A MODEL FOR BOUNDED TECHNOLOGICAL PROGRESS

by

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ABSTRACT

This paper presents a simple model capable of handling empirical cases where technology is bounded. The theoretical model is illustrated using empirical data from the Swedish Pulp Industry 1937-1978.

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INTRODUCTION

Technological progress is generally introduced into economic models by adding a time component to the static production function. By far the most common specification of this component is an exponential term which makes output grow at a fixed rate with a given amount of physical inputs.

Although the smooth exponential model for technological progress may be adequate for aggregate or long run analyses it becomes an oversimplification at the micro level such as, for example, when the development within a well defined process technology is studied. Various engineering restrictions enter at the micro level which open up the possibility of the saturation or exhaustion of a technology. This "saturation effect" is produced by the limits to progress which, in general case, exist at the micro level of technology. Two kinds of limits can easily be identified:

- (i) thermodynamic and chemical limits for the unit consumption of materials and energy;
- (ii) engineering limits given by the technical principle which underlies the process technology.

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It is usually relatively straightforward to calculate thermodynamic and chemical limits from engineering information. If, for example, we consider the smelting of iron ore, the richest ore has the chemical formula Fe_3O_4 and, we can ascertain from the atomic weights, that the minimum amount of ore required per ton produced iron must be about 1400 kg. This can be compared with the present practice level of about 1600 kg. In the same way it is possible to estimate that at least 1890 kg of Al_2O_3 are required to produce one ton of aluminium and about 1550 kg of material to produce one ton of cement. Also, the lower limit of energy input can be calculated, by noting that aluminium reacts with coal according to the formulae:

This list of examples can easily be expanded.¹

Engineering limits relate to a specific technology, defined by a set of engineering principles which form the foundation of the process. Nowadays, for example, most iron is produced in a shaft furnace, using coke as fuel. However it is also possible to use an electric furnace, a rolling furnace or a fluid bed with electricity, oil or gas as the energy source. Each technique involves a specific combination of engineering principles and its weaknesses and strengths are inherent in this combination.² For instance, a technique may be extremely demanding on the energy side and progress may be hard to achieve without altering the basic engineering solution. This is the case in the Blast furnace technology where the engineering limit for energy consumption is about 375 kg per ton iron whereas the theoretical minimum figure for the ideal engineering solution is around 225 kg.³

1 See for example Gyftopoulos et al. (1974).

2 For a discussion of these 'technical sets', see Wibe (1984), Ch. 4.

3 See Wild (1962).

The combined effect of both types of limits is that a specific technology may be exhausted; and that the amount of output which can be produced from a given set of physical inputs is restricted. This kind of limitation is taken account of in the model presented below.

A SIMPLE MODEL

There are many ways of modelling the path of progress when technology is bounded. The simplest is probably to expand the ordinary exponential formulation. Let us assume the existence of a production function:

$$q = f(v_1, \dots, v_n) \quad (2)$$

where q denotes quantity output and v_i quantities of inputs. Assuming progress to be Hicks-neutral⁴ (generalized to the case with n inputs) we can introduce technological progress by multiplying $f(v)$ with a factor $A(t)$ where t is time:

$$q = A(t) \cdot f(v_1, \dots, v_n) \quad (3)$$

$$(dA/dt > 0)$$

$A(t)$ is a numerical representation of the state of technological knowledge. If $A(t)$ has an exponential form, its time derivative can be written:

$$dA/dt = \alpha_0 \cdot A(t) \quad (4)$$

where the rate of technological change α_0 is a constant, $\alpha_0 > 0$.

⁴ See Gehrig (1980).

The choice of an exponential form for $A(t)$ could be based on the following argument: Growth of technological knowledge, dA/dt , is proportional to the state of knowledge $A(t)$ and the more this knowledge grows, the broader is the base from which new discoveries can be made and, hence, the flow of new knowledge per unit of time increases.

Suppose that technology is bounded and that $A(t)$ has an upper value C .⁵ The amount of undiscovered possibilities could then be measured with $C - A(t)$. If this factor is multiplied with (4), we obtain:

$$dA/dt = \alpha_0 \cdot A(t) \cdot (C - A(t)) \quad (5)$$

$$(\alpha_0 > 0)$$

Technological progress is now determined both by the state of knowledge, $A(t)$, and the amount of undiscovered possibilities, $C - A(t)$. The flow of new knowledge increases as a result of the greater knowledge base, but growth is hampered due to the exhaustion of possible discoveries. The resulting curve for dA/dt is bell-shaped. In the first phase, the growth of $A(t)$ dominates (5) and dA/dt increases. However, dA/dt starts to decline in the second phase when the effects of the decreasing term, $C - A(t)$, outweigh the effects of the increasing $A(t)$.

