed within the model formulation by comparative simulations.

The technique in the new capacity may not only be given by input-output coefficients in the existing best practice, but may also be a function of the scale of operation and should then ideally be endogenously determined in the model at the time of investment. If techniques, but also capital costs and investment coefficients depend on the size of new capacity, new nonlinearities are introduced into the model. In this case we can write $v_i(\tau^*, q(\tau^*))$, $d_i(q(\tau^*))$ and $k_i(q(\tau^*))$. Within a linear programming approach, this may either be modelled as a problem with optional, but mutually exclusive, techniques or a step function. With a nonlinear algorithm more efficient solutions may be tested.

In this section we have assumed that only one new technique is chosen in each sector and that it is treated as a single "block" of new capacity. An alternative approach could be used to develop a more realistic model, whereby new capacity is distributed over existing vintages and expectations on the character of the best techniques vary among the investors due to nonuniform information. In the next section such distributions of both exit of old and entry of new capacities are discussed.

3.4   EXIT AND ENTRY OF CAPACITIES

The vintage model presented in section 3.2 gave solutions where the activity in units with a negative gross profit were zero. This is, as was observed in subsection 3.2.5, a simplification of reality since we could observe, at least in the short-run, active vintages with negative gross profits. In the previous section, the time horizon was longer, because of the gestation lag in new capacity. A negative gross profit would then be possible to interpret as an exit and a scrapping of the vintage. In this section, our aim is to discuss the realism of this exit function further and to present some

empirical studies of capacity exit. Thereafter, our interest is turned towards the realism in the treatment of new capacity as a single vintage.

A couple of studies have investigated the empirical validity of the zero quasi-rent exit function.[29] Although those studies give a varied picture, a common conclusion is:

* The exit function in the vintage model is a considerable simplification. Without exception, all investigations showed that a negative gross-profit neither is a necessary, nor sufficient condition to close a vintage.

However, closed vintages were on average less profitabile compared with active units. Strömqvist (1983) estimated exit functions, with an exponentially increasing probability for exit as a function of the labour cost share $\theta_i(\tau)$, with some success. Those functions have a positive exit probability even in the most profitable vintages, which is in accordance with empirical observations. On the other hand it was observed that establishments remained active for years although the profit was negative. Movements within the structure between profitability classes were shown to be as important as exit of old units and entry of new units.

The fact that profitable establishments are closed down may be explained by various arguments. One explanation is the use of gross profits instead of net profits in the data base, forced by lack of information on the latter. The gross profit in an establishment covers overhead costs, depreciation, cost of repairs etc. Hence, a positive but not large enough gross profit may give a negative net profit. Government subsidies as part of a regional policy program, for example, may instead improve the financial result in an establishment, beyond what is shown by the gross profit measure. Strategic behaviour and nonuniform expectations among firms about future market and technical changes, may be other explanations.

The results of the studies do not completely contradict the quasi-rent activity rule, but suggest that refinements are possible. The use of a probabilistic exit function may be one way to improve the model. Such a function can operate at the national

---

29   Johansson and Strömqvist (1980), Strömqvist (1983), Stålhammar (1985), Hjalmarsson and Eriksson (1986).

level or with regionally differentiated exit probabilities. The latter would reflect differences in the regional environment which increase or moderate the exit probability. If the function also includes a time-dependent shift term, dynamic changes in such environmental factors or in firm strategies which not are reflected in the gross profit share may be taken care of.

In Figure 3.8, a zero quasi-rent and an exponential exit function are depicted.[30] The functions may be compared with what Johansen (1972) called "capacity utilization functions". The former is associated with uniform myoptic expectations in homogeneous sectors, while the latter reflects nonuniform formation of expectations.

EXIT PROBABILITY
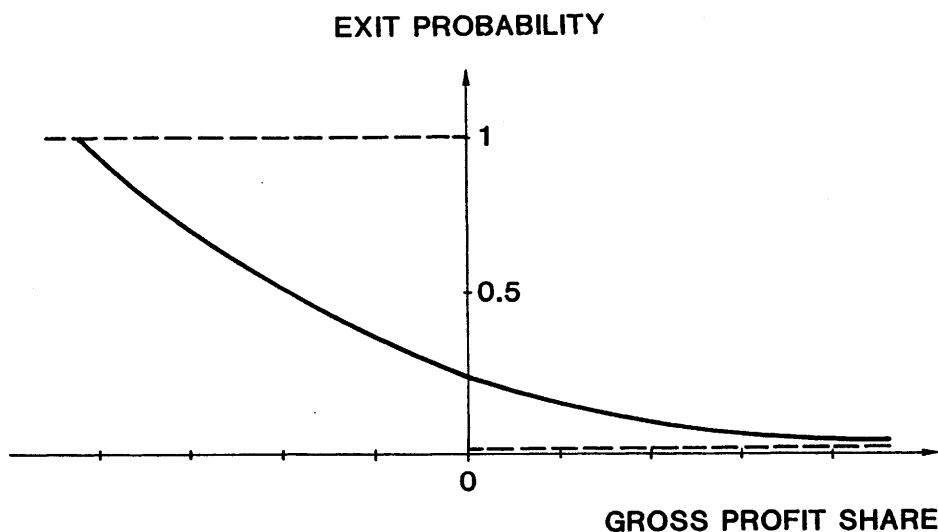
Figure 3.8   Two exit functions. [Adapted from Johansen (1972) and Strömqvist (1983).]

---

30   Time-dependent physical depreciation functions are alternatives to the exit functions. The problem then is to obtain information on the physical age, instead of the economic age used in the above functions.

A way to rationalize the quasi-rent approach may be to regard it as the long-run effect of an isolated disturbance while the short-run effect due to time "lags" in the formation of expectations more closely follows the exponential function.[31]

When dealing with entry of new capacity, the studies referred to earlier show that entry of establishments is not constrained to the most profitable part of a sector. Instead, unprofitable establishments are also introduced, and entry is almost equally distributed over the gross profit distribution [Strömqvist (1983)]. The most obvious explanation for this is diverse expectations on relative prices and the behaviour among rivals, but also on expected locational advantages or disadvantages of a plant, as a result of a dynamic analysis of future demand.

An interesting pattern is revealed if, as in Figure 3.9, investments per employee are studied instead of entry of capacity over gross profit shares. An explanation of the convex shape of the distribution may be suggested in relation to the product cycle theory. Product-oriented investments are made in advanced establishments which obtain high profits. On the other hand process-oriented, and often labour saving, investments are made in low profit classes to exploit returns to scale and to extend the lifetime of an establishment. The first type may be called offensive and the second type defensive investments. By analysis of such investment patterns, the future labour market situation in a region may be forecasted. In a dynamic vintage model, changes between the two types of investments may be introduced by means of a master equation. This equation then changes the entry probabilities in relation to the business cycle [Compare Haag et al. (1986)].

The type of entry discussed so far may be called embodied improvements in new or existing units. Disembodied improvements would be revealed by a decrease over time in the input coefficients even without investments, as a result of learning by doing etc. This may be introduced in the vintage model by an adjustment of the input-output coefficients. The speed of adjustment may then be a function of the labour skill in the vintages.

---

31 Compare Førsund et al. (1987) and the "optimal (in)efficiency" in a sector where units with negative quasi-rents are active.
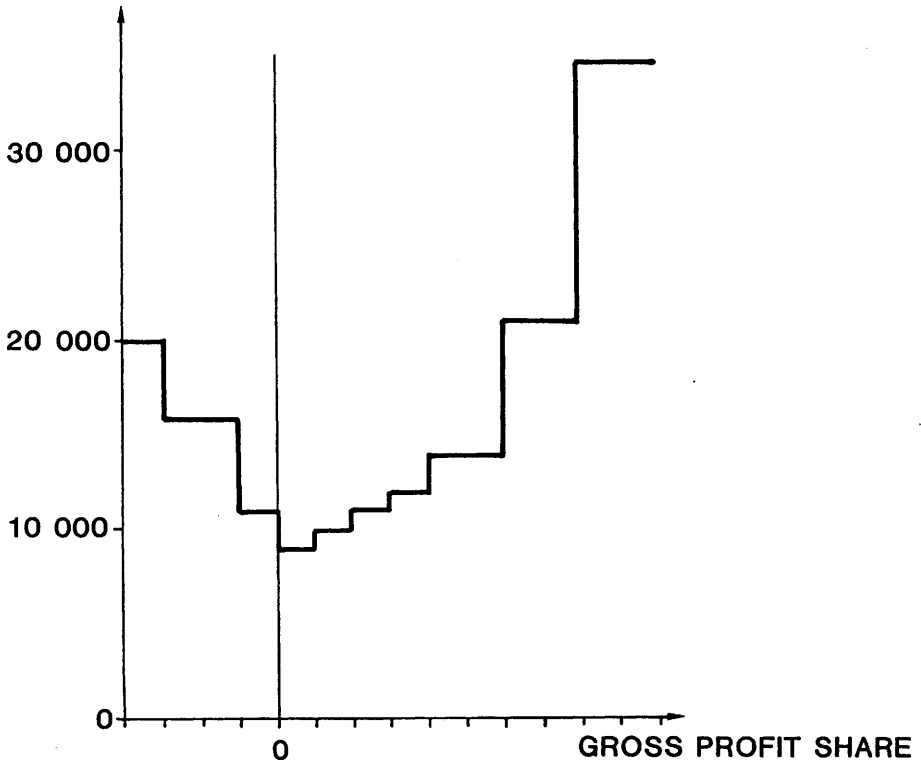
Figure 3.9   Investments per employee [SEK] over gross profit shares in the
                    Swedish manufacturing industry. Averages for the years 1972 - 1978
                    in 1975 prices. [From Strömqvist (1983).]

3.5 TECHNICAL CHANGE AND LABOUR SKILL

So far, labour has been assumed to be a homogeneous input factor. However, both in Chapter 2 and the previous section, the mutual interdependences between labour skill and economic development was emphasized. In the vintage model, the interaction between structural change and heterogeneous labour is introduced by a division of labour into skill dependent categories, denoted by $h = 1, \ldots; NH$. The unit labour demand in each vintage is thus divided into categories so that,

$$l_i(\tau) = \sum_h l_{ih}(\tau) = \sum_h (L_{ih}(\tau)/q_i(\tau)). \qquad (3.5.1)$$

Labour supply is divided into categories with the aggregation condition,

$$\sum\sum_{ih} \bar{L}_{ih} = \sum_i \bar{L}_i = \bar{L}. \qquad (3.5.2)$$

From this, the labour market constraint (3.2.9) is extended to,

$$\sum_\tau l_{ih}(\tau)q_i(\tau) \le \bar{L}_{ih}, \qquad \text{all } h. \qquad (3.5.3)$$

The market situation for each labour category gives rise to differentiated wages. Hence, the cost and profit functions have to be extended to take care of the increase in costs that a utilization of a scarce labour category may involve.

Introduction of heterogeneous labour introduces new options into the model framework. One may think about "on the job training" that changes the skill profile in a vintage and increases the overall labour productivity, or a change in the employment policy which give a similar effect. Also different effects of synergism between groups of employees may be modelled. Links between the educational sector and the labour supply may be introduced. However, the complexity of this sector, which combines processes with very different time scales, make any modelling attempt very demanding. This implies that generally the labour supply should be treated as exogenously given. In Westin (1987), a logistic model of substitution

between the supply of different labour categories over time is suggested. The speed of substitution may then be increased by investment in the educational system.

When different labour groups are introduced, it also becomes appropriate to discuss them in terms of household groups with diversified preferences. We are returning to this topic in Chapter 6.

3.6   CONCLUDING REMARKS ON THE VINTAGE FUNCTION

In this chapter we have given a systematic presentation of the single sector vintage model. The model may be used to study robustness of a sector in relation to exogenous disturbances. It is easily extended to take care of competition from firms in other regions [countries], which may be introduced by [un]constrained vintages with a production cost equal to the import price.

All models make simplifications of reality. The vintage model has the following limitations,

* The model does not treat the price spread around the average sector price and heterogeneous product design in a sector.

* The formation of expectations is underdeveloped. Myoptic or static expectations are assumed. System behaviour, with multi-plant firms, strategic decisions and synergisms is not analyzed.

* Production between zero and full capacity is not modelled. If suitable information is available, this could be treated by a further division of each vintage into discrete steps approximating a nonlinear cost function. In a (CP) representation more general nonlinear formulations are possible.

The vintage formulation gives, despite those limitations, advantages in analyses of problems related to structural change in the short and medium term. Among the advantages one should mention,

* It emphazises the rigidities in a production structure and focuses on the forces of change in the competition between new and old units and the possibility to obtain temporary monopolies.

* It has, as Johansen and Hildenbrand have argued, a less constrained functional form, compared with traditional production functions.

* It gives S-shaped supply functions which may be motivated both by theoretical and empirical arguments.

In long-term analyses, other functions may be more adequate, for instance functions which directly relate production and location to processes with a slower time scales [i.e. changes in the infrastructure].

4 SPATIAL MULTISECTOR VINTAGE MODELS WITH A SUPPLY-SIDE ORIENTATION

4.1 INTRODUCTION

The previous chapter contained the nonspatial single sector vintage model as a common denominator. In this chapter the model is extended in spatial multisector directions. The final vintage model is developed for analyses of the interaction between accessibility, technical change, trade, and location, with exogenously given final demand or price constraints. Spatial features such as spatially heterogeneous commodities and explicit link costs are by this introduced in the Scandinavian vintage modelling tradition.

The treatment of the transport sector is a central issue when spatial flows are introduced in multisector models. In order to study this topic, the vintage function is introduced in two classical archetype models, the interregional input-output model by Isard (1951) and the transport model by Koopmans (1949). While doing this, we observe that an explicit treatment of link costs, the interdependences between traded flows and the resource utilization in the regional transport sector, as well as the direct, indirect and induced income effects in the economy, generated from changes in transportation technique and infrastructure are important aspects of such an introduction.

Calibration and assessment of the models introduced in the sequel may be made with the help of the previously mentioned SIND database together with the transport statistics. The regional index connected with each establishment in the SIND database is then used in the spatial identification of each vintage. Spatial differences in the productivity structure at different points in time are thus revealed, as was shown by Figures 3.5 and 3.6. Such differences reflect, among other things, sectoral specialization and technical advantages on the supply side.

The role to which a region has been assigned [or has chosen], in the national and international network is furthermore revealed by its spatial trade pattern. Information regarding such flows between Swedish counties may be obtained from the Swedish commodity transportation survey [See e.g. Transportrådet (1983)]. The net flows of large commodity groups during 1980 from the county Norrbotten are shown in Table 4.1 [From Westin (1986)].

Table 4.1   Net flows of transported commodities from Norrbotten 1980.
             [Data from TPR, 1000 tons]

COMMODITY GROUP	NET FLOW
Ore	+11 605
Iron/Steel	+1 115
Pulp/paper	+882
Wood products	+365
Other manufacturing goods	+27
Building materials	-3
Agriculture products	-9
Manufacturing products	-11
Chemical products	-50
Containers etc	-54
Food	-132
Roundwood	-461
Minerals	-546
Fuel	-1 094
All commodities	+11 634

A large part of the flows in Table 4.1 are explained by the traditional orientation towards natural resources in the Norrbotten industry. This gives a further explanation of the large shifts in the supply function for the manufacturing industry in Norrbotten during the late seventies, which were revealed in Figure 3.6.

Natural resources dominate a measure in quantities as in Table 4.1. If the table was measured in value terms, deliveries of commodities for final demand (e.g. consumer and investment goods), would obtain an increased significance due to higher value per unit weight. This would then result in a more balanced trade pattern since those commodities are to a larger degree imported to Norrbotten. However, the fundamental structure of the flows is not changed by a value representation.

The models in this chapter, as well as the above information, are supply-side and commodity-oriented. The pattern of final demand is supressed and at most is introduced as exogenously given constraints. Passenger flows, tourism and business travel may be introduced in a crude way, but are easier to model within the network framework dicussed in Chapter 6 below. Prices are either obtained as cost determined, shadow prices or given exogenously as fixed prices. However, although the models are restricted in their behavioral content on the demand side, they may be used for simulation of the response to changes in exogenous demand and prices, among the sectors of a spatial economy. The main limitation of the models in the chapter is the lack of endogenous income effects. Those may be neglected if the analysis only considers a minor subset of the economy or if the disposable income is heavily influenced by exogenous forces, which may not be included in the model. However, the longer the time period of the analysis becomes and the larger the deviations are from the initial state, the more important it becomes to capture the system effects which act through incomes and taste on the demand side.

The models are as such special cases of equilibrium models and belong, as is shown in the sequel, to a rich class of nonlinear models, with more elaborate formulations of the demand side, the transportation sector and the spatial network. In order to emphasize this relation, complementarity formulations of the models are given in the chapter. The vintage model is a set of linear programs and the models in this chapter may be formulated as linear programs. However, in relation to transportation and investments, it is also shown where the limits of the linear formulation occur, and when nonlinear formulations are obtained.

The chapter has the following contents. In section 4.2, three spatial multisector models are introduced. Formulations with spatially homogeneous and heterogeneous commodities are compared and various ways to treat the transport sector are

assessed. The cost minimization problem, which gives the inverse spatial multisector supply function, is then introduced as a variable interregional input-output model. The profit maximization problem, which gives the ordinary multisector supply function, is also formulated in the section.

A multisector model with investments in an economy with spatially heterogeneous commodities is introduced in section 4.3. This model gives rich possibilities to simulate spatial exit and entry processes within the linear programming representation.

Spatial advantages are reflected in the skill composition of the labour supply, and the development possibilities of a region may thus be revealed by the educational profile. For this reason, introduction of heterogeneous labour into the spatial multisector model is discussed in section 4.4, while an evaluation of advantages and limitations of the models in the chapter is found in section 4.5.

4.2   TRADE AND TRANSPORTATION IN SPATIAL MULTISECTOR MODELS

4. 2. 1   INTRODUCTION

Spatial interaction activities compete with other activities for scarce resources. The cost of those resources influence the pattern of interaction and impose constraints on supply and demand in different locations. The link capacities, and accessibility costs given by the infrastructure are thus important for the understanding of structural change. Introduction of spatial interaction in applied models may be carried through in a number of ways.[1] In this section, the following aspects are emphasized as important in an integrated analysis of spatial trade and economic structural change;

*   A treatment of commodities as spatially heterogeneous.

---

1   See Kuenne (1963) for a discussion of different approaches during the fifties. Batten and Boyce (1986) and Batten and Westin (1990) provide overviews of models during the time after that.

*   Explicit recognition of link costs.

*   Modelling the competition for resources between the transport sector and other activities.

While discussing those aspects, linear programming formulations of vintage models with both endogenous and fixed prices are provided. Initially, the vintage formulation is introduced in two classical spatial models. First a vintage model with spatially heterogeneous commodities is developed. The formulation may be seen as an extension of Isard's (1951) "ideal" interregional model. In the model, the transport sector is treated as an ordinary input-output sector. In this way, the competition for resources between the transport sector and the rest of the economy is emphasized, but explicit link costs are not introduced. Subsequently, a vintage model is introduced with spatially homogeneous commodities and with trade flows determined by explicit link costs. This formulation has its origins in Koopmans (1949) transport cost minimization model.

The fundamental difference, as regards the network structure between models with spatially heterogeneous and homogeneous commodities, is that the former only has a single source of each commodity. This difference is illustrated by the two trade networks in Figure 4.1 below.

However, from a modelling point of view, the two types of networks may be seen as special cases of each other, since the difference is reducible to a question of notation and choice of spatial substitution elasticities. A homogeneous commodity model obviously contains the heterogeneous case if there is only a single source of each commodity. Heterogeneous commodity models, which allow for perfect substitution between commodities with different origins, are in fact treating those as spatially homogeneous. We are returning to a formalization of this in Chapter 7.
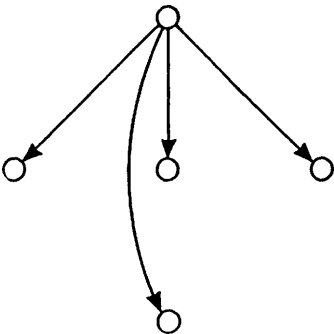
SPATIALLY HETEROGENEOUS
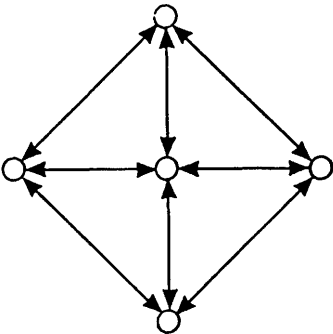
SPATIALLY HOMOGENEOUS

Figure 4.1.   Trade networks with spatially heterogeneous and homogeneous commodities.

4. 2. 2   INTERREGIONAL VINTAGE MODELS IN THE ISARD TRADITION

In this subsection, two models which extend the interregional input-output model in Isard (1951) into vintage formulations are developed. The approach implies that an interregional input-output vector is associated with each vintage. The transport sector is treated as an ordinary input-output sector, located over regions, which deliver transport margins to other sectors in their own and other regions.

The spatial vintage input-output matrix has the form,

$$
\mathbf{A^{rs}}(\tau) =
\begin{bmatrix}
a_{11}^{11}(1) & \cdots & a_{11}^{11}(NV) & \cdots & a_{1NS}^{1NR}(NV) \\
\cdot & & \cdot & & \cdot \\
a_{NS1}^{11}(1) & \cdots & a_{NS1}^{11}(NV) & \cdots & a_{NSNS}^{1NR}(NV) \\
\cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot \\
a_{NS1}^{NR1}(1) & \cdots & a_{NS1}^{NR1}(NV) & \cdots & a_{NSNS}^{NRNR}(NV)
\end{bmatrix} .
\tag{4.2.1}
$$

$$[(NRxNS) \ x \ (NRxNSxNV)]$$

Regions are denoted by $r, s = 1, \ldots,$ NR, and sectors by $i, j = 1, \ldots,$ NS. An element in the matrix is described by,

$$
a_{ij}^{rs}(\tau) q_j^s(\tau) = q_{ij}^{rs}(\tau) .
\tag{4.2.2}
$$

The assumption of a Leontief production function is thus retained for each vintage production function. This formulation allows for the interpretation that producers treat commodities as spatially heterogeneous. This interpretation is supported by empirically estimated input-output matrices, where goods bought from a sector are, in general, delivered from more then a single region. The model by Isard thus represents an early case of the "Armington" assumption. The observed heterogeneity may

be explained by various arguments. Kuenne (1963) suggested in relation to Isard's model that commodities from different sources either are not perfect substitutes from a technological point of view, that the intraregional structure implicitly is taken into account, or that other monopolistically competitive elements enter the model tacitly.

Other explanations may be that capacity constraints on suppliers, information deficits, and risk aversion among purchasers/sellers make it necessary or advantageous to trade with more then one actor, even if commodities are homogeneous. Long-term contracts and bargains to establish a stable clientele also explain rigidities and lack of adjustment. Since input-output tables reflect aggregated information on flows during a time period, changes in the direction of flows over time may also be reflected as cross-hauling and heterogeneity in the empirical material. The length of the time period of a model and the rigidity of the involved markets thus have to be treated explicitly in order to obtain reasonable elasticities and degrees of heterogeneity. Spatial heterogeneity is also a generic outcome in complex economies with demand for variety and product differentiation.

However, one may observe that the matrix $\mathbf{A}^{rs}(r)$ and Isard's matrix contain matrices with spatially homogeneous commodities as special cases, i.e. when all vintages in a sector take all inputs in each market from a single region. In a vintage model, heterogeneity at the sector level may thus be obtained either if each vintage treats commodities as spatially heterogeneous, or if commodities are treated as homogeneous, but vintages do not trade with the same suppliers for other reasons.

In volume terms, the balance of resources for a multiregion-multisector economy is formulated as,

$$\mathbf{q}^r - \mathbf{A}^{rs}(r)\mathbf{q}^r(r) + \mathbf{y}^r. \tag{4.2.3}$$

The left hand side of (4.2.3) gives gross outputs and the right hand side describes the intermediate and final demand over all sectors and regions. In this version, the interregional flows of final demand are solved exogenously. The two vectors of supply and final demand are written as,

$$\mathbf{q^r} = \begin{bmatrix} 1 \\ q_1 \\ \cdot \\ 1 \\ q_{NS} \\ \cdot \\ NR \\ q_{NS} \end{bmatrix}, \qquad \mathbf{y^r} = \begin{bmatrix} 1 \\ y_1 \\ \cdot \\ 1 \\ y_{NS} \\ \cdot \\ NR \\ y_{NS} \end{bmatrix}. \qquad (4.2.4)$$

[(NRxNS) x 1]   [(NRxNS) x 1]

The spatial multisector version of the vintage capacity constraint on output is,

$$\mathbf{q^r}(\tau) \le \mathbf{\bar{q}^r}(\tau), \qquad\qquad\qquad (4.2.5)$$

where the output and capacity vectors are specified as:

$$\mathbf{q^r}(\tau) = \begin{bmatrix} 1 \\ q_1(1) \\ \cdot \\ 1 \\ q_1(NV) \\ \cdot \\ 1 \\ q_{NS}(NV) \\ \cdot \\ NR \\ q_{NS}(NV) \end{bmatrix}, \qquad \mathbf{\bar{q}^r}(\tau) = \begin{bmatrix} 1 \\ \bar{q}_1(1) \\ \cdot \\ 1 \\ \bar{q}_1(NV) \\ \cdot \\ 1 \\ \bar{q}_{NS}(NV) \\ \cdot \\ NR \\ \bar{q}_{NS}(NV) \end{bmatrix}. \qquad (4.2.6)$$

[(NRxNSxNV) x 1]   [(NRxNSxNV) x 1]

The following cost minimization problem solves the interregional multisector vintage model with exogenous demand,

(B:V):

$$\text{Min}_{\{q^r(\tau)\}} \quad [1^r(\tau)^T w^r]^T q^r(\tau) \qquad (4.2.7)$$

$$\text{s.t.} \quad [S^r - A^{rs}(\tau)] q^r(\tau) \geq y^r \qquad (4.2.8)$$

$$q^r(\tau) \leq \overline{q}^r(\tau) \qquad (4.2.9)$$

$$q^r(\tau) \geq 0 \qquad (4.2.10)$$

A difference, compared with the single sector problem (B:II) is that only labour costs are given exogenously, while costs of intermediate commodities are determined endogenously. The exogenously determined labour costs in the objective are composed by wages which are given as,

$$w^{rT} = \left[\begin{array}{ccccc} w_1^1 & \cdot & w_{NS}^1 & \cdot & \cdot & w_{NS}^{NR} \end{array} \right].$$

$[1 \times (NR \times NS)]$

$$(4.2.11)$$

and the matrix of labour coefficients in the objective is written as,

$$1^r(\tau) = \left[\begin{array}{cccccc} 1_1^1(1) & \ldots & 1_1^1(NV) & \ldots & & \\ & & & \ldots & & \\ & & & & & \\ & & & \ldots & 1_{NS}^{NR}(1) & \ldots & 1_{NS}^{NR}(NV) \end{array} \right].$$

$[(NR \times NS) \times (NR \times NS \times NV)]$

$$(4.2.12)$$

The summation matrix, which aggregates vintage output into sector output, has the form,

$$
\mathbf{S^r} -
\begin{bmatrix}
\mathbf{s} & & & & \\
 & \cdot & & & \\
 & & \cdot & & \\
 & & & \cdot & \\
 & & & & \mathbf{s}
\end{bmatrix}, \quad
\mathbf{S} -
\begin{bmatrix}
\mathbf{s_1} & & & & \\
 & \cdot & & & \\
 & & \cdot & & \\
 & & & \cdot & \\
 & & & & \mathbf{s_1}
\end{bmatrix}, \quad (4.2.13)
$$

$$[\,(NR \times NS) \ \times \ (NR \times NS \times NV)\,] \qquad\qquad [\,NS \ \times \ (NS \times NV)\,]$$

and is composed of the summation vector, $\mathbf{S_1}$, defined in (3.2.34). The linear complementarity problem, which corresponds to problem (B:V), contains the mapping,

$$
\mathbf{F(z)} -
\begin{bmatrix}
\mathbf{1^r}(r)^T \mathbf{w^r} \\[4pt]
-\mathbf{y^r} \\[4pt]
\overline{\mathbf{q}}^{\,r}(r)
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{0} & -[\mathbf{S^r} - \mathbf{A^{rs}}(r)]^T & \mathbf{I} \\[4pt]
\mathbf{S^r} - \mathbf{A^{rs}}(r) & \mathbf{0} & \mathbf{0} \\[4pt]
-\mathbf{I} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q^r}(r) \\[4pt]
\sigma^r \\[4pt]
\overline{\sigma}^{\,r}(r)
\end{bmatrix},
$$

$$(4.2.14)$$

where σ^r [$(NR \times NS) \times 1$] is a vector of imputed prices and $\overline{\sigma}^{\,r}(r)$ [$(NR \times NS \times NV) \times 1$] a vector of vintage quasi-rents. The skew-symmetric form of the matrix is, as it should, retained. The model may be seen as a simple equilibrium model with inelastic final demand. In the model, prices are based on marginal costs and producers absorb the transport costs and apply uniform pricing over space. By repeated solution of (B:V) under variation of the final demands, the form of the inverse supply function for each sector may be obtained.

The variable input-output formulation is appropriate when changes in spatial price, flow and supply structures, of a variation in final demand are analysed. The reversed problem, occurring especially in small open economies, implies analysis of effects on the production structure from changes in exogenously determined prices. Let such a vector of market prices faced by the producers in each region at the solution year be given as, $\mathbf{p^r}$, where,

$$\mathbf{p}^{\mathbf{r}\mathbf{T}} = \begin{bmatrix} P_1^1 & \cdot & P_{NS}^1 & \cdot & \cdot & P_{NS}^{NR} \end{bmatrix} .$$ (4.2.15)

[1 x (NRxNS)]

A spatial multisector vintage profit maximization problem may then be formulated as,

(A:VI):

$$\underset{\{\mathbf{q}^{\mathbf{r}}(\tau)\}}{\text{Max}} \quad [[\mathbf{S}^{\mathbf{r}} - \mathbf{A}^{\mathbf{rs}}(\tau)]^{\mathbf{T}}\mathbf{p}^{\mathbf{r}} - \mathbf{1}^{\mathbf{r}}(\tau)^{\mathbf{T}}\mathbf{w}^{\mathbf{r}}]^{\mathbf{T}}\mathbf{q}^{\mathbf{r}}(\tau)$$ (4.2.16)

$$\text{s.t} \qquad\qquad\qquad \mathbf{q}^{\mathbf{r}}(\tau) \leq \overline{\mathbf{q}}^{\mathbf{r}}(\tau)$$ (4.2.17)

$$\mathbf{q}^{\mathbf{r}}(\tau) \geq \mathbf{0}$$ (4.2.18)

The first term in the objective, where the intermediates enter the formulation, represents value added in each vintage. Since it is exogenously given, information about spatial flows and prices is not always necessary. The model may thus easily be used in simple analyses of structural change where a sector or region is assumed to be triggered by changes in wages or a reduction in the value added share.

The endogenous part of problem (A:VI) is thus a minor extension of problem (A:III'), so it is easy to imagine and interpret the corresponding (LCP). The mapping $\mathbf{F}(\mathbf{z})$ has the form,

$$\mathbf{F}(\mathbf{z}) = \begin{bmatrix} -\pi^{\mathbf{r}}(\tau) \\ \overline{\mathbf{q}}^{\mathbf{r}}(\tau) \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}^{\mathbf{r}}(\tau) \\ \overline{\sigma}^{\mathbf{r}}(\tau) \end{bmatrix}$$ (4.2.19)

where the vintage gross profits, defined in the objective of problem (A:VI), constitute the vector,

$$\pi^{\mathbf{r}}(\mathbf{r})^{\mathrm{T}} = \left[\begin{array}{ccccccc} \pi^{1}_{1}(1) & . & \pi^{1}_{1}(\mathrm{NV}) & . & \pi^{\mathrm{NR}}_{\mathrm{NS}}(1) & . & \pi^{\mathrm{NR}}_{\mathrm{NS}}(\mathrm{NV}) \end{array} \right] \qquad (4.2.20)$$

$[1 \times (\mathrm{NRxNSxNV})]$

The vintage activity conditions, the inverse multisector supply function, $\tilde{p}^{\mathbf{r}}(q^{\mathbf{r}})$, and the ordinary multisector supply function, $q^{\mathbf{r}}(p^{\mathbf{r}})$, obtained from the spatial multisector problems (B:V) and (A:VI), are thus straightforward extensions of the nonspatial single sector counterparts in the previous chapter. Those multivalued functions are continuous, monotone and have an inverse relation.

In (B:V) and (A:VI), the final demand deliveries were treated exogenously. The endogenously determined pattern of trade is thus given by the activity of vintages and their demand for intermediate commodities. This endogenous part of the flows of commodities produced in sector i, from region r to region s, is obtained as,

$$x^{rs}_{i} = \sum_{j}\sum_{r} a^{rs}_{ij}(r)q^{s}_{j}(r), \qquad\qquad \text{all } r,s,i. \qquad (4.2.21)$$

The cost of transportation in the models is given by the supply prices in the transport sectors and the transport demand coefficients of the vintage. Let the subscript, T, denote the transport sector in the input-output matrix. The price of a unit transportation produced by the transport sector in region s is then obtained from the cost in the transport sector as,

$$p^{s}_{T} = \sum_{r}\sum_{j} p^{r}_{j}a^{rs}_{jT} + w^{r}_{T}1^{r}_{T}. \qquad (4.2.22)$$

The formulations in models (B:V) and (A:VI) thus imply that;

* Each provider of transportation is explicitly assigned to a location.

* There is an explicit relation between the transportation technique, the resource utilization, and the cost of transportation.

