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Abstract—A topology optimization technique is used for complete layout optimization of the radiating element of a planar monopole antenna. The design objective is to find a conductivity distribution that maximizes the energy received by the planar monopole over the frequency band 1-10 GHz. The finite difference time domain method (FDTD) is used for the numerical calculations and an adjoint problem is derived to calculate the corresponding sensitivities. Numerical results show a promising use of topology optimization techniques for the systematic design of ultrawideband monopoles.

Index Terms—topology optimization; ultrawideband (UWB); Maxwell equations; FDTD; adjoint equations

I. INTRODUCTION

Ultrawideband (UWB) antennas play a major role in the current revolution of wireless communication systems, microwave imaging, as well as non-destructive evaluation and testing. A key candidate for UWB antennas is the planar monopole, which is considered to be a variant of the cylindrical and conical monopole. The study of a disc-shaped planar monopole antenna by Honda [1] attracted attention to this kind of antennas and introduced it as a promising antenna for UWB systems. Further studies on other shapes of planar monopole antennas were conducted by Agrawall [2], who proposed an approximate formula to predict the frequency corresponding to the lower limit of the impedance bandwidth for these antennas. The design of this antenna type typically focuses either on the design of the radiating element, the feeding mechanism, or the finite-size ground plane. Classical design methods start with an existing geometry and introduce modifications of certain aspects of the design, for instance a trimming of the edge near the ground plane (beveling) or additions of shorting posts and cutting slots. One approach to design antennas in a systematic way is to use evolutionary algorithms. In the literature, many antenna types, including the UWB monopoles [3], have been extensively studied using evolutionary algorithms. However, evolutionary algorithms adopt random strategies to update the design variables. Since each suggested design requires a separate evaluation of the objective function, which typically includes a solution of Maxwell’s equations, evolutionary algorithms are expected to be too costly to handle optimization problems with a large number of design variables.

In this work, we attempt a more systematic approach that is better suited for a large number of design parameters. The material distribution approach to topology optimization, a technique originally developed to design load-carrying elastic structures [4], will be used for a complete layout optimization of the radiating element of UWB monopole antenna.

II. PROBLEM STATEMENT

As a reference, towards which the optimized designs can be compared, we use a standard square planar monopole. The geometry of the reference monopole, residing on an infinite simulated ground plane, is shown in Fig. 1. A 75 × 75 mm² radiating element is located a distance $h = 0.75$ mm above the ground plane and is fed by a 50 Ω coaxial cable at the middle of its bottom side. The reflection coefficient, $|S_{11}|$, of the reference monopole is calculated by the FDTD method and shown in Fig. 1 for the frequency band 0.5 to 10 GHz. The reference monopole radiates well at three narrow frequency bands centered around 1, 5, and 9 GHz, respectively. However, in the remaining bands, it reflects most of the power back to the coaxial cable.

Topology optimization techniques have been applied for the design of magnetic devices [5] and dielectric resonator antennas (DRA) [6]. However, to the best of our knowledge, the use of topology optimization methods for the design of metallic antennas has only been reported by Erentok and Sigmund [7], who use the finite element method and optimize the design for a single frequency. Here, in contrast, we apply the technique in time domain in order to facilitate a broadband design objectives.
Fig. 2. Illustration of the system energy balance.

When using the material distribution approach to topology optimization, the design domain is divided into small elements, and a design variable is assigned to each element. During the optimization process, the design variables change according to updates computed by a gradient-based optimization algorithm. In this study, the design variables determine the conductivity distribution, \( \sigma(x) \), over a design domain \( \Omega \), where \( x \) denotes a position in the design domain \( \Omega \). More precisely, in the numerical experiments, the conductivity at each Yee cell edge in the region where the radiating element can be situated is a design variable. The UWB monopole will be designed based on its receiving mode, using the reciprocity theorem, instead of its transmitting mode, and the objective function, \( W_{\text{coax}}(\sigma) \), is the energy received by the monopole and transmitted to the coaxial cable. The system energy balance is

\[
W_s = W_{\text{coax}} + W_{\text{loss}} + W_{\text{ref}},
\]

