ENGINEERING PRODUCTION FUNCTIONS – a survey

by

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I INTRODUCTION

More than 30 years ago H B Chenery published his pioneering article "Engineering Production Functions" (Chenery, 1949). Its purpose was to demonstrate how engineering information could be used to improve the empirical studies of production and to provide a bridge over the gap between the theoretical and empirical analyses of production.

This paper reviews the post-Chenery literature on engineering production functions. It is a fairly simple task because of the small number of articles on this issue, for an engineering approach has not often been applied in empirical production analysis. The more difficult problem involved in this task is in getting hold of useful information from the studies that have been carried out. Several writers seem to have forgotten that Chenery's method was advanced as a way to improve the quantitative aspect of the economic analyses. The aim of this method was not - as it sometimes appears to be - to transform economists into engineers, but rather to teach economists to apply engineering to economic issues. In those cases where a writer, trying to adapt to Chenery's basic idea, has constructed (part of) an economic production function, its quantitative properties have rarely been elucidated. In trying to compile and consider the results concerning substitution and scale properties, I had to, for example, resort to measuring the elasticity of substitution exclusively from figures, tables and text which did not deal explicitly with these phenomena. It

is too bad that this is the case, since the results (for example, the fact that engineering studies show an elasticity of substitution between labour and capital varying around the value 1) seem to have some general interest.

If the main point of Chenery's idea has sometimes been forgotten, this is true to an even greater extent of the formal model. It is true that the majority of writers take account of Chenery's basic model, but no one has applied it in a concrete analysis. Chenery's model, which is still the only formal model of engineering studies presented so far, seems to be too narrow a frame for the richness variety of ideas offered by the concrete cases.

II THE EMPIRICAL RESULTS

The studies analyzed here are briefly described in Table 1. Since the degree of engineering in production studies varies continuously from 0 to 1, the delimitation of the area presents a problem. However, I think that Table 1 contains the most important engineering studies, and that the presentation is representative of the area. A large group of studies concerning agricultural problems has not been included, because I do not believe that the substitution elasticities between proteins and vitamins in the breeding of chickens can be of much interest to economists.

The terms applied in Table 1, (engineering, economic and statistical production function) ought to be explained. A complete analysis based on Chenery's ideas comprised in the following steps:

(i) Construction of the engineering production function.

Let us assume that the production quantity is designated q and the engineering variables (for example length, speed, viscosity etc) describing the process are designated $x_1 \dots x_n$. Then the first problem is to find the relation:

$$q = e(x_1 \dots x_n) \quad (1)$$

where $e(x)$ is the engineering production function.

(ii) Construction of the input functions

A certain set of engineering variables also determines the the extent to which the economic factors are used (labour, capital etc). If the latter are marked $v_1 \dots v_m$, the problem is thus to find the relations:

$$v_j = g_j(x_1 \dots x_n) \quad j=1 \dots m \quad (2)$$

where $g_j(x)$ is the input functions.

(iii) Construction of the economic production function

Knowledge of (i) and (ii) allows us to transform the production function from the technical to the economic space by solving the problem:

$$\begin{aligned} \text{Max } q &= e(x_1 \dots x_n) \\ \text{ST} & \\ v_j &= g_j(x_1 \dots x_n) \quad j=1 \dots m \end{aligned} \quad (3)$$

The solution $q = E(v)$ is the economic production function. Since normally only input-output points are obtained, it is sometimes fruitful to proceed further and make:

(iv) A statistical functional adaptation to discrete input-output data

This step requires no special comment.

Table 1 Engineering production functions 1935-1980.