* There is an explicit relation between the provider and the customer of the transportation. The total transportation cost for a vintage in sector i, located in region r, is related to the level of output in the vintage in the following way,

$$\sum_s p_T^s a_{Ti}^{sr}(\tau) q_i^r(\tau).$$ (4.2.23)

This generates, through (4.2.22), direct and indirect effects on transportation and other sectors in the economy.

* At the vintage level, there is no substitution between different transport producers since the Leontief function in each vintage both determines the production technique and the spatial pattern of input sources. However, such substitution is possible at the sector level.

* There is no explicit relation between the delivery of a commodity on a link and the demand for transport, since transport demand not is directly related to the transportation of each commodity between two regions.

The last observation implies that the model cannot be used in analysis of infrastructure investments on specific links without exogenous assumptions about the relation between link flows and the regional transport producers.

As regards the fourth observation, it seems reasonable to assume that efficiency increases and factor substitution ex post of investment, is to some degree associated with spatial changes in the deliveries of slightly heterogeneous commodities, without a change in the technique. The speed of adjustment would thus be faster in the commodity flows than in the production techniques. With the introduction of "flexible manufacturing" such differences becomes even more important. However, in this case the technique may change as fast as the flows. Separation of technical and spatial substitution elasticities is discussed further in Chapter 7.

The application of a model of the Isard type is, as is well known, a data demanding task. The multiregional models by Moses (1955) and Chenery (1953) introduced simplifications, which may be adopted in the formulations of this section.[2] Each vintage is then assumed to have a unique technique, but the trade pattern is common to all vintages. This pattern is determined as the average flow for the region. However, if this approach is utilized, a part of the substitution possibilities are also assumed away. This gives an argument for replacement of the Leontief function in each vintage by a two-level nested production function, where the second level allows for spatial substitution, e.g. by use of a CES function. Such a nested function makes it possible to apply simulation with different spatial substitution elasticities.

The model by Isard is extreme since factor and trade substitution is prohibited. The vintage models reduce this inelasticity to some degree. In the following section, an alternative formulation with perfectly elastic spatial substitution is introduced. This creates instabilities in the spatial flows, which is a limitation of the model. However, explicit transport costs on each link which directly influences the trade pattern is another feature, and the main advantage, of the model.

4. 2. 3   A VINTAGE MODEL IN THE TRADITION OF KOOPMANS

In the previous models there were no explicit relations between transport costs on a specific link and the trade of a commodity on the link. The multisector transport assignment model by Koopmans (1949), contains such connections since the inter-regional transport flows are determined from a transport cost minimization problem. The aggregate costs of transportation, and thus the value of production in the transport sector, is obtained from a cost factor on each link. However, there are no connections from trade flows to the resource utilization in the transport sector and further back to the activity in the economy.

The model assumes commodities to be spatially homogeneous, which implies that a vintage immediately reacts to relative price shifts and changes sources of interme-diate deliveries. In a vintage version of the model, spatially homogeneous commodi-ties are introduced by a regional specification of each vintage input-output vector

---

2   Moses (1955) also has an interesting discussion on the stability conditions for commodity flows.

but without identification of the input source region. The "homogeneous commodity" matrix, $\mathbf{A}^{r}(\tau)$, with input-output coefficients, $a^{r}_{ij}(\tau)$, has the shape,

$$
\mathbf{A}^{r}(\tau) = \begin{bmatrix}
a^{1}_{11}(1) & . & a^{1}_{1NS}(NV) & & & \\
. & . & & & & \\
a^{1}_{NS1}(1) & . & a^{1}_{NSNS}(NV) & & & \\
. & . & . & . & & . \\
& & & a^{NR}_{11}(1) & . & a^{NR}_{1NS}(NV) \\
& & & . & & . \\
& & & a^{NR}_{NSNS}(1) & . & a^{NR}_{NSNS}(NV)
\end{bmatrix} ,
$$

$$
[\,(NRxNS) \ x \ (NRxNSxNV)\,] \tag{4.2.24}
$$

where,

$$
a^{r}_{ij}(\tau)q^{r}_{j}(\tau) = q^{r}_{ij}(\tau). \tag{4.2.25}
$$

The unit transportation costs are fixed and exogenously set for each commodity and link as $t^{rs}_{i} \geq 0$. The vector of such costs is denoted by $\mathbf{t}^{rs}$ and has the following composition,

$$t^{rs} = \begin{bmatrix} t_1^{11} \\ \cdot \\ t_{NS}^{11} \\ \cdot \\ t_1^{NR1} \\ \cdot \\ \cdot \\ t_{NS}^{NRNR} \end{bmatrix} \qquad x^{rs} = \begin{bmatrix} x_1^{11} \\ \cdot \\ x_{NS}^{11} \\ \cdot \\ x_1^{NR1} \\ \cdot \\ \cdot \\ x_{NS}^{NRNR} \end{bmatrix} \qquad (4.2.26)$$

$[(NR^2 xNS) \times 1]$ $\qquad\qquad$ $[(NR^2 xNS) \times 1]$

Above, the vector x^{rs} of inter- and intraregional flows of commodities is also defined. Final demand and unit labour cost are, as in the interregional input-output model, exogenously given. The vintage model with homogeneous commodities and explicit link costs is then formulated as the cost minimizing problem,

(B:VII):

$$\underset{\{q^r(r), \, x^{rs}\}}{\text{Min}} \quad [1^r(r)^T v^r]^T q^r(r) + t^{rs T} x^{rs} \qquad (4.2.27)$$

$$\text{s.t.} \quad \hat{G}^r x^{rs} - A^r(r)q^r(r) \geq y^r \qquad (4.2.28)$$

$$S^r q^r(r) - \tilde{G}^r x^{rs} \geq 0 \qquad (4.2.29)$$

$$q^r(r) \leq \overline{q}^r(r) \qquad (4.2.30)$$

$$x^{rs}, \, q^r(r) \geq 0 \qquad (4.2.31)$$

The objective is to minimize exogenously given labour and transportation costs. Two conservation of flow conditions are introduced as (4.2.28) and (4.2.29). Those contain the two summation matrices,

$$
\hat{G}^r - \begin{bmatrix}
1 & 1 & .. & 1 & & & & & & & \\
 & 1 & 1 & .. & 1 & & & & & & \\
 & & . & & . & .. & . & & & & \\
 & & . & & . & .. & . & & & & \\
 & & 1 & 1 & .. & 1 & & & & & \\
 & & & & & & 1 & 1 & .. & 1 & \\
 & & & & & & 1 & 1 & .. & 1 & \\
 & & & & & & . & . & .. & . & \\
 & & & & & & . & . & .. & . & \\
 & & & & & & 1 & 1 & .. & 1 &
\end{bmatrix} \qquad (4.2.32)
$$

$[\,(\text{NRxNS}) \; \times \; (\text{NR}^2\text{xNS})\,]$

and,

$$
\tilde{G}^r - \begin{bmatrix}
1 & & & & & & 1 & & & \\
 & 1 & & & & & & 1 & & \\
 & & 1 & & & & & & 1 & \\
 & & & 1 & & & & & & 1 \\
 & & & & . & & & & . & \\
 & & & & & 1 & & & & 1 \\
 & & & & & & 1 & & & 1 \\
 & & & & & & & 1 & & 1
\end{bmatrix} \qquad (4.2.33)
$$

$[\,(\text{NRxNS}) \; \times \; (\text{NR}^2\text{xNS})\,]$

The first flow condition, (4.2.28) guarantees that inflows to each region plus production, exceed demand in the regions, while in condition (4.2.29), local demand and outflows from a region is constrained by the local supply. The two vectors of shadow values to conditions (4.2.28) and (4.2.29), $\hat{\sigma}^r$ and $\tilde{\sigma}^r$, are written as,

$$
\hat{\sigma}^r -
\begin{bmatrix}
\hat{\sigma}^1_1 \\
\cdot \\
\hat{\sigma}^1_{NS} \\
\cdot \\
\hat{\sigma}^{NR}_{NS}
\end{bmatrix},
\qquad
\tilde{\sigma}^r -
\begin{bmatrix}
\tilde{\sigma}^1_1 \\
\cdot \\
\tilde{\sigma}^1_{NS} \\
\cdot \\
\tilde{\sigma}^{NR}_{NS}
\end{bmatrix}.
\qquad (4.2.34)
$$

$$[(NRxNS) \times 1] \qquad\qquad\qquad [(NRxNS) \times 1]$$

Those are interpreted as imputed demand and supply prices in each region. The conditions for optimal flows are,

$$\tilde{G}^{rT}\tilde{\sigma}^r + t^{rs} - \hat{G}^{rT}\hat{\sigma}^r \geq 0, \; x^{rs} \geq 0,$$

$$[\tilde{G}^{rT}\tilde{\sigma}^r + t^{rs} - \hat{G}^{rT}\hat{\sigma}^r]^T x^{rs} - 0. \qquad (4.2.35)$$

This is the spatial price equilibrium condition. It implies that positive flows on a link between two locations only is obtained when the difference between the supply and demand prices equals the cost of transportation between the locations. Hence, each customer pays the transportation cost from the producing region. The previous interregional model did not contain this explicit condition on each flow. In the model, flows and choices of location are sensitive to minor price changes. The model, and extensions of it, does in that respect represent an unstable model.

In (B:VII), the vintage activity conditions are obtained as,

$$-S^{rT}\tilde{\sigma}^r + A^r(\tau)^T\hat{\sigma}^r + 1^r(\tau)^T w^r + \overline{\sigma}^r(\tau) \geq 0, \; q^r(\tau) \geq 0,$$

$$[-S^{rT}\tilde{\sigma}^r + A^r(\tau)^T\hat{\sigma}^r + 1^r(\tau)^T w^r + \overline{\sigma}^r(\tau)]^T q^r(\tau) - 0. \qquad (4.2.36)$$

A vintage is active as long as the value added equals labour costs and gives a non-negative quasi-rent. The transportation cost is, via the spatial price equilibrium con-

dition and the difference between demand and supply prices, included in the cost of intermediate commodities.

The homogeneous commodity problem (B:VII), introduced two conservation of flow constraints and link costs as new features. Hence, it may be instructive to look at the mapping $F(z)$ in this case,

$$
F(z) = \begin{bmatrix} t^{rs} \\ 1^r(r)^T w^r \\ -y^r \\ 0 \\ \overline{q}^r(r) \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\hat{G}^{rT} & \tilde{G}^{rT} & 0 \\ 0 & 0 & A^r(r)^T & -S^{rT} & I \\ \hat{G}^r & -A^r(r) & 0 & 0 & 0 \\ -\tilde{G}^r & S^r & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x^{rs} \\ q^r(r) \\ \hat{\sigma}^r \\ \tilde{\sigma}^r \\ \overline{\sigma}^r(r) \end{bmatrix}
$$

$$(4.2.37)$$

The skew-symmetric form of the coefficient matrix and the two quadratic zero sub-matrices confirm that this problem is a linear program. It is also apparent, when comparing with the matrix in (4.2.14), that the link costs give explicit spatial cost differences and a possible separation between demand and supply prices in each location.

In relation to the transport sector, the model has the following properties;

* There are explicit, although exogenously fixed, link costs and an explicit spatial price condition connected with each flow.

* There is no endogenous relation between the link cost and the resource utilization in the transport sector.

* There is no endogenous relation between the flows on a link and the localization of the transport sector.

The last two points are the drawbacks of the formulation in economy-wide studies of structural change. An interpretation would be that the share of the resources which are utilized in transportation may be calculated exogenously. This would also imply that the transport sector should be moved from the input-output matrix to the final demand part of the model.

The transportation [or interaction] cost on each link in the model is linear and no other capacity constraints have been introduced as regards the infrastructure. Link flow constraints may be introduced as a vector of link capacities,

$$x^{rs} \leq \bar{x}^{rs}. \qquad (4.2.38)$$

The dual variables are then interpreted as congestion costs which increase the cost of transportation. However, such constraints are very crude measures of link capacities, since congestion costs are usually obtained before the capacity limit is reached. Nonlinear transport cost functions are thus preferable. Such functions may be represented as,

$$t^{rs} = t^{rs}(x^{rs}, \bar{x}^{rs}). \qquad (4.2.39)$$

The cost is then nonnegatively related to the actual flows and nonpositively related to the link capacity. This relation reflects the travel time on a link. Such functions may obviously be represented by step functions if the linear programming format should be retained. Smooth nonlinear functions bring the classical transport cost minimization model beyond the (LP) format.

4.2.4   TOWARDS AN INTEGRATED REPRESENTATION OF TRANSPORT NETWORKS IN MODELS OF ECONOMIC STRUCTURAL CHANGE

As was discussed previously, the models above deal with commodity flows and costs of interaction without an endogenous relation between link-related transportation and a localized transport sector. Hence, there was no endogenous relation between

link costs and the production costs in the transport sector. Other features of the models are:

* The transport sector does not involve modal choice and the network is simple without alternative paths.

In many applications, those features are not a problem but represent simplifications and a surpression of less important parts. Introduction of a disaggregated transport sector becomes important when the effects on the spatial structure of investments in specific modes are analysed. A simple disaggregation into modes has been suggested by Liew and Liew (1984). The model contains an extended interregional input-output table with a further division of the intermediate demand for transportation by modes. In the interregional vintage input-output model (B:V), this would imply that,

$$a_{ij}^{rs}(r) = \sum_{m} a_{ij}^{rsm}(r), \qquad\qquad (4.2.40)$$

where modes are denoted $m = 1, \ldots,$ NM. Still this only gives a crude representation of the transport network. More detailed, network-oriented descriptions of the transport sector may be found in freight network models [Friesz and Harker (1985)] and logit models [Domencich and McFadden (1975), Williams (1977), McFadden (1978), Fisk and Boyce (1984)], which contain detailed formulations of links, modes and actors on the network. The rest of the economy in those models is instead represented in a simplified way or treated exogenously.

A separation of the transport sector from the rest of the economy is only possible if the interdependences between the two parts are minor. Those interdependences are represented by,

* Direct, indirect, income-induced, technical and localizational effects on the transportation sector from changes at the demand and supply sides, for instance of technique, taste, and income flows, in the rest of the economy.

* Direct, indirect, income and technical effects on the spatial supply of commodities as well as factors, from changes in transportation demand and supply. Such

effects may be initiated by changes in transport techniques, and by infrastructure investments.

The strengths of those interdependences increase with the time horizon of a study. A separation is thus only possible in short-run analyses. A model of the system-effects generated by the interaction between the network-oriented transport sector and the rest of the economy should thus contain link related transport cost functions of the following type,

$$t^{rs} = t^{rs}(a_T^{rs}(x^{rs}, \bar{x}^{rs}),$$

$$l_T^{rs}(x^{rs}, \bar{x}^{rs}), p^r, w^r, t^{rs}). \qquad (4.2.41)$$

A link cost is then connected to the cost in localized transport sectors. This was not the case in (4.2.39). The link costs are here given by the production costs for the transportation of each commodity, which is related to the type of transported commodity, the transport technique, the congestion on the links which determine the demand for input factors, and the demand price of intermediates and labour. Infrastructure investments change the link related characteristics, while a technical development in the transportation sector affects the resource utilization in the sector.

An endogenous solution of such system-wide interaction implies that the model becomes a (NCP) problem. Such a model is introduced in Chapter 6. A first step towards this type of model is to iteratively solve a network model and a spatial multi-sector model where the transport sector is represented by both a link cost and an input-output sector. The link cost would then at each iteration be recalculated, given the prices in the transport sector. Such a connection between the link costs and the production cost in the transport sector gives consistency between the price of transports in the objective and the supply prices in the transport sector.

The spatial allocation and transportation model by Lefeber (1958a, 1958b), includes this explicit connection between the link flows and a transportation sector which demands resources in the economy. However, in the model by Lefeber, the transport

sector is common for all regions. The utilized resources for each transport are thus taken from the cheapest source, independent of where a link is located. Isard and Ostroff (1958) introduce a localized transport sector in their existence proof of an interregional equilibrium. However, their abstract model does not include congestion explicitly.

4.3   A SPATIAL MULTISECTOR MODEL WITH EXOGENOUS PRICES AND ENDOGENOUS INVESTMENTS IN NEW CAPACITY

The spatial framework in section 4.2 makes it possible to introduce the combined problem of investments and choice of location for new vintage capacities. The spatial dynamics, caused by differences in "best practice" techniques, constraints on the choice of location, and network effects of flows of investment goods, are then possible to analyze in an extended way.

Diffusion of information regarding the technological development is generally, due to accessibility frictions, uneven over regions. The choices of new techniques are thus not indifferent to location. It may be possible to observe a set of local functions or best-practice techniques. Johansen (1972) introduced the term "efficient ex ante function" to the envelope of such local ex ante functions. A similar division may be made between local and global "best-practice" sets. Investments in the communication system which improves the information flows between two nodes would then reduce the differences between their future "best-practice" sets and also change the relation between those and the global set.

In the model outlined below, an endogenous introduction of new capacities is modelled. The model has a profit maximization formulation with heterogeneous commodities and explicit link costs. It thus merges features from both the models by Isard and Koopmans. The following term for profits in new capacity is included in the objective function,

$$[[I^\Gamma - A^{\Gamma S}(r*)]^T p^\Gamma - \underset{A}{T^{\Gamma S}}(r*) - 1^\Gamma(r*)^T w^\Gamma$$

$$- \delta^\Gamma[B^{\Gamma S T} p^\Gamma + \underset{B}{T^{\Gamma S}}(r*)]]^T q^\Gamma(r*) - \pi^\Gamma(r*)^T q^\Gamma(r*) \qquad (4.3.1)$$

Above, new capacities in each region and sector are denoted by an asterisk attached to the input-output matrix, $A^{\Gamma S}(r*)$ [(NRxNS)x(NRxNS)], and the labour-input matrix, $1^\Gamma(r*)$ [(NRxNS)x(NRxNS)]. Those techniques are given exogenously. The matrix I^Γ is an identity matrix. The output vector of new capacities $q^\Gamma(r*)$, and the matrix of annuity factors related to new techniques, δ^Γ, are written as,

$$q^\Gamma(r*) = \begin{bmatrix} q_1^1(r*) \\ \vdots \\ q_{NS}^{NR}(r*) \end{bmatrix} \qquad \delta^\Gamma = \begin{bmatrix} \delta_1^1 & & \\ & \ddots & \\ & & \delta_{NS}^{NR} \end{bmatrix}. \qquad (4.3.2)$$

[(NRxNS) x 1]   [(NRxNS) x (NRxNS)]

The accelerator matrix $B^{\Gamma S}$, with the coefficients of investment deliveries per unit of new capacity, is written,

$$B^{\Gamma S} = \begin{bmatrix} b_{11}^{11} & \cdots & b_{1NS}^{11} & \cdots & b_{1NS}^{1NR} \\ \cdot & & \cdot & & \cdot \\ b_{NS1}^{11} & \cdots & b_{NSNS}^{11} & \cdots & b_{NSNS}^{1NR} \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ b_{NS1}^{NR1} & \cdots & b_{NSNS}^{NR1} & \cdots & b_{NSNS}^{NRNR} \end{bmatrix} \qquad (4.3.3)$$

[(NRxNS) x (NRxNS)]

The value of investment deliveries to new capacity, i.e. the investment cost, in region r, is thus,

$$Z^r - \sum_i Z_i^r - \sum_i k_i^r q_i^r (\tau *), \qquad \text{all } r. \qquad (4.3.4)$$

The vector of unit transport costs of intermediate inputs for new capacity, $T_A^{rs}(\tau *)$ is given as,

$$T_A^{rs}(\tau *) - \begin{bmatrix} T_{1,A}^1(\tau *) \\ \cdot \\ T_{NS,A}^1(\tau *) \\ \cdot \\ T_{NS,A}^{NR}(\tau *) \end{bmatrix} \qquad (4.3.5)$$

$$[(NRxNS) \times 1]$$

where an element $T_{j,A}^s(\tau *)$ is calculated as,

$$T_{j,A}^s(\tau *) - \sum_{ri} t_i^{rs} a_{ij}^{rs}(\tau *), \qquad \text{all } s, j. \qquad (4.3.6)$$

The vector of unit transport costs of investment commodities for new capacity, $T_B^{rs}(\tau *)$ is defined as,

$$T_B^{rs}(\tau *) - \begin{bmatrix} T_{1,B}^1(\tau *) \\ \cdot \\ T_{NS,B}^1(\tau *) \\ \cdot \\ T_{NS,B}^{NR}(\tau *) \end{bmatrix} \qquad (4.3.7)$$

$$[(NRxNS) \times 1]$$

where an element $T^s_{j,B}(\tau*)$ is calculated as,

$$T^s_{j,B}(\tau*) - \sum_{ri}\sum t^{rs}_i b^{rs}_{ij}(\tau*), \qquad \text{all } s, j. \qquad (4.3.8)$$

In existing vintages, the unit gross profits are obtained as,

$$\pi^r(\tau) - [S^r - A^{rs}(\tau)]^T p^r - T^{rs}_A(\tau) - 1^r(\tau)^T w^r. \qquad (4.3.9)$$

where the elements in the transport cost vector for existing capacities, $T^{rs}_A(\tau)$ [(NRxNSxNV) x 1] is the new part compared with (4.2.16). Those are calculated in a similar way as $T^{rs}_A(\tau*)$. A solution of the model is obtained from,

(A:VIII):

$$\underset{\{q^r(\tau),q^r(\tau*)\}}{\text{Max}} \quad \pi^r(\tau)^T q^r(\tau) + \pi^r(\tau*)^T q^r(\tau*) \qquad (4.3.10)$$

$$\text{s.t. } [S^r - A^{rs}(\tau)]^T q^r(\tau)$$

$$+ [I^r - A^{rs}(\tau*) - B^{rs}]^T q^r(\tau*) \le y^r \qquad (4.3.11)$$

$$q^r(\tau) \le \overline{q}^r(\tau) \qquad (4.3.12)$$

$$q^r(\tau) \ge 0 \qquad (4.3.13)$$

To bound the problem, a final demand constraint has been included by (4.3.11). With investments in new capacities, the constraint must contain the intermediate deliveries to the new vintage and the flows of investment goods for the installation of the new equipment. The Kuhn-Tucker activity condition for new capacity is,

$$[[I^r - A^{rs}(r*) - B^{rs}]^T[p^r - \sigma^r] - T_A^{rs}(r*) - 1^r(r*)^T w^r$$

$$- \delta^r[B^{rs T}p^r + T_B^{rs}(r*)]]^T q^r(r*) - 0, \quad q^r(r*) \geq 0. \qquad (4.3.14)$$

Since the market prices are included in the objective function, the shadow values to condition (4.3.11.) are interpreted as the decreases in prices necessary to obtain zero profit in the marginal vintage. Observe that this adjustment of prices does not change the payments to the rentier. One interpretation would be that rents are determined, together with the choice of technique, before the period, while the adjusted market prices are set within the period. A solution with consistence between endogenously determined investment costs and payments to rentiers gene-rates a (NCP). Such a model is discussed in Chapter 6. Although simple, the model in this chapter captures central features in the spatial dynamics of structural change. Observations in relation to the formulation are;

*   If a new capacity is introduced in a region, it becomes a marginal vintage, since it is not constrained by a capacity limit.

*   A new vintage has to cover the annualized payments to rentiers, in order to make introduction economically feasible. However, those costs are not affected by the decrease in prices due to the zero profit condition on the marginal vintage. But the cost of the commodities used to create the new capacity is reduced.

*   Introduction of new capacities in a region is constrained by the transport cost associated with the transportation of inputs and outputs. Infrastructure invest-ments may thus make new techniques feasible for new regions.

*   If the risk, as reflected by δ_i^r, associated with process investments in a mature product is lower then the risk with investments in the development of a new pro-duct, this lower risk may compensate for high link costs. Mature products would thus be given more remote locations.

Besides the constraints set by available techniques and transportation costs, a spatial labour supply constraint may also affect the location of new capacity. The following section deals with such constraints.

4.4   HETEROGENEOUS LABOUR IN SPATIAL MULTI-SECTOR MODELS

A central problem in the analysis of spatial structural change is the matching of exit and entry of techniques in a region with the supply of labour categories with various skills. The existing labour skill structure reflects the present and historical speciali-zation and technical level in a region. Introduction of a new technique thus often both requires and gives rise to unbalanced demand for specific categories of labour. Rigidities in the educational system and in the migration flows may thus, together with an inappropriate infrastructure, obstruct the development of a region in a desired direction.

In the spatial multisector vintage model, such constraints on the labour demand in sector i and region r are introduced as,

$$\sum_r L_{ih}^r(\tau) - \sum_r q_i^r(\tau) l_{ih}^r(\tau) \le \overline{L}_{ih}^r, \qquad \text{all } h, \qquad (4.4.1)$$

where the notation, besides the addition of the spatial index, follows the notation in section 3.5. The labour demand is in this case constrained to labour from the region in which the vintage is located. Alternative formulations with explicit demand for labour from other regions are also possible. Alternative constraints on the supply-side (e.g. only a total constraint over all regions for a category) give implicit com-muting or migration flows in the model.

On the demand side, this corresponds to a division of the final demand vector into a set of household-differentiated vectors which reflect the heterogeneity among labour. In the models discussed in this chapter, this have been an entirely exogenous matter, but the idea will be further developed when multihousehold demand systems are introduced in Chapter 6.

Spatial differences in the wage level of a given labour category may then also provide a further advantage or disadvantage for a region. With spatial wage variations, the unit profit in a vintage is given by,

$$\pi_i^r(\tau) = p_i^r - \sum_{sj} p_j^s a_{ji}^{sr} - \sum_h w_{ih}^r l_{ih}^r(\tau). \qquad (4.4.2)$$

Such differences would in the model, ceteris paribus, reduce production in regions with relatively high wages or with tight bounds on the labour supply. The unit gross profit of a vintage is in the latter case reduced by the increase of labour cost given by the shadow price to the labour supply constraints,

$$\pi_i^r(\tau) = \pi_i^r(\tau) - \sum_h \mu_{ih}^r l_{ih}^r(\tau). \qquad (4.4.3)$$

However, the effects of wage differences and labour shortages on the decision to introduce or to remove a technique from a region is dependent on the stage of the product cycle. Relatively high wages may be a signal of unique competence, which attracts firms in the initial product-developing stages. Often, this also implies that the shadow price of the category is positive. In an early product-developing part of the cycle, such costs should be possible to cover by increases in the price of the product. On the other hand, in the later stages of the cycle, a relatively high wage in a region may be a signal for labour-saving investments and relocation of the production to other regions.

The above observation has implications for the choice of technique in new capacities. The "best-practice" concept does not make a distinction between process- and product-oriented changes in techniques. The composition of the demand coefficients for various labour categories would be one important difference between process and product-oriented improvements. Another would be the elasticity in demand with respect to price changes.

The best practice sets should thus both have a spatial and a product cycle related identification. The choice of a product- or process-oriented technique in a region is further constrained by the existing labour supply. Transformation of a region from a

natural resource- or manufacturing-oriented region to a research intensive, or product-developing region is thus an extremely difficult and lengthy task because of rigidities within labour supply, existing capacities and infrastructure. Since differences in wages, shadow values and accessibility also motivate migration and govern decisions regarding education, interdependences between the spatial changes in labour supply and the technical development are obtained.

4.5   ASSESSMENT OF THE SUPPLY-SIDE ORIENTED MODELS

In the chapter we have dealt with supply-oriented input-output and exogenous price models with inelastic private demand. The integration of link costs, endogenous investments, multicategory labour and spatially heterogeneous commodities with the spatial multisector vintage formulation in (A:VIII) give a simple tool for structural change analysis.

The advantage of the linear programming formulations presented so far is the ease by which a solution may be obtained. The drawbacks are well known. Extreme specialization is easily obtained, since the number of nonzero variables in a solution never exceeds the number of constraints. The "Armington" assumption and the vintage formulation eliminate some of those effects. The Armington assumption reduced the instability of the trade flows in response to minor price changes, while it still allows for substitution. The vintage function have fixed capacities and thus nonmeable capital. This provides the stepwise industry supply function which introduces important nonlinearities in the linear programming representation.

The demand side has been characterized by price and income inelastic final demand. Hence, final demand will not be changed in response to changes in the transportation costs. A first step to introduce a price sensitive demand side, may be to represent the demand side by step functions. Endogenous income formation and budget constraints on each household are impossible features if the linear programming formulation shall be preserved [Ginsburgh and Waelbroeck (1981)]. In the models discussed in this chapter, exogenous assumptions thus determine the development of

incomes. However, income effects are probably not negligible for the understanding of regional growth.

We have also observed that attempts to integrate transport and spatial economic models easily bring the formulations beyond the possibilities set by the linear programming representation. In analyses of the long-run interaction between investments in infrastructure and the spatial growth in the economy, the induced income effects may be important. There is thus a need for a development of network models with an endogenous interaction between incomes, prices, demand, and supply.

5 VINTAGE MODELS IN THE "SPATIAL PRICE EQUILIBRIUM" TRADITION

5.1 INTRODUCTION

Spatial equilibrium models are characterized by endogenous price formation in markets where actors in a spatial network are separated by nonnegative interaction costs.[1] The equilibrium prices fulfil the spatial price equilibrium conditions. Hence, excess demand for each commodity is nonpositive. Moreover, the equilibrium demand price at a sink equals the sum of the equilibrium supply price at each source delivering to the sink, and the cost of interaction between the source and the sink.[2] A minimum requirement of a spatial equilibrium model is that it endogenously determines supply and demand quantities, prices at each location, and the quantities traded on the network. Those conditions where also fulfilled by the models in the previous chapter. However, an important extension in this chapter, compared with Chapter 4, is the price elastic final demand side.

There exist, as was discussed in Chapter 2, two mainstreams of models in the applied spatial equilibrium category. The earlier of the two, the Spatial Price Equilibrium (SPE) tradition, was introduced by an extremal formulation in Samuelson (1952), where the sum of producers and consumers surpluses was maximized. Takayama and Judge (1964, 1971) solved the applied case with elastic linear functions by a quadratic programming formulation.

It was then convenient to formulate the model as a "Marshallian equilibrium" with inverse demand and supply functions. By such a Marshallian equilibrium is meant the equilibrium solution of an adjustment process where supply and demand "bid-

---

1 This characterization allows for models with zero link costs, for example the model by Liew (1984).
2 The conditions apply to "deterministic" equilibrium models. In models of "stochastic" spatial equilibrium, e g. multinominal logit models in the tradition of McFadden (1975) and Williams (1977), the condition holds at the sink for the weighted average price of commodities from different origins.

prices" are given for each quantity, and where prices adjust faster than quantities. In a nonspatial, single commodity model, one may assume a Marshallian law of motion of the following excess price type.[3]

$$dq_i/dt = \Psi[\hat{p}_i(q_i) - \bar{p}_i(q_i)].$$ (5.1.1)

The traded quantity of the commodity i is denoted by q_i while $\hat{p}_i(q_i)$ and $\bar{p}_i(q_i)$ are inverse demand and supply functions. The sign-preserving function, Ψ, may be given as a constant, so that,

$$dq_i/dt = \kappa[\hat{p}_i(q_i) - \bar{p}_i(q_i)], \quad \kappa > 0.$$ (5.1.2)

As a complementarity condition, the equilibrium of (5.1.1) is written as: Find the equilibrium quantity that fulfils,

$$\bar{p}_i(q_i) - \hat{p}_i(q_i) \geq 0,$$ (5.1.3)

$$[\bar{p}_i(q_i) - \hat{p}_i(q_i)]q_i = 0. \quad q_i \geq 0.$$ (5.1.4)

For a positively traded quantity, the equilibrium price equals the supply and demand bid prices, thus $p_i = \hat{p}_i = \bar{p}_i$.

A Walrasian equilibrium, with ordinary supply and demand functions, is on the contrary price-oriented and presupposes quantities to adjust faster than prices. It is a common first approximation to assume that commodity markets are cleared due to the excess demand law by means of a "tatonnement" process, where each market price adjusts according to,[4]

$$dp_i/dt = \phi[d_i(p_i) - q_i(p_i)].$$ (5.1.5)

---

3   See e.g. Samuelson (1947), Negishi (1962), Kuenne (1963), Hansen (1966), and Beckmann and Wallace (1967) for discussions on Marshallian and Walrasian processes.
4   See e.g. Hahn (1982) for a discussion of tatonnement and non-tatonnement processes.

Above, the market price is denoted by p_i, and ϕ is a sign-preserving function.[5] The complementarity formulation is in this case: Find the equilibrium price that fulfils,

$$[q_i(p_i) - d_i(p_i)] \geq 0, \qquad (5.1.6)$$

$$[q_i(p_i) - d_i(p_i)]p_i = 0, \quad p_i \geq 0. \qquad (5.1.7)$$

The two complementarity formulations (5.1.3)-(5.1.4) and (5.1.6)-(5.1.7) give the core of the Marshallian and Walrasian models. In both cases the equilibrium assumptions, $dq_i/dt = 0$ and $dp_i/dt = 0$, imply that the future state of prices and quantities is studied by an adiabatic approximation of the dynamics of a system in continuous time. In the equilibrium models discussed in the sequel, where both prices and quantities are endogenous variables, the equilibrium approximation implies that only minor differences in the adjustment speeds are possible. In this chapter we are discussing models built around both types of market assumptions, while in Chapter 6 only Walrasian models are dealt with.

SPE models have, although the inverse formulation traditionally has dominated, been given both Walrasian and Marshallian formulations.[6] In applied analysis, the difference mainly is connected to the use of ordinary or inverse functions. In the latter case, the existence of the inverse has to be addressed. In programming solutions, a utilization of the former also has advantages if price floors and ceilings are used, while the latter is more appropriate when quantitative restrictions such as quotas should be introduced.