where \( W_s \), \( W_{\text{coax}} \), \( W_{\text{loss}} \), and \( W_{\text{ref}} \), as illustrated in Fig. 2, are the energy supplied from excitation sources, the energy received at the coaxial feed, the ohmic losses caused by intermediate values of \( \sigma \), and the energy reflected by the monopole, respectively. From energy balance (1), we see that maximizing \( W_{\text{coax}} \) for a fixed set of excitation sources is equivalent to minimizing the ohmic losses and the reflected signal. So, the optimization problem is conceptually

\[
\text{maximize } \sum_{\sigma(x) \in \{0, \sigma_{\text{max}}\}} W_{\text{coax}}(\sigma) \quad \text{subject to } \quad \Omega \subset \Omega_{\infty},
\]

The intermediate values of the design conductivities introduce losses. Since there are no ohmic losses for \( \sigma = 0 \) or \( \sigma = \infty \), any element with an intermediate value of \( \sigma \) will quickly be driven by any gradient-based optimization algorithm to either 0 or \( \sigma_{\text{max}} \). This property is in a sense desirable, since in the end we want a lossless antenna. However, for an element in the design domain to change from conductor to air, or vice versa, it needs to pass through a “barrier” of ohmic losses. Thus, in the basic formulation of the optimization problem (2), any gradient-based algorithm will quickly be trapped in a local minimum of the objective function with bad performance. This problem does not occur for such gradient-free optimization algorithms, for instance a genetic algorithm, that directly can handle the binary problem, where \( \sigma(x) \in \{0, \sigma_{\text{max}}\} \).

However as discussed earlier, such algorithms are extremely computationally costly, particularly for design domains of large dimensionality. The classical topology optimization problems for elastic structures do not suffer from this problem of lossy intermediate designs, but a similar issue has been reported for topology optimization problems applied to viscous fluids [8].

To handle the problem of lossy intermediate designs in the framework of gradient-based optimization, we rely on filtering of the design variables, an approach that is classic in topology optimization, but for other reasons [4, §1.3]. The conductivity \( \sigma \) used in the Maxwell equations is obtained by \( \tilde{\rho} = K_{R} \ast p \), where \( \tilde{\rho} = K_{R} \ast p \) is a local averaging of the design variables \( p \) that are actually updated by the optimization algorithm. The kernel of the integral operator \( K_{R} \) has support in a disk of radius \( R \). The filter leads to a “blurring” of the design variables \( p \), which imposes a certain amount of ohmic losses. We start with an initial filter radius \( R_0 \) and successively reduce the filter radius by setting \( R_{n+1} = \gamma R_n \), where \( \gamma < 1 \), while performing a number of iterations of the optimization algorithm for each filter radius. We iterate until a selected convergence criteria based on the first-order necessary conditions is met—this typically takes about 10–20 iterations. The algorithm thus progresses through a succession of less and less lossy designs until, for small values of the filter radius, the radiating element will almost entirely consist of elements with \( \sigma \) being either 0 or \( \sigma_{\text{max}} \).

III. DISCRETIZATION

We consider Maxwell’s equations in a region of space that is source free but may have a medium with variable material properties

\[
\nabla \times \tilde{H} - \left( \mu \frac{\partial }{\partial t} + \sigma \right) \tilde{E} = 0, \quad (3a)
\]

\[
\nabla \times \tilde{E} + \mu \frac{\partial \tilde{H}}{\partial t} = 0, \quad (3b)
\]

where \( \mu, \epsilon, \) and \( \sigma \) are the medium’s permeability, permittivity and conductivity, respectively. In this study, we aim to find the conductivity distribution, \( \sigma(x) \), over a design domain \( \Omega \subset \Omega_{\infty} \), where \( \Omega_{\infty} \) represents the whole computational domain. The values of \( \mu \) and \( \epsilon \) are constant during the design for \( \sigma(x) \).

We use the standard FDTD method to discretize Maxwell’s equations. The free space radiation condition is simulated, where it is needed, using a uniaxial perfectly matched layer. The total-field scattered-field technique simulates a set of excitation sources that surround the antenna from four sides. The excitation sources radiate vertically polarized plane waves that are synchronized to reach the feeding point at the same time. To have a precise control of the frequency band of interest, we use a truncated sinc pulse as the feeding signal.

