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Tax Policy and Present-Biased Preferences: Paternalism under International Capital Mobility*

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Abstract

This paper deals with tax-policy responses to quasi-hyperbolic discounting. Earlier research on optimal paternalism typically abstracts from capital mobility. If capital is mobile between countries, it may no longer be possible for national governments to control domestic savings via capital taxation (as in a closed economy). In this paper, we take a broad perspective on public policy responses to self-control problems by showing how these responses vary (i) between closed and open economies, (ii) between small open and large open economies, and (iii) depending on whether or not both source based and residence based capital taxes can be used.

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1 Introduction

Much research effort has been put into studying savings behavior as well as the effects of tax policy on the incentives to save.¹ A major reason is, of course, that savings play a crucial role for economic growth and, therefore, ultimately also for future welfare. Concerns have also been raised about the level of savings, where a frequent argument is that the savings rates may be "too low" in many countries and, in particular, in the U.S., where the savings rates have been quite low for a long time (by historical comparison).² One argument emphasized in earlier research as to why individuals may save too little is that they suffer from bounded rationality in the sense of having "present-biased" preferences, i.e. a time-inconsistent preference for immediate gratification. A mechanism that generates this behavior is quasi-hyperbolic discounting, where the individual at any time uses a higher utility discount rate for intertemporal tradeoffs in the near future compared to the utility discount rate attached to intertemporal tradeoffs in a more distant future.^{3,4}

¹See Bernheim (2002) for a literature review.

²See, e.g., Guidolin and Jeunesse (2007) and Feldstein (2008).

³See, e.g., Thaler (1981), Kirby and Marakovic (1995), Kirby (1997), Viscusi, Huber and Bell (2008), and Brown, Chua and Camerer (2009) for experimental evidence pointing in this direction. See also Fredrick, Loewenstein and O'Donoghue (2002) for a review of empirical research on intertemporal choice.

⁴Bernheim, Skinner and Weinberg (2001) use data from the Panel Study of Income Dynamics and Consumer Expenditure Survey, and find that the conventional life-cycle model is unable to explain observed variation in retirement wealth in the U.S. They argue, instead, that their data is consistent with rules of thumb, mental accounting or hyperbolic discounting. A similar argument is presented by Mastrobouni and Weinberg (2009), who find (on the basis of data from the Continuing Survey of Food Intake) that retirees with

The behavioral failure that quasi-hyperbolic discounting gives rise to is a self-control problem, where the preference for immediate gratification makes the individual's current self impose an externality on his/her future selves (sometimes referred to as "internality") which, in turn, provides an argument for policy intervention by a paternalistic government. A capital subsidy to correct the incentives to save was considered by Laibson (1996),⁵ who assumed that the government aims at implementing a savings-target. This policy response is interpretable as being designed for a closed economy, since Laibson did not consider the possibility that capital is mobile between tax-jurisdictions. To our knowledge, there are no studies analyzing the corresponding policy problem under international capital mobility. Such an extension of the literature is potentially very important because if the consumers can invest their savings both at home and abroad, then domestic capital taxes/subsidies may no longer constitute perfect instruments for influencing the incentives to save faced by the domestic residents. The reason is that international capital mobility may imply restrictions on the domestic post-tax interest rate, which render capital taxes ineffective; or at least less effective than in a closed economy. This will be described in greater detail below. Therefore, the optimal policy response to quasi-hyperbolic discounting derived for a paternalistic government in a closed economy may actually be misleading if applied to an open economy. The present paper examines how a paternalistic government can use the income tax instruments available in an open economy to address the undersavings-problem caused by quasi-hyperbolic discounting, and the analysis is based on a general equilibrium model.

little pension savings, whose income mainly comes from social security, consume much less the week before they receive the paycheck than the week after.

⁵Other literature on public policy responses to quasi-hyperbolic discounting includes sin taxes attached to unhealthy commodities (e.g., Gruber and Köszegi, 2004; O'Donoghue and Rabin, 2003, 2006), health capital subsidies (Aronsson and Thunström, 2008) and public investment (Aronsson and Granlund, 2011).

To further explain why capital mobility is important in this particular context, it is useful to distinguish between a small open economy whose government treats the world-market interest rate as exogenous, and a large open economy where the government recognizes that it may influence the world-market interest rate through public policy, as well as between the source-principle and residence-principle for capital taxation. According to the former principle, capital income is taxed at source irrespective of whether it accrues to domestic or foreign residents, whereas the latter principle means that the government taxes the domestic residents irrespective of whether they earn their capital income at home or abroad. In a small open economy, a source based capital income tax would be completely ineffective as a means of influencing the incentives to save: a change in the tax rate just leads to an inflow or outflow of capital until the domestic post-tax interest rate returns to the equilibrium level given by the ("exogenous") foreign rate. Similarly, in a large open economy, neither the source based nor the residence based tax alone constitutes a perfect instrument for influencing the incentives to save, since the capital tax is also a strategic instrument for influencing the world-market interest rate. As such, to exercise perfect control over the savings behavior, both an unrestricted source based tax and an unrestricted residence based tax are needed; otherwise, the optimal tax policy may also feature adjustments of other broad-based taxes.

We take a broad perspective on optimal income taxation under quasi-hyperbolic discounting by (i) distinguishing between closed and open economies with mobile capital, (ii) addressing the policy implications of time-consistent (sophisticated) versus time-inconsistent (naive) consumers, and (iii) focusing on the simultaneous use of two tax instruments that governments typically have at their disposal; labor and capital income taxes. The distinction between naive and sophisticated consumers is arguably important: whereas a naive consumer behaves in a time-inconsistent way by erroneously expecting the self-control problem to vanish in the future, a sophisticated consumer

recognizes that the future selves are also subject to the same self-control problem.⁶ Also, since countries typically differ quite much in terms of resources and size, we examine the tax policy responses to quasi-hyperbolic discounting both in the context of small open economies (applicable to a number of European countries) and large open economies (such as the U.S.). To do so, we develop an overlapping generations (OLG) model with endogenous labor supply and savings, where each consumer lives for three periods (at least three periods are required to model quasi-hyperbolic discounting). The purpose is to analyze how a paternalistic government - which does not share the consumer-preference for immediate gratification - uses the capital and labor income taxes to correct for the behavioral failure that quasi-hyperbolic discounting gives rise to.

The income tax system is assumed to be nonlinear, which gives a reasonably realistic description of the tax instruments that many countries have at their disposal. This implies that the use of distortionary taxes is a consequence of optimization by the government and not due to the necessity to raise revenue per se. It also means that tax competition and the associated problem of under-provision of public goods does not arise.⁷ Furthermore,

⁶The behavioral implications of quasi-hyperbolic discounting may depend on whether consumers are naive or sophisticated (e.g., O'Donoghue and Rabin, 1999, 2001; Diamond and Köszegi, 2003). In an experimental study, Hey and Lotito (2009) find behavioral patterns consistent with both naivety and sophistication, even if naivety seems to be a more common type of behavior. See also the review by DellaVigna (2009).

⁷Based on the seminal work by Zodrow and Mieszkowski (1986), Wilson (1986), Bucovetsky (1991) and Bucovetsky and Wilson (1991), several studies have examined capital tax policy in the presence of mobile capital and the implications for public good provision. The bulk of this literature assumes away lump-sum taxation as a supplementary tax instrument. However, as pointed out by, e.g., Stiglitz (1987) and Kay (1990), there is no justification for this assumption. Huber (1999), therefore, examines the role of capital taxes in an open economy with nonlinear taxation (based on Stern's (1982) and Stiglitz' (1982) two-type version of Mirrlees' (1971) optimal income tax model). In that case, the only reason for implementing nonzero marginal capital tax rates is that such policies may contribute to relax the self-selection constraint (which constrains the redistribution policy

we consider both the source-principle and residence-principle for capital income taxation: both these options are practically plausible, although not equivalent from the point of view of influencing the private incentives to save.

Our benchmark model presented in Section 2 refers to a large open economy, in which the consumers may either invest at home or abroad, and where the government can use any desired combination of source based and residence based capital income taxes as well as the labor income tax for purposes of correction, revenue collection and redistribution. Furthermore, the government recognizes (and incorporates into its decision problem) that it may affect the world-market interest rate through public policy. Section 3 deals with tax policy in the benchmark model and, in particular, how the government may use tax policy to influence the private incentives to save and, therefore, correct for the self-control problem. We show how the government may implement the first best by a combination of residence based and source based capital taxes: access to both instruments enables the government to target the incentives to save *and* the world-market interest rate. We also compare this tax policy with that of a small open economy and closed economy, respectively.

Although the results derived from the benchmark model are expected (due to that the principle of targeting applies), they provide, nevertheless, a useful reference case by which to compare the results to be derived in later parts of the paper. In reality, there are many bilateral agreements against double taxation; yet, effective taxation is often the result of a mix between the residence-based and source-based principles.⁸ Furthermore, the residence-principle relies on an information sharing system where source-countries assist in the collection of revenue.⁹ This suggests to us that it

due to informational asymmetries between the government and the private sector).

⁸We are grateful to one of the referees for suggesting this argument.

⁹This information exchange problem has been addressed by Baccetta and Espinosa (1995) and Eggert and Kolmar (2002).

is useful to relax the assumption that the government can freely use both principles for capital taxation. In Section 4, we consider situations where the government uses the labor income tax in combination with either the residence based or source based capital income tax. Two results are particularly interesting here: (i) in the absence of source-based taxation, the paternalist government's optimal tax policy in a large open economy does not necessarily feature marginal saving-subsidies (while the corresponding policy choice of a small open economy always includes saving-subsidies), and (ii) in the absence of residence-based taxation, in which case the principle of targeting no longer applies, marginal labor taxes/subsidies serve as indirect instruments to influence the incentives to save faced by the consumers. Section 5 presents a short summary and gives some suggestions for future research, while proofs are presented in the Appendix.

2 The Model

In this section, we present an OLG-economy in which each consumer lives for three periods and is subject to a self-control problem generated by quasi-hyperbolic discounting. We also present the production sector of the economy as well as the decision problem faced by the government.

2.1 Consumers

We assume that each consumer works in the first and second period of life and becomes a pensioner in the third. As the number of consumers is not important for the qualitative results to be derived below, the size of each cohort will be normalized to one. This means that one new consumer enters the economic system in each time period and that the population is constant. The consumers have identical preferences for consumption, c , leisure, z , and a public good, g . The instantaneous utility faced by a consumer of age $i = 0, 1, 2$ ($0 =$ young, $1 =$ middle-aged and $2 =$ old) in period t

can be written as $u_{i,t} = v(c_{i,t}, z_{i,t}) + \phi(g_t)$. The instantaneous utility function is increasing and strictly concave in each argument. We also add the (conventional) assumptions that c and z are normal goods, and that consumption and leisure are complements in the utility function in the sense that $\partial^2 u_{i,t} / \partial c_{i,t} \partial z_{i,t} \geq 0$. Since the available time in each period can be used either for work or leisure, or a combination of them, the consumer also faces a time constraint, $H = l_{i,t} + z_{i,t}$, where H is a fixed time endowment and $l_{i,t}$ is the hours of work.

Following the approach developed by Phelps and Pollak (1968), and later used by, e.g., Laibson (1997) and O'Donoghue and Rabin (2003, 2006), the intertemporal objective in period t faced by generation t (i.e. individuals born in period t) can be written as follows:

$$U_{0,t} = u_{0,t} + \beta \sum_{i=1}^2 \Theta^i u_{i,t+i} \quad (1)$$

where Θ is a conventional (exponential) utility discount factor while the parameter $\beta \in (0, 1)$ reflects the preference for immediate gratification.

