The Hazard of Tanning
Intertemporal Choices, Social Status, and Corrective Policy

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Abstract

Skin cancer is the most rapidly increasing cancer form today, our changed tanning behavior is often argued to be the main source of this development. Today a tanned skin is often associated with good health and status. If tanning is a status good and individuals have positional preferences for a suntan our own tanning-decision will create negative positional externalities on others, causing us to over-consume artificial tanning sessions. But why is it that we keep exposing ourselves to UV-light even though we know how dangerous it can be to us? The similarities between tanning and smoking are rather straightforward. Not only do studies indicate that frequent tanners show addictive-like behavior, the predominant similarity is the postponed health costs, in this case risk of skin cancer. An individual’s tanning choice is hence based on current benefits and discounted future health costs. Individuals that have present-biased preferences for immediate gratification will consume tanning sessions in a way that the future self will disagree with. Giving too little weight to future health cost will thus create internalities on the future self.

This study analyses optimal taxation on artificial tanning, correcting for behavioral failures generated from time-inconsistent preferences as well as the externalities caused by positional preferences for a suntan. The results indicate that sizing the tan tax equal to the health costs overlooked and the marginal externalities created generates over all improvement of welfare.
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1. Introduction

Generations ago the shade of our skin was an expression of wealth and social status, heavily tanned skin signaled incapability to avoid sun exposure when performing outdoor manual labor (Hunt et al., 2012; Loh, 2008). The Western society we observe today still considers a suntan as sign of status and wealth, although the associated attributes are somewhat the opposite. Individuals that are tanned are perceived as attractive, healthy and even successful since he or she is having the free time to enjoy the sun (Banerjee, 2008; Rawe, 2006). Improved appearance seems to be one of the main reasons why we expose ourselves to sunlight (Pagoto and Hillhouse, 2011), nevertheless there is also literature arguing that the behavior of extreme tanning might be founded in reinforcements that go beyond those of improved appearance (Harrington et al., 2011). Tanorexia is sometimes used to describe the obsession and addiction-like behavior an individual might experience from tanning (Kravitz, 2010). Studies recurrently find evidence suggesting similarities with addictions such as smoking or other substance use disorders, among others see for example Heckman et al. (2014), Harrington et al. (2011), Zeller et al. (2006), Warthan et al. (2005) or Kaur et al. (2005).

The Swedish population is found at the frontline when measuring risky attitudes and behaviors concerning tanning, both referring to usage of protection against UV-light exposure but also the amount of sun we expose ourselves to. Studies indicate that in comparison to the rest of the European countries Swedish tanners are not only less cautious, they are also more heavily tanned-oriented. (Cancerfonden, 2012; Swedish Radiation Safety Authority, 2015). Skin cancer in general and malignant melanoma in particular is rapidly increasing among the Swedish population and the connection to tanning is comparable with that of smoking and lung cancer (Cancerfonden, 2016).

Despite the last decade’s attempts to suppress the amount of UV-light we expose ourselves to, from sunlight and artificial tanning beds, this evolution of continuously increasing cases of skin cancer has taken place and is becoming a severe concern of the Swedish government. A comparison of social costs reveals a 14 percent increase in real expenditures between 2005 and 2011, mainly originating from increased costs of healthcare and losses in production when absent from work. The cost of 1,69 billion krona measured in 2011 is presumably even
an understatement of what the society actually will pay because of the consequences of skin cancer. (Swedish Radiation Safety Authority, 2015)

The World Health Organization (WHO) stresses the relationship between artificial tanning at a young age and development of skin cancer, pointing out the importance of public health policies concerning the issue (World Health Organization, 2003). WHO clearly advises against tanning for cosmetic purposes but nevertheless artificial tanning devices remains available to the public. For the purpose of reducing the risk of skin cancer, a federal tan tax was implemented in the U.S. in 2010. Sweden has no such governmental policy of tan taxation, however, the Swedish ministry of environment and energy recently proposed a legislation of age restrictions for usage of cosmetic sunbeds (Promemoria Miljö- och energidepartementet, 2015). A proposition that takes a strong standpoint against artificial tanning at a young age and is clearly in line with the recommendations of WHO.

In similarity with smoking, the damage UV-light causes does not reveal itself until many years later; this delay between exposure and consequences is an important aspect when addressing this matter. When deciding on UV-light exposure, the individual weighs the current benefits of a suntan against the discounted future health costs. An individual that chooses immediate gratification in a way that she (or he) over time will oppose herself of is said to have time-inconsistent preferences, such behavior might be generated by hyperbolic discounting. The optimal plan today hence collides with what the optimal plan would be from tomorrow’s perspective (Laibson, 1996). Present optimization choice would then be based on an underestimation of the costs borne by one’s future self, a form of externality in the literature often referred to as internality (Herrnstein et al., 1993; Laux and Peck, 2009). An additional matter that is important to take into account is the fact that a nice tan might be viewed as a sign of status (Yaniv and Siniver, 2015), which implies that if an individual has a nice tan he or she will create externalities on others. One individual getting more tanned, everything else unchanged, will make every one else a little less tanned by comparison. In a study written by Frank (2005) the importance of positional preferences is highlighted when discussing social welfare. The author argues that concerns for relative position should not be overlooked, basing his arguments on the evidence found in literature concerning happiness and subjective well-being. Relative consumption due to positional preferences for a suntan causes a conflict between social and individual welfare, a welfare-loss Frank among others argues to be preventable.
This thesis takes a behavioral economic approach to artificial tanning and examines some policy implications thereof. The analysis will be conducted in two different contexts. Initially I will study the behavioral failures that follow from self-control problems. Addressing intertemporal choices and the effects on welfare due to “sin taxes” on unhealthy goods, being artificial tanning sessions. Secondly, I will discuss whether taxation might be appropriate when attempting to correct externalities caused by positional preferences for a suntan.

To the best of my knowledge the only previous study examining optimal taxation of tanning is written by Yaniv and Siniver (2015). The authors aim to meet what they call “the applauding audience”, arguing that even if a tan tax might decrease consumption of artificial tanning it might also increase riskier practice of sunbathing. The authors assume individuals to derive utility from a tanned look, using a rational-choice model then enables them to analyse the individual’s indoor and outdoor tanning choice. The southern states are sun-flooded and give a readily available alternative to artificial tanning through sunbathing, while in the northern states this is only an alternative during limited periods of the year. The results found indicate that depending on external circumstances due to climate-conditions, a tan tax where sunbathing is continuously an option might even intensify the progression of skin cancer. The Swedish climate is similar to that of the northern states, sun-blessed only during the summer months. Yaniv and Siniver oppose themselves to a tan tax when states are flooded by the sun but are restrictive in their critique when not. When performing the analysis the authors have assumed that individuals exhibit time-consistent preferences, implying that the optimal consumption plan will look the same from both today’s and tomorrow’s perspective. Their model does include differences in awareness of risk (some are assumed fully aware and others are either unaware or ignoring the risk), referring to individuals as either cancer-conscious or non-cancer-conscious. In their study they assume all individuals to make fully rational decisions. No attention is then given to behavioral failures since the authors are ignoring the fact that possible health consequences might be overlooked or that positional goods tend to be over-consumed.