The discrete objective function is formulated as the energy received by the antenna and flowing in the direction indicated in Fig. 2,

\[
W_{\text{coax}}(\sigma) = \frac{1}{4Z_c} \sum_{n=0}^{N} (V_n^{n+1} - Z_c I^{n+\frac{1}{2}})^2 \quad (4)
\]
where \( V^{n+1} \) and \( I^{n+\frac{1}{2}} \) are the potential difference and the current in the coaxial cable calculated using the FDTD method, \( Z_c \) is the coaxial cable characteristic impedance, \( N \) is the total number of time steps used in simulation, and \( \sigma = [\sigma_1, \sigma_2, \cdots, \sigma_M] \) represent the discrete conductivities located at each Yee edge in the design domain. The potential difference \( V^{n+1} \) and the current \( I^{n+\frac{1}{2}} \) inside the coaxial cable, are implicit functions of the conductivities \( \sigma \).

We use the adjoint variable method (AVM) [9], [10] to derive the gradient of objective function given by equation (4) with respect to the conductivities \( \sigma \). The derivation is carried out in the fully discrete case, including the coaxial feed and the perfect matched layer. The derivation results in an adjoint problem which is an FDTD discretization of a version of Maxwell’s equations running backward in time. The derivation reveals that the excitation source for the adjoint equations will be located at the coaxial feed, instead of in the surrounding space. The source will be of the form

\[
\left( \tilde{V}^{n+\frac{1}{2}} - Z_c \tilde{I}^{n+\frac{1}{2}} \right) = \Delta^3 \left( V^{n+1} - Z_c I^{n+\frac{1}{2}} \right),
\]

where \( \tilde{V}^{n+\frac{1}{2}} \) and \( \tilde{I}^{n+\frac{1}{2}} \), are the potential difference and the current in the coaxial cable for the adjoint problem, respectively, and \( \Delta \) is the spatial discretization step used with the FDTD method. The final expression for the gradient is

\[
\frac{\partial W_{\text{coax}}}{\partial \sigma_i} = -\sum_{n=1}^{N} E_n^i \frac{\tilde{E}_{i-\frac{1}{2}}^{n-\frac{1}{2}} + \tilde{E}_{i+\frac{1}{2}}^{n+\frac{1}{2}}}{2}.
\]

where \( E_i \) and \( \tilde{E}_i \) are the discrete electric field of the forward and the adjoint problem, respectively, at the Yee edges of the design domain.

To evaluate the gradient, first the forward problem is solved and the electric field in the design domain as well as the outgoing signal through the coaxial cable are stored for all times. Then the adjoint equations are solved using the stored coaxial signal from the forward problem as excitation signal according to equation (5). Finally, the forward and the adjoint electric fields are used to calculate the gradient according to equation (6). Note that, only two FDTD simulations are required to find the gradient for any number of design variables.

Given the gradient of the objective function in equation (6), we use the Method of Moving Asymptotes (MMA) [11] to solve the following discretization of problem (2).

\[
\text{maximize} \quad W_{\text{coax}}(p) \\
\text{subject to} \quad \text{Maxwell’s equations}.
\]

where \( A = \{ p \in \mathbb{R}^M | \ p_i \in [0,1] \ \forall i \} \) is the set of admissible design variables.

The complete optimization process is illustrated in the flowchart shown in Fig. 3. The optimization process starts with an initial distribution of the design variables \( p_0 \) over the design domain \( \Omega \). Then the design variables are filtered and mapped to the physical conductivities. The forward FDTD solver is used to evaluate the objective function and store the electric field in the design domain. The same FDTD solver is used to solve the adjoint problem. Then the calculated forward and adjoint electric fields are used to evaluate the gradient given in expression (6). Next, chain rule is used and the gradient is filtered to get the correct gradient of the design variables, and the convergence criterion based on the first-order necessary condition is tested. If the convergence criterion is not satisfied, the optimization process continues to a new cycle where the MMA algorithm use the gradient and the objective function values to update the design variables. If the the convergence criterion is reached but the filter radius is still greater than the FDTD spatial step size \( \Delta \), the radius is updated, \( R_{n+1} = \gamma R_n \), and a new cycle starts. Finally, the optimization problem terminates if the convergence criterion is satisfied and the filter radius is smaller than \( \Delta \).