As we mentioned above, the consumer has the option to invest his/her savings at home or abroad. At any time t , r_t denotes the domestic before-tax interest rate, while R_t^* denotes the rate of return before residence based taxation that domestic consumers may attain by investing abroad.¹⁰ Capital income is taxed according to a mixed system, which contains a source based and a residence based part with marginal tax rates θ_t^s and $\theta_{i,t}^r$, respectively. Note that the source based tax is a proportional tax, where the tax rate at any time t is common to all consumers irrespective of age-group. It would be very difficult (if not impossible) to differentiate the source based tax rate among consumers, since those faced by the higher rate would invest their savings abroad instead of at home. The net (after-tax) interest rate faced

¹⁰To be more specific, if other countries only use residence based capital income taxes at time t , R_t^* may be thought of as the foreign gross rate of return; if other countries use source based capital income taxes, we can interpret R_t^* as the foreign rate of return net of the source based capital income tax.

by domestic consumers, if investing one dollar at home at time t , can then be written as

$$r_{i,t}^n = \underbrace{(1 - \theta_{i,t}^r)(1 - \theta_t^s)}_{=(1-\theta_{i,t})} r_t \quad (2)$$

where $\theta_{i,t} = \theta_{i,t}^r + \theta_t^s - \theta_{i,t}^r \theta_t^s$ will be referred to the *total marginal capital income tax rate* faced by age-group i in period t . If investing abroad, on the other hand, the consumer obtains the net return $(1 - \theta_{i,t}^r) R_t^*$. We assume that capital is perfectly mobile between countries, in which case the capital market equilibrium must obey the following condition:¹¹

$$(1 - \theta_t^s) r_t = R_t^*. \quad (3)$$

To simplify the notation, we abstract from bequests and assume that the initial wealth faced by each consumer is zero. The consumer earns labor income when young and middle-aged, and capital income when middle-aged and old. The gross wage rate is allowed to correlate with the age of the worker, meaning that the young and middle-aged worker may face different gross wage rates. Let $w_{i,t}$ denote the gross wage rate facing age-group i in period t , and $s_{i,t}$ denote savings. The marginal net (after-tax) wage rate can then be written as $w_{i,t}^n = w_{i,t}(1 - \tau_{i,t})$, where $\tau_{i,t}$ is the marginal labor income tax rate. Note that the marginal income tax rates (attached to both labor and capital) are allowed to vary over time and across age-groups. The tax system also contains lump-sum components, $T_{i,t}$ ($i = 0, 1, 2$), which

¹¹The capital market equilibrium condition can also be written as

$$(1 - \theta_{i,t}^r)(1 - \theta_t^s)r_t = R_t^*(1 - \theta_{i,t}^r)$$

for $i = 1, 2$. Equation (3) then follows by eliminating $(1 - \theta_{i,t}^r)$ on both sides. An alternative specification would be

$$(1 - \theta_{i,t}^r - \theta_t^s)r_t = R_t^*(1 - \theta_{i,t}^s)$$

for $i = 1, 2$. This formulation is clearly more restrictive than equation (3), as it would imply $\theta_{1,t}^r = \theta_{2,t}^r$.

may also vary over time and across age-groups. This flexible tax system provides a simple framework for studying the corrective and strategic use of taxation, as it implies that non-zero marginal income tax rates attached to labor and/or capital follow from optimization by the government and are not due to arbitrary restrictions on the tax instruments or the necessity to raise revenue per se.¹² The intertemporal budget constraint faced by an individual of generation t can then be written as

$$c_{0,t} = w_{0,t}^n l_{0,t} - T_{0,t} - s_{0,t} \quad (4)$$

$$c_{1,t+1} = w_{1,t+1}^n l_{1,t+1} + (1 + r_{1,t+1}^n) s_{0,t} - T_{1,t+1} - s_{1,t+1} \quad (5)$$

$$c_{2,t+2} = (1 + r_{2,t+2}^n) s_{1,t+1} - T_{2,t+2}. \quad (6)$$

Now, recall from the discussion in the introduction that although the consumers may suffer from self-control problems, it is not clear whether we should expect them to act in accordance with naivety or sophistication. We will, therefore, consider both these possibilities in the analysis below. Since the individual first order conditions as well as the policy rules for optimal taxation under naivety are interpretable as technical special cases of the corresponding first order conditions and policy rules, respectively, that follow under sophistication, we derive the results under the assumption that agents are sophisticated and then comment upon how the results are modified if agents instead were naive.

To arrive at a time-consistent solution for the sophisticated agents, their decision problems will be solved sequentially. We begin by briefly examining the labor supply and savings behavior of the middle-aged consumer, and then analyze the labor supply and savings behavior of the young sophisticated consumer who acts as a strategic leader vis-a-vis his/her middle-aged self. This strategic leadership motive is absent for naive consumers, who (erroneously) expect not to be facing the self-control problem in the future.

¹²Similar tax systems have also been examined in other literature on optimal income taxation in dynamic economies; see, e.g., Brett (1997) and Aronsson and Johansson-Stenman (2010).

When middle-aged, the consumer chooses $l_{1,t+1}$ and $s_{1,t+1}$ to maximize $U_{1,t+1} = u_{1,t+1} + \beta\Theta u_{2,t+2}$ subject to equations (5) and (6), where the level of savings chosen when young, $s_{0,t}$, is treated as fixed. The first order conditions are

$$w_{1,t+1}^n \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{1,t+1}}{\partial z_{1,t+1}} = 0 \quad (7)$$

$$\beta\Theta (1 + r_{2,t+2}^n) \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} = 0. \quad (8)$$

Note that quasi-hyperbolic discounting does not modify the atemporal trade-off between consumption and leisure, which means that the labor supply condition in equation (7) takes the same form as in a standard model, whereas $\beta < 1$ in equation (8) means that the consumer saves less than he/she would have done without any self-control problem, *ceteris paribus*. Since there is no incentive for the sophisticated middle-aged consumer to constrain his/her old self in our model (the old consumer makes no intertemporal choice), equations (7) and (8) take the same form independently of whether the consumer is naive or sophisticated. We can use equations (7) and (8) to derive the labor supply and savings functions

$$l_{1,t+1} = l_1(w_{1,t+1}^n, r_{1,t+1}^n, r_{2,t+2}^n, T_{1,t+1}, T_{2,t+2}, s_{0,t}) \quad (9)$$

$$s_{1,t+1} = s_1(w_{1,t+1}^n, r_{1,t+1}^n, r_{2,t+2}^n, T_{1,t+1}, T_{2,t+2}, s_{0,t}). \quad (10)$$

The sequential decision process means that $l_{1,t+1}$ and $s_{1,t+1}$ will be functions of $s_{0,t}$. As a consequence, equations (9) and (10) can be viewed as reaction functions via which the young consumer may influence the behavior of his/her middle-aged self. As an increase in $s_{0,t}$ typically means that more resources become available for consumption and saving when middle-aged, all results below will be interpreted under the assumption that $\partial s_{1,t+1}/\partial s_{0,t} > 0$.¹³

¹³This condition always applies except in the somewhat unlikely situation where an increase in $s_{0,t}$ leads to such a large reduction in $l_{1,t+1}$ that the resources available for consumption and savings when middle-aged actually decrease (recall that leisure is assumed to be a normal good).

The young sophisticated consumer maximizes the objective defined in equation (1) subject to the life-time budget constraint presented in equations (4) - (6) as well as subject to the reaction functions defined by equations (9) - (10). The first order conditions for $l_{0,t}$ and $s_{0,t}$ can be written as

$$0 = w_{0,t}^n \frac{\partial u_{0,t}}{\partial c_{0,t}} - \frac{\partial u_{0,t}}{\partial z_{0,t}} \quad (11)$$

$$0 = \beta \Theta (1 + r_{1,t+1}^n) \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} + \frac{\partial U_{0,t}}{\partial s_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \quad (12)$$

in which

$$\frac{\partial U_{0,t}}{\partial s_{1,t+1}} = \frac{(1 - \beta)}{\beta} \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} > 0.$$

The final term on the right hand side of equation (12) captures how the savings by the young consumer, $s_{0,t}$, affects the savings by his/her middle-aged self. With $\partial s_{1,t+1} / \partial s_{0,t} > 0$, this effect constitutes an incentive for the young sophisticated consumer to save more than he/she would otherwise have done to counteract the tendency to undersave when middle-aged (which the sophisticated young consumer is fully aware of).¹⁴ This effect would be absent for a naive consumer, who erroneously expects to have time-consistent preferences in the future, meaning that the first order condition for savings faced by the young naive consumer takes the same general form as equation (8) above. Equations (11) and (12) implicitly define the following labor supply and savings functions:

$$l_{0,t} = l_0(w_{0,t}^n, w_{1,t+1}^n, r_{1,t+1}^n, r_{2,t+2}^n, T_{0,t}, T_{1,t+1}, T_{2,t+2}) \quad (13)$$

$$s_{0,t} = s_0(w_{0,t}^n, w_{1,t+1}^n, r_{1,t+1}^n, r_{2,t+2}^n, T_{0,t}, T_{1,t+1}, T_{2,t+2}). \quad (14)$$

¹⁴If we were to allow agents to be partially naive in the sense of O'Donoghue and Rabin (2001), the first order condition for savings would still take the form of equation (12); although based on an underestimation of the $(1 - \beta)$ component of the final term on the right hand side (since the partially naive consumer underestimates the magnitude of the future self-control problem).

2.2 Production

Output is produced by identical competitive firms, the number of which is normalized to one. We assume that young and middle-aged workers are imperfect substitutes in the production and define the "effective labor" supplied in period t as follows; $L_t = l_{0,t} + al_{1,t}$, where a is a positive constant. If middle-aged workers are more (less) productive than young workers, then $a > 1$ (< 1). The production function is given by $F(L_t, K_t)$, which is increasing and strictly concave in its respective argument as well as characterized by constant returns to scale. By using the normalizations $f(k_t) = F(L_t, K_t)/L_t$ and $k_t = K_t/L_t$, we obtain the standard first order conditions

$$r_t = f_k(k_t) \quad (15)$$

$$w_{0,t} = f(k_t) - r_t k_t \quad (16)$$

together with $w_{1,t} = a w_{0,t}$.

2.3 Equilibrium

The aggregate savings in period $t-1$, $s_{0,t-1} + s_{1,t-1}$, earns interest in period t . Each consumer may either invest his/her savings at home (in the form of domestic capital) or abroad (in the form of foreign capital), or may use a combination of these two options. Let Q_t denote the net export of capital in period t . It will then follow from the national accounts that¹⁵

$$s_{0,t-1} + s_{1,t-1} = K_t + Q_t. \quad (17)$$

If our home country is a large open economy, its net export of capital will influence the foreign rate of return on capital. Therefore, we can write the foreign rate of return as a function of the net export of capital from "our"

¹⁵Recall that the size of each cohort has been normalized to one.

country, $R_t^* = R(Q_t)$, and we assume that $dR_t^*/dQ_t < 0$. The latter is interpretable to mean that an increase in Q will increase the foreign capital stock and, therefore, reduce the foreign interest rate, *ceteris paribus*. The capital market equilibrium condition given by equation (3) can then be specified as follows:

$$(1 - \theta_t^s) r_t = R(Q_t). \quad (18)$$

Now, by using the identities $k_t = K_t/L_t$ and $L_t = l_{0,t} + a l_{1,t}$ in combination with equations (15)-(18), we can derive the following equations for the domestic factor prices and net export of capital:

$$r_t = r(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}) \quad (19)$$

$$w_{0,t} = w_0(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}) \quad (20)$$

$$Q_t = Q(\theta_t^s, l_{0,t}, l_{1,t}, s_{0,t-1}, s_{1,t-1}). \quad (21)$$

In equations (19)-(21), the variables $l_{0,t}$, $l_{1,t}$, $s_{0,t-1}$ and $s_{1,t-1}$ are, in turn, determined by equations (9), (10), (13) and (14). Therefore, equations (19)-(21) provide the channels through which public policy affects the domestic factor prices and net export of capital.