No	Author	Year	Process studied	Result presented as*	Quantitative information on**		
					Substitution	Scale properties	Technical progress
1	R Frisch	1935	Chocolate production	E, EE	R-OC	Yes	No
2	H B Chenery	1953	Gas transmission	E, EE	C-OC	Yes	No
3	"	"	Evaporation	E	-	No	No
4	"	"	Electrolysis	E	-	No	No
5	"	"	Electrical transmission	E	-	No	No
6	A R Ferguson	1953	Air transportation	E	-	No	Yes
7	V L Smith	1957	Trucking industry	E, EE	-	No	Yes
8	"	1961	A number of studies on electrical and gas transmission, chemical reactions etc	E	-	No	No
9	A D Hurther	1962	Petroleum refining	E, EE	C-OC	Yes	No
10	G K Boon	1964	Producing grain	E, EE	C-L	No	No
11	"	"	Ploughing	E, EE	C-L	No	No
12	"	"	Earth-moving	E, EE	C-L	No	No
13	"	"	Metal turning	E, EE	C-L	Yes	No
14	"	"	Metal facing	E, EE	C-L	Yes	No
15	"	"	Wood working	E, EE	C-L	Yes	No
16	E D Heady, J D Dillon	1966	Corp production	EE	C-L	No	No
17	M Kurtz, A S Manne	1963	Metal machining	S	C-L	No	No
18	B W Brown	1966	Water pumping	E, EE	-	Yes	No
19	A E Belinfante	1969	Steam electric power generating	S	-	No	No
20	P W Mac Avoy	1969	Nuclear breeder reactors	E, EE	C-E	Yes	No
21	J S de Salvo	1969	Rail linehaul operation	E, EE	C-E	Yes	No
22	R J Anderson	1969	Asphalte production	E, S	See text	Yes	No
23	T G Cowing	1970	Steam electric generation	E	-	Yes	Yes
24	JR Marsden et al	1974	Chemical reactions	E	-	No	No
25	D I Pearl, J L Enos	1975	Oil transportation	E, EE	C-OC	Yes	Yes
26	T Lamyai	1978	Head lug production	E, EE, S	C-L	Yes	No
27	E Eide	1979	Production of oil tankers	E, EE, S	E-OC	Yes	Yes
28	S Wibe	1980	Iron production	E, EE, S	E-C	Yes	Yes

* E = Engineering relations
 EE = Economic production function
 S = Statistical estimation of economic production function

** R = Raw material, OC = Other costs (= sum costs of other inputs at fixed prices)
 C = Capital, L = Labor, E = Energy

Five out of the 28 studies (no 3-5, 8, 24) contain only engineering relations. The motive for these varies. Chenery (no 3-5) provides examples to illustrate his basic engineering model. Smith (no 8) uses engineering relations in order to demonstrate the empirical relevance of his stock-flow production model and to show that substitution between capital and variable inputs (mainly energy) are important in many important industrial processes. Marsden et al (no 24) want to demonstrate that the Cobb-Douglas and the CES-production function can be derived from the physical properties of some general biological and chemical reactions. It is not possible to obtain quantitative economic information from any of these studies.

Five other studies are incomplete from our point of view. Smith (no 6) uses engineering to derive a model for fuel consumption only. Brown (no 18) presents a lot of engineering relations, but his empirical engineering function is an extreme simplification and has only one input (energy). His statistical function is based on observable - and not engineering - data. Belinfante (no 19) uses engineering only as general information. His empirical results comes solely from ordinary statistical examination. Mac Avoy (no 20) does show substitution and scale variation, but only with three data points. Cowing (no 23), finally, uses engineering information only to derive a general form of the production function. His empirical results stem from a statistical examination (using these forms) in which observable data are used.

The remaining eighteen studies (no, 1, 2, 6, 9-17, 21, 22, 25-28) can be used to obtain direct engineering information on the properties of the production function. All provide data on different kinds of substitution, eleven provide data on scale properties (where more than one output level was studied) and four on technical progress. The most interesting aspect of these studies is of course engineering measures of substitution.

Measures of the elasticity of substitution

From the data points provided by tables, (no 10-17, 27-28) by graphs, (no 2, 9, 22, 25) or given implicitly in engineering relations (no 1, 21) it was possible to calculate the elasticity of substitution (ES). However, the isoquants were in most cases given by points which were connected by straight lines. Hence, the ordinary derivative measure of ES could not be calculated. In the case of two factors of production X_1 and X_2 we have

$$ES = \frac{d(\ln X_1/X_2)}{d(\ln dX_1/dX_2)}$$

For three successive points $(X_1^i; X_2^i)$, $(X_1^{i+1}; X_2^{i+1})$, $(X_1^{i+2}; X_2^{i+2})$ I then calculated ES as the log difference in factor proportions divided by the log difference in slope. Thus:

$$ES_{i,i+2} = \frac{\ln(X_1^i/X_2^i + X_1^{i+1}/X_2^{i+1}) - \ln(X_1^{i+1}/X_2^{i+1} + X_1^{i+2}/X_2^{i+2})}{\ln((X_1^i - X_1^{i+1})/(X_2^i - X_2^{i+1})) - \ln((X_1^{i+1} - X_1^{i+2})/(X_2^{i+1} - X_2^{i+2}))}$$

