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Electromagnetic surface modes in a magnetized quantum electron-hole plasma

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The propagation of surface electromagnetic waves along a uniform magnetic field is studied in a quantum electron-hole semiconductor plasma. A forward propagating mode is found by including the effect of quantum tunneling. In the classical limit ($\hbar \rightarrow 0$), one of the low-frequency modes found is similar to an experimentally observed one in n -type InSb at room temperature. The surface modes are shown to be significantly modified in the case of high-conductivity semiconductor plasmas where electrons and holes may be degenerate. The effects of the external magnetic field and the quantum tunneling on the surface wave modes are discussed.

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The propagation of surface electromagnetic waves along an external magnetic field in conducting solids has been a topic of important research over the last 40 years. In fact, there was excellent agreement between the theoretical predictions based on simple models and the experimental observations. In 1972, Baibakov and Datsko [1] experimentally observed a low-frequency surface wave mode along a constant magnetic field in electron-hole (EH) plasmas in n -InSb samples at room temperature. This surface mode was, however, theoretically explained by Flahive and Quinn [2] and later by Uberoi and Rao [3] with the predictions of additional surface modes. In contrast to a single-component plasma, two additional characteristic frequencies, namely the plasma and the cyclotron frequencies of holes, exist in EH plasmas. Thus, certain types of excitations may result in the propagation of surface waves with interesting behaviors.

On the other hand, even though the particle number density in semiconductors is lower than that in metals, the high degree of miniaturization of today's electronic components opens up the possibility that the thermal de Broglie wavelength of charge particles may be comparable to, or even larger than, the spatial variation of the doping profiles. Thus, one could expect the typical quantum mechanical effects, such as tunneling, to play important roles in electronic devices to be constructed in the future. A recent review of quantum collective phenomena and typical quantum effects on wave-particle and wave-wave interactions can be found in the literature [4]. Furthermore, in recent years there has been a considerable interest in the investigation of various surface wave modes in classical (see, e.g., Refs. [5–12]) as well as in quantum plasmas (see, e.g., Refs. [13–17]). However, most of these studies until now have been restricted to single-component magnetized or unmagnetized quantum plasmas.

In this Brief Report, we investigate the propagation of electromagnetic surface waves at the EH plasma-vacuum interface parallel to an applied magnetic field. We consider the quantum tunneling effect to be associated with the Bohm potential, which provides a dispersion due to the wavelike nature of particles [4]. In addition to a surface mode, which in the nonretarded

limit ($k \gg \omega$) is given by $\omega \approx (1 + k/\sqrt{1 + k^2 + 1/m\delta})^{-1/2}$ (in nondimensional form), where $k(\omega)$ is the wave number (frequency), m is the electron-to-hole mass ratio, and δ is the ratio of the hole to electron number densities, we find other surface modes with frequencies below the hole-cyclotron frequency as well as between hole and electron-cyclotron frequencies. Furthermore, a quantum surface mode is found to exist as a forward propagating wave by the quantum tunneling effect. In the classical limit, $\hbar \rightarrow 0$, one of the low-frequency modes is found to have similar properties to the experimentally observed wave in Ref. [1]. We consider a Cartesian geometry where the plane $x = 0$ separates the half space $x > 0$ filled by a quantum plasma consisting of electrons and holes (to be denoted respectively by $\alpha = e$ and h) and vacuum ($x < 0$). We also assume that the electron and hole densities are, in general, not equal [18]. In a uniform magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$, the dynamics of electrons and holes are governed by [4]

$$\partial_t n_\alpha + \nabla \cdot \mathbf{v}_\alpha = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}_e}{\partial t} = -(\mathbf{E} + \omega_{ch} \mathbf{v}_e \times \hat{\mathbf{z}}) - \kappa \nabla n_e + \frac{H^2}{4} \nabla \nabla^2 n_e, \quad (2)$$

$$\frac{\partial \mathbf{v}_h}{\partial t} = \mathbf{E} + \omega_{ch} \mathbf{v}_h \times \hat{\mathbf{z}} - \sigma \kappa \nabla n_h + \frac{mH^2}{4} \nabla \nabla^2 n_h, \quad (3)$$

whereas the electromagnetic wave fields are described by the following Maxwell-Poisson equations.