For further use, note also that our model nests two interesting special cases. First, if R_t^* is treated as exogenous for all t by the national government (instead of as a function of the net export of capital), our model describes a small open economy. Second, if $Q_t \equiv 0$ for all t , we have a closed economy. Both these special cases will be analyzed below along with the results of the more general model.

2.4 The Government

The government acts as first mover vis-a-vis the private sector (by recognizing how private agents respond to policy) and aims to correct for the self-control problem described above as well as raise revenue and achieve

redistribution. Following earlier literature on optimal paternalism,¹⁶ we assume that $\beta = 1$ from the point of view of the (paternalistic) government, meaning that the government wants to impose the following present value utility function on generation t : $\tilde{U}_{0,t} = u_{0,t} + \sum_{i=1}^2 \Theta^i u_{i,t+i}$. Note that this function differs from the actual utility function faced by generation t in equation (1) above due to the consumer preference for immediate gratification. The social welfare function can then be written as

$$W = \sum_t \Theta^t \tilde{U}_{0,t}. \quad (22)$$

To simplify the analysis, we ignore public debt.¹⁷ By using equations (2) and (17), and that the marginal unit taxes of labor and capital facing the consumer of age i are given by $w_{i,t} - w_{i,t}^n = \tau_{i,t} w_{i,t}$ and $r_t - r_{i,t}^n = \theta_{i,t} r_t$, the budget constraint facing the government can be written as

$$G_t = \sum_{i=0}^2 T_{i,t} + \sum_{i=0}^1 (w_{i,t} - w_{i,t}^n) l_{i,t} + \sum_{i=0}^1 (r_t - r_{i,t}^n) s_{i,t-1} - \theta_t^s r_t Q_t \quad (23)$$

for all t , where r_t , $w_{0,t}$ and Q_t are given by equations (19), (20) and (21) above, and $w_{1,t} = aw_{0,t}$. The final term on the right hand side of equation (23) follows because the government can only levy source based taxes on the domestic capital stock, i.e. $r_t K_t = r_t \sum_i s_{t-1}^i - r_t Q_t$ is the tax base for the source based tax.

The public decision problem is to choose $w_{0,t}^n$, $w_{1,t}^n$, $r_{1,t}^n$, $r_{2,t}^n$, $T_{0,t}$, $T_{1,t}$, $T_{2,t}$, θ_t^s and g_t for all t to maximize the social welfare function in equation (22) subject to the budget constraint presented in equation (23).¹⁸ The whole time sequence of each policy instrument is decided upon, and announced, at

¹⁶See, e.g., O'Donoghue and Rabin (2003, 2006), Aronsson and Thunström (2008) and Aronsson and Granlund (2011).

¹⁷Although this assumption limits the scope for redistribution over time, it is not important for the qualitative results derived below with respect to how the government uses marginal income taxation to correct for the self-control problem.

¹⁸Note that marginal tax rates and marginal factor prices (net of tax) are equivalent policy instruments.

time zero.¹⁹ The Lagrangean associated with this policy problem and the corresponding first order conditions are presented in the Appendix. Here, we concentrate on the implications of these first order conditions for optimal tax policy.

3 Tax Policy in the Benchmark Model

In this section, we examine the optimal tax policy in our benchmark model for a large open economy, whose government recognizes that it may influence the world-market interest rate. We will then turn to the special cases mentioned above, i.e. the small open economy and the closed economy, respectively. The tax policy used by the paternalistic government in a large open economy with the full set of instruments described above is summarized as follows:

Proposition 1. *In the benchmark model for a large open economy, the optimal tax policy can be characterized as follows for all t :*

(i) *Marginal labor income tax rates:*

$$\tau_{0,t} = \tau_{1,t} = 0;$$

(ii) *Source based capital income tax rate:*

$$\theta_t^s < 0 \text{ if } Q_t > 0, \theta_t^s > 0 \text{ if } Q_t < 0, \text{ and } \theta_t^s = 0 \text{ if } Q_t = 0;$$

(iii) *Total marginal capital income tax rates:*

$$\theta_{1,t} < 0 \text{ and } \theta_{2,t} < 0, \text{ where } |\theta_{1,t}| < |\theta_{2,t}| \text{ if the consumers are sophisticated, while } \theta_{1,t} = \theta_{2,t} \text{ if they are naive;}$$

(iv) *Residence based capital income tax rates:*

$$\theta_{1,t}^r < 0 \text{ and } \theta_{2,t}^r < 0 \text{ if } Q_t \leq 0, \text{ while } \theta_{1,t}^r \geq 0 \text{ and } \theta_{2,t}^r \geq 0 \text{ if } Q_t > 0.$$

¹⁹Note that the problem of time-inconsistent public policy does not arise here. In a more general model with information asymmetries between the government and the private sector, on the other hand, a policy based on commitment may no longer be time-consistent; see Aronsson and Sjögren (2012) for a study of time-consistent optimal taxation without commitment under quasi-hyperbolic discounting in a closed economy.

Proof: see the Appendix.

Part (i) of Proposition 1 reflects the principle of targeting. The intuition is that for the large open economy examined here, the two capital tax instruments can be used simultaneously to exercise control over the private incentives to save *and* exercise market power to influence the world-market interest rate. As a consequence, there is no reason to use the labor income tax as a supplemental instrument for correction.

Part (ii) describes how the government uses the source based tax to affect the foreign rate of return. This gives rise to the pecuniary international externality highlighted in DePater and Myers (1994).²⁰ In line with their results, we find that if the country is a net exporter of capital at time t , so $Q_t > 0$, an increase in the foreign rate of return will lead to a higher domestic national income. This can be accomplished via a source based capital subsidy implemented at home: this reduces the net export of capital which, in turn, contributes to increase the foreign rate of return. If the country is a net importer of capital, the argument for a positive source based tax is analogous. These results follow immediately from the tax formula for θ_t^s , which is given by

$$\frac{\theta_t^s}{1 - \theta_t^s} = \frac{Q_t dR_t^*}{R_t^* dQ_t}. \quad (24)$$

Equation (24) is a variant of the standard inverse elasticity rule for optimal taxation.

The total marginal capital income tax rates described in part (iii) reflect correction for the self-control problem (that would otherwise manifest itself in terms of too little saving). Note that the distinction between naivety and sophistication is important here. If consumers are naive, $\theta_{1,t}$ and $\theta_{2,t}$ satisfy

$$\theta_{1,t} = \theta_{2,t} = \left(\frac{\beta - 1}{\beta} \right) \left(\frac{1 + r_t}{r_t} \right) < 0, \quad (25)$$

²⁰DePater and Myers (1994) also show how a global planner may use Pigouvian correction to internalize this pecuniary externality.

whereas the corresponding policy with sophisticated consumers can be summarized as

$$\theta_{1,t} = \left(\frac{\beta-1}{\beta}\right) \left(\frac{1+r_t}{r_t}\right) - \left(\frac{\beta-1}{\beta}\right) \frac{1}{r_t} \frac{\partial s_{1,t}}{\partial s_{0,t-1}} < 0 \quad (26)$$

$$\theta_{2,t} = \left(\frac{\beta-1}{\beta}\right) \left(\frac{1+r_t}{r_t}\right) < 0. \quad (27)$$

The difference between equations (25) and (26) follows because the young sophisticated consumer acts strategically, i.e. chooses a higher level of $s_{0,t}$ to stimulate increased savings by his/her middle-aged self. Therefore, the second term on the right hand side of equation (26) will counteract - although it does not fully offset - the tendency to undersave caused by the preference for immediate gratification.²¹ This "strategic leadership effect" is absent for naive consumers, which explains why the government needs to subsidize the savings by young consumers at a higher rate under naivety than under sophistication, *ceteris paribus*. The subsidy rate attached to the savings by the middle-aged does not depend on whether the consumers are naive or sophisticated in our model, as the middle-aged consumer has no incentive to act strategically vis-a-vis his/her old self.²²

Observe that since the two capital tax instruments are linked via the total marginal capital income tax rate, part (iv) is interpretable to mean that the residence based tax serves as a "residual" such as to make the source based tax compatible with the total marginal capital income tax rate (which is the tax measure of relevance for private saving). By using the expression for the total marginal capital income tax rate presented in Section 2, we can solve for the residence based tax, i.e. $\theta_{i,t}^r = (\theta_{i,t} - \theta_t^s)/(1 - \theta_t^s)$ for $i = 1, 2$.

²¹Comparative statics based on equations (9) and (10) imply $\partial s_{1,t+1}/\partial s_{0,t} < (1+r_t)$.

²²This mix of residence-based and source-based taxation implies that the tax system is neither capital export neutral (capital export neutrality is achieved when there is no source-based taxation, in which case the tax system does not affect tax payers' decisions about whether to invest at home or abroad) nor capital import neutral (a tax system is capital import neutral when all investors in a jurisdiction face the same after-tax returns).

Since $\theta_{i,t} < 0$, we have $\theta_{i,t}^r < 0$ if $0 < \theta_t^s < 1$ (applicable to a net capital importer), while $\theta_{i,t}^r$ can be either positive or negative if $\theta_t^s < 0$ (applicable to a net capital exporter). If the self-control problem were absent, such that $\beta = 1$, we can immediately see that $\theta_{i,t}$ should be equal to zero, in which case the residence based tax would be used to fully offset the savings tax-wedge created by the source based tax. Note also that the motive for paternalism reflected in the total marginal capital income tax rates is purely corrective. As such, if we were to allow for demographic changes, this would not change the incentives behind the paternalist government's use of corrective taxation.

In the Appendix, we present the policy rule for optimal provision of the public good and show that this rule is given by the Samuelson condition. The reason is again the principle of targeting whereby the capital taxes are sufficient instruments to correct for the behavioral failure as perceived by the paternalist government. A similar result was derived by Aronsson and Granlund (2011) for a closed economy.

3.1 Two Useful Special Cases of the Benchmark Model

The benchmark model set out in Section 2, and examined above in this section, refers to a large open economy. As we mentioned before, this benchmark model nests two interesting special cases: first, if the foreign rate of return is treated as fixed by the national government (instead of as a function of the net export of capital), the model reduces to that of a small open economy and, second, if the net export of capital is equal to zero in each period, our model corresponds to a closed economy. These two special cases share common policy-elements, which are summarized in Proposition 2:

Proposition 2. *In a small open economy where R_t^* is treated as exogenous by the domestic government for all t , and in a closed economy where $Q_t \equiv 0$ for all t , the optimal tax policy satisfies conditions (i) and (iii) in Proposition 1.*

The proof of Proposition 2 follows immediately from the proof of Proposition 1 and is, therefore, omitted. The intuition is straight forward. If applied to a small open economy, the inverse elasticity rule in equation (24) means that the government will not use the source based tax instrument ($\theta_t^s = 0$, $\theta_{1,t}^r = \theta_{1,t}$ and $\theta_{2,t}^r = \theta_{2,t}$ for all t). By analogy, the source based tax is a redundant instrument in the closed economy, simply because the world-market interest rate is no longer an interesting target variable for the domestic government. In either case, therefore, the marginal capital income tax structure is characterized solely by equation (25) if the consumers are naive, and solely by equations (26) and (27) if they are sophisticated, which constitute perfect instruments for internalizing the internal externalities generated by quasi-hyperbolic discounting.

