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This is the published version of a paper published in *International Economic Review*.

Citation for the original published paper (version of record):

Aronsson, T., Johansson-Stenman, O. (2017)
Genuine saving and positional externalities.
International Economic Review, 58(4): 1155-1190
<https://doi.org/10.1111/iere.12248>

Access to the published version may require subscription.

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GENUINE SAVING AND POSITIONAL EXTERNALITIES*

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Much evidence suggests that people are concerned with their relative consumption. Yet, positional externalities have so far been ignored in savings-based indicators of sustainable development. This article examines the implications of relative consumption concerns for measures of sustainable development by deriving analogues to genuine saving when people are concerned with their relative consumption. Unless the positional externalities have been internalized, an indicator of such externalities must be added to genuine saving to arrive at the proper measure of welfare change. We also show how relative consumption concerns affect the way public investment ought to be reflected in genuine saving.

1. INTRODUCTION

How to measure social welfare, and correspondingly welfare change over time, is a classical and much discussed question in economics that has received increased attention recently, e.g., through the report of the Stiglitz-Sen-Fitoussi Commission (Stiglitz et al., 2009), initiated by French president Sarkozy, and its aftermath.² According to this report (p. 8), “it has long been clear that GDP is an inadequate metric to gauge well-being over time particularly in its economic, environmental, and social dimensions, some aspects of which are often referred to as *sustainability*.” The idea of sustainability, or sustainable development, has also grown in importance over time and is highlighted by the 17 sustainable development goals (Sachs, 2015) adopted by the United Nations in September 2015. Although there are many definitions and interpretations of sustainable development, it is often, as expressed by the World Commission on Environment and Development, defined to reflect a development that meets “the needs of the present without compromising the ability of future generations to meet their own needs” (Our Common Future, 1987, p. 54).³

The concept of genuine saving, which is at the core of this article, plays a key role in the measurement of both social welfare change and sustainable development. The genuine saving of a country is a measure of comprehensive net investment, i.e., the value of all capital formation undertaken by society over a time period. In other words, it summarizes the value of the net investment in all relevant capital stocks, potentially including net investments in man-made capital and human capital, changes in natural resource stocks and environmental capital, as well as biodiversity. Earlier research shows that the genuine saving constitutes an exact measure of economy-wide welfare change over a short time interval if the resource allocation is first best.⁴ Furthermore, in the aftermath of the World Commission on Environment and

*Manuscript received June 2015; revised March 2016.

¹ The authors would like to thank three very constructive anonymous referees, an associate editor, Sofia Lundberg, and Karl-Gustaf Löfgren for helpful comments and suggestions as well as Catia Cialani for helpful research assistance. Research grants from the Swedish Research Council (ref 421-2010-1420) are gratefully acknowledged. Please address correspondence to: Thomas Aronsson, Department of Economics, Umeå School of Business and Economics, Umeå University, SE – 901 87 Umeå, Sweden. E-mail: Thomas.Aronsson@econ.umu.se.

² See in particular Fleurbaey and Blanchet (2013) for a thorough and rigorous analysis of social welfare measures beyond GDP.

³ This report is often referred to as *The Brundtland Report*.

⁴ The seminal contributions are Pearce and Atkinson (1993) and Hamilton (1994, 1996). See also Hamilton (2010) for an overview of the literature and van der Ploeg (2010) for a political economy analysis of genuine saving. A

Development some 30 years ago, genuine saving has also become an indicator of sustainable development. Indeed, a natural and frequently used economic interpretation of sustainable development is that welfare must be nondeclining over time, meaning that genuine saving becomes an exact indicator of sustainable development over a short time interval.⁵ Another possible definition of sustainable development is that the instantaneous utility must not exceed its maximum sustainable level, on the basis of which Pezzey (2004) shows that nonpositive genuine saving constitutes an indicator of unsustainable development, although positive genuine saving does not necessarily imply that development is sustainable.⁶ For each of these definitions, genuine saving gives information of clear practical relevance for economic welfare, which is further emphasized by the attention paid to genuine saving by the World Bank, which regularly publishes estimates on genuine saving (referred to as *adjusted net saving*) for a large number of countries (see also Section 6).

Yet, the literature dealing with genuine saving has so far focused on traditional neoclassical textbook models, where people derive utility solely from their own absolute consumption of goods and services (broadly defined). Thus, it neglects the possibility discussed in behavioral economics literature that people also enjoy consuming more, and dislike consuming less, than others—an idea that appeared rather obvious to many leading economists of the past, including Adam Smith, John Stuart Mill, Karl Marx, Alfred Marshall, Thorstein Veblen, and Arthur Pigou, before it became unfashionable in the beginning of the 20th century (see Mason, 1998).

The purpose of this article is to examine how relative consumption concerns, through a preference for “keeping up with the Joneses,” affect the principles for measuring welfare change. It will then present a correspondingly modified measure of genuine saving. Arguably, such a study is relevant for several reasons. First, there is now a large body of empirical evidence that people are concerned with their relative consumption (and not just their absolute consumption as in standard economic models).⁷ Questionnaire–experimental research often concludes that up to 50% of an individual’s utility gain from increased consumption may actually be due to increased relative consumption (e.g., Alpizar et al., 2005; Solnick and Hemenway, 2005; Carlsson et al., 2007). Similarly, many happiness-based studies find that a large (or sometimes even dominating) share of consumption-induced well-being in industrialized countries is due to relative effects (e.g., Easterlin, 2001; Luttmer, 2005; Easterlin et al., 2010). In turn, this may distort the incentives underlying consumption and capital formation. Second, based on the estimates referred to above, wasteful conspicuous consumption generates positional externalities (often referred to as *consumption externalities* in the macroeconomic literature). These are likely to result in significant welfare costs, which—if not properly internalized—may change the principles for calculating welfare change-equivalent measures of saving. Indeed, we show that the more positional people are on average, the more the conventional measure of genuine saving (where people are assumed not to have positional preferences) will overestimate the true welfare change. Third, recent literature shows that optimal policy rules for public expenditure

public economics approach is taken by Aronsson et al. (2012), who derive a second-best analogue to genuine saving in a representative-agent economy with distortionary taxes and public debt accumulation, whereas Li and Löfgren (2012) address genuine saving in an economy where growth is stochastic. See also the recent empirical application by Geasley et al. (2014) and Fleurbaey (2015) for an examination of relationships between social welfare and measures of sustainable development (including genuine saving).

⁵ See, e.g., Arrow et al. (2003a).

⁶ See also Asheim (1994) and Pezzey (1994). This false-message problem is further analyzed by Valente (2008), who derives conditions under which a long-run measure of genuine saving is negative (which it may be even if the current genuine saving is positive). Relationships between genuine saving and consumption (or instantaneous utility) are also examined by, e.g., Aronsson et al. (1997) and Hamilton and Hartwick (2005).

⁷ See, e.g., Easterlin (2001), Johansson-Stenman et al. (2002), Blanchflower and Oswald (2004), Ferrer-i-Carbonell (2005), Luttmer (2005), Solnick and Hemenway (2005), Carlsson et al. (2007), Clark and Senik (2010), and Corazzini et al. (2012). See also Fliessbach et al. (2007) and Dohmen et al. (2011) for evidence based on brain science and Rayo and Becker (2007) for an evolutionary approach.

are modified in response to relative consumption concerns,⁸ suggesting that such concerns may also affect the value of public investment in the context of genuine saving. This will be further discussed below.

The analysis is based on a dynamic general equilibrium model where each consumer derives utility from his/her own consumption and use of leisure, respectively, and from his/her relative consumption compared with a reference consumption level (reflecting other people's current consumption). Our main contribution is that we show how positional concerns, and corresponding positional externalities, influence the way welfare change-equivalent savings ought to be measured. We distinguish between a social optimum where all externalities are internalized and market economies without externality correction. We also distinguish between first-best and second-best social optima by extending the benchmark model to allow for asymmetric information between the consumers and the social planner (or government). Furthermore, by using insights developed in the literature on tax and other policy responses to relative consumption concerns, we are also able to relate genuine saving to empirical measures of "degrees of positionality," i.e., the extent to which relative consumption is important for individual well-being.

The paper closest in spirit to ours is Aronsson and Löfgren (2008). They consider the problem of calculating an analogue to Weitzman's (1976) welfare-equivalent net national product in an economy where the (identical) consumers are characterized by habit formation. Their results show that if the habits are fully internalized through consumer choices, habit formation does not change the basic principles for measuring welfare (except that the individual's own past consumption affects his/her current instantaneous utility). However, with external habit formation, i.e., if the habits partly reflect other people's past consumption, the present value of this marginal externality also affects the welfare measure. Our study differs from Aronsson and Löfgren (2008) in at least four distinct ways: We (i) consider measures of genuine saving (or analogues thereof) instead of comprehensive net national product measures, (ii) focus attention on the empirically well-established keeping-up-with-Joneses type of comparison,⁹ (iii) allow for redistributive aspects by considering a case with heterogeneous consumers, and (iv) introduce public investments into the study of welfare change-equivalent savings.¹⁰

The article is outlined as follows. In Section 2, we present the benchmark model where each individual derives utility from consuming more than other people. We also present useful indicators of the extent to which relative consumption matters for individual well-being. Following earlier literature on genuine saving, we assume in the benchmark model that individuals are identical and that the population size is fixed. In Section 3, we use the benchmark model to analyze economy-wide measures of welfare change. In addition, we present two extensions: First, in Subsection 3.3 we analyze a case of endogenous population growth. Second, in Subsection 3.4 we examine the consequences of replacing the conventional welfarist social objective (which reflects all aspects of consumer preferences) with a nonwelfarist objective that does not reflect the consumer preference for relative consumption. Section 4 extends the benchmark model by analyzing the role of public investments. As a further extension, Subsection 4.2 analyzes the case with more than one externality. This means that the model encompasses a case where private consumption gives rise to environmental externalities beyond the positional ones. Section 5 examines a more general model with two ability types that differ in productivity, where

⁸ See Ng (1987), Brekke and Howarth (2002), Aronsson and Johansson-Stenman (2008, 2014b), and Wendner and Goulder (2008), who analyze different aspects of public good provision in economies where people are concerned with their relative consumption.

⁹ In a background working paper, Aronsson and Johansson-Stenman (2016), we also analyze the implications of catching-up-with-the-Joneses comparisons, where the measure of reference consumption refers to other people's past consumption. The results show that the associated externalities affect the welfare change measure in the same general way as the externalities following from keeping-up-with-the-Joneses comparisons.

¹⁰ Yamaguchi (2014) uses the model developed in Aronsson and Löfgren (2008) to derive an indicator of genuine saving. He examines conditions under which the value of investment in physical capital and the value of investment in the stock of habits jointly contribute to increased welfare in a social optimum. Thus, his analysis has a very different focus from ours.

productivity is private information not observable to the social planner. Such a model allows us to extend the welfare analysis to a second-best model that includes both redistribution and externality correction subject to an incentive constraint. Section 6 presents a numerical example based on World Bank data for all OECD countries, China, and India, and Section 7 concludes the article.

2. THE BENCHMARK MODEL AND EQUILIBRIUM

Consider an economy with a constant population comprising identical individuals, whose number is normalized to one.¹¹ The assumption of identical individuals is made for purposes of simplification; all qualitative results that we derive for this model would carry over in a natural way to a framework with heterogeneous consumers, as long as the redistribution policy can be implemented through lump-sum taxation. We will first, in Subsection 2.1, define the instantaneous utility function, which forms the basis for the subsequent measures of welfare change when people care about relative consumption. In Subsection 2.2, we consider the production side of the economy, where output is a function of labor and capital, before dealing with the dynamic optimization problem of individuals as well as the social planner.

2.1. Instantaneous Utility Function and the Degree of Positionality. Let c denote private consumption and z leisure. Similarly as in the models analyzed in Aronsson and Johansson-Stenman (2010), the instantaneous utility function faced by the representative individual takes the form

$$(1) \quad U_t = u(c_t, z_t, \Delta_t) = v(c_t, z_t, \bar{c}_t),$$

where the variable $\Delta_t = c_t - \bar{c}_t$ denotes the individual's relative consumption and is defined as the difference between the individual's own consumption and a reference measure, $\bar{c}_t$.¹² Although the reference consumption is an endogenous variable (see below), each individual behaves as an atomistic agent and treats $\bar{c}_t$ as exogenous. As we show below, the endogenous labor-leisure choice will not itself affect the welfare change measure derived in the benchmark model; it just means that the consumption externality is present also in a long-run steady state (and not just during a transitory phase).¹³ It plays a distinct role in Section 5, where the consumers are assumed to differ in labor productivity.

The assumption that the individual's relative consumption reflects a difference comparison is made for technical convenience. All qualitative results derived below will also follow—yet with slightly more complex mathematical expressions—if the difference comparison is replaced with a ratio comparison (in which the relative consumption would become $c_t/\bar{c}_t$).¹⁴ The function $u(\cdot)$ defines the instantaneous utility in terms of the individual's absolute consumption and use of leisure, respectively, as well as in terms of the individual's relative consumption compared with the reference measure, whereas the function $v(\cdot)$ is a convenient reduced form allowing us to shorten some of the notation below. We assume that the function $u(\cdot)$ is increasing in each argument and strictly concave, implying that $v(\cdot)$ is increasing in its first two arguments and

¹¹ The assumption of a constant population simplifies the analysis; it is not of major importance for the qualitative results derived below. In Subsection 3.3, we relax this assumption and extend the analysis to accommodate endogenous population growth.

