On Mathematical Reasoning
- being told or finding out

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Abstract

School-mathematics has been shown to mainly comprise rote-learning of procedures where the considerations of intrinsic mathematical properties are scarce. At the same time theories and syllabi emphasize competencies like problem solving and reasoning. This thesis will therefore concern how task design can influence the reasoning that students apply when solving tasks, and how the reasoning during practice is associated to students' results, cognitive capacity, and brain activity. In studies 1-3, we examine the efficiency of different types of reasoning (i.e., algorithmic reasoning (AR) or creative mathematically founded reasoning (CMR)) in between-groups designs. We use mathematics grade, gender, and cognitive capacity as matching variables to get similar groups. We let the groups practice 14 different solution methods with tasks designed to promote either AR or CMR, and after one week the students are tested on the practiced solution methods. In study 3 the students did the test in and fMRI-scanner to study if the differing practice would yield any lasting differences in brain activation. Study 4 had a different approach and focused details in students' reasoning when working on teacher constructed tasks in an ordinary classroom environment. Here we utilized audio-recordings of students' solving tasks, together with interviews with teachers and students to unravel the reasoning sequences that students embark on. The turning points where the students switch subtask and the reasoning between these points were characterized and visualized. The behavioral results suggest that CMR is more efficient than AR, and also less dependent on cognitive capacity during the test. The latter is confirmed by fMRI, which showed that AR had higher activation than CMR in areas connected to memory retrieval and working memory. The behavioral result also suggested that CMR is more beneficial for cognitively less proficient students than for the high achievers. Also, task design is essential for both students' choice of reasoning and task progression. The findings suggest that: 1) since CMR is more efficient than AR, students need to encounter more CMR, both during task solving and in teacher presentation, 2) cognitive capacity is important but depending on task design, cognitive strain will be more or less high during test situations, 3) although AR-tasks does not prohibit the use of CMR they make it less likely to occur. Since CMR-tasks can emphasize important mathematical properties, are more efficient than AR-tasks, and more beneficial for less cognitively proficient students, promoting CMR can be essential if we want students to become mathematically literate.

Keywords

mathematics education, creative reasoning, reasoning, cognitive proficiency, fMRI

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