4 Restricted Capital Taxation

The above analysis of tax policy in (large and small) open economies was carried out under the assumption that the government may implement both source based and residence based capital taxes. As we mentioned in the introduction, it may in reality be difficult for national governments to fully implement any of these two instruments. It is, therefore, useful to consider each principle for capital taxation separately, and analyze how the government in each such case uses the mix of labor and capital taxation to correct for the self-control problem generated by quasi-hyperbolic discounting.

4.1 Residence Based Taxation

If the residence based tax constitutes the only available instrument for capital taxation, it follows that all capital income faced by domestic residents will be taxed at home independently of source. In this case, where $\theta_t^s \equiv 0$ for all t by assumption, the capital market equilibrium condition given by equation (3) simplifies to read $r_t = R_t^*$, which means that the gross domestic

interest rate is equal to the rate of return that consumers attain if investing abroad.

In a small open economy, whose government treats R_t^* for all t as exogenous, it follows immediately from Proposition 2 that the optimal mix of labor income taxation and residence based capital income taxation satisfies conditions (i) and (iii) of Proposition 1. The intuition is, of course, that even if it were optional to use source based taxation (in addition to the other tax instruments), the small open economy would not implement such a tax. Therefore, the additional restriction that $\theta_t^s \equiv 0$ for all t introduced here is of no practical relevance.

Instead, let us focus attention on the large open economy. As we mentioned above, the government of such an economy treats R_t^* as a function of the domestic net export of capital, Q_t , which is, in turn, determined by domestic tax policy according to equation (21). A restriction on the use of source based capital taxation may in this case have important implications for how the government uses its other instruments. By solving the public decision-problem set out in subsection 2.4 under the additional restriction that $\theta_t^s \equiv 0$ for all t , we can derive the following result:

Proposition 3. *In a large open economy, whose government does not have access to the source based capital tax, i.e. $\theta_t^s \equiv 0$ for all t , the optimal tax policy can be characterized as follows for all t :*

(i) *Marginal labor income tax rates:*

$$\tau_{0,t} < 0 \text{ and } \tau_{1,t} < 0 \text{ if } Q_t > 0, \tau_{0,t} > 0 \text{ and } \tau_{1,t} > 0 \text{ if } Q_t < 0, \text{ and} \\ \tau_{0,t} = \tau_{1,t} = 0 \text{ if } Q_t = 0;$$

(ii) *Residence based marginal capital income tax rates:*

$$\theta_{1,t}^r < 0 \text{ and } \theta_{2,t}^r < 0 \text{ if } Q_t \leq 0, \text{ and } \theta_{1,t}^r \leq 0 \text{ and } \theta_{2,t}^r \geq 0 \text{ if } Q_t > 0.$$

Proof: see the Appendix.

The policy presented in Proposition 3 differs from that of Proposition 1, due to that the source based tax instrument is no longer available for influencing the foreign rate of return. Instead, both the labor income tax and the residence based capital income tax will in this case partly serve as (imperfect) instruments for exercising this market power. In the Appendix, we derive the following expression for the marginal labor income tax rate faced by age-group i in period t :

$$\tau_{i,t} = -\frac{Q_t}{w_{i,t}} \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial l_{i,t}}. \quad (28)$$

Note that $\partial Q_t / \partial l_{i,t} < 0$ due to complementarity between labor and domestic capital: increased use of labor in the domestic production will, therefore, contribute to reduce the net export of capital. A decrease in the net capital export leads to an increase in the rate of return that domestic consumers obtain if investing abroad, which is desirable if the country is a net exporter of capital. This decrease in the net capital export can be accomplished by subsidizing labor. The argument for a positive marginal labor income tax rate in case the country is a net importer of capital is analogous.

Turning to capital income taxation, note that a change in the level of savings by any age-group affects the net export of capital and, therefore, the foreign interest rate. This effect is captured by introducing the following variable:

$$\zeta_{i,t} = -\frac{Q_t}{\beta r_t} \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial s_{i,t-1}} \text{ for } i = 0, 1.$$

One can show that equation (21) implies $\partial Q_t / \partial s_{i,t-1} > 0$, while $dR_t^* / dQ_t < 0$ by the assumptions made earlier. The residence based marginal capital income tax rates (which are equal to the total marginal capital income tax rates) can then be characterized as follows if the consumers are sophisticated:

$$\theta_{1,t}^r = \theta_{1,t} = \left(\frac{\beta - 1}{\beta} \right) \left(\frac{1 + r_t}{r_t} \right) - \left(\frac{\beta - 1}{\beta} \right) \frac{1}{r_t} \frac{\partial s_{1,t}}{\partial s_{0,t-1}} + \zeta_{0,t} \quad (29)$$

$$\theta_{2,t}^r = \theta_{2,t} = \left(\frac{\beta - 1}{\beta} \right) \left(\frac{1 + r_t}{r_t} \right) + \zeta_{1,t}. \quad (30)$$

With naive consumers, the only modification would be that the second term on the right hand side of equation (29) vanishes. Therefore, and by analogy to the results derived in Section 3, pure correction for the self-control problem would necessitate that the savings by young consumers be subsidized at a higher rate under naivety than under sophistication.

The new aspect in equations (29) and (30) is that the total marginal capital income tax rates (the rates of relevance for the incentives to save) no longer only reflect policies to correct for the self-control problem, as they did in Section 3; instead, the final term on the right hand side is due to that the government also uses the residence based tax instrument to exercise market power in the international capital market (due to that the source based tax instrument is absent here). Since part of a given increase in the level of savings may be invested abroad, it follows that increased savings by domestic consumers contributes to reduce the foreign rate of return, *ceteris paribus*. This is desirable for a net importer of capital and undesirable for a net exporter. As a consequence, $\zeta_{i,t} < 0$ (> 0) for a net capital importer (exporter), which explains why $\theta_{1,t}$ and $\theta_{2,t}$ are both negative if the country is a net capital importer, and ambiguous in sign if it is a net capital exporter. In other words, if the large open economy is a net capital exporter, it may actually be optimal for the paternalistic government to implement *positive* marginal capital income tax rates, despite that this government does not share the consumer preference for immediate gratification.

4.2 Source Based Taxation

The residence based capital income tax constitutes a direct instrument for influencing the savings by domestic residents. Access to residence based taxation, therefore, means that the government uses this (and no other) instrument to correct for the self-control problem, which would otherwise manifest itself in terms of too little saving, although it may also use the residence based tax for other purposes (as we saw in equations (29) and

(30) in the previous subsection). However, without the residence based tax, there is no longer a direct instrument by which the government can target the incentives to save, in which case the labor income tax and - if the economy is large in the sense described above - the source based capital income tax might be used as indirect instruments to correct for the self-control problem.²³

Small Open Economy

Consider first the tax policy of a small open economy, whose government treats the foreign rate of return, R_t^* , in equation (3) as exogenous for all t . If the government does not have access to the residence based capital income tax, how should the labor income tax be used in response to the preference for immediate gratification? Basic intuition suggests that a marginal labor subsidy might accomplish this task; such a subsidy typically leads to increased hours of work and income which, in turn, leads to increased savings. We start by showing that this argument is correct in a simplified version of the model, irrespective of whether the consumers are naive or sophisticated. The simplified version of the model means that the labor supply is fixed for the middle-aged generation although flexible for the young, allowing us to avoid intertemporal labor supply responses to changes in $\tau_{0,t}$. We will then return to the general model where also $l_{1,t+1}$ is flexible.

Therefore, by solving the public decision problem in subsection 2.4 subject to $\theta_{i,t}^r \equiv 0$ for $i = 1, 2$ and all t , as well as subject to the additional restrictions that R_t^* and $l_{1,t+1}$ are exogenous for all t , we have derived the following result:

²³A possible argument against this approach is that other means of influencing the the incentives to save, such as information campaigns, may be more useful than the blunt instruments considered here. However, information campaigns are less likely to be successful in reaching their intended effects if the intertemporal choices are governed by a preference for immediate gratification. It is, therefore, of clear value to understand how the tax system ought to be modified in response self-control problems, even if direct instruments for targeting savings behavior are absent.

Proposition 4. *Suppose that the residence based tax instrument is not available, so $\theta_{i,t}^r \equiv 0$ for $i = 1, 2$ and all t . If the labor supply is fixed for the middle-aged, then the optimal tax policy in a small open economy satisfies (i) $\tau_{0,t} < 0$ for all t , and (ii) $\theta_t^s = 0$ for all t .*

Proof: see the Appendix.

The intuition behind part (ii) is the same as before. To interpret part (i), let us introduce the following compensated labor supply and savings responses to an increase in the marginal wage rate:

$$\frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} = \frac{\partial l_{0,t}}{\partial w_{0,t}^n} + l_{0,t} \frac{\partial l_{0,t}}{\partial T_{0,t}} > 0 \quad (31)$$

$$\frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} = \frac{\partial s_{0,t}}{\partial w_{0,t}^n} + l_{0,t} \frac{\partial s_{0,t}}{\partial T_{0,t}} > 0, \quad (32)$$

where the right hand side of equation (32) is positive due to complementarity between consumption and leisure in the utility function. Then, by using $\gamma_t > 0$ to denote the Lagrange multiplier associated with the government's budget constraint (i.e. the marginal cost of public funds measured in terms of utility) as well as the short notation

$$\alpha_{0,t} = \frac{\partial \tilde{s}_{0,t} / \partial w_{0,t}^n}{\partial \tilde{l}_{0,t} / \partial w_{0,t}^n} > 0$$

one can show that the marginal labor income tax rate implemented for the young naive consumer in period t is given by

$$\tau_{0,t} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta} \right) \left(\frac{\partial u_{0,t}}{\partial c_{0,t}} + \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \right) < 0. \quad (33)$$

With sophisticated consumers, the corresponding formula becomes

$$\tau_{0,t} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta} \right) \frac{\partial u_{0,t}}{\partial c_{0,t}} < 0. \quad (34)$$

Equation (33) contains an additional negative term by comparison with equation (34), which suggest that the subsidy may be larger with naive

than sophisticated consumers. The intuition is that sophisticated consumers partly internalize the behavioral failure themselves.²⁴ However, notice that the magnitudes of the labor supply and savings responses following this subsidy, as reflected in the variable $\alpha_{0,t}$, may also depend on whether the consumers are naive or sophisticated, which renders the comparison inconclusive without further assumptions.