Of course, this elasticity varies depending on the chosen pair of points. In computing the tables below, I have started from the point on the isoquant with the lowest capital - (or energy-) figure, and calculated the ES to successive points. Thus, I present $ES_{1,3}$ $ES_{1,4}$ etc in the table below.

Table 2 The elasticity of substitution (σ) in engineering studies (see table 1 for abbreviations).

Study no	σ_{C-L}	Study no	σ_{C-OC}	Study no	$\sigma_{C-E}, \sigma_{E-OC}$
10	9.08, 1.81	2	0.20, 0.28, 0.33, 0.31	21 ^c (C-E)	0.44, 0.34
11	0.54, 3.36	9	0.67, 0.91, 0.70, 0.82, 0.79, 0.69		(0.33, 0.21)
12 ^a	0.29 (0.76)	25	0.25, 0.36, 0.32	27 ^d (E-OC)	0.38, 0.41, 0.61, 0.54
13	0.92			29 ^e (C-E)	0.19, 0.21, 0.06
14	1.37				
15	1.83				
16	0.83, 0.93, 1.02, 1.29				
17	0.91				
26 ^b	0.52				
26'	0.60				

Notes: a. Output levels 6500 units and () 18 500 units.

b. Lamyai considers two basic technologies for head lug production. I label these 26 and 26'.

c. Output levels 100 000 ton miles/hrs and () 50 000 ton miles/hrs.

d. Output level 2.0.

e. The same for all output levels.

Two figures could be added to table 2. Calculations from Frisch's study (no 1) show an ES between raw material and other costs of about 0.06. The ES for emissions - other costs resulting from Anderson's study was 2.07.

Several conclusions can be drawn from table 2. Looking first at the measure for capital-labor substitution, we find a great dispersion. Depending on the process and the chosen factor ratio, the elasticity can vary between 0.3 and 9. Considering this dispersion, an a priori

assumption of $\sigma = 1$ does not seem to be too bad. (A simple average excluding the two extremes yields $\sigma \approx 1.2$. However, counting the number of processes yields $\sigma < 1$ for 5 processes and $\sigma > 1$ for 3). The table indicates that even if $\sigma = 1$ is a reasonable assumption for aggregate studies, the elasticity on the micro level can vary greatly. Turning to the substitution between capital and other costs, we find the elasticity is significantly less than one. $\sigma \approx 0.25$ for studies (2) and (25) and $\sigma \approx 0.75$ for study no (9). It is interesting to note the similarities in studies no (2) and (25). Both study pipe line transportation and the basic engineering substitution is the same (larger pipes vs stronger pumping engines). This of course increases the credibility of the estimates. Turning to the energy substitution, in general we find measures well below unity. The very low value for raw material substitution in Frisch's early study (no 1) is not surprising, both because we are dealing with ex post conditions, and because we would not expect much variation in raw material input. The overall impression is thus that the ES between capital and labor varies greatly, but with $\sigma \approx 1$ as an approximate average, whereas ES between other factors is far below 1.

Measures of scale properties

Information on scale properties could be calculated in eleven studies. The elasticity of scale (ESC) was defined as the log difference in output divided by the log difference in input. ESC was measured along factor rays through the origin (FR) or along expansion lines (EL). Input was measured as the euclidean distance to the origin in the first case and as the total cost in the second. The results are given in the table below, where the question of whether or not the production function is homothetic (if the isoquants were radical projections of each other) is commented upon.

