Formative assessment: Teacher knowledge and skills to make it happen

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Licentiand thesis, Educational works no 15
Naturvetenskapernas och matematikens didaktik
Umeå 2014

ISBN: 978-91-7601-104-1
Elektronisk version tillgänglig på http://umu.diva-portal.org/
Umeå, Sverige 2014

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Abstract

Several studies have demonstrated that substantial learning gains are possible when teachers use formative assessment in their classroom practice. At the heart of most definitions of formative assessment lies the idea of collecting evidence of students' thinking and learning, and based on this information modifying teaching to better meet students' needs. Such regulation of learning processes would require skills to elicit the thinking underlying students' oral and written responses, and the capacity to make suitable instructional decisions based on this thinking. When the continuation of the teaching is contingent on the information that appears in such assessments additional knowledge and skills are required compared with a more traditional approach to teaching.

Today, sufficient knowledge about how to help in-service teachers and pre-service teachers develop their formative classroom practice is lacking. In the pursuit of gathering research evidence about the specific content and design of professional development programs and teacher education courses in formative assessment, it is important that we know what kinds of skills and knowledge teachers need to successfully orchestrate a formative classroom practice.

The aim of this study is to identify activities and characterize the knowledge and skills that a teacher of mathematics uses in her formative assessment practice during whole-class lessons.

The study is a case study of a teacher's formative assessment practice during mathematics lessons in year 5. The data were analysed by identifying a) the formative assessment practice b) the teacher's activities during the formative assessment practice and c) the teacher knowledge and skills used during the activities.

The main result of the study shows that the formative assessment practice is a very complex, demanding and difficult task for the teacher in several ways. For example, during short term minute-by-minute formative assessment practice the teacher uses knowledge and skills to eliciting, interpreting and use the elicited information to modify instruction to better meet student learning needs. She also helps students' to engage in common learning activities and take co-responsibility of their learning. In the minute-by-minute formative assessment practice the teacher also handles new mathematics (to her), unpredictable situations and makes decisions about teaching and learning situations in a matter of seconds.

Sammanfattning

Ett flertal studier har visat att implementering av formativ bedömning ökar elevers lärande i matematik. I hjärtat av de flesta definitioner av formativ bedömning finns iden om att samla belägg om elevers tänkande och lärande, och utifrån beläggen anpassa undervisningen så att den möter elevernas behov. En sådan reglering av lärandeprocessen kräver kunskap om hur man framkallar den typ av svar som visar hur elever tänker. Det krävs också förmåga att välja ett lämpligt sätt anpassa undervisningen. När undervisningen kontinuerligt anpassas efter den typ av bedömning som beskrivits, så behövs ytterligare kunskap jämfört med den kunskap som behövs för att genomföra en mer traditionell undervisning.

Idag saknas tillräcklig kunskap om hur man kan utbilda och stödja lärare och blivande lärare att utveckla sin formativ bedömningspraktik. För att veta vilket innehåll och vilken design kompetensutbildningar i formativ bedömning bör ha så är det viktigt att vi vet vilka specifika kunskaper och förmågor en lärare behöver för att kunna genomföra en framgångsrik formativ bedömning.

Studiens syfte är att identifiera aktiviteter och att karaktärisera de kunskaper och förmågor en matematiklärare använder i sin formativa bedömningspraktik under lektioner i helklass.

Fallstudien följer en lärares formativa bedömningspraktik under matematiklektioner i årskurs 5. Data analyserades med avseende på a) lärarens formativa bedömningspraktik b) lärarens aktiviteter under denna praktik och c) de kunskaper och förmågor som läraren använde för att genomföra dessa aktiviteter. Studiens huvudresultat visar att den formativa bedömningspraktiken är mycket komplex, krävande och svår på flera sätt för läraren att göra. Till exempel, under en formativ bedömningspraktik som sker minut-för-minut så använder läraren kunskaper och förmågor för att framkalla, tolka och använda den informationen för att bättre kunna möta elevens behov. Läraren hjälper också eleverna att engagera sig i och ta ansvar för sitt lärande under lärandeaktiviteter. Under denna praktik så hanterar läraren även för henne ny matematik, oförutsägbara situationer och fattar beslut om undervisning inom loppet av sekunder.

Acknowledgement

Writing a licentiate thesis is not a one-person work, even if it is only one name on the front page. It's like a long ride in a diversified landscape. Many are those who have been with me on the tour. Thank you all.

You who brought me to the horse; thank you, colleagues and students during my teacher deed, especially from my first location at Pengsjö Skola who fostered me as a teacher, Catrin, Jonna, Maria, Mia, Elin. Thank you, headmasters, who gave me the possibility to develop: Carina, Elisabet, Christer. Inga-lill, who gave me the courage to get up on the horse apply for a position as a postgraduate student in Pedagogical work thank you.

Following and supporting me on my ride: All members of NMD, who always are ready to discuss, support and to share a fika. The incredible milieu for graduate students that UMERC creates has inspired me with meetings with a mix of experienced and novice researchers and among them, my supervisors Torulf Palm and Tomas Bergqvist. They were always helping me up again when I fell off the horse, keeping faith in my success, pointing out possible orientations and challenging me to see what's behind the words. Thank you. I will also acknowledge all members of DME graduate school, especially Janne, Helena and Maria, who have shared the struggling, challenging and developing years together with me. The formative assessment research group within UFM, Erica, Catarina and Sara, thanks for all enlightening discussions on formative assessment and life in general.

Without informants no study, so thank you, Teacher Eva and class 5 in a school in the north of Sweden, for letting me share your community and learn from you for a while.

Finally, Hasse, Maja, Robin, Ville, thank you for bringing me to waterholes to drink during the ride.

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1 Introduktion

I was really pleased. I had worked out my first matrix where the students easily could compare their own solutions in relation to criteria for problem solving. I carefully explained that the three-level matrix treated the goals for year six. So the students in year four would have time to work it out. “Feel no pressure” I told them. They had plenty of time to learn. At least, now they knew what to aim for. My own aim was to be a teacher practicing formative assessment. Twenty minutes later I was seated in the sofa together with two students in year four that were crying because they thought I would be disappointed since they did not know all the stuff in the matrix. I realized, professional development programs and getting inspired is good, but when you are about to change your practice you have to take small steps. And yes, we got better in using matrices, the students and I.

Still, it was not Key strategy 1 of formative assessment, *clarifying and sharing learning intentions and criteria for success* that made my interest for formative assessment. Formative assessment helped me to put words on, and understand, theories and processes behind the dialogue that my students and I had about learning, how their learning developed and the role they and I had in that process. It was this interest to develop the understanding of teachers’ practice that made me apply for a position as a postgraduate student in Pedagogical work (pedagogiskt arbete).

Since 1999, it has been possible to be admitted as PhD student in the subject pedagogical work. This was arranged since studies had shown that a small part of the dissertations produced by PhD’s within subject didactics and content didactics in education research had until then dealt with the agents of the practice and in the practice. The aim with Pedagogical work as a PhD subject is to connect school to scientific research by giving teachers the possibility to formulate questions and problems connected to their own practice (Vinterek, 2004). In the Education Act (SFS 2010: 800), it is stated that the work in school should be scientifically and evidence based. One way that the Swedish government supports this development is to support postgraduate educational programs for teachers. I am one of the teachers who have the opportunity to participate in one of these PhD programs.

The negative trend in results from national and international tests for Swedish students in mathematics, and the students’ decreasing lack of interest for mathematics, indicate that there are reasons to research how to improve education and teacher education in mathematics. There are no countries where the decrease in results has been as evidently negative as in Sweden in the dominating areas of education (Skolverket, 2014).

One possible way to improve instruction in mathematics is to implement formative assessment. Several studies have demonstrated that substantial

learning gains are possible when teachers use formative assessment (FA) in their classroom practice (Black & Wiliam, 1998a; Wiliam, Lee, Harrison, & Black, 2004). What do mathematics education research know about what mathematics teachers need to know to be able to implement formative assessment successfully?

Research has confirmed that teachers need to know the mathematics they are teaching (Askew, 2008). However, research has also shown that there is no clear relationship between the teachers' formal mathematical education and students' learning (Askew, 2008; Brookhart, 2011; Hattie, 2009). It might be the case that such a relationship has not been found because measuring teachers' mathematical knowledge just in terms of their level of formal education is not fit enough because there are probably aspects of such knowledge that are more important than others (Askew, 2008). If we add formative assessment to the question of what mathematics teachers specifically need to know the question becomes more difficult to answer (Brookhart, 2011; Schneider & Randel, 2010).

The knowledge and skills to implement formative assessment might not be entirely content specific. Research on mathematics teacher knowledge and skills in some specific parts of formative assessment (i.e. teachers' ability to assess tasks, to interpret students' answers, to provide feedback or to plan the next step in instruction) has been conducted during teacher education (Son, 2013, Schneider, 2013) but not in authentic teachers practice. There is research from authentic science classrooms investigating the knowledge and skills used, indicating that implementing formative assessment is not an easy quick fix of general aspects of teacher knowledge (Cowie & Bell, 1999; Ruiz-Primo & Furtak, 2007). Overall, this indicates the importance to investigating the knowledge and skills teachers actually use in authentic classroom practice. The purpose of this study is to add to that knowledge by investigating, describing and characterizing the activities, knowledge and skills that a teacher use when practicing formative assessment. A more detailed aim and specified research questions that include theoretical aspects will be accounted for in section 2.3.

In the next section I will outline research related to my study, including formative assessment, types of formative assessment and teacher knowledge and skills for teaching.

2 Background

In this section I will outline the research, theories and framework linked to, and used in, the study on formative assessment (Subsection 2.1) and teacher knowledge and skills (Subsection 2.2). Aim and research questions is introduced in Subsection 2.3.

2.1 Formative assessment

In the following subsection the definitions of formative assessment (Subsection 2.1.1) and different types of formative assessment (Subsection 2.1.2) will be introduced. In subsection 2.1.3 formative assessment during instructional dialouges will be introduced. Research on the impact of formative assessment on student achievement will be outlined. in Subsection 2.1.4. Teacher knowledge and skills in formative assessment are discussed and in Subsection.Finally, research on professional development programs will be outlined in Subsection 2.1.5.

2.1.1 Definitions of formative assessment

In 1967, Michael Scriven used the term formative evaluation to describe the role evaluation could play in curriculum improvement and contrasted it with summative evaluation, which use is to determine whether the chosen curriculum gives credit for the expenses (Scriven, 1967). Further, Sadler, (1989) described formative assessment as a systematic process to continuously gather information and provide feedback about learning while instruction is underway.

> *Formative assessment is concerned with how judgments about the quality of student responses (performances, pieces, or works) can be used to shape and improve student´s competence by short-circuiting the randomness and inefficiency of trial-and-error learning (Sadler, 1989, p 120)*

Summative assessment on the other hand, concerns summing up the achievement of a student, for example in the end of a course. The primary distinction between summative and formative assessment is related to purpose and effect (Sadler, 1989).

In 1998, Black and Wiliam, took on the task to review literature on formative assessment in the purpose to survey how and if improvement of classroom assessment could improve classroom learning (Black & Wiliam, 1998a), resulting in their first definition of formative assessment.

> We use the general term assessment to refer to all those activities undertaken by teachers-and by their students in assessing themselves-that provide information to be used as feedback to modify teaching and learning activities. Such assessment becomes formative assessment when the evidence is actually used to adapt the teaching to meet student needs (Black & Wiliam, 1998b, p 140)

The activities that were proven in the review to enhance student learning were; sharing success criteria with learners, classroom questioning, comment-only marking, peer- and self-assessment (Black & Wiliam, 1998b).

At this time, formative assessment was becoming a concept, first to contrast summative assessment (earlier noted formative and summative

evaluation) that changed the view of assessment, from thinking of assessment as a system to a process, and also the student involvement in this process. Brookhart (2011), points out that the Standards from 1990, only mention the teachers´ use of assessment information. The Standards, *Standards for Teacher Competence in Educational Assessment of Students* (1990) was developed with the purpose to guide teacher educator developing teacher education programs, work-shop instructors and an impetus for educational measurement instructors to conceptualize student assessment more broadly.

In order to provide a better theoretical grounding for formative assessment, Black and Wiliam, drew on Ramaprasad's three key processes (as cited in Wiliam & Tompson 2007) and crossed them with the three active agents, teacher, peer and student, creating the five key strategies for formative assessment and the the big idea:

1. *Clarifying and sharing learning intentions and criteria for success*
2. *Engineering effective classroom discussions, questions, and learning tasks that elicit evidence of learning*
3. *Providing feedback that moves learners forward*
4. *Activating students as instructional resources for one another*
5. *Activating students as owners of their own learning*

The big idea is that evidence about student learning is used to adjust instruction to better meet student needs – in other words, that teaching is adaptive to the student needs. (D. Wiliam & Thompson, 2007 p. 64)

Cowie and Bell, (1999) adopted a more narrow definition of formative assessment by using the definition "the process used by teachers and students to recognize and respond to student learning in order to enhance that learning, during the learning" (ibid p.537). This definition requires formative assessment to take place during learning. That is, focusing the unplanned an unpredictable situations during lessons when practicing formative assessment. Other definitions focus the feedback process and outcome, for example:

> *The purpose of formative assessment is to provide feedback to teachers and students during the course of learning about the gap between students' current and desired performance so that action can be taken to close the gap* (Heritage 2008, in Filsecker & Kerres, 2012, p 4)

Critic against this way of perceiving formative assessment points out the fact that this leaves the assessment itself open to the possibility to be just any random information. And as a reaction, Broadfoot introduced the term assessment for learning pointing out that the term *formative* implies no

more than assessment is carried out frequently and is planned at the same time as teaching (as cited in Wiliam, 2010)

On the other hand, the change of word used in Black & Wiliams definitions, "assessment evidence" in 1998, to "elicited information" 2009, implies that it is not only information gathered from tests, workbooks, nor coincidences that the teacher in a formative practice receives valuable information from students. Black & Wiliams definition from 2009, below, is the definition used in this study

> *Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited* (Black & Wiliam, 2009, p.9).

The term instruction is used to describe any activity that intends to create learning, denoting both teaching and learning. (Wiliam, 2010) The definition also clarifies that a formative use of elicited information might be to just carry on with the planned instruction, if the elicited evidence about student achievement gives that kind of information. Still, the use of information is crucial to the definition, pointed out by the use of the notion *formative,* in formative assessment.

Drawing on social-cultural theory, meta-cognitive theory and self-regulation theory a number of researches (Sadler 1989; Black & Wiliam 1998; Chappuis & Stiggins, 2002) pointed out that the students have a key-role in the assessment process. The view of assessment as exclusively a teacher activity was changed due to the influence of formative assessment (Brookhart 2011).

It is not only the definitions used that influences the nature of formative assessment. Formative assessment will be acted out differently depending on all the variables in the educational environment. One example is that teachers' beliefs about learning and mathematics will affect the way teachers take on formative assessment (Cowie & Bell, 1999; Marshall & Drummond, 2006; Watson, 2006). Another example comes from Torrance (2012) investigating formative assessment in higher education. Based on his findings he conceptualize formative assessment as being confirmative rather than divergent which would be appropriate considering the purpose of higher education to producing independent and critical learners.

2.1.2 Types of formative assessment

Ways to talk about formative assessment is to consider when and how it happens. Heritage, (2007) bases the conceptualization of formative assessment on the eliciting phase. On-the-fly is her notion on formative

assessment when the teacher uses spontaneous, unpredictable or unplanned situations during sessions to gather information. Planned formative assessment, is the notion when the teacher have planned questions, tasks or activities to elicit information and finally, curriculum-imbedded formative assessment where the elicited information might be elicited from diagnostic tasks in the textbook or likewise (Heritage, 2007).

In the study of Cowie and colleague (Cowie & Bell, 1999) ten science teachers in year 7-10 identified two types of formative assessment using the notions of planned and interactive formative assessment. When formative assessment is planned, the phases of formative assessment are typically generated from the teacher's initiative. It is a cyclic process of eliciting, interpreting and acting, carried out with all students in the class and relies on teachers' professional knowledge. In interactive formative assessment on the other hand, the information is noticed by the teacher during student talk or student questions and tends to be carried out with some individual student or small groups.

Riuz-Primo (2011) follow and develop Cowie's use of notions of formative assessment but they use the designations informal and formal formative assessment when studying science teachers' informal formative assessment. They describe formative assessment as a continuum from formal to informal. Formal formative assessment usually starts with a planned activity by the teacher, designed to focus some aspect of learning. The purpose of the activity is to check student understanding to plan the next step of instruction in order to move their students' learning forward.

In the given examples, informal formative assessment, on-the-fly and interactive formative assessment is assessment imbedded in instruction, for example when the teacher walks in the classroom listening to student talk or looking in their notebooks. The use-phase is verbal, with an individual student or as a whole-class discussion. Informal formative assessment uses everyday learning activities as potential assessments that provide evidence of students' learning in different modes.

Wiliam (2006) bases notions of formative assessment on time cycles (i.e. long, medim length and short cycle) depending on the responsiveness of the feedback system. That is, how long time it takes for the system to use the gathered information. He also indicates the formal-informal aspect when describing the continuum. Wiliam brings in other agents than the teacher and other forums than classrooms. For example, long-cycle formative assessment is described as a supervisor using results from state-tests to inform a plan of workshops for teachers (Wiliam, 2006). Medium length cycle formative assessment has a timescale of one to four weeks and is exemplified by a teacher using a pre-test to make adjustment in instruction for individuals or class before the actual test (Wiliam & Thompson, 2007).

Short-cycle formative assessment is described as either day-by-day or minute-by-minute. Formative assessment day-by-day is when the elicited information is used between lessons with a responsiveness up to two days. This enables the teachers to use the elicited information as a base when planning next lesson. Formative assessment minute-by-minute is when the feedback system's responsiveness is immediate, within the lesson. In Wiliam's description the assessment can be either planned or spontaneous.

For the episodes analysed in this study, Wiliam's notions of cycles will be used to select the episodes since they fitted the episodes in the study. Formative assessment is described and researched as consisting of three phases, eliciting-interpreting-use. The three phases of formative assessment is derived from the definition from Black & Wiliam 2009 (see also section 2.1.1) and formative assessment is in this study defined as sequences when information of student understanding of a learning object is elicited, interpreted and used by teacher, learners or peers.