In this thesis I will present an alternative method, considering the fact that a suntan might be viewed as a status good and a sin good. Consequently, I will allow for positional and time-inconsistent preferences to be present. Applying a standard framework of taxation both on externalities and internalities, I will by mathematical derivations show that an artificial tan tax can generate overall improvements of welfare.
2. Theoretical Background

2.1 Internalities, intertemporal choices and hyperbolic discounting

An intertemporal decision refers to a choice that affects more than just the instance; present choice will also impact future welfare (White and Dow, 2015). A decision of tanning today will both affect today’s utility in terms of instant satisfaction but future utility in terms of possible health consequences. The intertemporal decision will be based on a tradeoff between the two. That individuals always respond optimally when making decisions is a frequent assumption in the standard economic approach of rational choice theory. Maximizing utility given the individuals set of resources and specific preferences for a good is hence perceived as rational behavior. Individuals will choose a consumption plan that will maximize lifetime utility and is doing so based on rational expectations about the future (White and Dow, 2015).

How an individual adjusts future utility (or disutility) to present value is determined by a discount rate \((\varphi)\), a higher discount rate indicates a lower net present value (Severens and Milne, 2004). From the point of view of time 0, the discount factor referring to time \(s\) can then be written as

\[
\delta^s = \frac{1}{(1 + \varphi)^s}
\]

Consider a simple utility model (with exponential discounting) where utility \(u(x_s)\) is a function of consumption of a vector of goods \(x\) in each period. Lifetime utility is the sum of utility in period one and the discounted value of utilities in periods two trough \(T\).

\[
U = u(x_0) + \delta u(x_1) + \delta^2 u(x_2) + \ldots + \delta^T u(x_T),
\]

\[
U = \sum_{s=0}^{T} \delta^s u(x_s). \tag{1}
\]

In this standard model (with full information) preferences for a good are considered time-consistent, from this it follows that the tradeoff between present and future satisfaction is persistent regardless of the time perspective. An individual is then said to discount exponentially, implying that the discount factor between every two periods is the same.
throughout the lifetime. The discount factor ($\delta$) being constant over time is a key property for exponential discounting.

Measured at time zero,

the discount factor between period two and period one

$$\frac{\delta^2}{\delta} = \delta$$

the discount factor between period six and period five

$$\frac{\delta^6}{\delta^5} = \delta$$

Using the standard utility model all individuals are assumed to have time-consistent preferences, an assumption that will be slightly relaxed in this study. While still using a simple framework I will instead assume that there exist a self-control problem among the population. This self-control problem displays a propensity for immediate gratification, a short-term desire the individual will disapprove of at later stages in life. A consumer with a self-control problem will choose consumption of hours tanned in a way that the future self does not approve of and negative internalities are created as a consequence of the health costs overlooked (Herrnstein et al., 1993).

That individuals not always act in a way that is in their own best interest is widely found in the literature, often described as behavioral failures (O’Donoghue and Rabin, 2003). In this specific case, making an error is associated with over-consumption of artificial tanning sessions due to present-biased preferences. With no self-control problem no negative internalities are created from current consumption, O’Donoghue and Rabin (2006) describe this as individuals fully internalizing the internalities created on the future self. Future costs are thus given full weight in the present. In comparison, those with present-biased preferences only partially weigh future costs and all internalities are not internalized, a matter of hyperbolic discounting. Intertemporal tradeoffs in the near future are discounted at a higher rate than are those in the distant future (Laibson, 1997).

What characterizes a hyperbolic discounting consumer is the inconsistent preferences over time (Laibson, 1997). Because of these dynamically inconsistent preferences the optimal consumption path may have to be continuously revised over the time periods. We make a decision of tanning today but plan not to tan tomorrow. When tomorrow comes we modify our plan deciding to behave as yesterday and instead plan to change the day after. Continuous revision of our optimal plan might even make us maintain bad habits for long periods of our
lives despite the fact that we know it is in our best interest to quit this behavior (White and Dow, 2015).

Once again using the simple utility model from equation (1) although slightly modified through a quasi-hyperbolic discount function\(^1\). \(\beta\) is included in the model representing a measurement of individuals bias for the present.

\[
U = u(x_0) + \beta \delta u(x_1) + \beta \delta^2 u(x_2) + \ldots + \beta \delta^T u(x_T),
\]

\[
U = u_0 + \beta \sum_{s=1}^{T} \delta^s u(x_s). \tag{2}
\]

If \(\beta = 1\) no self-control problem exists, the individual will discount exponentially and preferences are time consistent. If \(\beta < 1\) the individual exhibits a self-control problem and weighs future time periods too little compared to the present, the future costs overlooked is then described by actual costs times \((1 - \beta)\).

Examining the discount factor between today and tomorrow we find

\[
\frac{\beta \delta}{1} = \beta \delta \tag{3}
\]

In comparison, measured at time zero, we find a different discount factor between period one and period two

\[
\frac{\beta \delta^2}{\beta \delta} = \delta \tag{4}
\]

As we can conclude from equation (3) and (4), bias for the present gives inconsistent preferences over time. Basing decisions on a quasi-hyperbolic discount function will make individuals undervalue future costs and over-consume in the present.

Reviewing the literature studying human behavior, it continuously suggests that people behave in a way consistent with hyperbolic discounting (see for example Strotz, 1955 or Ainslie, 1992). Thaler (1981) questions the normative economic theory of a perfect capital

\(^1\) A quasi-hyperbolic discount function is a discrete case of hyperbolic discounting, a method further discussed in Laibson (1997)
market where all individuals will lend or borrow up to the point where the marginal rate of substitution between today’s and tomorrow’s consumption equals the interest rate. His findings suggest that the discount rate for intertemporal tradeoffs both differs with the size of the reward and the length of time to be waited. Laibson (1996) emphasizes these findings, examining the relationship between time-inconsistent preferences and undersaving. Modeling an individual’s consumption problem as a number of intertemporal decisions he finds that undersaving will occur if preferences are inconsistent. The author explains this by the individual not being committed to the optimal plan. In the study individuals recurrently report themselves to save much less than what they think is appropriate given their incentives, implying that self-control plays an important part of explaining undersaving. In a related study Laibson (1998) argues that quantification of welfare costs due to undersaving is possible when using a hyperbolic model, showing that lacking commitment causes a significant welfare cost.