Returning to the general model in which both $l_{0,t}$ and $l_{1,t+1}$ are flexible, the subsidy result presented in Proposition 4 no longer necessarily applies. The reason is that public policy has intertemporal consequences, and a labor tax/subsidy when middle-aged, if expected when young, may influence the hours of work and savings behavior both when young and when middle-aged. We exemplify by considering the marginal labor income tax rates implemented for the sophisticated generation t , although the qualitative results also apply to naive consumers. We show in the Appendix that the labor income tax structure can be characterized as

$$\tau_{0,t} = \frac{\beta - 1}{\beta} \left[a_{0,t} \frac{\partial u_{0,t}}{\partial c_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} - b_{0,t} \frac{\partial u_{1,t+1} \Theta}{\partial c_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} \right] \quad (35)$$

$$\begin{aligned} \tau_{1,t+1} &= \frac{\beta - 1}{\beta} a_{1,t+1} \left\{ \frac{\partial u_{1,t+1} \Theta}{\partial c_{1,t+1}} \frac{\partial \tilde{s}_{1,t+1}}{\partial w_{1,t+1}^n} + \frac{\partial u_{0,t}}{\partial c_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} \right\} \quad (36) \\ &\quad - \frac{\beta - 1}{\beta} b_{1,t+1} \frac{\partial u_{0,t}}{\partial c_{0,t}} \frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n}, \end{aligned}$$

in which $a_{0,t} = [\partial \tilde{l}_{1,t+1} / \partial w_{1,t+1}^n] / \varphi_{0,t}$, $b_{0,t} = [\partial \tilde{s}_{1,t+1} / \partial w_{1,t+1}^n] / \varphi_{0,t}$, $a_{1,t+1} = [\partial \tilde{l}_{0,t} / \partial w_{0,t}^n] / \varphi_{1,t+1}$ and $b_{1,t+1} = [\partial \tilde{s}_{0,t} / \partial w_{0,t}^n] / \varphi_{1,t+1}$, while (for $i = 0, 1$)

$$\varphi_{i,t+i} = \gamma_{t+i} w_{i,t+i}^n \left[\frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial w_{1,t+1}^n} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} \right) - \frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} \right].$$

In general, neither equation (35) nor equation (36) can be signed unambiguously, since the intertemporal, compensated changes in $l_{0,t}$ and $s_{0,t}$ following a labor subsidy to the consumer's middle-aged self, i.e.

²⁴This is analogous to the result derived in Section 3 that the government ought to subsidize savings at a higher rate if the consumers are naive than if they are sophisticated.

$$\begin{aligned}\frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n} &= \frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} + l_{1,t+1} \frac{\partial l_{0,t}}{\partial T_{1,t+1}} \\ \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} &= \frac{\partial s_{0,t}}{\partial w_{1,t+1}^n} + l_{1,t+1} \frac{\partial s_{0,t}}{\partial T_{1,t+1}}\end{aligned}$$

can be either positive or negative. The changes in $\tilde{l}_{0,t}$ and $\tilde{s}_{0,t}$ captured by these compensated derivatives have direct effects on $\tau_{1,t+1}$ in equation (36), as well as indirect effects on both $\tau_{0,t}$ and $\tau_{1,t+1}$ through the variable $\varphi_{i,t+i}$. However, if $\partial \tilde{l}_{0,t} / \partial w_{1,t+1}^n$ and $\partial \tilde{s}_{0,t} / \partial w_{1,t+1}^n$ are small in absolute value by comparison with their atemporal counterparts, i.e. $\partial \tilde{l}_{0,t} / \partial w_{0,t}^n$ and $\partial \tilde{s}_{0,t} / \partial w_{0,t}^n$, then Proposition 4 continues to apply with the qualification that also $\tau_{1,t+1} < 0$. In that case, we can derive the following generalization of Proposition 4:

Proposition 5. *If $\partial \tilde{l}_{0,t} / \partial w_{1,t+1}^n$ and $\partial \tilde{s}_{0,t} / \partial w_{1,t+1}^n$ are sufficiently small in absolute value for all t , and if the residence based tax instrument is not available, so $\theta_{i,t}^r \equiv 0$ for $i = 1, 2$ and all t , the optimal policy mix in the small open economy satisfies $\tau_{0,t} < 0$ and $\tau_{1,t+1} < 0$ for all t .*

Proposition 5 follows directly from inspection of equations (35) and (36).

Large Open Economy

By comparison with the small open economy examined above, the large open economy constitutes a much more complex framework, as the government in such an economy typically implements a source based capital income tax in addition to the labor income tax. Therefore, to be able to concentrate on basic intuition, and avoid unnecessarily complicated policy rules, we will again consider a simplified version of the model where the hours of work are held constant for the middle-aged.

Let us once again focus on sophisticated consumers, although the qualitative results derived in Proposition 6 below also apply under naivety. To

simplify the comparison with the small open economy, we use $\tau_{0,t}^{small}$ as a short notation for the marginal labor income tax formula derived for the small open economy in equation (34), i.e.

$$\tau_{0,t}^{small} = \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\frac{\beta - 1}{\beta} \right) \frac{\partial u_{0,t}}{\partial c_{0,t}} < 0,$$

although it is, in this case, evaluated in the large open economy being examined here. We show in the Appendix that the marginal labor income tax facing the young consumer in period t and the source based tax implemented in time period $t + 1$, respectively, can be written as

$$\tau_{0,t} = \tau_{0,t}^{small} - \frac{\alpha_{0,t}}{\gamma_t w_{0,t}} \left(\theta_{t+1}^s \gamma_{t+1} r_{t+1} + \theta_{t+2}^s \gamma_{t+2} r_{t+2} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \right) \quad (37)$$

$$\frac{\theta_t^s}{(1 - \theta_t^s)} = \frac{Q_t}{R_t^*} \frac{dR_t^*}{dQ_t} + \frac{\Gamma_t}{\gamma_t R_t^*} \frac{dR_t^*}{dQ_t}. \quad (38)$$

Let us begin by interpreting the formula for the source based capital income tax, θ_{t+1}^s , given in equation (38). Since $dR_t^*/dQ_t < 0$, the first term on the right hand side contributes to decrease the source based tax if the country is a net exporter of capital ($Q > 0$) and increase the source based tax if it is a net importer of capital ($Q < 0$). This mechanism corresponds to the inverse elasticity rule presented in equation (24): as such, it reflects an incentive facing the domestic government to influence the foreign interest rate.

The second term arises because the government lacks a direct instrument for influencing the savings behavior. In the absence of a residence based tax, the source based tax (as well as the labor income tax discussed below) will, therefore, also serve as an indirect instrument for correction of the internal

externalities that quasi-hyperbolic discounting give rise to. The variable

$$\begin{aligned}\Gamma_t = & \left(\frac{\partial u_{1,t}}{\partial c_{1,t}} - \gamma_t \right) s_{0,t-1} + \left(\frac{\partial u_{2,t}}{\partial c_{2,t}} - \gamma_t \right) s_{1,t-1} \\ & + \mu_{0,t-2} \frac{\partial s_{0,t-2}}{\partial R_t^*} + \mu_{0,t-1} \frac{\partial s_{0,t-1}}{\partial R_t^*} + \mu_{1,t-1} \frac{\partial s_{1,t-1}}{\partial R_t^*} + \mu_{1,t} \frac{\partial s_{1,t}}{\partial R_t^*} \\ & + \eta_{t-1} \frac{dR_{t-1}^*}{dQ_{t-1}} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}} \frac{\partial l_{0,t-1}}{\partial R_t^*} + \gamma_{t-1} (w_{0,t-1} - w_{0,t-1}^n) \frac{\partial l_{0,t-1}}{\partial R_t^*} \\ & - \gamma_{t-1} \left[(1 - \theta_{t-1}^s) K_{t-1} \frac{\partial r_{t-1}}{\partial l_{0,t-1}} + \theta_{t-1}^s r_{t-1} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}} \right] \frac{\partial l_{0,t-1}}{\partial R_t^*}\end{aligned}$$

reflects this additional incentive, and the sign of Γ_t is, in general, ambiguous (see the Appendix).

Equation (37) shows how the optimal marginal labor income tax (or subsidy) rate implemented in the large open economy differs from that of a small open economy (presented in equation (34) and discussed in Proposition 4). We have derived the following result:

Proposition 6. *In a large open economy, whose government does not have access to the residence based capital income tax, i.e. $\theta_t^r \equiv 0$ for all t , the marginal labor income tax rate satisfies:*

- (i) $\tau_{0,t} < \tau_{0,t}^{small}$ if $\theta_{t+1}^s > 0$ and $\theta_{t+2}^s > 0$, which means that $\tau_{0,t} < 0$,
- (ii) $\tau_{0,t} > \tau_{0,t}^{small}$ if $\theta_{t+1}^s < 0$ and $\theta_{t+2}^s < 0$, which means that $\tau_{0,t}$ can be either positive or negative.

To facilitate the interpretation of Proposition 6, suppose that the sign of the source based tax is driven by the first term on the right hand side of equation (38), meaning that its sign depends on whether the country is a net importer or net exporter of capital (as in the absence of any restriction on the residence based tax described in Proposition 1). The first part of Proposition 6 is then interpretable in terms of a net importer of capital, where the government implements a positive source based capital tax to push down the foreign rate of return. This will exacerbate the undersavings problem

due to quasi-hyperbolic discounting and provides, therefore, an additional incentive for the government to subsidize labor at the margin (in addition to the incentive to subsidize labor in a small open economy), which explains why $\tau_{0,t} < 0$, and $\tau_{0,t} < \tau_{0,t}^{small}$. By analogy, a net exporter of capital implements a source based capital subsidy ($\theta < 0$), which pushes up the foreign rate of return. As such, this counteracts the undersavings problem caused by quasi-hyperbolic discounting, which explains why the marginal labor subsidy rate (that serves to increase savings) needs not be as large here as for the capital importing country; in fact, $\tau_{0,t}$ may be either positive or negative here depending on the relative size of the terms in equation (37).

5 Concluding Remarks

To our knowledge, this is the first paper dealing with the optimal mix of labor and capital income taxation in an OLG model of an open economy with mobile capital, where the consumer preferences are characterized by a self-control problem caused by quasi-hyperbolic discounting. As such, we make a distinction between (i) small and large open economies, (ii) naivety and sophistication from the perspective of consumer behavior, and (iii) the instruments available for taxing or subsidizing savings, i.e. residence based and source based capital income taxes/subsidies; all of which are motivated based on earlier literature on savings, taxation and self-control problems.

We would like to emphasize four broad conclusions. First, and given a full set of tax instruments (that contains both the residence based and source based taxes), the residence based capital income tax plays a residual role: it is set such that the total marginal savings subsidy - the effective subsidy rate attached to savings - corrects for the self-control problem (that would otherwise manifest itself in terms of too little savings). The source based tax will only be used to affect the foreign interest rate (large open economy) or not used at all (small open economy). Furthermore, note that the labor

income tax serves no corrective purpose if the government can tax capital income both according to the residence-principle and source-principle. Second, the total marginal capital subsidy faced by a young consumer is larger under naivety than under sophistication. The intuition is that the young sophisticated consumer internalizes part of the internal externality himself/herself, through strategic interaction vis-a-vis his/her middle-aged self, while the young naive consumer (who erroneously expects not to be time-inconsistent in the future) does not. Third, in the absence of the source based instrument, it is not necessarily optimal for the government of a large open economy to subsidize savings: instead, the optimal marginal capital income tax rates may be positive despite that the government does not share the consumer preference for immediate gratification. Fourth, in the absence of the residence based instrument (or if this instrument is subject to a binding restriction), the labor income tax plays a distinct role as an (imperfect) instrument to correct for the self-control problem. In that case, our results show that labor ought to be subsidized at the margin, which leads to higher income and, therefore, increased savings, *ceteris paribus*, and the marginal labor subsidy is likely to be larger under naivety than sophistication.