2.1.3 Formative assessment during instructional dialogues

In a regular classroom practice, when the focus not is on how the students think, but on what the students can and cannot do, the teacher might listen evaluative to student answers, only interested in the correctness of the answer. The pattern of such dialogues is described as Initiative-Response-Evaluation, IRE, (Mehan, 1979). The questions the teachers pose in these dialogues are mostly questions about facts that the teacher already knows. Student answers are only evaluated as correct or incorrect. If a student gives a wrong answer the question goes to a peer. This type of interaction is the most common in classrooms (Cazden, 2001).

When a teacher uses formative assessment the purpose for posing questions is to understand students' thinking, and the dialogue that unfolds can be described as Initiative-Response-Follow up (IRF). However, both IRE and IRF dialogues have been viewed as limiting the dialogue between teacher and students since it positions the teacher as the only one with opportunity to take initiative on what to talk about.

When formative assessment is embedded in instructional dialogues (i.e. interaction that intends to create learning, denoting both teaching and learning) the students' intentional participation is one of the fundaments. The purpose of such instructional dialogues is to elicit information and to use this information to better meet learning needs. The instructional dialogue can then be described as Eliciting-Response-Interpretation-Use (Ruiz-Primo & Furtak, 2006). Here, the student can take the initiative by asking a question or posing a comment. Student, peer or teacher can act upon the elicited information. For example, a student can give a counter argument or add to a peer's answer, or a quiet student can use other

students' contributions for her own learning. The E-R-I-U instructional dialogue can have multiple iterations before coming to a closure of which the purpose is joint understanding of the learning object.

Cowie's study of ten science teachers describes their interactive formative assessment as interaction during activities as for example science experiments. The teachers interact with individuals or groups while going around in the classroom, and changes to whole-class discussions if they notice a common misunderstanding of a concept (Cowie & Bell, 1999). When Cowie talks about interactive formative assessment it is mainly about the interaction between the teacher and an individual or a small group.

Duschl & Gitomer (1997) use the notion of *assessment conversation* described as

> ...a specially formatted instructional dialog that embeds assessment into the activity structure of the classroom. The intent of an assessment conversation is to engage students in the consideration of a diversity of ideas or representations produced by class members and then to employ evidence and age appropriate adaptions of scientific ways of knowing to foster a dialog about what does and does not fit with the emerging thematic structure of the lesson (Duschl & Gitomer, 1997, p 39).

Ruiz-Primo (2011), use and develop Duschl and Gitomer's definition of assessment conversation and its qualities in science practices. Their framework for exploring informal formative assessment through assessment practices in science inquiry teaching builds on identifying ESRU cycles containing three phases (Eliciting-Student Response-Recognize-Use) and analyse those in three aspects; epistemic, conceptual and social. Except for the difference in subject (science/mathematics), there are differences between the interpretations of the phases, for example, in Ruiz-Primo & Futak's study (2006; 2007), a teacher who clarifies a student utterance is analysed in the recognizing phase, while in this study, clarifying is analysed in the Eliciting phase. I see clarifying as an additional way to elicit information.

The episodes where formative assessment is examined in this study occur when the teacher instructs the whole group, creating interactive dialogues with students, between students or orchestrating a discussion. To do this, I use Wiliam's terms of formative assessment, medium length cycle and short cycle. I have adjusted Ruiz-Primo & Furtak's phases of formative assessment, due to the difference in interpretation of the phases. To underline that there are a difference, I use the notions Eliciting-Interpretation-Use (EIU) in the analysis (see also section 3.5.2).

2.1.4 Impact of formative assessment on student achievement

The review from Black and Wiliam (1998a) had the purpose to survey how and if improvement of classroom assessment could improve classroom learning (ibid). They reviewed 250 articles that concerned some of the activities connected to formative assessment, that is, activities that provide information to be used as feedback to adapt the teaching and learning activities in which they are engaged. Since the studies in the article were conducted with different kinds of data, implicit assumptions and so forth, no meta-analyses were appropriate to do. Still, Black & Wiliam concluded in another article (Black & Wiliam, 1998b) that the review points to an average effect size of 0.40-0,70 standard deviations, for the use of formative assessment and also pointed out that the result from the review were a set of guiding practices that, incorporated into the teacher's own practice, would take its own shape. One example of a study included in the review is a Portuguese study of 25 teachers that participated in a 20-week part-time course, training self-assessment methods. Pre- and post-test of students' mathematical achievement showed twice as high gain in performance as the control group.

The review from Black & Wiliam stimulated new investigations on the effect of formative assessment. One example is Black and Wiliam's own project, the Kings-Medway-Oxfordshire Formative Assessment Project (KMOFAP). This was a two-year-long project that involved 24 teachers implementing and exploring formative assessment in practice. In this study the effect size on student achievement was .32 (Wiliam, Lee, Harrison, & Black, 2004). Other studies, for example a quasi-experimental study from the South Carolina Department of Education show changes in teachers' assessment knowledge but the changes gave no significant difference in student achievement on tests (Schneider & Bruce, 2010).

One example that has close connection to this study in terms of method is a study of teachers who practice informal formative assessment. The results from the study indicates that indications that the teachers who fulfill complete informative formative assessment cycles (eliciting-recognize-use) are associated with higher student performance on embedded assessment (Ruiz-Primo & Furtak, 2006; Ruiz-Primo & Furtak, 2007). Wiliam (2006) argues that formative assessment based on medium length cycles have only modest impact on student learning but it is not clear why it is so.

There is a debate on the grade of efficacy of formative assessment, currently from Kingston and Nash (2011) with respect to the review Black & Wiliam conducted (Black & Wiliam, 1998a). The critique was based on the selection of the studies (i.e. non-comparable results) and the method for comparison. Kingston and Nash's meta-study suggests an effect size of formative assessment of 0.20 standard deviations (Kingston & Nash, 2011).

In the following debate (Briggs, Ruiz-Primo, Furtak, Shepard, & Yin, 2012; Filsecker & Kerres, 2012; McMillan, Venable, & Varier, 2013) there is an agreement that the lack of use of a common definition is one of the reasons why comparisons and therefore meta-analyses or reviews on effect sizes of student achievement are difficult to claim without much uncertainty. However, as Filsecker & Kerres (2012) point out, this diversity is an effect of the different research paradigms educational research that lead to the use of different ideas of what formative assessment is and with what methods to investigate. There is for example the question of quality of formative assessment, which is even more difficult to grasp with a matrix or a definition. There are many qualitative aspects involved in formative assessment of which some will be discussed later in this thesis.

Since formative assessment takes different shape depending on the context, how it is designed and used, and how the students take on their role as participants in the assessment process, formative assessment will most likely have different student outcome in different contexts.

So, how to design a professional development program and what do teacher need to know to be able to orchestrate formative assessment in their daily classroom practice? The research in those areas will be outlined in the following sections, starting with what is known about teacher knowledge and skills connected to formative assessment.

2.1.5 Teacher knowledge and skills for using formative assessment

Wiliam (2006) shows through examples of earlier studies that the quality of elicited information about student thinking is crucial for the formative assessment process of using it to guide the next steps in the teaching and learning process, and that this information often lacks the properties needed for such guidance. However, Watson's (2006) research on two teachers practicing informal formative assessment indicates that teachers' beliefs about mathematics and their students are more important for how the teacher interprets students' understanding than the influence of the actual elicited information. The results from an earlier study showed that raised awareness of processes of formative assessment might at least increase the sources of evidence of which teachers base their judgments. In that sense, formative assessment can work as a tool for increased equity in assessment practice since it is more likely that more frequent assessment based on multiply sources gives better assessment information (Morgan & Watson, 2002; Watson, 2006).

When investigating pre-service teachers' ability to interpret students' conceptual errors, Son (2013) found that a majority of them identified the errors as stemming from procedural aspects. Identifying errors does not help

to determine what to do about them. Research on sixth-grade teachers ability to determine what mathematic idea a task contains, how to interpret the students answer and how to respond or plan the next step of instruction to interpreting written student answers to mathematics tasks show that using assessment information to plan the next step of instruction tends to be the most difficult step (Heritage, 2007; Heritage, Kim, Vendlinski, & Herman, 2009). Schneider & Gowan (2013), who made similar investigations on elementary mathematics teachers, found that these teachers were equally skilled in the three investigated areas; identifying what an item measured, analysing student work, and determining the next step in instruction. In addition, providing students with targeted feedback was the most difficult task for the teachers in the study. These findings indicate that the interpretation and adjustment phase of formative assessment is not an easy task to take on for teachers. A study of science teachers in professional development using students' authentic answers to tasks Falk (2012) concluded that through this collaborative work the teachers used pedagogical content knowledge as an integral part of teachers' formative assessment practice. Knowledge of curriculum and of instructional strategies was the most common used and knowledge of student understanding was used when interpreting student work.

Teachers who practice formative assessment are also expected to support students to acquire the skills needed to enhance learning (Dixon & Haigh, 2009) designed a professional development program to change teachers' conceptualization and practice of assessment and feedback. One way to practice these skills is to invite students to participate and engage in the phases of formative assessment. For example, the students can contribute with questions, ideas and thoughts and listen to peers and discuss peers' ideas. The teacher's role is to help and create situations where the students can practice as participants so they can acquire the habits of mind that will enable them to share responsibility for learning and assessment (Cowie, 2005). One way to support students to be willing to reveal thoughts in front of the teacher and peers is for the teacher not to directly talk about the answers as right or wrong, but to acknowledge student contribution in other ways. (van Zee & Minstrell, 1997). Ruiz-Primo (2011) suggests several ways to acknowledge student contribution in a neutral way, to repeat, rephrased, clarify or summarize student utterances; to relate a student utterance to another students utterance; displaying students answers; promoting and responding reflectively to students questions.

Sufficient knowledge about the character and use of mathematics teachers' knowledge and skills when practicing formative assessment is lacking (Brookhart, 2011; Heritage, Kim, Vendlinski, & Herman, 2009). Pedagogical content knowledge is one area of teacher knowledge that has

been identified as playing an important role in teachers' formative assessment practices, but the ways in which pedagogical content knowledge contributes have not been empirically investigated in classrooms (Falk, 2012). However, studies of the development of teachers' pedagogical content-specific knowledge for teaching suggest that this knowledge can be developed through activities within or similar to formative assessment practices (Drageset, 2010). One teacher-scholar who has investigated her own teaching has also written extensively about the ways she constructed new understandings of students' mathematical strategies and thinking through careful attention to students' responses (Lampert, 2001). While these various studies demonstrate that there might be aspects of content knowledge that develop through close attention to students' thinking, Falk (2012), notes that there are no studies that explicitly examine the ways that teachers develop particular pedagogical content knowledge through particular formative assessment practices

In this section, previous research on teachers' role, and the difficulties that teachers meet, in practicing formative assessment have been presented. Furthermore, earlier research on teacher knowledge and skills used when practicing formative assessment have been introduced. I have outlined that sufficient scientific insights about what knowledge and skills teachers use when practicing formative assessment during their every-day practice is lacking.

2.1.6 Teacher professional development in formative assessment

The characteristics that are shown to be especially important in professional development programs (PDPs) in formative assessment for teachers with the aim of subsequently improve student learning include; a)administrative support, b) the possibility for teachers to be active participants in their learning and the decisions about the content of the PDP, c) time (i.e. contact hours and duration of the program), d) collaboration with colleagues, coherence between the PDP and other reform ideas, f) content knowledge (i.e. increasing teacher knowledge of the content they teach, how to teach the content and how student learn the content) (Schneider & Randel, 2010

What content is supposed to be most efficient? According to Guskey (2003) helping teachers to deeper understanding of the content they teach and students' learning progression of the content appears to be vital for effective professional development (Guskey, 2003). Knowledge of how student learn the content are especially important (Kennedy, 1998). More specific examination of the characteristics of effective PDPs of formative assessment is required (Schneider & Randel, 2010).

2.2 Teacher knowledge and skills

The lack of specific knowledge of what aspects of mathematic knowledge teachers need to know to help student learning (Askew, 2008; Hattie indicates the importance to investigate and characterize the knowledge and skills teacher uses. First, research of teacher knowledge and skills in the field of mathematics will be outlined in Subsection 2.2.1. Second, the framework used in this study for analysing such knowledge and skills is described in Subsection 2.2.2. Third, some complications when analysing teacher knowledge and skills are addressed in Subsection 2.2.3 and finally, the concept knowledge and skills for this study is defined in subsection 2.2.4.

2.2.1 Teacher knowledge and skills in the field of mathematics education research

Teacher knowledge is a widely researched area. Before 1986 research focused on characteristics describing general knowledge in teaching without regard to if there where special pedagogical skills that was needed for different subjects. The four first points in Figure 1 describe those general characteristics. Shulman (1986) named the lack of attention to the content as the "missing paradigm" (Ibid, p. 6). In addition to the four earlier mentioned categories he introduced and described three categories of knowledge attached to the content; teacher knowledge as containing content knowledge, knowledge of curriculum, and pedagogical content knowledge. With this, Schulman did not intend to limit the importance of general pedagogical knowledge but stated that teachers needed content knowledge that allows them to go beyond knowledge of facts or concepts to understanding of the structures of the domain.

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

(Shulman, 1987, p. 8)

Figure 1. Shulman's framework, Major Categories of Teacher Knowledge (Shulman, 1987 p. 8).

Shulman (1986) described *content knowledge* as "the amount and organization of knowledge per se in the mind of the teacher" (Schulman, 1986 p 9). He defined *curriculum knowledge* as knowledge about instructional materials available as well as topics and ways they ordinary are addressed during previous and subsequent years. *Pedagogical content knowledge* is described as "A second kind of content knowledge is pedagogical content knowledge, which goes beyond knowledge of subject matter knowledge per se to the dimension of subject matter *for teaching*" (Shulman, 1986, p.9).

After Shulman, a range of research has been conducted, following, developing or criticizing his work. Some of the critique have been on that the conceptualizing of the categories did not clarify the relations between the categories (Ruthven, 2011). Fennema & Franke (1992) modified Shulman's framework by suggesting that the knowledge teachers need is dynamic and interactive in its nature and that it often develops through interactions with the subject matter and the students in the classroom. Ball, Thames, & Phelps (2008) found the distinction between different categories in Shulman's framework too weak to be operationalized. Ball et al. (2008) aimed to develop a practice-based theory of content knowledge needed for teaching mathematics, starting with identifying what teachers did when they was teaching and which mathematical knowledge they needed to teach effectively. Their suggestion was to transform Shulman's category on content into subject matter knowledge (SMK) containing three categories: Common content knowledge, Specialized content knowledge and Horizon content knowledge (see fig 2. In addition, Pedagogical content knowledge (PCK) was also divided into three categories: Knowledge of content and students, Knowledge of content and teaching and Knowledge of curriculum.

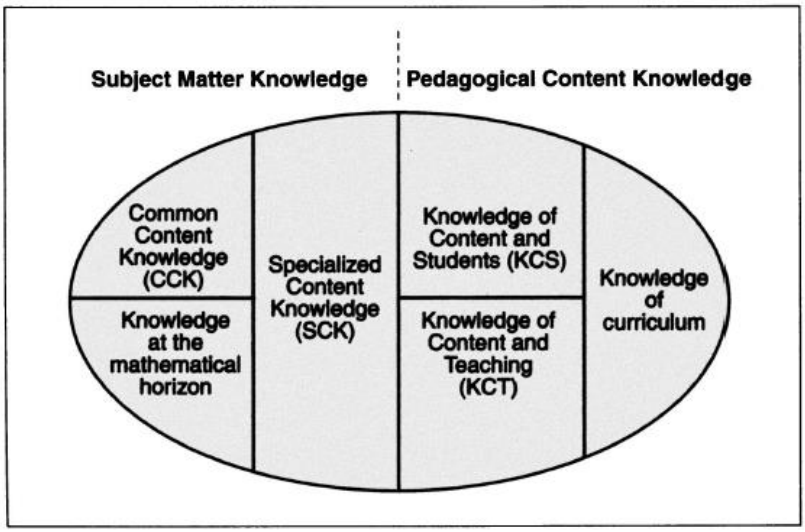

Figure 2. Content knowledge for teaching. (Ball, Thames, & Phelps, 2008)

Heritage (2007) suggests types of knowledge and skills teacher who practice formative assessment need. She describes four basic elements of teacher knowledge (including a number of skills) and in addition, four specific skills that pertain to pedagogical content knowledge that is not a basic knowledge in her framework. The basic elements of teacher knowledge are:

Domain knowledge. This includes concepts, knowledge and skills within the domain to be taught, including, being able to define learning progression, which could be used to clarify success criteria and to provide substantive feedback. Heritage also includes teachers' understanding of students' metacognition as it relates to assessment and students' motivational beliefs within the domain knowledge. I understand Heritage's notion of domain knowledge as corresponding or close to Ball's and colleagues (i.e. Hill, Ball, & Schilling, 2008; Ball, Thames, & Phelps, 2008) definition of subject matter knowledge.

Pedagogical content knowledge. In Heritage's view, pedagogical content knowledge includes familiarity with multiple models of teaching to be used to adapt instruction to student learning. This knowledge base also includes teachers' familiarity with multiple models of metacognitive processes and self-assessment skills.

Students' previous learning. Including teacher knowledge of students' level of knowledge and skills in the content area. It also includes knowledge of the

developing student' attitudes to the subject and their level of language proficiency.

Assessment knowledge. Including knowledge of formative assessment strategies to elicit information and knowledge of how to align formative assessment with instructional goals. Further, teachers must know that there are multiple sources of evidence on student learning, for example from student or peer assessment.

The four specific skills that Heritage (2007) suggests are all defined as pedagogical knowledge. The specific skills for practicing formative assessment that teachers need to be able to do are, according to Heritage; creating the conditions for the formative practice, using student self-assessment, being able to interpret evidence of student learning and matching instruction to the gap.

Brookhart (2011) suggests the *Standards for Teacher Competence in Educational Assessment of Students* to be updated due to formative assessment. These standards were developed with the purpose of guiding teacher educators developing teacher education program and work-shop instructors as well as functioning as an impetus for educational measurement instructors to conceptualize student assessment more broadly. (AFT & NCME & NEA 1990) The influences from formative assessment is the importance of students' as active participants in the assessment process, participants who need information that helps them make productive decisions (Brookhart, 2011).