A well-known and often cited article written by Michael Grossman (1972) presented a new perspective on health. Grossman suggested similarities to the capital investment market where health is a capital stock that depreciates with time and increases with investment. Consumers make choices of consumption to maximize utility, optimal investment in health is then achieved when the marginal benefit equals the marginal cost of this investment. A hyperbolic discounting consumer may plan to invest in health one day but these future plans might be undone when the time comes. Cutler et al. (2003) uses this framework when studying obesity among the American population. Analysing the effects of self-control problems when making eating decisions, reasoning that a hyperbolic discounting consumer will always want to postpone a diet and start tomorrow instead.

A wide range of literature is found on the subject of time-inconsistent preferences and the public policy implications appropriate for the matter. Taxes on items where consumption are believed to be harmful are well known; cigarettes and alcohol are probably the most common examples. The classic work “A Theory of Rational Addiction” written by Becker and Murphy (1988) implied that the key application of taxation on cigarettes was to correct interpersonal externalities. Arguing that individuals make rational decisions of smoking since they are recognizing their addictive behavior, and any decision to continue smoking is made if the benefit of consuming the addictive good exceeds the costs that follow. From this, Becker and
Murphy argued that governmental actions should be based merely on the overlooked costs imposed on others.

Gruber and Köszegi (2001) added important implications to the work of addiction by revising the standard model for “rational addiction”. The authors still assumed individuals being forward-looking but replaced the key assumption of time-consistent preferences with inconsistent ones. Their findings show that a tax on addictive goods such as cigarettes should not solely be based on the externalities caused, smoking will impose costs on others but the internal costs created are significantly higher. Acknowledging internalities generated by time-inconsistency will imply a much higher optimal tax according to Gruber and Köszegi.

A good that gives immediate gratification but future costs is often labeled a “leisure good”, evidently both a suntan and a cigarette fits well into this description. “Leisure goods” are goods that we want to consume with moderation (maybe not even at all) but that an individual with a self-control problem instead will tend to overconsume. O’Donoghue and Rabin (1999) presents an important distinction between those who are aware of their self-control problem and those who are not. Economic literature often argues individuals to be sophisticated when aware that the future self will meet the same self-control problem as the present self is doing. Sophisticated individuals will take this into account, acting like a strategic leader against the future self and choose a consumption plan that will be followed in coming periods. Even though a sophisticated individual might have time-inconsistent preferences the behavior is time-consistent. Individuals that are impatient and unaware of their problem of self-control are instead described as naive. When naive, you believe your preferences to be time-consistent while in reality they are present-biased. Unawareness that also the future self will meet self-control problems will make the individual behave time-inconsistently, reoptimizing in every period. Following the arguments of O’Donoghue and Rabin (1999) it is found that naifs will overconsume an addictive good. That heavy tanners repeatedly make the decision of toasting themselves in a tanning booth even though they are aware of the risk that comes with it might be explained by present biased preferences and unawareness of self-control problems. Conclusively, the findings of O’Donoghue and Rabin show that even if bias for the present is small welfare could be negatively affected.
Aronsson and Thunström (2008) add perspective to optimal paternalism when modeling an internality related to the health capital stock rather than continuous investments in health. Their findings imply that a subsidy directed to health capital rather than a “sin tax” should be imposed to correct divergences between social and private optimal. Similar studies are performed by O’Donoghue and Rabin (2003; 2006), the authors then address the matter of optimal paternalism on unhealthy items when the population consists of both rational and irrational agents. Assuming that individuals might make errors when decision-making they conclude that imposing a “sin tax” on unhealthy goods such as potato chips, and returning revenues to consumers, can create improvements of the social surplus. Taxation of internalities can therefore create improvements of welfare, the main challenge faced is though to distinguish recognized costs from overlooked ones. O’Donoghue and Rabin (2003, 2006) and Gruber and Koszegi (2001) are coherent in their conclusions, suggesting a Pigouvian tax to be useful when addressing internalities due to self-control problems. Using the approach of paternalistic policy by imposing a “sin tax” on artificial tanning could then help individuals make better choices.

2.2 Status consumption, externalities and policy implications

Late 19th century the pioneer Thorstein Veblen (1899) addressed the matter of status when introducing the concept of conspicuous consumption. Veblen argued that if we consume a good in order to gain status we are consuming that good conspicuously. Even though his work is perceived as classic it would take researchers many years before showing serious concerns for status consumption and relative standing. In 1949 James Duesenberry introduced the relative income hypothesis, proposing that utility of consumption is not only derived from own consumption but that it also depends on other peoples’ current consumption. The implications of his theory were not widely accepted until Richard Easterlin (1974, 1995) once again directed the spotlight on the matter. Easterlin showed that happiness in society has remained stable over time despite real economic growth. Explanations were found in relative considerations, if economic growth enables everyone to consume more, relative consumption would stay unchanged. In the book “Social Limits to Growth”, Fred Hirsch (1976) presented the term positional goods when attempting to explain the modest effects of economic growth on self-reported well-being. He pointed out the fact that although growth stimulates consumption, the additional consumption did not generate an equivalent increase in welfare. Hirsch explained this by introducing valuation of a good trough relative position, that consumption matters both in relative and absolute measure. Many studies have found results
that are consistent with those of Duesenberry and Hirsch, showing that also relative position matters. Well-being of an individual is positively affected when own income or consumption increases relative to those of others (see for example Blanchflower et al., 2001; Johansson-Stenman et al., 2002 or Clark and Senik, 2010).

In the literature consensus is that relative standing is an important aspect of utility, with higher income comes not only increased consumption but also higher status (Clark et al., 2008). Robert Frank explores the importance of relative standing in his work “Choosing the right pond” (1985), performing a comprehensive analysis of relative position. From his work it is evident that peoples’ behavior becomes much more complex when abandoning the general utility model that is independent of relative standing. Frank suggests that the importance of relative status is partially explained by envy and because of our interpersonal preferences we over-consume these positional goods. Frank and Heffetz (2008) argue that status is inherently positional, that an increase in relative status, hence increased consumption of a status good, automatically imposes a decrease in relative status for at least some others. The authors show similarities to prisoners’ dilemma where inefficient equilibria may result from individuals attempting to improve relative status. Consistent with the previous study of Frank (1985) the findings of the authors imply over-consumption of the status good. If utility was solely based on social rank society would appear like a zero sum game (Burkett, 2006). From society’s point of view uses are then wasted since trying to win a zero sum game will divert resources away from efforts that are welfare improving (Heffetz and Frank, 2008).