Table 3 The elasticity of scale (ESC) in engineering studies

Study No	Variation of output	ECS calculated along	Value of ESC	Variation with output	Homothetic production functions
1	1-4	FR	1.0	-	Yes
2	1-4	FR	1.13-1.40	Raising	Yes
9	1-8	FR	≈ 1	-	No
14	1-3	FR	1.0-1.70	-	No
15	1-45	FR	2.58	-	No
16	1-8	FR	1.08-1.97	-	No
21	1-65	EL	1.27-1.75	Falling	?
26	1-1000	EL	1.58	Raising	No
26'	1-1000	EL	2.14	Constant	No
27	1-4	FR	1.08-1.46	Falling	Yes
28	1-15	EL	1.12-0.81	Falling	No

As could be expected, ESC is > 1 in most cases. It is remarkable that only three out of eleven studies support the usual assumption of a homothetic production function. Thus a change of scale also implies a change of factor proportions (at given factor prices). Lamayai's study is the most detailed one on this point, demonstrating that factor proportions change drastically with scale. The change was furthermore not uniform with respect to factor prices. In one case (no 26') the capital - labour ratio could fall to onetwentieth or increase to twenty times the initial value when the output increased. Lamyai's main conclusion was that scale matters more than prices when it comes to factor ratios.

Statistical estimations of engineering relations

If the underlying technology is non-homothetic, what about the fit of ordinary statistical production functions? This question has been examined in studies 18, 26 and 28. Lamyai tried a translog production which behaved quite well ($R^2 \approx 0.99$). The estimated parametres did not support an assumption of homotheticity, and ES was (quite correctly)

found to vary with the capital/labor-ratio. The only relation not truly captured by the function was that between ESC and scale (Lamyai p 140).

Eide (no 27) obtained a very good fit with a simple Cobb-Douglas production function using fuel and other costs as factors of production ($R^2 = 0.998$). Eide concluded that

"... a C-D function may be used with more confidence than before for the type of aggregation that is considered".
(p 128)

This conclusion seems a bit odd indeed in comparison with my results. The ES is unity in C-D function, but inspection of Eides engineering data do show that $ES \approx 0.5$. The C-D estimate also yielded $ESC = 1.34$, whereas my inspection showed a varying and slightly falling ESC. Thus, the C-D specification did not capture the true scale and substitution properties despite the goodness of fit.

Wibe (no 28) tried a transcendental formulation of the C-D function, where ESC decreased with the output. The degree of fit was impressive ($R^2 \approx 0.99$) with all coefficients significant on 99% confidence level. However, as $ES = 1$ in this function, the substitution properties were misspecified (see Table 1). Also, the estimate of ESC was incorrect. The estimate predicted an optimal level ($ESC = 1$) at 30 000 units/day whereas cost minimum was obtained at ≈ 10 000 units. Wibe also compared the marginal productivity of labour which was found to be about ten times smaller (!) with engineering data when compared to the statistical estimate (p 186-187).

The conclusion seems to be that a good statistical fit (measured in R^2 , F-values etc) is no guarantee of a true representation of scale and substitution properties. The errors regarding substitution seem to be more serious than those of scale, probably due to the fact that usually more scale than factor ratio variation is found in production data.

Technological progress

Engineering studies of technical progress identify the sources of progress. For instance, in Wibe (no 28) it was found that higher process temperature and higher working pressure were major factors behind progress. Pearl and Enos (no 25) identified better control of operation and faster construction as major causes of the decrease in unit cost, etc. The engineering method explains progress, thus increasing our understanding of this complex phenomenon.

Unfortunately, there is little connection between economics and engineering here. Except for Cowing (no 22) who used engineering to test the vintage hypothesis, none has tried to classify the progress in engineering according to economic theory (whether progress is neutral, capital augmenting etc). Engineering studies of technological progress are - so far - of very little interest to economic theory.

Methodology

There is no study that has entirely followed the ideal schema described on pp 2-3. It is most common procedure to formulate the problem as one concerning minimization of costs, given the level of output. Since prices usually vary with the engineering variables $p = p(x_1 \dots x_n)$ the problem to be solved is

$$\begin{array}{ll} \text{Min } \sum_j v_j p_j \\ \text{ST} \end{array}$$

$$v_j = g_j(x_1 \dots x_n) \quad j=1 \dots m$$

$$q = e(x_1 \dots x_n) = \bar{q}$$

$$p_j = p_j(x_1 \dots x_n) \quad j=1 \dots m$$

Since the price equations contain shift parameters, different relative prices can be chosen, and in this way a number of points on the economic production function can be obtained.

