The Role of Gold in an Investment Portfolio

An empirical study on diversification benefits of gold from the perspective of Swedish investors

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Acknowledgments

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Abstract

Human interaction with gold can be traced far back in history, and throughout history, the metal has been both worshipped and fought for. People almost intuitively place a high value on this yellow metal and gold has always had a special place in the human heart. Nonetheless, the question many irresolute investors seek the answer to today is whether gold deserves a special place in their investment portfolios. The main purpose of this quantitative study was to determine whether gold is an appropriate diversifier for Swedish investors, and to find the optimal weight of gold in a Swedish equity portfolio. Corresponding properties of other precious metals silver and platinum were also investigated for comparative purposes. One of the reasons for augmented interest in investing in gold is the perceived risk in the economy. The theoretical framework for the study was Modern Portfolio Theory (MPT). The insight of MPT is that efficient diversification handles the risk better than individual assets. Risk management is especially crucial in an era of heightened economic, financial and political uncertainty as today. At the same time, rising correlation among traditional diversifiers make diversification more difficult. Gold, influenced by its history as a currency, has often taken the role as an inflation hedge and a portfolio stabilizer during turbulent financial markets. Inflation hedge assets like gold should be negatively correlated with the market and should give the best diversification benefits in a portfolio. This indicates that gold may be an appropriate diversifier in an equity portfolio. The study took the perspective of Swedish investors, and a Swedish equity index was therefore used as proxy for a well-diversified portfolio. Registry data for past prices of assets over a period of 47 years were obtained via a study published on the official website of the Swedish central bank and Thomson Reuters Datastream. Excess returns were then calculated and processed to obtain descriptive and inferential statistics. The optimal weights of gold and the other precious metals in an investment portfolio were calculated under the optimization framework of maximization of the Sharpe ratio where reward to volatility is highest. The calculations were performed for eight different holding periods. Results show near 0, or weak positive correlations between Swedish domestic equity and gold during the examined periods. On stand-alone basis, gold is superior to other precious metals in most of the studied periods, but all three precious metals have potential to function as diversifiers in an investment portfolio that is only devoted to Swedish domestic equities. Therefore, weightings of 9% gold, 12% silver and 9% platinum are preferred to improve the performance of the Swedish equity portfolio. However, the Sharpe ratio does not take into account the ethics of investing and possible environmental and social consequences. Therefore, the suggested allocation of gold in this study may not be a sustainable investment at long term.
Glossary

Bear market: a market condition where the price of an asset declines substantially over a period of time

Bearish: being under the assumption that the price of an asset will have a downward trend

Bull market: a market condition where the price of an asset increases substantially over a period of time

Bullish: being under the assumption that the price of an asset will have an upward trend

Diversifier: an asset with a correlation between minus one and plus one to another asset

Hedge: an asset with a correlation of minus one to another asset

Repo rate: Overnight borrowing/loan backed by the government securities

Safe haven: an asset that is uncorrelated with the market in times of extreme market conditions

Short selling: sale of a borrowed asset to profit from a downturn in a market
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1 Introduction

“There is no sensible reason not to own gold. I think gold should be a portion of everyone’s portfolio to some degree because it diversifies the portfolio” (Investment manager of Bridgewater, Ray Dalio, 2012).

“What motivates most gold purchasers is their belief that the ranks of the fearful will grow. During the past decade that belief has proved correct. Beyond that, the rising price has on its own generated additional buying enthusiasm, attracting purchasers who see the rise as validating an investment thesis. As "bandwagon" investors join any party, they create their own truth -- for a while.” (Chairman of Berkshire Hathaway, Warren Buffett, 2012).

1.1 Background

In modern financial history the global market has experienced numerous speculative bubbles and crises; the market crash (1987) followed by the Asian crisis (1989) and the dotcom crises that peaked in 2000. The economy is still not fully recovered from the latest global financial crisis that was caused by the collapse of the United States real estate bubble (2008) and associated outbreak of the Eurozone sovereign debt crisis (2010). The world’s central banks have taken numerous steps to overcome the adverse effects of this crisis and to mitigate the severe economic downturn. To battle the rising threat of deflation, the central bank of Sweden (Riksbanken) has lowered its key interest rate, the repo rate, to below zero percent for the first time in history (2015). The central banks of Denmark, Switzerland and Japan have followed the same path. In addition, Riksbanken along with Major central banks, such as the European Central Bank (ECB), and the central bank of the United States (FED; Federal Reserve), has even introduced a quantitative easing (QE) program as one of the last resort actions.

Both the repo rate and QE are designed to stimulate the economy by forcing down interest rates and boosting investments with the resulting excess liquidity. While a low repo rate indirectly expands the monetary supply by promoting the extension of credits, QE is a more aggressive tool that expands the monetary base. This ultimately leads to depreciation of currency of the implementing country and enhances the demand for export. Consequently, these expansionary monetary policies eventually create a bull market and an uptrend in asset prices. Therefore, the low interest rates can as well fuel an asset price bubble and provide a trigger for further financial turbulence (Bordo & Landon-Lane, 2013; Gerlach & Assenmacher-Weshe, 2008; Pagano et al., 2010). This of course is alarming in a country like Sweden (SE) that is in the midst of a property boom and has a high household debt that threatens the long-term stability of the economy (Chan & Fraser, 2017). Concerns about long-term stability are also reinforced by the impact of rising political risks in developed countries, i.e., Brexit (UK leaving the EU) and the Donald Trump victory in the US presidential election. Risk management has grown in importance as a consequence of this uncertain financial environment, and investors seek different strategies to mitigate their portfolio risks (Pullen et al., 2014, p. 77). Irresolute investors take a fresh look at their investments and are drawn to safe haven investments to hedge against the financial and economic uncertainties, or to build a more sustainable portfolio through diversification.
Traditionally, investors have diversified their investment portfolios with core asset classes; equities, i.e., stocks, and fixed incomes, i.e., bonds (Idzorek, 2006, p.1). Unfortunately, in an era of globalization, recent studies have found that the correlations within and between these primary asset classes are rising steadily over time (e.g. Idzorek, 2005; Johnson & Soenen, 1997). What is worse is that the correlation between these financial assets tends to move towards 1 in periods of economic turbulence, which was experienced by many investors during the crisis in 2008. As a result, investors experience shrinking diversification benefits and lack of protection when they need it at most (Ratner & Klein, 2008; Bernhart et al., 2011). This is of course disturbing in an era that is characterized by financial instability. Economic and political crises that overtake one another, and the measures taken to counter the crises, force investors to seek assets that are less correlated with traditional asset classes to diversify their investment portfolios. While stocks, bonds and financial instruments are ways for investors to include financial assets in portfolios, asset classes such as real estate and commodities are ways for investors to include real assets in their portfolios. Since factors that derive the prices of real assets differ from those of traditional assets, at least theoretically, their returns are expected to be less correlated with the traditional asset classes. Therefore, it is a common belief that one should include real assets in a portfolio for higher performance. Then again, investors have adopted a cautious view on investing in real estate as a result of the recent crash in the real estate market (subprime mortgage crisis in 2008) and the associated financial crises. On top of this, real estate prices in Sweden have skyrocketed since the 1990’s, with accompanying high household debts. This has sounded the alarm for a potential housing bubble.

Many commodity advocates claim that the asset class commodity is the answer to many of the problems the investment world faces today. They believe that, due to the low correlation with equities, bonds and inflation, it makes the asset class an invaluable component in investment portfolios (Idzorek, 2006, p. 1; Conover et al., 2010). Particularly the recent uptrend in the commodity gold has caught the attention of investors, specifically as the entrance of Exchange Traded Funds (ETF) into the market has improved the accessibility of the asset. Commodity gold is often utilized as a safe haven asset to offset the losses and preserve wealth as a result of its low to negative correlation with the market (Clapperton, 2010, p. 66-68; Conover et al., 2009). In times of global uncertainty, many investors retreat to gold as influenced by its history. During the classical gold standard period, almost every country’s currency was directly or indirectly pegged to gold at a fixed rate. To many investors this history rationalizes the metal as an alternative to paper currency. For centuries gold has likewise held the investment role as an inflation hedge asset due to its low/negative correlation with inflation/currency devaluation. As a hedge asset, the price of gold in general increases to balance the inflationary consequences and creates an uptrend in the gold price, and is presumed to maintain the purchasing power (Johnson & Soenen, 1997; Erb & Harvey, 2006, p.82, 2012, p. 3-4). Besides the investment role as a safe haven and hedge asset, the recent bull-run of gold prices from the end of 1999 to the beginning of 2012 has produced an annual gain of more than 15.4% in US dollars. This is far higher than the reported gains for US equity (1.5%) and bond (6.4%) markets for the same period. This has provoked a debate on the investment role of gold and awoken an interest in this precious metal as an alternative investment and a portfolio diversifier (Erb & Harvey, 2012, p. 69-97; Emmrich & McGroarty, 2013, p. 1553). This in turn has divided the investment world into two different teams: gold-bugs, who are bullish on gold and tend to over-weight the very metal in their investment portfolios, and gold-sceptics, who are bearish on gold and more or less banish the metal from their investment portfolios. On
the one side, the chairman of Berkshire Hathaway (a multinational conglomerate holding company), Warren Buffett, sees gold as an unproductive asset that rides on fear of investors, and he compares the latest price movement of metal to bubbles such as the tulip mania in the 17th century, the dot-com bubble in the 90s and the housing bubble in 2008. Many gold-sceptic investors agree with Buffett on this view, and see gold as a metal that lack both intrinsic value and producing power. Consequently, many investors as well believe that the industrial precious metals silver and platinum are to prefer as investments. On the other side, Ray Dalio, the founder and investment manager of Bridgewater, referring to the history and economics of the metal, disagree with Buffett. According to him, at least a modest amount (about 10%) of gold ought to be in a financial portfolio. Encouraging many bullish investors, successful institutional investor managers like George Soros and John Paulson seem to share the view of Dalio and increase their allocation to gold (Bloomberg 2012 August). Just as those different views and statements, the internet and many newspapers almost daily publish articles with contradicting content and advice for investing in gold. Most of these articles do not seem to stem from academic studies, and are usually biased to one side or the other. However, the statements from Buffett and Dalio cannot easily be discarded or neglected. They are successful in their own fields and hold long and extensive experience in investing. Yet, the fact remains; these are subjective opinions and not objective advice.

