Less information, more thinking

How attentional behavior predicts learning in mathematics

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I would like to express my sincere gratitude to the following people whom without this work would not have been possible. To Bert Jonsson for his supervision, guidance, and patience, together with the other members of the LICR research team for their stimulating, creative, and friendly support: Johan Lithner, Yvonne Liljekvist, Mathias Norqvist, Carina Granberg, Jan Olsson, Peter Vester gren, and Hendrik Van Steenbrugge.
It has been shown in experiments that a method of teaching where students are encouraged to create their own solution methods to mathematical problems (creative mathematically founded reasoning, CMR) results in better learning and proficiency than one where students are provided with solution methods for them to practice by repetition (algorithmic reasoning, AR). The present study investigated whether students in an AR practice condition pay less attention to information relevant for mathematical problem solving than students in a CMR condition. To test this, attentional behavior during practice was measured using eye-tracking equipment. These measurements were then associated with task proficiency in a follow-up test one week after the practice session. The findings support the theory and confirm previous studies in that CMR leads to better task performance in the follow-up test. The findings also suggest that students within the CMR condition whom focus less on extraneous information perform better.

**Background**

An issue regarded as central in teaching mathematics is the development of mathematical competences: the ability to understand, judge, do, and use mathematics across a variety of mathematical situations (Niss, 2003). Among those competences are problem solving abilities and conceptual understanding.

Following a line of empirical research, Jonsson, Norqvist, Liljekvist, & Lithner (2014) and Karlsson, et al. (2015) that relates these competences to learning conditions that are encouraging either a more creative reasoning or a procedural-based learning (Lithner, 2003; Lithner, 2008; Øystein, 2011), the present study explores attentional behavior, by proxy of measured eye movements (Yarbus, 1967), in relation to the learning conditions and problem solving ability as well as conceptual understanding. In addition, the
mathematical task solving and eye fixations during practice are considered in relation to test task performances.

Algorithmic and creative mathematical founded reasoning

A major theme in mathematics education today is the emphasis on rehearsing algorithms, or predefined procedures, as means of learning mathematical skills (Boesen, Helenius, Lithner, Bergqvist, Bergqvist, Palm, Palmberg, 2014; Hiebert, 2003). Typically, the teacher or textbook provides students with a task solution template that can be used to solve mathematical problems. This is followed by, on the students’ part, an extensive amount of repeated use of the provided template. While predefined templates, such as algorithms, provide an efficient and reliable method for solving the tasks they are designed for, it is not obvious that this rehearsal fosters conceptual understandings (Hiebert, 2003). Thus, conceptual understanding is not a prerequisite for using algorithms (Øystein, 2011). The extensive rehearsal of the provided algorithms may instead lead to an unreflective use of those algorithms (Boesen et al., 2014; Lithner, 2008). Instead of attending to the intrinsic properties of the task, which facilitate a conceptual understanding, the use of provided algorithms constrain pupils attention to superficial features of the algorithm which subsequent also affect their mathematical reasoning (Hiebert, 2003; Lithner, 2008; Vinner, 1997).

The present study assumes the model of imitative and creative reasoning articulated by Lithner (2008) which posits and defines a distinction between two types of reasoning in mathematical learning: Algorithmic Reasoning (AR) and Creative Mathematically founded Reasoning (CMR). Lithner (2008) defines CMR as fulfilling all of the following criteria:

- **Novelty.** A new (to the reasoner) reasoning sequence is created, or a forgotten one is re-created.
- **Plausibility.** There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible.
- **Mathematical foundation.** The arguments are anchored in intrinsic mathematical properties of the components involved in the reasoning.

In contrast to CMR, AR, as defined in the present study, only involves using known or supplied solution algorithms. Thus, all parts that are conceptually difficult are taken care of by the algorithm and only the easy parts are left to the user of the algorithm. Failure occurring while attempting algorithmic reasoning can come from either recollection error, misapplication of a solution algorithm, or a careless mistake.
In consequence, focusing on an AR teaching methods, where students mainly try to remember the superficial properties of the provided algorithms, does not give opportunities for students to develop the appropriate mathematical competences.