The studies analyzing "heavy" processes, where labour is not included in the economic substitution possibilities, have more or less followed the scheme described above. The starting point here has been an engineering description of the process, sometimes involving over one hundred relations. When the substitution labour capital-labour has been analyzed, the starting-point has been different, namely, a mapping of discrete, possible methods, where each method has been characterized by constant input coefficients. In those cases where the entire process has consisted of several operations carried out in succession, first the various methods for each operation have been mapped, and then a minimization of total costs has indicated which combination of operations has been connected with a certain price vector. (Lamyai's study number 26 is an example of this method).

III THE FORMAL MODEL

In an article on development in production theory (Solow 1967), R Solow commented on engineering studies as follows:

"... it appears that very little of general interest has been done since the early papers of Chenery."
(p 26)

This is still a valid observation. Chenery's basic model even now the main theoretical contribution to the field, is a classification of the engineering structure of production. The factors of production are classified according to the technical function ("materials" and "processing factors"). The production model consists of relations representing the technological structure: the "material transformation

function", (relating energy input and quality of material to quality and quantity of output) the "energy supply function" (relating quality of inputs to energy required), and the "input functions" (relating qualities of input to their quantity). Substituting the "energy" function into the "material" function yields the engineering production function.

Chenery's model is powerful in many respects. It is clear and easy to understand. The relations involved are truly fundamental to every production process and the classification scheme is relevant to the engineers' conception of production. That no one has tried to replace (or even modify) Chenery's basic model indicates its essential value. However, not only is there a lack of criticism of the model, but there is also a lack of application. Almost every empirical work cited here has referred to Chenery's model but no one has used it. To my knowledge there exists not one empirical "material transformation function" or an "energy requirement function". The engineering structure presented by Chenery seems inadequate for most empirical cases.

I believe that the problem is as follows: If we want an engineering theory of production, we must work at a high level of abstraction. We must look for features common to all (or at least most) processes. Chenery realizes that material transformation with the help of energy is the most representative physical description, and accordingly ... "An industrial plant may be represented by pattern of material flows and application of energy" (1953, p 299). His basic model is built to reflect this physical structure. But engineering is not concerned with general and abstract descriptions, but with concrete information for specific problems. Accordingly, the engineering information is never given in terms of a "material" or an "energy" function. Of course, in some cases, it would be possible to transform the given relations and information into Chenery's "material" or "energy" functions. But the question is: Why should an economist bother to conduct complicated transformations if his prime interest is not these transformed relations but the economic production function? If we answer this question on the basis of the existing literature, the answer is

clear. The route through the "material" and "energy" functions makes an unnecessary and complicated detour.

The lack of a common structure does not mean that all kinds of engineering information is that specific. The laws of thermodynamics (as well as other laws of physics and chemistry) applies to all production problems and hence, some pieces of information are bound to be the same. It is for instance possible to calculate a physically determined minimum energy requirement figure for many processes. Accordingly

$$E > E_0 \cdot q$$

E = energy consumption, E_0 = physical minimum of energy consumption per unit, q = production, is an engineering restriction typical for many processes.¹

III CONCLUSIONS

The conclusions that can be drawn from the empirical results contained in the studies may be summarized as follows:

(i) The substitution elasticity between capital and labour varies strongly but has an average value around 1. For other factors the elasticity is definitely below 1.

(ii) The scale elasticity is generally larger than 1 for all output levels.

(iii) The production function is generally non-homothetic. The proportions of various factors vary according to the scale when factor prices are fixed. A statistical function adaptation often gives misleading results, particularly as far as the substitution possibilities are concerned.

1 See Wibe (1982) for a discussion of these "scientific laws of production" and the implications for the production function.

The result may appear to be meagre, but on the other hand the exactness and reliability for example in the measurements of the substitution elasticities are much greater than the ones that are found in statistical analyses. There is no doubt in my mind that the quantitative results presented here are among the most reliable in the empirical production theory. It is also interesting to note that the engineering approach has been successfully applied in the study of the substitution between labour and capital as it has been generally believed that this method cannot be used in the study of labour intensive technologies. [See Chenery (1949, p 10). Muyskens (1979, p 113-115)].