¹² Note also that leisure is assumed to be completely nonpositional. This is, of course, questionable, yet the limited empirical evidence available suggests that private consumption or income is much more positional than leisure (Solnick and Hemenway, 2005; Carlsson et al., 2007).

¹³ See Liu and Turnovsky (2005).

¹⁴ Mujcic and Frijters (2013) compare different specifications in a “beauty contest” based on questionnaire-experimental data. They find that a specification with difference comparisons (together with absolute income) explains choice data best. However, specifications based on ratio comparisons or on the ordinal rank in the income distribution perform almost equally well. Moreover, each of these three measures performs substantially better than specifications based on measures of inequality and poverty.

decreasing in the third. To be more specific, following Equation (1), the relationships between the functions $v(\cdot)$ and $u(\cdot)$ are $v_c = u_c + u_\Delta$, $v_z = u_z$, and $v_{\bar{c}} = -u_\Delta$, where the subscripts denote partial derivatives.

We follow Johansson-Stenman et al. (2002) and define the *degree of positionality* as a measure of the extent to which relative consumption matters for an individual's marginal utility of consumption. To be more specific, the degree of positionality represents the share of the overall instantaneous utility gain from increased consumption that is due to increased relative consumption. By using the function $u(\cdot)$, which distinguishes between absolute and relative consumption, the degree of positionality at time t can be written as

$$(2) \quad \alpha_t = \frac{u_\Delta(c_t, z_t, \Delta_t)}{u_c(c_t, z_t, \Delta_t) + u_\Delta(c_t, z_t, \Delta_t)} \in (0, 1) \quad \text{for all } t.$$

Therefore, $1 - \alpha_t$ measures the degree of nonpositionality, i.e., the extent to which the instantaneous utility gain of increased consumption is due to increased absolute consumption—an entity that is always set to unity in standard economic models. As indicated in the introduction, empirical evidence from questionnaire–experimental methods suggests that the degree of positionality on average is in the interval 0.2–0.5 for income (which is interpretable as a proxy for overall consumption) in industrialized countries, although it may be even higher for certain visible goods such as houses and cars. Wendner and Goulder (2008) argue that the degree of positionality, as estimated based on various methodologies, is typically found to be in the interval 0.2–0.4.

2.2. Production and Intertemporal Maximization. We assume that production is determined by labor and capital. Let l denote the hours of work, defined by a time endowment, $\bar{l}$, less the time spent on leisure, and k denote the physical capital stock. To begin with, k is the only capital stock in the economy. We realize, of course, that our one-dimensional capital concept disregards several aspects of clear practical relevance such as the formation of human capital and environmental capital. Yet, our more narrow definition of capital is not in itself important for the qualitative relationship between genuine saving and welfare change, which does not in any way depend on the number of capital stocks in the economy. Therefore, to simplify the benchmark model as much as possible, we refrain from considering other types of capital than physical capital. In Section 4, we extend the model by incorporating public investment to show how the treatment of such investment in genuine saving reflects the policy rule for contributions to a state-variable public good.

Output is produced by a constant returns to scale technology with production function $f(l, k)$, which is such that $f_l > 0$, $f_k > 0$, $f_{ll} < 0$, and $f_{kk} \leq 0$.¹⁵ We suppress depreciation of physical capital, as it is of no concern in our context. This means that $f(\cdot)$ is interpretable as net output, or that the depreciation rate is zero. The net investment at time t is then written in terms of the resource constraint as

$$(3) \quad \dot{k}_t = f(l_t, k_t) - c_t,$$

where the initial (time zero) capital stock, k_0 , is fixed. The terminal condition can be written as $\lim_{t \rightarrow \infty} k_t \geq 0$.

¹⁵ Note that the possibility of $f_{kk} = 0$ means that the model is consistent with an AK structure, such that the economy grows at a constant rate in the steady state.

The objective faced by each consumer is to maximize the present value of future utility. If expressed in terms of the function $v(\cdot)$, the intertemporal objective function can then be written as (if measured at time 0)

$$(4) \quad \int_0^{\infty} U_t e^{-\theta t} dt = \int_0^{\infty} v(c_t, z_t, \bar{c}_t) e^{-\theta t} dt,$$

where θ is the utility discount rate.

The social decision problem is then to choose c_t and l_t for all t to maximize the present value of future utility given in Equation (4), subject to the resource constraint in Equation (3), the initial capital stock, and the terminal condition. In doing this, and in contrast to individual consumers, the social planner also takes into account changes in the reference consumption $\bar{c}_t$. The corresponding current value Hamiltonian of this problem is given by (if written in terms of the utility formulation $v(\cdot)$ in Equation (1))

$$(5) \quad H_t = v(c_t, z_t, \bar{c}_t) + \lambda_t [f(l_t, k_t) - c_t],$$

where λ denotes the costate variable attached to the capital stock. Note also that we are considering an economy with identical individuals, where $\bar{c}_t = c_t$. In addition to Equation (3) and the initial and terminal conditions, the social first-order conditions include

$$(6a) \quad v_c(c_t, z_t, \bar{c}_t) + v_{\bar{c}}(c_t, z_t, \bar{c}_t) = \lambda_t$$

$$(6b) \quad v_z(c_t, z_t, \bar{c}_t) = \lambda_t f_l(l_t, k_t)$$

$$(6c) \quad \frac{d\lambda_t e^{-\theta t}}{dt} = -\frac{\partial H_t}{\partial k_t} e^{-\theta t} = -\lambda_t f_k(l_t, k_t) e^{-\theta t},$$

where subscripts attached to the instantaneous utility and production functions denote partial derivatives. For further use, we also assume that the transversality conditions

$$(6d) \quad \lim_{t \rightarrow \infty} \lambda_t e^{-\theta t} \geq 0 \quad (= 0 \text{ if } \lim_{t \rightarrow \infty} k_t > 0)$$

$$(6e) \quad \lim_{t \rightarrow \infty} H_t e^{-\theta t} = 0$$

are fulfilled.¹⁶ Note that the left-hand side of Equation (6a) reflects the social marginal utility of consumption, $v_c + v_{\bar{c}} = u_c$, since the social planner recognizes that relative consumption is social waste.

In an unregulated economy where the consumption externality is uninternalized, the social first-order condition for private consumption given by Equation (6a) is not satisfied. Instead of introducing the decision problems faced by consumers and firms in the unregulated economy and then characterizing the general equilibrium, we just note that the outcome of such an economy would be equivalent to the special case of the model set out above where the social planner (erroneously) treats $\bar{c}_t$ as exogenous for all t . The first-order condition for consumption would then change to

$$(7) \quad v_c(c_t, z_t, \bar{c}_t) = \lambda_t,$$

¹⁶ For a more rigorous analysis of transversality conditions in optimal control theory, see Michel (1982) and Seierstad and Sydsaeter (1987).

whereas the first-order condition for work hours and the equation of motion for the costate variable remain as in Equations (6b) and (6c), respectively.

3. MEASURING WELFARE CHANGE IN THE BENCHMARK MODEL

This section presents measures of welfare change based on the benchmark model set out above. We begin by considering a welfare change measure under first-best conditions in Subsection 3.1 and continue with the unregulated economy in Subsection 3.2. Some extensions of the benchmark model are discussed in Sections 4 and 5.

3.1. First-Best Resource Allocation. As a point of departure, consider first the problem of measuring welfare change along the first-best optimal path that obeys Equations (6a)–(6c), where the externalities associated with relative consumption concerns are fully internalized. This constitutes a natural reference case, although it is presumably not very realistic. We use $*$ to denote the socially optimal resource allocation, such that

$$\{c_t^*, l_t^*, k_t^*, \lambda_t^*\} \quad \forall t$$

satisfy Equations (3) and (6a)–(6e) along with the initial and terminal conditions for the capital stock, and then define the corresponding optimal value function at time t as follows:

$$(8) \quad V_t^* = \int_t^{\infty} v(c_s^*, z_s^*, \bar{c}_s^*) e^{-\theta(s-t)} ds.$$

The welfare change over the short time interval $(t, t + dt)$ is given by the time derivative of Equation (8), i.e.,

$$(9) \quad \frac{dV_t^*}{dt} \equiv \dot{V}_t^* = \theta V_t^* - v(c_t^*, z_t^*, \bar{c}_t^*).$$

Defining genuine saving at any time t as $\lambda_t \dot{k}_t$, our first result is summarized as follows (and formally derived in the Appendix).

OBSERVATION 1. In a first-best optimum, genuine saving constitutes an exact measure of welfare change such that

$$(10) \quad \dot{V}_t^* = \lambda_t^* \dot{k}_t^*.$$

Observation 1 is a standard result that reproduces the welfare change-equivalence property of genuine saving in the context of the benchmark model. The left-hand side of Equation (10) is the welfare change over the short time interval $(t, t + dt)$, whereas the right-hand side is interpretable as the genuine saving for the model set out above measured in units of utility at the first-best social optimum. Although our model for simplicity only contains a one-dimensional capital concept (or state variable), namely, the physical capital stock, a generalization to several capital stocks is straightforward: The right-hand side of Equation (10) would then simply be the sum of changes in the value of all relevant capital stocks (see also Sections 4 and 6 below).

With reference to sustainable development, we make three broad observations based on earlier research. First, as indicated in the introduction, if sustainable development is interpreted to mean nondeclining intertemporal welfare, then $\lambda_t^* \dot{k}_t^* \geq 0$ is a necessary and sufficient condition for local sustainable development, i.e., sustainable development over the short time interval $(t, t + dt)$, and $\lambda_t^* \dot{k}_t^* < 0$ is the corresponding necessary and sufficient condition for local unsustainable development.

Second, negative genuine saving means that the instantaneous utility must eventually decline, whereas nonnegative genuine saving is less informative about the equilibrium path of future instantaneous utilities. By using Weitzman's welfare measure, $H_t^* = \theta V_t^*$ (see the Appendix), and then integrating Equation (8) by parts, we can follow Aronsson et al. (1997) in rewriting the relationship between genuine saving and welfare as follows:

$$(10a) \quad \lambda_t^* \dot{k}_t^* = \int_t^\infty \frac{dv(c_s^*, z_s^*, \bar{c}_s^*)}{ds} e^{-\theta(s-t)} ds.$$

Accordingly, $\lambda_t^* \dot{k}_t^* < 0$ means that the instantaneous utility must decline during some future time interval (for the sum on the right-hand side to be negative), whereas $\lambda_t^* \dot{k}_t^* \geq 0$ only means that the future discounted changes in the instantaneous utilities must sum to a nonnegative number; it does not require that all these future changes in instantaneous utilities are nonnegative. Equation (10a) is particularly interesting if we interpret the model in terms of a continuum of perfectly altruistic generations of consumers (instead of in terms of consumers with infinite planning horizons), in which case the instantaneous utility is interpretable as a measure of generational well-being. Third, and related to the second point, by assuming that the instantaneous utility is nonconstant along the optimal path and that this path is unique, Pezzey (2004) shows that nonpositive genuine saving at time t means unsustainable development in the sense that the (actual) instantaneous utility at time t exceeds the maximum sustainable instantaneous utility level.¹⁷ Therefore, and irrespective of which perspective we take, negative genuine saving contains a strong message: Neither the current instantaneous utility nor the current intertemporal welfare level is sustainable.

3.2. Unregulated Economy. Here we analyze the probably more realistic case where the externalities associated with relative consumption concerns are not internalized, implying that the first-order condition for private consumption is given by Equation (7) instead of Equation (6a) and that Equation (10) is no longer valid. Let

$$\{c_t^0, l_t^0, k_t^0, \lambda_t^0\} \quad \forall t$$

denote the resource allocation in the unregulated economy. As indicated above, the basic assumption underlying Equation (7) is that consumers maximize the discounted stream of future utilities in a world where the positional externalities are not internalized (since all of them treat the reference consumption as exogenous). Let the corresponding value function at time t be given by

$$(11) \quad V_t^0 = \int_t^\infty v(c_s^0, z_s^0, \bar{c}_s^0) e^{-\theta(s-t)} ds.$$

Also, let $r_t = f_k(l_t, k_t)$ denote the interest rate at time t , and $R_{s-t} = \int_t^s r_\tau^0 d\tau$ denote the sum of interest rates from t to s (where $s > t$). Now, by using

$$(12) \quad \dot{V}_t^0 = \theta V_t^0 - v(c_t^0, z_t^0, \bar{c}_t^0),$$

we can derive the following result.

¹⁷ Following Pezzey (2004), note that $\lambda_t \dot{k}_t \leq 0$ implies $H_t \leq v_t$. Then, using Weitzman's (1976) welfare measure $H_t = \theta V_t$, we have $\frac{1}{\theta} H_t = \int_t^\infty H_t e^{-\theta(s-t)} ds = \int_t^\infty v_s e^{-\theta(s-t)} ds$. Let v_t^m be the maximum instantaneous utility at time t that can be sustained forever. If the optimal path is unique and nonconstant, we have $H_t > v_t^m$ and, therefore, $v_t \geq H_t > v_t^m$.

PROPOSITION 1. *In an unregulated economy with externalities caused by relative consumption concerns, the measure of welfare change takes the form*

$$(13) \quad \dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 - \int_t^\infty \alpha_s^0 \exp(-R_{s-t}) \dot{c}_s^0 ds \right],$$

$$\text{where } \alpha_t^0 = \frac{u_\Delta(c_t^0, z_t^0, \Delta_t^0)}{u_c(c_t^0, z_t^0, \Delta_t^0) + u_\Delta(c_t^0, z_t^0, \Delta_t^0)}.$$

PROOF. See the Appendix.