This study focuses the use of knowledge and skills when the teachers practice formative assessment. The intention is neither to measure the teacher's knowledge nor investigate the relations between the knowledge the teacher has, nor is the intention to seek how the teachers knowledge develop or cause of the knowledge. I am interested in what knowledge and skills teachers use when practicing formative assessment. From these aspects, Shulman and Ball and colleagues framework are suitable. Heritage's framework is adapted to formative assessment and in that sense would be the most suitable framework to use. However, Ball and colleagues framework is widely spread in the field of mathematics and the framework is organized in the same way as teacher education (i.e. content courses, pedagogical content courses and general pedagogic courses). The framework is more developed and exemplified and describes how the different parts are related to each other. From these reasons a combination of the frameworks of Ball and colleagues and Schulman was chosen as the framework for this study. The framework is used to describe the character of the knowledge and skills used when a teacher practices formative assessment.

2.2.2 The framework used in this study for analysing teacher knowledge and skills for teaching mathematics

As described in Section 2.2.1, the base for the framework used to categorize teacher knowledge and skills in this study comes from Shulman (1986, 1987) and the framework *Content knowledge for teaching* (Hill, Ball, & Schilling, 2008; Ball, Thames, & Phelps, 2008). Shulman's framework is used to frame the general aspects of teaching. When defining the teacher's use of content knowledge, I use the framework Mathematical knowledge for teaching from Ball et al. (2008) since she has further developed and fine-grained the differences between content knowledge and pedagogical content knowledge from Shulman's framework (Hill et al, 2008; Ball et al, 2008). Subject matter knowledge (SMK) is divided into the subcategories specialized Content knowledge, Common content knowledge and Knowledge at the mathematical horizon. Within the category Pedagogical content knowledge the subcategories Knowledge of content and students (KCS) and Knowledge of content and teaching (KCT) is represented in the study. The subcategory knowledge of curriculum in the framework of Ball et al (2008) is beyond the scope of this study since the curricula artefacts not were included in the data collection. The category is therefore not represented in our framework. Thus, the categories of the framework used in this study are the following, which are also displayed in Figure 3 and further described in Section 2.2.2.1-2.2.2.6:

Common content knowledge (CCK)
Specialized content knowledge (SCK)
Knowledge of content and students (KCS)
Knowledge of content and teaching (KCT)
Knowledge of pedagogic, students and context (KPSC)

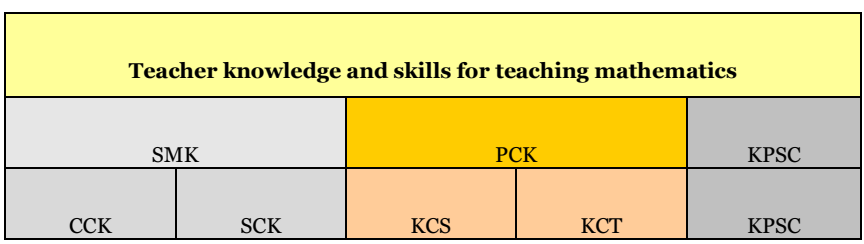

Figure 3. The figure shows the framework used in this study for analysing *teacher knowledge and skills for teaching mathematics*. The categories in the bottom row are used in analysis.

2.2.2.1 Common content knowledge (CCK)

CCK is defined as the mathematical knowledge and skill used in settings other than teaching, including

- Knowledge of the content that is required for the instructed content including calculating, reasoning, and justifying the content that is instructed. For example, when determining that 1,5 equals 3/2 the teacher uses her skills in calculating. An additional example is when a teacher is using her ability to reason when trying to justify a claim she never heard before.

2.2.2.2 Specialized content knowledge (SCK)

SCK is defined as mathematical knowledge and skill unique to teaching. This means such knowledge that are known in a self-conscious way that goes beyond the kind of tacit understanding of the content that is needed by most people. Including:

- Knowledge of how to accurately represent mathematical ideas and the skill to provide mathematical explanations for those. For example, the teacher can explain a mathematical concept in different ways and with different representations, such as representing fractions with drawings where each part are equal of size and being able to tell why they have to be that. Another example, when the teacher has got an incorrect answer she thinks about the different representations of the learning object she knows of and can choose among.
- Knowledge and skill to formulate productive mathematical questions, that is, questions that have the quality to indicate what action to take if the answer is incorrect.

2.2.2.3 Knowledge at the mathematical horizon.

Horizon knowledge is defined as an awareness of how mathematical topics over the span of mathematics are related. In this study no such use of knowledge where identified.

2.2.2.4 Knowledge of Content and Students (KCS)

KCS is defined as an amalgam between knowledge of content and knowledge of how students learn the content, including:

- Knowledge and skill on how students think about, know, or learn mathematics. This means knowledge of common conceptions and misconceptions including knowledge and skill of students pre-knowledge in mathematics. For example, a teacher recognizes the common misconception that students believe that the bigger denominator the bigger value of the fraction. She uses her

knowledge of the students' familiarity with the fractions ½ and ¼ to create a question.

- Knowledge and skills of how to interpret when student express their emerging and incomplete thinking. That is, when in dialogue with students', dealing with incorrect or fuzzy answers, the skill to recognize desirable concepts or misconception. For example, the teacher recognizes a student's common misconception during a fuzzy explanation that also includes incorrect use of notion.
- Knowledge and skill to decide which of several errors a student are most likely to make. That is, knowledge and skill to make a question easier or more difficult for the students to solve.

2.2.2.5 Knowledge of Content and Teaching (KCT)

In this study, KCT is defined as the knowledge of how to teach mathematics, including:

- How to sequence particular content for instruction. For example, the use of a logic learning progression in instruction or questions posed.
- Decision on sequencing of activities, including decisions on how to best build on students' prior knowledge. For example, the teacher knows that the students can determine if a fraction is bigger or lesser than a half. She uses that knowledge to challenge the idea that two fractions can be added by denominators and nominators.
- Design of instruction (and the advantages to use one representation instead of another). For example, when the teacher chooses to explain the size of the fraction 15/100 as an s a jigsaw with only 15 pieces, when the teacher chooses to draw the fraction instead of writing. The skill to design instructions also includes using a students' opinion to make a mathematical remark. For example, to let students' explanation correspond the teacher explanation.

2.2.2.6 Knowledge of pedagogic, students and context (KPSC)

In this study, KPSC is defined as knowledge and skills that appears to transcend subject matter, including,

- How to manage and organize the activities in the classroom. For example, the skill to organize activities to acquire valid information.
- Knowledge of students and their characteristics, for example when acknowledge contribution of student thoughts, the teacher uses her knowledge and skill of what and how to say so the students experience and accept the acknowledgement.
- Knowledge of educational contexts, ranging from working of the group to classroom. This includes skills to develop the context and to act according to it. For example, the skill to know when and how to

invite students as participants during the phases of formative assessment so the participation act makes sense, and is accepted of students. Knowledge of educational context in the classroom also includes teachers' approach to their own learning. For example, to take the opportunity to learn from students is one example of how a teacher can express the local educational context.

2.2.3 Complications

Since Ball's framework is framed from practice it brings in some of the "natural messiness and variability of teaching and learning" (Ball, Thames, & Phelps, 2008 p 403). She gives this as one reason to the problems with boundaries in the framework. "It is not always easy to discern where one of our categories distinguishes from the next" (ibid, p 403). Another problem is that which category that is used might be personal, For example, when a teacher analyse a student error the teacher might figure out the nature of a misunderstanding mathematically by searching for a pattern in the answer (using Specialized content knowledge), and another teacher might figure it out from her experience of what students usually struggle with (knowledge of content and students). A way to solve the problems with the boundaries between Subject matter knowledge (SMK) and Pedagogical content knowledge (PCK) is to investigate teacher knowledge and skills only from either the content perspective, as for example Drageset (2010) who focus his investigation only SMK. Son, (2013) investigates only pedagogical content knowledge PCK. In this study I use the formative purpose of the activities to identify the categories of the knowledge and skills used by the teacher. The method will be further described in section 3.5.3.

2.2.4 Conceptualization and operationalization of knowledge and skills

In this section I will first outline some epistemological issues when defining knowledge and skills. Second, I will describe how some of my main references have conceptualized or used knowledge and skills in their research. Third and finally, I will define knowledge and skills for this study.

What is knowledge and how can it be conceptualized? The two main epistemological paradigms are described as knowledge and learning as acquisition or participation. One of the theories from the acquisition paradigm is radical constructivism, which builds on Piaget's work and later developed by Glasersfeld. Constructivists see knowledge as not passively received, but actively constructed by the cognizing object. Knowledge is about organizing the world through experience into schemes (Glasersfeld, 1995). One problematic consequence with this view is to explain the problems of transfer knowledge from one context to another. The notion of

transfer of knowledge is one of the big issues discussed in this theory. Participation theories emphasize the socially and culturally situated nature of mathematical activity. To know is to be able to do (know-how) in a social community, or to participate in a discourse (Cobb, 1994).

> *There is currently a dispute over both whether the mind is located in the head or in the individual-in-social-action, and whether mathematical learning is primarily a process of active cognitive re-organization or a process of enculturation into a community of practice.* (Cobb, 1994, p.13)

Is it necessary to choose between the two perspectives? Cobb argues that "mathematical learning can be seen as both a process of individual construction and a process of enculturation into the mathematical practices of wider society" (Cobb, 1994, p.13). He encourages mathematics education research to use the theoretical perspective that gives the unit of analysis the best applicability, functionality and usability. He suggests using theory as a bricoleur, with no intension to achieve grand theoretical synthesis, neither contributions to the grand theories (Cobb, Stephan, McClain, & Gravemeijer, 2001).

Formative assessment itself is not positioned into any of the perspectives. It is, as Wiliam says, "an adequate account of classroom assessment must support any and all of these conflicting views of mathematics education, rather than impose a certain set of views" (Wiliam, 2007, p. 1055).

In the Standards, (AFT & NCME & NEA, 1990) the concept of teacher knowledge and skills are not defined. Knowledge and skills are used first by describing them in terms of activities, which are said to scope the teacher's professional role and responsibilities for student assessment. Second, from these activities, each of the seven standards is an expectation for assessment knowledge or skill that a teacher should possess in order to perform well in the activities. The Standards use the term skills as the use of knowledge in practice, for example "*Teacher will be skilled in planning*" (AFT & NCME & NEA, 1990, ¶ 5). The notion knowledge is used for example when they mention personal knowledge of facts. The standards also use notions of teachers' knowledge and skills as being "aware" and being "*able to*" (ibid, ¶ 7)

In this study, I follow Ball in her broad definition of knowledge in the purpose to build bridges "between the academic world of disciplinary knowledge and the practice world of teaching [...] by defining knowledge in broad terms, including skill, habits of mind, and insight, and by framing knowledge in terms of its use – in particular tasks of teaching" (Ball et al., 2008 p. 398).

In the definition, knowledge is the overall term used to frame all forms of knowledge. In the text in my study, the term knowledge is used in the same

way the term skill is used when referring to the use of knowledge in practice, as my interpretation of how my main references use the term (AFT & NCME & NEA, 1990; Heritage, 2007; Hill, Ball, & Schilling, 2008; Ball, Thames, & Phelps, 2008).

In the analysis I first analyse and categorize the teacher's actions into *activities* and then analyse the knowledge and skills that are used to make the activity happen, the same procedure as in the presentation of the Standards (AFT & NCME & NEA, 1990). In this study the teacher's actions is the starting point to identify the knowledge and skills used. That is, sometimes the identified teacher knowledge and skills in the study are implicit and more of habits of mind, and other time the knowledge and skills are explicit.

2.3 Aim and research questions

In the background I have outlined research on formative assessment and teacher knowledge and skills connected to the study. It is also coated that formative assessment practice gain student learning. From the background it is clear that specific knowledge of what mathematics teachers need to know in terms of knowledge and skills when practicing formative assessment is lacking. I have also found specific lack of empirical studies on the use of activities, knowledge and skills of teachers implementing formative assessment in authentic environments.

The purpose of this study is to add to that knowledge by investigating, describing and characterizing a) the formative practice b) orchestrated activities and c) the knowledge and skills she used, by a teacher practicing formative assessment during whole-class sessions in mathematics. The following are the more specific research questions in the study:

RQ 1 What types of activities constitute the teacher's formative assessment practice during whole-class lessons?

RQ 2 What are the similarities and differences in the activities used in the different cycles?

RQ 3 What is the character of the knowledge and skills the teacher uses for these activities?

RQ 4 What are the similarities and differences in the use of knowledge and skills used in the different types of cycles (responsiveness) in the formative assessment practice?

3 Method

In this section I will first introduce the research approach of the study (Subsection 3.1) and a description of the participants of the study (Subsection 3.2) and in section 3.2 after the data collection will be described. In Subsection 3.4 the proceeding of analyzes is presented. Finally, the studies credibility and the ethics are presented.

3.1 Research approach of the study

Qualitative research is driven by the wish to explain events or phenomena in real, every-day life (Yin, 2011). In this study the phenomenon is represented by one teacher's activities, knowledge and skills, which character I describe. The lessons is not designed or consciously affected by the researcher. In qualitative research, the researchers are considered part of the observed world and consequently, might change what they try to observe (Bassey, 1999). In this study, I experienced that in the beginning of the observing period I could sit and take notes without noticeably disturbing the teacher and students, but as the time went on the students became curious about what I wrote and both students and the teacher, Eva, involved me and invited me as participant during the lessons.

One example is when Eva asked if I understood a method that a student had "invented" for expanding fractions, another example is when a student asked me if 8 times 9 is 72. Merriam (2009) describes this change from almost complete observer in the beginning of a study to a participant observer as time passes during observation (Merriam, 2009). There is no way to tell exactly in what way my presence in the classroom has affected Eva and students. However, there is no indication that my presence has affected the choice of activities, knowledge and skills during the observation period.

Is the study a case study? Some would say that and others would disagree. In fact, researchers from many disciplines and paradigms call their work case studies and the descriptions of what characterizes a case study vary. However, some of the characteristics are generally agreed on (Hancock, 2011). These characteristics are, first, case study research generally focuses on an individual representative of a group, organization or phenomena. In my study I focus on how one teacher uses knowledge and skill so this criterion is fulfilled. Second, the object of study is researched in its natural context. This criterion is fulfilled in my study since I follow the ordinary lessons planned by the teacher. Third, a case study is richly descriptive since it is based on deep and various sources of information. The design of this study may not completely fulfill the criterion. The study is intended to be relatively richly descriptive, but does not involve several data sources. My main source is the transcript from the audio-files from the mathematics

lessons. Video-files and field-notes are used in the study for clarifying of utterances. For example, if the teacher says “look at this fraction”, the video-file is used to identify what fraction she refers to. Critics may claim that the teacher’s and students’ own voices are missing if I should suggest that this is a case study. Finally, during the act of collecting and processing data, the research questions in a case study develop in the act of investigating a topic in detail. This criterion corresponds with the research process during the period of observation and data processing described below. I refer to my study as a single-case study with the reservation mentioned above.

3.2 Participants and the context of the study

To provide a sense of the context in which the study take place, I’ll give a presentation of the school and how the teacher described herself, the class, and her view of the formative practice and how it works. For this description, interviews with the teacher and field notes are used.

The study is conducted at a school situated in a town in the north of Sweden. It is a small school. There are usually two parallel classes for each school year. The teacher and main informant in the study, Eva, is a legitimated teacher in mathematics and two other subjects. She has worked as a teacher for more than 20 years. Eva participated in a professional development program (PDP) on formative assessment in spring 2010 together with other mathematics teachers from a municipality in the north of Sweden. Eva was one of the teachers who changed her teaching a lot due to the program. Four of these teachers were asked if they had the opportunity to participate in the study, but Eva was the only one that had the possibilities and was also positive to participate in the study. The PDP was due to an earlier study in our research group that was designed and carried out by researchers in our research group (Andersson, Vingsle, & Palm, 2013). Eva considers the PDP as the best professional development she has ever had during her years as a teacher.

The class in the study is big, so it is divided in smaller groups at many occasions and the teacher has chosen mathematics to be one subject were the class should be divided. The teacher has organized the students in pairs, with a math peer with whom they are supposed to collaborate during the lessons. This means that they are supposed to discuss questions from the teacher during whole class sessions, they are supposed to first turn to their math peer if they are stuck when working and to listen and help the math peer that is stuck. To be able to do that, the teacher has instructed and trained the students in how to listen and ask specific question when they need help. Also, on the classroom wall there are posters where reminders on what kind of questions to ask as helper or help-seeker. There are also a whiteboard and a smart board in the classroom.

The teacher says that her strong side as a teacher is that she is not prestigious and is not afraid to make mistakes in front of the students. This is not something she learned during the professional development program (PDP) in formative assessment. It was during the PDP she realized that she herself contributes to the "*climate of positive failure*" which she thinks exists today. Before the PDP, she describes her responses to incorrect answers as comments like "Yes that was almost right". Even if she feels that she still can become quite consolidated sometimes, she feels that she can respond to students' incorrect answers in a more positive way now. The teacher uses a plurality of the activities and strategies known within formative assessment. For example, she uses tests made after 2/3 of the time for a topic. Instruction after the test is individualized for the students to focus the parts not fully understood. After a test, the students get comments instead of points. The comments are often formulated as two remarks about what the student has done well and one suggestion about how to improve. The students have a 10-minutes protocol where they are to assess their grade of mathematical activity at some occasions during one week. The purpose of this activity is to make the students reflect over their use of time in relation to their results.

The teacher's experience is that the students are more active during the whole class sessions since she started to use formative assessment. She mentions the students' small whiteboards on which they write their answers and the use of math peers as the cause of the change. She says:

> *"When the students have to answer on the whiteboards, it is not voluntary to participate or not, everybody has to endeavour and everybody get aware of that they'll have to deliver, even if there is only a question mark."*

Generally, the lessons start with a teacher-student interactive session. This means that the students sit with their math peer and a small whiteboard available. They answer the teacher's questions on the whiteboard and show their answers when the teacher rings a bell. The content of the sessions treats the goal for the week, which generally focuses the content of the students' work in their mathematics textbooks. Other times the topics have been chosen based on that the teacher has noticed that the students struggle with some specific content during individual work, or based on the students' answers to the three tasks she sometimes gives them in the end of a lesson. She describes the value of continuously assessing students understanding and using this information for decisions about the next step in her teaching:

> *"If I notice something, I want to address it at once. Lately I have noticed that some students do not know if a fraction is bigger or lesser than one. It benefits further understanding to have to be faced with arguments and counterarguments".*

3.3 Data collection

The study was conducted over a period of two and a half months, during their mathematics lessons. The teacher was audio-recorded during whole-class lessons. Eight to twelve students were audio-recorded during the lessons. Two thirds of the lessons were video-recorded. The teacher's audio files have been transcribed using the software Nvivo. If a student's response were un-audible in the teacher's audio-file, the student's audio-file was transcribed at that particular sequence. The researcher's field notes and video-files have supported the transcription with clarifications, for example, when the teacher points on or refers to something written on the whiteboard during interaction. The transcripts were translated to English by the researcher.