$A(t)$ now becomes the logistic curve familiar in the literature on process and product diffusion and in many other fields in the social and natural sciences:⁶

5 The problem of measuring C empirically is dealt with in the next section of this paper.

6 For a more information on applications of the logistic curve, see Ayres (1969) and Bright (1968).

$$A(t) = \frac{C}{1 + \alpha_1 \cdot e^{\alpha_2 t}} \quad (6)$$

(Where α_1 and α_2 are new constants with $\alpha_1 > 0$ and $\alpha_2 < 0$).

$A(t)$ grows from zero to C when time changes from minus to plus infinity. C is a numerical measure of total potential knowledge. The ratio $A(t)/C$ is a measure of the degree of technological saturation. From (6) we can calculate that $A(t)/C = 0.5$ when $t = -\log(\alpha_1)/\alpha_2$. At this point the technology has exhausted 50 % of its potential.

If we normalize so that $C = 1$ and combine (6) with the production function, we obtain:

$$q(t) = \frac{f(v_1, \dots, v_n)}{1 + \alpha_1 \cdot e^{\alpha_2 t}} \quad (7)$$

If we hold the amounts of inputs constant, production will increase over time as a result of technical progress. However as we have the following equation for the rate of technical progress:

$$(dq/dt)/q = \frac{-\alpha_1 \alpha_2 \cdot e^{\alpha_2 t}}{1 + \alpha_1 \cdot e^{\alpha_2 t}} = \frac{-\alpha_1 \alpha_2}{e^{\alpha_2 t} + \alpha_1} \quad (8)$$

we conclude that the rate of progress will fall constantly over time (if $\alpha_1 > 0$ and $\alpha_2 < 0$).⁷

7 The rate of progress increases if $\alpha_1 < 0$ and $\alpha_2 > 0$. Hence, the model (6) can be used to estimate the direction of the rate of progress in general. However, in this paper we only discuss the case when the rate of progress is declining.

Solving for dq/dt , we obtain:

$$dq/dt = \frac{-\alpha_1 \alpha_2 \cdot e^{\alpha_2 t} \cdot f(v_1, \dots, v_n)}{(1 + \alpha_1 \cdot e^{\alpha_2 t})^2} \quad (9)$$

which, when $f(\)$ is constant, yields a bell-shaped curve (dq/dt can be interpreted as the absolute gains from technological progress). (These are greatest when $A(t)/C = 0.5$, i.e. when $t = -\log(\alpha_1)/\alpha_2$).

AN EMPIRICAL ILLUSTRATION

In order to illustrate the model and to indicate the possible areas of application, we have conducted a simple empirical test using data from the Swedish pulp producing industry 1937-1978. We must however stress that our purpose is only to illustrate the model and not to provide empirical evidence of production relations in the pulp industry.⁸

The data base is presented in the appendix. We use four kinds of inputs: labour (L, measured using the number of hours worked by productive workers), raw material consumption (M, measured as wood input), energy (E, measured as a weighted sum of all different energy inputs; see appendix for weighted index), and capital (K, measured as the power of installed machinery). Q is the symbol for output and t for time. The units of measurement are described in the appendix.

Using a Cobb-Douglas specification of the production function, we analyzed two kinds of progress functions $A(t)$:

8 Our data base was not detailed enough for a reliable empirical analysis. For example, capital input was measured by the power of the installed machinery ('horsepowers'); and the effect of an increasing degree of integration between pulp- and paper-producing plants was not considered. However, work on the empirical application of our model to Swedish industrial sectors is in progress.

$$\begin{aligned}
 \text{(i)} \quad A(t) &= e^{\alpha_1 t} && \text{(Unbounded model)} \\
 \text{(ii)} \quad A(t) &= 1/(1 + \alpha_1 e^{\alpha_2 t}) && \text{(Bounded model)}
 \end{aligned}
 \tag{10}$$

The first specification is the ordinary exponential growth with constant rate of progress = α_1 ; and the second is our bounded model for technical advancement with a steadily declining rate of progress. The two functions were estimated using the logarithmic form:

$$\begin{aligned}
 \text{(i)} \quad \ln(Q) &= \ln\beta_0 + \alpha_1 t + \beta_1 \ln(K) + \beta_2 \ln(L) + \beta_3 \ln(M) + \beta_4 \ln(E) \\
 \text{(ii)} \quad \ln(Q) &= \ln\beta_0 - \ln(1 + \alpha_1 e^{\alpha_2 t}) + \beta_1 \ln(K) + \beta_2 \ln(L) + \beta_3 \ln(M) + \\
 &\quad + \beta_4 \ln(E)
 \end{aligned}
 \tag{11}$$

The first function was estimated using ordinary least-squares regression and the second using the maximum-likelihood method. The empirical results are presented in table 1 below:

Table 1 Estimated parameter values (Eqs. 11).
(t-ratios and asymptotic t-ratios below estimate).