The right-hand side of Equation (13) is written as the product of the real welfare change in consumption units (the expression in square brackets) and the marginal utility of consumption. Proposition 1 implies that the conventional measure of genuine saving, $\lambda_t \dot{k}_t$, does not generally constitute an exact measure of welfare change in an unregulated economy.¹⁸ The second term on the right-hand side is interpretable as the value of the change in the marginal positional externality. To see this more clearly, note that α_t measures the instantaneous marginal positional externality per unit of consumption at time t , implying that $\alpha_t \dot{c}_t$ denotes the change in this externality over $(t, t + dt)$. Integrating forward along the economy's general equilibrium path gives the final component in Equation (13). In particular, note that this component is forward looking, since the welfare function at any time t is intertemporal and reflects future utility. It arises because the consumer overvalues consumption at the margin compared with the social planner, i.e., $v_c = u_c + u_\Delta > u_c = v_c + v_\bar{c}$ for a given c , which is seen by comparing Equations (6a) and (7). As a consequence, in the unregulated equilibrium, the relationship between c and $\bar{c}$ is not internalized, implying, in turn, that the model is *nonautonomous time dependent* through the equilibrium path for $\bar{c}$. Note also that in the economy with identical individuals analyzed so far, $\bar{c}$ is always equal to c , implying that their growth rates will also be the same. Thus, in a growing economy in which $\dot{c}$ is predominantly positive, the second term on the right-hand side of Equation (13) will be negative, and genuine saving will overestimate the welfare change. Also, the more positional the consumers are, i.e., the larger the α , the greater the discrepancy between the conventional measure of genuine saving and the welfare change, *ceteris paribus*. Therefore, empirical estimates of the degree of positionality, along with estimates of changes in the average consumption, are important for calculating the second term on the right-hand side of Equation (13). Such calculations are presented in Section 6 based on data for the OECD countries, China, and India.

The right-hand side of Equation (13), i.e.,

$$\lambda_t \left[\dot{k}_t - \int_t^\infty \alpha_s \exp(-R_{s-t}) \dot{c}_s ds \right],$$

plays exactly the same role here as the conventional genuine saving did in Subsection 3.1. Thus, it constitutes an exact measure of welfare change as well as giving the same qualitative information on local (un)sustainable development as the conventional genuine saving measure does in a first-best resource allocation. However, since the second term on the right-hand side of Equation (13) is typically unobserved, an important question is that of the remaining informational content of $\lambda_t \dot{k}_t$, i.e., the conventional genuine saving measure. In general, if we

¹⁸ The insight that externalities may change the principles of measuring welfare and welfare change is, of course, not new. Earlier research on green national accounting shows that technological change and environmental externalities add additional components to measures of welfare and welfare change in unregulated economies (see Aronsson et al., 1997, and Aronsson and Löfgren, 2010, as well as references therein). The novelties of this article are that it (i) shows how positional externalities modify the principles of measuring welfare-equivalent saving and (ii) relates the welfare change measure to empirical measures of positional concerns.

erroneously were to apply the genuine saving formula given by Equation (10) in an unregulated economy, it would give little information for assessing the welfare change or of relevance for local sustainable development. However, in a growing economy where consumption typically increases along the equilibrium path, such that the second term on the right-hand side of Equation (13) is negative, $\lambda_t \dot{k}_t$ can still be used for one-sided tests of welfare decline and local unsustainable development. More specifically, $\lambda_t \dot{k}_t < 0$ then implies (i) welfare decline and the corresponding interpretation in terms of local unsustainable development, and (ii) that the instantaneous utility exceeds the maximum sustainable instantaneous utility level, meaning that it will eventually decline. The latter can be understood from an analogue to Equation (10a), which in this setting is given by

$$\lambda_t \left[\dot{k}_t - \int_t^\infty \alpha_s \exp(-R_{s-t}) \dot{c}_s ds \right] = \int_t^\infty \frac{dv(c_s, z_s, \bar{c}_s)}{ds} e^{-\theta(s-t)} ds,$$

where $\lambda_t \dot{k}_t < 0$ is a sufficient (yet not necessary) condition for the left-hand side to be negative and, as a consequence, that the instantaneous utility will decline sometime in the future.

3.3. Extension to a Model with Population Growth. In this subsection, we discuss how the benchmark model can be extended to include population growth and some implications thereof. Genuine saving under exogenous population growth is addressed by, e.g., Pezzey (2004). Such population growth makes the underlying optimal control problem nonautonomous time dependent and leads to a forward-looking term in the genuine saving measure. Endogenous population growth has been examined by Arrow et al. (2003b) in the context of genuine saving, and we follow their approach here by introducing the population as a state variable such that the population growth depends on the population stock

$$\dot{N}_t = h(N_t),$$

where $h(\cdot)$ is interpretable as a population growth function.¹⁹ We can then define $K = kN$ and write the capital accumulation equation as follows:

$$\dot{K}_t = f(K_t, N_t l_t) - N_t c_t.$$

Since all consumers living at the same time are identical by assumption, we follow Arrow et al. (2003b) and Pezzey (2004) by considering a utilitarian instantaneous social welfare function, $N v(c, z, \bar{c})$, meaning that the intertemporal objective at any time t can be written as

$$V_t = \int_t^\infty N_s v(c_s, z_s, \bar{c}_s) e^{-\theta(s-t)} ds.$$

Let ψ_t denote the current value costate variable corresponding to N_t . The social first-order conditions are given by Equations (6a)–(6c) above (where the right-hand side of Equation (6c) should now be interpreted as a derivative with respect to K) and

$$(14) \quad \frac{d\psi_t e^{-\theta t}}{dt} = -[v(c_t, z_t, \bar{c}_t) + \lambda_t \{f_L(K_t, N_t l_t) l_t - c_t\} + \psi_t h(N_t)] e^{-\theta t},$$

¹⁹ A more general formulation would be $h(N, c, l)$, allowing for a specific relationship between population growth and economic decisions, or $h(N, x)$ where x is a fertility-flow variable.

where $L = Nl$. Genuine saving in utility units for this economy is given by $\lambda_t \dot{K}_t + \psi_t \dot{N}_t$, since population is a state variable and population growth thus measures how this capital stock increases over time. If the resource allocation is first best (satisfying Equations (6a)–(6e) and (14) and the equations of motion for N and K), we can use the same technique as above to derive a straightforward analogue to Equation (10),

$$(15) \quad \dot{V}_t^* = \lambda_t^* \dot{K}_t^* + \psi_t^* \dot{N}_t^*,$$

where $*$ denotes social optimum (as before). Therefore, genuine saving continues to be an exact measure of welfare change, meaning that Observation 1 applies with the modification that genuine saving is measured in a different way here.

The unregulated market economy is more complicated here, since population growth in such an economy is not necessarily optimal from society's point of view. Nevertheless, if we assume that the shadow price of population develops according to Equation (14) also in the decentralized economy (conditional on the equilibrium trajectories for consumption, labor supply, and the capital stock), we can measure welfare change in the same general way as in Proposition 1, i.e.,

$$(16) \quad \dot{V}_t^0 = \lambda_t^0 \left[\dot{K}_t^0 + \frac{\psi_t^0}{\lambda_t^0} \dot{N}_t^0 - \int_t^\infty N_s^0 \alpha_s^0 \exp(-R_{s-t}) \dot{c}_s^0 ds \right].$$

Equation (16) is derived in the same way as Equation (13) and means that the welfare change is given by the sum of genuine saving (the first two terms) and the value of the change in the marginal positional externality (the third term). The intuition is, of course, that the positional externality constitutes the only market failure here, as it also did in Subsection 3.2.

However, if ψ_t^0 develops in a different way compared with Equation (14)—a possibility that strikes us as quite realistic—Equation (16) no longer applies. Instead, to measure welfare change, the right-hand side of Equation (16) must in this case be augmented by another forward-looking term, reflecting the welfare contribution of the change in the marginal externality of population growth. Although it is beyond the scope of this article to explore different possible scenarios with regard to such externalities, future research in this direction might be very fruitful.

3.4. A Nonwelfarist Perspective on the Social Objective. Harsanyi (1982) and others have argued that a government should not respect antisocial preferences such as jealousy. Since relative consumption concerns may be interpreted as reflecting jealousy, some have in turn argued that an appropriate objective function of the government should not necessarily reflect such concerns. In this subsection, we will show that the results obtained above remain the same irrespective of whether the social planner shares the consumer preference for relative consumption or not.²⁰ Arguably, this strengthens the conclusions drawn from the benchmark model in Subsections 3.1 and 3.2.

To simplify the presentation, we consider a separable instantaneous utility function such that²¹

$$U_t = u(c_t, z_t, \Delta_t) = \tilde{u}(c_t, z_t) + \varphi(\Delta_t) = v(c_t, z_t, \bar{c}_t).$$

Following the recent literature on optimal taxation under paternalism referred to above, suppose that the social planner does not share the consumer preference for relative consumption

²⁰ A related recent literature examines tax policy implications of relative consumption concerns from the point of view of a paternalist government that does not share the consumer preference for relative consumption (e.g., Eckerstorfer and Wendner, 2013; Aronsson and Johansson-Stenman, forthcoming).

²¹ The separable structure is not needed for the results to go through. Its role is merely to simplify the presentation by allowing us to distinguish the utility of relative consumption from other aspects of utility.

(reflected in subutility function $\varphi(\Delta_t)$) while sharing all other aspects of consumer preferences (reflected in subutility function $\tilde{u}(c_t, z_t)$). The social objective is thus based on the subutility function $\tilde{u}(c_t, z_t)$ such that (recall that $l = \bar{l} - z$)

$$(17) \quad V_t^* = \text{Max}_{\{c_s, l_s\}} \int_t^{\infty} \tilde{u}(c_s, z_s) e^{-\theta(s-t)} ds,$$

subject to Equation (3) and initial and terminal conditions. The first-order conditions of this problem coincide with Equations (6a)–(6e). By using the same technique as in the proof of Observation 1 (see the Appendix), it is straightforward to show that the welfare change measure coincides with Equation (10), i.e., $\dot{V}_t^* = \lambda_t^* \dot{k}_t^*$ for all t , with the only modification that V_t^* is now given by Equation (17). This result has the same interpretation here as in Subsection 3.1, despite that the underlying policy objective is now different.

In the unregulated economy, the behavior is, of course, governed by the consumers' own objective function and satisfies Equations (6b)–(6e) and (7), whereas the social welfare function is now given by (where superscript 0 is used to denote unregulated equilibrium, as before)

$$(18) \quad V_t^0 = \int_t^{\infty} \tilde{u}(c_s^0, z_s^0) e^{-\theta(s-t)} ds.$$

Based on Equations (6b)–(6e), (7), and (18), we can use the same technique as in the proof of Proposition 1 to derive

$$\dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 - \int_t^{\infty} \alpha_s^0 \exp(-R_{s-t}) \dot{c}_s^0 ds \right],$$

which coincides with Equation (13). Yet, the interpretation is slightly different here, since a nonwelfarist social planner (who does not share the consumer preference for relative consumption) has no intention to correct for positional externalities. Instead, the second term on the right-hand side is now interpretable as the welfare cost of a behavioral failure caused by the consumer preference for relative consumption (which is not corrected in the unregulated economy).

The reason why the welfare change measures take the same form as in the benchmark model in Subsections 3.1 and 3.2 is that the positional externality that the welfarist social planner would like to internalize coincides with the behavioral failure that the nonwelfarist social planner would like to correct for. Both of them are internalized in a social optimum (regardless of whether the planner is welfarist or nonwelfarist) and remain uninternalized in the unregulated equilibrium.

4. PUBLIC INVESTMENTS

This section extends the benchmark model by analyzing the role of public investments.²² The implications of positional externalities for the optimal provision of public goods have been addressed in several studies (e.g., Ng, 1987; Aronsson and Johansson-Stenman, 2008, 2014b). Relative concerns for private consumption affect the optimal policy rule for public good provision via two channels: (i) an incentive to internalize positional externalities through increased public provision (which reduces the private consumption) and (ii) an (indirect) incentive to reduce the public provision as relative consumption concerns lower the consumers' marginal

²² Aronsson (2010) examines the related problem of valuing public goods in the context of analogues to Weitzman's (1976) welfare measure, although based on models without consumer preferences for relative consumption.

willingness to pay for public goods, *ceteris paribus*. In turn, this has a direct bearing on the way public investment ought to be reflected in the context of genuine saving, which motivates the following extension.

4.1. The Value of Public Investment. We will, consequently, add a public good of stock character, which means that Equation (1) changes to

$$(19) \quad U_t = u(c_t, z_t, g_t, \Delta_t) = v(c_t, z_t, g_t, \bar{c}_t),$$

where g_t denotes the level of the public good at time t . The public good accumulates through the following differential equation:²³

$$(20) \quad \dot{g}_t = q_t - \gamma g_t,$$

where q_t denotes the flow expenditure directed toward the public good at time t , i.e., the instantaneous contribution, and γ denotes the rate of depreciation. We also impose the initial condition that g_0 is fixed and the terminal condition $\lim_{t \rightarrow \infty} g_t \geq 0$. Finally, since part of output is used for contributions to the public good, the resource constraint slightly changes such that

$$(21) \quad \dot{k}_t = f(l_t, k_t) - c_t - \rho q_t,$$

where ρ is interpretable as the (fixed) marginal rate of transformation between the public good and the private consumption good.