Meanwhile, numerous academic studies have shown that gold is an attractive investment vehicle to diversify a financial portfolio. Studies (e.g., Conover et al., 2009; Daskalaki & Skiadopoulos, 2011; Hillier et al., 2006) have found that, if compared to a standard equity portfolio, a portfolio that contains precious metals such as silver, gold and platinum performs significantly better than one without them. Particularly gold seems to improve the performance of the equity portfolio. As an investment, gold is also shown to be superior to other precious metals at stand-alone basis (Hillier et al., 2006; Conover et al., 2009). Low correlation in the returns of gold to most of the common investment assets suggests that it can be a good resource for lessening the portfolio’s exposure to fluctuations in the market value of investments, and therefore being a valuable component in a financial portfolio. In addition to low correlation with traditional assets, the commodity gold even show no relationship with real estate returns (Jaffe, 1989). In contrast to these studies, Johnson & Soenen (1997) claimed that diversification benefits of gold are almost nonexistent. This is especially the case for US- and Canadian investors. Consequently, the ultimate question, whether gold is a beneficial diversifier or not, is yet to be answered.

1.2 Purpose and Perspective
While studies (e.g. Jaffe, 1989; Chua et al., 1990; Sherman, 1982; Dempster & Artigas, 2010; Lucey et al., 2006; Hillier et al., 2006; Conover et al., 2009) have provided empirical evidence for diversification benefits of gold, a drawback in these studies is that they all exclusively used the US stock market indicator S&P 500 and focused on US investors, expect for the few studies that added a few other countries’ investors’ perspectives (e.g., Johnson & Soenen, 1997; Hillier et al., 2006). But then again, none of them took the perspective of Swedish investors. Sweden does not have a culture of investing in gold. However, the current economic environment together with the uptrend in prices has stimulated an interest in investing in gold (Dagens Industri, 2010, Nov 16). Hence, there is a growing need for studies that focus on the potential diversification benefits of gold in a portfolio from a Swedish investor point of view.
Sweden has managed the recent and ongoing crises surprisingly well, and even to the degree that “The Swedish model” is demonstrated as an example of a healthy fiscal framework. However, as an export dependent small economy, crises outside the border are sooner or later imported into the country. As a matter of fact, the ongoing and long-term crisis in the Eurozone has already slowed the growth rate in the country. Acknowledging this, Swedish investors are very much in need of investments that increase the diversification benefits in their investment portfolios.

A market portfolio, as it is stated in the mutual funds theorem, equals to the well-diversified portfolio that everyone should hold (Bodie et al., 2011, p. 311). This means, in theory, that including the factor of globalization, a market portfolio should replicate a global equity index. However, this theory is not always put into practice. Despite the fact that financial globalization has increased the integration between countries, and made it possible for investors to easily move their capital across countries, investors still tend to allocate the most part of their financial portfolios in the domestic equity market. This over-weight of domestic funds in portfolios is also called “home bias”, and can partly be explained by formal barriers such as taxes and restrictions, and partly by obstacles such as agency problems (conflicts of interest) developed due to the ownership concentration in domestic corporations (Lewis, 1999; Stulz, 2005, p. 1596-98). This implies that the market portfolio and the investors’ portfolios differ from one country to another. As a result of home bias, the findings from studies that focus on the diversification aspects of gold from the viewpoint of US investors cannot be applied straightforwardly to investors outside the United States. Furthermore, since gold is traded in US dollars, Swedish investors also face the volatility in exchange rate in addition to the volatility in gold prices.

In a “normal volatile” market, an equity portfolio should generate better returns than a diversified portfolio. Diversification is mainly a tool to handle the “abnormal volatile” market. This suggests that a tactical asset allocation strategy (market timing) should perform better than a strategic asset allocation strategy (long term oriented). But then again, this contradicts the efficient market hypothesis. Due to the transparency in an efficient market, prices of securities reflect all relevant information. This makes an active portfolio management to outsmart the market a wasted effort and unnecessary expense. Therefore, the efficient market hypothesis advocates a passive portfolio management (Bodie et al., 2011, p. 39, 378). Hillier et al. (2006) examined these hypotheses. They divided the examining period data into different classes depending on how closely returns were clustered around the mean, and by this they isolated the volatility and market uncertainty factors. Their finding was that generally a “buy and hold” portfolio performed better than a “switching” portfolio. One reason for this can be the difficulty of finding switching indicators for tactical asset allocation. These results are reinforced by another study (Conover et al., 2009), where tactical allocation was examined by using an ex-ante indicator based on monetary policy. According to the authors, tactical allocation guided by monetary policy shifts provides similar results as the strategic allocation, without providing additional value to the results.

Despite the strong integration between Swedish and US markets, how gold affects the portfolios of Swedish investors differs from that of US investors. Thus, the perspective of this study took the point of view of a Swedish investor, particularly the individual investor. The study was extended to include the after-the-global-financial-crisis period in which an expansionary monetary policy was implemented. In addition, most of the earlier studies did not use modern portfolio theory (MPT) to examine optimal allocation...
of gold in a financial portfolio (e.g., Conover et al., 2009). The studies that have used MPT calculations (e.g., Chua et al., 1990) used beta as a risk measure instead of standard deviation. While this has pros and cons, standard deviation is suggested as a risk measure by the MPT. Therefore, standard deviation was used as risk measure in optimal allocation calculations in this study. However, beta calculations for gold were also carried out to obtain a wider perspective.

While prior studies have investigated and provided many insights about gold as an investment, the aim of this study was to extend the literature regarding several important aspects of gold as an investment:

- The standpoint of Swedish investors by using Swedish market indices
- The use of a recent and extended dataset that includes the post 2008 crises period
- Optimal allocation using mathematics in modern portfolio theory
- Comparisons with the industrial precious metals silver and platinum
- Direct exposure produced by the investment channel gold bullion
- Strategic allocation, with both long- and short-term holding periods

The purpose of this study was to determine whether gold is an appropriate diversifier for Swedish investors, and to find the optimal weight of gold in a Swedish equity portfolio. Since gold mining stocks neither provide a direct exposure nor a pure exposure to gold, and exhibit characteristics more like equities, this study focused on the direct exposure produced by the investment channel gold bullion (Gorton & Rouwenhorst, 2006). In lack of identifiable valid switching indicators, the aim of this study was also strategic allocation, with both long- and short-term holding periods.

1.3 Research question and objectives

Based on the problem background and drawbacks in earlier literature, the aim of this study was to answer the question: Can Swedish investors benefit from gaining exposure to gold? In other words:

Is gold an appropriate diversifier in a financial portfolio for a Swedish investor, and can gold enhance the risk adjusted return in Swedish investors’ financial portfolios?

To answer this question, several sub questions should be answered. These are:

- Is the correlation between Swedish equities prices and gold prices less than plus 1?
- Does an allocation of gold improve the risk return characteristics of an investment portfolio that is only devoted to equity?
- What is the optimal allocation of gold in a Swedish equity portfolio?
- Is the optimal allocation of gold constant or change over time?
- Is gold a more beneficial diversifier than the other industrial precious metals silver and platinum?
2 Methodology part I – scientific approach

2.1 Preconceptions

Preunderstanding, with a basis in prior knowledge, experience and insight, can be a source of intrusion of values that can bias research results. This is most pronounced with some qualitative research methods where knowledge is created in a dialogue between the researcher and the subject (e.g., an open interview). Moreover, in observation studies, researchers can develop an affinity to participants that bias their perceptions. My personal background, having grown up in a developing country with a fondness for gold, and subsequently spending my adult life in a developed country, may have influenced the choice of topic. However, this background likely does not influence the results as this study was not designed with a qualitative research strategy. Collected data for historic prices cannot be affected by my preunderstanding. However, research cannot be value free as the preunderstanding affect how and what the researcher sees (Bryman & Bell, 2015, p. 40-41). Hopefully, my prior working experience as a quality and statistical controller of industrial diamond production, and academic experience, has added critical thinking, reflexivity and professionality to the research. I have studied at the Business Administration and Economics Program held at Umeå University, and I chose the program with the emphasis on service management. With this program I am able to get a Degree of Master of Science in Business Administration and Economics with in-depth knowledge of service management. Besides the core subjects within economic and business administration, the composition of courses within this program as well gives an understanding of management, statistics, law, psychology, marketing, sociology and design (www.umu.se). I also chose the field of finance and studied the courses given in this filed at advanced level. Although it was not a requirement, I also studied statistics at an advanced level. All these choices of courses have built a stable foundation for my profession as a financial administrator, and for the writing of this thesis.
2.2 Research paradigm

It is a common view that the nature of the research process should be logical and systematic to draw reliable conclusions. Therefore the underlying philosophical assumptions that underpin the research method are of utmost importance in a research process.

![Research paradigm diagram]

Figure 1. Research paradigm for this study (indicated by the dashed line)
Source: own construction based on Bryman & Bell (2011).