Documents used by the teacher during the lesson (i.e. lesson plans on power-points and student tasks) were gathered. These documents are used to assist in the description of the episodes and to support the transcriptions and analysis with information or clarification. For example, when the teacher gave the students three tasks in the end of a lesson to use in the formative practice next lesson, the students' answers were collected to the data. These documents, the students' answers, are used to interpret the teacher's interpretation of students' conceptions and misconceptions. When data collection ended, the work to analyze the field notes, documents, audio and video-files started with multiple review of the material, known as immersion, to get a perception of the overall meaning and sense in the data.

3.4 Selection of episodes

The four episodes that are analyzed in this study are chosen from instructional dialouges during whole-class lessons orchestrated by the teacher. I chose to study the instructional dialouges because this was where the teacher regularly used formative assessment on group-level during the time I conducted the study and that the collected data comprised.

Chein (1981,as cited in Merriam, 2009) argues that the most appropriate strategy when choosing sample in qualitative resarch is to choose the ones we could learn the most from To meet this aim, three steps of selection criteria were used; 1) character of formative assessment 2) content-rich episodes and finally 3) episodes that differ the most whithin Criterion 1 and 2. The characters found were medium length cycles, day-by-day short-cycles, and minute-by-minute short-cycles.(See Section 2.1 for further information of characters of formative assessment). A content-rich example contains a) at least two methods of eliciting information about student understanding and b) a dialogue with at least three turns between a student and a teacher or argumentation between student and student.

Twenty episodes including formative assessment were identified during the whole-class sessions. One of the episodes was categorized as medium length cycle (Episode 4) and one as short cycle day-by-day (Episode 3). These two episodes were chosen to be included in the study. The remainig eighteen episodes were identified as short cycle minute-by-minute. To choose which of the eighteen minute-by-minute episodes to analyse I used the criteria for content-rich episodes and indentified five of them as content-rich. One of these episodes, Episode 1, was finally chosen to be analyzed since this episode included the largest number of eliciting methods. This episode is characterized as *planned* using the terminology of Cowie and Heritage (Cowie 1999; Heritage 2007). One more episode was also chosen. The EIU-phases in this episode (Episode 2) differed the most from the first chosen short-cycle minute-by-minute episode, Episode 2 is also characterized as on-the-fly (Heritage, 2007) or interactive (Cowie, 1999).

3.5 Method for analysis

The methods that are used for analysis can be described as pragmatic from the aspect that I do not look for underlying causes to the teacher's actions. I only look for the actions themselves and the knowledge and skills used to make the actions happen. The action itself is analysed.

Important notions used in the analysis are defined in Subsection 3.5.1. In the next three sections, the three methods that are used for analysis in the study are presented. First, the method for identifying the three phases (Eliciting-Interpreting-Use) of a EIU-sequence will be introduced in section 3.5.2. Second, the method for identifying the teacher's activities during the phases is presented in Subsection 3.5.3. Finally, the method for categorizing the activities into the framework *Teacher knowledge and skills for teaching mathematics* is presented in Subsection 3.5.4.

In Subsection 3.5.5 there are a summary of the method of analysis.

3.5.1 Notions used in the analysis

The notions used in the analysis are:

Cycle Describes the type of formative assessment, based on the responsiveness of the feedback system (see also Section 2.1.1). The three different cycles used in the study are, short cycle minute-by-minute, short cycle day-by day and medium length cycle.

Episode An episode is the part of the lesson that is analyzed in this study. There are four episodes from four different lessons.

EIU-Sequence In the analysis, when an Eliciting-Interpreting-Use (EIU) instructional dialogue is identified during an episode it is noted as an EIU-sequence.

Phase Each EIU-sequence consists of the three phases; Eliciting phase, Interpreting phase and Use phase.

Action Teachers' acts that are identified as having a property assessed as belonging to formative assessment

Activity Actions that are identified in the same phase and have the same categorization in the framework *Knowledge and skills for teaching mathematics* were pooled together as an activity

(Mis)conception The notion (mis)conception is used to underline that in teachers knowledge of how students understand the learning object, it is equally important to recognise both desired conceptions and undesired conceptions (misconceptions) when practicing formative assessment. Misconceptions are in this study defined as the misapplication of a rule, an over- or under-generalization, or an alternative conception of the learning object.

3.5.2 Identification and analysis of EIU-sequences

The first step in the analysis is to identify EIU-sequences. Each EIU-sequence contains three phases; E (eliciting), I (interpretation), and U (use). I identify EIU-sequences by two criteria.

First criterion: The activities that start an EIU-sequence are supposed to have the potential to give information about what students know or don't know about learning objects (Eliciting phase). For example, the teacher might pose written or oral questions that give information about what students understand of mathematical procedures or concepts. In addition, students might take initiative to the EIU-sequence by posing a question. A concrete example of a teacher-initiated EIU-sequence is when the teacher asks the students to write the number one in as many forms they can on their small whiteboards. When they hold them up the teacher can see how everybody have answered the question. An example of a student-initiated EIU-sequence is when the student Kajsa has discovered a pattern from the examples the teacher has shown. She accepts the teacher's invitation to reflection by posing a question to the teacher about her discovery.

The second criterion used to identify the EIU-sequence is that the elicited information needs to be used by the teacher to create a student activity. For example, the activity might be that the teacher offers an explanation or poses a new question with the elicited information as a base.

The phase of interpretation is implicit and is therefore neither a criterion to identify the EIU-sequence nor identified as utterances in the sequence. In the analysis, the students' responses are also identified. This is necessary to be able to analyze the other phases in the sequence. So in the process of identification and of the phases the EIU-sequence becomes ESIU-sequence due to student response.

When the two criteria are identified, every utterance in the sequence is analyzed belonging to either of the phases E (eliciting), S (student response), or U (use). The Interpretation-phase (I-phase) is interpreted based from

what the teacher says and does in the use-phase. For example, after listening to a student's utterance, the teacher says, "Right now Kajsa said something unplanned, but I want you to take a look at this because it's a very good thing to check". Then she poses the question ½ + 2/3 = 3/6, is the statement false or true, and what are your arguments for your answer? This utterance and the act is the base for the interpretation of the teacher's interpretation in the interpretation phase.

The interaction between the teacher and students is not always a straight ESIU-sequence. When the teacher poses clarifying questions or checks her preliminary interpretation with a student the pattern of interaction might for example be E-S-I-E-S-I-U. The ESIU-sequence might vary in other ways also. One example is when the teacher uses information to pose a new question. Then the consequence will be an embedded ESIU-sequence since the new question might work both as a U-phase and an additional ESIU sequence. At such an occasion the instructional dialogue can be expressed ESI –U (ESIU).

Table 1. The table shows a short version of one EIU-sequence from Episode 1.

Short description of one EIU-sequence	Phase
Teacher's question: *What number is this? Write in as many forms as you can.*  The students are given time to think together with their math peer.	Eliciting-phase
Students hold up their answers on the white board.	Students' response
The teacher writes the students' answers on the big board; 5/4; 1,25; 1 ¼; 125/200; 125 %. The teacher interprets the answers and plan how to use the information.	Interpreting phase
Teacher: *What do you say about these suggestions? Do you have any questions on the answers?* One student questions the answer 125/200, and the student who gave the answer explains how he was thinking. The sequence ends with the teacher's explanation (see the full description in Section 4.1.1*)*	Use-phase/ (embedded EIU-sequence)

The table shows a an EIU-sequence were the interaction is shown as a straightforward E-S-I-U instructional dialogue. In the Eliciting phase, the teacher poses a planned question to the students, see left column The students answer the question on a small mini-whiteboard. Thus the teacher recieves answers (information) from all students. The answers are interpreted by the teacher and Eva decides to use the information to pose a new question, that is, to invite the student to interpret which answers that might be incorrect. Posing a new question is also to embed a new EIU-sequence since additional information is elicited. In this example, the new information elicited is that at least one student interprets 125/200 as incorrect, and the student who gave the incorrect answer describes how he thought when he answered the question. A full describtion of the use-phase/ embedded EIU-sequence is found in section 4.1.1.

For the four episodes, a total of 15 EIU-sequences are identified.The analysis is conducted on the transcribed text for the lessons and documents (e.g exit-passes and the teacher's powerpoint presentations) used during the episodes. This means that for the short cycle day-by-day and medium length cycle, the Eliciting phase as such is not a part of the analysis since they were elicited at another lesson.

3.5.3 Identification of the teacher's actions and activities during the practice of formative assessment

Step two in the analysis begins when the phases of the EIU-sequence is defined. That is, when it is clarified that it is formative assessment as defined in this study. Step two is the identification of the actions the teacher takes in every phase of the EIU-sequence (i.e. actions taken during the practice of formative assessment.

An action should have certain properties to be assessed as belonging to one of the formative assessment phases. It must meet at least one of the following criteria:

- Provides the teacher with valid information about what the student can or cannot about the learning object
- Allows, encourages and promotes student participation
- Influencing students' willingness to share their thoughts
- Processes elicited information
- Uses elicited information

An action might have one or more properties connected to formative assessment. Those are described in Empirical data and analysis (Section 4) as *connection to formative assessment.* In addition, effects or qualitative aspects of these properties are described.

Furthermore, actions that are identified in the same phase used in the same type of episodes and have the same categorization in the framework *Knowledge and skills for teaching mathematics* were pooled together as an activity. Some activities only include one action and in those cases, action and activity are identical. During categorization, Activity 5 parted into two activities (activity 5A and 5B) due to different coding.

3.5.4 Categorization of the teacher's use of knowledge and skills

When the activities are identified and described they are categorized into the framework *Teacher knowledge and skills when teaching mathematics*. This framework has been introduced and defined more thoroughly in Section 2.2.2. The categories of the framework that are used in the study are:

- Common content knowledge (CCK). CCK is defined as the mathematical knowledge and skill used in settings other than teaching.
- Specialized content knowledge (SCK). SCK is defined as mathematical knowledge and skill unique to teaching. This means such knowledge that are known in a self-conscious way that goes beyond the kind of tacit understanding of the content that is needed by most people.
- Knowledge of content and students (KCS). KCS is defined as an amalgam between knowledge of content and knowledge of how students learn the content, including knowledge of their common conceptions and misconceptions.
- Knowledge of content and teaching (KCT). KCT is defined as the amalgam between knowledge of teaching and knowledge of content, including knowledge of how to design instruction.
- Knowledge of pedagogic, students and context (KPSC). KPSC is defined as knowledge and skill that appears to transcend subject matter.

The categorization of teacher knowledge is based on the activities connection to formative assessment, noted as the purpose of the activity. This purpose (connection to formative assessment) is presented together with the categorization (interpretation and categorization) in connection to the activities, Sections 4:1.3; 4.2.2; 4.3.2.

The interpretation I do for the categorization is not necessary the teacher's conscious purpose with the activity.

In some cases categories might be overlapping. That is, a particular knowledge might belong to two categories. In the cases where formative assessment won't be decisive, argumentation on the choice of category is presented.

In the categorization of knowledge and skills, categorization of the same activity into two different categories is only made when the teacher uses knowledge of two different categories. For example, when the teacher listens to a student's answer she first uses Common content knowledge to recognize that the answer is, for example, incorrect. The next step for the teacher using formative assessment is to interpret the nature of the mistake. That is, to collect information about, not only that the student could not do this mathematics, but also gather information about the student's thinking. This requires using Specialized content knowledge. Double categorization for activities is made in such cases.

3.5.5 Summary of the method of analysis

To be able to answer the research questions of what activities, knowledge and skills the teacher used during her formative assessment practice, the data were analysed in four steps;

The first step is to identify the EIU-sequences and the three phases of Eliciting Interpreting and Using, and establish that there is a formative assessment practice. The second step is to identify the teacher's actions during the phases. In the third step the actions are categorized into the framework for teacher knowledge and skills for teaching mathematics. In the fourth step actions that have the same overall purpose and are categorized the framework *teacher knowledge and skills* as belonging to the same type of knowledge and skill in the framework, are pooled to an activity. A more detailed description of additional properties assessed as belonging to formative assessment, effect or quality for the action or general aspects of the activity is presented under the rubric connection to formative assessment in Section 4:1.3; 4.2.2; 4.3.2. This is presented together with activities and the interpretation and categorization.

3.6 Credibility

In case studies, credibility is evident by well-specified and well-described boundaries (settings, participants, procedures and technical tools) of the research (Hancock, 2011).

In this study the requirements described above have been addressed in what follows; its participant's procedures and technical tools are described in this section. In the background, the context of the study is described from the teacher's view, through interviews and field notes, and from the researcher's observations. Important concepts used in the study are defined and exemplified. I have endeavored to be transparent and clear about the decisions and procedures in the research process, when appropriate I have exemplified to clarify my meaning.

The purpose of a single-case study is not to find out what is generally true of the many, and generalizability in the statistical sense cannot occur in qualitative studies. In a single case study, the general lies in the particular; that is, what we learn in a particular situation we can transfer or generalize to similar situations subsequently encountered (Merriam, 2009). Yin (2011) develops the thoughts of generalizability in qualitative studies by arguing that instead of generalizing the results to other contexts, a qualitative study should try to develop and then discuss how the conclusions might contribute to a better understanding of the concepts or the phenomenon it studies.

In this study the above means that the conclusions that this study presents is the existence and character of teacher knowledge and skills when the participating teacher practices formative assessment. The conclusions are discussed and compared with earlier findings in the field of research. Altogether, this gives the reader the possibility to assess how the findings are applicable to their own context.

To avoid that my own unconscious biases would affect the findings, I regularly wrote intuitive findings. The purpose of this was that I experienced that since I already had wrote them down I would not search them out in the process of analysis and by that risk to bias the findings.

3.7 Ethics

The study is based on voluntary participation and is conducted under the principles from Sweden's Science Councils on good research ethics that aim to provide standards for the relationship between researchers and informants (Vetenskapsrådet 2011). In order to fulfill the information requirements, prior to the entry of the study the teacher and students were informed of the purpose of the study, how it would be implemented and how the data would be used. The data collected during and after the study is handled in accordance with the confidentiality requirement to protect the participants. To prevent identification of school, teacher or students the informants has received fictitious names in the study.

4 Empirical data and analysis

Empirical data and analysis is presented in three sections (4.1-4.3), one section for each type of cycle.

4.1 Short cycle (minute-by-minute), Episode 1 and 2

In this section, first a description of Episode 1 and Episode 2 (an episode is the analyzed part of a lesson) will be presented (Subsection 4.1.1-4.1.2). After that, the teacher's activities during the practice of formative assessment will be presented (Subsection 4.1.3). Along with the activities, the categorization

of the activities into the framework of *Teacher knowledge and skills for teaching mathematics* is presented.

4.1.1 Description of the teacher's instruction, Episode 1

The area that the class works on is about fractions and today Eva connects fractions to the earlier section, decimals. She presents a power point on the smart board and the first slide shows a blue square. On the slide it says, "*The blue square represents the number One, a Whole*". The square is used throughout the lesson as a Whole. Eva asks questions such as "*How do you write ¼ as decimals"*. The students write their answers on the whiteboards and show the whiteboards to Eva. All answers are correct.

The first Eliciting phase where Eva elicits information that she interprets and decides to act on starts when Eva shows the blue square, which they earlier defined as the number One, and asks the students to *"write corresponding numbers in as many forms as you can"* on the whiteboards. The students are given time to think and discuss and when Eva rings a bell, the pairs hold up their Whiteboards with the answers. Eva displays the different answers from the students; 1/1,0; 4/4; 1,0; 100 %; 1 on the big whiteboard and says that there is one answer she is curious about, pointing at 1/1,0. Tom, who has given the answer, says: "*That was mine and Annie's!"* Tom explains that he has written the answer in mixed form.

Eva confirms that Tom has done a mixed form of fractions and decimal form and she also tells him that it is mathematically correct.

Eva: That was a smart mix of different number forms, but I'm not sure anyone uses that mix. There is a form called mixed form, but then you mix integers and fractions.

In the next Eliciting phase Ewa shows a new slide to the students, Figure 4. She reminds the students that the blue square still represents a whole.

Write corresponding numbers in as many forms as you can.

Figure 4. The figure displayed on the whiteboard.

Eva displays the students' answers on the big board; 5/4; 1,25; 1 ¼; 125 %; and 125/200 and this time she asks the students if there are any of the answers they want to discuss. The following dialogue takes place, starting with Harald getting the word from Eva.

Harald: 125/200. I don't really understand.

Patrick: I can argue for that.

Eva: Yes, let's hear your arguments Patrick.

Patrick: Eh, we thought that there is one whole. And then there is twenty-five, or I thought. And then that a whole…if, there are only room for four, or three more cubes there, I thought that if, if you fill out with three, then you have another of them, and then I thought that twenty-five is, is divided by four, of one. That is, let' say that one is hundred.

Eva: I think you thought that you should do one more of these? [Pointing at the square representing a whole]

Patrick: Yes. I thought that there where one, well, that was what you were supposed to do.

Eva: Yes, exactly, I do understand how you. […] You thought that you had a hundred there and you had a hundred there. Then you would have two hundreds. We had 100 hundreds colored there but only twenty five hundreds there. Obvious, did you all get that explanation? [Turning to the class]

Now Eva has elicited information from Patrick about the way he was thinking, which she interprets and acts on by offering an explanation.

Eva: What Patrick did which you cannot do, what you have to think about is that you are not allowed to, there were two different figures there. There was one Whole there and the beginning of another one there but that was not finished. You cannot put them together and think of them as two hundreds, so that they make one figure.