These extensions mean that we have added the control variable q and the state variable g to the benchmark model; otherwise, the model is the same as in Section 2. Thus, the social decision problem is to choose c_t , l_t , and q_t for all t to maximize the present value of future utility,

$$\int_0^{\infty} U_t e^{-\theta t} dt = \int_0^{\infty} v(c_t, z_t, g_t, \bar{c}_t) e^{-\theta t} dt,$$

subject to Equations (20) and (21) along with initial and terminal conditions. The current value Hamiltonian can then be written as

$$(22) \quad H_t = v(c_t, z_t, g_t, \bar{c}_t) + \lambda_t [f(l_t, k_t) - \rho q_t - c_t] + \mu_t [q_t - \gamma g_t],$$

where μ_t denotes the current value shadow price of the public good at time t . Note that Equations (6a)–(6e), if written in terms of the instantaneous utility function examined here (i.e., Equation 19), are necessary conditions also in this extended model. In addition to (the modified) Equations (6), the first-order conditions for a social optimum also include an efficiency condition for q , an equation of motion for μ , and an additional transversality condition, i.e.,

$$(23a) \quad \lambda_t \rho = \mu_t$$

$$(23b) \quad \frac{d\mu_t e^{-\theta t}}{dt} = -\frac{\partial H_t}{\partial g_t} e^{-\theta t} = -[v_g(c_t, z_t, g_t, \bar{c}_t) - \mu_t \gamma] e^{-\theta t}$$

$$(23c) \quad \lim_{t \rightarrow \infty} \mu_t e^{-\theta t} \geq 0 \quad (= 0 \text{ if } \lim_{t \rightarrow \infty} g_t > 0).$$

²³ If we interpret the public good in terms of environmental quality, a possible (and realistic) extension of Equation (20) would be to assume that increased consumption (or output) leads to lower environmental quality. Such an extension is discussed in Subsection 4.2 below.

Equations (6a)–(6e) and (23) characterize the social optimum. For further use, let us also solve the differential Equation (23b) forward subject to the transversality condition (23c),²⁴ which gives

$$(24) \quad \mu_t = \int_t^{\infty} v_g(c_s, z_s, g_s, \bar{c}_s) e^{-(\theta+\gamma)(s-t)} ds.$$

As in Subsection 3.2, if the positional externality has not become internalized, Equation (6a) should be replaced with Equation (7) (again modified to reflect the instantaneous utility function (19) that contains the public good). This *partly regulated equilibrium* can be implemented in a decentralized setting (where the consumers choose their consumption and work hours at each point in time) by assuming that the planner (or government) raises revenue to finance the public good through lump-sum taxation, although it does not use the tax system to correct the individual first-order conditions for externalities.

We are now ready to present the main results of this section. As before, to separate the social optimum from an allocation with uninternalized positional externalities, we use * to denote the socially optimal resource allocation and superscript 0 to denote the economy with uninternalized externalities. The value function (social welfare function) at any time t will be defined in the same way as above, i.e.,

$$V_t = \int_t^{\infty} v(c_s, z_s, g_s, \bar{c}_s) e^{-\theta(s-t)} ds.$$

Then, by recognizing that genuine saving now reads $\lambda_t \dot{k}_t + \mu_t \dot{g}_t$ (since the capital concept is two-dimensional here), we can use the same procedure as in Observation 1 and Proposition 1 to derive an analogue to Equation (10) as follows:

$$(25a) \quad \dot{V}_t^* = \lambda_t^* [\dot{k}_t^* + \rho \dot{g}_t^*],$$

and a corresponding analogue to Equation (13):

$$(25b) \quad \dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \rho \dot{g}_t^0 - \int_t^{\infty} \alpha_s^0 \exp(-R_{s-t}) \dot{\bar{c}}_s^0 ds \right].$$

Except for the public investment component, Equations (25a) and (25b) are interpretable in exactly the same way as their counterparts in the simpler benchmark model, i.e., Equations (10) and (13).²⁵

Equations (25a) and (25b) together imply the following treatment of public investment:

PROPOSITION 2. *Irrespective of whether the resource allocation is first best or characterized by uninternalized positional externalities, the accounting price of public investment is given by the marginal rate of transformation between the public good and the private consumption good, ρ .*

Proposition 2 has a strong implication, as it means that the valuation of public investment might be based on observables and that the same valuation procedure applies in a social optimum

²⁴ We assume that $\lim_{t \rightarrow \infty} g_t > 0$.

²⁵ If we extend the model by allowing ρ to vary over time, e.g., due to disembodied technological change, an additional (forward-looking) term representing the marginal value of this technological change must be added to Equations (25a) and (25b).

and in a distorted market economy. Although convenient, this result may seem surprising at first sight. Yet, note that the information content in ρ differs between the two regimes due to differences in the underlying policy rules for public provision. To see this more clearly, let

$$MRS_{gc} = \frac{v_g}{v_c} = \frac{u_g}{u_c + u_\Delta}$$

denote the marginal rate of substitution between the public good and private consumption measured with the reference consumption, $\bar{c}$, held constant, and, following Aronsson and Johansson-Stenman (2008), let

$$CMRS_{gc} = \frac{v_g}{v_c + v_{\bar{c}}} = \frac{u_g}{u_c}$$

denote the corresponding marginal rate of substitution measured with the relative consumption, Δ , held constant. Therefore, MRS_{gc} refers to a conventional marginal willingness to pay measure with other people's consumption held constant. This means that an increase in the public good will not only imply reduced (absolute) consumption, the individual will also take into account the fact that his/her relative consumption decreases. $CMRS_{gc}$, on the other hand, reflects each respondent's marginal willingness to pay for the public good conditional on that other people have to pay the same amount, implying that relative consumption is held fixed. Thus, in this case there is no additional cost in terms of reduced relative consumption. Indeed, it is straightforward to show that $CMRS_{gc} = MRS_{gc}/(1 - \alpha)$.

The policy rule for public provision in the first-best optimum can now be written as

$$(26a) \quad \frac{\mu_t^*}{\lambda_t^*} = \int_t^\infty CMRS_{s,gc}^* e^{-(R_{s-t} + \gamma(s-t))} ds = \int_t^\infty \frac{MRS_{s,gc}^*}{1 - \alpha_s^*} e^{-(R_{s-t} + \gamma(s-t))} ds = \rho,$$

whereas the corresponding policy rule implicit in the economy with uninternalized externalities becomes

$$(26b) \quad \frac{\mu_t^0}{\lambda_t^0} = \int_t^\infty MRS_{s,gc}^0 e^{-(R_{s-t} + \gamma(s-t))} ds = \rho.$$

Equations (26a) and (26b) are different variants of the Samuelson condition for a state variable public good. In a social optimum where all positional externalities are internalized, the marginal rate of substitution between the public good and private consumption is given by $CMRS_{gc}$, which recognizes that relative consumption is pure waste. In the economy with uninternalized externalities, on the other hand, agents behave as if others' consumption, $\bar{c}$, is exogenous, meaning that the marginal rate of substitution implicit in the first-order conditions is given by MRS_{gc} . In either case, however, it is optimal to Equate the weighted sum of marginal rates of substitution with the marginal rate of transformation, which explains Proposition 2. There is an important practical implication here: Regardless of whether we are in a first-best economy or a market equilibrium with positional externalities, public investment can be valued by the instantaneous marginal cost, such that there is no need to estimate future generations' marginal willingness to pay for current additions to the public good.

4.2. A More General Model with Multiple Externalities. The model examined in the previous subsection describes a conventional state-variable public good, where accumulation is determined by public sector gross investments and natural depreciation, respectively. Such a model lacks a direct relationship between the evolution of the public good and the choices made by private agents (other than through the trade-off implied by the resource constraint),

which makes it less useful as a description of how many environmental public goods evolve over time.²⁶ Yet, the treatment of state-variable public goods in genuine saving as described in Proposition 2 carries over to more general models, where choices made by the private sector directly affect the accumulation equation.

To see this, let us interpret g as an indicator of environmental quality dependent on public investments *and* private consumption and replace Equation (20) with the following more general accumulation equation:

$$(20') \quad \dot{g}_t = p(q_t, c_t) - \gamma g_t,$$

where $p(q, c)$ denotes a production function for environmental quality such that $p_q(q, c) > 0$ and $p_c(q, c) < 0$. Equation (20') means that the production of the public good depends on the public sector resource flow (as in the previous subsection) and on the private consumption (which works as an “externality production factor” here). The social first-order condition for private consumption now becomes

$$v_c(c_t, z_t, g_t, \bar{c}_t) + v_{\bar{c}}(c_t, z_t, g_t, \bar{c}_t) = \lambda_t - \mu_t p_c(q_t, c_t),$$

whereas the social first-order condition for the government’s instantaneous contribution to the public good changes to read

$$\lambda_t \rho = \mu_t p_q(q_t, c_t).$$

The other social first-order conditions remain as in Subsection 4.1, and the first-order condition for private consumption in the absence of any externality correction will still be given by $v_c(c_t, z_t, g_t, \bar{c}_t) = \lambda_t$ as long as the individual consumer treats $\bar{c}$ and g as exogenous (as before).

The welfare change measures given in Equations (25a) and (25b), respectively, will now change to read

$$(27a) \quad \dot{V}_t^* = \lambda_t^* \left[\dot{k}_t^* + \frac{\rho}{p_q(q_t^*, c_t^*)} \dot{g}_t^* \right]$$

$$(27b) \quad \dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \frac{\rho}{p_q(q_t^0, c_t^0)} \dot{g}_t^0 + \int_t^\infty \left\{ \rho \frac{p_c(q_s^0, c_s^0)}{p_q(q_s^0, c_s^0)} - \alpha_s^0 \right\} \exp(-R_{s-t}) \dot{c}_s^0 ds \right],$$

where we have used $c_t = \bar{c}_t$ for all t in Equation (27b). There are two differences compared with Equations (25a) and (25b). The social marginal cost of the public investment is now given by $\rho/p_q(q, c)$, since the marginal productivity of this investment is equal to $p_q(q, c)$ (instead of being equal to 1 as in Subsection 4.1). In addition, Equation (27b) implies that the value of the marginal externality in curly brackets now contains two parts: (i) the value of the marginal environmental externality (captured by the first term in curly brackets) and (ii) the value of the marginal positional externality (captured by the second). The former effect arises because the private consumption directly affects the accumulation equation for the public good, while the latter takes exactly the same form as before.

Note that the implications for genuine saving are the same as in the simpler model examined in Subsection 4.1. First, genuine saving constitutes an exact measure of welfare change under first-best conditions according to Equation (27a). Second, in the absence of any externality correction, Equation (27b) means that the conventional genuine saving overestimates the true welfare change in a growing economy (where consumption predominantly increases along the

²⁶ We are grateful to a referee for suggesting the extension discussed in this subsection.

general equilibrium path), since the two externalities are negative. Finally, the principle of valuing additions to the public good in genuine saving presented in Proposition 2 continues to hold, with the only modification being that the social marginal cost of the public investment (in consumption units) now reflects that the marginal productivity of this investment may differ from unity.²⁷

5. HETEROGENEITY, REDISTRIBUTION POLICY, AND GENUINE SAVING

In the preceding sections we consistently considered model economies where the individuals are identical, which constitutes the typical framework used in earlier literature on genuine saving. In this section, we extend the analysis to a model where consumers are heterogeneous in terms of productivity and productivity is private information. As explained in the introduction, this enables us to generalize the study of genuine saving to a second-best economy where the social planner redistributes and internalizes externalities subject to an incentive constraint.²⁸ Such an extension is, of course, not without costs in terms of greater complexity and less transparency. However, the benefits are also considerable in terms of greater realism. Our analysis recognizes that an indicator of social welfare may reflect a desired distribution of welfare among individuals and that this objective is costly to reach. In turn, the incentive constraint directly affects the marginal social value of positional externalities, which further motives this extension.

We make two simplifying assumptions. First, we do not consider public investments. Since Proposition 2 can be shown to apply also in the model set out below, little additional insight would be gained from studying such investments here as well. Second, to avoid unnecessary technical complications with many different consumer types, we use the two-type setting originally developed by Stern (1982) and Stiglitz (1982). The consumers differ in productivity, and the high-ability type (type 2) is more productive (earns a higher before-tax wage rate) than the low-ability type (type 1).²⁹ The population is constant and normalized to one for notational convenience, and there is a constant share, n^i , of individuals of type i , such that $\sum_i n^i = 1$.

5.1. Preferences and the Social Decision Problem. We allow for type-specific differences in preferences. The instantaneous utility function facing each individual of type i can then be written as

$$(28) \quad U_t^i = u^i(c_t^i, z_t^i, \Delta_t^i) = v^i(c_t^i, z_t^i, \bar{c}_t),$$

where $\Delta_t^i = c_t^i - \bar{c}_t$ denotes the relative consumption of an individual of type i . The functions $u^i(\cdot)$ and $v^i(\cdot)$ in Equation (28) have the same general properties as their counterparts in Equation (1). Also, and similarly to the benchmark model presented in Section 2, the relative consumption concerns in Equation (28) solely reflect comparisons with other people's current consumption, i.e., we abstract from the catching-up mechanism here as well. To begin with, the reference level is assumed to be the average consumption in the economy as a whole,

²⁷ None of these general conclusions depend on the assumption that the environmental externality is driven by the private consumption instead of by the production in the above example. In fact, if we were to replace c in the function $p(q, c)$ in Equation (20') with a measure of output, the only important difference would be that the externality part of Equation (27b) takes a slightly different form. The proper accounting price for additions to the public good would still take the same form as in Equations (27a) and (27b).

²⁸ To our knowledge, the paper by Aronsson et al. (2012) is the only earlier study dealing with genuine saving in a second-best economy with distortionary taxation. Their study is based on a representative-agent model (and as such does not consider redistribution policy) and assumes that individuals only care about their own absolute consumption and use of leisure (meaning that relative consumption is not dealt with at all).