However, it is also easy to point to some basic weaknesses in the studies referred to here. First of all, it does not appear to have been clearly understood that the engineering approach is supposed to be used to answer economic questions, concerning for example, substitution and scale relations. A change in this respect would improve the nature of those works, in particular as far as studies of technical development are concerned. Second, there is reason to believe that the studies would have a greater degree of clarity if they were more independent of Chenery's formal model. The most important thing for engineering studies, it seems to me, is to disregard all ready-made models and to adapt the methods to the concrete problems under examination.

There appears to be an increasing scepticism concerning the results that are produced by statistical production analyses today. The other side of this scepticism is a positive attitude towards the engineering approach and an often explicitly expressed desire to have more engineering in empirical studies. [See Bosworth (1976) p 15, Johansen (1972) p 186-187, Walters (1963) p 11 ff and Solow (1967) p 26]. The results of the studies done so far are definitely not without interest, but they are not sufficiently impressive to justify a general programme for engineering studies of all types of processes. Chenery's method is time-consuming and intricate, which should be compared to

the simplicity and swiftness of statistical studies. The optimal state of affairs is to perhaps let engineering studies continue to play the same role in production analysis: i.e. as necessary sample tests, of whether central quantitative assumptions are correct. One example of the utility of this role, given recently by Brendt and Wood (1979), described a statistical study done in (1975), which found that capital and energy were complements in production. Engineering information showing that these factors were substitutes forced them to reconsider the form of the production function. The result was a much more realistic production function.

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APPENDIX

Data description

Study no 1, Frisch (1935)

Frisch article is a study of ex post substitution in the chocolate industry. When adding fat (which is more expensive than ordinary raw material) product flow can increase, thus decreasing all other costs per unit. Frisch production function is presented in a graph (p 24) with "fat addition" as one factor of production. It was possible (from the text) to add other raw materials, thus obtaining the following table.

<u>Raw material</u> (index)	<u>Other costs</u> (index)
1343	121
1348	113
1355	107
1362	103
1367	101
1378	99
1398	98

(This table is independent of output level).

Study no 2, Chenery (1953)

This study is commented much elsewhere. The table below is taken from Chenery's graphical illustration on p 525. It refers to output level 50.

<u>Capital</u> (index)	<u>Other costs</u> (= current inputs. Index)
54	16
44	17
35	20
29	25
26.5	30
25.5	37

Taking distances to origin along the factor ray current input/capital
= $20 \cdot 10^{-3}$ (fig p 525) yields for scale calculations:

<u>Output</u>	<u>Input</u> (index)
50	33
100	61
200	100

Studies no 3-5, Chenery (1953)

These studies are purely theoretical and can not be used to obtain quantitative information.

Studies 6, Ferguson (1953)

This is actually a more complete engineering study than Chenery's (no 2). Ferguson starts with an engineering production function $q = e(x_1, \dots, x_n)$ and calculates input functions $v_j = g_j(x_1, \dots, x_n)$ for labour, energy and capital. Unfortunately, the study stops with engineering, and Ferguson does not try to obtain an economic function. He can however explain the changes in economic inputs (v) over time by identifying changes in engineering variables x_i .

Study no 7, Smith (1957)

Smith derives an input function for fuel in this study. This function is used to forecast energy consumption.

Study no 8, Smith (1961)

Smith's book is full of examples of engineering production functions. They all illustrate the importance of substitution between capital and current inputs (mainly energy). However, like Ferguson, Smith presents only the engineering relations.

Study no 9, Hurther (1962)

This is one of the most detailed engineering studies made. Hurther studies two subprocesses in an oil refinery: distillation and thermal cracking. The substitution involved is (essentially) between capital and energy. Hurther does not transform his many hundred engineering relations into one economic production function but he presents the cost minimizing solution to thermal cracking in a graph (p 229). The factors are capital and current inputs (= other costs) and the figures I have taken from this graph are:

<u>Capital</u> (index)	<u>Other costs</u> (index)
35	1693
50	1645
80	1597
115	1555
155	1523
200	1501
310	1469
380	1453
455	1446

These figures are taken from the lowest isoquant (25 000 tons). Inspection of ESC along factor rays yield a value in the neighbourhood of 1 Hurther's calculation show that the ratio of capital to current input decreases with output.