Future research may take several directions and we briefly discuss four of them. First, our analysis assumes nonlinear taxation, where the tax schedule contains slope as well as intercept components that are subject to choice by the government. With a more restrictive tax system, such as a system with linear taxation, the results may differ from those derived above. Second, since the government in our model can use lump sum taxes to raise tax revenue, the basic motive for tax competition is eliminated. Therefore, an interesting topic for future research would be to analyze paternalistic tax policy in a setting where the motive for tax competition is present. Third, we have neglected other forms of capital accumulation than physical capital. For instance, quasi-hyperbolic discounting is also likely to affect the incentives underlying human capital accumulation faced by the consumers.

In that case, the (corrective) role of the labor income tax will also differ from that described here. Fourth, it would be very interesting to use numerical analysis to identify important consumer characteristics behind the optimal tax policy, especially under the restricted tax regimes discussed in section 4. A thorough numerical analysis (designed to give some practical guidance for policy design) involves a variety of important and technically challenging issues such as parameterization and calibration based on real world data, suggesting that such a study is worth a paper of its own. We hope to address these (and other related) questions in future research.

6 Appendix

Proof of Proposition 1

The Lagrangean of the policy problem in the Benchmark Model can be written as

$$\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \Theta^t \tilde{U}_{0,t} + \sum_{t=0}^{\infty} \sum_{i=1}^2 \alpha_{i,t+i} \Theta^{t+i} [(1 - \theta_{i,t+i}^r) R_{t+i}^* - r_{i,t+i}^n] \\
& + \sum_{t=0}^{\infty} \gamma_t \Theta^t \left[\sum_{i=0}^1 (w_{i,t} - w_{i,t}^n) l_{i,t} + \sum_{i=1}^2 (r_t - r_{i,t}^n) s_{i-1,t-1} + \sum_{i=0}^2 T_{i,t} - \theta_t^s r_t Q_t \right] \\
& + \sum_{t=0}^{\infty} \sum_{i=0}^1 \mu_{i,t} \Theta^t [s_{i,t}(\cdot) - s_{i,t}] + \sum_{t=0}^{\infty} \eta_{t+1} \Theta^{t+1} [R_{t+1}^*(Q_{t+1}) - R_{t+1}^*] \quad (\text{A.1})
\end{aligned}$$

where a , γ , μ and η are current value Lagrange multipliers. The functions $s_{i,t}(\cdot)$ are given in equations (10) and (14). Let $S_t = s_{0,t-1} + s_{1,t-1}$ and define the short notation

$$\begin{aligned}
A_{t+1} &= (1 - \theta_{t+1}^s) Q_{t+1} \left(\frac{\partial r_{t+1}}{\partial s_{0,t}} + \frac{\partial r_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \right) \\
&\quad - \theta_{t+1}^s r_{t+1} \left(\frac{\partial Q_{t+1}}{\partial s_{0,t}} + \frac{\partial Q_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \right) \\
A_{t+2} &= (1 - \theta_{t+2}^s) Q_{t+2} \frac{\partial r_{t+2}}{\partial s_{1,t+1}} - \theta_{t+2}^s r_{t+2} \frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \\
B_{t+j} &= (1 - \theta_{t+j}^s) Q_{t+j} \frac{\partial r_t}{\partial l_{j,t+j}} - \theta_{t+j}^s r_{t+j} \frac{\partial Q_{t+j}}{\partial l_{j,t+j}} \text{ for } j = 0, 1 \\
\frac{\partial \Psi_t}{\partial s_{0,t}} &= \Theta (1 + r_{1,t+1}^n) \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} \\
\frac{\partial \Psi_t}{\partial s_{1,t+1}} &= \Theta^2 (1 + r_{2,t+2}^n) \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}}.
\end{aligned}$$

The first order conditions can then be written as

$$\begin{aligned}
w_{0,t}^n : \quad 0 &= l_{0,t} \left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t \right) + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial w_{0,t}^n} \\
&\quad + \gamma_t B_t \frac{\partial l_{0,t}}{\partial w_{0,t}^n} + \mu_{0,t} \frac{\partial s_{0,t}}{\partial w_{0,t}^n} + \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial w_{0,t}^n} \quad (\text{A.2a})
\end{aligned}$$

$$\begin{aligned}
T_{0,t} : \quad 0 &= \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}} \right) + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial T_{0,t}} \\
&\quad + \gamma_t B_t \frac{\partial l_{0,t}}{\partial T_{0,t}} + \mu_{0,t} \frac{\partial s_{0,t}}{\partial T_{0,t}} + \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial T_{0,t}} \quad (\text{A.2b})
\end{aligned}$$

$$\begin{aligned}
w_{1,t+1}^n : \quad 0 &= l_{1,t+1} \left(\Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \gamma_{t+1} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial w_{1,t+1}^n} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial w_{1,t+1}^n} \\
&\quad + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} + \gamma_t B_t \frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} \\
&\quad + \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n) \frac{\partial l_{1,t+1}}{\partial w_{1,t+1}^n} + \gamma_{t+1} B_{t+1} \frac{\partial l_{1,t+1}}{\partial w_{1,t+1}^n} \\
&\quad + \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} + \eta_{t+1} \frac{\partial R_{t+1}^*}{\partial Q_{t+1}} \frac{\partial Q_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial w_{1,t+1}^n} \quad (\text{A.2c})
\end{aligned}$$

$$\begin{aligned}
T_{j,t+j} : \quad 0 = & \left(\gamma_{t+j} - \Theta^j \frac{\partial u_{j,t+j}}{\partial c_{j,t+j}} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial T_{j,t+j}} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial T_{j,t+j}} \\
& + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial T_{j,t+j}} + \gamma_t B_t \frac{\partial l_{0,t}}{\partial T_{j,t+j}} \\
& + \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n) \frac{\partial l_{1,t+1}}{\partial T_{j,t+j}} + \gamma_{t+1} B_{t+1} \frac{\partial l_{1,t+1}}{\partial T_{j,t+j}} \\
& + \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial T_{j,t+j}} + \eta_{t+1} \frac{\partial R_{t+1}^*}{\partial Q_{t+1}} \frac{\partial Q_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial T_{j,t+j}} \quad (\text{A.2d})
\end{aligned}$$

$$\begin{aligned}
r_{j,t+j}^n : \quad 0 = & \left(\Theta^j \frac{\partial u_{j,t+j}}{\partial c_{j,t+j}} - \gamma_{t+j} \right) s_{j-1,t+j-1} + \mu_{0,t} \frac{\partial s_{0,t}}{\partial r_{j,t+j}^n} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial r_{j,t+j}^n} \\
& + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial r_{j,t+j}^n} + \gamma_t B_t \frac{\partial l_{0,t}}{\partial r_{j,t+j}^n} - \alpha_{1,t+1} \\
& + \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n) \frac{\partial l_{1,t+1}}{\partial r_{j,t+j}^n} + \gamma_{t+1} B_{t+1} \frac{\partial l_{1,t+1}}{\partial r_{j,t+j}^n} \\
& + \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial r_{j,t+j}^n} + \eta_{t+1} \frac{\partial R_{t+1}^*}{\partial Q_{t+1}} \frac{\partial Q_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial r_{j,t+j}^n} \quad (\text{A.2e})
\end{aligned}$$

$$\theta_{j,t+j}^r : \quad 0 = -\alpha_{j,t+j} R_{t+1}^* \quad (\text{A.2f})$$

$$\begin{aligned}
\theta_t^s : \quad 0 = & \eta_t \frac{\partial R_t^*}{\partial Q_t} \frac{\partial Q_t}{\partial \theta_t^s} + \gamma_t (s_{0,t-1} + s_{1,t-1}) \frac{\partial r_t}{\partial \theta_t^s} \\
& + \gamma_t \left[L_t \frac{\partial w_{0,t}}{\partial \theta_t^s} - r_t Q_t - \theta_t^s r_t \frac{\partial Q_t}{\partial \theta_t^s} - \theta_t^s Q_t \frac{\partial r_t}{\partial \theta_t^s} \right] \quad (\text{A.2g})
\end{aligned}$$

$$R_{t+1}^* : \quad 0 = \alpha_{1,t+1} (1 - \theta_{1,t+1}^r) + \alpha_{2,t+1} (1 - \theta_{2,t+1}^r) - \eta_{t+1} \quad (\text{A.2h})$$

$$\begin{aligned}
s_{0,t} : \quad 0 = & \mu_{0,t} - \left[\frac{\partial \Psi_t}{\partial s_{0,t}} + \gamma_{t+1} (r_{t+1} - r_{1,t+1}^n) \right] - \gamma_{t+1} A_{t+1} - \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \\
& - \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n) \frac{\partial l_{1,t+1}}{\partial s_{0,t}} - \eta_{t+1} \frac{\partial R_{t+1}^*}{\partial Q_{t+1}} \frac{\partial Q_{t+1}}{\partial s_{0,t}} \quad (\text{A.2i})
\end{aligned}$$

$$s_{1,t+1} : \quad 0 = \mu_{1,t+1} - \left[\frac{\partial \Psi_t}{\partial s_{1,t+1}} + \gamma_{t+2} (r_{t+2} - r_{1,t+2}^n) \right] - \gamma_{t+2} A_{t+2} \\ - \eta_{t+2} \frac{\partial R_{t+2}^*}{\partial Q_{t+2}} \frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \quad (\text{A.2j})$$

$$g_t: \quad 0 = \frac{\partial u_{0,t}}{\partial g_t} + \frac{\partial u_{1,t}}{\partial g_t} + \frac{\partial u_{2,t}}{\partial g_t} - \gamma_t = 0 \quad (\text{A.2k})$$

for $j = 1, 2$.

To derive the formula for the source based capital income tax, use the first order conditions for $\theta_{1,t+1}^r$, $\theta_{2,t+1}^r$ and R_{t+1}^* together with the identity $s_{0,t-1} + s_{1,t-1} = K_t + Q_t$, and the zero profit condition. Note that the zero profit condition means

$$0 = K_t \frac{dr_t}{dx} + (l_{0,t} + al_{1,t}) \frac{dw_{0,t}}{dx} \quad (\text{A.3})$$

for any policy variable, x . The first order condition for θ_t^s in equation (A.2g) can then be rewritten as

$$0 = (1 - \theta_t^s) Q_t \frac{\partial r_t}{\partial \theta_t^s} - r_t Q_t - \theta_t^s r_t \frac{\partial Q_t}{\partial \theta_t^s}. \quad (\text{A.4})$$

Differentiating the capital market equilibrium condition presented in equation (3) with respect to θ_t^s and substituting the resulting expression into equation (A.4) gives

$$0 = \frac{\partial R_t^*}{\partial Q_t} Q_t - \theta_t^s r_t. \quad (\text{A.5})$$

Using equation (3) to substitute for r_t and rearranging gives in equation (24), which implies part (ii) of Proposition 1.