Evidence that utility partly depends on relative income is presented in the work of Luttmer (2005). His findings show that higher income of neighbors translate into lower levels of happiness (self-reported). When utility depends negatively on other peoples’ consumption, increased consumption will create negative externalities on others since relative consumption of others is lowered. That relative position can be important for many reasons is suggested by Solnick and Hemenway (1997). Appreciation, respect and power can be yielded from high rank in society. According to the authors an individual’s perception of own accomplishment is typically more affected by relative position than absolute wealth. Their findings show that positional concerns are strongest for physical appearance and intelligence and in general relative position are found to be more important for goods than for bads. Solnick and Hemenway conclude that ignoring our keen concern for being above average can yield defective directions of policy. In addition Wolf (1991) argues that our relentless interest in
relative physical appearance can create increases in resources spent, only for the relative position to remain unchanged. Although Wolf does not argue out of empirical evidence she pinpoints an important insight of human behavior.

Carlsson et al. (2006) also find evidence of the importance of relative consumption on at least some goods. For example their findings imply stronger positional preferences for a car than for safety equipment and that income is found to be more positional than leisure. Also Frank (2003) finds evidence of income being more positional than leisure. In addition his results imply a stronger positional concern for clothing and housing than it does for health and safety.

In the early work of Robert Frank (1985) he raised the matter of policy implications created by positionality. His findings imply that taxation of positional consumption could correct the externalities created in the economy. These are results that are coherent with a large literature (e.g. Boskin and Sheshinski, 1978; Oswald, 1983; Dupor and Liu, 2003; Aronsson and Johansson-Stenman, 2008; Wendner and Goulder, 2008) showing that positional concerns typically motivate higher marginal taxation. Frank (2005) once again addresses the conflict between social and individual welfare when arguing that welfare losses created by externalities are preventable. Libertarian economists tend to argue that relative position is not a legitimate reason for public policy, raising concerns for more widespread economic and social regulations. Frank reasons that society does have legitimate reasons to prevent envy, arguing that taxation is a necessary measure to prevent the welfare losses created.

The findings of Carlsson et al. (2007) and Solnick and Hemenway (2005), implying that if we desire to signal status positional preferences are more profound for goods that are clearly visible than for those that are less visible, are reviewed in the study of Aronsson and Mannberg (2015). The tax policy implications found by the authors’ are characterized for a durable good and clearly visible good, exemplified by housing. They show that a first best optimum is attained trough a tax on housing wealth equal to the marginal positional externality created.

Positional preferences have well founded implications for taxation, Pigou (1920) argued that status is an important explanation of consumer behavior and should not be overlooked in an analysis of social welfare. We feel less well-off when our reference group is doing better in comparison (Clark and Senik, 2010) and it is more important for us to signal status when a
good is visible. That a nice tan has become a sign of status emerged during the 20th century (Martin et al., 2009). A suntan is a good that is undoubtedly visible; following the literature this would induce that a suntan is a positional good. The argument of individuals having positional preferences for a suntan is also strengthened by the findings of Solnick and Hemenway (1997), showing that positional preferences are strongest for physical appearance. As above discussed consumption of status goods imposes externalities on others, it creates distortions in the economy and is followed by welfare-losses that could be eliminated by taxation. Imposing a tan tax could hence correct over-consumption caused by positional externalities and yield efficient economic solutions.

3. Models and Results

This section will contain a presentation of the two consumption-models that will be used to derive the main results. Initially I will discuss the model and assumptions made when preferences for a suntan are time-inconsistent, additionally some basic results will be presented. The following discussion will concern status consumption and interdependent preferences, also here some basic results will be derived. In the final section I will use a model capturing both effects, showing that it is necessary for the government to address both matters in order to determine optimal policy responses to artificial tanning.²

3.1 Internalities and optimal sin tax

Using a simple consumption-model the framework applied resembles that of O’Donoghue and Rabin (2006). An individual will derive utility from two goods, a “sin good” and a composite good. Naturally, in this model the “sin good” is referred to as artificial tanning. What defines a “sin good” is that consumption today gives immediate pleasure but will translate into negative consequences such as health costs in the future. For tanning the axiomatic health consequence is skin cancer.

Both goods are produced with constant returns to scale and in order for them to have identical marginal cost units will be normalized. Additionally I will assume markets to be competitive,

² Complete derivations of results can be found in Appendix.
that the price of the composite good is normalized to one and hence also the marginal costs of
the goods are one. If no tax is present the pricing of the goods equals the (constant) marginal
cost. Time is discrete, consumption is made in every period and individuals are neither
allowed to borrow nor save. Allowing consumers to do so would not create fundamental
changes in optimal policy implications except that a saving subsidy would be suggested as
optimal correction of saving distortions (e.g. Laibson, 1997).

I will base the analysis on the assumption of homogenous consumers. In addition I will
assume a one-to-one relationship between a tanning session in an artificial tanning booth and
the suntan received. Assumptions made in this model follow those of O’Donoghue and Rabin
(2006), highlighting the effect of time-inconsistent preferences. \( x_t \) denotes consumption of
artificial tanning and \( z_t \) consumption of a composite good (reflecting all other consumption)
in period \( t \). The utility function will be formed quasi-linearly, instantaneous utility in period \( t \)
will depend on current consumption of tanning sessions and the composite good as well as a
negative health effect from consumption of tanning sessions in the previous period.

\[
\begin{align*}
    u_t &\equiv f(x_t) - c(x_{t-1}) + z_t. \\
    f(x_t) &\text{ is the immediate enjoyment from tanning. The consequences on health is imposed by}
\end{align*}
\]
\( t \text{-tanning in the past, described by } c(x_{t-1}), \) this could also be interpreted as the future health
costs from tanning in period \( t \).

I assume the marginal benefits from tanning to be positive and decreasing, such that \( f_x > 0, \)
\( f_{xx} < 0 \). For every extra tanning session the pleasure of the extra suntan is a little smaller than
what was experienced in the previous session. The health costs of tanning are also assumed to
increase with consumption, hence \( c_x > 0 \). The literature implies a strong relationship between
frequency of tanning and the risk of skin cancer (Lazovich et al., 2010). I will assume
marginal health costs to increase \( c_{xx} > 0 \), thus the added risk of skin cancer will increase with
an additional time tanned. Also these assumptions are coherent with those of O’Donoghue
and Rabin (2006), in their model the sin good is represented by potato chips. The similarities
in some of the characteristics of the goods are rather straightforward, postponed health
consequences in particular, hence their assumptions concerning properties are also
appropriate here.
Based on the reasoning discussed in section 2.1 the functional form of an individual’s utility will allow for people to have time-inconsistent preferences, a framework in the literature often represented by the $\beta - \delta$ model.

\[ U^t(u_t, \ldots, u_T) \equiv u_t + \beta \sum_{s=t+1}^{T} \delta^{s-t} u_s. \]  

(5)

The direct enjoyments from artificial tanning will be weighted against the discounted value of future health costs. $\beta$ represents an individual's consistency in preferences of a suntan and as earlier stated; if $\beta < 1$ an individual has a time-inconsistent preference for immediate gratification.