The SPE model is mainly used with a given income distribution and without factor markets. The model is an extension of Koopmans classical transport problem, but with price elastic demand and supply. It explicitly takes care of link-related transportation costs, but has limitations if used as a model of economy wide structural change. Commodities are assumed to be homogeneous within sectors and over space. Hence, in a single-period model, cross-hauling is normally excluded by defi-

---

5 Compare the tatonnement process discussed in Chapter 2.
6 Models, which may be the most realistic ones, where Walrasian and Marshallian equilibrium processes interact simultaneously have been studied, e.g. by Brody (1970). Brody then formulated a dynamic Leontief model as an optimal control problem.

nition. The dimension becomes extremely large if a complete economy with diversified products is modelled. Because of this, the model has above all been used in studies of raw materials and other homogeneous commodity markets, in which commodity exchange markets may be established. However, even in homogeneous commodity markets, cross-hauling may be observed. Besides the fact that only a small amount of commodities originating from different locations in reality are homogeneous, this may be explained by deviations from the assumptions of perfect competition and perfect information, and from aggregation over time, as was discussed in Chapter 4.

In the present chapter, the aim is to show how the vintage supply-side may be introduced in a couple of SPE models, to relate those to each other, and to discuss the advantages and disadvantages of the formulations. The chapter is composed as follows. In section 5.2 the vintage approach is introduced in a single commodity Marshallian SPE model with spatially homogeneous commodities. This model is then, in section 5.3, extended into a spatial multisector model. Conditions for solution by quadratic programming are discussed in both sections. In section 5.4, we present a nonspatial single commodity and a spatial multisector formulation of the Walrasian SPE model with spatially homogeneous commodities. In section 5.5, the chapter is closed with a summary of the vintage models in the SPE category. It is also shown how spatial heterogeneity may be introduced, which gives a bridge over to the Walrasian "Armington" models, presented in Chapter 6.

5.2   A SINGLE COMMODITY MARSHALLIAN PRICE
EQUILIBRIUM

The following conditions define a Marshallian nonspatial single commodity equilibrium with a vintage supply-side and price elastic demand (E:1). The inverse demand function was introduced above in section 5.1;[7]

(E:1.1) No excess demand for the commodity. [Marshallian case] A positive equilibrium price implies zero excess demand.

---

7   Each equilibrium model, denoted by (E:model number), is formulated as a set of conditions, connected to the model number.

$$S_i q_i(\tau) - d_i \geq 0, \tag{5.2.1}$$

$$[S_i q_i(\tau) - d_i] p_i = 0, \quad p_i \geq 0. \tag{5.2.2}$$

(E:1.2) Demand equilibrium. A positive demanded quantity implies equality between the demand bid-price and the equilibrium price.

$$p_i - \hat{p}_i(d_i) \geq 0, \tag{5.2.3}$$

$$[p_i - \hat{p}_i(d_i)] d_i = 0, \quad d_i \geq 0. \tag{5.2.4}$$

(E:1.3) Producer equilibrium. A positive supply implies equality between the supply bid price and the equilibrium price.

$$\bar{p}_i(q_i) - p_i \geq 0, \tag{5.2.5}$$

$$[\bar{p}_i(q_i) - p_i] q_i = 0, \quad q_i \geq 0. \tag{5.2.6}$$

The vintage version of the producer equilibrium is given by the following conditions:

(E:1.3′) No active vintage earns a quasi-rent below what is earned by the marginal vintage. A vintage generating losses is not allowed to remain active.

$$\mathbf{v}_i(\tau) + \bar{\sigma}_i(\tau) - S_i^T p_i \geq \mathbf{0}, \tag{5.2.7}$$

$$[\mathbf{v}_i(\tau) + \bar{\sigma}_i(\tau) - S_i^T p_i]^T q_i(\tau) = 0, \quad q_i(\tau) \geq \mathbf{0}. \tag{5.2.8}$$

(E:1.4) No production above the capacity of any vintage. A vintage not used at full capacity, may not earn positive quasi-rents.

$$\bar{q}_1(r) - q_1(r) \geq 0, \tag{5.2.9}$$

$$[\bar{q}_1(r) - q_1(r)]^T \bar{\sigma}_1 - 0, \quad \bar{\sigma}_1 \geq 0. \tag{5.2.10}$$

The last condition is easily extended to include other quantitative resource constraints besides vintage capacities. The possibility to obtain positive quasi-rents, emphasizes that the model describes a sector with imperfect access to new techniques and with locational advantages. Quasi-rents are due to the scarcity of such favourable techniques and locations. However, the sector is competitive in the sense that all vintages are price-takers. A solution of (E:1) thus diverges from what one would obtain if the supply-side was characterized by constant returns-to-scale or if a multi-plant monopolist controlled the sector.

The following profit maximization problem may be shown to satisfy the equilibrium conditions above,[8]

(A:IX):

$$\underset{\{d_i, \ q_1(r)\}}{\text{Max}} \quad \hat{p}_i(d_i)d_i - v_1(r)^T q_1(r) \tag{5.2.11}$$

$$\text{s.t.} \quad d_i - S_i q_i(r) \leq 0 \tag{5.2.12}$$

$$q_1(r) \leq \bar{q}_1(r) \tag{5.2.13}$$

$$S_1^T \sigma_i - \bar{\sigma}_1(r) \leq v_1(r) \tag{5.2.14}$$

$$\hat{p}_i(d_i) - \sigma_i \leq 0 \tag{5.2.15}$$

$$d_i, \ q_1(r) \geq 0$$

The dual variable to the excess demand constraint (5.2.12), is the imputed equilibrium price. Hence, $\sigma_i - p_i$. This dual variable is introduced in the primal in

---

(5.2.14) and (5.2.15). In (5.2.14) it constrains the supply bid-price, given by the cost of production and the quasi-rent in the marginal active vintage. Furthermore, in condition (5.2.15) the bid demand price is constrained by the imputed equilibrium price. The problem (A:IX) is, with this mix of primal and dual variables, not possible to solve as an extreme problem. It has to be treated as an equilibrium problem, and solved by complementarity algorithms. The complementarity problem would then include the following vector and vector function,

$$
z = \begin{bmatrix} d_i \\ q_i(r) \\ p_i \\ \bar{\sigma}_i(r) \end{bmatrix} \qquad F(z) = \begin{bmatrix} p_i - \hat{p}_i(d_i) \\ v_i(r) + \bar{\sigma}_i(r) - S_i^T p_i \\ S_i q_i(r) - d_i \\ \bar{q}_i(r) - q_i(r) \end{bmatrix} \qquad (5.2.16)
$$

However, in the special case where the inverse demand function is linear, $\hat{p}_i = \theta_i - \Omega_i d_i$, a solution which satisfies (E:1) is obtained from the following quadratic programming (QP) problem,[9]

(A:X):

$$
\underset{\{d_i, \ q_i(r)\}}{\text{Max}} \quad [\theta_i - \tfrac{1}{2}\Omega_i d_i] d_i - v_i(r)^T q_i(r) \qquad (5.2.17)
$$

$$
\text{s.t.} \qquad d_i - S_i q_i(r) \leq 0 \qquad (5.2.18)
$$

$$
q_i(r) \leq \bar{q}_i(r) \qquad (5.2.19)
$$

$$
d_i, \ q_i(r) \geq 0
$$

The trick here is to multiply by $\tfrac{1}{2}$ in the inverse demand function. This forces the solution from one where the marginal cost equals the marginal revenue, to a

9   Compare Takayama-Judge (1964, 1971), Kennedy (1974), Mathiesen (1977), Uri (1978).

solution where the marginal cost equals the imputed equilibrium price. That is, monopolistic pricing, which has the revenue function $[\theta_i - \Omega_i d_i] d_i$, is prohibited. The objective thus captures the interaction between many actors on both the supply and demand sides of the market.

The equality between (A:X) and (E:1) with a linear demand function may be shown if the dual variables σ_i and $\bar{\sigma}_i(\tau)$ are associated with constraints (5.2.18) and (5.2.19). If we introduce the Lagrangean to (A:X) and differentiate with respect to demand and vintage supply, the following Kuhn-Tucker conditions are obtained,[10]

$$[\theta_i - \Omega_i d_i - \sigma_i] d_i = 0, \qquad\qquad (5.2.20)$$

$$[\mathbf{S_i}^T \sigma_i - \mathbf{v_i}(\tau) - \bar{\sigma}_i(\tau)]^T \mathbf{q_i}(\tau) = 0. \qquad\qquad (5.2.21)$$

From (5.2.20) and (5.2.21) it is obvious that the imputed demand price equals the quasi-rent plus the unit cost in the marginal vintage for positive demand. If the latter is interpreted as the supply bid-price, $\tilde{p}_i$, the following results are obtained,

$$d_i > 0 \qquad \Rightarrow \sigma_i = \theta_i - \Omega_i y_i = \hat{p}_i, \qquad\qquad (5.2.22)$$

$$q_i(\tau) > 0 \Rightarrow \sigma_i = v_i(\tau) + \bar{\sigma}_i(\tau) = \tilde{p}_i, \qquad \text{all } \tau, \qquad (5.2.23)$$

$$\sigma_i > 0 \qquad \Rightarrow d_i = \mathbf{S_i} \mathbf{q_i}(\tau). \qquad\qquad (5.2.24)$$

It is thus evident that a solution of (A:X) fulfils both the conditions in (E:1) and those of (A:IX). Problem (A:X) gives a complementarity problem with the following vector and vector function,

---

10 The Kuhn-Tucker conditions on the primal variables in a maximization problem are nonpositive, while complementarity problems are characterized by nonnegativity conditions. This explains the reverse of signs.

$$
z = \begin{bmatrix} d_i \\ q_i(r) \\ p_i \\ \bar{\sigma}_i(r) \end{bmatrix} \qquad F(z) = \begin{bmatrix} p_i - \theta_i + \Omega_i d_i \\ v_i(r) + \bar{\sigma}_i(r) - S_i^T p_i \\ S_i q_i(r) - d_i \\ \bar{q}_i(r) - q_i(r) \end{bmatrix} . \qquad (5.2.25)
$$

A further study of $F(z)$ shows, as expected, that this (CP) is an (LCP) with a matrix composed of the sum of a skew-symmetric and a symmetric matrix. The latter has the slope of the demand function, Ω_i, in the upper left corner and zero's elsewhere,

$$
F(z) = \begin{bmatrix} -\theta_i \\ v_i(r) \\ 0 \\ \bar{q}_i(r) \end{bmatrix} + \begin{bmatrix} \Omega_i & 0 & 1 & 0 \\ 0 & 0 & -S_i^T & I \\ -1 & S_i & 0 & 0 \\ 0 & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} d_i \\ q_i(r) \\ p_i \\ \bar{\sigma}_i(r) \end{bmatrix} . \qquad (5.2.26)
$$

If the inverse demand function in (5.2.26) is replaced by a fixed demand, the problem is reduced to the vintage cost function obtained from the linear programming formulation in problem (B:II).[11]

In the section we have shown, in the simplest possible model, how price elastic functions generally create a (CP), but also how the special case with linear functions may be solved by (QP). In the following section, the above nonspatial single commodity model is extended into a spatial multisector formulation and the conditions for solution by a quadratic programming algorithm are determined.

---

11   The supply function is thus in this case interpreted as an inverse function.

5.3   A MARSHALLIAN SPATIAL MULTISECTOR PRICE EQUILIBRIUM

The spatial multisector model is characterized by a distinction between interregional deliveries of intermediate goods, and goods for final demand. The multisector framework also conveys, as was discussed in Chapter 4, a necessity to be careful when specifying the transportation sector. In this section, transportation is assumed to be conducted by an exogenous sector, also excluded from the input-output matrix.

The model is traditional with an assumption of spatial homogeneity. This implies that the matrix $A^r(r)$, is used to represent intermediate demand. To simplify notation, the fixed unit labour costs are suppressed, but are easily introduced in the objective as in Chapter 4.

Generally, an ordinary nonlinear, price-dependent, multicommodity final demand function is written as,

$$y^r - y^r(p^r) . \qquad (5.3.1)$$

In a Marshallian SPE model, the following inverse demand function is assumed to exist,

$$\hat{p}^r - \hat{p}^r(y^r) , \qquad (5.3.2)$$

The vectors of market and demand bid-prices for the sectors are denoted by,

$$p^r - \begin{bmatrix} p_1^1 \\ \cdot \\ \cdot \\ p_{NS}^1 \\ \cdot \\ p_{NS}^{NR} \end{bmatrix}, \qquad \hat{p}^r - \begin{bmatrix} \hat{p}_1^1 \\ \cdot \\ \hat{p}_{NS}^1 \\ \cdot \\ \hat{p}_{NS}^{NR} \end{bmatrix} . \quad (5.3.4)$$
$$[(NR \times NS) \times 1] \qquad\qquad [(NR \times NS) \times 1]$$

Generally, the existence of an inverse multisector demand function is not guaranteed without further conditions. Existence of a one-to-one inverse function requires that the ordinary demand function be strictly monotone, which slightly constrains the use of the inverse function.

Given those definitions, a Marshallian multicommodity spatial equilibrium with spatially homogeneous commodities, (E:2), contains the following conditions.

(E:2.1) No excess demand for any commodity in any region. A positive demand price in a region implies zero excess demand in the region.

$$\hat{G}^r{}_x{}^{rs} - A^r(r)q^r(r) - y^r \geq 0, \tag{5.3.5}$$

$$[\hat{G}^r{}_x{}^{rs} - A^r(r)q^r(r) - y^r]^T\hat{p}^r = 0, \quad \hat{p}^r \geq 0. \tag{5.3.6}$$

(E:2.2) Possibility for excess supply of a commodity in a region. A positive supply price of a commodity in a region implies zero excess supply.

$$S^r q^r(r) - \tilde{G}^r{}_x{}^{rs} \geq 0, \tag{5.3.7}$$

$$[S^r q^r(r) - \tilde{G}^r{}_x{}^{rs}]^T\tilde{p}^r = 0, \quad \tilde{p}^r \geq 0. \tag{5.3.8}$$

(E:2.3) Spatial price equilibrium for each positive flow of a commodity.

$$\tilde{G}^{rT}\tilde{p}^r - \hat{G}^{rT}\hat{p}^r + t^{rs} \geq 0, \tag{5.3.9}$$

$$[\tilde{G}^{rT}\tilde{p}^r - \hat{G}^{rT}\hat{p}^r + t^{rs}]^T x^{rs} = 0, \quad x^{rs} \geq 0. \tag{5.3.10}$$

(E:2.4) Regional demand equilibrium for each commodity in each region.

$$\hat{p}^r - \hat{p}^r(y^r) \geq 0, \tag{5.3.11}$$

$$[\hat{p}^r - \hat{p}^r(y^r)]^T y^r = 0, \quad y^r \geq 0. \tag{5.3.12}$$

(E:2.5) Regional producer equilibrium for each commodity and region [Vintage formulation].

$$\mathbf{A^r}(r)^T \hat{\mathbf{p}}^r + \overline{\sigma}^r(r) - \mathbf{S}^{rT} \tilde{\mathbf{p}}^r \geq \mathbf{0}, \tag{5.3.13}$$

$$[\mathbf{A^r}(r)^T \hat{\mathbf{p}}^r + \overline{\sigma}^r(r) - \mathbf{S}^{rT} \tilde{\mathbf{p}}^r]^T \mathbf{q}^r(r) - 0, \quad \mathbf{q}^r(r) \geq \mathbf{0}. \tag{5.3.14}$$

(E:2.6) No excess utilization of a vintage in any region. A vintage which is not used at full capacity does not earn a positive quasi-rent.

$$\overline{\mathbf{q}}^r(r) - \mathbf{q}^r(r) \geq \mathbf{0}, \tag{5.3.15}$$

$$[\overline{\mathbf{q}}^r(r) - \mathbf{q}^r(r)]^T \overline{\sigma}^r(r) - 0, \quad \overline{\sigma}^r \geq \mathbf{0}. \tag{5.3.16}$$

The complementarity problem (E:2) contains the vector and vector function,

$$\mathbf{z} - \begin{bmatrix} \mathbf{x}^{rs} \\ \mathbf{y}^r \\ \mathbf{q}^r(r) \\ \hat{\mathbf{p}}^r \\ \tilde{\mathbf{p}}^r \\ \overline{\sigma}^r(r) \end{bmatrix} \quad \mathbf{F}(\mathbf{z}) - \begin{bmatrix} \tilde{\mathbf{G}}^{rT} \tilde{\mathbf{p}}^r - \hat{\mathbf{G}}^{rT} \hat{\mathbf{p}}^r + \mathbf{t}^{rs} \\ \hat{\mathbf{p}}^r - \hat{\mathbf{p}}^r(\mathbf{y}^r) \\ \mathbf{A^r}(r)^T \hat{\mathbf{p}}^r + \overline{\sigma}^r(r) - \mathbf{S}^{rT} \tilde{\mathbf{p}}^r \\ \hat{\mathbf{G}}^r \mathbf{x}^{rs} - \mathbf{y}^r - \mathbf{A^r}(r)\mathbf{q}^r(r) \\ \mathbf{S}^r \mathbf{q}^r(r) - \tilde{\mathbf{G}}^r \mathbf{x}^{rs} \\ \overline{\mathbf{q}}^r(r) - \mathbf{q}^r(r) \end{bmatrix}. \tag{5.3.17}$$

This problem may be compared with the (LP) problem in (4.2.37). Final demand is now an endogenous variable. The general form of this function implies that this is a nonlinear (CP). However, if the demand functions are linear and the matrix of substitution coefficients is symmetric, the spatial multicommodity model may also be formulated as a quadratic programming problem. Assume, that the demand for pri-

vate consumption is given as a set of affine functions of prices, which can be written as,

$$c_i^r = \bar{c}_i^r + \sum_j \beta_{ij}^r p_j^r, \qquad \text{all i.} \qquad (5.3.18)$$

In matrix notation (5.3.18) becomes,

$$c^r = \bar{c}^r + \beta^r p^r, \qquad (5.3.19)$$

where the consumption vector and the vector of intercepts are denoted,

$$
c^r = \begin{bmatrix} c_1^1 \\ \vdots \\ c_{NS}^{NR} \end{bmatrix}, \qquad \bar{c}^r = \begin{bmatrix} \bar{c}_1^1 \\ \vdots \\ \bar{c}_{NS}^{NR} \end{bmatrix}, \qquad (5.3.20)
$$

$$[(NRxNS) \times 1] \qquad\qquad [(NRxNS) \times 1]$$

and the substitution matrix is written,

$$
\beta^r = \begin{bmatrix} \beta^1 & & \\ & \cdot & \\ & & \cdot \\ & & & \beta^{NR} \end{bmatrix}, \quad \beta^r = \begin{bmatrix} \beta_{11}^r & \cdots & \beta_{1NS}^r \\ \cdot & & \cdot \\ \beta_{NS1}^r & \cdots & \beta_{NSNS}^r \end{bmatrix} \qquad (5.3.21)
$$

$$[(NRxNS) \times (NRxNS)] \qquad [NS \times NS]$$

Each submatrix, β^r, $r = 1, \ldots, NR$, is a region-specific matrix, where the origin of the demanded commodity is not specified.

Moreover, let the part of final demand which is given exogenously be denoted by $\bar{g}_i^r$ and the vector, $[(NRxNS) \times 1]$, of this demand be denoted $\bar{g}^r$. It includes government expenditures and investment demand. A linear final demand function is then determined as,

$$\mathbf{y}^r = \bar{\mathbf{y}}^r + \beta^r \mathbf{p}^r = \bar{\mathbf{g}}^r + \bar{\mathbf{c}}^r + \beta^r \mathbf{p}^r. \qquad (5.3.22)$$

Above, $\mathbf{y}^r$ is a vector of final demands and $\bar{\mathbf{y}}^r$ a vector of intercepts. The inverse linear demand function is in a spatial model with spatially homogeneous commodities written as,[12]

$$\hat{\mathbf{p}}^r = \Theta^r + \Omega^r \mathbf{y}^r, \qquad (5.3.23)$$

where the vector of intercepts and the matrix of substitution terms are written,[13]

$$\Theta^r = \begin{bmatrix} \theta_1^1 \\ \cdot \\ \cdot \\ \cdot \\ \theta_{NS}^{NR} \end{bmatrix}, \qquad \Omega^r = \begin{bmatrix} \Omega^1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot & \\ & & & & \Omega^{NR} \end{bmatrix}. \qquad (5.3.24)$$

$[(NRxNS) \times 1]$ $[(NRxNS) \times (NRxNS)]$

With inverse demand functions, the spatial multisector problem may be formulated as the following profit maximization problem,

---

12   This formulation was used by e.g. Kennedy (1974) and Takayama (1978) in their quadratic programming energy models. Also Norén (1987) has recently utilized the inverse linear function in a nonspatial multisector model.

13   The substitution matrix consists of submatrices in a similar way as the matrices in (5.3.21).

(A:XI):

$$\text{Max}_{\{x^{rs},\ y^r,\ q^r(r)\}} \quad [\theta^r - \tfrac{1}{2}\Omega^r y^r]^T y^r - 0^T q^r(r) - t^{rs} T_x^{rs} \qquad (5.3.25)$$

$$\text{s.t.} \quad y^r + A^r(r)q^r(r) - \hat{G}^r x^{rs} \leq 0 \qquad (5.3.26)$$

$$\tilde{G}^r x^{rs} - S^r q^r(r) \leq 0 \qquad (5.3.27)$$

$$q^r(r) \leq \overline{q}^r(r) \qquad (5.3.28)$$

$$x^{rs},\ y^r,\ q^r(r) \geq 0$$

Above, the zero vector, 0 [(NRxNSxNV) x 1], is also included in the objective. The vector function $F(z)$ is in this case written as,

$$
F(z) = \begin{bmatrix} t^{rs} \\ -\theta^r \\ 0 \\ 0 \\ 0 \\ \overline{q}^r(r) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & -\hat{G}^{rT} & \tilde{G}^{rT} & 0 \\ 0 & \Omega^r & 0 & I & 0 & 0 \\ 0 & 0 & 0 & A^r(r)^T\!-\!S^{rT} & I \\ \hat{G}^r & -I & -A^r(r) & 0 & 0 & 0 \\ -\tilde{G}^r & 0 & S^r & 0 & 0 & 0 \\ 0 & 0 & -I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x^{rs} \\ y^r \\ q^r(r) \\ \hat{\sigma}^r \\ \mathfrak{d}^r \\ \overline{\sigma}^r(r) \end{bmatrix}
$$

$$(5.3.29)$$

The equilibrium prices are obtained as the shadow values to constraints (5.3.26) and (5.3.27). The substitution matrix is the important difference, compared with the

linear programming case in (4.2.37). The matrix Ω^Γ satisfies the following condition only when it is symmetric,

$$\tfrac{1}{2}[\Omega^\Gamma + \Omega^{\Gamma T}] = \Omega^\Gamma. \tag{5.3.30}$$

Hence, the above "integrability condition" gives a necessary condition, for a solution of (A:XI) as a quadratic programming problem.[14]

Comparing the vector function in the complementarity formulation of the multisector transport cost minimization problem (B:VII) and the above vector function, reveals that the transport assignment model is a special case of the SPE formulation with price-inelastic final demand. One may furthermore notice that if the inverse linear demand function contains a zero matrix of substitution terms and a positive intercept, the problem becomes a combined price-quantity constrained sales-value maximization problem, where the difference between the exogenously given prices and the imputed price may be used as a signal in a dynamic price process[15].

5.4 WALRASIAN FORMULATIONS OF THE SPE VINTAGE MODEL

The models in sections 5.2 and 5.3 were formulated with inverse demand functions. The "bid supply prices" were obtained from inverse vintage supply functions. The models allowed for positive intraregional transport costs and differences between the supply and demand equilibrium prices in a region.

In this section, Walrasian models with ordinary demand functions are introduced together with the vintage supply side. In the spatial version, the assumption about spatial homogeneity will be retained. However, intraregional transport costs are assumed to be zero. The last assumption allows for simplified formulations, since an equality between supply and demand prices is obtained in each region.

---

14 See Takayama and Labys (1986).
15 Compare the discussion in Chapter 3 in relation to problem (A:IV).

To begin with, let us define a nonspatial Walrasian single commodity equilibrium with a vintage supply-side (E:3) as,

(E:3.1) No excess demand for the commodity. [Walrasian case.] A positive equilibrium price implies zero excess demand.

$$s_i q_i(\tau) - d_i(p_i) \geq 0, \tag{5.4.1}$$

$$[s_i q_i(\tau) - d_i(p_i)]p_i = 0, \quad p_i \geq 0. \tag{5.4.2}$$

(E:3.2) Producer equilibrium with a vintage supply function. Those are similar to conditions (E:1.3′) in the Marshallian model above.

(E:3.3) No production above the capacity in any vintage. Those conditions were given in (E:1.4).

Compared with the Marshallian, quantity-oriented formulations, the ordinary demand function in (5.4.1) and (5.4.2) is characteristic for this price-oriented formulation. Observe also that the explicit demand equilibrium (E:1.2) not has a similar counterpart in the Walrasian case.

Generally, (E:3) is a nonlinear (CP). However, different (LCP) formulations of the model with linear demand functions have been suggested. Takayama and Woodland (1970) noticed that the dual of the (QP) formulation of a Marshallian equilibria contained both price and quantity variables, which made it difficult to interpret the shadow variables as prices. The dual of the dual did not become the primal problem. Furthermore, such a dual still contains the inverse functions. Takayama and Woodland introduced a "purified dual", formulated in price terms only and with ordinary demand functions. However, the linear functions in the price problem may cause "irregularities" with negative quantities in the solution for some prices. A set of slack variables was thus included in the formulation to take care of such outcomes.[16]

---

16 See Takayama and Judge (1971). This purified model with slacks has recently been used by
    Kjellman (1988) in analyses of trade in steam-coal.

An alternative to the "purified dual" model is a programming formulation with ordinary functions and quantity constraints [Mathiesen (1977)]. If the demand function is linear, and $d_i - \bar{d}_i - \beta_i p_i$, problem (E:3) may be formulated as the following optimization problem in both prices and quantities,

(A:XII):

$$\text{Max} \atop \{p_i, q_i(r)\} \quad - \tfrac{1}{2} p_i \beta_i p_i - \mathbf{v_i}(r)^T \mathbf{q_i}(r) \qquad (5.4.3)$$

$$\bar{d}_i - \beta_i p_i - \mathbf{S_i q_i}(r) \leq 0 \qquad (5.4.4)$$

$$\mathbf{q_i}(r) \leq \bar{\mathbf{q}}_i(r) \qquad (5.4.5)$$

$$p_i, \ \mathbf{q_i}(r) \geq 0$$

Let σ_i and $\bar{\sigma}_i(r)$ be the dual variables to constraints (5.4.4) and (5.4.5). The following Kuhn-Tucker conditions are associated with the primal variables,

$$[\beta_i \sigma_i - \beta_i p_i] p_i - 0, \qquad (5.4.6)$$

$$[\mathbf{v_i}(r) + \bar{\sigma}_i(r) - \mathbf{S_i}\sigma_i]^T \mathbf{q_i}(r) - 0. \qquad (5.4.7)$$

So that,

$$p_i > 0 \Rightarrow p_i - \sigma_i. \qquad (5.4.8)$$

Hence, the market price in the demand function equals the imputed equilibrium price. This price also equals the market price faced by the producers,

$$q_i(r) > 0 \Rightarrow \bar{v}_i(r) + \bar{\sigma}_i(r) - p_i. \qquad (5.4.9)$$

Observe that the intercept in the linear demand function is not a part of the objective in this Walrasian formulation. Furthermore, both the price and the supplied quantities [but not the demanded quantity] are primal variables. In any case, the dual variables may be interpreted as prices and the market price obtained as the dual variable, σ_i, equals the price met by the consumers. The problem is neither a pure cost minimization nor a pure profit maximization problem. If β_i equals zero, the problem becomes a cost minimization problem in the sense of (B:II). It may thus be seen as an augmented cost minimization problem, but the objective does not directly depend on the demanded quantity but on the price of the output, i.e. a feature that characterizes a profit maximization problem.

An (LCP) formulation of problem (A:XII) contains a mapping $\mathbf{F}(\mathbf{z})$ with the structure,

$$
\mathbf{F}(\mathbf{z}) \; = \;
\begin{bmatrix}
\mathbf{v_1}(r) \\[4pt]
-\overline{\mathbf{d}}_i \\[4pt]
\overline{\mathbf{q}}_1(r)
\end{bmatrix}
\; + \;
\begin{bmatrix}
\mathbf{0} & -\mathbf{S_i}T & \mathbf{I} \\[4pt]
\mathbf{S_1} & -\beta_i & \mathbf{0} \\[4pt]
-\mathbf{I} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q_1}(r) \\[4pt]
\mathbf{p}_i \\[4pt]
\overline{\sigma}_1(r)
\end{bmatrix} . \quad (5.4.10)
$$

Demand no longer is the explicit entity it was in the Marshallian formulation (5.2.26). When the demand function in (E:1) is the inverse of the function in (E:3) the two solutions are equal since $\widetilde{p}_i \; = \; \hat{p}_i \; = \; p_i$. The advantages of this formulation compared with both the Marshallian and the purified dual are the reduction in the size of the problem, and the use of the ordinary demand function. As will be made clear in the next chapter, the formulation also has a direct relation to the Walrasian formulation used in ICGE models.

In the following this equilibrium formulation is extended to a Walrasian spatial multisector model. Although the assumption is not necessary, it is assumed that intraregional transport costs are zero, i.e. $t_i^{rr} \; = \; 0$, for all r, i. In this way, a single equilibrium price in each region is obtained and the formulation becomes simplified. The assumption of spatially homogeneous commodities is retained. The excess demand functions are thus given as ordinary functions of a single market price,

$$d_i^r(p^r) - q_i^r(p^r), \qquad\qquad \text{all r, i,} \qquad (5.4.11)$$

instead of the formulation,

$$d_i^r(\hat{p}^r) - q_i^r(\tilde{p}^r), \qquad\qquad \text{all r, i,} \qquad (5.4.12)$$

which would be used in a model with intraregional transport costs. The equilibrium conditions to this Walrasian spatial multicommodity model with spatially homogeneous commodities (E:4) are,

(E:4.1) No excess demand for any commodity in any region. [Walrasian formulation without intraregional transportation costs]. A positive regional equilibrium price, implies zero excess demand.

$$[S^r - A^r(r)]q^r(r) - y^r(p^r) - [\tilde{G}^r + \hat{G}^r]x^{rs} \geq 0, \qquad (5.4.13)$$

$$[[S^r - A^r(r)]q^r(r) - y^r(p^r) - [\tilde{G}^r + \hat{G}^r]x^{rs}]^T p^r = 0,$$

$$p^r \geq 0. \qquad\qquad (5.4.14)$$

Above, the following excess demand condition has been introduced,

$$q_i^r - \sum_j q_{ji}^r - y_i^r + \sum_s x_i^{sr} - \sum_s x_i^{rs} \geq 0, \text{ all } r,i. \qquad (5.4.15)$$

This simplification, compared with (E:2.1) and (E:2.2) is possible since the intraregional transport costs were assumed to be zero.

(E:4.2) Spatial multisector price equilibrium with a condition on positive flows.

$$[\widetilde{G}^{rT} - \hat{G}^r]^T p^r + t^{rs} \geq 0, \tag{5.4.16}$$

$$[[\widetilde{G}^{rT} - \hat{G}^r]^T p^r + t^{rs}]^T x^{rs} - 0, \quad x^{rs} \geq 0. \tag{5.4.17}$$

(E:4.3) Regional multisector producer equilibrium. Vintage formulation with a profitability condition on each vintage.

$$\bar{\sigma}^r(\tau) + [A^r(\tau) - S^r]^T p^r \geq 0, \tag{5.4.18}$$

$$[\bar{\sigma}^r(\tau) + [A^r(\tau) - S^r]^T p^r]^T q^r(\tau) - 0, \quad q^r(\tau) \geq 0. \tag{5.4.19}$$

(E:4.4) No production above capacity in any vintage. These conditions were given in (E:2.6).

The complementarity formulation contains the vector function $F(z)$,

$$F(z) - \begin{bmatrix} t^{rs} \\ 0 \\ 0 \\ \bar{q}^r(\tau) \end{bmatrix} + \begin{bmatrix} 0 & 0 & [\widetilde{G}^r - \hat{G}^r]^T & 0 \\ 0 & 0 & [A^r - S^r]^T & I \\ -\widetilde{G}^r + \hat{G}^r & S^r - A^r & -y^r(p^r) & 0 \\ 0 & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} x^{rs} \\ q^r(\tau) \\ p^r \\ \bar{\sigma}^r(\tau) \end{bmatrix}$$

$$\tag{5.4.20}$$

The exclusion of intraregional transport costs and the ordinary demand function thus considerably reduces the dimension of the problem, compared with (5.3.29). Assuming a linear demand function, with a symmetric negative semidefinite matrix β^r, the following quadratic multiregional multisector problem satisfies the conditions in (E:4),

(A:XIII):

$$\underset{\{p^r,\ x^{rs},\ q^r(\tau)\}}{\text{Max}} \quad - \tfrac{1}{2}p^{rT}\beta^r p^r - t^{rsT}x^{rs} - 0^T q^r(\tau) \qquad (5.4.21)$$

$$\text{s.t.} \quad \bar{y}^r - \beta^r p^r + [A^r(\tau) - S^r]q^r(\tau)$$

$$+ [\hat{G}^r - \hat{G}^r]x^{rs} \le 0 \qquad (5.4.22)$$

$$q^r(\tau) \le \bar{q}^r(\tau) \qquad (5.4.23)$$

$$p^r,\ x^{rs},\ q^r(\tau) \ge 0$$

Derivation of the complementarity conditions as was made in relation to problem (A:XII), reveals that at optimum the primal and dual prices are equal. The symmetry condition on β^r is the important constraint on the model if (QP) algorithms shall be used.

The formulation (E:4) gives a "Walras-Koopmans" model which gives a direct link between the traditional SPE formulation discussed earlier, and the ICGE models in the next chapter.