Because you see this number, the number we had here, it was more than one. We had filled one whole and then we had some more. But this fraction [points at 125/200], it's almost just a half, and then this [points at the figure] is not it.

But you're thought, your logic was very, it was a logic thought, and it shows that you can think just right but still misunderstood for example that one could add the hundreds and make them two hundreds. And so on. And that was a very good example. I thank you very much for the explanation, Patrick.

Eva asks Stefan: "*Can you determine if this fraction [pointing at 5/4] is bigger or lesser than one by looking at it*? Stefan says that he can see that it is bigger, but he cannot explain why. Eva asks his math peer if he can help out which he does, and Stefan adds, "*That is how I thought.*" Eva reminds the class that this issue was a question on previous week's homework and

asks Ida, "*Can you tell me how you determine if a fraction is bigger or lesser than one?* Ida answers that if the nominator is bigger than the denominator, it's more than one. Lena agrees and fills in with explaining how to determine that a fraction is lesser than one.

Now Eva goes back to the planned lesson. She shows a statement 5/4 = 1 ¼. She tells the students that she is going to use the randomize devise, a necklace with the students' names on, to choose who to answer. She asks the students " *When you are going to write 5/4 in mixed form, how do you know that you're going to write it as 1¼?*". After think time and time to discuss, Anna is randomly chosen by the randomize devise.

Anna:	Well, it was like, the Whole, that was the four, so it was a bigger number, so one more, five, up there, then it becomes one whole and then another one, one more piece, one fourth.

Eva rephrases and clarifies Anna's words, starting with "Anna said that..."and continues with drawing and showing on the board how 5/4 can be transformed to 1 ¼. She ends the explanation by asking Patrick if he is "with her". He answers yes.

4.1.2 Description of the teacher's instruction, Episode 2

The teacher, Eva, has prepared the session with a power point presentation on the smart board. Eva poses questions to elicit information on how the students understand the content they have been working on for the past week, to expand fractions by multiplying numerator and denominator. Eva uses both whiteboards and the randomize devise to elicit information. When Eva poses the last questions they have expanded ½ by 2 to 2/4 and 2/4 by 2 to 4/8. The information she has elicited so far has not given anything to act on until she poses her last planned question. The episode begins when the planned presentation is finished.

Eva:	Is there anyone who has some reflections on expanding fractions? Kajsa?
Kajsa:	Hang on, can't you, three sixths, can you also, it's just as much as...?
Eva:	Yes, it works just fine!

Eva gives an oral description on how to expand ½ to 3/6 by multiplying with 3 and describes that it is possible to extend with any number. Kajsa is not satisfied with the explanations.

Kajsa:	If you add on with the first number, if you take two fourths and then you add on the first number we had.
Eva:	Add one half?
Kajsa:	But with, add the two with the one.

Eva: You add two and one?

Kajsa: Mm, and four and two.

Eva: All right, yes, then you'll also get… you think that, you could take one…

Kajsa. Yes, and three sixths.

Eva: Mm, yes.

The dialogue continues a bit and students in the class give signs that they do not understand what Kajsa and Eva is talking about and Stefan interrupts.

Stefan: I don't understand anything!

Eva: That's ok; you do not have to understand anything right now. The best with math, it's that the starting point is that you are supposed to deal with things you don't understand yet. Things you are on the way to solve and figure out. That's why it's normal not understanding in math. Otherwise we would deal with stuff that is much too easy.

Eva believes that she now has enough information to interpret Kajsa's thoughts. Eva uses her interpretation of Kajsa's thoughts to formulate a statement for all students to work on. She writes ½+2/4 = 3/6 on the board.

Eva: Right now Kajsa said something unplanned, but I want you to take a look at this because it's a very good thing to check. I want you to discuss with your math friend, is it correct or not that one half and two fourths is three sixths? If it's true, then what proof do you have that it's true? And if it's not true, what is the proof for that? Take a moment to discuss.

Eva tells the students that she will use the randomizer devise, the necklace with the students' names on, to choose whom to answer. After a couple of minutes she has raffled Ida and Robin to answer. Ida claims the statement to be false since a half is a half and you are supposed to add two fourths to get three sixths which are a half, so you will end up with more than three sixths.

Eva has got the wrong interpretation once more. After a long dialogue with Kajsa where Eva follows every step in Kajsa's description, it shows that Kajsa has found a method to expand fractions unknown to Eva. You can add denominator to denominator and nominator to nominator for two fractions of the same value and get a third fraction with the same value

(½ +2/4 =1+2 in the denominator, and 2+4 in the nominator= 3/6). Eva tries this statement for additional fractions. Once more, peers get impatient.

Robin: I do not understand a thing!

Eva:	Yes, [laughing] but it's very exciting!
Robin:	Well, I do understand but still, I do not understand, I think it's totally wrong!
Eva:	Yes, yes, that's all wrong (points on 1/2 + 2/4=3/6 on the board) because you cannot add the nominators with each other, you know, I'll let you know, this is like saying one cow and two birds is three elephants. You know, this is like different kind of pieces that you cannot mix. If you are going to add a half and two fourths you can use the method of expanding fractions or thinking like you did.

The four minutes that follow, Eva tries Kajsa's method on additional fractions and concludes that it seems to work for every fraction. If you add two fractions of the same value in the way that Kajsa did, you´ll get an additional fraction of the same value. Eva cannot explain to Kajsa and the class why it works other than that it probably has to do with "*multiplication and addition is connected.*" Finally Eva says, "*The thought of Kajsa was very exciting and this is something that I and Lotta will have to work on and come back to.*"

4.1.3 The teacher's use of activities, knowledge and skills

In these sections, Eva's *activities* (e.g. actions that are identified in the same phase and have the same categorization in the framework knowledge and skills for teaching) found during short cycle minute-by-minute EIU sequences are presented. An EIU sequence contains at least one Eliciting phase, at least one interpretation phase and one Use phase where elicited information is used. In Episode 1 there are three EIU sequences. In Episode 2 there are four EIU sequences. The EIU-sequences are not presented.

Most activities occur regularly, and are exemplified with *actions (i.e.*. teachers' acts that are identified as having a property assessed as belonging to formative assessment). For example, the activity *Engage students as participants in the phases of EIU* occurs regularly but is exemplified and categorized with one action for Episode 1 and 2, and with other actions in Episode 3 and in Episode 4.

An action might have one or more properties connected to formative assessment. Those are described below in *connection to formative assessment*. The interpretation of the purpose with the activity and categorization into the framework *Teacher knowledge and skills for teaching mathematics* is also presented.

Notions used in the analysis are defined in Section 3.5.1.

4.1.3.1 Eliciting phase

Activity 1 Creating eliciting situations
Action A: Eva uses an all-response-system, consisting of mini-whiteboards, to elicit information about all students' knowledge and thinking at the same time. Through the use of these whiteboards all students simultaneously provide responses to her questions and tasks. One example of this is in the beginning of Episode 1 when the students are asked to come up with as many ways as possible to write the number one, and write them on their whiteboards. Within two minutes Eva knows how all students have answered. In this case the answers were all correct, however one answer were in an unusual form 1/1,0. With answers on the whiteboards Eva quickly knows how all the students (pairs) have answered.

Connection to formative assessment: The use of an all-response system provides the teacher with information about all students' thinking on a task and not only a single student's thinking. High-quality formative assessment entails modifying instruction to meet students' learning needs. This requires eliciting valid information from all students in the class. Valid information is essential to be able to make appropriate interpretations and adjust instruction in such a way that it helps the students.

Action B: Eva uses a device that randomly selects a student to answer the question. The device is a necklace with one student name on each pearl, which is used in both episodes. Eva uses the randomizer device in Episode 2 when she draws Ida and Robin to answer if 1/2+2/4 = 3/6 and what their arguments are for their answer.

Connection to formative assessment: The use of a randomizer devise signals to students that the teacher is interested in all students' thoughts, not specifically the ones who can come up with the right answer (e.g. hand-raisers in the regular classroom) and can therefore result in an increase in students' willingness to show their thoughts, even if they are unsure. All questions are not possible to answer quickly in written form, as with whiteboards, but still important to pose. For the questions where the students are to justify their answer with arguments the teacher uses the randomizer devise to still engage all student to think about the task.

Action C: Eva in the study provides the students with sufficient time to think before they give their answers. Eva also urges the students to discuss with their math peer. For the two first questions with whiteboard in Episode one, the students get a little more than one minute to think, discuss and write the

answers on the board. When she uses the randomizer device in Episode 2 she waits two minutes before a student is randomly chosen to answer the question. One example that indicates that this think and discuss time also engages students to participate in arguing is when Patrick has answered 125/200 in Episode 1. He is eager to explain to Harald how he thought when Harald questioned Patrick's answer.

Connection to formative assessment: Time to think gives the student opportunity to reflect on their prior knowledge Research shows that the typical wait-time between questions asked and students' answers is one second (Rowe, 1974). That amount of time allows no deep thinking or reflections. The possibility to discuss with a math peer might renders students to not be afraid of arguing for their view or presenting faulty answers since they know they were not alone with this thinking. The interpretation based on students' answers will also be better founded since the answers will be based on extended thinking. Consequently, the adaptation of the teaching, based on the information about student thinking, is more likely to better meet the students' learning needs. In addition, all students will have better opportunity to learn from the task solving since they all have had time to engage in thinking about the question.

Interpretation and categorization Activity 1. The purpose of this activity is to provide valid information on what students know or don't know of the learning object and to encourage students to participate by engaging in thinking on task, discussing with a peer and sharing thoughts with teacher and peers. To be able to implement the actions described above, Eva used knowledge of the activity described above to organize and handle the eliciting actions. This is knowledge that not require mathematical knowledge and consequently categorized as knowledge of pedagogic, students and context (KPSC).

Activity 2 Using rephrasing and clarification to elicit verbal information from individuals

Action: During the dialogue with Kajsa in Episode 2 Eva rephrases Kajsa's words, to direct the dialogue. There are also examples of when she clarifies students' utterances, meaning both sorting out less important information and controlling her interpretation of the students' utterances. One example of that is during Episode 1 when Patrick described how he was thinking when he answered 125/200 and Eva controlled her interpretation with Patrick by clarifying the utterance: *"You thought that you had a hundred there and you had a hundred there. Then you would have two hundreds. We had 100*

hundreds colored there but only twenty five hundreds there". Patrick confirms her clarification.

Connection to formative assessment: To be able to elicit information from students who might have difficulties to express their emerging or immature thinking rephrasing and clarifying can be used to maintain and orient the interaction between student and teacher and to check the interpretation (Ruiz-Primo, 2011). This increases the possibility to elicit valid information. Rephrasing and clarifying students' utterances are also a way to verify students' contributions and effort without judging them directly (Primo-Ruiz, 2011).

Interpretation and categorization Activity 2: Rephrasing and clarifying have the purpose to direct the dialogue in a desired mathematical way to provide Eva with valid information, which poses the activity as requiring content knowledge. Rephrasing requires use of Common content knowledge (CCK) as when Eva recognizes an incorrect notion and changes to a correct notion when rephrasing the student's utterance. When Eva clarifies the fuzzy student utterance she sorts out important information on how student understand the concept, which is use of knowledge of content and students (KCS).

Activity 3 *Using open-ended questions to reveal thoughts on mathematics*
Action: Eva uses open-ended questions to reveal students thoughts on mathematics in Episode 1. For example, when Eva asks the student to write the number 1 in as many forms as possible she elicits the answer 1/1.0, which represents a conception of the term that the student had constructed himself or met in another context. Questions are also used to adjust instruction, see Activity 8.

Connection to formative assessment: Questions, which elicit information, might have different focuses (see also Activity 4). To elicit information that represent students' prerequisites or mathematical thinking that is unknown to Eva, questions that have several possible correct answers may be used. Such open-ended questions are known to engage students in thinking and thus provoke answers based on prior knowledge unknown for the teacher (Zee, 1997). In the episodes Eva elicits information by asking questions. Questions might have the two-folded purpose to both elicit information and to create learning situations (Araceli Ruiz-Primo & Furtak, 2006). The latter aspect is connected to the use-phase and consequently analyzed in Activity 8.

Interpretation and categorization Activity 3: When the teacher formulates open-ended questions in the Eliciting phase the purpose is to reveal students' misconceptions, prerequisites or mathematical thinking not known for the teacher. The teacher knowledge that students might create their own number forms, and that it is a common difficulty with connection of number forms is knowledge of content and students (KCS).

Activity 4 *Using how and why-questions to reveal thoughts on mathematics*
Action: Eva uses how or why-questions in both episodes. For example, in Episode 2, Eva asks the students to determine if the statement ½ + 2/4 = 3/6 is true and to justify their answers.

Connection to formative assessment: Questions that require the students to justify their answers can provide the teacher with valuable information about misconceptions student hold, and thus give indication on how to adjust instruction to better meet the student needs. When answering how and why questions, students have to explain how they were thinking which gives the teacher the possibility to make a more direct inference about students' understanding. When answering the questions in the example, the students have the possibility to use their prior knowledge of fractions size (halves) to prove that the statement is false.

Interpretation and categorization Activity 4: One purpose with how and why-questions is to reveal misconceptions, and this type of questions requires the students to justify their answers. Hence, the activity requires the use of teachers' knowledge of content and students (KCS) to formulate the question on issues that Eva knows challenges what students often struggle with. The teacher knowledge to represent the addition of fractions with a false statement of known fractions, and do it correct, when formulating the question is Specialized content knowledge (SCK)

4.1.3.2 Interpreting phase

Activity 5 was separated during the categorization into the framework Teacher knowledge and skills when teaching mathematics. The activity is labeled 5A (for these episodes) and 5B for coming episodes because of the difference in coding.

Activity 5A Using knowledge of common misconceptions to interpret incorrect answers

Action: One example when the teacher recognizes misconceptions are when she sorts the information she gets from Patrick's fuzzy answer in Episode 1. She interpreted that he had made one figure of the two (i.e. he defined the two figures together as a whole instead of one figure being the whole, which Eva had defined it to be). We assume that Eva must know about this kind of

misunderstanding, to make this interpretation from the information she gets from Patrick.

Connection to formative assessment: During the interpretation phase of formative assessment, the teacher interprets students' responses on how they understand the concept. We use the notion of (mis)conception to emphasize the dual purpose. This means, it is as important to be able to identify desirable concepts as well as unwanted to make better-founded decisions on next step of instruction.

Interpretation and categorization Activity 5A: First the teacher recognizes the answer as incorrect, using Common content knowledge (CCK). To be able to quickly interpret what Patrick's misconception is based on from the information she gets, Eva first have to sort out important information (that he makes one figure from two) from unimportant information (for example, that he names one figure both one and a hundred) from Patrick's talk to understand (using KCS). Second, knowledge of students' common misconceptions is usually helpful to be able to interpret what the student knows and not knows about the learning object (using KCS).

Activity 6 Handle interpretation of unknown mathematics
Action: In Episode 2, when Eva finally understands what Kajsa means when she describes her method to expand fractions Eva is in a situation where she does had not anticipated the method suggested by the student. The method is new to Eva and she does not know if and how it works. Still, to be able to interpret Kajsa's thoughts Eva has to understand the method so she uses her problem-solving skills, and her knowledge of mathematics in her effort to understand Kajsa.

Connection to formative assessment: Teachers who practice formative assessment seeks to elicit students' thoughts about mathematics. When doing so students may sometimes bring up mathematics that is neither anticipated by, nor familiar to, the teacher. Thus, to be able to use formative assessment adequately the teacher must be able to deal with mathematics that is new to her. This kind of situations is not likely to happen often for a regular teacher's practice, but more likely for those who practice formative assessment.

Interpretation and categorization Activity 6: The purpose of this activity is for the teacher to make sense of student answers. To be able to interpret Kajsa's method Eva tries to derive understanding of the method by using her knowledge of mathematics to reason and justify mathematics, knowledge that are part of Common content knowledge (CCK).

4.1.3.3 Use phase

Activity 8 Using elicited information to design instruction
Action: Eva has interpreted Patrick's thoughts when he answers 125/200 as that he made one figure out of the two (one figure was defined a whole). Eva addresses this in two ways. She wants him to use his pre-knowledge to recognize the value of the fraction 125/200 compared with the drawing of the fraction one and a fourth. She also tells him that he is not allowed to put together two figures to one, once they are defined.

Connection to formative assessment: The purpose of this action is to adjust instruction to better meet learning needs from elicited information. In the example Eva uses explanations when she uses the elicited information. In other cases in the episode she also uses questions to adjust instruction.

When a teacher poses questions in the use-phase the main purpose is to create a learning opportunity with a question. Still, a question is also an opportunity to elicit more information, That is, an embedded EIU sequence in which the use-phase accounts for students explaining for their peers (see also Activity 8, section 4.3).

Interpretation and categorization Activity 8. Eva addresses Patrick's thoughts in two ways. She wants him to recognize the value of the fraction 125/200 compared with the drawing of the fraction one and a fourth. This is KCS, since she uses knowledge of his pre-conception of fractions lesser than one. She also tells him that you are not allowed to put together two whole to one during operation, which is Specialized content knowledge. Finally, to decide the design of adjustment, thinking of advantages to use one way instead of another, is to use knowledge of content and teaching (KCT). Posing questions is analyzed in the Eliciting phase (Activity 3 and 4).

Activity 9 Introduce new topics.
Action: A topic new to the students is introduced when Eva poses the statement ½ + 2/4 = 3/6, true or false? Earlier, the students had only met addition with the same nominators, but based on the interpreted information, Eva challenge their conception of fraction size and their habit of addition with the statement.

Connection to formative assessment: When eliciting information unexpected topics might appear when listening to student thoughts. To act on student misconceptions, introduction of an additional learning object is sometimes required. Introducing a new learning object should be done to makes sense for the students, and the information that is given should follow a learning progression logical to the students.

Interpretation and categorization Activity 9: The purpose of this activity is to use the elicited information to meet learning needs. When Eva decides to introduce an additional topic based on the information she has elicited about the learning object she is using knowledge on how to best build on students' prior knowledge. She is also using her knowledge of a logical learning progression when students meet addition with fractions. Knowledge of student pre-knowledge and how to best build on this pre-knowledge is part of knowledge of content and student (KCS).