People will face a series of intertemporal decisions throughout the lifetime. Coherent with the convention of O’Donoghue and Rabin I have assumed that behavior in one period will only affect the individual in its subsequent period. As previously discussed, individuals can either be sophisticated or naive with respect to their self-control problem. For a sophisticated individual to be able to behave as a strategic leader against the future self his or her behavior needs to at least affect three periods. From this it is evident that whether an individual is naive or sophisticated about their self-control problem is negligible here.

Choices of consumption of artificial tanning sessions ($x$) and the composite good ($z$) are made in every period. Each individual faces the exact same decision problem in every period, they hence act as if they solve the following maximization problem

\[ u^*(x, z) \equiv f(x) - c(x) + z, \]  

(6)

In the literature it is commonly assumed that public policy is decided by a paternalistic government (e.g. O’Donoghue and Rabin, 2003; 2006). The government does not share individual’s time-inconsistent preferences for a suntan and aims to make individuals with self-control problems behave as if they do not have these preferences. The welfare function from the paternalistic governments’ perspective is presented by a function of long-run utility, expressed as

\[ u^{**}(x, z) \equiv f(x) - c(x) + z. \]  

(7)
Equations (6) and (7) differs from each other, individuals base their consumption choice upon the previous function, including possible self-control problems, while actual utility experienced is based on the latter. If no self-control problem exists ($\beta = 1$) the two functions are identical, an individual will choose consumption levels of the goods in a way that also maximizes welfare. If an individual does exhibit a self-control problem ($\beta < 1$) a decision of consumption based on equation (6) will create behavioral failures and long-run utility will not be maximized.

The maximization problem faced is solved subject to the individual’s budget constraint. I assume individual’s income ($I$) to be exogenously given and the same in all time periods, the resource constraint faced by the consumers can be written

$$I = x + z,$$  \hspace{1cm} (8)

The first best choice of consumption is denoted ($x^{**}, z^{**}$), this consumption bundle maximizes long-run utility $u^{**}(x, z)$ subject to the budget constraint. Optimal consumption of tanning ($x^{**}$) satisfies

$$f_x(x^{**}) - c_x(x^{**}) - 1 = 0,$$  \hspace{1cm} (9)

collection of the composite good is hence

$$I - x^{**} = z^{**}.$$

This scenario is ideal from the individual’s long-run perspective. In comparison, the solution when maximizing equation (6) subject to the budget constraint (8) we find optimal consumption of tanning sessions ($x^{*}$) satisfying

$$f_x(x^{*}) - \beta c_x(x^{*}) - 1 = 0,$$  \hspace{1cm} (10)

and

$$I - x^{*} = z^{*}.$$

If $\beta < 1$ the individual will over-consume tanning sessions and long-run utility will not be maximized.
Following the assumptions made, price of tanning is normalized to one ($P_x = 1$) and equal to the marginal costs with no tax present. If imposing a per unit tax, $\tau$, on artificial tanning the price can be written $p_x = \tau + 1$. If the government provides a lump-sum transfer ($\ell$) to the consumer that is independent of consumer’s own behavior the constraint faced in every period can be written
\[ I + \ell = x(1 + \tau) + z. \] (11)

Maximizing equation (6) subject to the budget constraint (11) we find that, when the model contains a tan tax and a lump-sum transfer, a different consumption allocation is chosen. Consumption level of tanning sessions $x^*(\tau)$ when a tax is imposed satisfies
\[ f_x(x^*(\tau)) - \beta c_x(x^*(\tau)) - (1 + \tau) = 0 \] (12)
and
\[ I + \ell - x^*(\tau)(1 + \tau) = z^*(\tau, \ell). \]

From these calculations it follows that when no self-control problem exist ($\beta = 1$), and tax is absent ($\tau = 0$) the first best consumption of tanning $x^{**}$ found in equation (9) equals the chosen consumption level $x^*$ found in equation (12). If an individual does have present-biased preferences ($\beta < 1$) and taxes are absent this individual will overconsume artificial tanning because too little weight is given to the future health costs. Choice of consumption in equation (12) and (10) will then be identical. Further, the chosen amount of tanning sessions $x^*$ found in equation (12) will be larger than the amount of sessions suggested by the first best scenario $x^{**}$ from equation (9).

These results show that, with no self-control problem the chosen amount of tanning sessions $x^*$ is equal to the first-best consumption of tanning $x^{**}$ and $\tau = 0$ would be optimal. In comparison, if an individual does have self-control problems he or she will over-consume artificial tanning when tax is zero, creating internalities on the future self that are not fully internalized and a tan tax larger than zero ($\tau > 0$) would hence be optimal.

The proportion of the health costs overlooked can be written ($1 - \beta$). If assuming homogenous consumers such that all individuals have self-control problems, a pigouvian tax can be useful to correct for this behavioral failure. As will be shown below, sizing the tan tax
equal to the health costs overlooked and the government returning the proceeds by a lump-
sum transfer \((\tau x = \ell)\) will lead all individuals to choose their first-best outcome.

\[
\tau = (1 - \beta)c_x(x^{**}). \tag{13}
\]

Substituting this into equation (12) and rewriting it gives

\[
f_x(x^*(\tau)) - c_x(x^{**}) - 1 + \beta[c_x(x^{**}) - c_x(x^*(\tau))] = 0 \tag{14}
\]

If \(x^*\) is a unique solution, it must satisfy \(x^* = x^{**}\), in which case the previous equation reduces to

\[
f_x(x^{**}(\tau)) - c_x(x^{**}) - 1 = 0.
\]

The optimal tan tax to internalize the internalities created can be written as

\[
\tau^{**} = (1 - \beta)c_x(x^{**}).
\]

The results found are based on the assumption of homogenous consumers, it seems unlikely to think that this is the case in reality. It might be more natural to assume that the population consists of people that are different from one another regarding the experienced benefits from a suntan. As follows we might draw the conclusion that they also experience different marginal health consequences from tanning. Expressing heterogeneity by the parameters \(\rho\) and \(\gamma\) and assuming a higher \(\rho\) corresponding to a higher marginal benefit from a suntan \(f_{xp} > 0\), and that a higher \(\gamma\) is correspondent to higher marginal cost from tanning \(c_{xy} > 0\) we find new implications of public policy.

If people are heterogeneous in \(f(x; \rho),c(x; \gamma)\) or \(\beta\), an individual tax and lump-sum transfer will be necessary to reach first-best outcomes. But a solution of individual-specific commodity taxation seems rather unrealistic. I will restrict governmental interventions such that they can only be used linearly, hence all consumers experience the same tax and lump-
sum transfer. The propensity of over-consuming artificial tanning sessions increases with more severe self-control problems, the larger those self-control problems are the more likely it is one would be helped by taxation. If no-one in the population had present-biased preferences for a suntan they would already consume optimally, a tan tax would then distort individuals away from their first best choice. In coherence with O’Donoghue and Rabin (2006) I argue these distortions to be second order, that a small uniformed tan tax would still be overall welfare-improving if at least some individuals do have self-control problems.
3.2 Tanning and social status

If we tan in order to gain or maintain social status our behavior can be described as conspicuous consumption of suntan. In this section I intend to show how a suntan is connected to social status and social comparison and that an individual with a nice tan will impose negative externalities on others.