²⁹ Aronsson (2010) uses a similar two-type model to derive a second-best analogue to the comprehensive net national product. However, his study neither addresses the implications of relative consumption concerns nor examines genuine saving, which are the main issues here.

$\bar{c}_t = n^1 c_t^1 + n^2 c_t^2$.³⁰ As before, each consumer is small relative to the overall economy and hence treats $\bar{c}_t$ as exogenous.

In a way similar to Sections 2 and 3, it is useful to be able to measure the extent to which relative consumption matters for individual utility. By using the function $u^i(\cdot)$ in Equation (28), we can define the degree of positionality of an individual of type i at time t such that

$$(29a) \quad \alpha_t^i = \frac{u_\Delta^i(c_t^i, z_t^i, \Delta_t^i)}{u_c^i(c_t^i, z_t^i, \Delta_t^i) + u_\Delta^i(c_t^i, z_t^i, \Delta_t^i)} \in (0, 1) \quad \text{for } i = 1, 2, \text{ all } t.$$

Therefore, it has the same interpretation as in the benchmark model except that α_t^i is type specific. We can then define the positional externality in terms of the average degree of positionality. The average degree of positionality measured over all individuals at time t can be written as (recall that the total population size is normalized to unity)

$$(29b) \quad \bar{\alpha}_t = n^1 \alpha_t^1 + n^2 \alpha_t^2 \in (0, 1),$$

which is interpretable as the sum of the marginal willingness to pay to avoid the positional consumption externality, measured per unit of consumption. Estimates of $\bar{\alpha}_t$ can be found in empirical literature on relative consumption concerns, as discussed above (see Subsection 2.1).

The social objective is given by a general social welfare function,

$$(30) \quad W_0 = \int_0^\infty \omega(U_t^1, U_t^2) e^{-\theta t} dt,$$

and the instantaneous social welfare function, $\omega(\cdot)$, is assumed to be differentiable and increasing in the instantaneous individual utilities. The resource constraint can now be written as

$$(31) \quad \dot{k}_t = f(\ell_t^1, \ell_t^2, k_t) - \sum_i n^i c_t^i,$$

where $\ell_t^i = n^i l_t^i$ is interpretable as the aggregate input of type i labor at time t . As before, the production function, $f(\ell_t^1, \ell_t^2, k_t)$, is characterized by constant returns to scale.

Following Aronsson and Johansson-Stenman (2010), we assume that the social planner (or government) can observe labor income and saving at the individual level, whereas ability is private information. We also assume that the social planner wants to redistribute from the high-ability to the low-ability type. To eliminate the incentive for the high-ability type to mimic the low-ability type (in order to gain from this redistribution profile), we impose a self-selection constraint such that each individual of the high-ability type weakly prefers the allocation intended for his/her type over the allocation intended for the low-ability type. Also, to simplify the analysis, we assume that the social planner commits to the resource allocation decided at time zero.³¹ One way to rationalize this assumption in our framework is to interpret the model in terms of a continuum of perfectly altruistic generations, i.e., dynasties (instead of single consumers with infinite lives),³² implying that the ability of agents living at any time

³⁰ This is the most common assumption in the literature dealing with optimal policy responses to relative consumption concerns. Although the definition of reference consumption at the individual level (i.e., whether it is based on the economy-wide mean value or reflects more narrow social reference groups) matters for public policy, it is not Equally important here, where the main purpose is to characterize an aggregate measure of social savings.

³¹ See Brett and Weymark (2008) for an analysis of optimal taxation without commitment based on the self-selection approach to optimal taxation.

³² We realize that this interpretation is not unproblematic, since each individual in a succession of generations faces his/her own objective and constraints; see Michel et al. (2006) for a discussion. However, our main concern here is to

t is not necessarily observable beforehand by the social planner. Therefore, the self-selection constraint is assumed to take the form

$$(32) \quad U_t^2 = v^2(c_t^2, z_t^2, \bar{c}_t) \geq v^2(c_t^1, \bar{l} - \phi_t l_t^1, \bar{c}_t) = \hat{U}_t^2$$

for all t . The left-hand side represents the utility of the high-ability type at time t and the right-hand side the utility of the mimicker (a high-ability type choosing the same income and saving as the low-ability type). The variable

$$\phi_t = w_t^1/w_t^2 = f_{\ell^1}(\ell_t^1, \ell_t^2, k_t)/f_{\ell^2}(\ell_t^1, \ell_t^2, k_t) = \phi(\ell_t^1, \ell_t^2, k_t) < 1$$

denotes the relative wage rate at time t , and $\phi_t l_t^1 < l_t^1$ is the hours of work the mimicking high-ability type needs to supply to reach the same labor income as the (mimicked) low-ability type.

5.2. Second-Best Optimum and Genuine Saving. The social optimum can be derived by choosing c_t^1 , l_t^1 , c_t^2 , and l_t^2 for all t to maximize the social welfare function given in Equation (30), subject to the resource constraint and self-selection constraint in Equations (31) and (32), respectively, and to initial and terminal conditions for the capital stock (as before). Also, the social planner recognizes that the reference consumption is endogenous and given by $\bar{c}_t = n^1 c_t^1 + n^2 c_t^2$.³³ The current value Hamiltonian at any time t is written as

$$(33) \quad H_t = \omega(U_t^1, U_t^2) + \lambda_t \dot{k}_t.$$

Adding the self-selection constraint gives the current value Lagrangian

$$(34) \quad L_t = H_t + \eta_t[U_t^2 - \hat{U}_t^2],$$

where η_t denotes the Lagrange multiplier. If written in terms of the reduced form utility formulation $v^j(\cdot)$ in Equation (28), and by using $\hat{v}_t^2 = v^2(c_t^1, \bar{l} - \phi_t l_t^1, \bar{c}_t)$ as a short notation for the instantaneous utility facing the mimicker at time t , the first-order conditions can be written as (in addition to Equations (31) and (32) and the initial and terminal conditions for the capital stock)

$$(35a) \quad \frac{\partial L}{\partial c^1} = \omega_{U^1} v_c^1 - \eta \hat{v}_c^2 - \lambda n^1 + \frac{\partial L}{\partial \bar{c}} n^1 = 0$$

$$(35b) \quad \frac{\partial L}{\partial l^1} = -\omega_{U^1} v_z^1 + \eta \hat{v}_z^2 \left[\phi + \frac{\partial \phi}{\partial l^1} l^1 \right] + \lambda n^1 w^1 = 0$$

$$(35c) \quad \frac{\partial L}{\partial c^2} = \omega_{U^2} v_c^2 + \eta v_c^2 - \lambda n^2 + \frac{\partial L}{\partial \bar{c}} n^2 = 0$$

examine the implications of the self-selection constraint for how positional externalities ought to be treated in savings-based measures of welfare change. Since the externality is atemporal in our model (i.e., a keeping-up-with-the-Joneses externality), the exact way in which successive generations interact is of no major importance for the qualitative results.

³³ We will not go into how such an allocation can be implemented by optimal nonlinear taxation. Here, the reader is referred to Aronsson and Johansson-Stenman (2010). Other literature on optimal taxation under relative consumption concerns includes Boskin and Sheshinski (1978), Oswald (1983), Corneo and Jeanne (1997, 2001), Ireland (2001), Dupor and Liu (2003), Wendner and Goulder (2008), and Aronsson and Johansson-Stenman (2014a). For important macroeconomic implications of relative consumption comparisons, see, e.g., Ljungqvist and Uhlig (2000) and Yamada (2008).

$$(35d) \quad \frac{\partial L}{\partial l^2} = -\omega_{U^2} v_z^2 + \eta \left[-v_z^2 + \hat{v}_z^2 \frac{\partial \phi}{\partial l^2} l^1 \right] + \lambda n^2 w^2 = 0$$

$$(35e) \quad \frac{d\lambda e^{-\theta t}}{dt} = -\frac{\partial L}{\partial k} e^{-\theta t} = - \left[\lambda r + \eta \hat{v}_z^2 \frac{\partial \phi}{\partial k} l^1 \right] e^{-\theta t}.$$

The optimal resource allocation also fulfills transversality conditions analogous to Equations (6d) and (6e). In Equations (35), we have suppressed the time indicator to avoid notational clutter (since all components refer to the same point in time), and subscripts attached to the instantaneous utility functions and social welfare function refer to partial derivatives (as before). The factor prices are determined by marginal products, i.e., $w^1 = f_{\ell^1}(\ell^1, \ell^2, k)$, $w^2 = f_{\ell^2}(\ell^1, \ell^2, k)$, and $r = f_k(\ell^1, \ell^2, k)$.

The final term on the right-hand side of Equations (35a) and (35c) reflects the positional consumption externality and measures the partial instantaneous welfare effect of increased reference consumption times the effect of each type's consumption on the reference measure. This partial welfare effect can be either positive or negative and is given as follows:

$$(36) \quad \frac{\partial L_t}{\partial \bar{c}_i} = \omega_{U_i^1} v_{i, \bar{c}_i}^1 + \omega_{U_i^2} v_{i, \bar{c}_i}^2 + \eta_t [v_{i, \bar{c}_i}^2 - \hat{v}_{i, \bar{c}_i}^2].$$

Although the first two terms on the right-hand side are negative, since $v_{i, \bar{c}_i}^j < 0$ for $i = 1, 2$ by the assumptions made earlier, the third term (which is proportional to the Lagrange multiplier of the self-selection constraint) can be either positive or negative. We will return to Equation (36) in greater detail below.

We are now ready to derive a measure of welfare change for the second-best economy set out above. The main differences compared with the model analyzed in Sections 2 and 3 are that (i) the individuals differ in productivity and preferences here, meaning also that they may differ in terms of relative consumption concerns (see Equation (29a) above), (ii) the social planner has an objective that may necessitate redistribution policy, and (iii) the social planner is unable to implement the first-best resource allocation (provided that the self-selection constraint binds). Despite these differences, we will also in this case use $*$ to denote social optimum (albeit a second-best optimum) such that

$$\{c_t^{1,*}, l_t^{1,*}, c_t^{2,*}, l_t^{2,*}, k_t^*, \lambda_t^*\} \quad \forall t$$

satisfy Equations (6d), (6e), (31), (32), and (35a)–(35e) as well as the initial and terminal conditions. The optimal value function at time t (based on the social welfare function) then becomes

$$(37) \quad W_t^* = \int_t^\infty \omega \left(v^1(c_s^{1,*}, z_s^{1,*}, \bar{c}_s^*), v^2(c_s^{2,*}, z_s^{2,*}, \bar{c}_s^*) \right) e^{-\theta(s-t)} ds.$$

We can then derive the following strong result regarding the change in social welfare over a short time interval, $(t, t + dt)$.

PROPOSITION 3. *If the resource allocation satisfies (6d), (6e), (31), (32), and (35a)–(35e), and irrespective of whether the self-selection constraint binds, genuine saving constitutes an exact measure of social welfare change such that*

$$(38) \quad \dot{W}_t^* = \lambda_t^* \dot{k}_t^*.$$

PROOF. See the Appendix.

Proposition 3 constitutes a remarkable result. It means that the procedure for measuring welfare change in an economy with identical individuals (as discussed in Section 3) carries over to the second-best economy analyzed here, where the agents differ in productivity and productivity is private information. The only difference is that the shadow price of capital is measured in a slightly different way (which is seen by comparing Equations (6c) and (35e)). The intuition behind Equation (38) is that the redistribution is optimal given the social welfare function and constraints, and the positional externality is internalized. As a consequence, there is no market failure or failure to reach the distributional objectives that may cause a discrepancy between the welfare change and the conventional measure of genuine saving.³⁴ This holds true irrespective of whether the resource allocation is first best or second best.

To take this discussion a bit further, note that the social welfare, as defined in Equation (37), is nondeclining (declining) over the time interval $(t, t + dt)$ if, and only if, $\lambda_t^* \dot{k}_t^* \geq 0$ (< 0). Consequently, nonnegative genuine saving can be interpreted as an indicator of local sustainable development for the same reason as in Subsection 3.1, with the only modification being that the social welfare function takes a different form here. Similarly, and again by analogy to the first-best model examined in Subsection 3.1, it is straightforward to show that negative genuine saving means that the instantaneous social welfare, $\omega(U^1, U^2)$, must eventually decline. Therefore, the qualitative results and interpretations are analogous to those derived and discussed under first-best conditions above. The strong implication is, of course, that the results derived from the model with identical individuals in Subsection 3.1 are more general than might be apparent at first thought.

5.3. Welfare Change and Positional Externalities. To be able to focus on the welfare contributions of uninternalized positional externalities without introducing any other discrepancy from the second-best optimum, we take the same type of shortcut to the imperfect economy as we did in Sections 2 and 3. More specifically, we assume that the social planner does not internalize the positional externality, although the redistribution among consumer types continues to be optimal (in this case conditional on the equilibrium path for $\bar{c}$) in terms of the social welfare function described above. Thus, instead of explicitly examining the decision problems faced by consumers and firms in an economy without externality correction and then characterizing the general equilibrium, we just note that this equilibrium would be equivalent to the special case of the model set out in Subsection 5.2 where the planner (erroneously) treats $\bar{c}_t$ as exogenous for all t . As a consequence, the allocation that this planner implements means that the private consumption, hours of work, and capital stock are chosen *as if* the consumer preference for relative consumption does not give rise to externalities.³⁵

In this case, the social planner's first-order conditions for l_t^1 and l_t^2 remain as in Equations (35b) and (35d), respectively, and the equation of motion for the shadow price remains as in Equation (35e), whereas the first-order conditions for c_t^1 and c_t^2 reduce to read

$$(39a) \quad \frac{\partial L}{\partial c^1} = \omega_{U^1} v_c^1 - \eta \hat{v}_c^2 - \lambda n^1 = 0,$$

³⁴ Aronsson and Löfgren (1999) show that if the distribution of resources among consumers is not optimal during a time interval, the current value Hamiltonian (which is decomposable into the sum of the comprehensive net national product and the consumer surplus) may for this reason fail as a welfare-level measure. In their study, a first-best resource allocation constitutes the reference case.