It is important for the FDTD solver to be fast, since it will be called many times during the optimization process, both for the evaluation of the objective function and the gradient. The FDTD method can benefit from using multi-core systems since all the spatial calculations are independent and can be parallelized. However, a major problem with the FDTD method is the low number of floating point operations per memory fetch, since at each time step all field values have to be updated. This property makes the graphics processing unit (GPU) a suitable multi-core architecture to implement the FDTD method, since GPUs have high memory bandwidth. We used the parallel computing platform CUDA (https://developer.nvidia.com/what-cuda) to implement our FDTD code. Double precision floating point arithmetic is used for all calculations. For the numerical
experiments we used a nVidia GeForce GTX 285 installed on a node with two AMD Opteron 8431 running at 2.4 GHz. The average calculation time required for solving the forward or the adjoint problems is less than one minute for the results presented in this work.

IV. UWB MONOPOLE DESIGN

We design a UWB planar monopole antenna in a design domain corresponding to the area of the reference square planar monopole 75 × 75 mm². We present two design examples that are optimized for use with 50 and 75 Ω coaxial cables, respectively. The design domain, where the radiating element can be situated, is discretized into 100 × 100 Yee cell faces, yielding a design domain of 20200 design variables (one conductivity component for each Yee edge). The algorithm aims to increase the received signal by the monopole over the frequency band 1–10 GHz, which is equivalent to reduce the reflection coefficient, |S₁₁|, over this band. The design starts from a uniform distribution of the design variables, P₀ = 0.7. The initial filter radius R₀ is 15 mm and the filter decrease variable γ = 0.75.

The progress of the filtered design variables ̄p over a number of optimization cycles for the design connected to the 50 Ω coaxial is shown in Fig. 4. As can be noticed, the early optimization cycles are characterized by thick layers of the intermediate design variables, and the antenna shape is essentially symmetric. By the end of the optimization process, the filtered design variables converge either to a good conductor or a good dielectric, which are represented by the red and the light blue colors, respectively. The non-symmetry is due to properties of the finite precision arithmetic that is, due to roundoff errors, the design will even after the first cycle be slightly asymmetric. Further, the non-linearity of the problem and the decrease of the filter radius may bias the design into becoming asymmetric, as can be shown at the end of the optimization process.

The final designs connected to the 50 and 75 Ω coaxial cables are shown in Fig. 5 and Fig. 6, respectively. The optimization algorithm required 132 and 149 cycles to converge to the final designs connected to the 50 and 75 Ω coaxial cables, respectively. The optimized designs use only around 50 × 45 mm² out of the available design area 75 × 75 mm². The performance of the final designs compared with the reference monopole are illustrated in Fig. 7. The frequency bands, where the |S₁₁| is blow −10 dB, are 1.2 – 8.5 GHz and 1.3 – 10 GHz for the designs optimized to be used with 50 and 75 Ω coaxial cables, respectively. Overall, the performance of the optimized monopoles are superior to the reference monopole and the reflection coefficient, |S₁₁|, of the optimized monopoles stays below −10 dB over a wide frequency band.

V. SUMMARYING REMARKS

We introduce the material distribution approach to topology optimization in the context of broadband metallic antenna design, using a large parameter space and Maxwell’s equations in time domain for performance evaluation. Since we rely on adjoint equations and gradient expressions derived in the fully discrete case, we obtain exact gradients, up to roundoff, of the discrete objective function. In our experience, this exactness property is valuable in order to ensure a robust behavior of the optimization algorithm. The perhaps most challenging feature of this design optimization problem is the “self-penalizing” nature of the optimization problem: gradient-based optimization algorithms will quickly drive any intermediate conductivity to one of the extreme values. We address this issue by a continuation approach, enforcing a certain amount of losses by a filtering technique, successively reducing the losses as the iterations proceed. Numerical examples shows the usefulness of this approach for the design of UWB planar monopole antennas. Currently, we are investigating the design of different types of metallic antenna structures using our proposed approach.
Fig. 6. The final design of the UWB monopole connected to a 75 Ω coaxial cable.

Fig. 7. $|S_{11}|$ of optimized monopoles compared with the reference square planar monopole.

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