As is indicated in section 2.2, the literature typically suggests a higher marginal tax when models include externalities generated from social comparison. Traditionally it is assumed that optimal policy implications are decided by a welfarist government (see for example Dupor and Liu, 2003 or Frank, 2008). The welfarist government is respectful of the populations’ preferences and aims to correct for the externalities that status consumption gives rise to. Although this has been the normative approach there is literature arguing that governments should not address such considerations. A paternalistic government does not share individuals’ preferences for social comparison. They want consumers to behave as if they do not have preferences for relative consumption and does not account for relative consumption in their social objective function. Aronsson and Johansson-Stenman (2014) analyses tax policy implications suggested by the different approaches. Showing that although reasons differ, their policy responses to interdependent preferences are similar and might even be identical.

In this analysis I will use the standard approach, assuming a welfarist government. Optimal tax policy then aims to internalize the externalities created. I will follow the framework presented by Aronsson and Mannberg (2015), using a model aiming to capture the possibility of utility being dependent on relative consumption. Their work examines externalities caused by conspicuous housing consumption. Although both housing and a suntan are visible goods the difference is that housing is a durable good while a suntan is not. The model presented in their article will hence be a bit modified here.

Once again assuming an individual to derive utility from two goods, suntan ($x$) and a composite good ($z$). The new aspect here is that suntan is assumed to be the positional good, implying that individuals compare their suntan to that of referent others. From this it is found that utility of a suntan is both derived from relative and absolute consumption. The assumption of the second good being non-positional is made to simplify derivations. Also in
this model I will assume a one to one relationship between absolute consumption of artificial tanning sessions and the suntan received. The utility function can be written as

$$ U = U(z, x, \bar{x}). $$

(15)

$\bar{x}$ will then denote the suntan among referent others, i.e., reference level of a suntan individuals compare themselves to. This level is treated as exogenously given by each individual consumer. It could be argued that an individual’s utility is positively affected by increasing their own suntan $u_x > 0$ and increased consumption of the composite good $u_z > 0$, while being negatively affected by others intensifying their tan $u_{\bar{x}} < 0$. The third property is interpretable in terms of jealousy. A referent individual intensifying his or her tan, everything else unchanged, will make others envy him or her (e.g., Dupor and Liu, 2003). It might also be the case that individuals derive utility from being more tanned than others, i.e., from $x - \bar{x}$. Individuals derive utility from comparing themselves to others and gain utility from increasing the difference to the reference level. This approach is used by Aronsson and Mannberg (2015) and although the models are similar and the implications of the results would not differ substantially I will use the less restrictive utility function in equation (15).

The standard assumption of a utility function containing both absolute and relative consumption is concavity in absolute consumption. Marginal utility from additional tanning sessions will approach zero and relative consumption hence becomes increasingly important the more tanned an individual gets (Heffetz and Frank, 2008). I assume that $U = U(z, x, \bar{x})$ is increasing in $z$ and $x$, decreasing in $\bar{x}$ and strictly concave. Normalizing the price to one and including a per unit tan tax, the price of tanning can be written $p_x = \tau + 1$.

Restricting individuals from being able to borrow or save, the budget constraint faced by the consumers can be expressed as

$$ z = I - x(1 + \tau). $$

(16)

$I$ represents individual income, an individual maximizes utility by choosing the optimal bundle of tanning sessions and the composite good subject to the budget constraint.
Note that individuals take $\bar{x}$ as exogenously given, the first order condition resulting from the private decision-problem of $x$ then looks like \(^3\)

$$x : -U_x(1 + \tau) + U_x = 0, \tag{17}$$

equation (17) can also be written

$$MRS_{x,z} = 1 + \tau. \tag{18}$$

The tax rate of artificial tanning then equals $\tau$. Optimal tax rates can be derived by comparing social and private first order conditions. By normalizing the population to one and assuming the social-welfare function to increase with the utility faced by each period we get $W = U$. Following Aronsson and Mannberg (2015), the social decision problem is to maximize $W$ subject to the budget constraint

$$z = I - x. \tag{19}$$

The welfarist social planner internalizes the externality by recognizing that $\bar{x} = x$ in equilibrium. Writing down the Lagrange function where $\lambda$ is the Lagrange multiplier of the resource constraint then gives

$$\mathcal{L} = W + \sum \lambda [I - x - z]. \tag{20}$$

The private marginal rate of substitution between tanning and consumption of the non-positional good can be defined as

$$MRS_{x,z} = \frac{U_x}{U_z} > 0.$$ 

Increased consumption of tanning will “cost” decreased consumption of the non-positional good.

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\(^3\) Throughout the analysis short notations for partial derivatives will be used, for example I will denote $\frac{\partial u}{\partial z} = U_z$ and $\frac{\partial u}{\partial x} = U_x$. 
The social first order conditions can be written as follows

\[ z: \quad U_z = \lambda, \]
\[ x: \quad \frac{U_x}{U_z} + \frac{U_{\bar{x}}}{U_z} = 1. \]

Suppose that \( x^{**} \) and \( z^{**} \) will satisfy this.

The private first order condition can then be written

\[ \tau = z^{**} = -\frac{U_{\bar{x}}}{U_z}, \]

evaluated in the point \( x^{**} \) and \( z^{**} \).

The degree of positionality (\( \alpha \)) for a suntan can be defined as the fraction of the overall utility gain from an extra krona spent on tanning due to increased relative consumption. Alternatively, it can be interpreted as the marginal externality per unit of artificial tanning consumption. If the degree of positionality equals say 0.3, 30 percent of the gained utility from a tanning session comes from getting relatively more tanned.

\[ \alpha = -\frac{U_{\bar{x}}}{U_x} \tag{21} \]

\[ \Rightarrow \quad \tau = -\frac{U_{\bar{x}}}{U_z} = -\frac{U_{\bar{x}}}{U_x} \frac{U_x}{U_z} \]
\[ = \alpha \frac{U_x}{U_z} = \alpha MRS_{x,z}. \tag{22} \]

First best optimal tax policy is expressed by equation (22). The marginal tax on artificial tanning is expressed by the degree of positionality of a suntan times the marginal willingness to pay for tanning. The size of \( \alpha \) thus represents the proportion of the marginal willingness to pay to avoid the positional externality. If striving to achieve efficient solutions this over-consumption should be taxed away. To conclude, the result indicates that consumers desiring to be more tanned than their referent others create externalities that by a tan tax sized as in equation (22) can be fully internalized.
3.3 Optimal tan tax

I will in this section try to capture effects of both self-control problems and conspicuous consumption of a suntan in the same model. Showing that optimal tax policy corrects behavioral failures generated from both relative consumption and self-control problems.