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad (4)$$

$$\nabla \times \mathbf{B} = \mathbf{v}_h - \mathbf{v}_e/\delta + \partial_t \mathbf{E}, \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (6)$$

$$\nabla \cdot \mathbf{E} = n_h - n_e/\delta, \quad (7)$$

where the number density n_α and the velocity $\mathbf{v}_\alpha$ for a α -species particle are normalized by the equilibrium value $n_{\alpha 0}$ and c_s , respectively. Here, c_s may be defined as either $c_s = \sqrt{k_B T_e/m_h}$ for moderate densities (using, e.g., an isothermal equation of state) or $c_s = \sqrt{2k_B T_{Fe}/m_h}$ for a relatively dense medium (where electrons and holes are degenerate and Fermi-Dirac pressure law pertaining to a three-dimensional zero-temperature Fermi gas is applicable), in which $T_{F\alpha}$ (T_α) is the Fermi (thermodynamical) temperature of electrons or holes with k_B denoting the Boltzmann constant. The electric and the magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are normalized

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by $m_h c_s \omega_{ph}/e$ and $m_h c_s \omega_{ph}/\sqrt{\epsilon_0 \mu_0} e$, respectively. Moreover, the space and the time variables are normalized by c_s/ω_{ph} and ω_{ph}^{-1} respectively. Also, $\omega_{p\alpha} = \sqrt{n_{\alpha 0} e^2 / \epsilon_0 m_\alpha}$ is the plasma frequency, $\omega_{c\alpha} \equiv e B_0 / m_\alpha \omega_{ph}$ is the normalized cyclotron frequency, $m = m_e / m_h$ is the electron-to-hole mass ratio, and $\delta = n_{h0} / n_{e0}$. Furthermore, the quantum coupling parameter H appearing in Eqs. (2) and (3) may be defined as $H = \sqrt{\delta \hbar \omega_{pe}} / k_B T_e$ for classical thermal spread or $H = \sqrt{\delta \hbar \omega_{pe}} / 2 k_B T_{Fe}$ for degenerate dense plasmas. In the latter case, which may be relevant for high-conductivity semiconductor plasmas, the temperature ratio will be related to the density ratio according to $\sigma \equiv T_{Fh} / T_{Fe} = \delta^{2/3} / m$, where $k_B T_{F\alpha} = \hbar^2 (3\pi^2)^{2/3} n_\alpha^{2/3} / 2 m_\alpha$ and $\hbar \equiv h / 2\pi$ is the reduced Planck's constant. Thus, $\kappa = 1$ corresponds to classical thermodynamical temperature or moderate density, where $\sigma = T_h / T_e$, and $\kappa = 1/3$ for the Fermi pressure or, relatively, the dense medium in which electrons and holes are degenerate. In what follows, we will find solutions that represent the surface waves propagating along the interface $x = 0$. To this end, we assume that the electromagnetic fields and the perturbed densities associated with the surface wave with the wave number k and the frequency ω ($< \omega_{ph}$) vary as $\Psi(x, y, t) = \Psi(x) \exp(iky - i\omega t)$. Thus, from Eqs. (1)–(3) and (7), we obtain the wave equations for the density perturbations as

$$\partial_x^2 n_\alpha - \gamma_\alpha^2 n_\alpha = 0, \quad (8)$$

where

$$\gamma_\alpha = \left[\frac{(k^2 \beta_2 + v_e + 1/\delta)(k^2 \beta_1 + v_h + 1) - 1/\delta}{3k^2 \beta_1 \beta_2 + (v_e + \mu_e) \beta_1 + (v_h + 1) \beta_2} \right]^{1/2}, \quad (9)$$

with $\beta_1 = \sigma \kappa + m H^2 k^2 / 4$, $\beta_2 = \kappa + H^2 k^2 / 4$, $v_e = m(\omega_{ce}^2 - \omega^2)$, $v_h = \omega_{ch}^2 - \omega^2$, $\mu_e = 1/\delta$, and $\mu_h = \delta$. In obtaining Eq. (8), the very slow nonlocal variations are neglected (i.e., $\partial^6 / \partial x^6$, $k^{-2}(\partial^4 / \partial x^4) \ll \partial^2 / \partial x^2 \ll k^2$). In this approximation [14,15], the existence of some particular modes (e.g., degenerate or singular waves [19]) not of current interest is disregarded. Next, the equation for the magnetic field is