5.5 ASSESSMENT OF THE SPE MODELS

In the chapter, vintage versions of a set of models in the SPE tradition have been derived. The relation between the Walrasian and the Marshallian formulations have also been demonstrated. The models have been formulated with spatially homogeneous commodities. Spatial heterogeneity is obtained in the models by treating each commodity originating from a given region as a specific commodity, when the elasticity of substitution between the same commodity from two different regions is less then infinite. The excess demand condition (5.4.15) is then simplified to,

$$q_i^r - \underset{s\ne r}{\Sigma}\ x_i^{rs} - \underset{j}{\Sigma}\ q_{ji}^r - y_i^r \ge 0, \qquad (5.5.1)$$

since $\Sigma \atop s$ $x^{sr} \atop i$ $-$ 0. In the extreme case, the number of commodities is increased from NS to NRxNS, and thus $i, j - 1, \ldots, NRxNS$.

Endogenous investments in new vintages, as formulated in Chapter 4, are possible to introduce in the SPE models of this chapter. Such a model is discussed in Johansson and Westin (1987). Labour market constraints for different skill categories are also easily introduced.

The models in Chapter 4 were supply-side oriented and based on a rudimentary treatment of private consumption. This gave these models some considerable short-comings, despite the simplified solution methods. Although such supply-side models can illustrate the interaction between actors within the industry, the important and easily observed interdependences between production, consumer behaviour and structural change are left out from the formulations. In this chapter price-dependent demand functions have been introduced, increasing the realism of the models especially in medium- and long-term applications.

However, income effects have been exogeneously determined. This has allowed for an extremal formulation and been motivated by focusing on a smaller subset of commodities in traditional SPE models. A nonlinear (CP) would also allow for an introduction of income effects in the multicommodity SPE model. In order to ana-lyze income effects, one has to treat a major part of the economy. Hence, Walras-Armington models have an advantage in this respect since they are formulated for more aggregated and heterogeneous commodities. In this chapter, the step from a "Marshall-Koopmans" to a "Walras-Koopmans" version of the SPE model was demonstrated. When we move from the "Walras-Koopmans" to a "Walras-Armington" model and also introduce income effects we arrive in the area of ICGE models. Before we leave the SPE tradition, it is again appropriate to empasize the important contribution of recent models in the SPE tradition.[17] The link related treatment of the transport sector has been extended to allow for a detailed description of the transport network in those models. We will introduce such a detailed network in an ICGE model in the following chapter.

---

17   See e.g. Friesz and Harker (1985), Florian and Los (1982), and Echenique et al. (1988).

6 INTERREGIONAL COMPUTABLE GENERAL EQUILIBRIUM MODELS WITH A VINTAGE FUNCTION

6.1 INTRODUCTION

In this chapter, vintage models which belong to the ICGE class are introduced. The models are thus of a Walrasian type. The Armington assumption is utilized to obtain spatially heterogeneous commodities. Income effects are treated endogenously through linear expenditure systems. In the chapter, the emphasize is placed on demonstrating how different behavioural assumptions can be combined in equilibrium models.

The proof of existence of a spatial general equilibrium for a productive economy was, although in an implicit way, given by Arrow-Debreu (1954) and Debreu (1959). Isard and Ostroff (1958), starting from the Arrow-Debreu model, gave a proof with spatially homogeneous commodities, explicit budget constraints and an explicit spatial price equilibrium formulation.

Negishi (1960) gave an alternative definition of a single region Walrasian equilibrium in terms of a social welfare function and a choice of welfare weights. This formulation was used by Hardley-Kemp (1966) to define an international equilibrium for two countries. Woodland[1] (1969) extended this to a multicountry model.

A traditional ICGE model normally contains the following features.[2] The economy consists of a specified number of regions with a number of consumers, each with an initially given endowment. Each consumer has a preference preordering and a budget constraint which together give demand functions for commodities and supply

---

1 Also found in Takayama and Judge (1971).
2 Early contributions of ICGE models may be found in Shoven and Whalley (1974), Manne et al. (1980), and Liew (1984).

functions for factors. Those are continuous, nonnegative, homogeneous of degree zero in prices and income, and satisfy Walras' Law for all prices. Hence, consumer incomes equal consumer expenditures. A pure ICGE model, in the Walras-Arrow-Debreu sense, thus contains an induced income effect through a spatial relation between factor costs, incomes, and private consumption. Consumers may be aggregated and represented by a set of households.

The production system, located over regions, mostly consists of nonincreasing returns to scale activities or production functions of CES or similar type.[3] Producers maximize profits and the profit functions are homogeneous of degree one in output and factor prices. This implies that demand functions are homogeneous of degree zero in the factor prices. The fact that demand functions are homogeneous of degree zero imply that only relative prices are of interest and prices may be normalized and related to a numeraire in some appropriate way.

Trade is usually introduced by means of the Armington assumption with spatial CES functions and demand systems. The underlying network is of the simplest possible type. In this respect, the approach is close to the interregional input-output model by Isard (1951). In ICGE models which are used in international trade, the interaction costs are mainly represented by tariffs and other trade controls, while transport costs are suppressed. However, link costs and detailed networks may be introduced in the models and an explicit fulfilment of the spatial price equilibrium condition may be obtained.[4]

An ICGE model is economy-wide and contains all commodities and factors in the economy which are considered as relevant for the equilibrium. This introduces, together with the feedback between factor incomes and consumer demand, complex nonlinearities in the model. Solution algorithms, more demanding than the algorithms associated with the equilibrium formulations in, at least the traditional, SPE models are thus needed. ICGE models are generally deeply rooted in this Walras-Arrow-Debreu tradition. The models suggested in the following have some features which give them a set of specific and different properties compared with the traditional set up. The following aspects are emphasized:

---

3   Harris (1984) discusses a model with scale economies. Further development in this direction may
     be expected.
4   Buckley (1987), Westin (1988).

* The vintage supply-side introduces rigidities and existence of temporary mono-polies.

* Endogenous demand for investment commodities related to capacity expansion, guarantee a consistency between the growth of the economy and the production possibilities in the existing structure.

* Existing rigidities in each region, together with the endogenous determination of exit and entry, bring about possibilities for analyses of the speed of change in the locational pattern within an economy.

* Multihousehold formulations associated with constraints on the supply of labour with different skill, allow for simulation and analyses of distributional effects of spatial structural change.

* Rich possibilities for simulation of different policy alternatives are obtained since the models may be formulated with or without the explicit connection between factor costs and incomes as well as with fixed or endogenously determined wages.

* Explicit link costs give a difference between origin and destination prices. Accessibility is thus important in the determination of the flows.

* A detailed network representation with network assignment and modal choice is introduced. Such a model is appropriate as a tool for analysis of investments in the transport system.

The chapter has the following contents. In section 6.2, an intermediate model is defined with a national demand side (as represented by a linear expenditure system) and spatially identified vintages. Although markets are cleared at the national level, an endogenous solution of the spatial exit and entry problem may be obtained in a setting with moderate data demands. Hence, the regional "break down" is endoge-nously determined. The approach is superior to approaches which make this "break down" exogenously.

A model with both spatial demand and supply sides, link-related transport costs, explicit commodity flows and a multihousehold/labour skill formulation is introduced in section 6.3. The spatial representation is thus more complete in this model compared with the model in 6.2. In section 6.4, endogenous investment is included. So far, the models are formulated without explicit factor cost-income interdependences and with a very simple network. Explicit income formation and network features are then introduced in section 6.5.

The presented spatial multisector vintage formulations represent important extensions of both the vintage modelling and the ICGE traditions. The advantages, and limitations, of this type of models are finally discussed in section 6.6.

6.2   A MULTISECTOR EQUILIBRIUM MODEL WITH A SPATIAL VINTAGE FUNCTION

In this section, a model in which commodity markets are cleared at the national level, but where a regional specification of each vintage provides an opportunity to analyze spatial effects of national scenarios is presented.[5] The model is specified in a complementarity setting and it is shown that the problem is a nonlinear (CP).

A regional "break down" of a national equilibrium solution is obtained endogenously in the model. An alternative way to model such a break down of a national equilibrium may be exemplified by the ORANI-model [Dixon et al. (1982)]. The "top-down" approach in the ORANI-model implies that the regional solution is not obtained endogenously in the main model, but in a submodel, solved with the national solution as a given datum. The advantage of the model presented here, is the endogenous treatment of some spatial feedbacks.

In the model, final demand is derived from a national linear expenditure system. Supply and intermediate demand is obtained from vintage functions with a regional identification. Both supply and intermediate demand are then aggregated to national

---

totals. As a consequence, the model does not include interregional commodity flows
and spatial interaction costs in an explicit way.

The model is formulated with an exogenously fixed spatial wage structure and the
national, for private consumption, disposable income, Y, as an endogenous variable.
The equilibrium disposable income may be interpreted as the income which, via the
linear expenditure system, gives full employment of the labour force. However, with
given capacities, this may not always be possible to achieve below full capacity uti-
lization. The solution thus identifies bottle-necks in the economy.

The linear expenditure system (LES) may, but need not, be rationalized by assuming
that households maximize a Klein-Rubin-Stone-Geary utility function under a bud-
get constraint.[6] A single region system is given as,

$$p_i c_i = p_i \bar{c}_i + \alpha_i [Y - \sum_j p_j \bar{c}_j], \qquad \text{all } i. \qquad (6.2.1)$$

The elements $\bar{c}_i$, were interpreted by Samuelson (1947) as "subsistence" demands.
The coefficients, $\alpha_i \geq 0$, are marginal budget shares which allocate consumption
on commodities from the "supernumerary" income. The positive linear relation
between expenditures and income implies that the behaviour of the household is
constrained so that all commodities are normal commodities. The system gives the
following set of demand equations,

$$c_i = \bar{c}_i + [\alpha_i/p_i][Y - \sum_j p_j \bar{c}_j], \qquad \text{all } i. \qquad (6.2.2)$$

The demand functions are homogeneous of degree zero in prices and income.
Commodities are also net substitutes in pairs and the compensated cross-price elas-
ticities are positive and symmetric [Green (1979)]. The linear expenditure system
thus has properties which allow for existence of an equilibrium. However, when

---

6   Alternative demand systems exist. The LES is chosen since it as been estimated on Swedish data.
     The system will be further discussed in a spatial, multihousehold version in section 6.3.

income increases at constant prices, the income elasticities of all commodities move toward unity. The income elasticity for commodity i is obtained as,

$$e_{iY} = \alpha_i / [p_i c_i (\mathbf{p}, Y)/Y] \qquad\qquad (6.2.3)$$

The derivative of the elasticity with respect to income is,

$$de_{iY}/dY = [\alpha_i / p_i c_i (\mathbf{p}, Y)][1 - e_{iY}] \qquad\qquad (6.2.4)$$

Hence, when income increases at constant prices, the income elasticities will increase/decrease dependent on if they initially are below/above unity. This feature, which implies that necessities becomes more of luxuries when income increases, is a drawback of the system. This especially becomes a problem when large income changes are generated.

In short, the system may be summarized as a vector-valued function of prices and incomes. If exogenous final demand is added to the function as a fixed quantity the function in (6.2.5) is obtained.

$$\mathbf{y} = \mathbf{y}(\mathbf{p}, Y) \qquad\qquad (6.2.5)$$

A combined national-regional Walrasian multisector equilibrium with fixed wages, is then defined by the conditions specified below in (E:5).

(E:5.1) No excess demand of any commodity at the national level. A commodity in excess supply has zero price.

$$\Sigma^r [\mathbf{S}^r - \mathbf{A}^r(r)] \mathbf{q}^r(r) - \mathbf{y}(\mathbf{p}, Y) \geq \mathbf{0}, \qquad\qquad (6.2.6)$$

$$[\Sigma^r [\mathbf{S}^r - \mathbf{A}^r(r)] \mathbf{q}^r(r) - \mathbf{y}(\mathbf{p}, Y)]^T \mathbf{p} = 0, \qquad \mathbf{p} \geq \mathbf{0}, \qquad (6.2.7)$$

where Σ^r is the summation matrix,

$$\Sigma^{\mathbf{r}} = \begin{bmatrix} 1 & . & . & 1 \\ & . & : & . \\ 1 & . & . & 1 \end{bmatrix},$$ (6.2.8)

[NS x (NRxNS)]

which aggregates the regional supply of each commodity to national totals.

(E:5.2) No active vintages earn a quasi-rent below what is earned by the marginal vintage at the national prices. A vintage generating losses is not active.

$$\mathbf{1}^{\mathbf{r}}(r)^{T}\mathbf{w}^{\mathbf{r}} + \overline{\sigma}^{\mathbf{r}}(r) + [\mathbf{A}^{\mathbf{r}}(r) - \mathbf{S}^{\mathbf{r}}]^{T}\Sigma^{\mathbf{r}T}\mathbf{p} \geq \mathbf{0},$$ (6.2.9)

$$[\mathbf{1}^{\mathbf{r}}(r)^{T}\mathbf{w}^{\mathbf{r}} + \overline{\sigma}^{\mathbf{r}}(r) + [\mathbf{A}^{\mathbf{r}}(r) - \mathbf{S}^{\mathbf{r}}]^{T}\Sigma^{\mathbf{r}T}\mathbf{p}]^{T}\mathbf{q}^{\mathbf{r}}(r) = \mathbf{0},$$

$$\mathbf{q}^{\mathbf{r}}(r) \geq \mathbf{0}.$$ (6.2.10)

(E:5.3) No excess utilization of any vintage. A vintage not used at full capacity will not earn positive quasi-rents. The conditions are similar to (E:2.6).

(E:5.4) No excess utilization of the national labour force. At the equilibrium income, either the labour force or the production capacity is fully utilized.

$$\min\{\overline{L}, \Sigma\Sigma^{\mathbf{r}}\mathbf{1}^{\mathbf{r}}(r)\overline{\mathbf{q}}^{\mathbf{r}}(r)\} - \Sigma\Sigma^{\mathbf{r}}\mathbf{1}^{\mathbf{r}}(r)\mathbf{q}^{\mathbf{r}}(r) \geq \mathbf{0},$$ (6.2.11)

$$[\min\{\overline{L}, \Sigma\Sigma^{\mathbf{r}}\mathbf{1}^{\mathbf{r}}(r)\overline{\mathbf{q}}^{\mathbf{r}}(r)\} - \Sigma\Sigma^{\mathbf{r}}\mathbf{1}^{\mathbf{r}}(r)\mathbf{q}^{\mathbf{r}}(r)]Y = 0,$$

$$Y \geq 0.$$ (6.2.12)

Above, Σ is a unit vector which adds the sectoral labour demand into a national aggregate. Aggregate labour supply is exogenously given by $\overline{L}$. The constraint on labour demand is here given either by the labour supply or by the labour demand at full capacity utilization. Both labour supply and available capacities are given exoge-

nously. Hence, a solution with full employment cannot be guaranteed. In equilibrium, either the labour force, the production capacity or both will be fully utilized.

The model may, if an unconstrained new capacity vintage is introduced in each sector, also be used to estimate the amount of investments needed to obtain full employment at the national level.

However, wages are exogenously given, while quasi-rents, prices, and the disposable income is determined endogenously in the model. In this closed economy there is an equality between aggregate factor costs and household income, and the solution gives the share of factor incomes which has to be consumed by the household in order to fulfil condition (E:5.4).[7]

The complementarity problem associated with (E:5) contains the following vector function,

$$
F(z) = \begin{bmatrix} 1^{r}(r)^{T} w^{r} \\ 0 \\ \bar{q}^{r}(r) \\ \min\{\cdot\} \end{bmatrix} + \begin{bmatrix} 0 & [A^{r}(r)-S^{r}]^{T}\Sigma^{r T} & I & 0 \\ \Sigma^{r}[S^{r}-A^{r}(r)] & -y(p,Y) & 0 & -y(p,Y) \\ -I & 0 & 0 & 0 \\ -\Sigma\Sigma^{r}1^{r}(r) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q^{r}(r) \\ p \\ \bar{\sigma}^{r}(r) \\ Y \end{bmatrix}
$$

$$(6.2.13)$$

This mapping may be compared with the mapping of the Walrasian model in (5.4.10). The linear expenditure system, and the income constraint in condition (E:5.4), introduce new nonlinearities into the complementarity problem. The symmetry conditions, necessary for a solution by linear and quadratic programming are destroyed via the right column and the bottom row of the matrix in (6.2.13). The problem is a nonlinear (CP), and other algorithms have to be used. The adjustment

---

7   This equality will be discussed further in section 6.5 below. The equality is obtained by summation
     of the equality part of the complementarity conditions in the model.

of incomes to obtain the employment-capacity target may, for example, be seen as an outer loop around a (QP), obtained by linearization or in a tatonnement algorithm.

A model of this type was reported and solved with a tatonnement algorithm in Persson and Westin (1985). The problem was then to analyze spatial effects of exogenously imposed constraints on labour supply and international trade. The main advantage of this combined national-regional formulation, is the simplified estimation procedure. The main disadvantages are that spatial differences in taste and income cannot be included. Moreover, the transport sector is treated as a national aggregated input-output sector instead of a link-related accessibility cost, and the spatial flows are suppressed.

6.3   A SPATIAL WALRAS-ARMINGTON MULTI-HOUSEHOLD VINTAGE MODEL

6.3.1   INTRODUCTION

In this section, the spatial vintage structure from the previous models is kept, but the demand side is given a regional and multihousehold formulation. Hence, each region consist of a set of households with different endowment of labour and taste. A spatial multihousehold linear expenditure system is used to model demand for private consumption. It is shown how different equilibrium solutions may be obtained either by fixing the income level, by adjustment of the wage levels or, at fixed wages, by adjustment of final demands or disposable incomes.

6.3.2   A SPATIAL MULTIHOUSEHOLD LINEAR EXPENDITURE SYSTEM

In order to obtain price- and income-dependent spatial demand, a multihousehold spatial linear expenditure system (MSLES) with spatially heterogeneous commodities is developed in this section.

A spatial multihousehold version of the utility function for household h, in region s, is given by,

$$U^s(h) = \sum_{r i} \sum \alpha_i^{rs}(h) \ln[c_i^{rs}(h) - \bar{c}_i^{rs}(h)], \text{ all } s,h, \qquad (6.3.1)$$

$$\alpha_i^{rs}(h) \geq 0, \ \sum_{r i} \sum \alpha_i^{rs}(h) = 1,$$

$$c_i^{rs}(h) \geq \bar{c}_i^{rs}(h) \geq 0, \qquad\qquad \text{all } r,s,i,h. \qquad (6.3.2)$$

In this formulation, commodities are assumed to be spatially heterogeneous. The problem for each household is to maximize $U^s(h)$ under the following budget constraint,

$$\sum_{r i} \sum p_i^{rs} c_i^{rs}(h) \leq Y^s(h). \qquad (6.3.3)$$

The price, p_i^{rs}, is the price in region s of commodity i produced in region r. With a fixed link-related transport cost, this price is obtained as,[8]

$$p_i^{rs} = p_i^r + t_i^{rs}, \qquad\qquad \text{all } r,s,i. \qquad (6.3.4)$$

The demand functions obtained from (MSLES) are,

---

8   An alternative to this fixed transportation cost is an ad valorem formulation of the following type, $p_i^r(1 + \hat{t}_i^{rs})$. In this case, the transport cost is a function of the price of a commodity. With an increase in the price, the transport cost would also increase. This may be motivated by changes of transport mode and cargo type as well as increases in insurance and storage costs, connected with an increased value of a commodity.

$$c_i^{rs}(h) - \bar{c}_i^{rs}(h) + [\alpha_i^{rs}(h)/p_i^{rs}][Y^s(h) - \sum_{rj} p_j^{rs}\bar{c}_j^{rs}(h)],$$

$$\text{all } r,s,i,h. \qquad (6.3.5)$$

After addition of exogenously given final demand,

$$y_i^{rs} - \sum_h c_i^{rs}(h) + \bar{y}_i^{rs}, \qquad (6.3.6)$$

is it possible to write the system as,

$$\mathbf{y^{rs}} - \mathbf{y^{rs}(p^r, Y^r(h))} \qquad (6.3.7)$$

where,

$$\mathbf{y^{rs}} - \begin{bmatrix} y_1^{11} & \cdot & \cdot & y_1^{1NR} \\ \cdot & & & \cdot \\ y_{NS}^{11} & & & y_{NS}^{1NR} \\ \cdot & & & \cdot \\ y_{NS}^{NR1} & \cdot & \cdot & y_{NS}^{NRNR} \end{bmatrix}, \quad \mathbf{Y^r(h)} - \begin{bmatrix} Y^1(1) \\ \cdot \\ Y^1(NH) \\ \cdot \\ Y^{NR}(NH) \end{bmatrix}. \quad (6.3.8)$$

[(NRxNS) x NR] [(NRxNH) x 1]

Although the above demand system is simple and includes some restrictive assumptions, an advantage is that the derivation from utility maximization is not a necessary assumption. The system may be rationalized by other assumptions, for instance habitutal behaviour among the consumers. Such differences between

regions and cities in the composition of households with different habitats ought to play a role in the explanation of spatial growth.

Utilization of a (MSLES) is no burden from a computational point of view. The demand for data to estimate the system is more crucial. Limited availability of data may create problems with the degrees of freedom in an estimation. This has constrained most applications to national systems with a limited number of composite commodities. However, recently an estimation of a multiregion linear expenditure system was published by Giles and Hampton (1987). In their work, other demand systems are also estimated and compared with the linear expenditure system. Their multiregional system is not equal to the "ideal" system discussed above, but represents a simplification where spatial heterogeneity is not taken care of. The system may be seen as a simplification in the tradition of Chenery-Moses where the spatial flow pattern is obtained by a set of average trade coefficients.

An important part of the study by Giles and Hampton, is that their material made it possible to reject the hypothesis that income and price elasticities are uniform over regions. The result is expected, and is in accordance with our discussion in Chapter 2 on the importance of spatial differences in the demand pattern. How important such differences are for the result of a simulation, compared with an approximation with equal elasticities over space, can only be answered after simulation experiments. While the research in the field of refinement and estimation of spatial demand systems develops, "expert guesses" of parameters in data-demanding systems may be used. Experience from such approaches are found in the literature of applied modelling in international trade.

In the case of Sweden, a national linear expenditure system is used in the long-term planning, but so far no system has been estimated at a lower spatial level. Information exists at the regional level from expenditure surveys among different household categories [Official Statistics of Sweden (1980)], which may be used in calibration of spatially and household-differentiated characteristics.

Deeper knowledge regarding such characteristics is important if distributional effects of, and the welfare conflicts associated with, structural change shall be modelled appropriately.

6.3.3   EQUILIBRIUM CONDITIONS

Earlier, in Chapter 4, demand for heterogeneous labour in a spatial vintage pro-
duction system was introduced. When each labour category corresponds to a house-
hold group, unit labour demand in each vintage is given by the matrix,

$$
\mathbf{1}^{\mathbf{r}}(\mathbf{h}, r) =
\begin{bmatrix}
1_1^1(1,1) & . & 1_1^1(1,NV) & & & \\
. & & . & & & \\
1_1^1(NH,1) & . & 1_1^1(NH,NV) & & & \\
& & & . & & \\
& & . & & 1_{NS}^{NR}(1,1) & . & 1_{NS}^{NR}(1,NV) \\
& & & & . & & . \\
& & & & 1_{NS}^{NR}(NH,1) & . & 1_{NS}^{NR}(NH,NV) \\
& & & & & . &
\end{bmatrix}
$$

[(NRxNSxNH) x (NRxNSxNV)]   (6.3.9)

The labour supply is now given by regional and household specified constraints. The
disposable income levels are thus adjusted to obtain full utilization of each labour
category or the existing capacities.

In the model below, we cling to the assumption that wages and labour supply are
exogenously given for each labour category, as specified in (6.3.10).

$$
\mathbf{w}^{r}(h) \;-\;
\begin{bmatrix}
\dfrac{1}{w}\underset{1}{(1)} \\[4pt]
\cdot \\
\dfrac{1}{w}\underset{NS}{(NH)} \\[4pt]
\cdot \\
\cdot \\
\cdot \\
\overset{NR}{\underset{NS}{w}}(NH)
\end{bmatrix}
\qquad
\overline{\mathbf{L}}^{r}(h) \;-\;
\begin{bmatrix}
\dfrac{1}{\overline{L}}\underset{1}{(1)} \\[4pt]
\cdot \\
\dfrac{1}{\overline{L}}\underset{NS}{(NH)} \\[4pt]
\cdot \\
\cdot \\
\overset{NR}{\underset{NS}{\overline{L}}}(NH)
\end{bmatrix}
\qquad (6.3.10)
$$

[(NRxNSxNH) x 1]   [(NRxNSxNH) x 1]

The Walras-Armington spatial, multisector, multihousehold equilibrium with fixed wages, exogenous interaction costs and full utilization of the labour supply or the production capacity is given by the conditions in (E:6).

(E:6.1) No excess demand of any commodity in any region [price and income-dependent version]. A positive price implies equality between demand and supply of a commodity.

$$
\Phi_{p}r \;-\; S^{r}q^{r}(r) \;-\; A^{rs}(r)q^{r}(r) \;-\; y^{rs}(p^{r},\; Y^{r}(h))\Sigma_{y}^{r} \;\geq\; 0, \quad (6.3.11)
$$

$$
\Phi_{p}r^{T}p^{r} \;-\; 0, \quad p^{r} \;\geq\; 0. \qquad (6.3.12)
$$

The summation matrix, Σ_{y}^{r}, [NR x 1] summarizes final demand of a commodity over regions.

(E:6.2) No active vintages earn a quasi-rent below what is earned by the marginal vintage. A vintage generating losses is not active [Version with spatial labour skill differences].

$$\Phi_q r - 1^r(h,r)^T w^r(h) + \bar{\sigma}^r(r) + A^{rs}(\cdot r)^T p^r +$$

$$T_A^{rs}(r) - S^{rT} p^r \geq 0, \qquad (6.3.13)$$

$$\Phi_q r^T q^r(r) - 0, \quad q^r(r) \geq 0. \qquad (6.3.14)$$

(E:6.3) No excess utilization of a vintage. A vintage not used at full capacity will not earn a positive quasi-rent. The conditions are similar to (E:2.6).

(E:6.4) No excess demand for a labour category in any region. At the equilibrium income levels, either the regional labour supply or the capacity is fully utilized.

$$\Phi_{Y^r(h)} - \min \left(\bar{L}^r(h), \Sigma_i 1^r(h,r) \bar{q}^r(r) \right)$$

$$- \Sigma_i 1^r(h,r) q^r(r) \geq 0, \qquad (6.3.15)$$

$$\Phi_{Y^r(h)}^T Y^r(h) - 0, \quad Y^r(h) \geq 0. \qquad (6.3.16)$$

The matrix Σ_1, given below, adds demand for each labour category in each sector and region to totals for each category and region.

$$\Sigma_1 - \begin{bmatrix} 1 & \dots & 1 & & & & & \\ & 1 & \dots & & 1 & & & \\ & & & \ddots & & & & \\ & & & & \ddots & & & \\ & & & & & \ddots & & \\ & & & & & 1 & \dots & 1 \\ & & & & & & 1 & \dots & 1 \end{bmatrix}. \qquad (6.3.17)$$

[(NRxNH) x (NRxNSxNH)]

If compared with our vintage formulation of Isard's interregional model, (B:V) and (4.2.14), this formulation includes link costs and a multihousehold, price- and income-dependent final demand. The assumption of spatial heterogeneity is however retained.

In the model, wages are exogenously given, while relative prices, the level and the distribution of the disposable incomes over households are endogenous variables. These endogenous variables force the equilibrium solution towards maximal utilization of the given resources. In this model, changes in the exogenously given transport cost on some links may also result in a different spatial resource utilization. This was not the case in the previous model. The complementarity formulation in R^n, where $n = 2(NRxNSxNV) + (NRxNS) + (NRxNH)$, now consists of,

$$
z = \begin{bmatrix} q^r(r) \\ p^r \\ \bar{\sigma}^r(r) \\ Y^r(h) \end{bmatrix}, \qquad F(z) = \begin{bmatrix} \Phi_{q^r} \\ \Phi_{p^r} \\ \Phi_{\bar{\sigma}^r(r)} \\ \Phi_{Y^r(h)} \end{bmatrix}. \qquad (6.3.18)
$$

Walras law, in the sense that income equals factor costs, is in this case not fulfilled for each household since there is no explicit connection between the factor payments to a labour category and the income obtained by a household group. This means that there is an implicit system of transfers between households in the economy.

Alternative model formulations may be obtained by replacement of (E:6.4) by other conditions, while the rest of the model is kept intact. Three such cases are discussed below.

Firstly, the model contains the special case when (E:6.4) is replaced by an exogenously given income for each household, directly introduced in (E:6.9). In this case, the solution gives labour demand of each category, which when compared with the available labour supply in each region, reveals the pattern of unemployment. How-

ever, since there are no binding constraints on labour demand, labour in this case implicitly is assumed to migrate to regions where there is excess demand.

Secondly, a labour constraint may be introduced in a model with given income levels. In such a formulation wages may be fixed downwards but flexible upwards. Consider the labour supply constraint set,

$$\Phi_{\mu}{}^{r}(h) - \overline{L}^{r}(h) - \Sigma_{i} l^{r}(h, r) q^{r}(r) \geq 0, \tag{6.3.19}$$

$$\Phi_{\mu}{}^{r}(h)^{T} \mu^{r}(h) - 0, \ \mu^{r}(h) \geq 0. \tag{6.3.20}$$

The wage entering the price condition (6.3.13) is with this constraint calculated as,

$$\tilde{w}^{r}(h) - \mu^{r}(h) + w^{r}(h). \tag{6.3.21}$$

To the right is the wage increase $\mu^{r}(h)$, given by (6.3.20). The complementarity condition implies that this is zero as long as unemployment exists for a category. Together with the exogenously given wage level, $w^{r}(h)$, the wage entering the vintage price conditions, $\tilde{w}^{r}(h)$, is obtained. Hence, instead of the migration flows, labour cost will increase for scarce categories in regions with an excess demand for labour. Production will thus be redistributed towards regions and categories with unemployment.

A third possibility would be to keep both income and wages as exogenously given but to adjust the exogenous part of final demand, given by the vector, $\overline{y}^{r}[$ (NRxNS) x 1], with elements,

$$\overline{y}_{i}^{r} - \Sigma_{s} \overline{y}_{i}^{rs}. \tag{6.3.22}$$

until the production capacity is fully utilized. The condition would in this case become,

$$\Phi_{\overline{y}}r - \overline{q}^r - q^r \geq 0, \tag{6.3.23}$$

$$\Phi_{\overline{y}}r^T \overline{y}^r - 0, \quad \overline{y}^r \geq 0. \tag{6.3.24}$$

With the above condition, only the level of the exogenous final demand for each commodity is adjusted, not the spatial distribution of the flows of this demand. In this version the model may reflect a policy aiming at full capacity utilization by increases in public demand.

The model thus offers a variety of possibilities to simulate different policy alternatives and to study the outcome on the spatial structure.[9] It contains a set of simpler model alternatives, of which some have been discussed previously. Although a nonlinear (CP), the nonlinearities are still quite well behaved and solution of problems with normal dimensions are generally easy. In the rest of the chapter, further extensions of the model are discussed which often need more demanding algorithms.

6.4   A SPATIAL WALRAS-ARMINGTON EQUILIBRIUM MODEL WITH ENDOGENOUS INVESTMENTS AND CHOICE OF LOCATION

A supply-oriented multisector vintage model with endogenous investment was introduced in section 4.6. Within the (LP) representation in that section it was impossible to obtain annualized investment costs based on equilibrium prices. The model in this section combines the accelerator function in that model with the Walrasian formulation in section 6.3, in which it was assumed that investments and capacities were given exogenously.[10] The (CP) formulation allows both for endogenously determined investment costs and a price and income elastic demand side.

Compared with the model in section 6.3, this formulation emphasizes that change is generated by the incentive to make investments in new techniques and the possibility

---

9   In a model of an open economy the average wage level or an exchange rate may furthermore be used to obtain an exogenously given current account condition.

10   Single region vintage models in the Walrasian equilibrium tradition with endogenous investments have previously been formulated by Johansson and Persson (1983, 1987).

for an investor to gain a temporary monopoly profit after introduction of a new technique in a node. The model also ascertains consistency between the increase of capacity and demand for investment commodities. The growth of the economy is thus constrained by the existing production possibilities.[11]

New vintages are denoted by an asterisk and are characterized by the set,

$$\{q_i^r(r*), \ a_{ji}^{sr}(r*), \ b_{ji}^{sr}, \ l_i^r(h,r*)\}, \text{all } r,s,i,j,h. \qquad (6.4.1)$$

An extension, compared with the model in Chapter 4, is the skill-differentiated labour demand coefficient. The spatial flows generated by the accelerator matrix is dependent on the choice of technique in each region. A Walras-Armington spatial multisector multihousehold equilibrium with exogenous wages and transport costs but endogenous investments, (E:7), is characterized by the following set of conditions.

(E:7.1) No excess demand for any commodity in any region. A positive price implies equality between demand and supply of the commodity [Endogenous investment version].

$$\Phi_p r - [I - A^{rs}(r*) - B^{rs}]q^r(r*) +$$

$$[S^r - A^{rs}(r)]q^r(r) - y^{rs}(p^r, Y^r(h))\Sigma_y^r \geq 0 \qquad (6.4.2)$$

$$\Phi_p r^T p^r - 0, \ p^r \geq 0. \qquad (6.4.3)$$

(E:7.2) No active vintages, existing or new, earn a quasi-rent below what is earned by the marginal vintage. A vintage generating losses is not active.