In two separate studies Jonsson et al. (2014) and Karlsson et al. (2015) showed that learning in mathematics through creative mathematically founded reasoning (CMR) resulted in better performance in a follow-up test than did learning through algorithmic reasoning (AR). In both studies students (n = 131, 16-17 years of age in Jonsson et al. (2014), and n = 73, aged 18-22 years in Karlsson et al. (2015), respectively) participated in a three-session intervention. In the first session individual differences were measured with regards to Raven’s advanced matrices (Raven, Raven, & Court, 1991) and Operation span (Unsworth & Engle, 2005), as well as demographic information, including age, sex and latest mathematics grade. These data were used to allocate participants into two balanced groups, a CMR-group and an AR-group that were considered equally in terms of cognitive proficiency, acquired mathematical knowledge and gender. In session two, participants individually practiced mathematics tasks designed to promote either CMR or AR. The tasks were administered by a computer program, which allowed for a precise amount of time for each task, and also provided feedback to the participants’ responses. After a one-week interim, the participants were tested on numerically similar tasks as they had practiced. In the case of Jonsson et al. (2014) the follow-up tests were conducted on a personal computer. The participants were also asked to provide a solution method in terms of an algebraic formula. Both the algebraic and the numerical answers were typed on the PC-keyboard. In Karlsson et al. (2015) the follow-up test was instead conducted in an fMRI scanner where the participants were scanned while completing the test tasks. The method of providing answers was adapted to this environment by temporally separating the act of numerical calculation from giving the response, as well as by using multiple-choice questions answered by a push of a button. A spelling error task interlacing the test tasks were used the baseline condition. In both studies, CMR was contrasted with AR on test task performance and the findings show that students who practice mathematics under a condition inviting CMR performs significantly better than the other group in solving the same type of tasks again, one week later. Thus, there is evidence that limiting the amount of information pertaining to the solution of a mathematical task that is presented to the learner is beneficial for learning the mathematics of the task. Without a provided solution, the learner has to construct it by him or herself by engaging in creative problem solving. To investigate whether these different training conditions cause the learners to attend the tasks differently during training is the focus for the present study. Looking further into these attentional characteristics may advance our understanding of the learning strategies used, which that in turn can provide information about performance differences at tests. To index those attentional characteristics eye tracking equipment was used.
Eye tracking

Eye tracking, i.e. measuring the point of gaze in relation to a visual stimulus, is an established way to determine what parts of the stimulus is being attended to, and for how long (Duchowski 2003; Rehder & Hoffman, 2005). Although it is well known that attention, to some extent, can dissociate from gaze during intentional covert attention (Posner, 1980) eye movements principally follow shifts in attention (Kowler, Anderson, Dosher & Blaser, 1995) and eye fixations are highly coupled to the locus of attention for all but the simplest stimuli (Deubel & Schneider, 1996). It is therefore argued that visual attention in terms of eye fixations can provide a window to the viewer’s cognitive processes (Yarbus, 1967) and their understanding of the task at hand (Jarodzka, Scheiter, Gerjets, & van Gog, 2010).

In the educational science literature, few studies have investigated if eye tracking can provide information about the attentional processes involved in mathematical task solving, and whether those processes differ with regard to learning approaches, task specificity, and successful and unsuccessful performances.

However, in a study by Hegarty, Mayer, & Monk (1995) on arithmetic word problems it was shown, by using eye tracking, that unsuccessful problem solvers based their solution on superficial properties of the problem formulation whereas the successful problem solvers use a meaningful approach that is based on an elaborated problem model. Participants were presented with arithmetic problems in textual form whilst eye-tracking equipment was used to measure to what parts of the text the participants attended to. In short, the unsuccessful problem solvers selected key relational terms (such as “more” and “less) and translated them directly into arithmetic operations such as “addition” and “subtraction” in an attempt to quickly solve the problem rather than trying to construct an integrated mental representation of the problem. This so called “short cut” approach taken by the unsuccessful problem solvers evades the effort of forming a conceptual understanding of the problem and is, as such, an ineffective strategy for acquiring a correct solution.