$$\partial_x^2 \mathbf{B} - \alpha_p^2 \mathbf{B} = 0, \quad (10)$$

where $\alpha_p = (1 + k^2 - \omega^2 + 1/\delta m)^{1/2}$. The solutions of Eqs. (8) and (10) are then given by

$$n_\alpha = A_\alpha \exp(-\gamma_\alpha x), \quad x > 0, \quad (11)$$

$$\mathbf{B} = \mathbf{F}_v \exp(\alpha_v x), \quad x < 0, \quad (12)$$

$$\mathbf{B} = \mathbf{F}_p \exp(-\alpha_p x), \quad x > 0, \quad (13)$$

where $\alpha_v = \sqrt{k^2 - \omega^2}$ is the decay variable of the wave into vacuum and A_α , $\mathbf{F}_{v,p}$ are arbitrary constants. Using the Maxwell Eqs. (4)–(6), the solutions for the electric field can be obtained as

$$\mathbf{E} = \mathbf{R}_v \exp(\alpha_v x), \quad x < 0, \quad (14)$$

$$\mathbf{E} = \mathbf{R}_p e^{-\gamma x} + \sum_{\alpha=e,h} \zeta_\alpha A_\alpha \Lambda_\alpha (-\gamma_\alpha \hat{\mathbf{x}} + i k \hat{\mathbf{y}}) e^{-\gamma_\alpha x}, \quad x > 0, \quad (15)$$

where $\mathbf{R}_{v,p}$ are arbitrary constants with $\zeta_{e,h} = \mp 1$ and

$$\gamma = \left[k^2 - \omega^2 - \frac{\omega^2}{\omega_{ch}^2 - \omega^2} - \frac{\omega^2}{m \delta (\omega_{ce}^2 - \omega^2)} \right]^{1/2}, \quad (16)$$

$$\Lambda_e = \left[1 + \frac{m \omega^2}{\omega_{ch}^2 - \omega^2} \left(\kappa + \frac{H^2}{4} (\gamma_e^2 - k^2) \right) \right] \times [1/\delta (\gamma_e^2 - \gamma^2)], \quad (17)$$

$$\Lambda_h = \left[1 + \frac{\omega^2}{\omega_{ch}^2 - \omega^2} \left(\kappa \sigma - \frac{m H^2}{4} (\gamma_h^2 - k^2) \right) \right] \times [1/(\gamma_h^2 - \gamma^2)]. \quad (18)$$

Surface waves are those solutions for which the wave number normal to the surface has a negative imaginary part, leading to an exponential decay away from the surface. Thus, in the above solutions in Eqs. (11)–(15), we have retained only those parts in both the regions that exponentially decay away from the interface. Next, we use the boundary conditions, namely, (i) the tangential component of $\mathbf{E}$ and $\mathbf{B}$ are continuous at $x = 0$, (ii) the normal component of the displacement vector is continuous at $x = 0$, and (iii) velocity components (along the x axis) vanish for both electrons and holes (hot species) (i.e., $\mathbf{v}_{ex} = \mathbf{v}_{hx} = 0$ at $x = 0$). Thus, we obtain a system of linear homogeneous equations that has nontrivial solutions only if the determinant of the resulting system vanishes. This leads to the following dispersion relation.

$$[\omega^2 \alpha_v + (\omega^2 - 1) \alpha_p] \left[\frac{[\omega^2 \gamma_e - (\omega^2 - 1) \alpha_p] \Delta_e}{\delta (\gamma_e^2 - \gamma^2) (\beta_2 - H^2 \gamma_e^2 / 4)} + (2\omega^2 - 1) \alpha_v + \frac{[\omega^2 \gamma_h - (\omega^2 - 1) \alpha_p] \Delta_h}{(\gamma_h^2 - \gamma^2) (\beta_1 - m H^2 \gamma_h^2 / 4)} \right] = 0, \quad (19)$$

where

$$\Delta_e = 1 + \frac{\omega^2}{m(\omega_{ce}^2 - \omega^2)} \left(\kappa - \frac{H^2 k^2}{4} \right) + \frac{H^2 \gamma_e^2}{4}, \quad (20)$$

$$\Delta_h = 1 + \frac{\omega^2}{\omega_{ch}^2 - \omega^2} \left(\sigma \kappa + \frac{m H^2 k^2}{4} \right) - \frac{m H^2 \gamma_h^2}{4}. \quad (21)$$