The paradigm that is adopted to design a scientific research process is derived mainly from three major dimensions: ontology, epistemology and methodology. Ontology deals with the nature of reality and with the perception of whether reality can be observed objectively (objectivism) or it is influenced by participants and thereby constructed (constructivism) by the observer (Bryman & Bell, 2011, p. 20). If the data for expected future gold prices and the optimal allocation of gold are collected via interviews, data are considered to be influenced by participants. This was not the case in this study. Data used in this study were derived from the historical prices of precious metals, stock market indicators and the risk-free rate. Even though stock market prices and precious metal prices are the results of investors’ behavior, historical data can no longer be influenced by participants and can be assumed to be independent from the observer. Objectivism leads to the research design epistemology of positivism, used mostly in natural science, whereas constructivism advocate the research design interpretivism that is usually associated with social and behavioral science. Positivism has a standpoint of an observable reality and interpretivism take a subjective viewpoint and applies to some research in social science (Saunders et al., 2012, p. 113-116).

While the prices of gold, silver and platinum are highly subjective to investors’ expectations and behavior, historical data can be considered as no longer influenced by the participants (investors) or observers (data collectors). Accordingly, the epistemology of positivism was adopted for this study. Interpretivism is associated with the inductive research methodology and is accordingly used with qualitative research methods like...
interviews and focus groups. A deductive research strategy is typically associated with quantitative data, and an inductive research strategy with qualitative data. Then again, this is a generalization and not an absolute (Bryman & Bell 2011, p. 26-28). As a result of the adopted reasoning of positivism, a deductive research approach was applied for this study. In addition, a quantitative method was applied due to the large set of numerical data for precious metals. This is a research strategy that is designed as a way of testing a theoretical proposition. An additional qualitative research method, whereby financial expertise answered a survey regarding the importance of gold to diversify an equity portfolio (which is not the case here) could have added more depth to this study.

![Diagram of research process]

**Figure 2. Deductive approach**

*Source: own construction based on Bryman & Bell (2011, p. 11).*

The deductive process is the dominant approach in most scientific research and starts with set of premises, and derives conclusions from logical reasoning from those premises (Saunders et al., 2012, p. 669). The inductive approach moves in the opposite direction and is a way of generating a theory (Saunders et al., 2012, p. 143-145). With the premises generated from academic literature and prior studies, this study was designed to test the applicability of the portfolio theory for domestic investors in Sweden. Thus, the theory is that gold is a suitable asset to improve the benefits of diversification for Swedish investors. The resulting hypothesis is that gold has a low correlation with the Swedish equity market. Correlations for gold are obtained and the hypothesis can thereby be confirmed or rejected, which can lead to revision of theory. However, the steps in a research process are not always linear, and depend on findings, and can shuttle back and forth combining different modes of reasoning. Therefore, it is not always possible to draw a clear line between the approaches, and consequently these strategies should be seen more as tendencies rather than a hard and fast rule (Bryman & Bell 2011, p.14).

### 2.3 Ethical considerations

Because of the methodology used, the needs for ethical considerations were limited when conducting this study. Specifically, the use of historical economic data that is publicly available should pose no threat to individual integrity or welfare. Ethical aspects do however come into play when considering the consequences that investments may have. Risk in investments is often only calculated on a short-term monetary basis, but a rising amount of investors are also concerned about long-term risks associated with social and environmental consequences of an asset. As a response, strategies like Socially Responsible Investment (SRI) that take environmental sustainability and social
aspects into account has grown exponentially lately (Boatright, 2010, p. 394). Gold mining is linked to environmental devastation. Chemicals like arsenic and mercury, which are often used in the extraction process of gold mining, impact the environment negatively by contaminating water and soil. This leaves long-term damaging footprints on the planet (Dooyema et al., 2012). Many argue that the SRI and return maximization cannot be combined without sacrificing portfolio performance. However, on the contrary, researches indicate SRI has a positive effect on portfolio performance (Margolis & Walsh, 2001). The Sharpe ratio used in this study does not take the ethics of investing into account, and therefore the suggested allocation of gold may not be a sustainable investment in the long term.

3 Theoretical Framework

3.1 Gold

Gold (Au) is a metal that is widely known around the world, and it is the most popular investment metal. Gold has many features that are attractive for investors. In contrast to mainstream investments like bonds and equity, physical gold carries no credit risks since it is not a liability of anyone, and therefore the uncertainty of a counterparty’s ability to meet its obligations are nonexistent. For bonds there is always a risk of default coupon payments, and for equity there is always a counterparty risk in the event of a bankruptcy. Gold, like equity and bonds, trades virtually 24 hours a day and creates a liquid market for investors, but gold’s liquidity is higher than most mainstream investments. This is partly due to the worldwide demand from many sectors and subsectors such as jewelry, medicine, industrial manufacturing, the technological sector, and financial intuitions. Another part is because of the availability of many official and unofficial investment channels, such as bullion bars, coins, jewelry, certificates, structured products, as well as futures, options and the newcomer, the “exchange traded funds”. The depth of the gold market suggests a narrower spread and a more rapid trading possibility. This in turn takes away the liquidity risk and makes gold more liquid than most diversifiers (WGC, 2013). Along with the high liquidity value, especially during periods of economic turbulence, physical gold also has a higher consumption value than stocks and bonds; this is because gold holdings can easily be converted into capital (Conover et al., 2010). Likewise, producers can benefit from the higher convenience yield (premium of holding an actual possession instead of a contract) of gold, especially in a time of uncertainty. Nonetheless, benefits usually come at a price. Hence, all these benefits of gold suggest that the returns of the metal probably are lower than a comparable asset with the same risks. For this reason, gold is often not seen as an asset that generates high returns. However, Jaffe (1989) showed that gold as well is able to generate high risk premiums. One explanation for this may be the institutional investors who avoid paying penalties to benefits, which have lower value to them (Jaffe, 1989, p. 54, 57).

Even though gold is a precious metal, it behaves remarkably different than other precious metals and commodities. To a degree some even question whether gold is a commodity at all. One of the main reasons for this remarkable behavior of gold is, besides its sheer rarity, that the demand is also affected by its symbolic value. The symbolic value of gold has almost created a general global obsession for the metal. And this of course stems from its historical and traditional link to the divine. In many religions, gold is embodied with divine qualities and considered as an appropriate
material to address the gods. As a result, in many cultures, gold is associated with glory, power and wealth. The ancient empire of the Incas, who also entitled themselves as the people of the sun, thought of gold as the sweat of the sun. Gold was therefore looked upon as a blessing from the sun god and a symbol of eternity. The ancient Egyptians, who also worshipped the sun, believed that the skin of gods was made of gold, and therefore the metal was credited with divine power (Clapperton, 2010, p. 69). These ancient symbolic values still affect the value of the metal in today’s modern world.

Lakshmi, the Hindu goddess of wealth, is often depicted with golden skin, and with a hand that eject an infinite stream of flowing gold coins. Hinduism is one of the oldest religions in the world and is still the main religion in modern India. But, even beyond the religion, gold is firmly embedded in Indian culture and tradition. This in turn can explain India’s world leading position in gold demand. With a population of 1.3 billion people (as of 2016), there is a huge demand from consumers. India has therefore a huge impact on the gold market. Especially since India’s gold demand is higher than the country’s mining supply. In fact, according to the Indian government, the rising amount of imported gold is adding to the trade deficits and hurting the Indian economy. In order to address this problem, the country has introduced restrictions in the form of import duty and sells taxes on gold (Union budget of India 2012-13). In contrast, China, the second largest market for gold, both increases its holdings in the country’s central bank and urges its citizens to invest in the metal (MetalMiner, 2011). Accelerating growth and the expanding middle class in these two major key emerging economies, exert a significant influence on the gold market (Bloomberg, 2012 Jul).

3.2 Literature review

To address the question, whether investors should include gold in their portfolio, it is essential to look at academic research performed in this area (Bryman & Bell, 2011, p. 110). Literature searches were mainly done by using the library web search tool at Umeå University Library. The advanced search was performed using the keywords gold, diversification and investments. The search resulted in 5 260 peer reviewed articles, and out of them, 4 523 articles had full text available. However, the literature search was refined several times to complement with literature referred to in prior studies, and to find the primary source. According to Hillier et al. (2006), literature that investigates the role of gold in financial markets can be roughly classified in to five different areas with varying degree of overlap; role of gold as a hedge, diversification properties of gold, market efficiency of gold, relationship of gold to macroeconomic factors, and characteristics of gold production (Hillier et al., 2006, p. 98). These five areas were narrowed down to three areas in a later study by Lucey (2013, p. 12):

- Economic and financial aspects of gold
- Gold as a currency and its historical use
- Nature and impact of gold mining on the environment and on society

Despite being delineated, these three areas are interconnected and directly or indirectly influence each other. As an example, gold as a currency and its historical use affect the demand for gold and therefore the price of gold, which is a dominant factor in financial aspects of the metal. This in turn pushes forward the need for gold mining in an aggressive way and thereby the consequences for the environment and on society (Dooyema et al., 2012). Literature relevant for this study is written in the main area of “Economic and financial aspects of gold”, which include the subcategories; Gold as a
hedge, safe haven and diversifier. Even though there is a considerable body of early literature written in these areas, recent academic literature that examine the area of gold as a diversifier is rather scarce.