In relation to Hegarty at el. (1995) and Jonsson et al. (2014) it is argued here that, to attain a conceptual understanding of the problem, the learner has to attend to all the relevant properties that are part of the problem. If always being provided with a solution (a short cut) the learner might not see the value in understanding the underlying properties that are required in order to attain a conceptual understanding. Instead, the strategy becomes simply to use the provided solutions, which is economic in terms of solving the task. In Jonsson et al. (2014) it was shown that training on tasks
in which a formula and example how to apply it was provided, did not facilitated a conceptual understanding. On the contrary, the result indicated that being provided with less information during training was more effective for later tests. In the CMR training condition no solution was provided, hence, the participants had to undertake the effort of constructing a solution by themselves. In relation to the findings of Hegarty et al. (1995), a CMR-training condition is therefore expected to directed participants attention to the relevant properties of the problems. Thus, participants training with CMR tasks acquire a conceptual understanding to a greater extent compared to participants training with AR task who are expected to mainly focus their attention on “short cut” approaches such as using the provided formula and/or the example how to apply it (Boesen et al., 2014; Jonsson et al., 2014; Lithner, 2008).

Purpose and hypotheses

The purpose of the present study is to extend the Jonsson et al. (2014) study by investigating whether the two training conditions used (AR and CMR) elicit different attentional behavior, as measured by eye tracking (Kowler et al., 1995), as well as to compare the performance in a follow-up test as a function of training condition and attentional behavior. If significant differences can be found in both attentional behavior and in test performance this could motivate further research into trying to understand learning strategies in relation to training conditions and subsequent tests.

By spatially partitioning the information presented to the subjects into areas of interest, the amount of attention allocated to each area while solving mathematical problems can be measured using eye-tracking equipment. A follow-up test can then be used to associate test results with subjects’ attention patterns during practice.

Three hypotheses were tested in the present study:

1. In line with the results from Jonsson et al. (2014) and Karlsson et al. (2014), it is expected that AR participants should outperform CMR participants at practice and that the reversed pattern should be seen at the subsequent test session.

2. In the second hypothesis, it is expected that an average group difference in eye fixations should emerge. More specifically it is expected that AR participants to a greater degree will attend to information (areas of interest) that, in the short term, is effective for solving the task at hand, i.e. formulas and the examples of how to apply them and that CMR participants, to a greater degree, will attend to the illustration and the question that contain relatively more meaningful information (Hegarty et al., 1995; Jonsson, 2014 &
Lithner, 2008).

3. Thirdly, within the CMR group, attentional behavior during training is expected to predict success in problem solving at both training and at the follow-up tests. Successful problem solvers are expected to pay more attention to the question statement and less attention to less relevant information in the provided example (Hegarty et al., 1995). Corresponding effects are not expected to be found for participants practicing AR.

Method

Participants

Twenty-three participants were recruited from an upper secondary school and from a university in Sweden. The participants were aged between 19 and 36 (mean 22.4, SD 3.85), and received monetary compensation for their participation. All had normal vision. Written informed consent was obtained in accordance with the Declaration of Helsinki, and the study was approved by the Regional Ethical Review Board, Sweden. The study was outside of the ordinary school curricula and students completed it within the school semester.

Cognitive measures

The participants were administered the Raven’s Advanced Progressive Matrices Test (APM) (Raven et al., 1991) and Operation Span (Unsworth & Engle, 2005) in counterbalanced order.

The Operation Span (O-span) test is a measure of working memory capacity (WMC) with a good test-retest reliability and high internal consistency (Conway, Cowan, Bunting, Therriault, & Minkoff, 2002; Engle, Tuholski, Laughlin, & Conway, 1999; Klein & Fiss, 1999) and correspond well with other measures of WMC and higher order cognitive tasks (Conway et al., 2002; Unsworth & Engle, 2005). Sequences of letters that are to be remembered are presented on a computer screen, one letter at a time. Interleaved between the letters are mental arithmetic tasks that must be correctly answered within a limited amount of time. The dependent measure used in the analysis of WMC was the total of letters in correct position. See Unsworth & Engle (2005) for a detailed description. Age, sex, and grades in mathematics were also collected.
Raven’s APM is designed to measure the subject’s non-verbal reasoning ability, or, fluid intelligence. In this study the second subset of tasks (Set II) was used, which comprises 36 tasks of increasing difficulty. Out of these, every odd numbered task in this set were administered with a total time limit of 20 minutes.