We note that the surface waves occur only if $k > \omega$. The first factor of the dispersion equation (19) gives ordinary surface mode (as mentioned in the introduction) independent of the magnetic field and the quantum correction term, which decays with the wave number k . However, it approaches a constant value $\omega \approx 0.7$ in the limit $k \gg \omega$. In order to analyze numerically the dispersion relation in Eq. (20), we consider two different density regimes relevant for nondegenerate and degenerate plasmas. We consider the fixed parameters $m_e = 0.01 m_0$, $m_h = 0.4 m_0$, and $T_e = T_h \approx 300$ K, where m_0 is the free electron mass. For nondegenerate particles (Figs. 1 and 2), we use the densities $n_{\alpha 0} \sim 10^{22} \text{ m}^{-3}$, as in Refs. [1,2], and for degenerate electrons and holes we consider $n_{\alpha 0} \sim 10^{26} \text{ m}^{-3}$ (Fig. 3). In the latter, the thermal de Broglie wavelength λ_B is greater than the average interparticle distance (i.e., $n_{\alpha 0} \lambda_B^3 \sim 17 > 1$) and so the quantum effect is no longer negligible. Thus, the typical quantum mechanical effects (e.g., tunneling) will certainly play an important role in the modification and/or generation of a new dispersive surface

FIG. 1. (Color online) The dispersion relation in Eq. (20) in nondimensional form is contour plotted to show different surface wave modes in the k - ω plane (The case of nondegenerate plasmas). The modes labeled A, B, and C are for $H = 0$ and those labeled Q are for nonzero H . The labels A1, B1, C1, and Q1 [thin (red) lines] and A2, B2, C2, and Q2 [thick (blue) lines] indicate the surface modes corresponding to the densities $n_{e0} = 4 \times 10^{22} \text{ m}^{-3}$, $n_{h0} = 2 \times 10^{22} \text{ m}^{-3}$, $n_{e0} = 1.35 \times 10^{22} \text{ m}^{-3}$, and $n_{h0} = 10^{22} \text{ m}^{-3}$, respectively. Other parameters are $m_e = 0.01m_0$, $m_h = 0.4m_0$, $T_e = T_h \approx 300 \text{ K}$, $n_{e0} = 1.35 \times 10^{22} \text{ m}^{-3}$, and $n_{h0} = 10^{22} \text{ m}^{-3}$.

mode. Notice that the quantum modified modes may not appear in the present case as we have disregarded the very slow nonlocal variations [14,15]. Instead, a mode appears by the quantum force that does not exist otherwise (i.e., in the limit $\hbar \rightarrow 0$).

In Figs. 1–3, the lines labeled A, B, and C correspond to the classical ($H = 0$) surface modes whereas those labeled Q are due to quantum tunneling effects. We find that three different modes appear for $H = 0$ whose frequencies lie in the regimes: $\omega_{\text{ch}} < \omega < \omega_{\text{ce}}$, $\omega_{\text{ch}} < \omega \approx \omega_1 < \omega_{\text{ce}}$ and $0 < \omega < \omega_{\text{ch}}$, where ω_1 depends on the parameters to be chosen. In the latter case, the low-frequency mode labeled A1 or A2 has similar behaviors to those experimentally observed in Ref. [1]. Its slope increases as the strength of the magnetic field increases and the wave-phase speed tends to zero at a nonzero $k = k_1$ (i.e., the existence of surface wave at $k < k_1$ is not possible). This particular mode was theoretically explained by Flahive *et al.* [2] and later by Uberoi *et al.* [3], however, there was no quantitative verification due to lack of experimental data in Ref. [1]. In Fig. 1, the surface modes corresponding to the magnetic field $B_0 = 1.6 \text{ T}$ appear as thick solid lines, and the thin lines are due to $B_0 = 0.6 \text{ T}$. From Fig. 1, we also find that as the magnetic field strength increases, the forward propagating quantum surface mode labeled Q1 or Q2 appears at higher values of k and the frequency domain in which it appears reduces. In contrast to Fig. 1, Fig. 2 shows that the slopes of the classical modes decrease with increasing the electron-to-hole density ratio or corresponding to a higher density regime ($\lesssim 10^{24} \text{ m}^{-3}$). Also, the quantum mode appears in a larger frequency domain with smaller wave numbers.