Activity 10 Using the unknown mathematics to model learning opportunities

Action: One way to handle situations where knowledge on how to respond to the content is lacking is to act on them to model learning opportunities. The example of how Eva handles the unpredictable as learning opportunities is when she in the spirit of creating and forming the session together makes no excuses when she does not have the answer to Kajsa's method of expanding fractions. On the contrary, through her behavior (see Activity 13) she models that she finds it exciting to learn something new.

Connection to formative assessment: When practicing formative assessment on-the-fly it is not possible to predict the topics for the lesson in detail. A consequence of formative assessment is that Eva will not always know answers to questions or directly know how to act in unpredictable situations. This creates the activity *using the unknown*, which makes many teachers feel insecure. Teachers who practices formative assessment on-the-fly needs to feel comfortable in those situations or they will probably start to avoid them (Cowie 1999). In the example, Eva does not avoid these situations, on the contrary she finds them "exciting".

Interpretation and categorization Activity 10: The purpose of the activity is to handle situations when teacher knowledge or experience is lacking. The skill to act on them as learning opportunities is an expression of the educational context that is created in this classroom and is therefore use of knowledge of pedagogic, student and context (KPSC).

4.1.3.4 Not phase specific

Activity 11 Engage students as participants in the phases of EIU

Action: One example of when Eva engages students as participants in the phases of EIU is when Eva displays the answers 1¼, 125%, 12.25 and 125/200 on the board during Episode 1. Eva lets the peers reflect on the answers, to see what they have understood, and then continue with how to

increase their understanding about the things they had not understood. In this example Harald says he does not understand the answer 125/200, and Patrick volunteers to tell Harald how he had been thinking.

Connection to formative assessment: For the teacher to be able to elicit information the students have to be willing to share their thoughts with their peers and teacher. Engagement as participating in the phases of EIU is therefore critical. Many of the students show this willingness, there is seldom a student who passes during the episodes and they often ensure that their answers are displayed on their mini-whiteboards.

Research shows that there are additional positive effects with students' participation. To engage students as active agents in the phases of formative assessment facilitates their possibilities to become self-regulating learners and increases motivation (Chappuis & Stiggins, 2002). The teachers' role to enable this is to help to create situations where the students can practice as participants so they can acquire the habit of mind that will enable them to share responsibility for learning and assessment (Cowie, 2005).

Interpretation and categorization Activity 11: The purpose of this activity is to make the students willing to share their thoughts with their peer and teacher. To be able to do that, Eva uses her knowledge of pedagogic, students and context (KPSC), knowing in what way students can participate (e.g. displaying answers on the big whiteboard). To practice this regularly is to use the knowledge of how to, first, develop the educational context that includes student participation, and then to act according to it. To be able to invite students in a way that works for the students (regarding their pre-knowledge is enough) to debate the answers, is knowledge of content and students (KCS).

Activity 12 Using acknowledgement of student contribution to engage students.

Action: In Episode 1 and 2, students who contribute with thoughts to teacher and peers get praise for their contribution as logical, obvious, smart or exciting. When student have contributed with thoughts and ideas, Eva does not talk about the ideas directly as right or wrong but acknowledge their contribution in different ways. She might tell what kind of mistake the student made, as in the beginning of Episode 1 when she told Tom that 1/1.0 not was wrong, but still not a common way to write the number one. The main idea that becomes visible from the teacher's way to acknowledge student contribution is that the body of knowledge increases as we listen to each other and incorrect answers is as useful when we learn as correct ones.

Connection to formative assessment: The purpose of this activity is to make the students willing to share their thoughts with their peer and teacher. To acknowledge contributions can make students experience that engaging in sharing of thoughts adds positive to the session. According to Zee (1977), this nonjudgmental acknowledging promotes student engagement.

Interpretation and categorization Activity 12: The purpose of this activity is to make the students willing to share their thoughts with their peer and teacher. Many of the students show this willingness, there is seldom a student who passes. The skill to know about and use acknowledgement of student contribution is a matter of manage teacher activities and as such knowledge of pedagogic, students and context (KPSC). In the example with Tom, to do this in a way that the student understands and accepts, the teachers refers to Tom's uncommon answer not as incorrect but as "a smart way to mix, and then describes what mixed form is. This requires knowledge of content and students (KCS).

Activity 13 Practicing formative assessment on-the-fly
Action: When Eva invites students to reflect on the instruction in Episode 2, she has no idea what to expect, except that she is set to listen to the students' thoughts. However, the dialogue with Kajsa takes time and other students get impatient when they do not understand. In this situation Eva has to make priorities, should she listen to the group or to the individual? How long can she listen to Kajsa without losing the group completely? In this case, Eva continues the dialogue, consistent with the purpose to understand the individual student's thoughts. Also, she leaves the dialogue with Kajsa two times to instruct the group. One time when Eva tells the group, that not understanding is how it is to learn mathematics and that learning mathematics is an exciting, she gets the opportunity to show the students that she finds it exciting to not understand. This is when she has to work hard to understand the method that Kajsa has seen.

Connection to formative assessment: When the teacher practices formative assessment there are situations where the teacher has to make decisions and do priorities that usually do not occur in a regular classroom. When formative assessment occurs on-the-fly during whole-class different kinds of decisions have to be made that are unplanned, unpredicted and under time-pressure.

Interpretation and categorization Activity 13: The purpose of this activity is to adjust instruction to meet learning needs as quickly as possible and to create situations where as many as possible of the individuals in the class are

engaged on task. This requires a lot of decisions about designing the content to meet learning needs for the episode (using KCT) balance listening to Emma and keep the group patient and keep track on lesson time (using KPSC) for the unpredictable learning opportunities. This makes the activity categorized both as knowledge of content and teaching and knowledge of pedagogic, students and content.

4.2 Short cycle (day-by-day), Episode 3

In this section, first a description of Episode 3 will be presented in Subsection 4.2.1 (an episode is the analyzed part of a lesson). After that, teacher activities during the practice of formative assessment will be presented. Along with the activities, the categorization of the activities into the framework of Teacher knowledge and skills for teaching mathematics is presented (Subsection 4.2.2).

4.2.1. Description of the teacher's instruction, Episode 3

In the end of Lesson 1, where Eva has introduced *from fractions to mixed form,* she gave the students an exit pass in the form of three tasks that the students solved individually before they left the classroom.

Tasks on the exit pass:
These fractions are bigger than 1. How many whole are there and how many parts will be left over?
Write the fractions in mixed form: 6/5= 11/7 = 8/3=

A total of 32 students worked with the tasks and 11 of them had one or more incorrect answer. The last task had the most incorrect answers. The incorrect answers were 1 5/3; 1 7/4; 1 3/3; 2 1/3; 5/3 and 5 .

The following lesson, Lesson 2, Eva tells the students that there where many wrong answers and the most common wrong answer was that they did not fully exchange the fractions. Eva said that she herself had been fuzzy that this was expected. However, this lesson they were going to do the tasks from the exit pass again. This time Eva draws figures of the fractions on the board. This means that the students have an image of the mixed form in front of them. In addition, Eva reads the task and underlines the words *whole* and *parts*. Eva invites the students to answer the tasks on the board on their mini-whiteboards. They are supposed to discuss with their math friend before answering. All answers are correct this time. As Eva goes through the answers she also uses the drawn figures on the board so the students can understand the conversion. Since no student came up with the incomplete exchange answer from the exit pass during Lesson 1 Eva writes it on the board herself, 8/3 = 1 5/3, and says:

Eva: How do you think, because this is logical, those who wrote like this, how do you think they thought? Because it is not wrong mathematically, but it's still not the answer I want to have.

Linus answers the questions after random choice with the devise. His answer is inaudible.

Eva: Do I understand you right now, Linus? First they take one whole and puts there [Eva points at the figure and then at the one in 1 5/3] and the parts that is left, is one, two, three, four, five, five thirds.

Eva adds that 8/3 is as much as 1 5/3 mathematically. Next task is to formulate what they think it is about when writing in mixed form. First they discuss the task together with the math friend, and Eric is then randomly selected to answer the question.

Eric: Well, you have to get as many whole together as possible.
Eva: Yes, that is what this is about! .[...] When you write in mixed form, first take as many whole as you can, and then write what's left.

The lesson continues with activities according to the instruction plan before the exit pass.

4.2.2 The teacher's use of activities, knowledge and skills

In this section, the teacher's activities found during the six EIU sequences in the episode are presented. One EIU sequence contains at least one Eliciting phase, one interpretation phase and one Use phase were elicited information is used. A full argumentation for the activities' connection with formative assessment is only presented for the new activity found in this episode, Activity 7. For the full argumentation of the other activities see Section 4.1.3.

4.2.2.1 Eliciting phase

Activity 1 *Creating eliciting situations*
Action A*:* In this episode Eva uses two all response-systems to elicit information. She bases the lesson planning on information from an *exit pass*. That is, she gave the students three written individual tasks in the end of the previous lesson. The tasks are supposed to be quick to do for the students and easy to assess for the teacher. By gathering the chosen information in written form from all students, Eva created time for interpretation and instruction planning before the next lesson. Whiteboards are used in the beginning of the episode to elicit information.

Action B: Random selection of students to answer her questions is used at two occasions in in the episode. For example, when she asks the students to answer how they think the students who answered 8/3 = 1 5/3 thought.

Action C: The teacher in the study provides the students with sufficient time to think before they give their answers. Eva also urges the students to discuss with their math peer. The students get extended waiting time and the possibility to discuss at three occasions. One example is when they are to transform fractions to mixed form and write on their mini-whiteboards. The students have three minutes to solve the tasks.

Connection to formative assessment: The purpose of this activity is to provide valid information on what students know or don't know of the learning object and to encourage students to participate by engaging in thinking on task, discussing with a peer and sharing thoughts with teacher and peers. Valid information is essential to be able to make appropriate interpretations and adjust instruction in such a way that it helps the students.

Interpretation and categorization Activity 1. To be able to implement the actions, Eva used knowledge of the activities described above to organize and handle the eliciting actions. This is knowledge that not requires mathematical knowledge and consequently categorized as knowledge of pedagogic, students and context (KPSC).

Activity 2 Using rephrasing and clarifying to elicit verbal information from individuals

Action: In this episode Eva uses the eliciting activity rephrasing and clarifying. For example, when the student said "the first number we had" she rephrases into "add on half?" and clarifies by saying "Do I understand you right now, Linus? First they take one whole and puts there [Eva points at the figure and then at the digit one in 1 5/3] and the parts that is left, is one, two, three, four, five. Five thirds."

Connection to formative assessment: To be able to elicit information from students who might have difficulties to express their emerging and sometimes immature thinking about the learning object, rephrasing and clarifying can be used to orient the interaction between student and teacher. Rephrasing and clarifying students' utterances are also a way to verify students' contributions and effort without judging them directly (Primo-Ruiz, 2011).

Interpretation and categorization Activity 2: The purpose of the activity is to elicit valid information from an individual student. Rephrasing requires use of Common content knowledge (CCK) as when Eva recognizes Linus's not outspoken notion (the first number) and changes to a half when rephrasing his utterance. When Eva clarifies Linus's fuzzy utterance she sorts out important information on how he understands the concept, which is use of knowledge of content and students (KCS).

Activity 4 Using how and why-questions to reveal thoughts on mathematics
Action: Eva uses how and why-questions during the episode when she asks "How do you think, [...] how do you think they thought?" referring to the ones who answered 8/3 = 1 5/3 when transforming from fractions to mixed form.

Connection to formative assessment: When answering how and-why questions, students have to explain how they were thinking which gives the teachers the possibility to make a more direct inference about their mathematical thinking.

Interpretation and categorization Activity 4: One purpose with how and why-questions are to reveal misconceptions. Hence, the activity requires the use of Eva's knowledge of content and students (KCS) to formulate the question with fractions familiar to the students. The knowledge to formulate a question that asks for the definition of mixed form is Specialized content knowledge (SCK).

4.2.2.2 Interpreting phase

Activity 5B Interpret and recognize the nature of incorrect answers using Specialized content knowledge to
Action: In this episode there are four different answers that might be interpreted as incomplete exchange of 8/3. The incorrect answers is 5; 5/3; 1 3/3 and 1 5/3. Incomplete exchange is the misconception Eva detects and chooses to adjust.

Connection to formative assessment: When facilitating the interpretation phase of the fundamental idea, Eva interprets students' responses to what understanding of the conception the student show to have. This means, it is as important to be able to identify desirable conceptions as well as unwanted. A teacher who works formative also needs to recognize different expressions of notions on the same kind of misconception.

Interpretation and categorization Activity 5B: Recognizing and size up the nature of an incorrect answer that not are known or directly recognized as a common misconceptions known for the teacher is using Common content knowledge and Specialized content knowledge. First, determine if the answer is correct or incorrect (using CCK); Second, the teacher searches for patterns in the incorrect answers, finding incomplete exchange as the common mistake (SCK).

Activity 7: Recognizing incorrect answers that not are based on mathematics

Action: Not reading thoroughly or not understanding what's read might cause incorrect answers. Eva's adjustment after the student answers on an exit pass shows that her interpretation of the answers 5 and 5/3 are answers that are both incompletely exchanged. These answers do not respond to the part of the question that asks for "*how many wholes*" but only "*how many parts are over*?". The student who answers only 5 might not have perceived that "*how many parts are over*" was supposed to be reported as a fraction. These kinds of answers are examples of incorrect answers that not clearly, and only, have mathematical sources. The fact that Eva is reading the task aloud with emphasis on *whole* and *parts* indicates that Eva has interpreted the answers that way.

Connection to formative assessment: Since teachers who practice formative assessment are interested in detail to understand how the students think, recognizing and sorting out incorrect answers that are not based on mathematics can be useful to be able to make well-founded decisions about what to do to help students learn.

Interpretation and categorization Activity 7: The skill to understand that students might answer only a part of the question, or are may misunderstand what is asked for, is knowledge of how students learn mathematics, KCS.

4.2.2.3 Use phase

Activity 8 Using elicited information to design instruction

Action: When Eva has decided how to act, the activity can take different forms. In this episode Eva uses two different forms to act. She offers explanation or she poses a new question. One example when she offers explanation is in the beginning of the lesson when Eva starts with the same questions as on exit pass but Eva also draws the figures of the fractions on the board. This drawings works as an explanation of how a fractions looks like in mixed form. This differs from the instructions and the exit pass during the previous lesson that only treated fractions on an instrumental

level. Eva also poses questions to the students to make them reflect on a topic, for example the question "How do you think, those who wrote like this, how do you think they thought?" Eva aims at the answer 8/3 = 1 5/3.

Connection to formative assessment: The purpose of this activity is to adjust instruction to better meet learning needs from elicited information.

Interpretation and categorization Activity 8: To decide the design of adjustment, to use one way instead of another, is to use knowledge of content and teaching (KCT). Eva has made the original question easier, by also using drawings together with the fractions, using KCS. The skill to represent the learning object in different ways is SCK. The skill of posing questions is analysed in Activity 3 and 4.

4.2.2.4 Not phase specific

Activity 11 Engage students as participants in the phases of EIU
Action: In this episode, Eva invites students to participate when she displays the students' answers in the Eliciting phase, using them as a starting point for a debate or additional eliciting. She also uses students' answers as explanations even though she might reformulate or clarify their answers. For example, when she clarifies Linus explanation on how the persons might have thought when they answered 1 5/3, she used his explanation, adding the drawing on the board and counted the parts not included in the whole one by one.

Connection to formative assessment: For the teacher to be able to elicit information the students have to be willing to share their thoughts with their peers and teacher. Engagement as participating in the phases of EIU is therefore critical.

Interpretation and categorization Activity 11: The purpose of this activity is to make the students willing to share their thoughts with their peer and teacher. To be able to do that, Eva uses her knowledge of pedagogic, students and context (KPSC). KPSC includes knowing in what way and when student can participate (displaying answers, using their explanations) and to practice this regularly is to first develop an educational context that includes student participation, and then to act according to it. To be able to invite students in a way that works for the students, with reference to their pre-knowledge, Eva uses her knowledge of how student think about and learn mathematics, that is, knowledge of content and students (KCS).

4.3 Medium length cycle, Episode 4

In this section, first a description of Episode 4 will be presented (Subsection 4.3.1). An episode is the analyzed part of a lesson. After that, the teacher activities during the practice of formative assessment will be presented. Along with the activities, the categorization of the activities into the framework of *Teacher knowledge and skills for teaching mathematics* is presented (Subsection 4.3.2

4.3.1 Description of instruction, Episode 4

This episode unfolds after a written test on topics instructed during the semester. The test contains mainly tasks on decimals and there are also tasks on the connection between different number forms.

Eva: I have chosen the task I expect to be the one that most of you either had difficulties with or maybe misunderstood. Some of you have some things right and some have been writing answers that do not hold, so to speak.

The task was: "Show Astrid how there is a connection between numbers written in decimal form and numbers written in other forms. Choose a number in decimal form and write the same number in fractions. (You can also use mixed form and percentage)".

Eva clarifies that the word connections here means that you are supposed to write a number with the same value, but in different forms. Eva draws a table on the board (Table 2) and fills some of the squares in the table (those are marked grey in Table 2) Eva starts by asking about the names of the different forms of numbers she has written in the top of the table. Eva chooses whom to answer and there is no special time to think together.

After that she instructs: "Now I want you to, together with your math friend, take your whiteboard, and write numbers that has the same value as one and a half. Write them in fractions, in mixed form and as percentage". Six minutes later the students' discussions are over and Eva has written their answers in the table (students' answers in the white squares).

Table 2 Students' incorrect answers on the exit pass. In the white cells, students' answers are gathered.

X, X	**x/x**	**X x/x**	**X%**
1.5	15/10 1 1/2 3/2 15/100	1 3/2 1 1/2 1 50/100 1 10/20	150% 50%

Eva tells the students that two of the fractions in the form x/x are correct and two are incorrect. The students get two minutes to consider ”Which are right and why, what are your arguments that your answers is worth one and a half? You are just to think, not to write on your whiteboard.”

Eva:	I take the necklace, and turn the balls. Let's see, the ball says Michael.
Michael:	But I do not want to
Eva:	Do you want to refrain from answering?
Michael:	[inaudible]
Eva.	Ok. I will come back to you later. Since you now will get the opportunity to listen to what the other says. Later you will decide if you agree or disagree to what they say. Then I'll just pick the next to the right. And that'll be Ana. Ana and Bengt have you agreed on suggestions or do you disagree with one and another? Which two have you chosen?