Studies 10-15, Boon (1964)

This study (together with no 26) is probably the most detailed work on capital-labor substitution. It is unfortunately, also one of the least commented studies. Boon studies various methods (of digging etc) in great detail, and arrives finally at a capital and a labor input figure for each method. Each (effective) method yields a point on the isoquant.

Study no 10, Producing grains 264 hectares

Capital	Labor hours
34 034	105 600
113 670	34 716
144 380	10 874
160 603	6 864

Source: Table 1 p 305

Study no 11, Ploughing 750 acres

Capital	Labor hours
7 350	8 475
10 050	4 875
10 650	4 500
18 920	263

Source: Table 2 p 305

Study no 12, Earth-moving. Field trench 2 400 m

Capital	Labor hours
45 000	602 400
308 000	37 200
554 400	30 000

Source: Table 3 p 306

Study no 13, Metal turning

(i) 6 500 units		(ii) 18 500 units	
Capital	Labor hours	Capital	Labor hours
10 000	2 000	30 000	5 698
18 000	1 274	36 000	3 626
50 000	702	50 000	2 000

Source: Table 4 p 306

Study no 14, Metal facing

(i) 5 320 units		(ii) 23 810 units	
Capital	Labor hours	Capital	Labor hours
12 000	2 000		
20 000	1 080	35 000	2 000
35 000	447		

Source: Table 5 p 306

Study no 15, Wood working; window frames

(i) 50 000 frames		(ii) 400 000 frames	
Capital	Labor hours	Capital	Labor hours
8 500	4 000	48 400	22 480
15 050	2 810	115 000	4 320
80 000	540		

Source: Table 6 p 306

Study no 15, Wood working; window frames

(i) 50 000 frames		(ii) 400 000 frames	
Capital	Labor hours	Capital	Labor hours
8 500	4 000	48 400	22 480
15 050	2 810	115 000	4 320
80 000	540		

Source: Table 6 p 306

Study no 16, Heady and Dillon (1966)

This book contains several examples of production functions relevant for agricultural production e g production of broilers (with inputs corn and soybean oil). However, the ES between soybean and corn are of

little interest for economists. But the authors also provide one example of capital-labor substitution referring to crop production on a 160 acre farm on Clariton-Webster Soil. The capital-labor figures refer to combinations of machine services and labor needed to produce an average crop. (Table 16.6 p 563).

Capital	Labor months
900	14.22
1 000	10.96
1 100	8.66
1 200	6.99
1 400	4.78
1 600	3.44
1 800	2.07

Study no 17, Kurtz and Manne (1963)

This study is the most commented study apart from Chenery's. Suffice it to say that Kurtz and Manne estimates a C-D and CES production function from engineering data for various metal machining processes. The original data are unfortunately not published, and the only result is from the statistical estimation.

Study no 18, Brown (1966)

This study contains a very ambitious engineering description of water pumping. This description is not, however carried over to the economic world, and the only economic function that is derived with engineering is an ex post relation between energy input and output. This relation has an ESC less than one.

Study no 19, Belinfante (1969)

This is essentially a very detailed statistical study. Engineering is used only as general information.

Study no 20, Mac Avoy (1969)

This is an ex ante study of nuclear breeder technique. Mac Avoy works with capital and fuel as inputs and obtains 3 input-output points from engineering data, two on which is on the same isoquant. He uses these 3 points to fix the parametres of a Cobb-Douglas production function.