Several studies have shown that WM and its executive functions are intimately related to arithmetic performances (Cragg & Gilmore, 2014). For that reason, the participants were matched into two groups, AR and CMR, equal in terms of WMC and mathematics grades. The Raven’s advanced matrices test was used as a measure of individual variation with regard to non-verbal problem solving and used in the analyses as a covariate.

**Training session**

Each participant was given a set of training tasks. All training was conducted in computer laboratory, the tasks were presented on a computer monitor, and a keyboard was used to type the answers. An Eyelink 1 eye tracker (SR research) recording at 500 Hz, using a corneal eye-reflection technology, was mounted on a fixed location in relation to the computer screen, making it possible to collect the participant’s eye movements and fixations without introducing any restriction on the participant’s head movements.

The training tasks consisted of 10 sets of tasks, referred to as task sets. Each subtask within a task set consisted of identical background information and the same question, aside from that the numerical value in the question differed. These mathematical tasks follow the principles for algorithmic reasoning (AR) and creative mathematically founded reasoning (CMR) as described in Lithner (2008), and Jonsson et al. (2014). The AR tasks are designed to be of the type commonly found in school, while tasks were students are required to construct solution methods (e.g. by CMR) are uncommon (Boesen, Lithner, & Palm, 2010; Hiebert, 2003; Jäder, Lithner, & Sidenvall, 2015; Palm, Boesen, & Lithner, 2011).

The task sets were practiced in succession in the same order for all participants. Each set was practiced for five minutes, after which the software would immediately go on to the next set. There were 10 subtasks within each task set, and if the student finished all of them within five minutes, the subtask sequence would restart from the beginning. Thus, all participants practiced 5 minutes on each task set.

For both AR and CMR practice tasks, each task was divided into five areas of interest. These areas were denoted as: (1) illustration (2) description, (3) formula (4) example (5) question. See Figure 1 (a & b) showing the five areas, left to right and top to bottom in
the order mentioned, for the same subtask in the AR and the CMR condition respectively.

Each area remained at the same fixed position for all tasks and for both training conditions. In both AR and CMR tasks the same areas were presented. However, do note that actual algebraic expressions in the areas for the example and the formula were only included in the AR condition, leaving CMR participants with less meaningful information for solving the task.

In Jonsson et al. (2014) less information corresponding to the example and the formula was available in the CMR tasks. However, in the interest of eye-tracking comparisons in the present study, the tasks were made visually identical for the two training conditions in all aspects but the inclusion or exclusion of algebraic expressions in the areas for the formula and the example. In addition, in Jonsson et al. (2014), at the end of each set
CMR participants were asked to generate an algorithmic formula based on the previous numerical practice tasks. This subtask was removed in the present study.

As an example of a mathematical problem given, consider the one presented in Figure one and two where the task is to find out how many matches that are needed to form a row of squares of a given length. One solution method is to ‘form a mental image of the squares and count the matches’. Only the AR group was provided with a formula and an example of how to apply the formula.

The target knowledge for both AR and CMR was solution methods for the 10 different sets of mathematical tasks. A solution method refers to a method that is applicable to any variation of natural numbers ($\mathbb{N}_1$) for the provided task parameters, such as the number of squares.