Utility will be derived from consumption of artificial tanning and a composite good, a suntan will both be interpreted as a sin good and a status good. Thus, the utility function will contain time inconsistent and interdependent preferences, the function is expressed as

\[ U(x, \bar{x}, z; \beta) = f(x, \bar{x}, z) - \beta c(x). \]  

(23)

With the exception of the utility function being quasi-linear from section 3.1, assumptions made in section 3.1 and 3.2 will still apply. We hence know that \( f_z > 0, f_x > 0, f_{\bar{x}} < 0 \) and \( c_x > 0 \). An individual’s utility is positively affected by own increased consumption of the composite good, by increased consumption of tanning, deriving direct pleasure from intensifying ones own suntan and by increased social status. Everything else unchanged utility will indirect be negatively affected by a referent other getting more tanned, causing the individual to feel envy. Increased consumption of tanning sessions will also increase health costs, adding to the risk of getting skin cancer.

Using the budget constraint from equation (11) the optimization problem faced by the individual is given by

\[ \mathcal{L} = f(x, \bar{x}, z) - \beta c(x) + \lambda (I + \ell - x(1 + \tau) - z). \]

The first order conditions can be expressed as

\[ \mathcal{L}_z = \quad f_z = \lambda, \]
\[ \mathcal{L}_x = \quad f_x - \beta c_x - f_{\bar{x}} (1 + \tau) = 0. \]

The private marginal rate of substitution between the two goods is defined as

\[ MRS_{x,z} = 1 + \tau. \]
In order to find first optimal tax taxation I once again compare private and social first order conditions.

Applying the standard assumptions from the literature, the government will use a mixture of a welfaristic and a paternalistic approach to correct for behavioral failures. The government accepts individuals’ preferences for jealousy and recognizes how social comparison is determined, by defining $x = \bar{x}$ in equilibrium externalities are internalized. Additionally the government is aware of, but does not share, individuals time inconsistent preferences, aiming to make consumers behave as if no self-control problem existed. The government realizes that $\tau x = \ell$, thus the social decision problem is defined as

$$\mathcal{L} = f(x, \bar{x}, z) - c(x) + \sum \lambda [I - x - z].$$

The social first order conditions can be expressed as

$$\mathcal{L}_z = f_z = \lambda,$$

$$\mathcal{L}_x = f_x - c_x + f_{\bar{x}} - \lambda = 0.$$ 

Assuming this is satisfied by $x^{**}$ and $z^{**}$ I combine private and social first order conditions, hence we find

$$\frac{f_x}{f_z} + \frac{f_{\bar{x}}}{f_z} - \frac{c_x}{f_z} = \frac{f_x - \beta c_x}{f_z} = \tau.$$ 

It then follows that

$$\tau = \frac{(1 - \beta)c_x}{f_z} - \frac{f_{\bar{x}} f_x}{f_x f_z}.$$ 

Using the notation for positionality in equation (21) the first best optimal tax tax can be written as follows

$$\tau = \frac{(1 - \beta)c_x}{f_z} + \alpha \frac{f_x}{f_z}. \quad (24)$$
The results indicate that we can separate first optimal taxation on artificial tanning into two parts. The first part reflects the health costs overlooked due to the self-control problem while the second part expresses how resources are wasted due to relative consumption.

Had I followed the assumption by O’Donoghue and Rabin (2006) assuming the utility function to be quasi-linear (such that \( f_x = 1 \)) the first part would reduce to \( (1 - \beta)c_x \). Internalities created on future selves could then be corrected for by sizing the tax equal to the health costs overlooked, identical to optimal tan tax found in section 3.1. The second part of the expression is defined as the marginal degree of positionality of a suntan times the marginal willingness to pay for a suntan, equivalent to what is found in section 3.2. The degree of positionality is defined as the marginal willingness to pay to avoid positional externality. It hence represents how much consumption that is wasted trying to win a zero sum game and for efficiency of economic solutions should be taxed away.

Note that without self-control problems (\( \beta = 1 \)) no behavioral failures would be generated from present-biased preferences. Optimal tan tax policy would hence only contain the second part that is correcting externalities imposed on others. On the contrary, if individuals did not compare their own suntan to that of others they would not be negatively affected by a referent other intensifying their tan (\( f_x = 0 \)). Optimal tan tax would then merely correct for the overlooked internalities imposed on the future self.

It is evident that the policy implication suggested by the government is both dependent on how an individual’s tanning choice affects own well-being, but also how it affects well-being of others. Equation (24) indicates that the government can fully internalize the internalities and externalities created from artificial tanning. The first best optimal tan tax corrects the behavioral failures that follow from time-inconsistent and interdependent preferences, generating overall welfare-improving solutions.
4. Concluding Remarks

This study focuses on behavioral failures concerning tanning decisions, arguing that individuals over-consume artificial tanning sessions. These failures are founded in preferences for a suntan that are both positional and present biased. An individual that chooses immediate gratification in a way that he or she will oppose herself of in the future is acting irrationally and said to have time-inconsistent preferences, often referred to as a self-control problem. When an individual has present-biased preferences he or she will over-consume artificial tanning sessions in the present because too little weight is given to future health costs and the internalities imposed on the future self are not fully internalized. Individuals that consider a suntan as a status good will be positively affected by intensifying ones own suntan but will on the other hand be negatively affected when someone else intensifies their. Own consumption of tanning sessions will be rational but it will impose negative positional externalities on referent others. Conspicuous consumption of a suntan tends to generate over-consumption and from a societal perspective resources are wasted trying to win a zero sum game.

Based on a model of “sin taxes” presented by O’Donoghue and Rabin (2006) and a model of relative consumption applied in Aronsson and Mannberg (2015) this study analyses optimal policy implication on artificial tanning. The results are rather straightforward, indicating that if individuals have self-control problems and positional preferences for a suntan the government can fully internalize the internalities and externalities generated, creating overall welfare improvements. Thus, sizing the tax equal to the health costs overlooked and the marginal positional externality created gives a first optimal policy implication.

There are some challenging aspects with the results derived that are important to acknowledge. Optimal taxation both in section 3.1 and section 3.3 are based on the health costs overlooked. The problematic part is hence to distinguish recognized costs from overlooked ones. Continued research of tanning behavior is necessary in order for solutions to be as efficient as is theoretically implied. O’Donoghue and Rabin (2006) discusses the complexity of policy implications in general, addressing the common concern that adults make better choices for themselves than the government would. They nevertheless conclude
that if addressing the problem assuming some individuals to be boundedly rational social surplus can be improved.