To derive the expressions for the marginal income tax rates, combine the first order conditions for $w_{0,t}^n$ and $T_{0,t}$; $w_{1,t+1}^n$ and $T_{1,t+1}$; $r_{1,t+1}^n$ and $T_{1,t+1}$; and $r_{2,t+2}^n$ and $T_{2,t+2}$, respectively. Using the first order conditions for $s_{0,t}$ and $s_{1,t+1}$ to eliminate $\mu_{0,t}$ and $\mu_{1,t+1}$ and rearranging gives the following

equation system:

$$0 = \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} + \chi_3 \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} + \chi_4 \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} \quad (\text{A.6})$$

$$0 = \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial w_{1,t+1}^n} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} \right] + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial w_{1,t+1}^n} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} \right) \quad (\text{A.7})$$

$$0 = \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial r_{1,t+1}^n} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial r_{1,t+1}^n} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial r_{1,t+1}^n} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r_{1,t+1}^n} \right] + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial r_{1,t+1}^n} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r_{1,t+1}^n} \right) \quad (\text{A.8})$$

$$0 = \chi_1 \frac{\partial \tilde{s}_{0,t}}{\partial r_{2,t+2}^n} + \chi_2 \frac{\partial \tilde{l}_{0,t}}{\partial r_{2,t+2}^n} + \chi_3 \left[\frac{\partial \tilde{s}_{1,t+1}}{\partial r_{2,t+2}^n} + \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r_{2,t+2}^n} \right] + \chi_4 \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial r_{2,t+2}^n} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial r_{2,t+2}^n} \right) \quad (\text{A.9})$$

where \tilde{l} and \tilde{s} denote compensated labor supply and savings function, respectively, and

$$\begin{aligned} \chi_1 &= \Theta (1 + r_{1,t+1}^n) \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \frac{\partial u_{0,t}}{\partial c_{0,t}} + \gamma_{t+1} (r_{t+1} - r_{1,t+1}^n) \\ \chi_2 &= \gamma_t (w_{0,t} - w_{0,t}^n) \\ \chi_3 &= \Theta^2 (1 + r_{2,t+2}^n) \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} - \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} + \gamma_{t+2} (r_{t+2} - r_{1,t+2}^n) \\ \chi_4 &= \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n). \end{aligned}$$

Note that equations (A.6) - (A.9) are satisfied if

$$\chi_1 = \chi_2 = \chi_3 = \chi_4 = 0. \quad (\text{A.10})$$

Clearly, $\chi_2 = 0$ and $\chi_4 = 0$ imply $\tau_{0,t} = 0$ and $\tau_{1,t+1} = 0$, respectively.

Finally, note that equations (A.6) - (A.9) imply $\mu_{0,t} = \mu_{1,t+1} = 0$ and

$$\gamma_t = \frac{\partial u_{0,t}}{\partial c_{0,t}}, \quad \gamma_{t+1} = \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}}, \quad \gamma_{t+2} = \Theta^2 \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}}. \quad (\text{A.11})$$

Using the private first order condition for $s_{0,t}$ together with equations (A.11) and $\theta_{1,t+1} r_{t+1} = r_{t+1} - r_{1,t+1}^n$ give equation (26). Similarly, the private first

order condition for $s_{1,t+1}$ together with equations (A.11) and $\theta_{2,t+2}r_{t+2} = r_{t+2} - r_{2,t+2}^n$ give equation (27). Parts (i), (iii) and (iv) of Proposition 1 immediately follow.

Notice that equations (A.11) imply $\partial u_{0,t}/\partial c_{0,t} = \partial u_{1,t}/\partial c_{1,t} = \partial u_{2,t}/\partial c_{2,t} = \gamma_t$. Substituting into equation (A.2k) gives the Samuelson condition for the public good

$$\sum_i \frac{\partial u_{i,t}/\partial g_t}{\partial u_{i,t}/\partial c_{i,t}} = 1.$$

Proof of Proposition 3

The Lagrangean associated with the government's decision-problem is identical to equation (A1) with the exception that $\theta_t^s = 0$ for all t . As a consequence, the first order conditions characterizing the public decision problem take the same form as in the Benchmark model with the exception that $\theta_t^s = 0$ for all t here.

By combining the first order conditions for $w_{0,t}^n$ and $T_{0,t}$; $w_{1,t+1}^n$ and $T_{1,t+1}$; $r_{1,t+1}^n$ and $T_{1,t+1}$; and $r_{2,t+2}^n$ and $T_{2,t+2}$, respectively, we can derive the following equation system:

$$0 = \mu_{0,t} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} + \gamma_t \left[(w_{0,t} - w_{0,t}^n) + Q_t \frac{\partial r_t}{\partial l_{0,t}} \right] \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} \quad (\text{A.12})$$

$$0 = \mu_{0,t} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} + \gamma_t \left[(w_{0,t} - w_{0,t}^n) + Q_t \frac{\partial r_t}{\partial l_{0,t}} \right] \frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n} + \mu_{1,t+1} \frac{\partial \tilde{s}_{1,t+1}}{\partial w_{1,t+1}^n} \\ + \gamma_{t+1} \left[(w_{1,t+1} - w_{1,t+1}^n) + Q_{t+1} \frac{\partial r_{t+1}}{\partial l_{1,t+1}} \right] \frac{\partial \tilde{l}_{1,t+1}}{\partial w_{1,t+1}^n} \quad (\text{A.13})$$

$$0 = \mu_{0,t} \frac{\partial \tilde{s}_{0,t}}{\partial r_{1,t+1}^n} + \gamma_t \left[(w_{0,t} - w_{0,t}^n) + Q_t \frac{\partial r_t}{\partial l_{0,t}} \right] \frac{\partial \tilde{l}_{0,t}}{\partial r_{1,t+1}^n} + \mu_{1,t+1} \frac{\partial \tilde{s}_{1,t+1}}{\partial r_{1,t+1}^n} \\ + \gamma_{t+1} \left[(w_{1,t+1} - w_{1,t+1}^n) + Q_{t+1} \frac{\partial r_{t+1}}{\partial l_{1,t+1}} \right] \frac{\partial \tilde{l}_{1,t+1}}{\partial r_{1,t+1}^n} \quad (\text{A.14})$$

$$0 = \mu_{0,t} \frac{\partial \tilde{s}_{0,t}}{\partial r_{2,t+2}^n} + \gamma_t \left[(w_{0,t} - w_{0,t}^n) + Q_t \frac{\partial r_t}{\partial l_{0,t}} \right] \frac{\partial \tilde{l}_{0,t}}{\partial r_{2,t+2}^n} + \mu_{1,t+1} \frac{\partial \tilde{s}_{1,t+1}}{\partial r_{2,t+2}^n} \\ + \gamma_{t+1} \left[(w_{1,t+1} - w_{1,t+1}^n) + Q_{t+1} \frac{\partial r_{t+1}}{\partial l_{1,t+1}} \right] \frac{\partial \tilde{l}_{1,t+1}}{\partial r_{2,t+2}^n} \quad (\text{A.15})$$

Note that equations (A.12) - (A.15) are satisfied if the following conditions hold:

$$0 = (w_{0,t} - w_{0,t}^n) + Q_t \frac{\partial r_t}{\partial l_{0,t}} \quad (\text{A.16})$$

$$0 = (w_{1,t+1} - w_{1,t+1}^n) + Q_{t+1} \frac{\partial r_{t+1}}{\partial l_{1,t+1}} \quad (\text{A.17})$$

$$0 = \mu_{0,t} \quad (\text{A.18})$$

$$0 = \mu_{1,t+1}. \quad (\text{A.19})$$

Differentiating the capital market equilibrium condition $R_t^*(Q_t(\cdot)) = r_t(\cdot)$ w.r.t. $l_{0,t}$ and $l_{1,t+1}$ and substituting the resulting expressions into equation (A.16) and (A.17), respectively, produces the marginal labor income tax formula in equation (28).

Finally, if equations (A.16) - (A.19) are satisfied, the first order conditions for $T_{0,t}$, $T_{1,t+1}$ and $T_{2,t+2}$ can be written as in equations (A.11). By using equations (A.11) together with (a) the private first order conditions for $s_{0,t}$ and $s_{1,t+1}$, (b) the expressions for unit savings taxes, $\theta_{1,t+1}r_{t+1} = r_{t+1} - r_{1,t+1}^n$ and $\theta_{2,t+2}r_{t+2} = r_{t+2} - r_{2,t+2}^n$, and (c) the capital market equilibrium condition, it is straight forward to derive equations (29) and (30).■

Proof of Proposition 4 and Derivation of Equations (35) and (36)

The small open economy with source based capital income taxation means that the capital market equilibrium condition is given by $(1 - \theta_t^s) r_t = R_t^*$, with R_t^* treated as fixed by the domestic government. Since we assume away residence based capital income taxes, i.e. $\theta_{i,t}^r \equiv 0$, the Lagrangean associated with the public decision-problem can be written as

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \Theta^t \tilde{U}_{0,t} + \sum_{t=0}^{\infty} \sum_{i=0}^1 \mu_{i,t} \Theta^t [s_{i,t}(\cdot) - s_{i,t}] \\ & + \sum_{t=0}^{\infty} \gamma_t \Theta^t \left[\sum_{i=0}^1 (w_{i,t} - w_{i,t}^n) l_{i,t} + \theta_t^s r_t K_t + \sum_{i=0}^2 T_{i,t} \right]. \quad (\text{A.20}) \end{aligned}$$

Starting with the source based capital income tax, the first order condi-

tion for θ_t^s is written as

$$0 = \gamma_t \left[\frac{dw_{0,t}}{d\theta_t^s} l_{0,t} + a \frac{dw_{0,t}}{d\theta_t^s} l_{1,t} + r_t K_t + \theta_t^s r_t \frac{\partial K_t}{\partial \theta_t^s} + \theta_t^s K_t \frac{dr_t}{d\theta_t^s} \right]. \quad (\text{A.21})$$

Now, by using equation (A.3), and then differentiating the capital market equilibrium condition with respect to θ_t^s and substituting into equation (A.21), we obtain

$$0 = \theta_t^s r_t \frac{\partial K_t}{\partial \theta_t^s}. \quad (\text{A.22})$$

Equation (A.21) implies $\theta_t^s = 0$.

To derive the expressions for the marginal labor income tax rates, we use the following first order conditions:

$$\begin{aligned} w_{0,t}^n : \quad 0 &= l_{0,t} \left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial w_{0,t}^n} \\ &+ \gamma_t \left[(w_{0,t} - w_{0,t}^n) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial w_{0,t}^n} \end{aligned} \quad (\text{A.23a})$$

$$\begin{aligned} T_{0,t} : \quad 0 &= \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial T_{0,t}} \\ &+ \gamma_t \left[(w_{0,t} - w_{0,t}^n) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial T_{0,t}} \end{aligned} \quad (\text{A.23b})$$

$$\begin{aligned} w_{1,t+1}^n : \quad 0 &= l_{1,t+1} \left(\Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} - \gamma_{t+1} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial w_{1,t+1}^n} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial w_{1,t+1}^n} \\ &+ \gamma_t \left[(w_{0,t} - w_{0,t}^n) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial w_{1,t+1}^n} \\ &+ \gamma_{t+1} \left[(w_{1,t+1} - w_{1,t+1}^n) + \theta_{t+1}^s r_{t+1} \frac{\partial K_{t+1}}{\partial l_{1,t+1}} \right] \frac{\partial l_{1,t+1}}{\partial w_{1,t+1}^n} \end{aligned} \quad (\text{A.23c})$$

$$\begin{aligned} T_{1,t+1} : \quad 0 &= \left(\gamma_{t+1} - \Theta \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial T_{1,t+1}} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial T_{1,t+1}} \\ &+ \gamma_t \left[(w_{0,t} - w_{0,t}^n) + \theta_t^s r_t \frac{\partial K_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial T_{1,t+1}} \\ &+ \gamma_{t+1} \left[(w_{1,t+1} - w_{1,t+1}^n) + \theta_{t+1}^s r_{t+1} \frac{\partial K_{t+1}}{\partial l_{1,t+1}} \right] \frac{\partial l_{1,t+1}}{\partial T_{1,t+1}} \end{aligned} \quad (\text{A.23d})$$

$$\begin{aligned}
s_{0,t} : \quad 0 = & \mu_{0,t} - \frac{\partial \Psi_t}{\partial s_{0,t}} - \gamma_{t+1} (w_{1,t+1} - w_{1,t+1}^n) \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \\
& - \gamma_{t+1} \theta_{t+1}^s r_{t+1} \frac{\partial K_{t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} - \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \quad (\text{A.23e})
\end{aligned}$$

$$s_{1,t+1}: \quad 0 = \mu_{1,t+1} - \frac{\partial \Psi_t}{\partial s_{1,t+1}} \quad (\text{A.23f})$$

where $\partial \Psi_t / \partial s_{0,t}$ and $\partial \Psi_t / \partial s_{1,t+1}$ take the same form as in the proof of Proposition 1 above; yet with the modification that $r_{1,t+1}^n = R_{t+1}^*$ and $r_{2,t+2}^n = R_{t+2}^*$.