---

11   This constraint may be weaker when trade with the rest of the world is allowed. However, if a condition on the balance of payments also is introduced, interesting macro balancing problems will occur.

$$\Phi_{q^r} - 1^r(h,r)^T w^r(h) +$$

$$\bar{\sigma}^r(r) + A^{rs}(r)^T p^r + T_A^{rs}(r) - S^T p^r \geq 0, \qquad (6.4.4)$$

$$\Phi_{q^r}{}^T q^r(r) - 0, \quad q^r(r) \geq 0. \qquad (6.4.5)$$

$$\Phi_{q^{r*}} - 1^r(h,r*)^T w^r(h) + \bar{\sigma}^r(r*) + A^{rs}(r*)^T p^r +$$

$$T_A^{rs}(r*) + \delta^r[B^{rs^T} p^r + T_B^{rs}(r*)] - p^r \geq 0, \qquad (6.4.6)$$

$$\Phi_{q^{r*}}{}^T q^r(r*) - 0, \quad q^r(r*) \geq 0. \qquad (6.4.7)$$

The condition contains, as model (A:VIII) also did, three vectors of unit transportation costs. Two vectors of costs for the transportation of intermediate commodities and one vector with costs for investment commodities.

(E:7.3) No excess utilization of a vintage in a region. A vintage not used at full capacity does not earn a positive quasi-rent. This condition equals condition (E:2.6).

(E:7.4) No excess utilization of labour. A positive household income implies full employment of the category in the region [Endogenous investment version].

$$\Phi_{Y^r(h)} - \bar{L}^r(h) - \Sigma_i 1^r(h,r) q^r(r) -$$

$$\Sigma_i * 1^r(h,r*) q^r(r*) \geq 0, \qquad (6.4.8)$$

$$\Phi_{Y^r(h)}{}^T Y^r(h) - 0, \quad Y^r(h) \geq 0. \qquad (6.4.9)$$

The condition implies that incomes will be adjusted and investments in new capacity will be accomplished until the labour supply is fully utilized at the given wages.

Observe that in this case, nonnegative quasi-rents are possible also in the new techniques since labour supply may constrain the solution at a point where the demand price exceeds the unit production cost. Secondly, the quasi-rents are here "net quasi-rents", since profits are reduced by the annual payment to the rentiers. Thirdly, in this equilibrium model, the value of the payments to rentiers is determined endogenously by the prices and is not, as in (A:VIII), set ex ante of the investment.

The equivalence between the equilibrium (E:7) and a complementarity problem is obtained by definition of,

$$
z = \begin{bmatrix} q^r(r) \\ q^r(r*) \\ p^r \\ \bar{\sigma}^r(r) \\ Y^r(h) \end{bmatrix}, \qquad F(z) = \begin{bmatrix} \Phi_{q^r} \\ \Phi_{q^r*} \\ \Phi_{p^r} \\ \Phi_{\bar{\sigma}^r(r)} \\ \Phi_{Y^r(h)} \end{bmatrix} \qquad (6.4.10)
$$

Clearly, both (E:6) and (A:VIII) are special cases of this model. Model (E:6) is obtained by exclusion of the new capacity condition, while (A:VIII) is obtained from an exclusion of the labour market income constraint and the price elasticity on final demand. The model may be given the following behavioral content. Incomes, and thus demanded quantities of final commodities, are adjusted until an equilibrium is reached in each labour market. New capacities are introduced in the regions where they are competitive and necessary for the achievement of equilibrium. In this process, demand is adjusted until the price structure has such a relation to the given wage level that an introduction of demand-induced new capacity becomes possible. The government is implicitly assumed to adjust net transfers to obtain the required income distribution.

Alternative formulations of the model without endogenous investments were discussed in section 6.3 and may also be introduced in this setting. The "full employment" formulation may thus be replaced by a formulation with an exogenously given

income development. Unemployment and migration would then be possible out-
comes of a simulation under a given trend of technological development.

Another extension which have been discussed previously would be to allow for some
amount of ex post substitution possibilities in each vintage, for instance by modelling
each vintage as a CES, instead of a Leontief technique. Introduction of such substi-
tution may be appropriate in some sectors and is possible in this (CP) format.

The capacity changes in the model are endogenously determined in commodity and
service producing sectors, but infrastructure capacities are exogenously given and
only represented through the constant transportation cost. Demand for investment
commodities associated with investments in the latter, also has to be determined
exogenously. An extension of the model, which is discussed in the sequel, is thus to
model the transport network in such detail that evaluations of mode- and link-
related investment programs are possible, and that congestion effects may be ana-
lyzed.

Despite the model being only solved for a single point in time, the solution of (E:7)
may be seen as the result of dynamic processes related to the product cycle theory.
On the micro level, this process of spatial-structural change is represented by
decisions to close down, initiate and relocate production. An understanding of the
phases of the product cycle thus facilitate a successful implementation of programs
for structural change. In (E:7), investments in new and exit of obsolete capacity is
modelled explicitly and spatial relocation is the outcome of those actions. In a single
commodity model by Johansson and Westin (1987), exogenously determined cost
changes cause discontinuities in the optimal location of an establishment producing
a maturing commodity. This process is reflected in model (E:7) by exit in nodes with
high wages, and entry of capacities in low wage nodes, when the price of the com-
modity is lowered due to process investments. According to the model, investments
in new capacity are made in nodes within which there are movable factors, advan-
tageous production costs or a high demand. A new technique has to compete with
existing techniques, without the investment costs, both in it's own sector and other
regions, and must also often attract labour from other sectors.

The model may also be used in simulations related to the Swedish "Rhen-Meidner" program [Rhen(1948), Lundberg (1950), Svennilson (1945)]. In short, this program is aimed at an equalization of wages over vintages in a sector and between sectors and regions. A Keynesian stimulation policy to obtain full employment would in such cases result in an inflation pressure, since regions with an advantageous vintage structure would be overstimulated in the attempts to secure full employment in the oldest vintage in regions with a unfavorable structure. In the program, it was instead suggested that vintages with obsolete techniques would be closed down and introduction of new techniques, with higher productivity, promoted. To avoid unemployment, programs for education, job training and migration for the employees in existing vintages should be initiated. Hence, economic growth would be associated with low unemployment and low inflation.

A program of the above type of must, in order to be successful, be governed by an understanding for the speed of the different processes involved, and thus the strength of the rigidities in the system. Model (E:7) gives opportunities to simulate and to study the spatial outcome of different assumptions on the speed and direction of technical development, migration and education efforts of such programs. Since external trade easily is introduced in the model, the sensitivity in the economy in relation to exogenous disturbances may also be studied.

6.5   INCOME-INDUCED EFFECTS, BUDGET CONSTRAINTS, AND SPATIAL FLOWS

6.5.1   INTRODUCTION

The theoretical Walras-Arrow-Debreu model, as well as some applied ICGE models, contains an explicit budget constraint connected with the factor costs for each household. This connection gives the income-induced effects, the "general equilibrium effects", which distinguish the models from models with incomes as exogenously given. Even if income flows are endogenously determined, the relation between household income and factor payments may still be implicit, as in models (E:6)-(E:7). The classical Walrasian formulation, with an explicit relation between

factor costs and incomes, was not followed in those cases. Instead a government was assumed to adjust incomes by transfers to obtain full employment.

Applied multisector equilibrium models have to be partial, since not all markets have similar adjustment times toward equilibrium. Hence, all markets cannot be specified as endogenously. The characteristic feature of an applied "general equilibrium" model should thus not, as sometimes is suggested, be that all markets are endogenously cleared, but that this endogenous connection between demand, income and factor costs exists.

A good reason to exclude connections between factor costs and incomes in a regional context is that rigorous information regarding such flows is not available at the moment. However, it is an important task to make assumptions about the pattern of those flows and to study the sensitivity of a model in relation to changes in such assumptions. It is not, especially in medium- and long-term analysis, possible to assume that induced effects, given by income elasticities and income changes, are a less important part of the system-related explanation of regional development and spatial structural change then the price effects. This motivates a discussion of how such interdependences may be formulated.

So far, the transport sector has been treated exogenously, in the sense that the flows on the network has not generated demand for resources and income flows in the nodes. Already our introduction of link-related transport costs has brought the model beyond the traditional applied general equilibrium approach. An endogenous treatment of the sector was initially discussed in Chapter 4. In section 6.5.3 a network model of the resource utilization and income effects generated by transportation is given.

The aim of this section is thus to compare different formulations of budget constraints in spatial models and to demonstrate how the income flows from a network-oriented transport sector may be included in this context. In applications, a mixture of the models in this section and in the previous section is possible. However, we stress that in each case it is important to clarify the explicit or implicit relation which exist between factor costs, income flows, transfers and final demand in a model.

Initially, in subsection 6.5.2, a single-region, single-household model is discussed along with alternative ways to close this model with respect to incomes. This illustrates some fundamental factor cost and income interdependences in multisector models. Thereafter, in subsection 6.5.3, it is shown how budget constraints may be introduced in a multihousehold network model with an endogenous multimode transport sector.

6. 5. 2   WALRAS LAW AND INCOME EFFECTS IN SINGLE-REGION, SINGLE-HOUSEHOLD MODELS

In this section, a Walrasian single-region, single-household model of a closed productive economy is introduced. Our aim is to analyze how Walras law implicitly constrains the solution in this simple formulation. The model follows the standard Walrasian model, but a completely wage-inelastic labour supply and a given set of vintage capacities are assumed. All markets are competitive and have a uniform medium-term adjustment time. However, wages are flexible and adjusted to obtain equilibrium in the factor markets through the labour demand. The equilibrium conditions (E:8) for this model are,

(E:8.1) Commodity market equilibrium

$$\Phi_p - [S - A(r)]q(r) - c(p,w,\bar{\sigma}(r)) \geq 0,$$

$$\Phi_p^T p - 0, \quad p \geq 0. \tag{6.5.1}$$

(E:8.2) Vintage activity conditions

$$\Phi_{q(r)} - A(r)^T p + 1(r)^T w + \bar{\sigma}(r) - S^T p \geq 0,$$

$$\Phi_{q(r)}^T q(r) - 0, \quad q(r) \geq 0. \tag{6.5.2}$$

(E:8.3) Vintage capacity constraints

$$\Phi_{\overline{\sigma}(\tau)} - \overline{q}(\tau) - q(\tau) \geq 0,$$

$$\Phi_{\overline{\sigma}(\tau)}^T \overline{\sigma}(\tau) - 0, \ \overline{\sigma}(\tau) \geq 0. \qquad (6.5.3)$$

(E:8.4) Labour market equilibrium

$$\Phi_{\mathbf{w}} - \overline{L} - 1(\tau)q(\tau) \geq 0,$$

$$\Phi_{\mathbf{w}}^T \mathbf{w} - 0, \ \mathbf{w} \geq 0. \qquad (6.5.4)$$

The interpretation of these conditions, and the related vectors and matrices, closely follows the interpretations in the previous models [e.g. (E:5)] and need not be repeated. The important deviations from previous models are that no final demand component except for private consumption is included in (6.5.1) and that wages, through condition (6.5.4) instead of income, are adjusted to reach full employment. The given capacity is assumed to allow for full utilization of the labour force. Private consumption is now also directly determined by wages, prices, and profits through a budget constraint and satisfies Walras' law for all prices.

With nonsatiation, Walras' law implies that the value of excess demand is zero and thus that final demand equals factor costs,

$$c(\mathbf{p}, \mathbf{w}, \overline{\sigma}(\tau))^T \mathbf{p} - \overline{L}^T \mathbf{w} + \overline{q}(\tau)^T \overline{\sigma}(\tau). \qquad (6.5.5)$$

This condition is also obtained by addition of the equality part of conditions (6.5.1) - (6.5.4) at the equilibrium point. Hence, the budget constraint is satisfied in the solution.[12]

---

12   Compare Mathiesen (1987).

If the private demand function in (E:8) is instead replaced by a demand system which fulfills the income constraint, but where income not is explicitly connected with factor costs, the equilibrium income - expenditure condition given by the demand system becomes,

$$c(p,Y)^T p - Y \qquad (6.5.6)$$

When the equality constraints in (E:8) are added, household incomes will still equal factor costs, since,

$$Y - c(p,Y)^T p - \overline{L}^T w + \overline{q}(r)^T \overline{\sigma}(r). \qquad (6.5.7)$$

Walras' law is thus fulfilled in the equilibrium solution of the single-region, single-household model, even if the connection between final demand and factor costs is only implicit. The complementarity formulation thus offers a strait forward way to show this well known relation.[13]

The result above is possible to extend to the model with final demand for commodities from an external sector and for capacity-increasing investments. In this case Walras' law implies,

$$c(p,Y)^T p + \overline{y}^T p + [Bq(r^*)]^T p -$$

$$\overline{L}^T w + \overline{q}(r)^T \overline{\sigma}(r) + \overline{q}(r^*)^T \overline{\sigma}(r^*) + [\delta Bq(r^*)]^T p. \qquad (6.5.8)$$

Gross income covers private consumption, external demand and investments. The income disposable for private consumption is thus obtained after reduction by net transfers to the external sector and investment outlays. The profits from new vintages are divided on "net quasi-rents" and a return to the rentiers.

---

13 As examples of Swedish single-region, single-household vintage models without an explicit formulation of the connection between factor costs and incomes, one may mention the models by Johansson and Persson (1983, 1987) and Persson (1983). Those, and the models by Bergman (1982, 1986), satisfy the condition implicitly by adjustment of the domestic cost level in relation to an external condition on the current accounts.

When, in such a model, wages are fixed at w but the disposable income is endogenously determined, a net transfer is assumed to be adjusted so that the disposable income for private consumption gives labour market equilibrium. Hence, an implicit net transfer, TR, exists, such that,

$$\tilde{Y} - \bar{L}^T\bar{w} + \bar{q}(r)^T\bar{\sigma}(r) + \bar{q}(r*)^T\bar{\sigma}(r*) + [\delta Bq(r*)]^T p -$$

$$Y + TR + s\tilde{Y} - c(p,Y)^T p + \bar{y}^T p + [Bq(r*)]^T p \qquad (6.5.9)$$

Above, $\tilde{Y}$ is the national income, and s the saving share. A fixed wage level constrains the solution but does not violate the consistency between factor costs, incomes and demand. If the model is extended to include two or more households, the above implicit conditions for fulfilment of each household's budget constraint are not available. Explicit budget constraints with spatial flows of factor earnings have then to be introduced. The overall consistency is nonetheless still kept.

6.5.3   INCOME EFFECTS IN A SPATIAL MULTIHOUSEHOLD NETWORK MODEL WITH ENDOGENOUS TRANSPORTATION COSTS

A complete introduction of income effects in a spatial equilibrium model implies that two types of repercussions have to be specified:

*   Relations between the localized transport sector and the link flows on the transport network.

*   Spatial distribution of factor costs between producers and households.

An extended representation of the transport network, which allows for an endogenous determination of the transportation costs and a link-related evaluation of infrastructure investment programs, furthermore implies that:

*   Different link types and their characteristics, as well as various link connections, have to be identified.

* Modal split and link assignment have to be modelled.

The interdependence between household incomes and the factor costs which are generated by transportation activities on network links, may be obtained in a number of ways. A first aspect of this concerns the delimitation of the transport sector. Isard and Ostroff (1958) introduced specialized transport firms operating on links with explicit regional ownerships of the firms. In their model only firms within the transport sector are allowed to transport commodities. In a reformulation of the Arrow-Debreu model into a model with an explicit spatial structure, Harris-Nadji (1987) noted that the Arrow-Debreu model allows each producer either to transport commodities themselves or to use a transport agency. Their formulation would thus be more general than the Isard-Ostroff formulation. This may also be the case. However, from both a practical modelling point of view and the fact that there seems to be a movement from self-produced transportation to market solutions, it is advantageous to assume that transportation is made by a specialized transport sector, in the Isard-Ostroff tradition.[14]

The network in the models has so far been simple and consists only of producing and demanding nodes. An enlarged network includes a detailed representation of alternative links and specialized nodes. Some nodes consists of the demand and supply regions previously introduced. Other nodes are connections between different link types.

In the model below, a detailed network is introduced into a model closely related to model (E:7) in section 6.4. However, transport costs are now determined endogenously on the network. Furthermore, in this formulation wages are endogenously determined and explicit budget constraints are introduced. Assume that network modes, $m = 1, \ldots, NM$, operate on the links, $n = 1, \ldots, NL$. Each network mode has access to a subset of the links. Furthermore, a chain of network modes may define a "supermode", $\hat{m} = 1, \ldots, NSM$.[15] For example, the supermode air consists of car and rail connections to the origin and destination airports. Given the

---

14   The movement towards market solutions of transportation is not self-evident. As long as the supply side of the transport market does not adapt to the new demand for logistic management and deliveries on time of small entities, this may be solved internally by the demand side.

15   Compare Echenique et al. (1988).

enlarged network, the following conditions characterize a spatial multihousehold network equilibrium with endogenous income formation (E:9).

(E:9.1) Transportation producer equilibrium. No transportation is carried out by any mode on a nonprofitable link.

$$\Phi_{q_{mn}} - A_{mn}^{rs}(q_{mn}, T_n, t_m^r)^T p^r + T_{mn} +$$

$$1_{mn}^r(h, q_{mn}, T_n, t_m^r)^T w^r(h) - t_{mn} \geq 0, \qquad (6.5.10)$$

$$\Phi_{q_{mn}}^T q_{mn} - 0, \quad q_{mn} \geq 0. \qquad (6.5.11)$$

Above, t_{mn} is a vector [(NMxNL) x 1] of transportation tariffs by mode and link, q_{mn} [(NMxNL) x 1], is the amount of transportation produced, and t_m^r [(NMxNR) x 1] is a vector of average tariffs related to each mode in each region. These conditions are contingent on changes of input coefficients due to technical change in the transport sector and in the infrastructure, as well as the prices of inputs. The transportation techniques used by each mode on each link are represented by the elements in the matrices for demand of intermediate commodities and labour,

$$A_{mn}^{rs} - \begin{bmatrix} a_{111}^{1.} & \cdot & a_{11NL}^{1.} & \cdots & a_{1NMNL}^{1.} \\ \cdot & & & & \cdot \\ a_{NS11}^{1.} & & & & \cdot \\ \cdot & & & & \cdot \\ a_{NS11}^{NR.} & \cdot & \cdot & \cdots & a_{NSNMNL}^{NR.} \end{bmatrix}, \qquad (6.5.12)$$

[(NRxNS) x (NMxNL)]

and,

$$
1^{r}_{mn}(h) = \begin{bmatrix} 1^{1}_{111}(1) & . & 1^{1}_{11NL}(1) & . & 1^{1}_{1NMNL}(1) \\ . & & & & . \\ 1^{1}_{111}(NH) & & & & . \\ . & & & & . \\ 1^{NR}_{NS11}(NH) . & & . & . & 1^{NR}_{NSNMNL}(NH) \end{bmatrix} . \quad (6.5.13)
$$

[(NRxNSxNH) x (NMxNL)]

The elements in the matrices are obtained in a hierarchical way described below,

$$
a^{r.}_{imn} = \sum_{s} a^{rs}_{imn} = \sum_{s} \xi^{s}_{mn}(q_{mn}, t^{r}_{m}) a^{rs}_{im}(q_{mn}, T_{n}) \quad (6.5.14)
$$

and,

$$
1^{s}_{imn}(h) = \hat{\xi}^{s}_{mn}(q_{mn}, t^{r}_{m}) 1^{s}_{im}(h, q_{mn}, T_{n}) \quad (6.5.15)
$$

The parameters ξ^{s}_{mn} and $\hat{\xi}^{s}_{mn}$, distribute the transportation supply over regions, according to the regional price of a mode and in relation to the flow pattern. The transportation by a mode on a link is thus connected with the transport sector in the producing nodes. The production in the transport sector and the incomes from transportation in a node by this become functions of the transport activities on the network. These parameters reflect the competition between actors in the transportation industry. The explicit formulation of this connection may be by fixed coefficients or by elastic functions e.g. of CES type. In simplified applications, a connection of all transportation may be made, either to the origin, the destination, or divided between the two. With the use of a price elasticity, a larger share of the transportation supply would be directed towards cheap producers.

The use of inputs may furthermore be dependent on the scale of the flows and the travel time on links, $T_n[NL \times 1]$. The elements in the latter vector are non-decreasing functions of the free flow time, $\overline{T}_n$, and the given capacity of the links, $\overline{U}_n$, but nonincreasing functions of their load, U_n,

$$T_n - T_n(\overline{T}_n, \overline{U}_n, U_n). \qquad (6.5.16)$$

The load is the sum over all network modes of the flows on a link, measured in equivalent units,

$$U_n - \sum_m \zeta_{mn} q_{mn}, \qquad \text{all n.} \qquad (6.5.17)$$

where ζ_{mn} are conversion factors. The cost of inputs for transportation by mode m on link n includes the transport cost for those inputs, given by,

$$T_{mn} - \sum_{rsi} a_{imn}^{rs} t_i^{rs}, \qquad \text{all m,n.} \qquad (6.5.18)$$

which gives the vector,

$$\mathbf{T_{mn}} - \begin{bmatrix} T_{11} \\ \cdot \\ \cdot \\ \cdot \\ T_{NMNL} \end{bmatrix}. \qquad (6.5.19)$$

$[(NMxNL) \times 1]$

The condition above describes the supply-side of the transport sector. The network-related demand side is introduced in the next condition.

(E:9.2) No excess demand for transportation. The mode- and link-related transport cost gives equilibrium between demand and supply of a mode on a link,

$$\Phi_{t_{mn}} - q_{mn} - d_{mn}(x^{rs}, u_{mn}) \geq 0, \tag{6.5.20}$$

$$\Phi_{t_{mn}}{}^T t_{mn} - 0, \quad t_{mn} \geq 0. \tag{6.5.21}$$

Demand for network modes on links, i.e the vector, $d_{mn}(x^{rs}, u_{mn})$, [(NMxNL) x 1] is determined through modal split and link assignment from the commodity flow distribution, x^{rs}, and the generalized transport cost on modes and links for each commodity, the vector, u^{mn} [(NMxNL) x 1]. The generalized cost is a [e.g. linear] function of the transport cost, and link- and mode-related disutilities including, for example travel time and mode-specific discomfort parameters.

Modal split is modelled by a hierarchic, multinominal logit function.[16] Choice of supermode is obtained from the probabilities which assign a flow of commodity i from origin r to destination s, to each supermode $\hat{m}$,

$$\text{prob}[\hat{m}|rsi] - \omega_{\hat{m}}\exp[-\lambda_{\hat{m}}u_{\hat{m}}|rsi]/\sum_{\hat{m}} \omega_{\hat{m}}\exp[-\lambda_{\hat{m}}u_{\hat{m}}|rsi]$$

$$, \text{ all } \hat{m},r,s,i \tag{6.5.22}$$

Each probability is a function of the generalized costs from using a supermode, for transportation of commodity i between r and s, $u_{\hat{m}}|rsi$, a spread parameter $\lambda_{\hat{m}}$, and the parameters $\omega_{\hat{m}}$. The latter are a priori weights, adding up to one, which may or may not be used to introduce exogenous information to the flows. As the spread parameter goes to infinity, the model is reduced to a model of minimum cost choice of the same type as some freight network models. The generalized cost of superode $\hat{m}$ is obtained as the log sum of the cost of all possible routes which contain the supermode, e.g. the composite cost,

---

16 See Domencich and McFadden (1975), Williams (1977), McFadden (1978), Daly and Zachary (1978), Lerman (1982), and Fisk and Boyce (1984) for the derivation of the logit model and calcu-lation of composite cost measures.

$$u_{\hat{m}}|rsi = -[1/\lambda_R]\ln[\sum_R \omega_R \exp[-\lambda_R u_R|\hat{m}|rsi]]. \qquad (6.5.23)$$

Above, λ_R is the spread parameter over routes, $u_R|\hat{m}|rsi$ is the generalized cost of utilizating route R, given that supermode $\hat{m}$ has been chosen, and ω_R are optional a priori weights. The generalized cost of each individual route for a supermode is, at the next level, the sum of the generalized costs of modes on the links which build the route. The flow on each link is in a similar way obtained by logit functions which assign the route flows to network modes and links. A formulation with minimum cost assignment, is a possible special case. This gives a model with clear properties, but often with too little dispersion of the flows when evaluations are made against real flows. Logit formulations give this heterogeneity of flows but may, on the other hand, not be independent of the formulation of the assignment algorithm.[17] Since a minimum cost path solution is also obtainable in the logit assignment algorithm, the properties of the algorithm in relation to the minimum cost path may be evaluated by parameter analysis.

The transport cost between node r and s for commodity i, t_i^{rs}, is, independent of the chosen assignment algorithm, built up from modes and links to supermodes. Hence, commodity-related costs may be calculated by use of the tariff-related part of the generalized cost,

$$t_i^{rs} = -[1/\lambda_{\hat{m}}]\ln[\sum_m \omega_{\hat{m}} \exp[-\lambda_{\hat{m}} t_{\hat{m}}|rsi]], \quad \text{all } r,s,i, \qquad (6.5.24)$$

which gives the difference between the supply and demand prices for commodities in the model. This also indicates how the previously introduced models with a simple network, are special cases of the model with a detailed network.

The remaining part of the model follows the model in section 6.4, with the exception that an endogenous determination of wages is now introduced in order to illustrate an alternative labour market equilibrium condition.

---

17 Williams (1977) and Barra and Pérez (1986) give examples of this.

(E:9.3) No excess demand for any commodity in any node. A positive price implies equality between demand and supply of a commodity. [Endogenous investment version given by condition (E:7.1)]

(E:9.4) No active vintages, existing or new, earn a quasi-rent below what is earned by the marginal vintage. A vintage generating losses is not active. [Given by condition (E:7.2)]

(E:9.5) No excess utilization of a vintage in any mode. A vintage which is not used at full capacity does not earn positive quasi-rents. [Given by condition (E:2.6)]

(E:9.6) No excess utilization of any labour category. A positive wage implies full employment of the category.

$$\Phi_{\mathbf{w}}^{\mathbf{r}}(\mathbf{h}) - \overline{\mathbf{L}}^{\mathbf{r}}(\mathbf{h}) - \mathbf{1}^{\mathbf{r}}(\mathbf{h}, r)\mathbf{q}^{\mathbf{r}}(r) - \mathbf{1}^{\mathbf{r}}(\mathbf{h}, r*)\mathbf{q}^{\mathbf{r}}(r*) \geq \mathbf{0}, \quad (6.5.25)$$

$$\Phi_{\mathbf{w}}^{\mathbf{r}}(\mathbf{h})^{\mathrm{T}}\mathbf{w}^{\mathbf{r}}(\mathbf{h}) - \mathbf{0}, \quad \mathbf{w}^{\mathbf{r}}(\mathbf{h}) \geq \mathbf{0}. \qquad (6.5.26)$$

The budget constraint which enters the expenditure system includes spatial flows of factor costs from all vintages $r' \in r$, $r*$ and has the following income side for household h in node r,

$$\widetilde{Y}^{\mathbf{r}}(\mathbf{h}) - \sum_{\mathbf{s}i\mathbf{r}'} \gamma_i^{\mathbf{s}\mathbf{r}}(\mathbf{h}, r') [w_i^{\mathbf{s}}(\mathbf{h}) L_i^{\mathbf{s}}(\mathbf{h}, r') +$$

$$\overline{\sigma}_i^{\mathbf{s}}(r') \overline{q}_i^{\mathbf{s}}(r') + \sum_{i j r} \delta_i^{\mathbf{s}} B_{ji}^{\mathbf{rs}} [p_j^{\mathbf{r}} + t_j^{\mathbf{rs}}]]. \qquad (6.5.27)$$

The first term reflects labour incomes from each vintage, in each producing node to node r and the following two terms are flows of profit and amortizations between nodes. Those flows have a structure reflected by the nonnegative parameters $\gamma_i^{\mathbf{s}\mathbf{r}}(\mathbf{h}, r')$. The factor costs include incomes from transportation modes. A poss-

ible generalization would be to allow each factor income category to have an individual set of share parameters. The disposable incomes entering the expenditure system are calculated with the saving parameters, $s^r(h)$, and the net transfer from each household as,

$$Y^r(h) = [1-s^r(h)]\tilde{Y}^r(h) - TR^r(h).$$ (6.5.28)

The following income and expenditure constraint is obtained by summation of the equality conditions in (E:9),

$$[y^{rs}(\cdot)\Sigma_y^r]^T p^r + T_y^{rs} + T_B^{rsT} q^r(r*) + [B^{rs} q^r(r*)]^T p^r =$$

$$[\delta^r B^{rs} q^r(r*)]^T p^r + \delta^r T_B^{rsT} q^r(r*) + \bar{\sigma}^r(r*)^T q^r(r*) +$$

$$\bar{L}^r(h)^T w^r(h) + \bar{\sigma}^r(r)^T \bar{q}^r(r)$$ (6.5.30)

where T_y^{rs} is the transportation cost associated with the flows of commodities for private consumption and exogenous final demand. On the left side of the equality sign are expenditures for final demand and their associated transportation costs. The income side at the right side is composed by return to rentiers, net profits from new capacities and the value added in existing, active vintages. In this case both transportation and investment costs are endogenously determined by prices on commodities and factors.

The equivalence between the above equilibrium model and a complementarity problem is obtained by definition of,

$$
z = \begin{bmatrix} q^r(r) \\ q^r(r*) \\ p^r \\ \bar{\sigma}^r(r) \\ w^r(h) \\ q_{mn} \\ t_{mn} \end{bmatrix}, \qquad F(z) = \begin{bmatrix} \Phi_{q^r} \\ \Phi_{q^r*} \\ \Phi_{p^r} \\ \Phi_{\bar{\sigma}^r} \\ \Phi_{w^r(h)} \\ \Phi_{q_{mn}} \\ \Phi_{t_{mn}} \end{bmatrix}. \tag{6.5.31}
$$

Existence of such an equilibrium is guaranteed under the ordinary monotonicity conditions on the supply and demand functions, as discussed in Chapter 2. In the model, the aggregate transport cost may be calculated in three different ways, since,

$$
x^{rsT}t^{rs} = T_{mn}{}^{T}q_{mn} + T_A^{rsT}q^r(r) + T_A^{rsT}q^r(r*) +
$$

$$
T_B^{rsT}q^r(r*) + T_y^{rs} = d_{mn}{}^{T}t_{mn}. \tag{6.5.32}
$$

The model gives an integrated representation of the system effects due to the interactions between a detailed transport system, and the growth of production capacities and incomes over space. Investments in a link in order to reduce the travel time may result in a direct substitution effect in the link flows and indirect changes as a result of the spatial exit process and the choice of location and techniques in the producing sectors.

One may in multiregion models with a single household in each region, as an alternative to the explicit income condition (6.5.27), include a current account condition on each region in order to obtain an implicit budget constraint on each household. In a spatial multihousehold model, there is obviously no possibility to obtain an implicit budget constraint on each household via a condition on the regional trade alone.

Passenger trips are generated by demand for business and household services as well as for recreation and culture. Those trips are obtained from the input-output structure and the household expenditure system. Passenger trips create demand both in transfer nodes, which are included in a route, and in the destination node. Such tourism effects have not been taken care of in the formulation above, but would be taken care of by a redistribution of demand in relation to the passenger flows between nodes.

Generally, each infrastructure investment object is, given the medium-term horizon, assumed only to have marginal effects on the interaction costs on the network. However, the model may be used to analyze the effects of investments in critical links, improvements of routes, or a technical change on the supply-side of the transportation market. The model may also be used to solve different network design problems. Such a problem implies that the network should be designed so that the resource utilization in the transport sector and for the creation and maintenance of the network gives optimal economic growth, a distribution of spatial flows, and a location of activities in accordance with political demands on the development.

6.6   ASSESSMENT OF THE ICGE MODELS

In this chapter, a set of ICGE models with endogenous income effects have been introduced. Models have been formulated both with and without an explicit connection between factor costs and incomes. The available empirical studies of the spatial pattern of financial flows are not complete and the results from spatial models with explicit connections have to be used carefully. The net effects on the flows of changes in savings, transfers and factor costs are not easily captured. Objections may thus be raised against formulation of explicit conditions on a detailed spatial level. However, and as has been argued earlier, the existence of income effects in economy-wide analyses makes it important to study the sensitivity of a result with respect to changes in the pattern of the income flows, and in relation to models with "implicit flows" and exogenous assumptions.

A similar argument may be used to the advantage of our endogenous investment formulation. Once a (CP) algorithm is developed for an application there are possi-

bilities to simulate the sensitivity in the solution of different investment functions. Such simulations may give important new information to modelers and decision makers.

Our introduction of a detailed network representation into the ICGE framework, both extends the range of analyses possible and gives a connection to other models of spatial flows. Logit formulations were used in the network representation. The advantage of the logit formulation, compared with the special case of minimum cost path-oriented modal split and link assignment, is the dispersion of flows which represent heterogeneities and uncertainty. The logit formulation allows for simple calibration of a model of such spatial dispersion. A considerably more data-demanding alternative to the logit model, in an "Armington" direction, would be to specify demand systems with functions for each route, consisting of choices of links and modes, and substitution possibilities between the accessible alternatives for each vintage and household. Less demanding would be to represent the network with nonlinear cost functions for aggregates of links. However, this would not allow for a direct introduction of link-related changes in the model.

The integrated network model represents the competitive forces in the transportation industry by endogenous substitution possibilities between carriers from different regions. We have suggested an approach to represent this competition. The transport market consists of different segments, from nationwide schedulized transport networks around nodes with terminals to specialized transport companies. Both price and quality, for instance "just in time" deliveries, are means of competition. New transport concepts spring out of the synergy between different modes of transportation. Changes in the amount and type of deliveries will affect the stability of the flows and the location of terminals will thus have impacts on the distribution of incomes and the accessibility over space. Hence there is scope for further theoretical and empirical research on the relation between transport sector activities and the spatial distribution of factor incomes.