	$\ln\beta_0$	α_1	α_2	β_1	β_2	β_3	β_4	R^2
Unbounded model	-1.503 (-1.03)	0.0189 (2.37)	-	-0.405 (-1.67)	0.299 (1.44)	0.792 (6.54)	0.061 (0.16)	0.994
Bounded model	1.596 (1.26)	19.35 (1.71)	-0.0209 (-2.83)	-0.402 (-2.10)	0.276 (2.40)	0.796 (8.53)	0.071 (0.28)	- *

* The curve fit measured with the corresponding R^2 was 0.992 and the M - L estimate of sigma-square was 0.00183.

Comparing the results in table 1, we find that the input elasticities with respect to output ($\beta_1 - \beta_4$) are very similar. The elasticity with respect to capital is negative in both cases, this is probably due to our very crude measurement of capital measure. With regard to the statistical measures, we find a slightly higher R^2 in the case with exponential progress, but generally higher t-ratios in the case with bounded progress. The general conclusion must be that neither model takes priority over the other judging from the statistical results alone.

The two models do, however, give radically different forecasts for the technical development within the industry. The exponential model gives an estimate of 0.0189 for the yearly rate of technical progress (rate of increase in output, given the amount of inputs), a rate which is constant throughout the period. Whereas the non-linear model assumes a declining rate of progress. The two models' predictions of the rate of progress are compared in figure 1.

Figure 1 The rate of technological progress in the Swedish Pulp Industry 1900-2100 according to model estimations (table 1).

According to the model with bounded, the rate of technological advancement is declining from about 2 % per year in 1900 to about 0.5 % per year by 2100.

The advancement of knowledge can also be illustrated using the progress functions (10). In our bounded model, the function $A(t) = 1/(1 + \alpha_1 e^{\alpha_2 t})$ can be used as a measure of the degree of technological saturation. (The concept of technological saturation is meaningless in the first model (10) since there is no limit to possible achievements in this model). If, at $t^x = t$, we know that $A(t^x) = 0.5$, then we also know that 50 % of the industry's technological potential is exhausted. By studying the time path for $A(t)$ we can, accordingly, obtain an estimate of the future technological possibilities of the industry. The function $A(t)$ is displayed in figure 2.

Figure 2 The Technological Saturation Level for the Swedish Pulp Industry according to the model's estimates (table 1).

Figure 2 shows that the industry (again according to the model) has utilized approximately 20 % of its "technical potential" by about 1980. This is contrary to a very wide spread belief that the industry has already entered the stage of technical maturity.

CONCLUSION

In this paper we have presented a simple model capable of handling cases when technological progress can be expected to be bounded. We have used this model on a data base consisting of figures from the Swedish Forest Industry 1937-1978. The results show that the statistical fit is as good as that obtained with with an unbounded, exponential, progress model.

The main advantage of our model is that it does not assume a constant rate of technological progress. By introducing the notion of a limit to possible progress, the model allows us to estimate whether or not a technology (or an industry) has reached a stage of technical maturity and exhausted most of its technical potential. It is, of course, not advisable to make such predictions from results based on our model alone, nevertheless estimations using this model can provide valuable information which can be of assistance to decision makers when formulating their views on a policy.

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APPENDIX

The Data Base.

C10 = Year

C2 = Production of pulp in Sweden (1000 tons, dry weight)

C3 = Labour input (1000 labour-hours)

C4 = Installed machinery (1000 horsepower)

C5 = Raw material consumption (wood, 1000 m³)

C6 = Oil consumption (1000 m³)

C7 = Electricity consumption (Gwh)

C8 = Consumption of fuel wood (1000 m³)

C9 = Coal + coke consumption (1000 tons)

C10	C2	C3	C4	C5
1937	3519	48908	441	17631
1940	1905	32687	469	8367
1943	1246	28980	426	5555
1946	2722	43227	519	11418
1950	3187	46979	556	13017
1953	3199	42110	638	13237
1956	4104	44830	778	16624
1959	4173	41467	877	17614
1963	5577	39165	1184	31277
1966	6386	34410	1349	36445
1969	7563	25132	1296	45037
1972	8200	23350	1548	51673
1975	8414	21140	1607	50383
1978	8484	17219	1516	52514

C10	C6	C7	C8	C9
1937	0	1366	2770	937
1940	0	1033	2500	456
1943	0	843	3900	230
1946	159	1523	3850	140
1950	249	1923	2000	373
1953	290	2266	1980	220
1956	540	2474	1740	81
1959	545	3100	1395	43
1963	630	3672	1815	35
1966	796	4120	1582	7
1969	882	3826	1600	0
1972	900	4368	1352	0
1975	653	4324	972	0
1978	510	4144	530	0

(Source: SOS Industri 1937-1978/Swedens Official Statistics: Industry).

Energy input is defined as $E = C7 + 11.627 * C6 + 1.581 * C8 + 9.302 * C9$. These weights are based on the relative prices of the different energy sources.