3.2.1 Gold as a hedge and safe haven
Due to their moderate use in the industrial sector, precious metals are less sensitive to the wellbeing of the economy, and accordingly are less correlated with stocks and bonds. An asset that is negatively correlated with another asset is considered as a hedge. Safe haven assets exhibit hedging properties during extreme market conditions. Pullen et al. (2014, p.76) and Baur & Lucey (2010) provided a statistical measure that verify that while an asset with low or negative correlation can act as a hedge, supplementary positive skewness might even add safe haven properties. Using this statistical measure, Baur & McDermott (2010) found that gold on average acts as a natural hedge against equities, but not against bonds. Nonetheless, the same study verified that gold is capable of acting as a safe haven asset to both equities and bonds, and among precious metals gold was proven to be least linked to equities and bonds. Notwithstanding, the metal is highly correlated with the Consumer Price Index (CPI), which is a measure of the inflation rate (Erb & Harvey, 2006, p.82, 2013, p. 3-4; Wang et al., 2010, p. 806). Inflation reduces the value of money and purchasing power. Gold, on the other hand, is presumed to maintain the purchasing power. Thus, for centuries investors have utilized gold as a natural hedge against inflation or currency devaluation. Studies (e.g., Johnson & Soenen, 1997; Worthington & Pahlavani, 2007; Wang et al., 2010) have shown that gold is an inflation hedge for US investors both in the short- and long run. General evidence is that gold has the ability to serve as a flight-to-safety or as a safe-haven investment and hedge against inflation and currency devaluation in the long run (Baur & Lucey, 2010; Conover et al., 2009; Ghosh et al., 2004; Capie et al., 2005; Joy, 2011). Research (Pullen et al., 2014) as well has confirmed gold’s ability to hedge against financial distress.

3.2.2 Gold as a diversifier
Low correlation in the returns of gold to most of the common investment assets suggests that it can be a good resource for lessening the portfolio’s exposure to fluctuations in the market value of investments, and therefore being a valuable component in a financial portfolio. Studies (e.g., Conover et al., 2009; Daskalaki & Skiadopoulos, 2011; Hillier et al., 2006) have found that, if compared to a standard equity portfolio, a portfolio that contains precious metals such as silver, gold and platinum performs significantly better than one without them. Particularly gold seems to improve the performance of the equity portfolio. As an investment, gold was also shown to be superior to other precious metals at stand-alone basis (Hillier et al., 2006; Conover et al., 2009).

Concluding approximately 16 years of monthly data in his studies (September 1971 to June 1987), Jaffe (1989) found that the correlation between the US equity market and gold bullion is low, i.e., 0.054. Jaffe (1989) as well provided evidence that beta, which is a measure of the systematic risk in an asset, for gold is low (0.09). Chua et al. (1990) confirmed the findings in the above study, and despite the additional year in this study, the correlation of 0.050 was almost the same as in Jaffe’s study. Further, Chua et al. (1990) confirmed that the systematic risk of the portfolio can be reduced by adding gold, since gold has a very low beta of 0.11. The study also showed that gold bullion as a diversifier is beneficial for investors in both short- and long-term investing (Chua et al., 1990). Considering the low correlation between equity and gold, Sherman (1982)
proposed 5%-weighting of gold in an equity portfolio. Jaffe (1989) confirmed that including 5% gold in a well-diversified portfolio improves the performance of the portfolio and reduces the risk. However, the author also found that an allocation of 10% gold in a portfolio is more beneficial than an allocation of 5%. In comparison to three other inflations hedges: S&P GSCI, TIPS and BB REIT, Dempster and Artigas (2010) found that gold proved to be the most effective diversifier, and suggested a 7-10% allocation to maximize the portfolio performance. Higher allocation of 25% gold in equity portfolios was suggested by Chua et al. (1990).

In contrast to above studies, Johnson & Soenen (1997) claimed that the diversification benefits of gold are almost nonexistent; this is especially the case for US investors and Canadian investors. Nevertheless, investors from France, Germany, Switzerland, and the UK could have benefited from adding gold in the period from 1978 to 1983. In the study, the authors used monthly data from January 1978 to December 1995, and this gave gold a correlation of 0.020 to the US equity market. According to data in the study, the authors claimed, there is no use of adding gold after 1984, since both stocks and bonds dominated gold after this period.

However, with a more recent and extended data set, Lucey et al. (2006), Hillier et al. (2006) and Conover et al. (2009) claimed the opposite. Hillier et al. (2006) provided significant evidence that adding gold to an equity portfolio is beneficial, especially during periods of financial and economic uncertainties. The time period for the Hillier et al. (2006) study stretched from January 1976 to September 2004, and the authors used daily data, giving gold a correlation of -0.030 to equities. The conclusion of this study was that adding 5-10% gold to a portfolio is beneficial, but adding 10% is more beneficial than adding 5%. Using the properties of low correlation combined with positive skewness, Lucey et al. (2006) found that a financial portfolio can be diversified by including an optimal weight of gold between 6% and 25% to lower the risk and achieve higher returns. Examining a broader time period (daily data from January 1973 to December 2006), Conover et al. (2009) confirmed that adding gold to a portfolio is beneficial. Even with the extended time period, gold got the same correlation value (-0.030) as in the Hillier et al. (2006) study. The authors even found that adding as much as 25% gold increased the performance of the US equity portfolio. A portfolio with 25% gold produced 1.65% higher annual returns and reduced the risks by 1.86% compared to a portfolio only devoted to US equity. However, Johnson & Soenen (1997) and Conover et al. (2009) provided evidence that benefits of gold vary depending on monetary policies. According to Conover et al. (2009) adding gold to a portfolio is slightly more beneficial during monetary tightening periods than easing periods. This is because the returns of equities drop during periods of restrictive monetary policies, due to actual or expected rise in inflation. Emmrich & McGroarty (2013) provided evidence that gold bullion is more beneficial than the other forms of gold investments like gold mining stocks and ETFs, as a result of the inflation expectations.

### 3.3 Modern Portfolio Theory (MPT)

The Economist and Nobel laureate Milton Friedman identified, that everything has a price and somebody always pays the price, i.e., everything costs something; consequently nothing is free. According to Friedman, “There's No Such Thing as a Free Lunch”. Thus “free lunch” is a misconception and a myth. Although he used this statement to highlight the relationship between government spending and its impact on individuals, the statement as well defines the tradeoff between risk and return (Friedman, 1975). Nonetheless, Modern Portfolio Theory (MPT) finds a loophole in
above hypothesis. The Nobel laureate Harry Markowitz, the father of the MPT, asserted the possibility of a “free lunch” for investors who hold multiple assets instead of a single asset (Chua et al., 1990). The key concept of Modern Portfolio Theory (MPT) is that combinations of assets can provide investors with better returns per risk unit than individual assets. The origin of MPT is the paper “Portfolio selection” published in 1952 by Nobel laureate Harry Markowitz. Markowitz introduced a mathematical technique to combine assets to optimize an investment portfolio. The method is called “Mean-variance optimization”. Markowitz recognized that investors are facing conflicting objectives when making an investment decision. That is, to maximize returns while minimizing risk. The conflict is the risk-reward tradeoff - the market usually rewards high risk with high returns and low risk with low returns. This means that maximizing returns gives high risk, while minimizing risk gives low returns. MPT is the solution to investors’ problems with conflicting objectives; that is, to diversify by investing in multiple securities instead of investing in just one. The reason is, according to Markowitz, that the co-movement of assets with each other is more crucial in returns and risk relationship than the characteristics of the individual assets (Sharpe et al., 1999, p. 139).

Calculation techniques in the Markowitz function do however present a challenge for investors because of the massive number of data estimates required for implementing the model. In order to choose the best possible portfolio among infinite possible alternatives of portfolios, the efficient frontier should first be defined. In a risk return space, this is a line that shows the asset combinations with the highest returns for a given level of risk. To define the efficient frontier, in addition to estimates of expected return and variance of every security, a covariance matrix with every asset must be calculated. As a result, the number of parameter estimates increases exponentially with increasing number of assets. Furthermore, since covariance calculations are performed for every asset, the values are mutually inconsistent and can lead to errors in estimations. This makes things complicated. Since an asset covaries differently with different assets, it is hard to value the riskiness of an asset. As a solution, a single-index-model derived from MPT standardizes the covariance by calculating the covariance of an asset with the market index. The Single Index Model replicates the core theory of MPT and simplifies the data tabulation required for portfolio analysis. Relating the returns of a security to a common macroeconomic factor, where a market index can be used as a proxy model, reduces the number of parameter estimates required to calculate the optimal combinations of assets. This in turn makes the Markowitz model more applicable in practice for portfolio analyses (Elton et al., 2007, p. 130).

### 3.3.1 Optimal portfolio

"If a portfolio is “efficient,” it is impossible to obtain a greater average return without incurring greater standard deviation; it is impossible to obtain smaller standard deviation without giving up return on the average" (Markowitz, 1959, p. 22).

The idea of Portfolio theory is that combinations of low correlation assets have the potential to provide the investor with better returns than a single asset relative to the risk taken by the investor. Correct proportions of these low correlation assets make the optimal portfolio. However, the optimal portfolio changes with goals and the risk averseness of the investors. Therefore, calculations of correct allocations can be done in different ways. Some investors may prefer a conservative portfolio with minimal possible risk to preserve wealth, and others may prefer a portfolio with minimum risk to
a given level of return in order to achieve set goals. Optimization can also be done to obtain maximum return with minimum risk. This is done by finding the allocation along the Capital Asset allocation Line (CAL), where every point on the line defines the optimal allocation of each asset. Optimal allocation is calculated with the purpose of maximizing the returns, or equivalently, minimizing the risk for every given level of expected return (Idzorek, 2006, p. 2).

The optimal complete portfolio is determined depending on the risk averseness of the investor. The objective of this paper was to find the optimal risky portfolio. A portfolio in the study contains the risky assets; equities and gold, where the market index is used as a proxy for a well-diversified equity portfolio. Optimal risky portfolio only determines the desirable asset allocations of risky assets, and as it is shown in figure 3 below, the optimal complete portfolio lies along the CAL. Depending on risk averseness of the investor, the risky portfolio is completed by adding allocation to the risky asset. Figure 3 shows the opportunity set of the risky asset G and E. The optimal risky portfolio where the Sharpe ratio is maximized is marked with the letter S, and this portfolio contains only the risky assets, i.e., zero allocation to the risk-free asset. The fraction of the total value held in the portfolio invested in risk-free assets is 1 - risky asset allocation. With increasing risk averseness the proportion allocated to the risk-free asset increases. This means that for a risk-natural investor (risk averseness of 0), portfolio S is also the optimal portfolio, while the optimal portfolio of an investor with a risk averseness >0 lies lower than portfolio S on the CAL line. The minimum variance portfolio where the risk is minimized is marked with the letter M.