A lot of assumptions made have been done out of simplification, although this does not have substantial effects on the results found there are some general limitations of the analysis performed. The results derived in this study are accounted for by assuming that the intertemporal decision of tanning in period $t$ would only impose health consequences one period forward. From this it was evident that the distinction between naive and sophisticated individuals was irrelevant. Although it was negligible in this model it might not be in reality. A subject for future studies would hence be to extend the model to allow for more general intertemporal relationships. A sophisticated individual would then be able to act strategically against the future self, setting up a consumption plan that would be followed through in all periods, which might result in optimal policy implications that differ from what is found here (Aronsson and Mannberg, 2013).

The legislation proposal of age restriction on artificial tanning presented by the Swedish ministry of environment and energy is an important step forward in the stressing matter of skin cancer evolution. I have in this study attempted to add additional implications, addressing the matter from a different perspective of governmental actions. The results indicate that the conflict between social and individual welfare is avertable by taxation. As far as I know this is the first study taking a behavioral economic approach to artificial tanning. The generated results might be based on a simplified environment but it nevertheless adds some valuable insight to our puzzling behavior of tanning.
5. References


6. Appendix

Derivations of section 3.1

\[ U^t(u_t, \ldots, u_T) \equiv u_t + \beta \sum_{s=t+1}^{T} \delta^{s-t} u_s \]

Budget constraint

\[ I = x + z \]

The paternalistic government’s optimization problem

\[ u^*(x, z) \equiv f(x) - c(x) + z \]
\[ x^* \text{satisfies} \]
\[ f(x^*) - c(x^*) - 1 = 0 \]
\[ I - x^* = z^* \]

Individuals’ optimization problem

\[ u^*(x, z) \equiv f(x) - \beta c(x) + z \]
\[ x^* \text{satisfies} \]
\[ f(x^*) - \beta c(x^*) - 1 = 0 \]
\[ I - x^* = z^* \]

Budget constraint with tax and lump-sum transfer

\[ I + \ell = x(1 + \tau) + z \]

Individuals’ optimization problem when tax gives

\[ x^*(\tau) \text{satisfying} \]
\[ f(x^*(\tau)) - \beta c(x^*(\tau)) - (1 + \tau) = 0 \]
\[ I + \ell - x^*(\tau)(1 + \tau) = z^*(\tau, \ell) \]

Using costs overlooked equaling tax

\[ \tau = (1 - \beta) c(x^*) \]

By substitution we find

\[ f(x^*(\tau)) - \beta c(x^*(\tau)) - (1 + (1 - \beta) c(x^*)) = 0 \]

Rewriting the equation gives

\[ f_x(x^*(\tau)) - c_x(x^*) - 1 + \beta \left[ c_x(x^*) - c_x(x^*(\tau)) \right] = 0 \]

If \( x^* \) is a unique solution, it must satisfy \( x^* = x^{**} \), then previous equation reduces to

\[ f_x(x^{**}(\tau)) - c_x(x^{**}) - 1 = 0 \]

Optimal tan tax

\[ \tau^{**} = (1 - \beta) c_x(x^{**}) \]
**Derivations of section 3.2**

Utility function  
\[ U = U(z, x, \bar{x}) \]

Budget constraint faced by the consumer  
\[ z = I - x(1 + \tau) \]

**Private first order conditions**  
\[-U_z(1 + \tau) + U_x = 0 \]

\[ \Rightarrow \frac{U_x}{U_z} = MRS_{x,z} = 1 + \tau \]

Social welfare function  
\[ W = U \]

Budget constraint faced by social planner  
\[ z = I - x \]

Social decision problem  
\[ \mathcal{L} = W + \sum \lambda [I - x - z] \]

**The social first order conditions**  
\[ \frac{\partial \mathcal{L}}{\partial z} = U_z - \lambda = 0 \]

\[ \Rightarrow U_z = \lambda \]

\[ \frac{\partial \mathcal{L}}{\partial x} = U_x + U_{\bar{x}} - \lambda = 0 \]

\[ \Rightarrow \frac{U_x}{U_z} + \frac{U_{\bar{x}}}{U_z} = 1 \]

Using private FOC gives  
\[ \frac{U_x}{U_z} + \frac{U_{\bar{x}}}{U_z} = \frac{U_x}{U_z} - \tau \]

\[ \tau = z^{**} = -\frac{U_{\bar{x}}}{U_z} \]

Note that  
\[ \alpha = -\frac{U_{\bar{x}}}{U_x} \]

\[ \Rightarrow \tau = -\frac{U_{\bar{x}}}{U_z} = -\frac{U_x}{U_x U_z} \]

\[ \tau = \alpha \frac{U_x}{U_z} = \alpha MRS_{x,z} \]
Derivations of section 3.3

Private decision problem

\[ \mathcal{L} = f(x, \bar{x}, z) - \beta c(x) + \lambda (I + \ell - x(1 + \tau) - z) \]

\[ \mathcal{L}_z = \quad U_z = \lambda \quad \text{or} \quad f_z = \lambda \]

\[ \mathcal{L}_x = \quad U_x - \lambda(1 + \tau) = 0 \quad \text{or} \quad f_x - \beta c_x - \lambda(1 + \tau) = 0 \]

Combining \( \mathcal{L}_x \) and \( \mathcal{L}_z \) and rewriting the expression we find

\[ \frac{u_x}{u_z} = 1 + \tau \quad \text{or} \quad \frac{f_x - \beta c_x}{f_z} = 1 + \tau \]

\[ \text{or} \quad \frac{f_x - \beta c_x}{f_z} - \tau = 1 \]

Social decision problem

\[ \mathcal{L} = f(x, \bar{x}, z) - c(x) + \lambda (I - x - z) \]

\[ \mathcal{L}_z = \quad U_z = \lambda \quad \text{or} \quad f_z = \lambda \]

\[ \mathcal{L}_x = \quad U_x + U_{\bar{x}} - \lambda = 0 \quad \text{or} \quad f_x + f_{\bar{x}} - c_x - \lambda = 0 \]

Combining \( \mathcal{L}_z \) and \( \mathcal{L}_x \) and dividing all elements by \( f_z \) gives

\[ \frac{f_x + f_{\bar{x}} - c_x - f_{\bar{x}}}{f_z} = 0 \quad \Rightarrow \quad \frac{f_x + f_{\bar{x}} - c_x}{f_z} = 1 \]

Combining private and social FOC:s

\[ \frac{f_x + f_{\bar{x}} - c_x}{f_z} = \frac{f_x - \beta c_x - \tau}{f_z} \quad \Rightarrow \quad \tau = \frac{f_x - f_{\bar{x}} + c_x - f_x - \beta c_x}{f_z} \]

\[ \tau = \frac{(1 - \beta)c_x}{f_z} \frac{f_{\bar{x}}}{f_z} \]

Optimal tax equals

\[ \tau = \frac{(1 - \beta)c_x}{f_z} - \alpha \frac{f_z}{f_z} \]