The ICGE models have their advantage in economy-wide studies of system effects caused by the interaction between supply, demand and transportation of commodities. In each region, the commodities may be in different stages of the product cycle with different techniques and labour demand. Investments in new products and processes have impacts on this interaction. Income and price effects redistribute

demand between commodities and generate structural change. Choice of technique and location for the supplier also affects incomes, prices and the spatial pattern of interaction. The multihousehold approach allows for analysis of how the available labour supply restricts, and how distributional impacts generate, such changes. Investments in the network give those processes spatial directions. This gives arguments for increased research in the field of integrated transportation - economic structure models.

7 COMPARISON AND ASSESSMENT OF MODELS IN THE VINTAGE NETWORK EQUILIBRIUM CLASS

7.1 INTRODUCTION

In the previous three chapters, the vintage function was introduced into the two main traditions of applied spatial equilibrium models. An extension of ICGE models into a network formulation was also suggested. The latter type of model allows for an integrated analysis of spatial multisector growth with a network representation of the transport system. In this chapter it is further emphasized that the various versions of vintage models presented. are special cases of a class of multisector network models.

Within this class, the choice of model for a specific application should be made in relation to modelling objectives and assumed or estimated elasticities. The latter is among other things governed by the degree of aggregation and the time period in the analysis.

Henceforth, the chapter consists of two sections. In section 7.2, a classification of spatial equilibrium models is made. It is shown how an appropriate choice of nesting and elasticities allow for representation of both spatially homogeneous and hetero-geneous commodities within a single framework. In section 7.3, the models in the class and their network structures are assessed.

7.2 A CLASSIFICATION OF SPATIAL EQUILIBRIUM MODELS

As illustrated in Figure 7.1 below, one may distinguish between four categories within the class of spatial equilibrium models. The classification is made according

to the use of inverse or ordinary functions and whether commodities are assumed to be spatially homogeneous or heterogeneous. Examples of modelling contributions are also given in the Figure.

Figure 7.1   Four categories of spatial equilibrium models.

	Marshallian Formulation	Walrasian Formulation
Spatially Homogeneous Commodities	Samuelson (1952) Takayama and Judge (1964)	Takayama and Woodland (1970) Friesz et al. (1983)
Spatially Heterogeneous		Shoven and Whalley (1974) Liew (1984) Buckley (1987)

Previously, in Chapters 5 and 6, the vintage function was introduced into three of the four approaches. The fourth and remaining, a "Marshall - Armington" model, with inverse functions and heterogeneous commodities is, as was discussed in Chapter 5, a special case of the Marshallian homogeneous commodity model.

An advantage of the Walrasian, compared with the Marshallian, formulations is the utilization of ordinary demand and supply functions. The use of ordinary functions implies that the question of existence of the inverse does not have to be raised. A second advantage is the ease by which price constraints may be introduced. This is especially an advantage when programming algorithms are utilized.

A fundamental difference between the two Walrasian formulations is the spatial homogeneity of commodities in traditional SPE models and their spatial hetero-geneity in ICGE models. The difference may, as has been discussed previously, be seen as a result of different levels of aggregation in the models, but may also be

explained by an emphasis on differentiated commodities in the latter. The difference also has consequences for the formulation of demand functions. In an ICGE model, the flows are determined by a priori given origin-destination demand functions, while in an SPE model, the flows are results of a derived demand for transportation in interaction with the commodity demand and supply.

A second difference is the treatment of income effects, which plays a central role in ICGE models but is exogenously determined in SPE models. Since this nowadays only has to do with the character of the problem and is not connected with major computational differences, the models may in this respect be treated as similar. The close relation between Walrasian ICGE and SPE models is further revealed by inspection of their spatial flow conditions. In equilibrium, the supply price in a single commodity ICGE model with zero intraregional interaction costs, should fulfil the following spatial price conditions for all r and s,

$$q_i^r(p_i^r) - \sum_s d_i^{rs}(p_i^{rs}) \geq 0, \qquad (7.2.1)$$

$$[q_i^r(p_i^r) - \sum_s d_i^{rs}(p_i^{rs})]p_i^r = 0, \; p_i^r \geq 0, \qquad (7.2.2)$$

$$p_i^r + t_i^{rs} - p_i^{rs} = 0. \qquad (7.2.3)$$

Where, d_i^{rs} is the demand in region s for the commodity from region r. Hence, in the "Armington" tradition, region r is the sole supplier of this specific version of commodity i. A Walras-Armington SPE model, comparable with the ICGE model above, comprehends the following equilibrium conditions for all r and s,

$$q_i^r(p_i^r) - d_i^r(p_i^r) - \sum_s x_i^{rs} + \sum_s x_i^{sr} \geq 0, \qquad (7.2.4)$$

$$[q_i^r(p_i^r) - d_i^r(p_i^r) - \sum_s x_i^{rs} + \sum_s x_i^{sr}]p_i^r = 0, \; p_i^r \geq 0, \qquad (7.2.5)$$

$$p_i^r + t_i^{rs} - p_i^s \geq 0, \qquad (7.2.6)$$

$$[p_i^r + t_i^{rs} - p_i^s]x_i^{rs} = 0, \ x_i^{rs} \geq 0, \tag{7.2.7}$$

However, since there is only a single source for each commodity in a Walras - Armington SPE model,

$$\sum_s x_i^{sr} = d_i^r, \tag{7.2.8}$$

while,

$$\sum_s x_i^{rs} = \sum_s d_i^{rs}, \tag{7.2.9}$$

so that,

$$- \sum_s x_i^{rs} + \sum_s x_i^{rs} = \sum_s d_i^{rs} - d_i^r = \sum_{s \neq r} d_i^{rs} \tag{7.2.10}$$

Introduction of (7.2.10) into (7.2.4) - (7.2.5) confirms the close relation between the two approaches. The remaining difference is that the spatial price condition is fulfilled by definition in the ICGE case, but fulfilled endogenously by the complementarity condition related to the derived demand in the SPE model.

In Figure 7.2 the explicit or implicit elasticities of substitution between commodities and regions in ICGE and SPE models are illustrated.
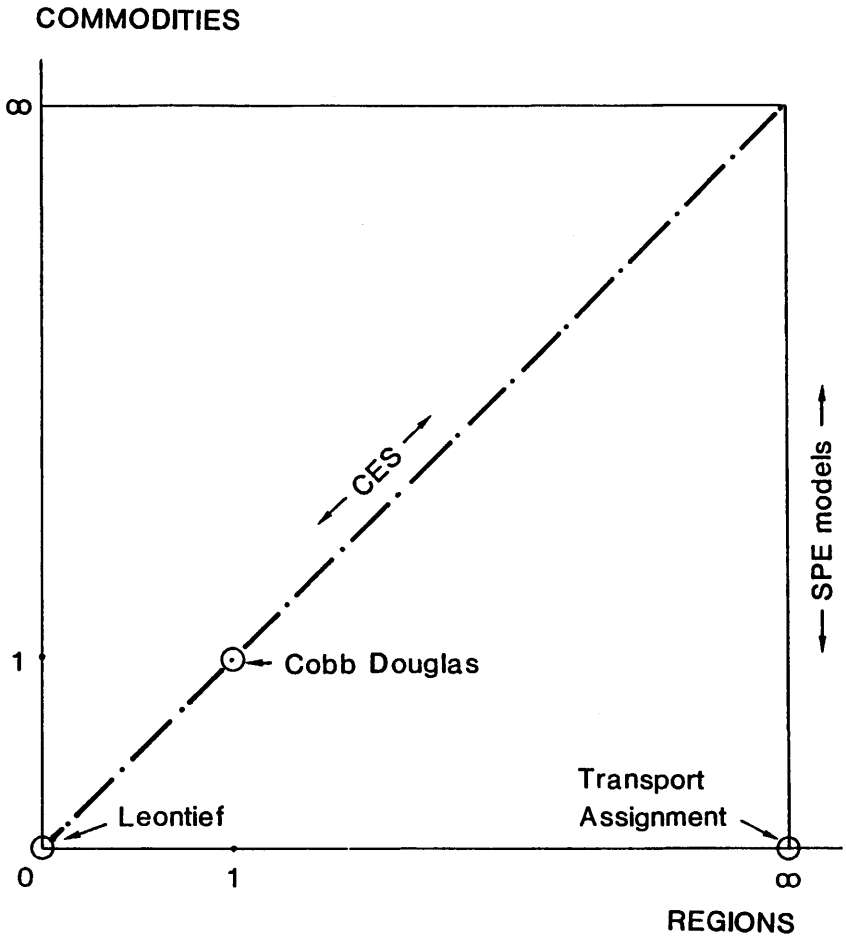
COMMODITIES

Figure 7.2   Elasticities between commodities from different regions in spatial equilibrium models.

The traditional SPE model with spatial homogeneous commodities covers one side of the box.[1] An ICGE model with a single-level CES function covers the diagonal, while nested CES functions, by a suitable choice of parameters and nesting, may be used to obtain various combinations of elasticities between commodities and regions.

Such nested, or hierarchical, functions have only recently been introduced in studies of interregional flows. In a nested function, each level consists of goods or aggregates (composite commodities) with similar characteristics and substitution elasticities. This allows for designing models which overcome constraints set by the uniform elasticity of substitution between all pairs of commodities and regions in the single level CES function. The function may be introduced as a production function or a utility function. When used at the producer side, the conditional factor demand functions are derived from the cost function by making use of Shephard's lemma. At the household side, utility maximization under a budget constraint gives an indirect utility function from which, by Roy's identity, price and income dependent demand functions are derived [Varian (1984)].

One may in a model with nested functions also exploit the fact that the CES function contains other common functions as special cases. Consider the spatial multisector CES function,

$$q_i^r = [\underset{sj}{\sum\sum} \delta_{ji}^{sr} q_{ji}^{sr}{}^{-\theta_i^r} + \delta_L^r L_i^r{}^{-\theta_i^r}]^{-1/\theta_i^r},$$
$$(7.2.11)$$
$$\underset{sj}{\sum\sum} \delta_{ji}^{sr} + \delta_L^r = 1,$$

with the elasticity of substitution,

$$\sigma_i^r = 1/(1 + \theta_i^r). \qquad (7.2.12)$$

When $\sigma_i^r = 0$, a Leontief production function with interregional flows is obtained,

---

1   The attempts to combine entropy, logit and gravity models with SPE models may be seen as a way
     to force the SPE model into the middle of the box.

i.e. the interregional model by Isard (1951). If $\sigma_i^r = 1$, the Cobb-Douglas case is generated. This case was utilized in the interregional model by Liew and Liew (1984). The factor shares in an input-output table of Isard type were then used to obtain point estimates of the output elasticities. At the other extreme, if $\sigma_i^r = \infty$, one obtains a linear production function with perfect substitution between all inputs from all regions.

A spatial model with different technical and spatial substitution elasticities on the supply side, may thus be obtained from a nesting of CES functions. At the first level, the function may be of Leontief type as in a traditional input-output model. Hence,

$$q_i^r = \min\{q_{1i}^r/a_{1i}^r, \ldots, q_{NSi}^r/a_{NSi}^r, L_i^r/l_i^r\}, \qquad (7.2.13)$$

where the composite commodities and their input coefficients are defined as,

$$q_{ji}^r = \sum_s q_{ji}^{sr}, \quad a_{ji}^r = \sum_s a_{ji}^{sr}. \qquad (7.2.14)$$

This Leontief function gives demand for inputs in fixed proportions, independent of the location of the source, as in a model with spatial homogeneous commodities. The demand for each composite commodity is thereafter, at the second level, divided between different spatial sources by a new CES function,

$$q_{ji}^r = [\sum_s \delta_{ji}^{sr} q_{ji}^{sr} {}^{-\theta_{ji}^r}]^{-1/\theta_{ji}^r}, \qquad (7.2.15)$$

which may be rewritten as,

$$q_{ji}^r = [\sum_s \delta_{ji}^{sr} q_{ji}^{sr} {}^{(\sigma_{ji}^r - 1)/\sigma_{ji}^r}]^{\sigma_{ji}^r/(\sigma_{ji}^r - 1)}. \qquad (7.2.16)$$

Conditional factor demand functions are then, since one may normalize the parameters so that,

$$\sum_s [\delta_{ji}^{sr}]^{\sigma_{ji}^r} - 1, \tag{7.2.17}$$

obtained from the cost function as [Hickman and Lau (1973)],

$$q_{ji}^{sr} - q_{ji}^r [\delta_{ji}^{sr}]^{\sigma_{ji}^r} [p_{ji}^{sr}/p_{ji}^r]^{-\sigma_{ji}^r}, \qquad \text{all } s. \tag{7.2.18}$$

The price of the composite commodity is in this case,

$$p_{ji}^r - [\sum_s \delta_{ji}^{sr\sigma_{ji}^r} p_{ji}^{sr(1-\sigma_{ji}^r)}]^{1/(1-\sigma_{ji}^r)}. \tag{7.2.19}$$

The following adding-up property may then be derived,

$$q_{ji}^r p_{ji}^r - \sum_s p_{ji}^{sr} q_{ji}^{sr}. \tag{7.2.20}$$

With this nesting, the elasticity of substitution between different regional sources becomes equal for each composite commodity. Further disaggregation and nesting is of course possible. The nested approach is illustrated by the tree, in Figure 7.3 below. At the top of the tree is the choice of commodites determined by a Leontief technique. For each such composite commodity, the supply region is at the second level determined by a CES function. Further levels are possible to introduce and different elasticities of substitution are of course possible at each ramification.

One may in a similar way, from a nested utility function, obtain demand for private consumption with a variety of substitution elasticities [Whalley (1985)].

Hence, with the nested approach both models with spatially heterogeneous and homogeneous commodities may be formulated. If at the second level, i.e. (7.2.15), a linear model is chosen instead (with $\theta_{ji}^r - -1$), the function represents a world
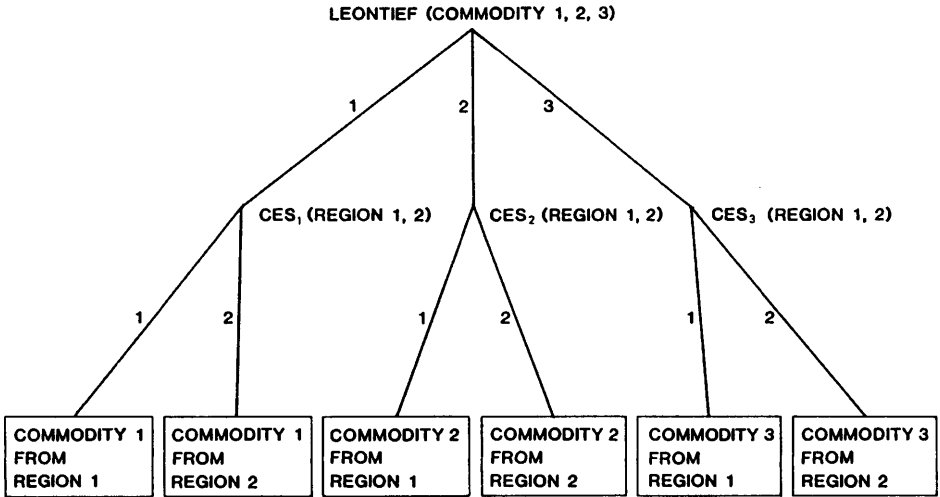
Figure 7.3   The tree structure of a nested CES function.

with a spatially homogeneous commodity. Such a nested ICGE model with a similar set of elasticities as the homogeneous commodity SPE model is thus written in two levels as,

$$d^s = CES^s[d_1^{\cdot s}, \ldots, d_i^{\cdot s}, \ldots, d_{NS}^{\cdot s}], \quad \text{all } s, \quad (7.2.21)$$

$$d_i^{\cdot s} = LINEAR_i^s[d_i^{1s}, \ldots, d_i^{rs}, \ldots, d_i^{NRs}], \quad \text{all } s, i. \quad (7.2.22)$$

This is obviously an extreme version of the model where the elasticity of substitution is infinite. As soon as the elasticity is finite, the model generates solutions with spatial heterogeneity. In an applied model, both types of commodities may be repre-

sented while the choice of parameters should be dependent on estimated elasticities. Introduced as a production function for each vintage, it allows for a representation of both the heterogeneous matrix (4.2.1) and the homogeneous case (4.2.24). In a similar way substitution possibilities may be introduced for labour categories within and between regions. If such cross-regional labour demand is interpreted as migration, a link to the private consumption in the region also has to be included.

7.3   ASSESSMENT OF THE MULTISECTOR NETWORK MODELS

This merging of ICGE and SPE models into a class of spatial equilibrium models implies that the experiences of detailed network formulations in the SPE tradition and the multicommodity formulations with intermediate demand, income feedbacks and endogenous investments in ICGE models may be integrated and taken advantage of in multisector network models.

The network in the traditional SPE and in the ICGE models are uncomplicated with a single-mode and a simple-path structure. Within the network extended SPE framework, in freight network models [Harker (1984)], networks with a considerable complexity and detail have been studied. So far, the networks in the ICGE models have not been extended beyond the simple single-path structure. Our network formulation thus implies such an extension of the ICGE model.

A freight network model assigns flows to links by a "deterministic" user-optimal equilibrium approach. Other approaches, which allow for stochastic assignment are gravity, logit, entropy and other more heuristic algorithms, which introduce stochastic elements and dispersion into freight network formulations. We have, from an applied point of view, argued above in favour of logit assignment algorithms. However, still a lot of work still remains in the field of algorithms for disperced assignment.

As regards the mathematical formulation, all our models are complementarity problems. In Chapter 4 it was also shown how linear programming formulations of the interregional input-output model and the classical transport assignment model are

special cases of this class of multisector network models. A general algorithm may thus be used to solve different variants of models in the model class. However, uniqueness of equilibrium and convergence of the algorithms are still only obtained under the restricted conditions on the chosen functional forms mentioned in Chapter 2.

The choice of functional form should be governed by elasticities, dependent on the time period for the modelled process. The methods for estimation of such dynamic elasticities are thus an important field which should be developed further. Meanwhile, evaluation of comparative simulations with different functional forms may give more precise experiences on the appropriate formulation for a given time period and the speed of adjustment for various processes.

8 ECONOMIC DYNAMICS AND MODELS OF ECONOMIC EQUILIBRIUM

8.1 INTRODUCTION

In Chapter 2 some important aspects of spatial structural change dynamics were emphasized. Structural change was then considered as a continuous process, in which differences in the adjustment speed of processes were caused by economic, physical and other rigidities. We also found that there is a need for applied models which may capture the dynamics of structural change.

However, in applied modelling there is a trade-off between the possibility to model details and fine structures, and to obtain an extensive dynamic specification. For this reason dynamic models in continuous time of the interaction between demand and supply in a spatial multisector world are rare, or heavily oriented toward studies of nongeneric balanced growth solutions. Even less frequently, dynamic models are applied with explicit price and quantity adjustment and with a treatment of the variation in adjustment times of different economic processes.

When details are introduced, the models are often formulated as quasi-dynamic sequences of single-period equilibrium models, where "ad hoc" invariants and allocation rules are given a considerable role. The endogenous development is then also often supported by an exogenous time series of "driving" forces or stabilizing structures. The properties of such quasi-dynamic models easily become either too linear or impossible to gain behavioral insights from. This gives a motivation for the type of single-period equilibrium models we have presented previously as approximations of the behaviour of a complex and interdependent economic system in applied work.

The natural question is under what conditions such an approximation is valid? In the following it is shown that an explicit recognition of time both constrains and clarifies the appropriate use of the equilibrium approach. We relate equilibrium models to the "slaving principle" and the principle of "adiabatic approximation". We argue that

those principles have to govern the further development of economic equilibrium modelling.

In section 8.2, a formalization of the relation between equilibrium models and models with continuous dynamics is given. Subsequently, in section 8.3, some approaches to multiperiod modelling are evaluated in order to point out weaknesses and promising directions for further research. The chapter is then closed in section 8.4 with comments on the further development of dynamic multisector models.

8.2   EQUILIBRIUM APPROXIMATIONS OF CONTINUOUS DYNAMICS

This section provides a more formalized discussion regarding the use of a single-period equilibrium model as an adiabatic approximation of a complex economic system in continuous time. Such an economic system may, with the processes $i = 1, \ldots, N$, be described by the following set of differential equations:

$$\dot{v}_1 = f_1(v_1, \ldots, v_N) + u_1(t)$$
$$\dot{v}_2 = f_2(v_1, \ldots, v_N) + u_2(t)$$
$$\begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array} \qquad\qquad (8.2.1)$$
$$\dot{v}_N = f_N(v_1, \ldots, v_N) + u_N(t)$$

Above, $f_i(\mathbf{v})$ are the endogenous and $u_i(t)$ the exogenous parts of the system. As a vector differential equation, (8.2.1) may be summarized as,

$$\dot{\mathbf{v}} = \mathbf{f}(\mathbf{v}) + \mathbf{u}(t). \qquad\qquad (8.2.2)$$

When $\mathbf{u}(t)$ is zero the system is closed. Such a conservative system corresponds to an "idealized" model of the economy since no exogenous forces are affecting the development. Mechanical systems are often approximated by such conservative sys-

tems. However, to be realistic, an applied economic model has to be described as an open subsystem of (8.2.2), and thus as a dissipative system.

A modelling of the economy as an open system reflects the limits of economic knowledge and the impossibility to estimate the complete system in applied models. Dissipativity reflects that the dynamics of a given economic system generally are influenced by an external exchange of energy, in terms of information, commodities, waste and financial resources. One may also observe that when the economy is modelled as a closed system, the option to study changes in the endogenous variables by other means than changes in the model parameters is eliminated [Allen (1964)].

Therefore, an applied analysis of (8.2.2) is constrained both by the complexity of the system and the demand for empirical information. Hence, direct analysis of systems or subsystems on this form are, as mentioned above, rare and an equilibrium approximation becomes motivated. We are then back to the question regarding when an equilibrium model represents a dynamic economy in a satisfying way?

The problem has been addressed previously in economics. When discussing the choice of endogenous and exogenous variables in a model, Samuelson (1947) accentuated the fact that some processes move slowly compared with others. Long-run tendencies should thus be distinguished from short-run tendencies and so forth in infinite regression. When a set of variables is stable and rapidly damped, the exact path from one equilibrium to another may be neglected. This discussion by Samuelson has strong similarities with the implication of the slaving principle and the use of equilibrium analysis as an adiabatic approximation of more complex processes [Haken (1983a)]. In the following we will discuss how the slaving principle and the adiabatic approximation may be used in relation to (8.2.2).

Suppose the variables in (8.2.2) are ordered according to the speed of adjustment, as given by $\mathbf{f}(\mathbf{v})$. The slaving principle implies that this inherent organization is utilized in the formulation of a model. Given a specified time period and a focus on the dynamics of faster processes, slow variables may be treated as almost fixed. In this way they will be reflected (directly or indirectly) by parameter values in the faster equations. If there also exist relatively faster processes, which may be assumed to

adjust instantaneously to changes in the system, the latter are embedded in the model.

Assume that the variables in (8.2.2) are grouped in vectors by the speed of adjustment as follows,

$$\dot{\mathbf{x}} = \mathbf{H}(\mathbf{x}, \ \mathbf{z}, \ \mathbf{y}) + \mathbf{h}(t), \tag{8.2.3}$$

$$\dot{\mathbf{z}} = \mathbf{G}(\mathbf{x}, \ \mathbf{z}, \ \mathbf{y}) + \mathbf{g}(t), \tag{8.2.4}$$

$$\dot{\mathbf{y}} = \mathbf{E}(\mathbf{x}, \ \mathbf{z}, \ \mathbf{y}) + \mathbf{e}(t). \tag{8.2.5}$$

Where $\mathbf{x}$ is a vector of slow, $\mathbf{z}$ of medium-fast, and $\mathbf{y}$ a vector of fast variables. The slow variables may for example be infrastructure and judicial systems, the medium-fast variables contain capacity exit and entry, while financial flows are among the fast variables. Since we are interested in the development during a medium-term horizon, the time period $t \in [0, T]$, it is relevant to concentrate on the variables in the vector $\mathbf{z}$. During this time period the slow variables in $\mathbf{x}$ are almost unchanged so one may assume,

$$\dot{\mathbf{x}} \approx \mathbf{0}, \tag{8.2.6}$$

and substitute $\mathbf{x}(t)$ for $\bar{\mathbf{x}}$,

$$\mathbf{x}(t) \approx \bar{\mathbf{x}} \ ; \ t \in [0, T]. \tag{8.2.7}$$

The values in $\bar{\mathbf{x}}$ should not be considered as equilibrium values, only as approximately unchanged. Observe that this implies that the change in the exogenous force $\mathbf{h}(t)$ is either minor, or is balanced against or absorbed by the development of $\mathbf{H}(\cdot)$, so that $\mathbf{H}(\cdot) \approx -\mathbf{h}(t)$. For example, this may be maintenance which balances wear and tear on the physical network.

Equation (8.2.7) gives a structure within which the two sets of faster processes develop. Mathematically, such a structure may be introduced by parametrization of the faster system. The medium-term system (8.2.4) is then rewritten as,

$$\dot{z} - G(\bar{x}, z, y) + g(t),\qquad (8.2.8)$$

which after introduction of a vector of parameters, such as elasticities, conversion factors and network properties, denoted by α, gives,

$$\dot{z} - \hat{G}(\alpha; z, y) + g(t).\qquad (8.2.9)$$

Before we study this system further, consider the fast variables in vector y. Assume that the speed of adjustment to the equilibrium among the variables in y is relatively high compared with the dampening among those in z. Then,

$$[E(\bar{x}, z, y) + e(t)]/y \ll [G(\bar{x}, z, y) + g(t)]/z,\qquad (8.2.10)$$

that is,

$$\dot{y}/y \ll \dot{z}/z.\qquad (8.2.11)$$

This means that if y is disturbed away from its equilibrium $y*$, it will return to an equilibrium quickly compared with the time it takes for z to reach an equilibrium. Hence, on the time scale of z, y is close to equilibrium almost all the time and one may set,

$$\dot{y} - 0.\qquad (8.2.12)$$

It is then possible to solve (8.2.5) for,

$$y* - \hat{E}(\bar{x}, z) + e(t).\qquad (8.2.13)$$

This approximation also requires that the exogenous force on the variables is absorbed by the system, i.e. $E(\cdot) = -e(t)$. The advantage gained from this is that the fast variables may, by introduction of (8.2.13) into (8.2.9), be embedded into the function $\hat{G}(\cdot)$. Hence, (8.2.9) may be written,

$$\dot{z} = \hat{G}(\alpha;\ z,\ \hat{E}(\overline{x},\ z) + e(t)) + g(t),\qquad (8.2.14)$$

which, since the fast variables are dominated by the slower, may be simplified to,

$$\dot{z} = \hat{G}(\alpha;\ z;\ t) + g(t)\qquad (8.2.15)$$

In the function $\hat{G}(\cdot)$, the slow variables are reflected by parameters (e.g. fixed structures), while the fast variables may be solved for in another step with the slower variables as parameters. The system in (8.2.15) may be compared with the system in (8.2.2) and the conclusion is obvious,

* A considerable reduction of the dimension of a problem is possible when the slaving principle may be applied.

The system of differential equations in (8.2.15) is in itself interesting to analyze. The stability of the model in relation to changes in parameter values and initial states are of particular interest. However, the difficulty to model such a system increases when the number of variables and the inherent complexity of the system increases. It may in some such cases, as a further simplification and reduction of the dimension, be appropriate to make an adiabatic approximation of the reduced system and constrain the analysis to equilibrium states. The condition for such an approximation is;

* If the system is damped, i.e. there exists a stable equilibrium to (8.2.15), then,

$$\dot{z} = 0 = \hat{G}(\alpha;\ z;\ T) + g(T).\qquad (8.2.18)$$

Furthermore,

* It is in this case, at least in principle, also possible to solve for the equilibrium vector $z*$ as a function of the parameters and the exogenous forces,

$$z* - \widetilde{\widetilde{G}}(\alpha; \ z; \ T) + g(T).\tag{8.2.19}$$

Hence, condition (8.2.6) on the changes in the slow variables also gives a condition for the use of unchanged parameters in equilibrium models. Changes in the slow variables may thus not be introduced during the time period, but before the initial point. The equilibrium solution may, as was discussed in Chapter 2, be obtained by formulation of an algorithm for solution of the complementarity problem,

(CP):

Find $z* \in \mathbf{R}^n$ that solves

$$\mathbf{F}(z*) \geq 0, \ z \geq 0, \ z*^T \mathbf{F}(z*) - 0.\tag{8.2.20}$$

The length of the time period in such a single-period equilibrium model is given by the processes involved in z. The stability of the equilibrium is guaranteed as long as assumption (8.2.18) is appropriate, while uniqueness is a question of functional forms and the values of the parameters in the problem. When the slow variables, and thus the parameters change, the model may reach bifurcation points where the number of equilibria changes.

In Figure 8.1 below, this ordering of a dynamic system according to differences in adjustment speed and the use of equilibrium analysis as an adiabatic approximation of a continuous model is illustrated.

The equilibrium approximation is, as is well known, a standard tool in economics. The function $\widetilde{\widetilde{G}}(\cdot)$ then contains the equilibrium conditions, i.e. behavioural rules and system constraints, on the endogenous variables. Traditionally, those have been represented by various versions of the law of diminishing returns, maximization
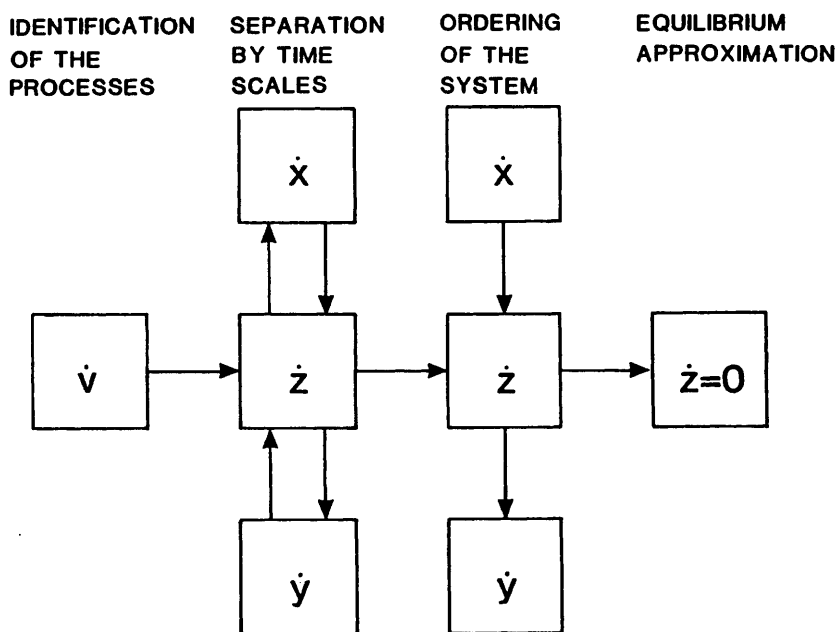
Figure 8.1   Separation, ordering, and equilibrium approximation of a dynamic system.

behaviour among actors, and gap closing in markets with excess demand, where the functional form has been chosen so as to guarantee existence of an equilibrium.

However, Kehoe (1985) emphasizes that in productive economies, uniqueness of equilibrium only has been proved under very restrictive assumptions. When strong non-convexities and non-monotonicities are generated, for instance by joint production, the equilibrium may be difficult to solve for and occurrence of multiple equilibria possible. Existence of multiple equilibria is however not an analytical problem when it may be interpreted as results of structural shifts, caused by changes in the slow variables. Nonexistence of a point equilibrium make modelling and esti-

mation more difficult or perhaps impossible, but it is also possible that other well behaved attractors than a point attractor may be found, e.g. a limit cycle.

The relevance of an equilibrium model in a given application is thus dependent on the model specification in relation to the assumed period length and the stability properties of the continuous model. Crucial points are the separation of processes in relation to their time scales and the stability of an equilibrium in relation to perturbations in parameters and intitial states.

Relevant critique in relation to an equilibrium approximation should thus be directed towards the adjustment speed of the variables, the chosen functional forms, and the elasticities. The discussion in this section also implies that applied equilibrium modelling generally is initiated from the wrong starting point. The traditional approach implies that functional forms are chosen so that an equilibrium exists, although not necessarily unique, while our analysis suggest that the starting point should be taken in a continuous time model, from which an equilibrium model is derived through simplifications. The limits set by existing solution algorithms may imply that further simplifications have to be introduced so that the final model comes close to what we have presented in the previous chapters. Ultimately, the value of such simplifications is determined by how well the results and the insights gained from the model are confirmed by empirical observations.

8.3   MULTIPERIOD MODELS OF DYNAMIC PATHS

A natural question which follows from the discussion in the previous section is to what extent quasi-dynamic models, consisting of sequences of single-period equilibria of the type discussed above, may give further insights into the dynamics of spatial structural change? In other words, what are the gains and losses if one solves a sequence of shorter periods $t - 1, \ldots, T$, instead of a single-period model with the period length T?

A quasi-dynamic model is a way to approximate a continuous model while obtaining some information regarding the adjustment path from the initial to the terminal point in time. It implies, in terms of the slaving principle, that each period consists of

variables which reach equilibrium within this period while the modelled sequence of periods also is determined by the adjustment of some slower variables towards (but perhaps not into) equilibrium at the terminal point. The adjustment of the slower variables then have impacts on, and may be affected by, the variables in the single-period model. The properties of such a quasi-dynamic model are dependent on the formulation of, and parameters in, the intertemporal linkages in between periods and the relation between the model and the exogenous forces.

In Figure 8.2 the relation between the endogenous variables, exogenous variables, and the intertemporal connection of such a four period quasi-dynamic model are illustrated. Above the horizontal time line are the exogenous variables for the initial state and each subsequent time period. At the bottom are the endogenously obtained equilbrium solutions for each time period. In between we have the inter-temporal connections which transform the solution from a given period, while taking care of the new exogenous information, to a new equilibrium problem.