Figure 3. Opportunity set of two risky assets
4 Methodology part II – data and procedure

4.1 Data and research process
The value of accurate and valid data cannot be over emphasized in order to ensure accurate results. Regardless of the accuracy of the logics used in the analyses, inaccurate data always produce inaccurate results. To secure the validity of the conclusions of this study, I have put effort into finding reliable sources and relevant data for the study. This study was based on quantitative monthly time series data. Since expected prices are not directly observable, past prices were projected into the future to forecast future prices. Therefore registry data for historical price observations were used in this study (Bodie et al., 2011, p. 145). While data collected via interviews and/or survey could have added more depth to the results, it would have been more biased by reflecting the mindset of investors today. Furthermore, given the time frame, a large effort would have been needed to expand the data set with additional methods. Along with the accessibility to a large data set, past price data also enable a testable hypothesis incorporating MPT. Therefore registry data for past prices was used in this study. Major parts of the data were obtained via a study published on the official website of the Swedish central bank Riksbanken (riksbank.se) and complementary data was obtained via Thomson Reuters Datastream. Data for prices was converted and analyzed in Swedish Krona (SEK) to focus on characteristics of gold for Swedish investors. The optimal weight of gold in an investment portfolio was calculated under the optimization framework of maximization of the Sharpe ratio. Returns were calculated from the collected data from Riksbanken and Datastream. Excess returns were calculated from the returns. These excess returns were then processed in Minitab to obtain descriptive statistics for all the assets and additional inferential statistics. Results were then processed in excel to find the optimum allocations of assets by applying the Excel solver function to the MPT equations to maximize the Sharpe ratio. Constraints that assets weights should sum up to one were used to exclude short selling that has complexity in regulations.
Figure 4. Schematic of the calculation procedure for the study

To ensure the accuracy, data were collected from the highly reliable sources Datastream and Riksbanken.se. Furthermore, outliers were carefully reviewed to safeguard the accuracy in data entries. Outliers can have a big influence on correlations. They can influence the results dramatically and lead to false conclusions. Therefore it is important to review outliers carefully to determine whether to include or exclude outliers from the analysis. Outliers in this case are returns that are extremely low or extremely high. This may be an outcome of incorrectly recorded prices for the studied assets. Or it can be an outcome of incorrectly included data values. In either case, outliers should be removed from the analysis. Outliers can also be unusual price data that is recorded correctly and belong to the analyzed dataset, as it was in the present study. Those outliers were not, and should not be removed (Anderson et al., 2014, p. 130). However, it should be mentioned that early price data was not stored in computers. The early data obtained via Riksbanken is a reconstruction of price data published in journals. Reconstruction can lead to potential mistakes, but the validity of the data set is confirmed by the accompanying research *Swedish stock and bond returns, 1856–2012* by the author (Waldenström, 2014). To ensure the quality of literature, mainly peer-reviewed studies were used in the literature review along with papers written by professionals in the field. An extended literature review is given in the theoretical chapter. In addition, the research process is thoroughly described, and a large data set (from 1969 to 2016) was used to improve the reliability of results. The large data set and transparency make it easier to follow and replicate the study as well as to minimize the errors (Bryman & Bell, 2011, p. 279; Saunders et al., 2012, p. 156, 274).

Along with the availability, researchers are given the choice whether to use monthly or daily data. Jaffe (1989), Chua et al. (1990), and Johanson & Soenen (1997) used monthly data in their studies, while Hillier et al. (2006) and Conover et al. (2009) used daily data in their studies. Then the question arises, does the frequency of data matter.
for the results? In a study regarding appropriate frequency of data, the authors of the study claimed that compared with monthly data, daily data is preferable because it minimizes the bias and maximizes the efficiency of estimates (Morse, 1984, p. 606). However, in order to extend the time period of the study, I was forced to use monthly data due to the lack of availability of reliable long term historical daily data. While this may be less preferable than daily data, the large data set compensates for the flaws in monthly data and maximizes the efficiency of estimates (Bryman & Bell, 2011, p. 279; Saunders et al., 2012, p. 156, 274).

4.1.1 Variables
This study took a Swedish investors’ point of view, and therefore a domestic risky stock market index and a risk-free rate was chosen for the study. Data collections for variables used in the study are thoroughly described below.

**Equity returns**
The stock market indicator represents a well-diversified equity portfolio. The stock index used (from 1968 to 2012) is a composite of indices from the Stockholm stock exchange and was calculated by authors Frennberg, Hansson, Asgharian and Waldenström. Prices were mainly calculated by using historical data published in the financial journals “Kommersiella Meddelanden” (1906-1918) and “Affärsvärlden” (1919-2006). By including dividends in the prices the authors have created a reliable total return index that reflects the domestic market in Sweden (Waldenström, 2007, p. 4). This is the longest time series of Swedish stock and bond indices available, and data are obtainable at the website of the Swedish central bank (riksbank.se; Waldenström, 2014). For the remaining time period, 2013 to 2016, SIX Return Index (SIXRX) with Datastream code AFFKAST was used. SIXRX is a market weighted capitalization index (total market value is measured by multiplying share price by number of outstanding shares) that is designed as an indicator of the Stockholm Stock Exchange (OMXS). The index is representative for the Swedish domestic market, because it measures the market equity performance in Sweden, and is more suitable than OMX, because it is a gross index, which OMXS is not (OMX is a price index). As a gross return index or a total shareholder return index, SIXRX, besides the capital gains, calculates the prices as dividends were re-invested at the ex-dividend day (six.se).

**Risk-free Rate**
A risk-free asset is an asset that generates actual returns equal to expected returns, without any variance around the mean returns. This indicates that the return of a risk-free asset is uncorrelated with the returns of the risky assets in the market (Damodaran, 2008, p. 4). Since the risk-free assets generate a certain return without any variance, the standard deviation of the risk-free asset should be zero (Elton et al., 2007, p. 85). This is true if the inflation risk is not regarded; the risk-free rate is assured in a certain currency, but that does not mean that it is certain in another unit, such as the unit consumer price index (CPI). Even the risk-free rate is subject to variance in purchasing power, unless the risk-free rate is given in real terms, i.e., adjusted for inflation, and often this is not the case (Bodie et al., 2011, p. 146). As a result of availability of real rates, most studies have used the nominal risk-free rate; even with the possibility to adjust for the inflation using CPI. This was also the case in this study. To make comparisons easier with other studies, all the rates used in this study were in nominal terms. Short term (maturity < 1 year) debt obligations backed by a government are often considered as safe investments, since the default risk is almost nonexistent. Treasury bills or T-bills issued by the US government often determine the risk-free rate. T-bills
backed by the Swedish government are called “statsskuldväxlar”, and the Swedish National Debt Office (Riksgälden in Swedish) issued its very first statsskuldväxel in 1983. Although government bonds lack the counterparty risk, choosing a government bond that does not match the investment time horizon can still bring risk in form of price risk and reinvestment risk (Damodaran, 2008, p. 6). This means that investors holding a risk-free asset with longer maturity than the investment time horizon cannot be certain of the value of the risk-free asset at the end of the holding period. Consequently this means a price risk, and investors holding a risk-free asset with shorter maturity than the investment time horizon cannot be certain of the reinvestment rate. This is the same for the coupon bonds; investors holding a coupon bond cannot be certain of the reinvestment rate for the coupon payments (Sharpe et al., 1999, p. 205).

Above facts suggest that the appropriate estimate of a risk-free rate for this study should be derived by a zero-coupon, long-term government bond. Since long-term bonds are often coupon bonds, the study “What is the risk-free rate? A Search for the Basic Building Block” suggested to strip the coupon bond to make it a zero coupon bond. The study suggested to separate the coupon payments from the principal and calculate the repayment as a zero coupon bond (Damodaran, 2008, p. 7).

Government bonds quoted in clean prices, i.e., prices without accrued interest can be obtained from Datastream for several countries, including Sweden. Even though this matches the requirement of a stripped bond, I have chosen to use the prices with reinvested coupon payment to estimate the risk-free rate. This is because I used ex-post values and further used total return prices for equity indices. Using the total return prices for bond indices as well give a fair image of the rate obtained by this asset. Further, due to the mismatch of availability of historical data for statsskuldväxlar with different maturities, the long-term bond index (Long-term government bond yield index) published at the Swedish Central Bank Website was used. The Bond index calculated and put together by Frennberg, Hansson and Waldenström represents the risk-free rate in this study for the period 1968-2012. The Long-run yield index was calculated with adding the coupon rate to the long-run government bond prices for more accurate data (Waldenström, 2007). Since the longest maturity for Swedish government bonds that exist today is 10 years, the 10+ year bond index constructed by Datastream with the code ASDGVG5 (RI) was used as the risk-free asset for the remaining period (2013 to 2016).

**Precious metals returns**

Metals, also called the hard commodities, are divided into two subcategories; base metals and precious metals. Metals with high economic values are referred to as precious metals. Gold, silver and platinum fall into this category and are most sought by investors (Benhamou & Mamalis, 2010). Due to their moderate use in the industrial sector, precious metals are less sensitive to the wellbeing of the economy, and accordingly are less correlated with stocks and bonds.