Study no 21, de Salvo (1969)

This study is one of the most complete, involving engineering, input and economic functions. Figure 2 on p 15 give output (ton-miles/hour) with "locomotive horsepower" and "number of cars" as inputs. His eq (40) on p 21 shows that horsepower is proportional to energy input. Eq (44) give locomotive capital cost as a function of horsepower and table 6 (p 22) the capital cost for a freight car. Thus, it is possible to obtain capital-energy points on the economic production function. (Inspection of his labor input function shows that labor is more or less proportional to output). The following data emerged:

(i) Output 50 000

(ii) Output 100 000

Energy	Capital	Energy	Capital
1 480	331	2 950	652
1 143	352	2 290	722
1 000	430	2 000	824
930	528	1 710	1 032

de Salvos long run cost curve on p 25 give the following table (index):

Output	Input (= total cost)
100	100
196	147
278	186
378	233
652	358

Study no 22, Anderson (1969)

This study concentrates on the cost of eliminating emissions. The basic model consist of over hundred engineering equations. A cost minimization procedure give variation of total cost with emission at 3 output levels. The graph on p 114 give the following table:

Total costs (index)	Emissions (index)
100	100
100.15	51
100.55	16
100.70	2.4

(The figures refer to output level 150 tons per hour. There is no change if other isoquants are considered. Inspection shows $ES = 2.07$ and $ESC = 1.0$).

Study no 23, Cowing (1970)

Cowing obtains information from engineering and estimates (with data from operating plants) how age and expected price of fuel/capital influence (i) the efficiency of machines (ii) the size of machines. The result is that age is significant for both, but price only for machine efficiency.

Study no 24, Marsden et al (1974)

The authors try to show how a Cobb-Douglas and a CES production function can be derived from chemical and biological laws characterizing some processes. The study is purely theoretical.

Study no 25, Pearl and Enos (1975)

This study departures from a study of Cookenboo (1955) on oil transportation by pipelines. Cookenboo's study referred to 1952 technology,

and the authors compare this with 1969 technology. From figure 3 on p 59, showing output 125 000 barrels/day (1 000 miles) as a function of "other costs" and "capital" the following figures were obtained:

Capital (index)	Other costs (index)
67	62
68	38
80	20
86	15
106	12

Study no 26, Lamyai (1978)

Lamyai studied the production of a minor part of a bicycle - the head lug. The production process consisted of many simple operations conducted in sequence. Each operation could be performed with several methods, each with a specific labor and capital input coefficient. Lamyai calculated with 6 different relative prices capital/labor ($P = P_0 \cdot 2^n$, $n = 1, \dots, 6$) and 8 output levels ranging from 250 to 250 000. He considered two basic technologies: full sharing and no sharing, and calculated least-cost combinations of methods for each technology, output level and price. The values of ES are given in table 4.6 p 134.

Average values of ES at output levels	Full sharing	No sharing
250, 500, 1 000	0.122	0.111
5 000, 10 000	1.024	0.551
25 000, 50 000	0.865	0.137
250 000	0.003	0.063
ALL*	0.518	0.597

(* This is the value I use).

The ES value are extreme point elasticity i e the change in factor proportions and slope between the end points of the isoquant are considered.

Values of ESC are found at the same page. I use the figure for the average for all output levels.

Study no 27, Eide (1979)

This is perhaps the most detailed engineering study made. Eide shows how the design variables of a tanker change with the relative price of fuel. (E g speed, lenght etc). The characteristics of an optimal ship (for turbine tankers) are given in tables (p 123) and in figure (p 127). The following figures were obtained from the table.

(i) Output level 2.0

Energy (index)	Other costs (index)
23 920	7 284
26 120	7 083
29 350	6 874
34 080	6 668
56 020	6 058
61 250	5 986

(ii) Output level 0.5

Energy	Other costs
6 440	2 613
6 930	2 571
7 530	2 531
8 440	2 495
10 120	2 450
16 690	2 301

ESC are calculated along factor rays in the figure.

Study no 28, Wibe (1980)

This is a detailed study of a very small process; iron production in blast furnaces. Wibe studies substitution, scale and technical progress. The possibility of substitution arises because pieces of equipment can be added to the fundamental construction, equipment that

lower energy consumption. This substitution is about the same for all output levels. (The table below is from p 190).

Capital (index)	Energy (index)
118	476
111.7	482
105.8	490
98.6	505
98.3	519
98.0	538
97.8	559

Economics of scale arises because (i) Labor is more or less independent of output. (ii) Production is (more or less) proportional to the volume of the furnace, whereas capital cost is proportional to the surface area. Diseconomies arise because larger furnaces are more difficult to operate and hence that the capacity utilization decrease.