Given the discussion in the previous chapter on the importance of the time scale of variables, the following observations may be made in relation to the Figure.

* If a single-period model is replaced by a multiperiod model covering the same time horizon, the variables in the equilibrium model may be replaced by variables with a faster adjustment speed.

* The endogenous variables in the single-period model may now be found in the intertemporal connections which give parameters to each equilbrium model. Those parameters are adjusted towards equilbrium, and thus become variables in the intertemporal transformer.

* If instead a set of single-period models are connected into a quasi-dynamic model with a longer time horizon, some of the parameters in the slow process now may be adjusted over time and should thus be moved to the intertemporal transformer.

Independent of which case we are considering, it is apparent that the speed of adjustment of the variables in the intertemporal connections has to be estimated in relation to both slower and faster variables.
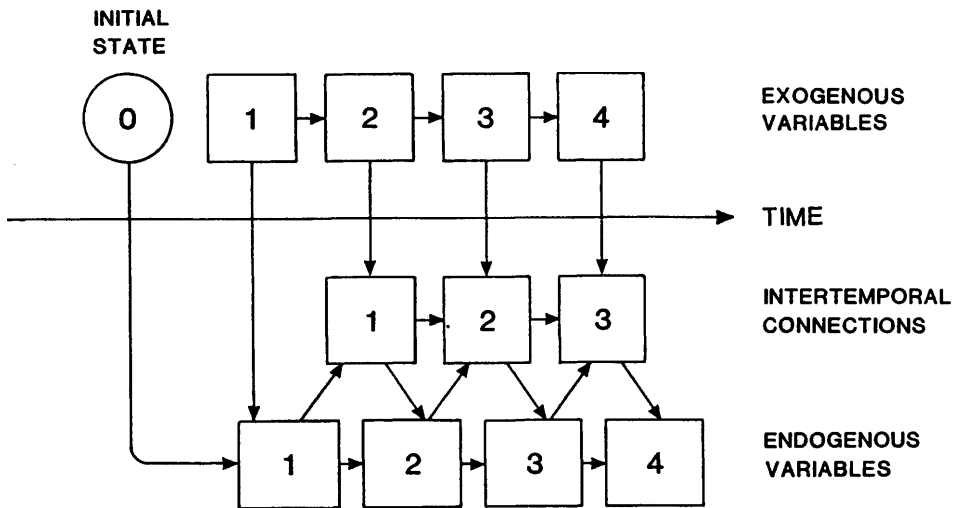
Figure 8.2.   Structure of a four-period quasi-dynamic model.

In economics, mainly two quasi-dynamic approaches may be identified, the recursive and the intertemporal approach. The quasi-dynamic model in Figure 8.2 is of a recursive type since the dynamics are irreversible. The recursive approach has been utilized and displayed by among others Day (1970), Day and Kennedy (1970a, 1970b), Nelson (1971) and Day and Cigno (1978). Recursive applied general equilibrium models have been suggested by Bergman (1982, 1986), Ballard et al. (1985), Adelman and Robinson (1978), Dervis et al. (1982), Persson and Johansson (1982), Johansson and Persson (1983), Haaland (1988). Other recursive formulations of

multisector problems may be found in Takayama and Judge (1971), Ysander (1986a) and Johansson and Snickars (1988).

One may also note that recursive economic dynamics have strong similarities with the work within what Ohlin (1937) chose to call "the Stockholm school". The problems they met, caused by the discrete division of time and the problem with the formulation of expectations have been discussed by Palander (1941), Jonung (1987), and Peterson (1987).

The advantage of the recursive approach is that it makes use of the fact that decisions and system behaviour are not optimal in the way suggested by, for instance, optimal control formulations, intertemporal equilibrium or dynamic programming approaches. Recursive models emphasize existence of genuine uncertainty regarding the future in each time period and stresses myopic and adaptive behaviour.

Besides the recursive approach, multiperiod formulations have also been represented by intertemporal equilibrium models with perfect foresight. All methods have their drawbacks, but the "clairvoyant" intertemporal equilibrium models [Takyama and Judge (1971), Takayama et al. (1984), Nagurney and Aronsson (1988)] assume perfect information and reversible time, properties which clearly constrain the use to planning exercises.

In the following, a couple of recursive models are reviewed, with the aim to illustrate how investments and formation of expectations may be treated in between two periods. These constitute important problems in relation to structural change dynamics. Modelling investment behaviour in an intertemporal transformer represents a way to connect the models with exogenously given investments and an alternative to the models with investment decisions endogenously determined within a period. Both types have been used in the previous vintage models.

A model, where a sequence of single-period CGE models with exogenous investments are solved is suggested by Dervis et al. (1982). Aggregate investments are given by aggregate savings, and in the simplest allocation function, a fixed coefficient approach is used. The coefficients are obtained from the bench-mark equilibrium as,

$$\Lambda_i - \sum_j p_j \hat{b}_{ji} Z_i / TS, \qquad \text{all } i, \sum_i \Lambda_i - 1, \qquad (8.3.1)$$

where Z_i are investments in sector i, and $\hat{b}_{ij}$ are investment coefficients which relate the demand for investment commodities to the capital increase in a sector. Together with prices, this gives the unit cost of investment. Given total savings, TS, and prices from the period before, the investments over sectors and the new stock of fixed capital in the next period may thus be obtained from (8.3.1).

In an extended formulation, the coefficients (Λ_i) are instead treated as variables and are thus assumed to have a faster adjustment time. Aggregate savings are now allocated in proportion to the share in aggregate profits of each sector given by the equilbrium model. Those proportions may furthermore be adjusted due to the relative profit rate of each sector compared with the average rate of profits in the economy. A higher relative profit rate gives a larger share of the investable funds to a sector. The investment coefficients may thus be represented as,

$$\Lambda_i - \Lambda_i(\pi(t-1), \alpha, r(t), r*(t)), \qquad \text{all } i. \qquad (8.3.2)$$

The first term is a vector of sectoral profits. The parameter α is an investment mobility parameter and an indicator of the responsiveness of the capital markets to the static market signals. Investments are, dependent on the value of α, either financed by retained profits in each sector or, alternatively, sectors with a high rate of return ($r_i(t)$) compared with the market rate $r*(t)$, may attract funds from low profit sectors. The variables in (8.3.2) may also be used with other time lags. The formulation exemplifies how elasticities with respect to the solution of the fast variables in the equilibrium model have to be introduced into the intertemporal transformer. The results obtained from the model are thus, to a large extent, dependent on the success of those estimates.

Another example of a quasi-dynamic model is Bergman (1982, 1986). In a growth model of a "small open economy" a sequence of medium-term CGE models are solved. The firms in this model have explicit price expectations at least in relation to the investment decision. Those are represented by,

$$\bar{p}(t) = \alpha p(t) + (1-\alpha)\bar{\bar{p}}(t). \tag{8.3.3}$$

Above the vector $\bar{p}(t)$ represents the prices firms expect will be average prices during the lifetime of the units under construction. In this case, the parameter α, gives an opportunity to model different types of expectations. If set to one, expectations are static (myopic) and the firms believe recent prices, $p(t)$, to prevail during the planning horizon. If set to zero, prices are determined by the vector $\bar{\bar{p}}(t)$, which either are adaptively formed by previous prices in the intertemporal transformer, or are obtained from some exogenous forecast.

In the model by Bergman, aggregate savings are determined by an exogenous saving rate out of value added and the foreign debt. This in turn gives the value of aggregate investment, which in between the periods is allocated over sectors with rules including expected unit cost, expected sectoral rate of returns and an exogenous market rate of interest. In short, the rules imply that the investment budget in a first step is allocated to those sectors whose rate of return exceed the market rate of interest. If investments after this are less than aggregate savings, investments are distributed also to sectors with a rate of return below the market rate of interest. Hence, the rules have much in common with the function (8.3.2). Scrapping of old plants are obtained by an exogenous annual sectoral rate of depreciation.

In the above two models by Dervis et al. and Bergman, the investment decisions are made in the interperiod transformer. An advantage, especially in short-run models, is that expectations and other exogenous information, and not solely the equilibrium prices, may then govern those decisions. However, the longer each time period is, the more advantageous an endogenous treatment of investments seems to be.

Alternatives to the above models are models with endogenously formed expectations regarding prices, demand and production based on a recursive-nested model. In Takayama and Judge (1971) such an adaptive SPE model is introduced. An intertemporal storage optimization problem is solved on the basis of expectations about the development of exogenous variables. At the end of the first period, new information is assumed to be available and the planning horizon is extended one period. A new optimal program is solved for the time from the second period until the

extended planning horizon, but only the first of those periods (i.e. the second of the original periods) is actually carried out. The approach represents one way to use nested models in order to introduce nonmyoptic expectations. The actors make decisions governed by what they believe are the future values of a variable. In this case, it is important that those expectations are formed by a model which equals the one used to model the actual development of the economy.

A model based on recursive nesting of multisector network equilibrium models may be given the following structure. The medium-term model with endogenously determined investments is used as a model for formation of investment plans, production and demand on a five year basis. This solution is then broken down into annual changes and the economy moves one year ahead. The annual development within each period may be calculated by a linear growth assumption, so that, for example, the annual capacity increase, $\Delta \bar{q}_i(t)$ in a model covering T years becomes,

$$\Delta \bar{q}_i(t) = \bar{q}_i(t+1) - \bar{q}_i(t) = [\bar{q}_i(T) - \bar{q}_i(0)]/T. \qquad (8.3.4)$$

Alternatively, the solution may be annualized by a geometric growth assumption, where,

$$\Delta \bar{q}_i(t) = r_i \bar{q}_i(t), \qquad (8.3.5)$$

or,

$$\bar{q}_i(t+1) = (1 + r_i)\bar{q}_i(t). \qquad (8.3.6)$$

The growth rate of capacity in sector i is then obtained from the medium-term model as,

$$r_i = [\bar{q}_i(T)/\bar{q}_i(0)]^{1/T} - 1 \qquad (8.3.7)$$

Similar approaches are used to obtain annual values for other variables. A new five year solution may then be obtained, and so forth until the final time horizon is obtained. The above approach represents a version of modelling which has been named "nested dynamics" [Persson and Johansson (1982)]. It represents, in addition to the recursive model by Takayama and Judge, an attempt to introduce expectations about future states by use of a complete model of the system itself. Investment decisions are based on expectations about the future state of the economy. The medium-term model is such an internal model of the economy which is supposed to be used to form expectations. Expectations are thus rational in the sense that actors make decisions about consumption and production based on the internal model, and no one is kept uninformed about the assumed behaviour of the economy. A criticism one may have against the outcome of such a model is that the annualization may give a false picture of the interdependences between the variables in the economy.

The above models give some insights in "the state of the art" as regards quasi-dynamic multisector models. Comparative evaluations of such models in relation to a similar empirical material ought to be made in order to make a discrimination between the intertemporal formulations.

Moreover, the properties and the sensitivity of the models, in relation to parameter changes, have not been studied. The dynamics of discrete time models may exhibit different archetype trajectories, from simple linear paths to nonlinear explosive, damped, cycling and chaotic behaviour. A small change in the value of a parameter may in some intervals cause a transition from one archetype behaviour to another. When this is combined with more or less "ad hoc" assumptions in the intertemporal transformer, it gives models of the above type unclear properties and their solution has to be treated very carefully. This gives further motivation for the single-period model when clear properties are urgent.

A related problem concerns if and how modifications of the interperiod transformer and the single-period model should be introduced when the economy learns from the actual outcome of it's own behaviour in relation to the exogenous environment. This may be seen as an endogenous adjustment of parameters given by slow processes in response to the development among the faster variables. An adaptive system has a predicative model of itself and the environment, and may furthermore

adjust the predicative model. Learning is thus an endogenous part of the model. Modelling of adaptivity should include such a combination of learning and formation of expectations, but this also represents a core problem when dealing with the dynamics of self-organization and living systems.

The crux is how to avoid an infinite hierarchy of overlapping models, each modifying the behaviour of the model one step above in order of precedence. Rosen (1972), and more recently, Casti (1987) have suggested the metabolism-repair system, as one attempt to solve the problem. Applied to an industrial process, the metabolism would correspond to the production system, while the repair system is investment, marketing and R&D functions which improve and keep the system viable. Instead of an introduction of a repair-repair system, new repair systems are generated from the metabolism, i.e. the production process itself, and a self-referential system is created.

Applied to a multisector model of the type discussed previously in the thesis, new input-output, $A^{rs}(r*)$, labour $1^r(h, r*)$, and investment, B^{rs}, coefficients should thus not only be determined endogenously within the production system. The parameters in the "frontier" or "best practice" functions for the determination of those coefficients, should also be determined in the model. An interesting and demanding task. One may have doubts about the possibilities to simulate such an adaptive system by point attractors.

8.4   TOWARDS AN EXTENDED REPRESENTATION OF DYNAMICS IN THE ANALYSIS OF STRUCTURAL CHANGE

We have in this chapter motivated a use of equilibrium models in the analysis of dynamic problems when adiabatic approximations are possible. We have also argued in favour of single-period equilibrium models, instead of multiperiod models, when clear model properties are urgent.

However, a quasi-dynamic model has a special advantage as a tool for analysing the process of adjustment from one state to another. The obvious problem is to give an

explicit formulation of how the slower variables adjust. An appropriate formulation, and the estimation of such adjustment processes, easily becomes complex. Such efforts are thus situated on the research frontier in fields related to evolution and self-organization.

Learning behaviour and robustness of quasi-dynamic models are topics were further research is necessary. In this work one may glance at results obtained from "heterodox" micro-simulation models of Schumpeterian competition and evolutionary development [Nelson and Winter (1982), Day and Eriksson, (1986), Silverberg (1987)] which also has some connections to our vintage model. However, so far the results from the micro simulation experiments are not easy to summarize into some general conclusions, relevant with regard to the formulation of multisector models.

Recently, some attempts have also been made to develop the old dynamic Leontief model in continuous time [Medio (1987), Anderson and Zhang (1988)]. This work has been inspired by the recent contributions to the knowledge of nonlinear dynamic models and is only in an introductary phase. The discovery, of the ease by which chaos may be obtained by parameter changes in existing dynamic models has raised the question of how suited those are for forecasting and simulation of real world phenomena. However, this will also inspire to a development of a new generation of dynamic models, and methods for their estimation.

The progress in the above fields have been supported by the new generations of fast and cheap computers. The computer development has, interestingly enough, also brought about extended possibilities of numerically specifying and solving applied multisector network equilibrium models. Hence, one may as a result of comparative simulation and empirical evaluation, expect a period of progress also in this area. This indicates a necessity of developing further insights in the relation between continuous time representation and equilibrium approximations of an economic system.

9   EPILOGUE

9.1   MODELLING SPATIAL STRUCTURAL CHANGE

A persistent motivation for carrying out this study has been a growing demand for assessment and scenario analyses using applied models of structural change and the evolution of spatial patterns. It may thus be appropriate to close with some comments on the relation between the model formulations presented in the thesis and this requirement.

In the thesis, we have introduced a class of discrete time models of spatial structural change. The vintage function, an important part of the Scandinavian modelling tradition, is a common attribute of this class. With the suggested models, the vintage model has been extended to allow for a deeper analysis of multisector network dynamics. Vintage functions take care of some of the heterogeneities among producers in a sector, and represent rigidities in the spatial and sectoral allocation of productive resources. Those rigidities constitute the existing pattern and constrain the creation of new structures at least in the medium term. Vintage functions also contain an inherent duality, since rigidities represent options to obtain temporary monopolies and quasi-rents. These market conditions motivate firms to develop, and invest in new structures and commodities.

Our choice of functional form is related to the time period of the models. The longer the time period is, the less rigid the production system becomes and the more appropriate it would be to replace the vintage approach by other approaches.

Still, the estimation of parameters, endogenous and exogenous variables in equilibrium models is a tough exercise, in which one usually has to use "benchmark" calibration in relation to a data set at a single point in time. Often only a subset of the parameters may be obtained by statistical analyses on larger data sets. Since the development of solution algorithms has been rapid, new calibration techniques is the field where improvements are most urgent.

The class of multisector models developed above may also be compared with the classical Heckscher-Ohlin model of international and interregional trade. The main difference is that vintage functions specify technical differences between regions, while the technique is assumed to be uniform over regions in the Heckscher-Ohlin model. Another difference is the possibility of introducing spatial heterogeneity into the model instead of the homogeneity which prevails in the Heckscher-Ohlin model.[1] One may also note that the Heckscher-Ohlin model does not contain explicit transportation costs, and the transport network is indeed implicit.

We have also argued that equilibrium models with spatially heterogeneous commodities are important tools in applied analysis, and should be developed further. While doing this, it was shown how the SPE model, which has dominated the field of spatial equilibrium analysis, may be seen as a special case of this larger class of models. Spatial heterogeneity reflects demand for variety and supply of differentiated products. We have used a spatial linear expenditure system with heterogeneous commodities. Although the system is simple, and includes some restrictive assumptions, an advantage is that the system may also be rationalized by, for example, habitual behaviour and rigidities among the consumers.

It is our belief that differences between regions and cities in the composition of households with different habitats will play an increased role in the explanation of spatial growth. It is also belived that product variety increases in a growing economy and that there is a connection between spatial variety and spatial growth. Models allowing for heterogeneity will therefore have increased relevance.

Previously, Jacobs (1984), and Johansson and Westin (1987), have stressed the importance of import substitution in the growth process. The more varied inflow on the links to a node (region), the larger is the possibilities for growth. This accentuates the need to integrate network development with changes in supply and demand. We have emphasized the importance of an explicit recognition of transportation systems, interaction costs and networks in the analysis of multisector structural change and growth.

---

1. Ohlin (1933) was aware of the heterogeneity problem but suppressed it in the trade model.

Growth and change are closely interlinked with the emergence of novel patterns, such as new system design concepts. The latter is a result of a complex dynamics, which the presented equilibrium models by no means give new insights into. However, the vintage formulation with endogenous investments gave insights into the motives for, and constraints on, the introduction of such novelties in an economy.

In the vintage model, a new product is only observed as a new technique generated by an exogenously given process. An allowance for joint production of commodities, economies of scope, and household demand functions with attributes in line with the models by Lancaster would both allow for a richer introduction of substitution processes and product competition. As suggested by product cycle theory producers of a commodity may shift from product to price competition as the product matures. Price equilibrium analysis would thus be most suited for sectors dominated by mature products. Introduction of other signals besides prices would reduce the bias towards analysis of price competition in contemporary equilibrium modelling. We may however note that technique-specific temporary monopoly features are natural elements of a vintage model.

Spatial differences due to different dynamic cultures, i.e. differences in the choice of strategies by, and the mix of, various actors are essential parts of structural change. An interesting task would be to classify regions and sectors under different epochs in relation to a taxonomy of adaptive behaviour. Such a taxonomy may reveal implicit connections between structural patterns, variety in flows, agglomeration in networks, growth of incomes and adaptive behaviour, i.e. factors which separate spatial growth points in the sense of Perroux (1964), from backward and stagnating nodes.

9.2   FURTHER RESEARCH

Applied equilibrium modelling seems to be moving out of a phase with technical problems regarding algorithms and computer capacity. Hence, although the solution of an equilibrium model is not a standard procedure, greater efforts and more energy may be laid on the behavioral and empirical content of the models. A number of suggestions for further research related to this study may obviously be made.

Let us here just mention a few of the most urgent fields, some of which previously have been discussed in the text.

* The conditions for equilibrium approximation of models in continuous time ought to be analyzed further with more structure in the continuous model. Movements in regions of instability and transitions between equilibria in response to parameter changes constitute an important field in this research.

* Development of network-oriented equilibrium models with an explicit interaction between the communication sector in a broader respect and the rest of the economy is important. This would give an integration of nonmaterial and material networks into a dynamic analysis of accessibility. A deeper understanding for short- and long-run effects on localization and production of investments in infrastructure has to be developed.

* The relation between spatial equilibrium models and entropy, gravity and logit models may be studied further. Those approaches have close connections to each other and represent alternative ways to simulate heterogeneity in flows on a network. Evaluation of the different approaches in relation to historical observations is an important task.

* Appropriate formulations of exit and entry functions in vintage models with rules for the distribution of aggregate sector investments over a vintage structure should be derived.

* Introduction of sectors with a concentrated market structure, where oligopoly and negotiations are dominating. Introduction of product competition and differentiated outputs.

* The spatial pattern of income flows, savings and transfers should be investigated further.

* Development of techniques for estimation of differences in adjustment time, of spatial demand functions, and of integrated equilibrium systems.

BIBLIOGRAPHY

Adelman, I. and S. Robinson (1978) Income Distribution, Import Substitution, and Growth Strategies in a Developing Country. In Day, R.H. and A. Cigno. **Modelling Economic Change: The Recursive Programming Approach.** Amsterdam: North-Holland.

Albegov, M., Å.E. Andersson and F. Snickars (eds.) (1982) **Regional Development Modeling: Theory and Practice.** Amsterdam: North-Holland.

Allen, R.G.D. (1964) **Mathematical Economics.** London: MacMillan.

Anas, A. (1983) Discrete Choice Theory, Information Theory and the Multinomial Logit and Gravity Models. **Transportation Research B,** 17B, pp. 13-23.

Anas, A. and L.-S. Duann (1985) Dynamic Forecasting of Travel Demand, Residential Location and Land Development. **Papers of the Regional Science Association 56,** pp. 37-58.

Andersson, Å.E. and B. Johansson (1984) Knowledge Intensity and Product Cycles in Metropolitan Regions. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA).** Working paper WP-84-13.

Andersson, Å.E. and H. Persson (1980) Integration of Transportation and Location Analysis: A General Equilibrium Approach. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA).** Research report RR-80-40.

Andersson, Å.E. and W.-B. Zhang (1988) Decision Centralization and Decentralization in the Dynamic Leontief System. **Umeå University.** Working Paper from CERUM 1988:6.

Armington, P.S. (1969) A Theory of Demand for Products Distinguished by Place of Production. **International Monetary Fund Staff Papers,** pp. 159-176.

Arrow, K.J. and G. Debreu (1954) Existence of an Equilibrium for a Competitive Economy. **Econometrica 22,** pp. 265-290.

Arrow, K.J. and F.H. Hahn (1971) **General Competitive Analysis.** San Francisco: Holden-Day.

Ballard, C.L., D. Fullerton, J.B. Shoven and J. Walley (1985) **A General Equilibrium Model for Tax Policy Evaluation.** Chicago, London: The University of Chicago Press.

Bardhan, P.K. (1966) International Trade Theory in a Vintage-Capital Model. **Econometrica 34,** pp. 756-767.

Barra, T de la and B. Perez (1986) Asymmetry in some common assignment algorithms: the dispersion factor solution. **Environment and Planning B 13,** pp. 293-304.

Batten, D. and D. Boyce (1986) Spatial Interaction, Transportation, and Interregional Commodity Flow Models. In Nijkamp, P.(ed.). **Handbook of Regional and Urban Economics,** vol I. Amsterdam: North-Holland.

Batten, D., Casti, J. and B. Johansson (1987) **Economic Evolution and Structural Adjustment.** International Conference on Mathematical Modelling at the University of California, Berkeley, USA. Berlin: Springer-Verlag.

Batten, D. and B. Johansson (1985) Industrial Dynamics of the Building Sector: Product Cycles, Substitution and Trade Specialization. In Snickars, F. et al. **Economic Faces of the Building Sector.** D20:1985. Stockholm: Swedish Council for Building Research.

Batten, D and L. Westin (1990) Modeling Commodity Flows on Trade Networks: Retrospect and Prospect. In Chatterji, M. and Kuenne, R. (eds.). **New Frontiers in Regional Science.** Essays in Honour of Walter Isard, vol. 1. London: Macmillan.

Beckmann M.J., C.B. McGuire and C.B. Winsten (1956) **Studies in the Economics of Transportation.** New Haven: Yale University Press.

Beckmann, M.J. and J.P. Wallance (1967) Marshallian versus Walrasian stability. **Kyklos 20,** pp. 935-949.

Beckmann, M.J. and T. Puu (1985) **Spatial Economics: Density, Potential and Flow.** Amsterdam: Elsevier Science Publ.

Bergman, L. (1978) Energy Policy in a Small Open Economy: The Case of Sweden. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA)** Research report RR-78-16.

Bergman, L. (1982) A System of Computable General Equilibrium Models For a Small Open Economy. **Mathematical Modelling 3,** pp. 421-435.

Bergman, L. (1986) ELIAS - A Model of Multisectoral Economic Growth in a Small Open Economy. In Ysander, B.-C. (ed.). **Two Models of an Open Economy.** Stockholm: Almqvist & Wiksell International.

Bliss, C. (1968) On Putty-Clay. **Review of Economics and Statistics 35,** pp. 105-132.

Brody, A. (1970) Optimal and time-optimal paths of the economy. In Carter, A.P. and A. Brody (eds.). **Contributions to Input-Output Analysis.** Amsterdam: North-Holland.

Bröcker, J. (1988a) Interregional Trade and Economic Integration. A Partial Equilibrium Analysis. **Regional Science and Urban Economics 18,** pp. 261-281.

Bröcker, J. (1988b) **Partial Equilibrium Theory of Interregional Trade and the Gravity Model.** Paper presented at the 28th European Congress of the Regional Science Association in Stockholm.

Buckley, P.A. (1987) An Interregional Computable General Equilibrium Model for National - Regional Policy Impact Analysis. **Umeå University: Working Papers from CERUM 1987:5.**

Carey, M. (1980) Stability of Competitive Regional Trade With Monotone Demand/Supply functions. **Journal of Regional Science 20,** no 4, pp. 489-501.

Carter, A.P. and A. Brody (eds.). **Contributions to Input-Output Analysis.** Amsterdam: North-Holland.

Casti, J.L. (1979) **Connectivity, Complexity and Catastophe in Large-Scale Systems.** Chichester: J. Wiley & Sons.

Casti, J.L. (1987) **The Theory of Metabolism-Repair Systems.** Technical University of Vienna: Institute for Econometrics & Operations Research. Mimeo.

Cavalieri, A., D. Martellato and F. Snickars (1982) A Model System For Policy Impact Analysis in the Tuscany Region. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA).** Collaborative paper CP-82.

Chenery, H.B. (1949) Engineering Production Functions. **Quarterly Journal of Economics 63,** pp. 507-531.

Chenery, H.B. (1953) Regional Analysis. In Chenery, H.B., P.G. Clark and V.C. Pinna (eds.). **The structure and growth of the Italian Economy,** pp. 97-129. Rome: U.S. Mutual Security Agency.

Christaller, W. (1966, 1935) **Central Places in Southern Germany.** Englewood Cliffs, N.J.: Prentice-Hall.

Cottle, R.W. and G.B. Dantzig (1968) Complementary Pivot Theory of Mathematical Programming. **Linear Algebra and its Applications,** pp. 103-125.

Cottle, R.W., F. Giannessi and J-L. Lions (eds.) (1980) **Variational Inequalities and Complementarity Problems.** Chichester, New York, Brisbane, Toronto: John Wiley & Sons.

Cournot, A.A. (1929, 1838) **Researches into the Mathematical Principles of the Theory of Wealth.** New York: MacMillan.

Dafermos, S. (1980) Traffic equilibrium and variational inequalities. **Transportation Science** 14, pp. 42-54.

Dafermos, S. (1983) An Iterative Scheme for Variational Inequalities. **Mathematical Programming** 26, pp. 40-47.

Daley, A.J. and S. Zachary (1978) Improved multiple choice models. In Hensher D.A. and Q. Dalvi (eds.): **Determinants of travel choice,** pp. 335-357. Westmead: Saxon House.

Day, R.H. (1970) Recursive programming models of industrial development and technological change. In Carter, A.P. and A. Brody (eds.). **Contributions to Input-Output Analysis.** Amsterdam: North-Holland.

Day, R.H. (1987) The General Theory of Disequilibrium Economics and of Economic Evolution. In Batten, D. et al. (eds). **Economic Evolution and Structural Adjustment.** Berlin: Springer-Verlag.

Day, R.H. and A. Cigno (1978) **Modelling Economic Change: The Recursive Programming Approach.** Amsterdam: North-Holland.

Day, R.H. and Eriksson (1986) **The Dynamics of Market Economies.** New York: North-Holland.

Day, R.H. and T. Groves (eds.) (1975) **Adaptive Economic Models.** New York: Academic Press.

Day, R.H. and P.E. Kennedy (1970a) Recursive Decision Systems: An Existence Analysis. **Econometrica** 38, pp. 666-681.

Day, R.H. and P.E. Kennedy (1970b). On a Dynamic Location Model of Production. **Journal of Regional Science** 10, pp. 191-197.

Deardorff, A.V. and R.M. Stern (1986) **The Michigan Model of World Production and Trade.** Cambridge, Mass.: The MIT Press.

Deaton, A. and J. Muellbauer (1980) **Economics and Consumer Behaviour.** New York: Cambridge University Press.

Debreu, G. (1959) **The Theory of Value.** New York: Wiley.

De Melo, J.A.P. and K. Dervis (1977) Modelling The Effects of Protection in a Dynamic Framework. **Journal of Development Economics** 4, pp. 149-172.

Dervis, K., J. de Melo and S. Robinson (1982) **General equilibrium models for development policy.** New York: Cambridge University Press.

Dial, R.B. (1971) A Probabilistic multipath traffic assignment model which obviates path enumeration. **Trans. Res.** 5, 83-111.

Dixit, A.K. and V. Norman (1986) **Theory of International Trade.** Cambridge University Press.

Dixon, P.B., B.R. Parmenter, J. Sutton and D.P. Vincent (1982) **Orani: A Multi-sectoral Model of the Australian Economy.** Amsterdam: North - Holland.

Domencich, T. and D. McFadden (1975) **Urban Travel Demand: A Behavioral Analysis.** Amsterdam: North-Holland.

Dorfman, R., P.A. Samuelson and R.M. Solow (1958) **Linear Programming and Economic Analysis.** New York: McGraw-Hill.

Duann, L.-S. (1982) **A Dynamic Simulation Model for Urban Residential Location, Travel Mode Choice, and Residential Land Development.** Evanston, Illinois: Northwestern University. Ph.D. thesis.

Echenique, M. et al. (1988) A Technical introduction to the MEPLAN system. (mimeo)

Enke, S. (1951) Equilibrium Among Spatially Separated Markets: Solution by Electric Analogue. **Econometrica** 19, pp. 40-48.

Fisk, C.S. and D.E. Boyce (1984) A modified composite cost measure for probabilistic choice modeling. **Environment and Planning A** 16, pp. 241-248.

Florian, M. and M. Los (1982) A New Look at Static Spatial Price Equilibrium Models. **Regional Science and Urban Economics** 12, pp. 579-597.

Friesz, T.L. and P.T. Harker (1985) Freight network equilibrium: a review of the state of the art. In Daughety, A.F.: **Analytical studies in transport economics.** Cambridge University Press.

Friesz, T.L., R.L. Tobin, T.E. Smith and P.T. Harker (1983) A Nonlinear Complementarity Formulation and Solution Procedure for the General Derived Demand Network Equilibrium Problem. **Journal of Regional Science** 23, pp. 337-359.

Friesz, T.L., P.V. Viton and R. Tobin (1985) Economic and Computational Aspects of Freight Network Equilibrium Models: A Synthesis. **Journal of Regional Science** 25, pp. 29-49.

Førsund, F. (1984) Om Kvasirente. (In Norwegian) **Sosialøkonomen** 1, pp 19-32.

Førsund, F., S. Gaunitz, L. Hjalmarsson and S. Wibe. (1980) Technical Progress and Structural Change in the Swedish Pulp Industry 1920-74. In Puu, T. and Wibe, S.: **The Economics of Technological Progress.** London: Macmillan Press.

Førsund, F. and L. Hjalmarsson (1987) **Analyses of Industrial Structure: A Putty - Clay Approach.** Stockholm: The Industrial Institute for Economic and Social Research.

Giles, D.E.A. and P. Hampton (1987) A Regional Consumer Demand Model for New Zealand. **Journal of Regional Science** 27, pp. 103-118.

Ginsburgh V.A. and J.L. Waelbroeck (1981) **Activity Analysis and General Equilibrium Modelling.** Amsterdam: North-Holland.

Granholm, A. (1981) Interregional Planning Models. **University of Gothenburg:** Department of Economics. Ph.D. thesis.

Green, H.A.J. (1979) **Consumer Theory.** London: MacMillan Press.

Haag, G. and W. Weidlich (1986) **The Schumpeter Clock.** Paper presented at the European Summer School in Umeå, Sweden.

Haaland, J.I., V. Norman, T. Rutherford and T. Wergeland (1987) VEMOD: A Ricardo-Heckscher-Ohlin-Jones model of world trade. **Scandinavian Journal of Economics** 3, pp. 251-270.

Haaland, J.I. (1988) Modelling General Equilibrium in a Small, Open Economy. A Norwegian Example. **Norwegian School of Economics and Business Administration.** Discussion Paper 12/88.

Hahn, F. (1984, 1982) Stability. In Arrow, K.J. and M.D. Intriligator (eds.). Handbook of Mathematical Economics I. Amsterdam: North-Holland.

Haken, H. (1983a) Synergetics. Berlin Heidelberg: Springer Verlag.

Haken, H. (1983b) Advanced Synergetics. Berlin Heidelberg: Springer Verlag.

Hansen, B. (1966) Lectures in Economic theory, Part one. Lund: Studentlitteratur.

Hardley, G. and M.C. Kemp (1966) Equilibrium and Efficiency in International Trade. Metroeconomica 18, pp. 125-141.

Harker, P.T. (ed.) (1985a) The State of the Art in the Predictive Analysis of Freight Transport Systems. Transport Reviews 5, no 2, pp. 143-164.

Harker, P.T. (ed.) (1985b) Spatial Price Equilibrium: Advances in Theory, Computation and Application. Lecture Notes in Economics and Mathematical Systems, 249. Heidelberg: Springer-Verlag Berlin.