*Gold* is considered to be an industrial precious metal to a lesser extent than both silver and platinum, and main demand drivers for gold are jewelry and investments (Hillier et al., 2006). Investors are able to include an allocation to gold within a portfolio, either by gaining exposure to price movements of the gold, or by holding physical gold. There are many complex financial products that track the price of gold, and along with the rising interest in gold, there are also many ETFs that track the price movement of gold, either by using derivatives, or holding the commodity gold, i.e., physical gold traded as gold
bullion. ETFs track the performance of an underlying benchmark, similar to an index fund, and are listed and traded just like a stock on exchange. Hence, ETFs are hybrids between regular stocks and mutual funds. ETFs make it easier for individual investors to gain exposure to many more alternative assets than it was possible before. Nevertheless, many investors still prefer coins and bullions or mining stock equities (Hillier et al., 2006, p. 99). According to Chua et al. (1990), gold bullion offers a better risk reduction than gold mining stocks, since the 1970s. One of the attractions of investing in precious metal lies in its storability, and gold, platinum and silver have a high degree of storability due to their high value and unchangeable quality characteristics, such as resistance to corrosion. This of course makes a physical possession of these metals more convenient, and direct exposure feasible (Fabozzi et al., 2008, p.7). Only gold bullions, or the ETFs that actually hold the commodity gold, allow investors to gain direct exposure to gold. Prices for gold used in the present study were derived from London gold bullion prices per troy ounce (31.1035g) and prices were converted from United States Dollar (USD) to Swedish Krona (SEK) directly in Datastream using up-to-date exchange rates. Datastream code GOLDBLN was used to download data. Excess returns for gold were calculated using downloaded data for gold bullion.

Silver is considered to be an industrial precious metal to a greater extent than gold, but to a lesser extent than platinum. Silver as well experience demand from the jewelry and investment sectors (Hillier et al., 2006). Prices for commodity silver were derived from Silver Fix London Bullion Market prices per troy ounce and prices were converted from USD to SEK directly in Datastream using up-to-date exchange rates. Datastream code SLVCASH was used to download data. Data for silver started in 1970. Excess returns for silver were calculated using downloaded data for silver.

Platinum is considered to be an industrial precious metal to a greater extent than silver and demand comes both from the investment and industrial sector (Hillier et al., 2006). Prices for commodity platinum were derived from London Platinum Free Market prices per troy ounce and prices were converted from USD to SEK directly in Datastream using up-to-date exchange rates. Datastream code PLATFRE was used to download data. Data for platinum started in 1976. Excess return for platinum was calculated using downloaded data for platinum.

4.2 Sample period
Prior studies also show that the time period chosen for data can very well affect the outcome of the study. Using data from 34 years, Conover (2009) showed that including precious metal as a portfolio component was beneficial throughout the entire period, but that if the studied period was changed to 1995-2000, the outcome would have been different. He found that studies using data for this period would conclude that adding precious metal to the portfolio is harmful, whereas studies using data from post 2000 would find that adding precious metal to the portfolio is tremendously beneficial. In contrast, Chua et al. (1990) showed that gold bullion is a valuable diversifier at both long- and short-term. In a “normal volatile” market an equity portfolio should generate better returns than a diversified portfolio. Diversification is mainly a tool to handle the “abnormal volatile” market. This suggests that a tactical asset allocation strategy (market timing) should perform better than a strategic asset allocation strategy (long term oriented). But then again, this contradicts the efficient market hypothesis. Due to the transparency in an efficient market, prices of securities reflect all relevant
information. This makes an active portfolio management to outsmart the market a wasted effort and unnecessary expense. Therefore, the efficient market hypothesis advocates passive portfolio management (Bodie et al., 2011, p. 39, 378). Hillier et al. (2006) examined these hypotheses. They divided the examining period data into different classes depending on how closely returns were clustered around the mean, and by this they isolated the volatility and market uncertainty factors. Their finding showed that generally a “buy and hold” portfolio performed better than the “switching” portfolio. One reason for this can be the difficulty of finding switching indicators for tactical asset allocation. These results are reinforced by another study (Conover et al., 2009), where tactical allocation was examined by using an ex-ante indicator based on monetary policy. According to the authors, tactical allocation guided by monetary policy shifts provided similar results as the strategic allocation without providing additional value to the results.

This study examined the investment properties of gold over a 47-year period with eight different holding periods as listed below. By this approach it was possible to examine the optimal allocation of gold in investments held in different periods of time. Periods are numbered from one to eight. Availability of historical data for gold, which has the starting point in 1968, set the time range of the historical data analyzed in this study. The data range from February 1968 to November 2016. First, the whole period was examined to determine optimal allocation of gold in a long holding period of 47 years (586 observations). Second, the time period was divided into two equal subperiods to examine if the optimal allocation based on the entire period was also valid for the subperiods, and whether the allocation was the same in the first and second half (293 observations each) of the examined time period. Third, the time period was divided into five different equivalent short term investment horizons and examined with and without adding the precious metals silver and platinum. Due to the availability of historical data for precious metals, only subperiods II (120 observations), III (120 observations), IV (120 observations) and V (119 observations) were examined with the inclusion of precious metals. Subperiod I (107 observations) was examined only with the inclusion of gold.

Table 1. Sample periods used in the study

<table>
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<th>Sample period</th>
<th>Start</th>
<th>End</th>
<th>Observations</th>
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<tbody>
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<td>1968 Feb</td>
<td>2016 Nov</td>
<td>586 months</td>
</tr>
<tr>
<td>Subperiod A</td>
<td>1968 Feb</td>
<td>1992 June</td>
<td>293 months</td>
</tr>
<tr>
<td>Subperiod B</td>
<td>1992 July</td>
<td>2016 Nov</td>
<td>293 months</td>
</tr>
<tr>
<td>Subperiod I</td>
<td>1968</td>
<td>1976</td>
<td>107 months</td>
</tr>
<tr>
<td>Subperiod II</td>
<td>1977</td>
<td>1986</td>
<td>120 months</td>
</tr>
<tr>
<td>Subperiod III</td>
<td>1987</td>
<td>1996</td>
<td>120 months</td>
</tr>
<tr>
<td>Subperiod IV</td>
<td>1997</td>
<td>2006</td>
<td>120 months</td>
</tr>
<tr>
<td>Subperiod V</td>
<td>2007</td>
<td>2016</td>
<td>119 months</td>
</tr>
</tbody>
</table>

The optimal weight of gold in an investment portfolio was calculated under the optimization framework for maximization of the Sharpe ratio. Returns were calculated from the collected price data from Riksbanken and Datastream. Risk premiums were calculated by using the risk-free rates. Returns adjusted for risk-free rates were processed in Minitab to obtain descriptive statistics for all the assets. Results were then
processed in excel to find the optimum allocations of assets by applying the Excel solver function to the MPT equations to maximize the Sharpe ratio. Constraints that assets weights should sum up to one were used to exclude short selling that has complexity in regulations.

4.3 Calculation of risk, returns and excess returns

The term returns refers to gains from an investment over a holding period, and for gold it is the price appreciation. For equities there are two components that contribute to the returns; price appreciation and dividends. After downloading closing prices, returns and excess returns were calculated as below (Sharpe et al., 1999, p. 140).

\[
Rate \ of \ return = r = \frac{(P_1 - P_0)}{P_0}
\]

(1)

\[
Excess \ return = Er = r - r_f
\]

(2)

\[
Arithmetic \ average \ (expected \ return) = \bar{r} = \frac{\sum r}{n}
\]

(3)

Where:
- \(P_1\) = ending price
- \(P_0\) = beginning price
- \(r_f\) = risk-free rate
- \(n\) = number of observations

The risk and return relationship is derived from the assumption that investors are risk-averse, meaning that they do not like risk, and therefore are unwilling to bear any risk unless they are compensated for the amount of risk they take. Accordingly, assuming that the risk of an asset is known to investors, the expected return of an asset reflects the risk of the same asset (Sharpe et al., 1999, p. 399).

Risk premium is the excess return that an asset provides over the risk-free rate (Bodie et al., 2011, p. 157). This tradeoff between risk and return suggests that investors who seek higher expected returns are forced to pay a penalty by bearing higher variability in expected returns. This variability in returns, or dispersion around the expected return, is measured by the statistical measure variance and denoted by \(\sigma^2\). Variance is the average of squared differences from the expected return or the mean. The standard deviation is the positive square root of variance and measures the dispersion around the average return and is often calculated by using historical returns. Thus, assets with historically higher standard deviation are expected to generate higher returns, and assets with historically lower standard deviations are expected to generate lower returns. This tradeoff that exists between risk and return confirms the “no free lunch” hypothesis (Bodie et al., 2011, p. 37, 130-131).
Formulas for variance and standard deviation used are shown below.

\[
\text{Variance} = \sigma^2 = \frac{\sum (r - \bar{r})^2}{n - 1} \tag{4}
\]

\[
\text{Standard deviation} = SD = \sqrt{\frac{\sum (r - \bar{r})^2}{n - 1}} \tag{5}
\]

Where:
- \( n \) = number of observations
- \( r \) = rate of returns
- \( \bar{r} \) = mean return

### 4.4 Calculation of correlations

The benefits of diversification vary depending on the degree of co-movement of portfolio components. Diversification varies in this study, depending on the co-movement of gold, silver, platinum and equity. This co-movement is determined by the statistic covariance or more frequently correlation (\( \rho \)). Correlation is the standardized covariance and takes values between -1 and +1. These statistics refer to the degree to which returns of one security move relative to the returns of another security.