They consider 15/10 and 15/100 as the two correct answers. After thanking Ana and Bengt for the contribution Eva invites other students to “either agree, or change fraction one wants to advocate. The main point is to motivate ones answer”.

The first student, Johan, says that 15/10 =3/2. He motivates by first bring one whole of the fraction, then state that there is one half left. This implies that the fraction 15/10 and 3/2 equals 1.5.

Eva helps Johan to pronounce *three halves* and participates in the dialogue by supporting questions as “How do you motivate”, “MM”, “Okay” and “Yes”.

Eva:	Now we have listened to Johan's argumentation. If you agree with him, you do not have to say anything, but if you do not agree with Johan or want to clarify something or to argue why the others are wrong. Sam?

Sam has another way to prove that 15/10 equals 3/2. He argues that the extension of 3/2 by five gives 15/10.

Eva:	Yes, and if you are convinced of 15/10 you know that this was of equal value. Thank you for your standpoint Sam.

Then Eva gives Tove the opportunity to argue.

Tove:	I want to talk about 15/100
Eva:	Yes, we want to listen to that. Because 15/100, that was what Ana and Bengt was thinking. Now Ana and Bengt, listen to Tove and see if you have some argument and if you want to protest.

Tove:	If you convert fifteen hundredths to decimal than you'll have cero point fifteen.
Eva:	Okay, and you do not think that cero point fifteen is the same as one point five? [...] Is there some other way you could reveal fifteen hundreds? Sofie?
Sofie:	Well, there it's hundred. Then it would have to, there, above, be hundred to become a whole.
Eva:	You think the numerator must be as big as the denominator to become a whole?
Sofie:	And now there is one and a half.
Eva:	Yes.
Sofie:	Then it'll have to be even more.
Eva:	Even more, how much do you think it would have to be to become one and a half?
Sofie:	One hundred and fifty, just as there, as percentage.

Kim adds that if you were allowed to add cero "up there", on 15/100 it would have been the same as 1.5, but, as he points out, "it won't work because then you would have to add cero down there as well".

Eva returns to the math pair who from the beginning thought 15/100 and 150/100 was equal and the same as 1.5. She asks them if they've got any hint on how to decide if a fraction is bigger or lesser than one.

Eva:	Can you see on this fraction for example [points at 15/100] that it was not more than a whole?

The pair gives a weak yes. Eva describes that if you are going to lay a jigsaw with 100 pieces you have to have all hundreds to make a whole picture. If you only have 15 pieces of the hundred you will only be able to lay a small part of the jigsaw.

The lesson continues with the answers in mixed form in the table.

4.3.2 The teacher's use of activities, knowledge and skills

In this section, the teacher's activities found during the two EIU sequences in the episode are presented. For the full argumentation, interpretation and categorization of activities identified also in the minute-by-minute short-cycle formative assessment practices see Section 4.1.3.

4.3.2.1 Eliciting phase

Activity 1 Creating eliciting situations

Action A: In this episode Eva uses the all-response systems written test and mini-whiteboards to elicit information. The elicited information that Eva

based the first activity on during the episode was elicited from an individually written test on students' knowledge on numbers in different forms with focus on decimal form. One example when she uses their mini-whiteboards is when Eva reformulates the question from the test and asks the students to write the number 1.5 as a fraction, in mixed form and as percentage on their mini-whiteboards. Within six minutes Eva knows how many, which and in what way students in the pair think on the task.

Action B: Eva uses a randomizer devise at two occasions in the episodes. For example, when she asks for possible reasons to answer $8/5 = 1\,5/3$.

Action C: Eva uses extended wait time to engage all students on task. When she uses the randomizer device in the episode Eva waits for two minutes before a student is randomly chosen to answer the question. In the episode Eva uses peer discussions at two occasions, when eliciting information with whiteboard and when eliciting information using the randomizer device. When the students have thought through and explained to a peer and somebody else has another opinion, the motivation to argue is likely to be high. One example is when Eva invites students to argue on the answer that Bengt gave on which answers that were correct and which were incorrect, five students wanted to participate with their thoughts.

Connection to formative assessment: In this episode Eva uses a written individual test as eliciting method. Medium length cycle response systems for acquiring information on students' thoughts and understanding of concepts are usually tests or diagnoses for a certain topic. One advantage with information from tests is that Eva has written information from all students, and the students are usually prepared when they answer the questions. Still, tests and diagnoses are often carried out to seldom to be effective as the only source for formative assessment since they do not occur so frequently and often are made in a summative purpose to find out what the student can.

Interpretation and categorization Activity 1. The purpose of this activity is to provide valid information on what students know or don't know of the learning object and to encourage students to participate by engaging in thinking on task, discussing with a peer and sharing thoughts with teacher and peers. To be able to implement the actions, Eva used knowledge of the activities described above to organize and handle the eliciting actions. This is knowledge that not requires mathematical knowledge and consequently categorized as Knowledge of pedagogic, students and context (KPSC).

Activity 2 Using rephrasing and clarifying to elicit verbal information from individuals
Action: During this episode Eva uses the eliciting activities rephrasing and clarifying. One example is when she rephrases Johan's "*three halves*" as a help with pronunciation. An example of clarifying is when she asks Tove "You do not think that cero point fifteen is the same as one point five?" when she has argued that fifteen hundreds only is cero point fifteen as a decimal.

Connection to formative assessment: To be able to elicit information from students who might have difficulties to express their emerging or immature thinking rephrasing and clarifying can be used to maintain and orient the interaction between student and teacher and to check interpretation (Ruiz-Primo, 2011). This increases the possibility to elicit valid information.

Interpretation and categorization Activity 2: Rephrasing requires use of Common content knowledge (CCK) as when Eva recognizes a notion that a student has difficulties to pronounce and changes to a correct notion when rephrasing the student's utterance. When Eva clarifies the fuzzy student utterance she sorts out important information on how the student possibly understands the conception, which is use of Knowledge of content and students (KCS).

Activity 4 Using how and why-questions to reveal thoughts on mathematics
Action: Eva used a why-question when she asked "Which answers are right and why, what are your arguments that the answers is worth one and a half? The answers that you think are incorrect, what are the arguments for that?". When answering the question the students used their prior knowledge of decimal form, fraction size, mixed form and expansion of fractions to prove other students' arguments false.

Connection to formative assessment: When answering how and why-questions students have to explain how they were thinking, which gives Eva the possibility to make a more direct inference about their understanding.

Interpretation and categorization Activity 4: One purpose of how and why-questions are to reveal (mis)conceptions and answering require the students to justify their answer. Hence, the activity requires the use of Eva's Knowledge of content and students (KCS) to decide that the students' pre-knowledge of the different number forms were mature enough for answering the question with good arguments. The skill to formulate the question that

produced information on students' conceptual thinking, and to do it correct, is Specialized content knowledge (SCK).

4.3.2.2 Interpreting phase

Activity 5B Using Specialized content knowledge to interpret and recognize the nature of incorrect answers
Action: From the starting point, the test, we have no data of student responses. However, from what Eva says in the beginning of the lesson and the actions she makes, the conclusion is that she has thought of several bases for the nature of incorrect answers. Probable interpretations for students' incorrect answers, beside not being able to identify or produce the four representations of the same value, are; students do not connect form to the notion, students chose a decimal number that were difficult to transform to fractions.

Connection to formative assessment: During the interpretation phase of formative assessment the teacher interprets students' responses on how they understand the concept. We use the notion of (mis)conception to emphasize the dual purpose. This means, it is as important to be able to identify desirable conceptions as well as unwanted to make better-founded decisions on next step of instruction.

Interpretation and categorization Activity 5B: Recognizing and assess the nature of an incorrect answer for answers that not are known or directly recognized as a common misconceptions known for the teacher is using Common content knowledge and Specialized content knowledge (SCK). First, determine if the answer is correct or incorrect (using CCK); second, Eva makes a lot of changes in the Use phase compared to the first question which indicates that she has been searching for patterns to find the base for the nature of the errors, indicating the use of SCK before KCS during interpretation. Another interpretation is that the information was so time-consuming to interpret since the students were to choose their own number to transform that Eva chose to make the eliciting once more.

Activity 7: Recognizing incorrect answers that not are based on mathematics
Action: From the way Eva adjusted instructions in the episode we can draw the conclusion that she thought that one of the possible obstacles to find the right answer was the way the question was formulated. Part of the question in the test was formulated as a suggestion in brackets and when she formulated the new question she re-formulated the question so it was clear

how many forms they were going to show and what they looked like (for example X x/x)
Connection to formative assessment: Since teachers who practice formative assessment are interested in detail to understand how the students think, reflections on the formulations of the posed questions is one of the interpreting skills teachers can use. When teachers interpret information there might be other obstacles than mathematical ones that cause students to give the wrong answer.

Interpretation and categorization Activity 7: The skill to understand that students might answer only a part of the question, or to misunderstand what is asked for, is Knowledge of how students learn mathematics, KCS.

4.3.2.3 Use phase

Activity 8 Using elicited information to design instruction
Action: In this episode Eva acts on the information from the written test to pose the same question again. This time the formulation is more specific. After that, she displays the answers and invites the students to discuss with arguments to justify their answers. Eva orchestrates the discussion and after the peer spontaneous arguments, she asks for a specific argument "Is there some other way you could reveal fifteen hundreds?", which she either wants to check if the students know of or wants the students to hear and reflect on. In short, from the elicited information from the test, Eva narrowed the focus to control a specific knowledge and from this information changed perspective on the learning object to reveal more specific information of (mis)conceptions by asking how to justify the answer.

Finally, Eva offers an explanation to the students who gave the wrong answer in the beginning when she uses an everyday example of a jigsaw to determine if a fraction is bigger or lesser than one.

Connection to formative assessment: The purpose of this activity is to adjust instruction to better meet learning needs. Formative assessment becomes formative first when the teacher has elicited and interpreted information and decides if and how and with what content to act on the information. The action should be such that it meets the students' learning needs

Interpretation and categorization Activity 8: To decide the design of adjustment, to use one way instead of another, is to use Knowledge of content and teaching (KCT). Another aspect of KCT is to keep track of different arguments that are shown. But the skill to search for the different representations in the students' answers is SCK. The skill to transform the

original question in the test to an easier is Knowledge of content and students (KCS). Formulation of questions (analyzed in activity 3 and 4) or formulation of explanations to make students reflect on a special content is use of Specialized content knowledge (SCK).

4.3.2.4 Not phase specific

Activity 11 Engage students as participants in the phases of EIU
Action: In this episode the students participate as interpreters to determine which of the displayed answers that are correct or incorrect and the most characteristic is the use of students' as the ones who offer explanations (see also Activity 12). This is another way of displaying students' answers and an opportunity to peer learning when students assess themselves in relation to peers' answers.

Connection to formative assessment: For Eva to be able to elicit information the students have to be willing to share their thoughts with their peers and teacher. One way to increase this is to invite students as participants.

Interpretation and categorization Activity 11: The purpose of this activity is to make the students willing to share their thoughts with their peers and teacher. To be able to do that Eva uses her Knowledge of pedagogic, students and context (KPSC). Knowing in what way (invite as interpreters) students can participate and to practice this regularly is to first develop an educational context that includes student participation, and then to act according to it. To be able to invite students in a way that works for the students Eva uses her knowledge of students pre-knowledge that is, Knowledge of content and students (KCS).

Activity 12 Using acknowledgement of student contribution to engage students
Action: In this episode students who contribute with thoughts get praise for their contribution. Eva's response to, and acknowledgment of, Sam's contribution is an example: "Yes, and if you are convinced of 15/10 you know that this was of equal value. Thank you for your standpoint Sam".

Connection to formative assessment: To facilitate students' engagement in the phases of formative assessment, and to make them experience that the sharing of thoughts adds positive to the session is to acknowledge student contributions. According to Zee (1977) this nonjudgmental acknowledging promotes student engagement (Van Zee & Mistrell, 1977).

Interpretation and categorization Activity 12: The purpose of this activity is to make the students willing to share their thoughts with their peers and teacher. Many of the students show this willingness, there is seldom a student who passes. The skill to know about and use acknowledgement of student contribution is a matter of managing teacher activities using Knowledge of pedagogic, students and context (KPSC). In this example Eva adds an argument to Sam's, showing him that she understood what he said and that it made sense to her, before acknowledging his contribution using her knowledge of how students think about and learn mathematics, Knowledge of content and students (KCS).

5 Results

The result from one research question will be presented for each section. Table 3 shows information from the analysis that is referred to in more than one research question.

Table 3: Activities, knowledge and skills during each cycle. The table displays from which phase and cycle each activity has been identified, and the knowledge and skills used in the activities.

Phase of FA	Activity	Activity occurrence in type of cycles			Teacher knowledge and skills used for activity.				
		Minute-By- minute	Day –by-Day	Medium length	CCK	SCK	KCS	KCT	KPSC
E-Phase	A1	X	X	X					KPSC
	A2	X	X	X	CCK		KCS		
	A3	X					KCS		
	A4	X	X	X		SCK	KCS		
I-Phase	A5A	X			CCK		KCS		
	A5B		X	X	CCK	SCK			
	A6	X			CCK				
	A7		X	X			KCS		
U-Phase	A8	X	X	X		SCK	KCS	KCT	
	A9	X					KCS		
	A10	X							KPSC
Non specific phase	A11	X	X	X			KCS		KPSC
	A 12	X		X			KCS		KPSC
	A13	X						KCT	KPSC

5.1 What types of activities constitute the teacher's formative assessment practice during whole-class lessons?

Table 3 shows that a total of fourteen activities (Activity 5 has two sub activities) were identified in the EIU-sequences in the episodes. They are equally distributed over the three phases of the EIU-sequences (4, 3, and 3 different types of activities were identified in the eliciting, interpreting and use-phase respectively) and three of them are not connected to a specific phase.

The activities in the Eliciting phases are all connected to eliciting information about student thinking and learning. Mini-whiteboards, a randomize devise, exit passes and tests are used to gather information. Extended waiting time and peer discussion are provided to obtain valid information and facilitate learning. Two different kinds of questions are identified as activities; open-ended questions where valuable unexpected information might turn up and how and why-questions where the students reasoning can inform the teacher of how the students understand the learning object. An additional identified activity is to repeat and clarify the students' utterances to orient the dialogue and check the interpretation. This activity takes place when a student is in dialogue with the teacher.

During the activities in the episodes Eva interpreted both answers that were misconceptions she had met before, answers that were correct but uncommon, and one student answer that she at first could not interpret. The interpretations were quickly made during the lessons. Also, in the interpretation of the students' answers to the test and the exit-pass, used as a foundation for the planning of Episode 3 and 4, she had recognized incorrect answers that might have been based on other kinds of problems than mathematical, such as reading problems.

Using the elicited information to design instruction was done in two different ways, to explain or to create a new question directing new learning situations to better meet students' learning needs. The interpretations of one of the misunderstandings lead Eva to introduce a new topic by a statement for the students to consider. Finally, Eva modelled problem solving after hearing the student's answer that she could not interpret.

Three of the activities identified in the episodes had the purpose of increasing students' willingness to reveal their thoughts on mathematics. The teacher acknowledged the students' contributions in different ways, engaged them as participants in the different phases of formative assessment and gave them time on task and a math peer to discuss with. The last activity was practicing formative assessment on-the-fly. This activity required the teacher to make all those decisions about when and how to act on

information about student thinking while considering both the group and the individuals and under time pressure.

5.2 What are the similarities and differences between the activities used in the different cycles in the formative assessment practice?

In this section I will present the activities that are common for all cycles with examples of activities from the episodes in Subsection 5.3.1. In Subsection 5.3.2 the differences between the cycles are presented, exemplified with examples from the specific activities in the day-by-day short cycle.

5.2.1 Activities that are common for all cycles in the formative assessment practice

From Table 3 we can see that six of fourteen activities are common for all types of cycles, A1, A2, A4, A5, A8, and A11. Table 3 also shows that all phases include at least one activity that is used in all cycles. The eliciting activities (Activity 1,2 and 4), include the use of all-response systems, random selection of students who answers questions, the use of extending wait time and peer discussions. Eva is also using how and why-questions, rephrasing and clarifying activities to elicit information on how the students understand the learning objects from individual students in all types of episodes. Activity 5 represents the common activity for the interpretation phase. Activity 8, which is about using information to design instruction, is also common for all types of cycles. In all episodes Eva also uses the not phase-specific activity to invite the students to participate in phases of formative assessment (Activity 11).The use of these common activities, creates the structure for the formative assessment practice (EIU) embedded in the instructional dialogues in whole-class lessons. I will exemplify this formative practice by the following examples from the episodes.

The transcript that follows is from Episode 1 when Eva has shown a figure equivalent to 1 ¼ and asked the students to write the value of the figure in as many number forms they can on their mini-whiteboards (Activity 1). Patrick volunteer's to explain how he was thinking when answering 125/200.

Patrick:	Eh, we thought that there is one whole, and then there are twenty-five, or I thought. And then that a whole, if, there are only room for four, or three more cubes there, I thought that if, if you fill out with three, then you have another of them, and then I thought that twenty-five is, is divided by four, of one. That is, let' say that one is hundred…
Eva:	You thought that you had a hundred there and you had a hundred there. Then you would have two hundreds. We had 100 hundreds colored there but only twenty five hundreds there. Obvious, did you all get that explanation? [

In situations like this, Eva elicits information from the student in dialogue by rephrasing and clarifying students' utterances (Activity 2).
The following transcript is from the beginning of Episode 1 when Tom answered 1/1.0 = 1.When she has interpreted the answer she explains (Activity8):

Eva: That was a smart mix of different number forms, but I'm not sure anyone uses that mix. There is a form called mixed form, but then you mix integers and fractions.

The next transcript is from Episode 4. Eva has invited the students to participate in the phases of EIU (Activity 11) by using their displayed answers as the base for the debate on which of the two fractions on the whiteboard that are correct and which are incorrect (Activity 1 and 2):
3/2 equals 15/100, 15/10; 1 1/2; 3/2. Eva wants the students to argue on which answers that are right or wrong and what are their reasons for their answers? Tove volunteers to answer:

Tove: I want to talk about 15/100
Eva: Yes, we want to listen to that. Because 15/100, that was what Ana and Bengt was thinking. Now Ana and Bengt, listen to Tove and see if you have some argument and if you want to protest.