**Test session**

Seven to eight days after the training session, the participants were tested on the same types of mathematical problems. No eye-tracking camera was used at this time. The test tasks were identical for both groups. The first two questions per task set had a time limit of 30 seconds while the final question allowed for an additional 300 seconds of reflection before an answer had to be provided. An incomplete or omitted answer counted as an incorrect response. For each mathematical problem practiced during the training session three test tasks, denoted as test task I, II, and III (see Figure 2a & b). In test task I the participants had a time limit on 30 sec and were asked to type the formula associated to the specific task solution method. This task was novel for all participants, irrespectively of group, and required them to retrieve the formula from memory since it was seen as highly unlikely to read the task, construct a new solution and write the answer in only 30 seconds. In Test task II (short numerical) the participants were given 30 seconds to complete a numerical calculation of the same type of mathematical task. The basic idea was that 30 seconds is too short to read the task, (re)construct a solution method and write the answer, but enough time to recall and apply a solution. In test task III the time limit was set to 300 seconds, providing enough time to also (re)construct a solution method. Test tasks II and III were similar to both AR and CMR practice tasks (cf. Figure 1, 2b), whereas test task I was novel in that participants had not been asked to provide algorithmic solving methods to the mathematical problems during practice.

An experimenter was present in the lab during both the practice and test sessions to monitor the procedure. During practice and test, no assistance was provided, except for answers regarding the use of the computer.
Data analyses, an overview

The eye fixation data was calculated as the ratio of the total number of eye fixations for each practice subtask across the five areas of interest on an individual level. Both eye fixations and test score performances were screened for outliers. However, no outliers were discovered. MANCOVA and ANCOVA were used to investigate group differences in eye fixations for the areas of interest. A MANCOVA was used to pursue whether areas of interests are predictive of training and test tasks performance. In order to control
statistically for fluid intelligence all analyses of group difference included Raven’s advanced matrices as a covariate. All statistical analyses were conducted using the Statistical Package for the Social Sciences, version 22 (SPSS 22).

Results

In Table 1 the mean values and standard deviations for Operation span and Raven’s advanced matrices are displayed. An initial t-test revealed that the AR and CMR groups are equal in with respect to WMC as measured by the Operation span test, \( t(21) = .62, p = .54 \) and individual variation in WMC is therefore expected to not confound the results. No significant difference was found for the Raven’s advanced matrices test score between the AR and CMR groups, \( t(21) = 1.67, p = .11 \). However, since the groups were not matched on non-verbal problem solving performance, the Raven’s matrices test was used as a covariate in all analyses.

<table>
<thead>
<tr>
<th>Training condition</th>
<th>Raven’s Advanced Progressive Matrices Score</th>
<th>Operation Span Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M )</td>
<td>( SD )</td>
</tr>
<tr>
<td>AR</td>
<td>13.0</td>
<td>2.34</td>
</tr>
<tr>
<td>CMR</td>
<td>11.2</td>
<td>2.89</td>
</tr>
</tbody>
</table>

In hypothesis one, it was expected that AR should outperform CMR at practice and that the pattern should be reversed at later test (Jonsson et al., 2014). An ANCOVA revealed that there was no significant main effect of group for practice tasks, \( F(1,20) = 2.39, p = .194 \).

However, for the test task performance A MANCOVA revealed a main multivariate effect of group, Pillai’s Trace = .56, \( F(3, 18) = 7.62, p = .002, \eta_p^2 = .56 \). Follow-up ANCOVAs for each test task showed that there was a significant effect of group for all three test tasks, formula: \( F(1,22) = 8.30, p = .009, 30s, \eta_p^2 = .29 \), short numerical: \( F(1,22) = 24.89, p < .001, \eta_p^2 = .55 \), and long numerical, \( F(1,22) = 7.73, p = .012, \eta_p^2 = .28 \), where CMR outperforms AR in all test tasks. Table 2 shows the mean values and standard deviations.
Table 2. Means and standard deviations on proportion of correctly answered training and test tasks as a function of training condition

<table>
<thead>
<tr>
<th>Training condition</th>
<th>training</th>
<th>30s, formula</th>
<th>30s, numerical</th>
<th>5min, numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>.89</td>
<td>.06</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>CMR</td>
<td>.70</td>
<td>.27</td>
<td>0.26</td>
<td>0.29</td>
</tr>
</tbody>
</table>