Correlation standardizes the strength of the linear association between assets during a period of time. A perfect positive correlation (+1) between equity and gold will indicate that returns of the assets move in tandem, and this gives a maximum portfolio risk and lowest returns per risk unit. Furthermore, the risk of the portfolio will be the same as the weighted average of the individual asset risks. Diversification thereby does not reduce the risk of the portfolio, and provide no diversification benefits (Bodie et al., 2011, p. 228). A perfect negative correlation (-1) indicates that gold move in completely opposite direction of equity, and this will provide maximum diversification benefits (Sharpe et al., 1999, p. 152, 178). However, a perfect negative correlation as well completely offsets the returns from correlated assets. In this case, when equity returns are high, gold returns will be low to match the increase and will completely offset the gains from equity. Therefore a correlation higher than -1 (lower than +1), where risk is reduced without completely offsetting the returns, is to prefer.
As displayed in figure 5, if the returns of gold and equity move in the same direction they are positively correlated; if they move in opposite directions they are negatively correlated. The free lunch in diversification improves with declining correlations. Thus, hedge assets that are negatively correlated with the other assets make the biggest contribution to portfolio diversification (Bodie et al., 2011, p. 229). Correlation measures the linear association between two variables, in this case between assets gold and equity. The correlation coefficient is symmetric (as can be seen from the formula below), meaning that the usual division into independent and dependent variables is not applicable. The purpose of this study was not to predict from one variable to the other as there is very likely any simple and direct causal relationship between prices of assets. Formulas for covariance and correlation between two assets (asset 1 and asset 2) that were used in the study are shown below.

\[
Covariance = \sigma_{1,2} = \frac{\sum (r_1 - \bar{r}_1)(r_2 - \bar{r}_2)}{n - 1} \tag{6}
\]

\[
Correlation = \rho_{1,2} = \frac{1}{n - 1} \sum \left( \frac{r_1 - \bar{r}_1}{\sigma_1} \right) \left( \frac{r_2 - \bar{r}_2}{\sigma_2} \right) = \frac{Cov_{1,2}}{\sigma_1 \sigma_2} \tag{7}
\]

Where:

- \( n \) = number of observations
- \( r \) = return
- \( \bar{r} \) = mean return
- \( \sigma \) = return
As shown in figure 6 above, when the correlation between assets decreases the effects of diversification increases. Therefore, to benefit from diversification, it is important to combine assets with correlation less than +1. In Minitab (minitab.com) the standard null and the alternative hypothesis for hypothesis testing of correlations are:

$$H_0: \rho = 0 \text{ (there is no correlation)}$$
$$H_1: \rho \neq 0 \text{ (there is correlation)}$$

Hypotheses in this study were tested with the significance level alpha ($\alpha$) set to 0.05. A statistically significant correlation in the empirical chapter means that the obtained $p$-value was smaller than the selected $\alpha$-level. The null hypothesis was thereby rejected in favor of the alternative hypotheses that there is a significant linear relationship between the variables (i.e., assets). An obtained $p$-value above the alpha level does not mean that the correlation coefficient is zero, which indicates that returns of investments are uncorrelated. Instead it means that there was not sufficient statistical evidence in this study for a linear relationship between assets.

### 4.5 Total risk

The Markowitz model treat the risk of a security as a one unit, while the single index model reasons that the risk of a security consists of two components; risk due to macroeconomic factors that affect all the securities simultaneously, and the risk that is specific for every individual security and therefore unique to every security. This indicates that the risks that arise due to the macroeconomic factors (such as inflation risk, business cycle risk, interest rate risk and exchange rate risk), also called the systematic risk of all securities, and are correlated with each other. The risks that are specific to every security, such as business risk, financial risk and operational risk, also
The role of gold in an investment portfolio

4.6 Unsystematic Risk

Assuming that investors are rational, a frequency plot of actual returns realized from a security takes a bell shape, and this is called a normal distribution. The mean of the normal distribution is the expected average return. In a normal distribution, 99.74% of data lie in a range of ± 3 standard deviations from the mean. The standard deviation, as mentioned above is a measure of volatility and therefore a measure of uncertainty. For investors, uncertainty of a cash flow is a risk. Higher standard deviation gives a flatter curve and smaller standard deviation gives a peakier curve (Bodie et al., 2011, p. 130-131). This suggests that security with a higher standard deviation is riskier than one with a smaller standard deviation. This is because the base of the bell curves gets wider and therefore uncertainty greater.

The theory holds that with increasing number of securities in a portfolio, unsystematic risk reaches zero and can be eliminated, provided that the correlations in included assets are < 1. With equal weighting of securities, the mean (expected average return) of the portfolio is exactly the same as the average of the un-weighted securities mean. At the same time, the standard deviation (risk) of the same portfolio is less than the average of the standard deviation of the individual securities. Consequently, with increasing number of securities in a portfolio, firm-specific risk decreases. In fact, a portfolio consisting of 30 or more securities practically take away the idiosyncratic risk. This is valid under the assumption that securities are randomly selected (uncorrelated) and equally proportioned (Sharpe et al., 1999, p. 187).
4.7 Calculation of systematic risk

While macroeconomic factors affect the whole market, the sensitivity to the movements in the market differs among securities. This sensitivity to the movements in the market is called the systematic risk or the market risk. Systematic risk is measured by the sensitivity coefficient beta, which is denoted by the Greek letter $\beta$. In contrast to unsystematic risk, this systematic risk cannot be diversified away. This means that beta of a portfolio is equal to the weighted average betas of securities included in the portfolio. While systematic risk is non-diversifiable, it can still be reduced by adding assets with lower betas to an investment portfolio. A security that has a beta equal to 1 mirrors the returns of the market index. A security that is more volatile than the index has a beta greater than 1, and is termed an aggressive asset. A security that is less volatile than the index has a beta less than one, and is termed a defensive asset. However, the market sensitivity of a security is not consistent and fluctuates over time, which means that beta of a security is not stationary over time (Sharpe et al., 1999, p.183-187, 236-240).
The formula for beta used in this study is shown below.

\[
Beta = \beta_s = \frac{\sum(r_m - \bar{r}_m)(r_A - \bar{r}_A)}{\sum(r_m - \bar{r}_m)^2} = \frac{Cov(r_A, r_m)}{Var(r_m)}
\]  \hspace{1cm} (8)

Where:
- \( r \) = returns
- \( \bar{r} \) = mean returns
- \( m \) = market
- \( A \) = asset

4.8 Calculation of kurtosis and skewness

In addition to standard deviation and beta, volatility is also measured with the additional statistics; kurtosis and skewness. When returns are normally distributed, standard deviation overestimates the risk by adding the extreme positive deviation to the risk. Extreme positive returns are a gain and not a loss. In the same way, if the returns are negatively skewed, the standard deviation underestimates the risk by cutting off the extreme negative deviation too early. If the distribution has a higher probability of occurrence of both extreme positive and negative values (fat tails), standard deviation will underestimate the probability of extreme losses as well as extreme gains. This is because a distribution with fat tails is still symmetric at the expense of the less probability mass at the center of the bell curve. Therefore the additional information provided by the skewness and the kurtosis is essential to estimate the risk more accurately. A normal distribution has a kurtosis of 3 and is termed mesokurtic. A kurtosis less than 3 (termed platykurtic) involves a less peaked curve with skinny tails. Kurtosis over 3 (termed leptokurtic) involves a more peaked curve with fat tails. Figure 9 illustrate different types of kurtosis.
The higher the kurtosis, the more likely returns will be extremely low or extremely high. That is, kurtosis measures the volatility of the volatility. Furthermore, a normal distribution has a skewness of 0. Positive skewness involves asymmetry to the right due to outliers with high values. Negative skewness involves asymmetry to the left due to outliers with low values (Bodie et al., 2011, p. 165). Formulas for skewness and kurtosis used in this study are shown below.

\[
Skewness = \frac{\sum (r - \bar{r})^3}{\sigma^3}
\]  
\[
Kurtosis = \frac{\sum (r - \bar{r})^4}{\sigma^4}
\]

Where:
- \(r\) = returns
- \(\bar{r}\) = mean returns
- \(\sigma\) = variance

### 4.9 Calculation of the Sharpe Ratio

The optimal portfolio lies along CAL. The optimal risky portfolio can be found by maximizing the slope of this line. This is also the point with the maximum Sharpe ratio. The Sharpe ratio is highest when the reward to volatility is highest. The formula for the Sharpe ratio (SR) for a two asset portfolio that was used in this study is shown below (Bodie et al., 2011, p. 236).
Maximize: \[ \text{SR}_p = \frac{E_{r_p} - r_f}{\sigma_p} = \frac{\left(W_1(E_{r_1} - r_f) + W_2(E_{r_2} - r_f)\right)}{\sqrt{(W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_{12})}} = \frac{\text{Portfolio Return}}{\text{Portfolio Risk}} \]

\[ W_2 + W_1 = 1 \]

Where:
- \( E_r \) = expected return
- \( \sigma \) = variance/covariance
- \( r_f \) = risk-free rate
- \( w \) = weight

In order to find the optimum weights of gold and equity in the portfolio, the Sharpe ratio for the portfolio should be maximized with respect to the weights. This is done by taking the partial derivative of this function equal to zero with respect to the weights. The formula used in this study is shown below (Bodie et al., 2011, p. 236).

\[ W_1 = \frac{(E_{r_1} - r_f)\sigma_2^2 - (E_{r_2} - r_f)\text{Cov}(r_1, r_2)}{(E_{r_1} - r_f)\sigma_2^2 + (E_{r_2} - r_f)\sigma_1^2 - (E_{r_1} - r_f + E_{r_2} - r_f)\text{Cov}(r_1, r_2)} \]

\[ W_2 = 1 - W_1 \]

Where:
- \( E_r \) = expected return
- \( r \) = return
- \( r_f \) = risk-free rate
- \( w \) = weight
5 Empirical Findings

Calculated monthly excess returns were processed in the software programs Minitab and Excel. Results are presented in tables and figures below and all the periods are compared further below.

5.1 Full period F (1968-2016)

Table 2 presents descriptive statistics for calculated monthly returns for gold and equity, and the risk free rate, for the entire period 1968-2016. Mean monthly return for gold was 0.010 and it was lower than the mean monthly return of 0.014 for the equities. The variability for gold (0.077) was higher than the variability for equities (0.072). The Risk-free rate for the period was slightly negative (-0.001). The correlation between gold and equities for the period was 0.36 and the p-value was 0.000.