³⁵ It is straightforward to implement this resource allocation in a decentralized setting with nonlinear labor and capital income taxation. The "optimal" marginal labor income tax rates would then take the same general form as those derived by Stiglitz (1982) for an economy without relative concerns. An alternative approach to the decentralized economy is to assume away government intervention completely, in which case none of the social first-order conditions would be fulfilled. With the social welfare function in Equation (30), it would then be difficult to separate the welfare consequences of uninternalized externalities from those of a suboptimal distribution among individuals.

$$(39b) \quad \frac{\partial L}{\partial c^2} = \omega_{U^2} v_c^2 + \eta v_c^2 - \lambda n^2 = 0.$$

Now, let us return to the partial instantaneous welfare effect of increased reference consumption in Equation (36), which is no longer internalized by the social planner. By using $v_c^i = u_c^i + u_\Delta^i$ and $v_c^j = -u_\Delta^j$ together with Equation (29a), we have $v_c^j = -\alpha^j v_c^j$, which means that Equation (36) can be written as

$$\frac{\partial L_t}{\partial \bar{c}_t} = -\omega_{U^1} v_{t,c_t}^1 \alpha_t^1 - \omega_{U^2} v_{t,c_t}^2 \alpha_t^2 - \eta_t [v_{t,c_t}^2 \alpha_t^2 - \hat{v}_{t,c_t}^2 \hat{\alpha}_t^2],$$

where $\hat{\alpha}_t^2$ denotes the degree of positionality of the mimicker at time t . Solving Equation (39a) for $\omega_{U^1} v_c^1$ and Equation (39b) for $\omega_{U^2} v_c^2$ and substituting into the above expression gives

$$(40) \quad \frac{\partial L_t}{\partial \bar{c}_t} = -\lambda_t \bar{\alpha}_t + \eta_t \hat{v}_{t,c_t}^2 [\hat{\alpha}_t^2 - \alpha_t^1].$$

Equation (40) is analogous to a result derived by Aronsson and Johansson-Stenman (2008) and implies that the partial welfare effect of an increase in $\bar{c}_t$ can be decomposed into two parts. The first term on the right-hand side depends on the average degree of positionality and is interpretable as the pure efficiency cost of increased reference consumption at time t , ceteris paribus. This is so because $\bar{\alpha}_t$ reflects the sum of marginal willingness to pay to avoid the externality, measured over all consumers, and the multiplication by λ_t converts this number into utility units. The second component in Equation (40) depends on the difference in the degree of positionality between the mimicker and the low-ability type. Note that this component is proportional to the Lagrange multiplier of the selection constraint, which means that it vanishes in a first-best setting where the differences in productivity are observed by the social planner. If the mimicker is more positional than the low-ability type at time t such that $\hat{\alpha}_t^2 > \alpha_t^1$, an increase in $\bar{c}_t$ contributes to higher welfare by relaxing the self-selection constraint, in which case the sign of the right-hand side of Equation (40) is ambiguous. In other words, the pure efficiency cost of a higher $\bar{c}_t$ is accompanied by a welfare gain through this relaxation of the self-selection constraint. On the other hand, if $\hat{\alpha}_t^2 < \alpha_t^1$, i.e., if the low-ability type is more positional than the mimicker, the right-hand side of Equation (40) is unambiguously negative, since an increase in $\bar{c}_t$ in that case contributes to tighten the self-selection constraint.

To distinguish the resource allocation analyzed here from the second-best optimum in Subsection 5.2, we shall once again use the superscript 0 to denote the economy with uninternalized positional externalities, i.e.,

$$\{c_t^{1,0}, l_t^{1,0}, c_t^{2,0}, l_t^{2,0}, k_t^0, \lambda_t^0\} \quad \forall t.$$

The corresponding optimal value function becomes

$$(41) \quad W_t^0 = \int_t^\infty \omega (v^1(c_s^{1,0}, z_s^{1,0}, \bar{c}_s^0), v^2(c_s^{2,0}, z_s^{2,0}, \bar{c}_s^0)) e^{-\theta(s-t)} ds,$$

which measures the social welfare at time t .

For notational convenience, define the instantaneous welfare effect of an increase in $\bar{c}_t$ measured in consumption units (by dividing Equation (40) by the shadow price of capital) such that

$$(42) \quad \Lambda_t = -\bar{\alpha}_t + \frac{\eta_t \hat{v}_{t,c_t}^2}{\lambda_t} [\hat{\alpha}_t^2 - \alpha_t^1].$$

We can then derive the following result.

PROPOSITION 4. *If the economy satisfies (6d), (6e), (31), (32), (35b), (35d), (35e), (39a), and (39b), the measure of welfare change can be written as*

$$(43) \quad \dot{W}_t^0 = \lambda_t^0 \dot{k}_t^0 + \int_t^\infty \lambda_s^0 \Lambda_s^0 \dot{\bar{c}}_s^0 e^{-\theta(s-t)} ds.$$

PROOF. See the Appendix.

Proposition 4 shows how the conventional measure of genuine saving should be modified to reflect the change in welfare over the short time interval $(t, t + dt)$ if the otherwise second-best optimal resource allocation contains uninternalized positional externalities. As in the analogous measure of welfare change derived in the simpler model with identical individuals in Section 3, the second term on the right-hand side of Equation (43)—which adjusts the conventional genuine saving measure for these externalities—is forward looking. To be able to write Equation (43) in exactly the same format as its counterpart in Equation (13), i.e., as the product of the welfare change in consumption units and the social shadow price of capital, we would need an additional, and simplifying, assumption, namely, that the relative wage rate, $\phi_t = w_t^1/w_t^2 = f_{\ell^1}(\ell_t^1, \ell_t^2, k_t)/f_{\ell^2}(\ell_t^1, \ell_t^2, k_t)$, is independent of k_t for all t (which it is for standard constant returns to scale production functions such as the Cobb–Douglas or constant elasticity of substitution production functions). In this case, the shadow price of capital takes the simple form

$$\lambda_t = \lambda_0 \exp \left(- \int_0^t r_\tau d\tau + \theta t \right)$$

and Equation (43) can be rewritten as

$$(44) \quad \dot{W}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \int_t^\infty \Lambda_s^0 \exp(-R_{s-t}) \dot{\bar{c}}_s^0 ds \right].$$

The only important difference between Equations (13) and (44) is that the partial instantaneous welfare effect of increased reference consumption is no longer necessarily negative here, i.e., Λ_t can be either positive or negative, whereas it would be unambiguously negative in a full information setting where the self-selection constraint does not bind (in which case $\eta_t = 0$ and $\Lambda_t = -\bar{\alpha}_t < 0$ for all t). Thus, the second term on the right-hand side of Equation (44) reflects not only the change in the pure efficiency cost of the positional externality (as represented by the average degree of positionality). It also reflects the fact that an increase in this externality may either facilitate or hinder redistribution.

To be more specific, if the low-ability type is always more positional than the mimicker such that $\alpha_t^1 > \hat{\alpha}_t^2$ for all t , then $\Lambda_t < 0$ for all t and Equation (44) is interpretable in the same general way as Equation (13). However, if the mimicker is more positional than the low-ability type such that $\hat{\alpha}_t^2 > \alpha_t^1$ during a certain time interval $t \in (t_1, t_2)$ and if this difference in degree of positionality is sufficiently large, we cannot rule out that Λ_t is positive during this time interval. As a consequence, an increase in the reference measure $\bar{c}_t$ through increased consumption along the equilibrium path may actually be associated with a net social benefit (instead of cost), meaning that the conventional measure of genuine saving underestimates (instead of overestimates) the change in social welfare.

What determines differences in the degree of positionality between the mimicker and the low-ability type? In our (quite general) model where the utility functions are allowed to differ between types, these positionality differences may arise for a variety of reasons. To simplify the interpretation, consider the special case where all individuals share a common utility function such that the functions $u^i(\cdot)$ and $v^i(\cdot)$ are the same for both types. In this case, the only difference between the mimicker and the low-ability type is that the mimicker spends more time on leisure. Therefore, $\hat{\alpha}_t^2 > (<) \alpha_t^1$ if the degree of positionality increases (decreases) with the use of leisure. Although this information is not directly observable, it is (at least in principle) recoverable via econometric methods.

An implication of the above is also that the first term on the right-hand side of Equations (43) and (44), $\lambda_t \dot{k}_t$, is less informative here than in the context of Equation (13). More specifically, even if we were to assume that the economy is growing such that $\bar{c}_t$ increases along the equilibrium path, we can no longer use $\lambda_t \dot{k}_t < 0$ for a one-sided test of welfare decline and unsustainable development (as we did in Subsection 3.2) without making additional assumptions. This is so because $\lambda_t \dot{k}_t < 0$ does not imply that the right-hand side of Equation (44) is negative. Since the instantaneous welfare effect of increased reference consumption can be either positive or negative here, $\lambda_t \dot{k}_t < 0$ implies neither declining intertemporal welfare nor that the current instantaneous social welfare is unsustainable in a growing economy. The applicability of the one-sided test of welfare decline presupposes that the second term on the right-hand side of Equation (44) is negative. If $\bar{c}_t$ increases along the equilibrium path, a sufficient condition for this term to be negative is that

$$\Lambda_t = -\bar{\alpha}_t + \frac{\eta_t \hat{v}_{t,c_t}^2}{\lambda_t} [\hat{\alpha}_t^2 - \alpha_t^1] < 0 \quad \text{for all } t.$$

This inequality is satisfied if the low-ability type is more positional than the mimicker (as explained above) or if the difference in the degree of positionality between the mimicker and the low-ability type is small enough in absolute value relative to the average degree of positionality. In either case, the information requirements are more demanding here.

5.4. An Alternative Reference Measure. Although based on much earlier literature, the average consumption in the economy as a whole is not necessarily a realistic measure of reference consumption at the individual level. For instance, Veblen (1899) and others argue in favor of upward comparisons in the income or wealth distribution, such that the consumption among richer persons carries a larger weight (or even the whole weight) in the reference measure. Consider the following general measure:

$$(45) \quad \bar{c}_t = \beta c_t^1 + (1 - \beta) c_t^2 \quad \text{for } \beta \in [0, 1].$$

Two special cases of this general formulation are predominantly upward comparisons ($\beta < n^1$) and predominantly downward comparisons ($\beta > n^1$), although the latter does not strike us as very plausible.

If we were to replace the mean-value comparison analyzed above with Equation (45), the optimal policy response to these comparisons will also change.³⁶ This is easily understood by recognizing that $\beta \neq n^1$ implies that the two ability types differ in their marginal contribution to the externality. For instance, $\beta = 0$ ($\beta = 1$) means that all positional externalities are generated by the high-ability (low-ability) type.

However, the implications for welfare measurement of Equation (45) are qualitatively very similar to those following from the mean-value comparison analyzed in Subsections 5.2 and 5.3. First, if the economy represents a social optimum, genuine saving constitutes an exact measure of welfare change because all positional externalities are internalized. This conclusion holds

³⁶ See, e.g., Aronsson and Johansson-Stenman (2010) and Eckerstorfer and Wendner (2013).

irrespective of how the reference consumption level is determined. Second, if the positional externality remains uninternalized, it is straightforward to show that the partial instantaneous welfare effect of increased reference consumption is given by Equation (40) also when the reference consumption level is measured as in Equation (45). This means that the welfare change measure can still be written in the same format as Equation (44), i.e.,

$$\dot{W}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \int_t^\infty \Lambda_s^0 \exp(-R_{s-t}) \dot{c}_s^0 ds \right],$$

with the only modification being that the time derivative of the reference consumption is now given by $\dot{c}_t = \beta \dot{c}_t^1 + (1 - \beta) \dot{c}_t^2$ instead of $\dot{c}_t = n^1 \dot{c}_t^1 + n^2 \dot{c}_t^2$. Therefore, despite that the assumptions underlying the reference consumption level may have important implications for policy, the strong message here is that the principles of measuring welfare change remain as in Propositions 3 and 4.

6. NUMERICAL ILLUSTRATION

In previous sections, we have shown that conventional measures of genuine saving do not generally constitute valid measures of welfare change (unless the positional externality has become fully internalized), and we have also derived exact welfare change measures for economies with uninternalized positional externalities. Although this is important per se, it is also important to be able to say something about likely orders of magnitude and whether the discrepancy between the true welfare change (as given by the model) and the conventional genuine saving is likely to be similar between countries. In this section, we illustrate, first, that the discrepancies between measures of genuine saving and welfare change are indeed likely to be substantial, and, second, that these discrepancies may also differ greatly across countries.

Our numerical simulation example is based on World Bank data and serves to provide a rough estimate of the right-hand side of Equation (44), i.e., the genuine saving adjusted for uninternalized positional externalities. To be able to manage this task without a lengthy numerical extension of the article, we make four additional, simplifying assumptions: (i) All consumers share a common utility function where leisure is weakly separable from the other goods, (ii) the positionality degree is constant over time for all consumers, (iii) the interest rate is constant over time, and (iv) the average consumption in the economy can be approximated by a consumption function with a constant growth rate.