Harker, P.T. (1988) Dispersed Spatial Price Equilibrium. Environment and Planning A 20, pp. 353-368.

Harris, C.C. (1973) The Urban Economies, 1985. Lexington: Lexington Books.

Harris, C.C. and M. Nadji (1985) The Spatial Content of the Arrow-Debreu General Equilibrium System. Journal of Regional Science 25, pp. 1-10.

Harris, R. (1984) Applied General Equilibrium Analysis of Small Open Economies with Scale Economies and Imperfect Competition. American Economic Review 74, pp. 1016-32.

Hashimoto, H. (1985) A Spatial Nash Equilibrium Model. In Harker, P.T. (ed.) Spatial Price Equilibrium: Advances in Theory, Computation and Application. Heidelberg: Springer-Verlag Berlin.

Heckscher, E.F. (1918) Svenska Produktionsproblem. (In Swedish). Stockholm: Bonniers förlag.

Helpman, E. (1976) Solutions of General Equilibrium Problems for a Trading World. Econometrica 44, pp. 547-559.

Helpman, E. and P.R. Krugman (1985) Market Structure and Foreign Trade. Cambridge, Massachusetts: The MIT Press.

Hewings, G.J.D. and R.C. Jensen (1986) Regional, Interregional and Multiregional Input-Output Analysis. In Nijkamp, P. (ed.) **Handbook of Regional and Urban Economics**, vol I. Amsterdam: North-Holland.

Hickman, B.G. and L.J. Lau (1973) Elasticities of substitution and export demands in a world trade model. **European Economic Review** 4, pp. 347-380.

Higgs P.J., B.R. Parmenter and R.J. Rimmer (1988) A Hybrid Top-Down, Bottom-Up Regional Computable General Equilibrium Model. **International Regional Science Review** 11, pp. 317-328.

Hildenbrand, W. (1981) Short-run Production Functions Based on Microdata. **Econometrica** 49, pp. 1095-1125.

Hirsh, M. and Smale, S. (1974). **Differential equations, dynamic systems, and linear algebra**. London: Academic Press.

Hitchcook, F.L. (1941) Distribution of a Product from Several Sources to Numerous Localities. **Journal of Mathematics and Physics** 21, pp. 224-230.

Hjalmarsson, L. and S.-G. Eriksson (1986) Choice of Technology and Energy Demand in a Vintage Framework. **University of Gothenburg:** Department of Economics. Memorandum 98.

Hotelling, H. (1929) Stability in Competition. **Economic Journal** 39, pp. 41-57.

Hotelling, H. (1932) Edgeworth's Taxation Paradox and the Nature of Demand and supply Functions. **The Journal of Political Economy** 40, pp. 577-616.

Houthakker, H.S. (1955) The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis. Review of Economic Studies 23, pp. 27-31.

Intriligator, M.D. (1981) Mathematical Programming with Applications to Economics. In Arrow, K.J and Intriligator, M.D.: **Handbook of Mathematical Economics I**. Amsterdam: North-Holland.

Isard, W. (1951) Interregional and Regional Input-Output Analysis: A Model of a Space-Economy. **The Review of Economics and Statistics** 33, pp. 318-328.

Isard, W. (1956) **Location and Space-Economy**. Cambridge, Mass.: The MIT Press.

Isard, W. and W. Dean. (1987) The Projection of World (Multiregional) Trade Matrices. **Environment and Planning A** 19, pp. 1059-1066.

Isard, W. and D.J. Ostroff (1958) Existence of a Competitive Interregional Equilibrium. **Papers And Proceedings of the Regional Science Association** 4, pp. 49-76.

Italianer, A. (1986) Theory and Practice of International Trade Linkage Models. **Groningen University. Ph.D. thesis.**

Iwai, K. (1981) **Disequilibrium Dynamics - A Theoretical Analysis of Inflation and Unemployment.** Cowles Foundation, Monograph 27, Yale University Press.

Jacobs, J. (1984) **Cities and the Wealth of Nations.** New York: Random House.

Johansen, L. (1960) **A Multisectoral Study of Economic Growth,** Amsterdam: North-Holland.

Johansen, L. (1972) **Production Functions.** Amsterdam: North-Holland.

Johansen, L. (1987, 1965) Regional Economic Problems Elucidated by Linear Programming. In Førsund, F.R.: **Collected Works of Leif Johansen I.** Amsterdam: North-Holland.

Johansson, B. (1986) A Structural Change Model for Regional Allocation of Investments. **TIMS Studies in the Management Sciences** 21, pp. 143-158.

Johansson, B. and D. Batten. (1983) Price Adjustment and Multiregional Rigidities in the Analysis of World Trade. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA).** Working paper WP-83-110.

Johansson, B. and I. Holmberg (1982) A Regional Study of the distribution of vintages and Profits of Industrial Establishments: A Stochastic Transition Model. In Albegov et al. **Regional Development modeling: Theory and Practice.** Amsterdam: North-Holland.

Johansson, B. and B. Marksjö (1984) An Interactive System for Regional Analysis of Industrial Sectors. Laxenburg, Austria: **International institute for Applied Systems Analysis (IIASA).** Working paper WP-83-54.

Johansson, B and H. Persson (1983) Dynamics of Capital Formation, Capacity Constraints and Trade Patterns in a Multisectoral Model. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA).** Working paper WP-83-3.

Johansson, B. and H. Persson (1987) **ISMOD - systemets flersektormodeller av den svenska ekonomin.** (In Swedish) The Swedish National Industry Board (SIND) PM 1987:3.

Johansson, B. and F. Snickars (1988) Modelling the Economic Dynamics of a Knowledge-Intensive Metropolis. **Sistemi Urbani**, pp 63-91.

Johansson, B. and U. Strömqvist (1980) **Vinster och sysselsättning i svensk industri**. (In Swedish) The Swedish National Industry Board (SIND) 1980:2. Stockholm: Liber förlag.

Johansson, B. and U. Strömqvist (1981) Regional Rigidities in the Process of Economic Structural Development. **Regional Science and Urban Economics** 11, pp. 363-375.

Johansson, B. and L. Westin (1987) Technical Change, Location and Trade. **Papers of the Regional Science Association** 62, pp. 13-25.

Jonung, L. (1987) Stockholmsskolan - vart tog den vägen. (In Swedish) **Ekonomisk Debatt**, no 4.

Karamardian, S. (1971) Generalized Complementarity Problem. **Journal of Optimization Theory and Applications** 8, pp. 161-168.

Karlqvist, A. (1987) Matematiska modeller och dynamiska system. (In Swedish) **Umeå University, CERUM** report 1987:1.

Karlqvist, A., R. Sharpe, D.F. Batten and J.F. Brotchie (1978) A Regional Planning Model and it's Application to South Eastern Australia. **Regional Science and Urban Economics** 8, pp. 57-86.

Karlqvist A. and U. Strömqvist (1982) **Var kommer krisen?** (In Swedish) The Swedish National Industry Board. (SIND) 1982:3. Stockholm: Liber förlag.

Kehoe, T.J. (1985) Multiplicity of Equilibria and Comparative Statics. **The Quarterly Journal of Economics**, February, pp. 119-147.

Kennedy, M. (1974) An Economic Model of the World Oil Market. **Bell Journal of Economics and Management Science** 5, pp. 540-577.

Kimbell L.J. and G.W. Harrison (1984) General Equilibrium Analysis of Regional Fiscal Incidence. In Scarf, H. and J.B. Shoven. **Applied General Equilibrium Analysis**. New York: Cambridge University Press.

Kimbell L.J. and G.W. Harrison (1986) On the Solution of General Equilibrium Models. **Economic Modelling** 3, pp. 197-212.

Kinderlerer, D. and G. Stampia (1980) **An Introduction to Variational Inequalities and Their Applications**. New York: Academic Press.

Kjellman, S. (1988) International Trade in Steam-Coal. **University of Stockholm.** Department of Economics. Ph.D. thesis.

Koopmans, T.C. (1949) Optimal utilization of the Transport System. **Econometrica** 17 suppl., pp. 136-146.

Kuenne, R.E. (1963) **The Theory of General Economic Equilibrium.** Princeton: Princeton University Press.

Lefeber, L. (1958a) General Equilibrium Analysis of Production, Transportation, and the Choice of Industrial Location. **Papers and Proceedings of the Regional Science Association** 4, pp. 77-86.

Lefeber, L. (1958b) **Allocation in space.** Production, Transport and Industrial Location. Amsterdam: North-Holland.

Lemke, C.E. (1965) Bimatrix Equilibrium Points and Mathematical Programming. **Management Science** 11, pp. 681-689.

Lemke, C.E. (1980) A Survey of Complementarity Theory. In Cottle et al.: **Variational Inequalities and Complementarity Problems.** Chichester, New York, Brisbane, Toronto: John Wiley & Sons.

Lerman, S.R. (1982) Recent Advances in Disaggregate Demand Modelling. Mimeo.

Leontief, W. (1960, 1951) **The Structure of the American Economy: 1919-1929.** New York: Oxford University Press.

Leontief, W. (1970) The Dynamic Inverse. In Carter, A. and A. Brody: **Contributions to Input-Output Analysis** 1. Amsterdam: North-Holland.

Liew, L.H. (1984) A Johansen Model for Regional Analyses. **Regional Science and Urban Economics** 14, pp. 129-146. Amsterdam: North-Holland.

Liew C.K. and C.J. Liew (1984) Multi-Modal, Multi-Output, Multi-Regional Variable Input-Output Model. **Regional Science and Urban Economics** 14, pp. 265-281.

Liew C.K. and C.J. Liew (1985) Measuring the Development Impact of a Transportation System: A Simplified Approach. **Journal of Regional Science** 25, pp. 241-258.

Luenberger, D. (1979) **Introduction to Dynamic Systems.** New York: John Wiley & Sons.

Lundberg, E. (1950) Lönepolitik under full sysselsättning, **Ekonomisk Tidskrift**. (In Swedish)

Lundgren, S. (1985) **Model integration and the Economics of Nuclear Power**. Stockholm School of Economics: The Economic Research Institute. Ph.D. thesis.

Lundqvist, L. (1980) A Dynamic Multiregional Input-Output Model for Analyzing Regional Development, Employment and Energy Use. Stockholm: **Royal Institute of Technology**. TRITA-MAT 1980:2.

Lösch, A. (1964, 1940) **The Economics of Location**. New Haven: Yale University Press.

Madden, J.R. (1987) The structure of the Tasmain Model. University of Melbourne: IAESR. Working paper.

Manne, A.S., S. Kim and T.F. Wilson (1980) A three-region intertemporal model of energy, international trade and capital flows. **Stanford University, Calif.**, Department of Operations Research. Working paper.

Manne, A.S. and P.V. Preckel (1985) A three-region intertemporal model of energy, international trade and capital flows. **Mathematical Programming Study** 23, pp. 56-74.

Marshall, A. (1961, 1920) **Principles of Economics**. London: Macmillan.

Mathiesen, L. (1977) Marginal Cost Pricing in a Linear Programming Model: A case with Constraints on Dual Variables. **Scandinavian Journal of Economics** 79, pp. 468-477.

Mathiesen, L. (1985a) Computation of Economic Equilibria by a Sequence of Linear Complementarity Problems. **Mathematical Programming Study** 23, pp. 144-162.

Mathiesen, L. (1985b) Computational Experience in Solving Equilibrium Models by a Sequence of Linear Complementarity Problems. **Operations Research** 33, pp. 1225-1250.

Mathiesen, L. (1987) An Algorithm Based on a Sequence of Linear Complementarity Problems Applied to a Walrasian Equilibrium Model: An Example. **Mathematical Programming** 37, pp. 1-18.

MacFadden, D. (1978) Modelling the Choice of Residential Location. In Karlqvist, A. et al. (eds.) **Spatial Interaction Theory and Planning Models**. Amsterdam: North-Holland.

Miller, R.E. and P.D. Blair (1985) **Input-Output Analysis. Foundations and Extensions.** New Jersey: Prentice-Hall.

Medio, A. (1987) A multisector model of the Trade Cycle. In: Batten, D. et al. (eds.). **Economic Evolution and Structural Adjustment.** Berlin: Springer-Verlag.

Moses, L.N. (1955) The Stability of Interregional Trade Patterns and Input-Output Analysis. **American Economic Review** 45, pp. 803-832.

Moses, L.N. (1960) A General Equilibrium Model of Production, Interregional Trade, and Location of Industry. **The Review of Economics and Statistics** 42, pp. 373-397.

Nagurney, A. (1987) Competitive Equilibrium Problems, Variational Inequalities and Regional Science. **Journal of Regional Science** 27, pp. 503-517.

Nagurney, A. and Aronson, J. (1988) A General Dynamic Spatial Price Equilibrium Model: Formulation, Solution and Computational Results. **Journal of Computational and Applied Mathematics** 22, pp. 339-357.

Nagurney, A. and Zhao, L. (1988) Diequilibrium and Variational Inequalities. **University of Massachusetts**: Department of General Business and Finance. Working paper.

Negishi, T. (1960) Welfare Economics and Existence of an Equilibrium for a Competitive Economy. **Metroeconomica** 5, pp. 22-30.

Negishi, T. (1962) The Stability of a Competitive Economy: A Survey Article. **Econometrica** 30, pp. 635-669.

Nelson, J.P. (1971) An interregional Recursive Programs model of Production, Investment, and Technological Change. **Journal of Regional Science** 11, pp. 33-47.

Nelson, R. and V. Norman (1977) Technological Change and Factor Mix Over the Product Cycle: A Model of Dynamic Comparative Advantage. **Journal of Development Economics** 4, pp. 3-24.

Nelson, R. and S.G. Winter (1982) **An Evolutionary Theory of Economic Change.** Cambridge Massachusetts: Harvard University Press.

Nickel, S.J. (1978) **The Investment Decisions of Firms.** Oxford: Cambridge University Press.

Nijkamp, P. (ed.) (1986) **Technological Change, Employment and Spatial Dyna-mics**. Berlin, Heidelberg: Springer Verlag.

Norén, R. (1987) Comparative Advantages Revealed. **University of Stockholm.** Department of Economics. Ph.D. thesis.

Official Statistics of Sweden (1980) **The Family Expenditure Survey 1978.** Stockholm: Statistics Sweden.

Ohlin, B. (1933) **Interregional and International Trade.** Cambridge: Harvard University Press.

Ohlin, B. (1937) Some Notes on the Stockholm Theory of Saving and Investment I-II. **Economic Journal** 47, pp. 53-69, 221-240.

Ohlsson, H. (1988) Cost-Benefit Analysis of Labour Market Programs. **University of Umeå**: Umeå Economic studies, no 182. Ph.D. thesis.

Ohlsson, O. and A. Granholm (1973) Regionalekonomiska modeller med tillämp-ning på Stockholmsregionen. (In Swedish) **Stockholms Generalplanebered-ning.**

Oosterhaven, J. (1984) A Family of Square and Rectangular Interregional Input-Output Tables and Models. **Regional Science and Urban Economics** 14, pp 565-582.

Ortega J.M. and W.C. Rheinboldt (1970) **Iterative Solution of Nonlinear Equations in Several Variables.** New York: Academic Press.

Palander, T. (1935) **Beiträge zur Stansdortstheorie.** Uppsala: Almqvist & Wiksell.

Palander T. (1941) Om "Stockholms skolans" begrepp och arbetsmetoder. Metodo-logiska reflektioner kring Myrdals "Monetary Equilibrium". **Ekonomisk tid-skrift** 43, pp. 88-143.

Pang, J.S. (1984) Solution of the General Multicommodity Spatial Equilibrium Problem by Variational and Complementarity Methods. **Journal of Regional Science** 24, no 3, pp. 403-414.

Pang, J.S. and D. Chan (1982) Iterative Methods for Variational and Complementa-rity Problems. **Mathematical Programming** 24, pp. 284-313.

Perroux, F. (1964) La notion de pôle de croissance. **L'économie du XX'eme siècle.** Paris: Presses Universitaires de France.

Persson, H. (1983) Theory and applications of Multisectoral Growth Models. University of Gothenburg. Department of Economics. Ph.D. thesis.

Persson, H. and B. Johansson (1982) A Dynamic Multisector Model with Endogenous Formation of Capacities and Equilibrium Prices. An Application to the Swedish Economy. Laxenburg, Austria: International Institute for Applied Systems Analysis (IIASA) Professional Paper PP-82-8.

Persson, H. and L. Westin (1985) En kortsiktig flersektoriell tillväxtmodell. (In Swedish) Umeå University. Department of Economics. Umeå Economic Studies 147.

Pettersson, J. (1987) Erik Lindahl och Stockholms skolans dynamiska metod. (In Swedish.) Lund University: Lund Economic Studies. Ph.D. thesis.

Phelps, E.S. (1963) Substitution, Fixed Proportions, Growth and Distribution. International Economic Review 4, pp. 265-288.

Ponsard, C. (1983) History of Spatial Economic Theory. Heidelberg: Springer-Verlag.

Poston, T. and I. Steward (1978) Catastrophe theory and its Applications. New York: Pitman.

Puu, T. (1987) A Two-Region nonlinear Multiplier-Accelerator model. Umeå University: Working Paper From CERUM 1987:24.

Puu T. and S. Wibe (eds.) (1980) The Economics of Technological Progress. London: The Macmillan Press Ltd.

Rehn, G. (1948) Ekonomisk politik vid full sysselsättning. Tiden (In Swedish.)

Ricardo, D. (1962, 1821) The Principles of Political Economy and Taxation. London: J. M. Dent & Sons LTD.

Rosen, R. (1972) Some Relational Cell Models: The Metabolism - Repair Systems. In Rosen, R. (ed.): Foundations of Mathematical Biology, vol. 2. New York: Academic Press.

Rosen, R. (1975) Biological Systems as Paradigms for Adaption. In Day, R. H. and T. Groves. Adaptive Economic Models. New York, San Francisco, London: Academic Press.

Salter, W.E.G. (1960) Productivity and Technical Change. Cambridge: Cambridge University Press.

Samuelson, P.A. (1947) **Foundations of Economic Analysis**. Cambridge, Massachusetts: Harvard University Press.

Samuelson, P.A. (1952) Spatial Price Equilibrium and Linear Programming. **American Economic Review** 42, pp. 283-303.

Samuelson, P.A. (1983) Thünen at Two Hundred. **Journal of Economic Literature** 21, pp. 1468-1488.

Scarf, H. (With T. Hansen) (1973) **The Computation of Economic Equilibrium**. New Haven, Conn.: Yale University Press.

Scarf, H. och J.B. Shoven (1984) **Applied General Equilibrium Analysis**. New York: Cambridge University Press.

Schumpeter, J.A. (1934, 1912) **Theory of Economic Development**. New York: Oxford University Press.

Seierstad, A. (1985) Properties of Production and Profit Functions Arising From the Aggregation of a Capacity Distribution of Micro Units. In Førsund et al.: **Production, Multi-Sectoral Growth and Planning**. Amsterdam, New York: Elsevier Science Publ.

Sheppard, E. and L. Curry (1982) Spatial Price Equilibria. **Geographical Analysis** 14, pp. 279-304.

Shoven, J.B. and J. Whalley. (1974) On the Computation of Competitive Equilibrium on International Markets With Tariffs. **Journal of International Economics** 4, pp. 341-354.

Shoven, J.B. and J. Whalley (1984) Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey. **Journal of Economic Literature** 22, pp. 1007-1051.

Silverberg, G. (1987) Modelling Economic Dynamics and Technical change: Mathematical Approaches to Self-Organisation and Evolution. To appear in Dosi, G. et al.: **Technical Change and Economic Theory**.

Snickars, F. and A. Granholm (1981) A Multiregional Planning and Forecasting Model with Special Regard to the Public Sector. **Regional Science and Urban Economics** 11, pp. 377-404.

Snickars, F., B. Johansson and T.R. Lakshmanan. (1985) **Economic Faces of the Building Sector**. D20:1985. Stockholm: Swedish Council for Building Research.

Snickars, F. and L. Lundqvist (1978) Investments and Transport in Interdependenent Regions - a Dynamic Regional Model. In Buhr, W. and P. Fredrich (eds.). **Competition among Small Regions.** Baden-Baden: Nomos.

Solow, R.M. (1962) Substitution and Fixed Proportions in the Theory of Capital. **Review of Economic Studies** 29, pp. 207-218.

Srinivasan T.N. and J. Whalley (eds.) (1986) **General Equilibrium Trade Policy Modelling.** Cambridge, Mass.: The MIT Press.

Strömqvist U. (1983) **Lönsamhetsstruktur och investeringsmönster.** (In Swedish) The Swedish National Industry Administration SIND 1983:1. Stockholm: Liber förlag.

Steedman, I. (ed.). (1979) **Fundamental Issues in Trade Theory.** New York: St. Martin's Press.

Stoneman, P. (1983) **The Economic Analysis of Technological Change.** Oxford: Oxford University Press.

Stålhammar, N.-O. (1985) En analys av sambandet mellan lönsamhet, produktivitet, och strukturomvandling inom den svenska tillverkningsindustrin. (In Swedish) Stockholm: **Trade Union Institute for Economic Research** (FIEF). Research report 3.

Suknam, K. and G.J.D. Hewings (1987) A Regional Computable General Equilibrium Model for Korea. **Korean Journal of Regional Science 2.**

Svennilson, I. (1938) **Ekonomisk planering. Teoretiska studier.** (In Swedish.) Uppsala: Almqvist & Wicksell. Ph.D. Thesis.

Svennilson, I. (1945) Industriarbetets växande avkastning i belysning av svenska erfarenheter. In: **Studier i ekonomi och historia tillägnade Eli F. Heckscher.** Uppsala: Almqvist & Wicksell. (In Swedish)

Takayama, T. (1978) An application of Spatial and Temporal Price Equilibrium Model to World Energy Modeling. **Papers of the Regional Science Association** 42, pp. 43-58.

Takayama, T. and G.G. Judge (1964) Equilibrium among Spatially Separated Markets: A Reformulation. **Econometrica 32,** pp. 510-524.

Takayama, T. and G.G. Judge (1971) **Spatial and Temporal Price and Allocation Models.** Amsterdam: North-Holland.

Takayama, T. and W.C. Labys (1986) Spatial Equilibrium Analysis. In Nijkamp, P. (ed.). **Handbook of Regional and Urban Economics**, vol I. Amsterdam: North-Holland.

Takayama, T., H. Hasimoto, and N.D. Uri (1984) Spatial and Temporal Price and Allocation Modeling: Some Extensions. **Socio-Economic Planning Sciences** 18, pp. 227-234.

Takayama, T. and N. Uri (1983) A Note on Spatial and Temporal Price and Allocation Modelling. **Regional Science and Urban Economics** 13, pp. 455-470.

Takayama, T and A.D. Woodland (1970) Equivalence of Price and Quantity Formulations of Spatial Equilibrium: Purified Duality in Quadratic and Concave Programming. **Econometrica** 38, pp. 889-906.

Talman, D. and G. van der Laan (eds.) (1987) **The Computation and Modelling of Economic Equilibria**. Amsterdam: Elsevier Science Publ.

Theil, H. (1980) **The System-Wide Approach to Microeconomics**. Oxford: Basil Blackwell.

Theil, H. and K.W. Clements (1987) **Applied Demand Analysis**. Cambridge, Mass.: Ballinger Publishing Company.

Thore, S. (1982a) The Takayama-Judge Spatial Equilibrium Model With Endogenous Income. **Regional Science and Urban Economics** 12, pp. 351-364.

Thore, S. (1982b) The Takayama-Judge Model of spatial Equilibrium Extended to Convex Production Sets. **Journal of Regional Science** 22, pp. 203-212.

Thore, S. (1986) Spatial Disequilibrium. **Journal of Regional Science** 26, pp. 661-675.

Thünen, J.H. von (1966, 1826) **von Thünen's Isolated State. An English Edition of Der Isolierte Staat**. Oxford: Pergamon Press.

Transportrådet (1983) **Transporter i Sverige**, del IIA. (In Swedish) Solna: Transportrådet Rapport 1983:3.

Uri, N.D. (1978) The Effectiveness of Regulation in the Electrical Energy Industry. **Regional Science and Urban Economics** 8, pp. 87-103.

Varian, H.R. (1984) **Microeconomics**. New York: Norton.

Walras, L. (1965, 1874) **Elements of Pure Economics**. Illinois: Richard Irwin.

Weber, A. (1962, 1909) **Theory of the Location of Industries.** Chicago: University of Chicago Press.

Westin, L. (1986) Norrbotten i det interregionala godstransportsystemet. Umeå Universitet: Arbetsrapport från CERUM 1986:7. (In Swedish.)

Westin, L. (1987) Industrial Change, Trade and Education in Norrbotten. In Wiberg, U. and Snickars, F. (eds.). **Structural Change in Peripheral and Rural Areas.** Stockholm: Swedish Council for Building Research, Document D12:1987.

Westin, L. (1988) A Short Run Inter-Regional Equilibrium Vintage Model. **Australian Journal of Regional Studies** 3, pp. 3-11.

Westin, L. (1989) Spatial dynamics and Equilibrium in Applied Economic Modelling. **Working paper from CERUM** 1989:4, Umeå Universitet.

Westin, L. (1990) Location, Transportation and Investments in a Multisector Network Model. In Anselin, L. and Madden, M. (eds.): **New Directions in Regional Analysis: Integrated and Multiregional Approaches.** Belhaven Press.

Westin, L., B. Johansson and M. Grassini (1982) Estimation of Capital matrices for Multisectoral Models: An application to Italy and Tuscany. Laxenburg, Austria: **International Institute for Applied Systems Analysis (IIASA)** Working paper WP-82-92.

Whalley, J. (1985) **Trade Liberalization among Major World Trading Areas.** Cambridge, Mass.: The MIT Press.

Whalley, J. and I. Trela (1986) **Regional Aspects of Confederation.** Toronto: University of Toronto Press.

Wibe, S. (1980) Teknik och aggregering i produktionsteorin. (In Swedish). Umeå University, Sweden. Ph.D. thesis.

Wibe, S. (1982) What is shown by a Macro Production Function? In Andersson, Å.E. et al. (eds.) A Nordic Workshop On Models For The Forest Sector. Umeå University, Swedish College of Forestry. Research Report 1982:1.

Wibe, S. (1985) Prisvariationer mellan företag - en empirisk studie. (In Swedish) Umeå University. Mimeo.

Williams, H.C.W.L. (1977) On the formation of travel demand models and economic evaluation measures of user benefit. **Environment and Planning A** 9, pp. 285-344.

Woodland, A.D. (1969) Competitive Equilibrium in International Trade. **University of New England, Australia.** Ph.D. thesis.

Ysander, B.-C. (1986a) ISAC - A Model of Stabilization and Structural Change in a Small Open Economy. In Ysander (ed.). **Two Models of an Open Economy.** Stockholm: Almqvist & Wiksell International.

Ysander, B.-C. (ed.) (1986b) **Two Models of an Open Economy.** Stockholm: Almqvist & Wiksell International.

Zhao, L. (1989) Variational Inequalities in General Equilibrium Analysis and Computation. **Brown University.** Ph.D. thesis.

UMEÅ ECONOMIC STUDIES
(Studier i nationalekonomi)

All the publications can be ordered from Department of Economics, University of Umeå, S-901 87 Umeå, Sweden.

Umeå Economic Studies was initiated in 1972. For a complete list, see Umeå Economic Studies No 221 or earlier.

* = out of print

180 Löfström, Åsa: Könsdiskriminering på arbetsmarknaden - en kvantitativ analys, 1988.

181 Andersson, Åke och Kobayashi, Kiyoshi: Some Theoretical Aspects of Spatial Equilibria with Public Goods, 1988.

182 Ohlsson, Henry: Cost-Benefit Analysis of Labor Market Programs - applied to a temporary program in northern Sweden, 1988. Diss.

183 Zhang, Wei-Bin: Limit Cycles in an Optimal Employment Policy Model, 1988.

184 Zhang, Wei-Bin: Oscillations of Population Migration in Developing Countries, 1988.

185 Karlsson, Charlie: Innovation Adoption and a Product Life Cycle, 1988. Diss.

186 Engström, Lars, Löfgren, Karl-Gustaf and Westerlund, Olle: Intensified Employment Services, Unemployment Durations, and Unemployment Risks, 1988.

187   Aronsson, Thomas: Progressive Income Taxation and the Supply of Round-wood, 1988.

188   Ohlsson, Henry: Beredskapsarbeten 1970-1987 - sysselsättning, anslag, utgifter och kostnader, 1988.

189   Löfgren, Karl-Gustaf: Buying and Selling Behavior in Stochastic Environments with Back-Stop Markets, 1988.

190   Zhang, Wei-Bin: Spatial and Temporal Urban Pattern: Interpretations of the FitzHugh-Nagumo Equations, 1989.

191   Zhang, Wei-Bin: Urban Structural Changes in Continuous Time and Space, 1989.

192   Löfgren, Karl-Gustaf, Ranneby, Bo and Sjöstedt, Sara: Forecasting the Business Cycle Using Min/Max Autocorrelation Factors, 1989.

193   Ohlsson, Henry: Job Creation Measures as Activist Fiscal Policy - an empirical analysis of policy reaction behavior, 1989.

194   Batten, David and Westin, Lars: Modelling Commodity Flows on Trade Networks: Retrospect and Prospect, 1989.

195   Bergman, Mats and Löfgren, Karl-Gustaf: Firm Behavior under Stochastic Input Supply: The Case of the Swedish Pulp and Paper Industry, 1989.

196   Löfström, Åsa: Diskriminering på svensk arbetsmarknad. En analys av löneskillnader mellan kvinnor och män, 1989. Diss.

197   Axelsson, Roger: Svensk arbetsmarknadsutbildning - en kvantitativ analys av dess effekter, 1989. Diss.

198   Zhang, Wei-Bin: Theory of Economic Development - Nonlinearity, Instability and Non-Equilibrium, 1989. Diss.

199   Aronsson, Thomas: Optimal Harvesting Behaviour and Nonlinear Income Taxation - An Application to Swedish Forestry, 1989.

200   Puu, Tönu: A Chaotic Model of the Business Cycle, 1989.

201   Puu, Tönu: On Growth and Dispersal of Populations, 1989.

202   Brännlund, Runar: Disequilibrium and Asymmetric Price Adjustment. The Case of the Swedish Timber Market, 1989.

203   Zhang, Wei-Bin: Conservation Laws in the Housing Market - A Further Investigation, 1989.

204   Zhang, Wei-Bin: Nonequilibrium in a Disequilibrium Keynesian System, 1989.

205   Hansson, Pär: Intra-Industry Trade: Measurements, Determinants and Growth - A Study of Swedish Foreign Trade, 1989. Diss.

206   Johansson, Börje: Vintage Analysis of Regional Economic Change: A Nordic Tradition, 1989.

207   Brännlund, Runar and Löfgren, Karl-Gustaf: A New Approach to Cyclical Dumping. An Analysis of the Canadian Forest Industry, 1989.

208   Kriström, Bengt: On the Calculation of Mean Willingness to Pay in Discrete Response Valuation Experiments, 1989.

209   Löfgren, Karl-Gustaf: The Measurement of Welfare Gains from Genetic Progress in Forestry: General Equilibrium and Disequilibrium Results, 1989.

210   Löfgren, Karl-Gustaf: The Use and Non-Use of Economics and Mathematics in Forest Economics. The Swedish Experience 1876-1976, 1989.

211   Engström, Lars, Löfgren, Karl-Gustaf and Westerlund, Olle: The Impacts of Intensified Employment Services on Unemployment Durations, and Unemployment Risks, 1989.

212   Jonsson, Bo och Larsson, Rolf: Utvärdering av Östra Norrbottenprojektet - Ett regionalt arbetsmarknadspolitiskt program, 1990.

213   Zhang, Wei-Bin: Planning with "National Pride" and Social Welfare - Investment Models with Heavy and Light Industries, 1990.

214   Andersson, Åke E. and Zhang, Wei-Bin: Quantity and Quality Control - A New Vision of Economic Growth, 1990.

215   Lundberg, Lars: Svensk industristruktur - basindustri eller hi-tech?, 1990.

216   Hedlund-Nyström, Torun, Jonung, Lars, Löfgren, Karl-Gustaf and Sandelin, Bo: An Unpublished Manuscript by Knut Wicksell: A Comment, 1990.

217   Bergman, Mats and Löfgren, Karl-Gustaf: Supply Risk Management under Imperfect Competition - Empirical Applications to the Swedish Pulp and Paper Industry, 1990.

218   Lundberg, Lars: Economic Integration, Inter- and Intra-Industry Trade: The Case of the EC and the Nordic Countries, 1990.

219   Kriström, Bengt: Valuing Environmental Benefits Using the Contingent Valuation Method: An Econometric Analysis, 1990. Diss.

220   Aronsson, Thomas: The Short-Run Supply of Roundwood under Nonlinear Income Taxation - Theory, Estimation Methods and Empirical Results Based on Swedish Data, 1990. Diss.

221   Löfgren, Karl-Gustaf: Another Reconciliation between Economists and Forestry Experts: OLG-Arguments, 1990.

222 Granberg, Björn and Ohlsson, Henry: Where in the Swedish Government Budget Can a Countercyclical Stabilization Be Found?, 1990.

223 Puu, Tönu: On Technological Progress in Cultural Production, 1990.

224 Hultkranz, Lars: Subsidies to Silvicultural Activities and Forest Road Construction in Northern Sweden: Effects on Forestry Employment and Seasonal Unemployment, 1990.

225 Batten, David: Intrastructure as a Network System: Mera Revisited, 1990.

226 Hultkrantz, Lars: The Role of Discount Rate, Amenity Values and a Negative Borrowing Rate on the Price of Swedish Forestland, 1965-1987, 1990.

227 Westin, Lars: Vintage Models of Spatial Structural Change, 1990. Diss.