In hypothesis two, it was expected that AR participants would, to a larger extent than CMR participants, attend to information (areas of interest) that in the short term is effective for them in solving the tasks, i.e. formulas and the examples. CMR participants, on the other hand, were expected to focus on the illustration and the question, which for them, contain information that is more meaningful. Figure 3 displays the average distribution of eye fixation with regard to each area of interests and group. A MANCOVA revealed a main multivariate effect of group on the five areas of interest, Pillai’s Trace = .72, F(4, 17) = 10.8, p < .001, ηp² = .72, and subsequent ANCOVAs that the groups significantly differed in terms of eye fixations with regard to the illustration, formula and the question; F(1,22) = 9.55, p = .006, ηp² = .32; F(1,22) = 11.45, p = .003, ηp² = .36; and F(1,22) = 26.74, p < .001, ηp² = .57, respectively. However, there were no significant group differences for the example and the description. The results support hypothesis two with respect to the relative amount of eye fixations on the illustration, formula and question, but not the example.
In Hypothesis three, it was expected that attentional behavior during training should predict success in problem solving at both the training sessions and the follow-up test for CMR but not for AR. To test this hypothesis four separate multivariate linear regression analyses were performed. In each analysis, the independent variables were the proportions of fixations for each of the five areas of interest, and the dependent variables were the proportion of correct answers. The results of the analyses, for each dependent variable, were as follows: (1) Practice score for CMR: Pillai’s Trace = .81, F(4, 6) = 6.35, p = .024. (2) Practice score for AR: Pillai’s Trace = .18, F(4, 7) = .382, p = .815. (3) Test score for CMR: Pillai’s Trace = .90, F(4, 6) = 14.0, p = .003. (4) Test score for AR: Pillai’s Trace = .07, F(4, 7) = .127, p = .968. The results showed that the distribution of fixations on the areas of interest during CMR training were significantly predictive of both training and test results. Table 3 (a & b) the results of the statistically significant independent predictors from analyses 1 and 3, respectively.
Table 3a. Summary of significant predictors of multivariate linear regression model for proportion of correctly answered training tasks for CMR.

<table>
<thead>
<tr>
<th>Area of interest</th>
<th>Coefficient (B)</th>
<th>SE</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>-.484</td>
<td>.112</td>
<td>.165</td>
<td>18.7</td>
<td>.002</td>
</tr>
<tr>
<td>Question</td>
<td>.513</td>
<td>.162</td>
<td>.186</td>
<td>10.0</td>
<td>.011</td>
</tr>
</tbody>
</table>

Table 3b. Summary of significant predictors of multivariate linear regression model for proportion of correctly answered test tasks for CMR.

<table>
<thead>
<tr>
<th>Area of interest</th>
<th>Coefficient (B)</th>
<th>SE</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>-.491</td>
<td>.081</td>
<td>.196</td>
<td>37.1</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Question</td>
<td>.592</td>
<td>.096</td>
<td>.285</td>
<td>38.3</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

In summary, the analyses confirm that practicing CMR is better than practicing AR for later test tasks. This is in line with Jonsson et al. (2014). There was a significant difference between AR and CMR in attentional behavior as measured by eye tracking. Participants in the CMR group attended with a greater degree to two of the predicted areas of interest: the illustration and the question, while AR attended more to the formula but not the example. Within the CMR group the relative amount of fixations on the question and the example were found to be significant predictors of task performance during practice and the follow-up test. Specifically, for the example the coefficient was negative and for the question the coefficient was positive, meaning that an increase in focus on the example decreased the probability of task success and vice versa for focus on the question. No corresponding predictors of task performance were found for the AR group.

Discussion

The results of the current study is in line with Jonsson et al. (2014) in showing that a learning situation that promotes the subject to engage in creative mathematically founded reasoning (CMR) results in a better problem solving when testing the same type of problems again one week later, when compared to a learning situation that promotes algorithmic reasoning (AR). The difference in the setup between the CMR and the AR training conditions was the supplied information. For AR participants every task was provided with a formula and an example how to apply it. For CMR participants those areas of interests contained approximately the same amount of information, the information was however not meaningful in relation to the task solutions.
The eye-tracking results confirm that participants in the AR group do attend to the information that is economically useful in the short term for solving the tasks whereas participants in the CMR group differ significantly with respect to attentional behavior and focus on information that support a more conceptual understanding. In their training condition, the areas for the formula and the example were irrelevant for solving the task, so instead they attend to a higher degree to the illustration and to the question itself. These areas are useful for constructing a conceptual model of the posed problem.