Assumption (i) means that the mimicker and the mimicked agent are equally positional, i.e., $\hat{\alpha}_t^2 = \alpha_t^1$, implying, in turn, that the second term on the right-hand side of Equation (42) vanishes.³⁷ Taken together, assumptions (i) and (ii) imply $\Lambda_t = -\bar{\alpha}_t = -\bar{\alpha}$ for all t ,³⁸ while assumption (iii) implies $r_t = r$ for all t . Finally, assumption (iv) is consistent with models used in literature on endogenous growth and implies that we can write the average consumption at any time $s > t$ as

$$(46) \quad \bar{c}_s = \bar{c}_t e^{\kappa(s-t)},$$

where κ is the growth rate.

Our aim is now to operationalize the welfare change measure given in Equation (44). Thus, the calculations below presuppose that the positional externality is uninternalized, although

³⁷ To our knowledge, there is no empirical evidence available here, which suggests that the relationship between the degree of consumption positionality and the use of leisure is a relevant issue for future research. Thus, lack of empirical evidence on this relationship motivates us to assume leisure separability.

³⁸ Assumption (ii) can alternatively be replaced with a functional form assumption on the instantaneous utility function, such that the degree of positionality is constant over time; see, e.g., Ljungqvist and Uhlig (2000).

all other aspects of public policy are optimally chosen conditional on the level of reference consumption. By using assumptions (i)–(iv), Equation (44) can be written in the following convenient way:³⁹

$$(47) \quad \begin{aligned} \dot{W}_{t\lambda_t} \frac{1}{\lambda_t} &= \dot{k}_t + \int_t^\infty \Lambda_s \exp(-R_{s-t}) \dot{\bar{c}}_s ds \\ &= \underbrace{\dot{k}_t}_{GS} - \underbrace{\frac{\bar{\alpha}\kappa}{r-\kappa} \bar{c}_t}_{ME}. \end{aligned}$$

The left-hand side of Equation (47) is a money-metrics version of the welfare change (through division by the marginal utility of consumption), whereas the right-hand side decomposes this into the conventional genuine saving (*GS*) and the value of the change in the marginal positional externality (*ME*). In particular, note that the higher the average degree of positionality, $\bar{\alpha}$, or the higher the consumption growth rate, κ , ceteris paribus, the larger the marginal positional externality will be, and the more the conventional measure of genuine saving will overestimate the welfare change.

To illustrate Equation (47), we use country level data from the World Bank Open Data (<http://data.worldbank.org/>) on genuine saving, referred to as the adjusted net saving. The adjusted net saving is defined by the World Bank as the net national savings plus education expenditure minus an estimated value of energy depletion, mineral depletion, forest depletion, carbon dioxide emissions, and particulate emissions damage. We use per capita variables to avoid possible welfare consequences of population size, and, to avoid yearly fluctuations, our calculations are based on mean values for the genuine saving per capita and aggregate private consumption per capita over the period 2010–14 expressed in 2014 USD.⁴⁰

We will thus use the World Bank measure of adjusted net saving as a conventional indicator of genuine saving that does not reflect any positional externalities (i.e., our *GS* above). We will then examine how this measure ought to be modified in response to uninternalized positional externalities through the second term on the right-hand side of Equation (47). The welfare change given by Equation (47), i.e., $GS - ME$, will then correspondingly be referred to as the *modified genuine saving* to distinguish it from the conventional genuine saving (given by *GS*). The magnitude of this modification will, of course, strongly depend on the assumptions on the variables involved, i.e., (in addition to the private consumption per capita) on $\bar{\alpha}$, r , and κ . Furthermore, for given levels of $\bar{\alpha}$, r , and κ , the change in the marginal positional externality based on Equation (47) is proportional to the average private consumption (the measure of reference consumption used here), meaning that the level of private consumption per capita drives the discrepancy between the two measures of genuine saving. In Figure 1, we plot both the conventional measure of genuine saving and the modified measure that takes the positional externalities into account against private consumption, based on assumptions that are far from extreme, for all OECD countries⁴¹ plus China and India.

The numbers reported in Figure 1 should be seen for what they are, i.e., the outcome of a computational example and not the result of empirical research. With this reservation in mind, we would like to make three broad observations from the figure. First, the modification of the

³⁹ Since assumption (i) eliminates any difference in the degree of positionality between the mimicker and the low-ability type, the second line of Equation (47) would also follow from the welfare change measure given in Equation (13).

⁴⁰ The World Bank reports both adjusted net saving and private consumption in current USD. We divide by the yearly population size for each country and use the U.S. GDP deflator to convert all figures to 2014 USD per capita before we take averages. For some countries we only have data for four out of the five years; in these cases we take the average over these four years. We do not include countries where we have data for less than four of these years. All data are from the database World Development Indicators of the World Bank: <http://data.worldbank.org/data-catalog/world-development-indicators>.

⁴¹ Except Iceland for which we lack measures of genuine savings for at least four out of the five years.

NOTE: The Measures of Modified Genuine Saving are Based on The Following Common for all and Fixed Assumptions: The Average Degree of Positionality Equals 0.3, the Real Annual Interest Rate is 2%, and the Real Annual Consumption Growth Rate is 1%. the Numbers Reflect Averages 2010–14.

FIGURE 1

CONVENTIONAL AND MODIFIED GENUINE SAVING PER CAPITA PLOTTED AGAINST THE PRIVATE CONSUMPTION PER CAPITA FOR ALL OECD COUNTRIES, PLUS CHINA AND INDIA (IN 2014 CURRENCY)

TABLE 1
PRIVATE CONSUMPTION AND CONVENTIONAL AND MODIFIED GENUINE SAVING PER CAPITA FOR ALL OECD COUNTRIES, PLUS CHINA
AND INDIA (IN 1000 2014 USD)

	Private Consumption	Conventional Genuine Saving	Modified Genuine Saving		
			Different Parameter Assumptions		
			$\bar{\alpha} = 0.2$ $r = 0.02$ $\kappa = 0.01$	$\bar{\alpha} = 0.3$ $r = 0.02$ $\kappa = 0.01$	$\bar{\alpha} = 0.3$ $r = 0.03$ $\kappa = 0.02$
Australia (AUS)	35.13	5.48	-1.55	-5.06	-15.60
Austria (AUT)	27.51	6.54	1.03	-1.72	-9.97
Belgium (BEL)	24.77	5.02	0.06	-2.41	-9.84
Canada (CAN)	29.35	3.25	-2.62	-5.56	-14.36
Chile (CHL)	9.36	0.68	-1.19	-2.13	-4.94
China (CHN)	2.29	2.13	1.67	1.44	0.75
Czech Rep. (CZE)	10.24	0.96	-1.09	-2.11	-5.19
Denmark (DNK)	29.78	8.52	2.56	-0.42	-9.35
Estonia (EST)	9.36	2.57	0.69	-0.24	-3.05
Finland (FIN)	27.40	4.07	-1.41	-4.15	-12.37
France (FRA)	24.29	3.10	-1.76	-4.19	-11.48
Germany (DEU)	25.78	5.99	0.84	-1.74	-9.48
Greece (GRC)	17.24	-2.06	-5.51	-7.23	-12.40
Hungary (HUN)	7.29	1.05	-0.40	-1.13	-3.32
India (IND)	0.89	0.30	0.12	0.03	-0.23
Ireland (IRL)	24.13	5.22	0.40	-2.02	-9.26
Israel (ISR)	19.83	4.84	0.87	-1.11	-7.06
Italy (ITA)	22.80	1.15	-3.41	-5.69	-12.53
Japan (JPN)	26.34	1.70	-3.57	-6.20	-14.10
Korea (KOR)	13.08	4.93	2.31	1.00	-2.92
Luxembourg (LUX)	36.74	11.34	3.99	0.31	-10.71
Mexico (MEX)	6.81	0.76	-0.60	-1.29	-3.33
Netherlands (NLD)	23.83	8.77	4.00	1.62	-5.53
Norway (NOR)	23.40	4.16	-0.52	-2.86	-9.88
New Zealand (NZL)	41.10	19.50	11.28	7.17	-5.16
Poland (POL)	8.56	1.27	-0.44	-1.30	-3.87
Portugal (PRT)	15.00	0.07	-2.93	-4.43	-8.93
Slovakia (SVK)	10.46	0.44	-1.66	-2.70	-5.84
Slovenia (SVN)	13.54	1.79	-0.91	-2.27	-6.33
Spain (ESP)	18.05	1.92	-1.67	-3.49	-8.90
Sweden (SWE)	27.64	11.38	5.85	3.09	-5.20
Switzerland (CHE)	46.61	16.16	6.83	2.17	-11.81
Turkey (TUR)	7.71	1.07	-0.47	-1.24	-3.56
UK (GBR)	27.99	1.69	-3.91	-6.71	-15.11
USA (USA)	36.31	2.61	-4.66	-8.29	-19.18

NOTE: The numbers reflect averages 2010–14.

genuine saving measure due to the change in the positional externality is likely to be substantial. Note that this is so despite our relatively conservative estimates of the average degree of positionality, growth rate, and interest rate. Indeed, as shown in Table 1, even based on the low estimate of the average degree of positionality of 0.2, with the same assumption on the growth and interest rates as in Figure 1, we still obtain substantial differences between the conventional and modified measures of genuine saving. This suggests that the conventional measure of genuine saving may largely overestimate the welfare change. Second, this discrepancy is far from similar across the countries. The reason is that the change in the marginal positional externality depends on the average private consumption (our measure of reference consumption), which varies greatly. As we can see from Figure 1, there is a positive relationship between the conventional genuine saving and the private consumption in the data, although there is no corresponding relationship between the modified genuine saving and the private consumption,

given the assumptions made above. These relationships are illustrated by the simple bivariate ordinary least squares regression lines in the figure. This, of course, also implies that the ranking between countries will, in general, be different depending on whether it is based on the conventional or modified measure of genuine saving. Third, the calculations also suggest that the conventional measure of genuine saving may give incorrect qualitative information as to whether the development is locally sustainable. In fact, the modified genuine saving is estimated to be negative for most of these countries during 2010–14, despite that the conventional (World Bank) measure of genuine saving is typically positive.

The results reported in Table 1, based on three different sets of parameters for the estimate of the modified genuine saving, reinforce all three observations discussed above. Yet, it is necessary to exercise caution in the interpretation of the results, especially with regard to the sign of the modified genuine saving. To be able to estimate the right-hand side of Equation (47), we have implicitly assumed that the World Bank numbers for genuine saving reflect all sources of intertemporal welfare change other than the positional externality. Thus, we have assumed away technical progress and possibly also other factors that work in the direction of higher welfare over time. This suggests that the actual welfare change in a world with technical progress may be positive even when our modified genuine saving measure is negative, as reported above. As a consequence, the extent to which the conventional measures of genuine saving convey a false message of local sustainable development may correspondingly be exaggerated. Instead, the take-home message of this example is that conventional measures of genuine saving (which are calculated as if the economy has reached a social optimum) may seriously overestimate the intertemporal welfare change when consumers engage in social comparisons, a conclusion that holds also in a world with technical progress.

With these qualifications in mind, we end this section by relating Equation (47) to country characteristics through econometric analysis. In the three regressions presented below, the dependent variables are, in turn, the conventional genuine saving (GS), the modified genuine saving ($GS - ME$), and the value of the change in the marginal positional externality given by the difference between the modified and conventional genuine saving ($-ME$) for all 122 countries for which the World Bank has calculated the conventional genuine savings measure. The explanatory variables are all from the same online World Bank data set and are based on five-year averages in the same way as described above (except for geographical indicators). The regressions are presented in Table 2.

The results in Table 2 solely reflect correlations and are not interpretable in terms of causal relationships. From the first and third columns, we can immediately see that an increase in GDP per capita (a commonly used indicator of average material standard of living) is associated with a higher level of conventional genuine saving and a larger (negative) marginal positional externality,⁴² respectively. Similarly, since these two effects influence the modified genuine saving in opposite directions, we can see from the second column that the GDP per capita is not significantly associated with the modified genuine saving. These results suggest that countries with a high material standard of living tend to have a higher level of conventional genuine saving and generate more positional externalities per capita than other countries, *ceteris paribus*. Having controlled for differences in GDP per capita, as well as other socioeconomic variables, most effects of geographical location are not statistically significant at conventional levels, except that the positional externality seems to be larger in North America than in Sub-Saharan Africa.

Somewhat surprisingly, we can also observe that large natural resource rents, defined as the sum of oil rents, natural gas rents, coal rents, mineral rents, and forest rents, are associated with larger (not smaller) genuine savings. One reason could be that gross savings of oil-rich countries are often very large; for example, the gross saving rates of both Kuwait and Qatar are above 50% of GDP. Yet, this result may also indicate the difficulties of measuring genuine saving accurately. Note that higher natural resource rents are also associated with smaller positional

⁴² Note that the marginal positional externality is always negative here according to Equation (47).