By combining equations (A.23a) and (A.23b), and equations (A.23c) and (A.23d), respectively, using equations (A.23e) and (A.23f) to eliminate Lagrange multipliers, as well as using that $\theta_t^s = 0$, we can derive

$$\begin{aligned}
0 = & \gamma_t \tau_{0,t} w_{0,t} \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} + \gamma_{t+1} \tau_{1,t+1} w_{1,t+1} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} \\
& + \left(\frac{\partial \Psi_t}{\partial s_{0,t}} + \frac{\partial \Psi_t}{\partial s_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \right) \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} \quad (\text{A.24a})
\end{aligned}$$

$$\begin{aligned}
0 = & \gamma_t \tau_{0,t} w_{0,t} \frac{\partial \tilde{l}_{0,t}}{\partial w_{1,t+1}^n} + \gamma_{t+1} \tau_{1,t+1} w_{1,t+1} \left(\frac{\partial \tilde{l}_{1,t+1}}{\partial w_{1,t+1}^n} + \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} \right) \\
& + \left(\frac{\partial \Psi_t}{\partial s_{0,t}} + \frac{\partial \Psi_t}{\partial s_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \right) \frac{\partial \tilde{s}_{0,t}}{\partial w_{1,t+1}^n} + \frac{\partial \Psi_t}{\partial s_{1,t+1}} \frac{\partial \tilde{s}_{1,t+1}}{\partial w_{1,t+1}^n}. \quad (\text{A.24b})
\end{aligned}$$

Solving equation system (A.24) gives the expressions for $\tau_{0,t}$ and $\tau_{1,t+1}$ in equations (35) and (36). In the special case where $l_{1,t+1}$ is fixed, we can use equation (A.24a) to derive equation (33), which proves part (i) of Proposition 4. ■

Derivation of Equations (37) and (38)

This is a restricted version of the Benchmark model where $\theta_{i,t}^r \equiv 0$. Therefore, since $R_{t+1}^*(Q_{t+1}) = R_{t+i}^* = r_{i,t+i}^n$, the restriction $R_{t+i}^*(1 - \theta_{i,t+1}^r) = r_{i,t+i}^n$

in the Benchmark model will be redundant here, and the Lagrangean can be written as

$$\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \Theta^t \tilde{U}_{0,t} + \sum_{t=0}^{\infty} \sum_{i=0}^1 \mu_{i,t} [s_{i,t}(\cdot) - s_{i,t}] + \sum_{t=0}^{\infty} \eta_{t+1} \Theta^{t+1} [R_{t+1}^* (Q_{t+1}) - R_{t+1}^*] \\
& + \sum_{t=0}^{\infty} \gamma_t \Theta^t \left[\sum_{i=0}^1 (w_{i,t} - w_{i,t}^n) l_{i,t} + \sum_{i=1}^2 (r_t - r_{i,t}^n) s_{i-1,t-1} \right. \\
& \left. + \sum_{i=0}^2 T_{i,t} - \theta_t^s r_t Q_t \right] \tag{A.25}
\end{aligned}$$

The first order conditions for θ_t^s and R_t^* are given by

$$\theta_t^s : 0 = \eta_t \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial \theta_t^s} + \gamma_t \left[r_t K_t - (1 - \theta_t^s) K_t \frac{\partial r_t}{\partial \theta_t^s} - \theta_t^s r_t \frac{\partial Q_t}{\partial \theta_t^s} \right] \tag{A.26a}$$

$$\begin{aligned}
R_t^* : 0 = & \frac{\partial u_{1,t}}{\partial c_{1,t}} s_{0,t-1} + \frac{\partial u_{2,t}}{\partial c_{2,t}} s_{1,t-1} - \eta_t \tag{A.26b} \\
& + \mu_{0,t-2} \frac{\partial s_{0,t-2}}{\partial R_t^*} + \mu_{0,t-1} \frac{\partial s_{0,t-1}}{\partial R_t^*} + \mu_{1,t-1} \frac{\partial s_{1,t-1}}{\partial R_t^*} + \mu_{1,t} \frac{\partial s_{1,t}}{\partial R_t^*} \\
& + \eta_{t-1} \frac{\partial R_{t-1}^*}{dQ_{t-1}} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}} \frac{\partial l_{0,t-1}}{\partial R_t^*} + \gamma_{t-1} (w_{0,t-1} - w_{0,t-1}^n) \frac{\partial l_{0,t-1}}{\partial R_t^*} \\
& - \gamma_{t-1} \left[(1 - \theta_{t-1}^s) K_{t-1} \frac{\partial r_{t-1}}{\partial l_{0,t-1}} + \theta_{t-1}^s r_{t-1} \frac{\partial Q_{t-1}}{\partial l_{0,t-1}} \right] \frac{\partial l_{0,t-1}}{\partial R_t^*}.
\end{aligned}$$

Note that the capital market equilibrium condition $R_t^*(Q_t) = r_t(Q_t)[1 - \theta_t^s]$ implies

$$\frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial \theta_t^s} = (1 - \theta_t^s) \frac{\partial r_t}{\partial \theta_t^s} - r_t. \tag{A.27}$$

Substituting equation (A.27) into equation (A.24a), while using equation (A.24b) to solve for η_t , gives equation (38).

To derive equation (37), we use the first order conditions for $w_{0,t}^n$, $T_{0,t}$, $s_{0,t}$ and $s_{1,t+1}$

$$\begin{aligned}
w_{0,t}^n : \quad 0 &= l_{0,t} \left(\frac{\partial u_{0,t}}{\partial c_{0,t}} - \gamma_t \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial w_{0,t}^n} + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial w_{0,t}^n} \\
&\quad - \gamma_t \left[(1 - \theta_t^s) K_t \frac{\partial r_t}{\partial l_{0,t}} + \theta_t^s r_t \frac{\partial Q_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial w_{0,t}^n} \\
&\quad + \eta_t \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial w_{0,t}^n}
\end{aligned} \tag{A.28a}$$

$$\begin{aligned}
T_{0,t}^n : \quad 0 &= \left(\gamma_t - \frac{\partial u_{0,t}}{\partial c_{0,t}} \right) + \mu_{0,t} \frac{\partial s_{0,t}}{\partial T_{0,t}^n} + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial l_{0,t}}{\partial T_{0,t}^n} \\
&\quad - \gamma_t \left[(1 - \theta_t^s) K_t \frac{\partial r_t}{\partial l_{0,t}} + \theta_t^s r_t \frac{\partial Q_t}{\partial l_{0,t}} \right] \frac{\partial l_{0,t}}{\partial T_{0,t}^n} \\
&\quad + \eta_t \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial l_{0,t}}{\partial T_{0,t}^n}
\end{aligned} \tag{A.28b}$$

$$\begin{aligned}
s_{0,t} : \quad 0 &= \frac{\partial \Psi_t}{\partial s_{0,t}} - \mu_{0,t} + \eta_{t+1} \frac{dR_{t+1}^*}{dQ_{t+1}} \frac{\partial Q_{t+1}}{\partial s_{0,t}} + \mu_{1,t+1} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \\
&\quad + \gamma_{t+1} \left[\theta_{t+1}^s r_{t+1} - \theta_{t+1}^s r_{t+1} \frac{dQ_{t+1}}{\partial s_{0,t}} - (1 - \theta_{t+1}^s) K_{t+1} \frac{\partial r_{t+1}}{\partial s_{0,t}} \right]
\end{aligned} \tag{A.28c}$$

$$\begin{aligned}
s_{1,t+1} : \quad 0 &= \frac{\partial \Psi_t}{\partial s_{1,t+1}} - \mu_{1,t+1} + \eta_{t+2} \frac{dR_{t+2}^*}{dQ_{t+2}} \frac{\partial Q_{t+2}}{\partial s_{1,t+1}} \\
&\quad + \gamma_{t+2} \left[\theta_{t+2}^s r_{t+2} - \theta_{t+2}^s r_{t+2} \frac{\partial Q_{t+2}}{\partial s_{1,t+1}} - (1 - \theta_{t+2}^s) K_{t+2} \frac{\partial r_{t+2}}{\partial s_{1,t+1}} \right].
\end{aligned} \tag{A.28d}$$

Combine equations (A.28a) and (A.28b) to derive

$$\begin{aligned}
0 &= \mu_{0,t} \frac{\partial \tilde{s}_{0,t}}{\partial w_{0,t}^n} + \gamma_t (w_{0,t} - w_{0,t}^n) \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} + \eta_t \frac{dR_t^*}{dQ_t} \frac{\partial Q_t}{\partial l_{0,t}} \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n} \\
&\quad - \gamma_t \left[(1 - \theta_t^s) K_t \frac{\partial r_t}{\partial l_{0,t}} + \theta_t^s r_t \frac{\partial Q_t}{\partial l_{0,t}} \right] \frac{\partial \tilde{l}_{0,t}}{\partial w_{0,t}^n}
\end{aligned} \tag{A.29}$$

Define $K_t = s_{0,t-1} + s_{1,t-1} - Q_t$, and observe that $\Gamma_t = \eta_t - \gamma_t (s_{0,t-1} + s_{1,t-1})$ and $\tau_{0,t} w_{0,t} = w_{0,t} - w_{0,t}^n$. Then, by using the capital market equilibrium condition $R_t^*(Q_t) = r_t(Q_t)[1 - \theta_t^s]$, and that θ_t^s can be written as

$$\theta_t^s = \frac{Q_t}{r_t} \frac{dR_t^*}{dQ_t} + \frac{\Gamma_t}{\gamma_t r_t} \frac{dR_t^*}{dQ_t},$$

equation (A.29) can be rewritten to read

$$\tau_{0,t} = - \frac{\mu_{0,t}}{\gamma_t w_{0,t}} \frac{\partial \tilde{s}_{0,t} / \partial w_{0,t}^n}{\partial \tilde{l}_{0,t} / \partial w_{0,t}^n}. \quad (\text{A.30})$$

Finally, use equations (A.28c) and (A.28d) together with the capital market equilibrium condition to substitute for $\mu_{0,t}$ and $\mu_{1,t+1}$, and define $\tau_{0,t}^{small}$ as in the text. This gives equation (37).

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