In the overall structure of the lessons, there is flexibility, a search for use of student answers to create teachable situations (i.e. formative assessment). For this to happen, the students have to be willing to share their thoughts.
From the analysis (Section 4.1.3; 4.2.2. and 4.3.2) it is shown that four of the activities (Activity 1,2, 11 & 12) make the students engaged in the formative assessment practice (i.e. engaged on task, willing to reveal their thoughts; participate in the learning process) Three of them are common in all cycles except short cycle day-by-day. The examples above show how.

5.2.2 Activities that differ between cycles in the formative assessment practice

The activities exclusive for the minute-by-minute short cycle are logically connected to each other; an open-ended question (Activity 3), can give unexpected answers that requires the teacher to introduce new topics (Activity 9) or be the reason for topics unknown to the teacher. To fulfil the formative assessment practice she has to use the information, and she does this by using the unknown to model learning situations (Activity 10). This

unpredictable situation puts Eva in situations where she has to make decisions on-the-fly (Activity 13).

The following transcript is from Episode 2 and shows as an example on how the teacher handles on-the-fly formative assessment. Eva is stuck, not understanding Emma's method for expanding fractions. Eva found herself in a situation where she did not understand why the mathematical model worked.

Robin:	I do not understand a thing!
Teacher:	Yes, [laughing] but it's very exciting!
Robin:	Well, I do understand but still, I do not understand, I think it's totally wrong!

Her strategy to handle the on-the fly situation (Activity 13) herself was to participate in the learning process, using the unknown mathematics to model a learning opportunity (Activity 10).

Activity 7 is specific for day-by-day and medium cycle. The activity concerns the teachers' skill to interpret and sort out mathematical incorrect answers from non-mathematical in students written answers.

5.3 The character of the knowledge and skills the teacher uses in the activities

The description of the character of the knowledge and skills used in the activities are described from the examples used in the analysis of actions (Section 4.1.3, 4.2.2 and 4.3.2). As the right half of Table 3 shows, all the different categories of teacher knowledge and skills appear at least in two of the fourteen activities. Nine of the activities is categorised as Knowledge of content and student, which is the most frequent category. Common content knowledge is more present in the two first phases (eliciting and interpreting). Knowledge of content and teaching appears in the Use-phase and activities that are non-specific. Specialized content knowledge occurs in all three phases. Knowledge of pedagogic, student and context occurs in all phases but the interpretation-phase.

Common content knowledge (CCK)

Teachers must know the material they teach. In the activities, all the CCK the teacher use has not been noted in the activities. For example, fractions, decimals and the connections between them are the topics of the presented episodes. Common content knowledge is a base for teaching mathematics, and Eva uses it in the episodes. But in this study we focus the special knowledge and skills used specifically in the activities of formative assessment. Eva's use of CCK during the activities is identified in the activities during the Eliciting phase and the interpreting phase.

CCK is also used to correct students' answers. When asking open-ended questions she gets many different answers, which she has to quickly determine if they are correct or not. She recognizes and corrects students' incorrect use of notions when she rephrases students' utterances and helps them with the pronunciation. At one occasion Eva meets a method for expanding fractions that is new to her. Then she uses her skills in problem solving to justify that the method works for every number.

Specialized content knowledge (SCK)

Specialized content knowledge is the special knowledge on mathematics that is only useful for teachers. This includes the detailed knowledge of representations of a mathematical idea. Eva uses this knowledge when assessing the nature of incorrect answers. When getting many different incorrect answers for the same question, she looks for a pattern in the incorrect answers, finding they are caused by incorrect expanding of fractions one time and probably end up interpreting that there are several causes for the incorrect answers another time. Furthermore, Eva uses her Specialized content knowledge when she chooses a specific representation of the mathematical idea to use when formulating questions or when explaining mathematical ideas to the students. SCK also includes making a correct representation of the mathematical idea. During the activities, we have seen her explain, in different ways, the content using knowledge of different representations for fractions, number form, and figures and explaining fractions in everyday context. The capability to provide different explanations and representations of the same mathematical idea is particularly crucial in formative assessment, since the information collected about student understanding is used to adapt the teaching to better meet the students' learning needs.

Knowledge of content and students (KCS)

When Eva formulates questions she uses her knowledge of how students learn the content and their pre-knowledge. When she gets a fuzzy answer from a student she uses KCS to determine if the answer represents a mature understanding enough to continue her way of teaching towards the learning object or if adjustment is needed. When interpreting the elicited information knowledge about common misconceptions is used, such as incomplete exchange, changing the whole during operation and adding nominators and denominators of two fractions. KCS is used during rephrasing and clarifying when she chooses what issue to rephrase or checking her interpretation with a clarifying response during dialogues with students. To be able to recognize the misconceptions she sorts out non-mathematical bias and mathematical incorrect answers that are of lesser importance for the moment. She adjusted

questions from a test and exit pass so they became easier for the students to understand, and thus provide more valid information about their mathematical understanding.

Knowledge of content and teaching (KCT)
For two of the episodes, Eva had designed specific eliciting situations, which gave her time to interpret and plan the next lesson. At the other two episodes, interpretation and designing the use-phase were decided on-the-fly. During the unplanned and unpredictable elements in the phases of EIU, decisions are to be made that involves both content and teaching. In addition, the decision of which representation to use to meet a specific identified learning need is a skill of KCT. During these on-the-fly moments she designed questions, explanations and used students' answers as a part of use-phases, and orchestrated student discussion.

Knowledge of pedagogic, student and context (KPSC)
The knowledge and skills with the character of KPSC used in the activities can be summarized as knowledge and skills when organizing the episodes for the group and the individuals, using eliciting activities, knowledge of how to develop the educational context and act in line with it.

First, Eva monitors how the episode unfolds, keeping track of group and individuals as the elicited information gives her different opportunities to act on. She uses knowledge of, and skills to use, eliciting activities as exit pass, mini-whiteboards, randomizer device and students' waiting time before answering questions. Second, Eva has together with the students developed an educational context in which the students are invited to participate as initiators in the Eliciting phase, interpreting peers' answers, argue for and against their point of view as a part of the use phase. Finally, Eva herself takes the learning opportunity when a student has something to learn her.

5.4 What are the similarities and differences between the teacher knowledge and skills used in the different cycles in the formative assessment practice?

What are the similarities?

Table 3 shows that all categories of teacher knowledge and skills appear in all cycles. From the analysis of the actions (Section 4.1.3, 4.2.2 and 4.3.2) we find that similarities that are common for Knowledge of pedagogic, context and students are the knowledge and use of eliciting activities that provides the teacher with valid information and knowledge of how to support students to engage as participants in the formative assessment practice.

The content-specific knowledge and skills that are used in all cycles are the skills to use questions where the students have to explain their thinking and the use of and to explain different representations of a mathematical idea. During interpretation of students (mis)conceptions Eva used both Specialized content knowledge and Knowledge of content and students to understand the nature of the incorrect answer.

What specific knowledge and skills does the teacher use during short cycle minute-by-minute cycle?

Eva sorts, interprets and decides how to act on the unexpected and unpredictable topics that occur. In these situations teacher knowledge and skills pertaining to many different categories were used.

Orchestrating formative assessment minute-by-minute requires high standards on the teacher's content knowledge, both pedagogical and the Specialized content knowledge that only teachers use. It also requires that Eva is prepared to meet situations where she does not have the specific knowledge that is asked for. She also needs to make decisions about how to answer and proceed within seconds, and base these decisions on the needs of both individual students and the whole class.

What knowledge and skills does the teacher use during short cycle day-by-day?

During formative assessment day-by-day, Eva elicited information by a pre-planned written test of three tasks. This means that she had to plan in advance what information that would be the most efficient to elicit at that particular moment. The interpretation phase is different from minute-by-minute phase since she only has written information to interpret and time to interpret between lessons. The skill to plan ahead, interpret and sort out information from written answers between lessons, adjust the level of difficulty of questions are the specific knowledge and skills she uses in formative assessment day-by-day cycle. The knowledge and skills required are non-subject specific pedagogical knowledge and subject-specific knowledge from all the categories used in the framework.

What knowledge and skills does the teacher use during medium length cycle?

In the medium-length formative assessment cycle, Eva displays the students' different answers on the big whiteboard and tells the student that there are two incorrect answers. She uses the randomise device to select the first student to answer and lets other students volunteer to the debate afterwards. In this episode, there was more of a discussion where students gave different arguments on why answers are correct or incorrect. When Eva knows that the students together have enough knowledge to contribute to a question

with different arguments, a discussion of this kind works both as eliciting of information and adjustment of teaching and learning activities. The specific knowledge and skills she uses during this cycle is mainly to orchestrate and orient the debate so the desired reasoning turns up. To be able to do this on an appropriate level of difficulty for the students, Knowledge of content and students' are required

6 Discussion

The discussion is separated into three parts: short summary of the most important results (Subsection 6.1), discussion of methods used in the study (Subsection 6.2), and discussion of results (Subsection 6.3)

6.1 Summary of the most important results

In this study I have identified fourteen activities and a number of actions used by one teacher during formative assessment in the three different cycles (short cycle minute-by-minute, short cycle day-by-day and medium-length cycle). Four of the thirteen activities have properties known to engage student on task, participate in the phases of formative assessment (EIU) or to keep them willing to reveal their thoughts on mathematics.

In a comparison between the cycles regarding which activities that are used the main difference is mainly between the minute by minute cycle and the two other phases. One reason for this is that the minute-by-minute formative assessment appears on-the-fly (i.e. under time pressure with regard to the needs of individuals and the group). Another reason for the difference is that the main part of the specific minute-by-minute activities are connected to each other through the phases of formative assessment. The properties in the cycle, for example the topics are un-predicable and sometimes un-known for the teacher. The knowledge and skills that the teacher uses are complex since many activities requires use of different categories of knowledge and skills simultaneously. In addition, for every cycle all the different categories in the framework are used.

The formative assessment practice is a very complex, demanding and difficult task for the teacher in several ways. For example, during short term minute- by-minute formative assessment practice Eva uses knowledge and skills to eliciting, interpreting and use the elicited information to modify instruction to better meet student learning needs. She also helps students to engage in common learning activities and take co-responsibility of their learning. In the minute-by-minute formative assessment practice she also handles new mathematics (to her), unpredictable situations and makes decisions about teaching and learning situations in a matter of seconds.

6.2 Discussion of methods used in the study

As described in Section 3.1, a case study is characterized as a study with several sources to enrich the description of the phenomena. During the data collection, my aim was to look for what actually happened, not what the teacher intended.

A way to enrich the data in had been to follow more than one teacher. But, to use only one teacher as informant has allowed me to deepen the examination on the way Eva and the class practiced formative assessment could also have interviewed her to enrich the description, but that would have been out of the scope of this study since I not investigated her intentions with the practice.

As mentioned in Section 2.2.4, the distinctions between the categories in the framework used in the study are not completely clear. This problem were raised by Ball at (2008) when they introduced the framework. Another problem with the framework is that to which category a particular knowledge belongs to might be personal. For example, when a teacher analyse a student error the teacher might figure out the nature of a misunderstanding mathematically by searching for a pattern in the answer (using Specialized content knowledge), and another teacher might figure it out from her experience of what students usually struggle with (Knowledge of content and students). I handled this problem by using my interpretation of the purpose with the formative assessment practice to determine how to categorize in unclear cases.

6.3 Discussion of results

The discussion of results is separated into five subsections. I discuss the difficulty of using formative assessment in practice (6.3.1). Next subsection (6.3.2), discusses the similarities and differences in the teacher's formative practice. Students' participation in formative assessment is discussed in (6.3.3) and the efficacy of the formative assessment is discussed in (6.3.4). Finally, how to support teacher to implement formative assessment is discussed in Subsection 6.3.5.

6.3.1 Formative assessment- How hard can it be?

Some readers would contend that the identified activities and actions are skills that all teacher use. And probably they are right. Most teachers use these activities sometimes. But the teacher in the formative assessment practice actively seeks the teachable moments by the use of the activities associated with each other in a regular structure (i.e. E-I-U). To be able to do this a teacher needs a lot of teacher knowledge and skills. The knowledge and skills that the teacher in the study used as are briefly outlined in Section 6.1.This includes to be flexible in planning and quick in decision making, to

be able to take up the un-expected topics that occur, to be prepared to handle topics that are unknown, to have Knowledge of the content and especially the Knowledge of how student learn mathematics or the skill to deduce the nature of misconceptions and to be prepared to meet this misconceptions with an activity that challenge and change the students conceptions.

As the students get used to and accept the formative assessment practice and experience the learning gains they have of participating in the activities, they will pose more questions that create even more on-the-fly situations. To practice formative assessment requires time to plan, courage and different kinds of content knowledge and pedagogical knowledge from the teacher. To implement this kind of instruction is complex and requires deeper and wider knowledge than the knowledge required in a regular teachers' practice who mainly uses the IRE pattern (Initiative-Response-Evaluation) during dialogues and who follows the plan for the lesson no matter what and leaves the responsibility for understanding to the students without specific support.

6.3.2 Why similarities and differences in activities

Why this similarity in activities regardless of cycles? The similar use of activities in all cycles when practicing formative assessment is due to the fact that that formative assessment is imbedded in the ordinary instruction during whole-class sessions. This regularity is consistent regardless of the length of the cycle, whether elicited information is planned or unplanned, based on information on test or if the formative assessment opportunity is initiated as a question from a student. The similarities are due to the regularity of the assessment practice and the differences occur when the teacher elicits information during the lesson that she decides to act on. Depending on the nature of the elicited information (unknown to the teacher or new to students) activities unique to minute-by-minute is required.

6.3.3 Students' participation in formative assessment

When conducting formative assessment the teacher elicits information, interprets and uses the information to modify instruction to better meet learning needs. However, Eva also supports the students to participate in the activities. Students' engagement (i.e. engaged on task, willing to reveal their thoughts; participate in the learning process) in their learning is an important ingredient in the formative assessment practice (Chappuis & Stiggins, 2002; Cowie,2005; Wiliam,2007; Ruiz-Primo 2011,)and the teacher uses several actions to support students' participation. One reason that this is important is that student engagement in the formative assessment practice supports their possibility to take co-responsibility for their own learning. Another reason for the importance of students' participation is because this increases the validity of the elicited information. The activities that support

student engagement in different ways are used regularly which gives the students the possibility to experience that they or their peers are being helped by revealing what they are insecure about. Also, that their contribution leads to developed knowledge for their peers, for example when their questions are reformulated as a question to the peers. To be able to reveal students' (mis)conceptions during instructional dialogues in whole class sessions, the students have to be willing to reveal what they feel insecure about in front of the peers and teacher. That is, to write an insecure answer on the whiteboard instead of a questions mark, to answer when the random devise has chosen you as answerer of the question or to volunteer to describe the thinking behind your answer. All of these examples are from the episodes in the study. This does not conclude that all students in the class were comfortable to tell even if they were insecure of the mathematics, but a good part of them were.

In the last episode, were the teacher lets the students do most of the explaining, the debate can be described as Durchl's definition of assessment conversations, that is, as *an instructional dialogue that embeds assessment into the activity structure in the classroom. […] to engage students in the consideration of diverse ideas or representations produced by class members (Duschl & Gitomer, 1997)*. Assessment conversation is far from IRE (Initiative-response-evaluation) dialogues and also distant from IRF (initiative-response-follow up), which could be considered as formative assessment practice since it contains the eliciting and Use-phase, but lacks the student participation part.

Formative assessment is a student-centred approach to teaching and the above shows how much also the students are involved in the classroom practice when formative assessment is conducted in the way this teacher does, and represents one of the changes in the view of assessment due to formative assessment as Brookhart (2011) pointed out. However, such student participation does not come of itself. All the activities the teacher has been shown to do in this study to engage them as active learners and actively participating in all of the phases of formative assessment are likely to have been decisive for the students' engagement.

6.3.4 The efficacy of the formative assessment practice

This study shows that the formative assessment practice conducted by the teacher included the use of a number of activities, knowledge and skills not required to the same extent in a teaching practice based on the IRE-cycle. In addition, many of these activities, knowledge and skills were used together and under time pressure and in new situations that were not predictable.

Previous research has shown that both in-service teachers and pre-service teachers often have difficulties with each step of the phases of formative

assessment (Heritage et al., 2009; Schneider & Gowan, 2013; Son, 2013). Heritage (2009) questions talking about effective formative assessment from the results of her study because of the lack of teachers' content knowledge and pedagogical content knowledge. Further, their investigations are not in the real world situations with all the complexity that it brings which probably not makes these activities easier. Thus, it is not an easy way to practice formative assessment. It is therefore reasonable to ask if it is worth the cost to develop teaching towards such a practice

Do we know that the formative assessment the teacher and the students in the study practiced was effective? Well, despite that it was not perfect she really conducted a lot. From the activities used by the teacher, we know that the students have had time to think on topics some of them had problems grasping. Sometimes they were asked to think about the learning object in another perspective, form or with a different content. The teacher might not have practiced formative assessment to perfection, but she did it and it was demanding and difficult. In addition, we know that the results from the study Eva participated in earlier show students' significant learning gains with Cohens d = 0.8 on post-test compared to the control group (Andersson et al, 2013). Some of the common activities that constitute Eva's formative assessment practice are common activities for the teachers in the study. In addition, the research of Primo-Ruiz (2007) showed that students increased their learning gains when the teachers fulfilled the EIU cycles, as Eva did in the study. We can therefore assume that the activities and the character of the knowledge and skills Eva used in this study are activities, knowledge and skills that do gain student learning.

6.3.5 How to support teachers' implementation of formative assessment?

Previous sections have pointed out that the formative assessment that the teacher practice is not easy task to implement since it requires deep teacher knowledge and skills. Further, the effort to implement formative assessment might still be worth the effort due to student learning gains. The activities, knowledge and skills that Eva uses when practicing formative assessment are likely of the types that gain student learning. To be able to implement such formative assessment, teachers need education and support. The result in my study indicates that such education partly can be framed by not subject specific content and partly should be subject specific. Previous research indicates that subject specificity is one of the five most important factors for professional development programs to be effective for students' learning (Schneider & Randel, 2010). That is understandable when we look at the complex use of subject specific knowledge and skills Eva uses when she practices formative assessment.

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