TABLE 2
OLS REGRESSION OF CONVENTIONAL AND MODIFIED GENUINE SAVING, AS WELL AS THE DIFFERENCE BETWEEN THEM, BASED ON 122 COUNTRIES FROM THE WORLD BANK OPEN DATA (STANDARD ERRORS IN PARENTHESES)

	Conventional Genuine Saving per Capita (1000 2014 USD)	Modified Genuine Saving per Capita ^a (1000 2014 USD)	Difference between Modified and Conventional Genuine Saving per Capita (1000 2014 USD)
GDP per capita (1000 2014 USD)	0.15*** (0.011)	0.022 (0.014)	-0.13*** (0.005)
Population density (individuals/km ²)	0.16*** (0.02)	0.20*** (0.03)	0.034*** (0.011)
Urbanization (% who live in urban areas)	0.001 (0.011)	-0.021 (0.014)	-0.022*** (0.006)
Natural resource rents (1000 2014 USD/capita)	0.61*** (0.06)	0.76*** (0.08)	0.15*** (0.03)
Carbon dioxide emission (100 metric tons per capita) ^b	-0.30*** (0.05)	-0.24*** (0.07)	0.057*** (0.028)
Percent females in Parliament	0.019 (0.014)	0.012 (0.018)	-0.007 (0.007)
East Asia and Pacific ^c	0.36 (0.54)	0.03 (0.70)	-0.33 (0.28)
Europe and Central Asia	0.16 (0.47)	-0.15 (0.61)	-0.31 (0.24)
Latin America & Caribbean	-0.30 (0.54)	-0.18 (0.70)	0.12 (0.28)
Middle East & North Africa	-0.46 (0.68)	-0.75 (0.87)	-0.11 (0.34)
North America	-1.44 (1.24)	-4.36*** (1.59)	-2.92*** (0.63)
South Asia	-0.30 (0.67)	-0.44 (0.86)	-0.14 (0.34)
R ²	0.89	0.70	0.95

NOTES: ** and *** represent significance at the 5% and 1% levels, respectively.

^aBased on parameters $\alpha = 0.3$, $r = 0.02$, and $\kappa = 0.01$.

^bAverage 2009–11 instead of 2010–14 as for all other variables.

^cSub-Saharan Africa constitutes the reference level.

externalities, which in our model can largely be explained by a stronger correlation between resource rents and GDP than between resource rents and private consumption. Countries with high population densities or low CO₂ emissions tend to have higher genuine saving and smaller positional externalities than other countries, whereas increased urbanization is associated with a larger positional externality, *ceteris paribus*. No other parameters are significant at conventional levels.

In summary, and irrespective of whether the conventional and modified genuine saving share the same sign, the intuition for the qualitative discrepancy between them as indicated in Figure 1 and Table 1 is that we are in a growing economy where the average consumption increases as time passes. As a consequence, the marginal positional externality is negative and will thus counteract the welfare gain that capital formation (broadly defined) would otherwise give rise to. This suggests that the conventional genuine saving does not in general provide a good estimate of the intertemporal welfare change in economies with uninternalized positional externalities. Table 2, finally, illustrates how the two measures of genuine saving and the change in the marginal positional externality correlate with some observable country characteristics.

7. DISCUSSION AND CONCLUSION

This article deals with the measurement of welfare change by developing an analogue to genuine saving in an economy where the consumers are positional. Recent empirical evidence

shows that people derive utility from their own consumption relative to that of referent others, and also that the corresponding negative externalities might be substantial, implying that measures of social savings ought to be modified accordingly. This is further emphasized by the role that genuine saving currently plays as an indicator of local sustainable development.

In the context of first-best economies with homogeneous consumers, earlier research shows that the genuine saving is an exact indicator of welfare change and is also interpretable as an exact indicator of local sustainable development. We show that this result carries over to a second-best economy with heterogeneous consumers based on asymmetric information about individual productivity and a binding self-selection constraint in the sense that the change in social welfare (measured by a general social welfare function) over a short time interval is exactly equal to the genuine saving.

However, in the arguably more interesting case where the positional externality has not become fully internalized, the equality between welfare change and genuine saving no longer applies, and we show how this discrepancy depends on the degrees of positionality (i.e., measures used in empirical literature to indicate the extent to which relative consumption matters for individual well-being). In a model with identical individuals, and if the consumption increases along the general equilibrium path (which the ideas of a positional arms race may suggest), uninternalized positional externalities typically imply that the genuine saving overestimates the change in social welfare. Consequently, negative genuine saving can still be used as an indicator of local unsustainable development; i.e., negative genuine saving implies that the social welfare declines. These qualitative results also carry over to a case with endogenous population growth as well as to the case where the social objective does not reflect the individual preference for relative consumption.

On the other hand, in an economy with productivity differences and asymmetric information, the conventional measure of genuine saving is less informative. Indeed, if the positional externalities are not internalized, we can in this case no longer use negative genuine saving as an indicator of local unsustainable development even if the measure of reference consumption increases along the general equilibrium path.

We also show how the value of public investment affects genuine saving and derive a simple accounting price that applies irrespective of whether or not the positional externalities have become internalized. Regardless of whether we are in a first-best economy or market equilibrium with positional externalities, a public investment can be valued by the instantaneous marginal cost. This means that there is no need to estimate future generations' marginal willingness to pay for current additions to the public good. Moreover, essentially the same pricing rule continues to hold in the more general case with multiple externalities.

Finally, we demonstrate in a numerical exercise based on data for a large number of countries that conventional measures of genuine saving (that do not reflect uninternalized positional externalities) are likely to largely overestimate the true welfare change. As a consequence, such measures may also incorrectly indicate welfare improvement and thus that the development is locally sustainable.

Future research may take several possible directions. First, recent empirical evidence shows that individuals do not only compare their own consumption with that of other domestic residents, they also engage in between-country comparisons (Becchetti et al., 2013). This suggests to us that a multicountry framework might be useful in future research on genuine saving. Second, although empirical evidence shows that people are concerned with their relative consumption, people are also found to have an aversion against too much inequality. This suggests that it might be relevant to extend the analysis of genuine saving by allowing also for other types of social comparisons. Finally, an interesting extension of the numerical analysis would be to consider a fully parameterized model. The obvious advantage of such a framework—compared with our approach of relying on World Bank numbers for the conventional genuine saving—is that it allows us to calculate both the conventional genuine saving and the positional externality using the same model. As a result, we would also be able to avoid the approximation errors that are most likely present in the World Bank numbers (e.g., due to the way in which investments in

human capital are measured by the World Bank). Yet, since this would clearly require a paper of its own, it is left for future research.

APPENDIX

A. *The Benchmark Model.* The value function is given by

$$V_t = \int_t^{\infty} v(c_s, z_s, \bar{c}_s) e^{-\theta(s-t)} ds.$$

Differentiating with respect to t gives

$$(A.1) \quad \dot{V}_t = \theta V_t - v(c_t, z_t, \bar{c}_t).$$

To write Equation (A.1) in the format of Equation (10) or (13), we must find an expression for θV_t .

A.1. *Proof of Observation 1.* It is convenient to start by using the present value Hamiltonian

$$(A.2) \quad H_t^p = v(c_t, z_t, \bar{c}_t) e^{-\theta t} + \lambda_t^p [f(l_t, k_t) - c_t],$$

where $\lambda_t^p = \lambda_t e^{-\theta t}$ is the present value costate variable. Differentiating the present value Hamiltonian with respect to time and using $\bar{c}_t = c_t$ gives

$$(A.3) \quad \frac{dH_t^p}{dt} = -\theta v(c_t, z_t, \bar{c}_t) e^{-\theta t} + \frac{\partial H_t^p}{\partial c_t} \dot{c}_t + \frac{\partial H_t^p}{\partial l_t} \dot{l}_t + \frac{\partial H_t^p}{\partial k_t} \dot{k}_t + \frac{\partial H_t^p}{\partial \lambda_t^p} \dot{\lambda}_t^p.$$

The first term on the right-hand side is the direct effect of time through the utility discount factor, whereas the remaining four terms represent indirect effects of time through the control, state, and costate variables, respectively. Equations (5), (6a), (6b), and (6c) imply $\partial H_t^p / \partial c_t = \partial H_t^p / \partial l_t = 0$, $\dot{\lambda}_t^p = -\partial H_t^p / \partial k_t$, and $\partial H_t^p / \partial \lambda_t^p = \dot{k}_t$, and Equation (A.3) reduces to

$$(A.4) \quad \frac{dH_t^p}{dt} = -\theta v(c_t, z_t, \bar{c}_t) e^{-\theta t}.$$

Solving Equation (A.4) forward subject to the transversality condition $\lim_{t \rightarrow \infty} H_t^p = 0$ implies

$$(A.5) \quad \theta \int_t^{\infty} v(c_s, z_s, \bar{c}_s) e^{-\theta s} ds = H_t^p.$$

Multiplying both sides of Equation (A.5) by $e^{\theta t}$ gives Weitzman's (1976) welfare measure $\theta V_t = H_t$. Substituting into Equation (A.1) and using $H_t = v(c_t, z_t, \bar{c}_t) + \lambda_t \dot{k}_t$ gives Equation (10). ■

A.2. *Proof of Proposition 1.* Differentiating Equation (A.2) totally with respect to time, while treating $\bar{c}_t$ as a separate variable, we can rewrite Equation (A.3) as

$$(A.6) \quad \frac{dH_t^p}{dt} = -\theta v(c_t, z_t, \bar{c}_t) e^{-\theta t} + \frac{\partial H_t^p}{\partial c_t} \dot{c}_t + \frac{\partial H_t^p}{\partial l_t} \dot{l}_t + \frac{\partial H_t^p}{\partial k_t} \dot{k}_t + \frac{\partial H_t^p}{\partial \lambda_t^p} \dot{\lambda}_t^p + \frac{\partial H_t^p}{\partial \bar{c}_t} \dot{\bar{c}}_t.$$

Then, by using Equations (6b), (6c), and (7), Equation (A.6) simplifies to

$$(A.7) \quad \frac{dH_t^p}{dt} = -\theta v(c_t, z_t, \bar{c}_t) e^{-\theta t} - \lambda_t e^{-\theta t} \alpha_t \dot{\bar{c}}_t,$$

where we have used $\partial H_t^p / \partial \bar{c}_t = v_{\bar{c}} e^{-\theta t} = -\alpha_t v_c e^{-\theta t} = -\alpha_t \lambda_t^p = -\alpha_t \lambda_t e^{-\theta t}$. Solving Equation (A.7) forward subject to $\lim_{t \rightarrow \infty} H_t^p = 0$ and multiplying by $e^{\theta t}$ to convert into current value gives

$$(A.8) \quad \theta V_t = H_t - \int_t^\infty \lambda_s \alpha_s \dot{\bar{c}}_s e^{-\theta(s-t)} ds.$$

Substitute Equation (A.8) into Equation (A.1) and use $H_t = v(c_t, z_t, \bar{c}_t) + \lambda_t \dot{k}_t$,

$$(A.9) \quad \dot{V}_t = \lambda_t \dot{k}_t - \int_t^\infty \lambda_s \alpha_s \dot{\bar{c}}_s e^{-\theta(s-t)} ds.$$

Using $\lambda_s / \lambda_t = \exp(-\int_t^s r_\tau d\tau + \theta(s-t))$ and substituting into Equation (A.9) gives Equation (13). ■

B. The Model with Heterogeneous Consumers. The social welfare function is given by

$$W_t = \int_t^\infty \omega(v^1(c_s^1, z_s^1, \bar{c}_s), v^2(c_s^2, z_s^2, \bar{c}_s)) e^{-\theta(s-t)} ds.$$

The welfare change measure can then be written as

$$(A.10) \quad \dot{W}_t = \theta W_t - \omega(v^1(c_t^1, z_t^1, \bar{c}_t), v^2(c_t^2, z_t^2, \bar{c}_t)).$$

B.1. Proof of Proposition 3. The present value Lagrangian is given by

$$(A.11) \quad L_t^p = \omega(U_t^1, U_t^2) e^{-\theta t} + \lambda_t^p \dot{k}_t + \eta_t^p [U_t^2 - \hat{U}_t^2],$$

where $\lambda_t^p = \lambda_t e^{-\theta t}$ (as before), and $\eta_t^p = \eta_t e^{-\theta t}$ denotes the present value Lagrange multiplier associated with the self-selection constraint. Differentiating Equation (A.11) with respect to time and using that the first-order conditions in Equations (35a)–(35e) are fulfilled gives

$$(A.12) \quad \frac{dL_t^p}{dt} = -\theta \omega(v^1(c_t^1, z_t^1, \bar{c}_t), v^2(c_t^2, z_t^2, \bar{c}_t)) e^{-\theta t}.$$

By solving Equation (A.12) in the same way as we solved Equation (A.4) above and using that $L_t^p = H_t^p$, the proof follows by analogy to the proof of Observation 1. ■

B.2. Proof of Proposition 4. Differentiating the present value Lagrangian in Equation (A.11) with respect to time while using the first-order conditions in Equations (35c), (35d), (35e), (39a), and (39b) gives

$$(A.13) \quad \frac{dL_t^p}{dt} = -\theta \omega(v^1(c_t^1, z_t^1, \bar{c}_t), v^2(c_t^2, z_t^2, \bar{c}_t)) e^{-\theta t} + \lambda_t e^{-\theta t} \Lambda_t \dot{\bar{c}}_t,$$

where we have used $\partial L_t^p / \partial \bar{c}_t = \lambda_t^p \Lambda_t = \lambda_t e^{-\theta t} \Lambda_t$. Solving Equation (A.13) forward subject to the transversality condition $\lim_{t \rightarrow \infty} H_t^p = 0$, using $L_t^p = H_t^p$, and multiplying by $e^{\theta t}$ to convert into current value terms gives

$$(A.14) \quad \theta \int_t^\infty \omega(v^1(c_s^1, z_s^1, \bar{c}_s), v^2(c_s^2, z_s^2, \bar{c}_s)) e^{-\theta(s-t)} ds = H_t + \int_t^\infty \lambda_s \Lambda_s \dot{\bar{c}}_s e^{-\theta(s-t)} ds.$$

Substituting into Equation (A.10) and using $H_t = \omega(U_t^1, U_t^2) + \lambda_t \dot{k}_t$ gives Equation (43). ■

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