In test task I participants were asked to type a formula as a solution to the corresponding problem previously practiced. It was intended to measure the memory of a formula representing task solutions at training. The time limit for this task (30 s) was set to be short enough so as to reduce the probability of a successful construction or reconstruction. Considering the repetitive exposure of the formulas during training for AR participants (five times for each tasks set) whereas the CMR group was never asked to consider an algebraic expression for solving the task during practice, nor was provided with one, it seems reasonable to assume that the AR group would perform at least as well as the CMR group on this task. However, as evident, the CMR group outperformed the AR group on this task. It seems as the repetitive use of the provided algebraic information does not create a long-term memories of the formulas. Extracting relevant information in CMR training tasks seems, however, to support memory consolidation - a consolidation process that in turn facilitate memory retrieval of appropriate formulas one week later.

In test tasks II and III, where a numerical answer had to be calculated, again participants in the CMR group performed significantly better than did those in the AR group. While the number of correct responses increased for AR from test task II to test task III it did not reach the same level as for CMR despite a five-minute time allotment per task. Thus, a significant effect was demonstrated from the difference in training conditions of the, per task type, five minute practicing one week earlier.

The significant difference between the groups as measured by eye tracking is an indication that the two learning conditions indeed elicited different attentional behaviors (Yarbus, 1967). During practice, the CMR group paid more attention to the illustration while the AR group instead focused on the formula. Whereas the illustration is arguably an essential piece of information for generating conceptual understanding, the formula – which was only provided to the AR group – only contains algorithmic information. The results, thus, support the notion that users of algorithms limit their attention to superficial features of the algorithm (Hiebert, 2003; Lithner, 2003; Vinner, 1997).
In Hegarty et al. (1995) it was demonstrated that unsuccessful problem solvers of arithmetic word problems base their solution plan on keywords searched for in the problem presentation, whereas successful problem solvers construct a mental model of the problem and base their solution plan on this model. By analogy, in hypothesis three of the present study successful CMR problem solvers were expected to pay more attention to the question itself and less so to the, less informative, example. The results confirmed the hypothesis. Within the CMR group, practice task performance was strongly predicted by the proportion of fixations on the example and the question, as well as for the follow-up test task performance. Thus, in line with Hegarty et al. (1995), successful problem solvers within the CMR group seem to be able to construct a model of the problem and base their solution successfully on that, whereas less successful problem solvers within the CMR group, to a greater degree, seem to look for clues in the example.

Limitations

The sample size is a potential limitation to the current study. While the sample size did prove sufficient for providing some significant findings, it is possible that a larger sample size would demonstrate different or even more significant differences between learning conditions. Furthermore, the homogeneity of the sample in terms of participants’ age and level of education might limit the degree of generalizability of this study to a broader population of students.

Conclusion

In conclusion, procedural based learning, as elicited by the AR training condition, was not as good a strategy as learning by applying creative mathematically founded reasoning in regards to acquiring and applying the target knowledge, as measured by the follow-up test. The participants in the CMR group seem to attain a greater amount of conceptual understanding, as measured by test task I, as well as a generally higher proficiency in solving numerical equivalents (task II, and III). The difference in attentional behavior during practice between the two groups was found to be significant. It is argued that the difference in training condition elicits different problem solving strategies, in the present study reflected by different attentional behavior, that in turn explain the divergence in performance in the follow-up test one week after practice. Even within the CMR group, the variation on task performance was compellingly explained by differences in problem solving strategy, probed by measures of attentional behavior. The difference between successful and unsuccessful problem solvers within the CMR group seems to be related to the ability to construct models of the given problem. The findings of this study may motivate further research into effects
and distinctions between attentional behaviors resulting from differences in training condition, as well as methods that can help learners in mathematics to engage in creative mathematical reasoning.
References