Next, we consider the case in which electrons and holes are degenerate. This may be relevant for high-conductivity semiconductors where the particle number density may exceed 10^{24} m^{-3} , for example. In this case, the pressures of degenerate electrons and holes can be described by the Fermi-Dirac pressure law. Thus, we consider the parameters $m_e = 0.02m_0$, $m_h = 0.4m_0$, the densities $n_{e0} \sim 10^{26} \text{ m}^{-3}$, and the magnetic field $B_0 \sim 1 \text{ T}$. We see from Fig. 3 that one low-frequency mode appears that approaches a constant value ($< \omega_{\text{ch}}$) at higher values of k . Note that the classical mode can not

FIG. 2. (Color online) The same as in Fig. 1, but for the effect of density variation. The modes labeled A, B, and C are for $H = 0$ and those labeled Q are for nonzero H . The labels A1, B1, C1, and Q1 [thin (red) lines] and A2, B2, C2, and Q2 [thick (blue) lines] indicate the surface modes corresponding to the densities $n_{e0} = 4 \times 10^{22} \text{ m}^{-3}$, $n_{h0} = 2 \times 10^{22} \text{ m}^{-3}$, $n_{e0} = 1.35 \times 10^{22} \text{ m}^{-3}$, and $n_{h0} = 10^{22} \text{ m}^{-3}$, respectively. Other parameters are the same as for the thick (blue) lines in Fig. 1.

be recovered in this case for $H = 0$, as the denominator of H will now be the Fermi energy. Furthermore, $H \rightarrow 0$ corresponds to extremely dense regimes which might not be relevant for semiconductor plasmas. For the quantum modes (labeled Q1, Q2, and Q3), there exists a critical value of the wave frequency, below which the phase speed increases with the increasing strength of the magnetic field, and above which it remains unchanged (lines labeled Q1 and Q2). In contrast to Fig. 2 (the case of nondegenerate particles), the enhancement of the electron density (keeping other parameters fixed) shows that the quantum mode appears in a smaller frequency domain at higher values of K .

To summarize, a quantum surface mode at a plasma-vacuum interface is shown to appear by the inclusion of the quantum mechanical (tunneling) effect in a magnetized electron-hole semiconductor plasma. Recent technological progress in the creation of smaller scales of plasma oscillations in electronic

FIG. 3. (Color online) The dispersion relation in Eq. (20) in nondimensional form is contour plotted to show different surface wave modes in the k - ω plane (the case of degenerate dense plasmas). The effects of the variations of both the magnetic field and the particle number density are shown. One low-frequency ($< \omega_{\text{ch}}$) mode appears (not labeled) whose slope increases with the magnetic field and decreases with the density enhancement. The lines labeled Q1 and Q2 are the quantum surface modes corresponding to $B_0 = 0.6 \text{ T}$ and $B_0 = 3 \text{ T}$, respectively with fixed $m_e = 0.02m_0$, $m_h = 0.4m_0$, $n_{e0} = 2 \times 10^{26} \text{ m}^{-3}$, and $n_{h0} = 10^{26} \text{ m}^{-3}$. The quantum mode (labeled Q3) is for $B_0 = 0.6 \text{ T}$, $n_{e0} = 3 \times 10^{26} \text{ m}^{-3}$, $m_e = 0.02m_0$, and $m_h = 0.4m_0$.

devices indicate that the thermal de Broglie wavelength can even be larger than the interparticle distance of the charge carriers, and so quantum mechanical effects (tunneling) may no longer be neglected. We note that such a surface mode due to quantum tunneling appears as a forward propagating wave up to a certain frequency below the hole-plasma frequency. Furthermore, the phase speed of these waves strongly depends on the external magnetic field and the electron-hole concentration in the plasma. Apart from that, several other low-frequency modes also appear in the limit $\hbar \rightarrow 0$, one of which has similar behaviors to that experimentally observed mode in n -InSb at room temperature [1]. On the other hand, when quantum statistical effects are taken into consideration along with the quantum tunneling, specifically for high-conductivity

semiconductors where electrons and holes are rather dense and may be degenerate, the quantum surface wave propagates in a different way in contrast to nondegenerate plasmas. The results may be useful for understanding the dispersion properties of quantum surface waves in semiconductor plasmas, which can be observed experimentally in the near future.

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