Table 2. Descriptive and inferential statistics for calculated monthly excess returns for gold and equity for the period 1968-2016

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Dev.</td>
<td>Variance</td>
<td>Skewness</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>Equity</td>
<td>0.014</td>
<td>0.072</td>
<td>0.005</td>
<td>-0.22</td>
<td>1.86</td>
</tr>
<tr>
<td>Gold</td>
<td>0.010</td>
<td>0.077</td>
<td>0.006</td>
<td>0.75</td>
<td>2.70</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inferential statistics</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.361 (P-Value = 0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta (for gold)</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 10. Histograms for monthly excess returns for gold and equity for the period 1968-2016

As can be observed in figure 10, the negative value of skewness for equity indicates negative return outliers, and positive skewness for gold indicates positive return outliers. Figure 11 below presents the results of the portfolio analysis for gold and equity for the entire period 1968-2016. A Maximum Sharpe ratio of 0.20 (optimal asset allocation) was obtained by computing a portfolio with 26% gold and 74% equities. This generated a portfolio return of 0.013 with a variability of 0.063.
Figure 11. Optimal asset allocation 1968-2016

5.2 Subperiod A (1968.02-1992.06)

For subperiod A, gold generated a monthly average return of 0.009 with a 0.074 risk, whereas equity had both a higher return and lower risk than gold. The correlation for the period was 0.18 with a significant p-value of 0.003. Results are given in table 3.

Table 3. Descriptive and inferential statistics for calculated monthly excess returns for gold and equity for the period 1968.02-1992.06

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.012</td>
<td>0.064</td>
<td>0.004</td>
<td>-0.25</td>
<td>1.37</td>
</tr>
<tr>
<td>Gold</td>
<td>0.009</td>
<td>0.074</td>
<td>0.005</td>
<td>1.05</td>
<td>3.55</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inferential statistics</th>
<th>Covariance</th>
<th>Correlation</th>
<th>Beta (for Gold)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.175 (p-value = 0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta (for Gold)</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Portfolio analysis results for subperiod A are presented in figure 12 below. Calculations suggested a portfolio composition of 70% equity and 30% gold. This gave a maximum Sharpe ratio of 0.21 and generated monthly average returns of 0.01 with a monthly average risk of 0.05.
The role of gold in an investment portfolio

Equity 70%  
Gold 30%  
Portfolio return 0.011  
Portfolio risk 0.053  
Sharpe ratio 0.210

Figure 12. Optimal asset allocation 1968.02-1992.06

5.3 Subperiod B (1992 Jul-2016 Nov)

Results for subperiod B are shown in table 4. Gold had almost the same risk (0.080) as equity (0.079), but generated a lower monthly average return (0.011) compared to equity (0.016).

Table 4. Descriptive and inferential statistics for calculated monthly excess returns for gold and equity for the period 1992.07-2016-11

<table>
<thead>
<tr>
<th></th>
<th>Descriptive statistics</th>
<th>Inferential statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Dev.</td>
</tr>
<tr>
<td>Equity</td>
<td>0.016</td>
<td>0.079</td>
</tr>
<tr>
<td>Gold</td>
<td>0.011</td>
<td>0.080</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>-0.004</td>
<td></td>
</tr>
</tbody>
</table>

As presented in figure 13, for subperiod B, an allocation of 20% gold improved the performance of a portfolio only devoted to equites.
The role of gold in an investment portfolio

Figure 13. Optimal asset allocation 1992.07-2016.11

5.4 Subperiod I (1968-1976)

Table 5. Descriptive and inferential statistics for calculated monthly excess returns for gold and equity for the period 1968-1976

<table>
<thead>
<tr>
<th>Descriptive statistics 1968-1976</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.006</td>
<td>0.047</td>
<td>0.002</td>
<td>-0.52</td>
<td>0.02</td>
</tr>
<tr>
<td>Gold</td>
<td>0.009</td>
<td>0.077</td>
<td>0.006</td>
<td>1.35</td>
<td>3.93</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inferential statistics</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.129</td>
<td>(p-value = 0.187)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta (for Gold)</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5 shows average return and variability as standard deviation for assets, and beta for gold, as well as the correlation with equity. As presented in figure 14, a portfolio with 38% gold performed better than a portfolio with 100% equities.
The role of gold in an investment portfolio

5.5 Subperiod II (1977-1986)

Table 6. Descriptive and inferential statistics for calculated monthly excess returns for precious metals and equity for the period 1977-1986

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.019</td>
<td>0.064</td>
<td>0.004</td>
<td>0.42</td>
<td>0.37</td>
</tr>
<tr>
<td>Gold</td>
<td>0.015</td>
<td>0.082</td>
<td>0.007</td>
<td>0.76</td>
<td>2.44</td>
</tr>
<tr>
<td>Silver</td>
<td>0.013</td>
<td>0.129</td>
<td>0.017</td>
<td>0.45</td>
<td>7.25</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.018</td>
<td>0.113</td>
<td>0.013</td>
<td>0.33</td>
<td>2.15</td>
</tr>
<tr>
<td>Risk-free</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Inferential statistics

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta (for Gold)</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correlation (p-value)

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Gold</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.162</td>
<td>(0.077)</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>0.114</td>
<td>(0.216)</td>
<td>0.805 (0.000)</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.191</td>
<td>(0.036)</td>
<td>0.770 (0.000)</td>
</tr>
</tbody>
</table>

Table 6 shows average return and variability as standard deviation for assets, and beta for gold. Table 6 also shows the correlations for the precious metals and equity. The p-value of 0.077 indicates that there is not a linear relationship between gold and equity. As presented in figure 15, the optimal asset allocation in the portfolio included an
allocation of 25% gold and 2% platinum. This hybrid portfolio with SR 0.3312 performed better than a portfolio with 100% equities (SR 0.3034) and marginally better than a hybrid portfolio with only gold and equity (SR 0.3311).

Figure 15. Optimal asset allocation 1977-1986

5.6 Subperiod III (1987-1996)

Table 7. Descriptive and inferential statistics for calculated monthly excess returns for precious metals and equity for the period 1987-1996

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.017</td>
<td>0.089</td>
<td>0.008</td>
<td>-0.35</td>
<td>1.13</td>
</tr>
<tr>
<td>Gold</td>
<td>0.004</td>
<td>0.061</td>
<td>0.004</td>
<td>0.13</td>
<td>1.29</td>
</tr>
<tr>
<td>Silver</td>
<td>0.005</td>
<td>0.088</td>
<td>0.008</td>
<td>0.67</td>
<td>1.09</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.003</td>
<td>0.070</td>
<td>0.005</td>
<td>0.19</td>
<td>1.01</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>-0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inferential statistics</th>
<th>Beta (for Gold)</th>
<th>Gold (p-value)</th>
<th>Silver (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
</tr>
<tr>
<td>Gold</td>
</tr>
<tr>
<td>Silver</td>
</tr>
<tr>
<td>Platinum</td>
</tr>
</tbody>
</table>

The role of gold in an investment portfolio
Table 7 shows average return and variability as standard deviation for assets, and beta for gold. Table 7 also shows the correlations for the precious metals and equity. As presented in figure 16, all the precious metals failed to add diversification benefits to the equity portfolio in this sub period.

<table>
<thead>
<tr>
<th></th>
<th>Exposure gold (gold)</th>
<th>Exposure gold &amp; precious metals (gold+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Gold</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Silver</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Platinum</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Portfolio return</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td>Portfolio risk</td>
<td>0.089</td>
<td>0.089</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.193</td>
<td>0.193</td>
</tr>
</tbody>
</table>

Figure 16. Optimal asset allocation 1987-1996

5.7 Subperiod IV (1997-2006)

Table 8. Descriptive and inferential statistics for calculated monthly excess returns for precious metals and equity for the period 1997-2006

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.017</td>
<td>0.071</td>
<td>0.005</td>
<td>0.06</td>
<td>-0.50</td>
</tr>
<tr>
<td>Gold</td>
<td>0.009</td>
<td>0.058</td>
<td>0.003</td>
<td>-0.04</td>
<td>0.99</td>
</tr>
<tr>
<td>Silver</td>
<td>0.014</td>
<td>0.077</td>
<td>0.006</td>
<td>-0.45</td>
<td>2.45</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.015</td>
<td>0.067</td>
<td>0.005</td>
<td>-0.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>-0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Inferential statistics**

| Beta (for Gold) | 0.3 |

**Correlation (p-value)**

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Gold</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.324 (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>0.310 (0.001)</td>
<td>0.655 (0.000)</td>
<td></td>
</tr>
<tr>
<td>Platinum</td>
<td>0.396 (0.000)</td>
<td>0.687 (0.000)</td>
<td>0.546 (0.000)</td>
</tr>
</tbody>
</table>
Table 8 shows average return and variability as standard deviation for assets, and beta for gold. Table 8 also shows the correlations for the precious metals and equity. As presented in figure 17, the optimal asset allocation in portfolio included an allocation of 19% silver and a high allocation (35%) of platinum. This portfolio performed better than a portfolio with 100% equities and a portfolio with gold and equity by increasing the returns for the same level of risk.

<table>
<thead>
<tr>
<th></th>
<th>Exposure gold (gold)</th>
<th>Exposure gold &amp; precious metals (gold+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>64%</td>
<td>46%</td>
</tr>
<tr>
<td>Gold</td>
<td>36%</td>
<td>0%</td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td>19%</td>
</tr>
<tr>
<td>Platinum</td>
<td></td>
<td>35%</td>
</tr>
<tr>
<td>Portfolio return</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>Portfolio risk</td>
<td>0.056</td>
<td>0.056</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.248</td>
<td>0.277</td>
</tr>
</tbody>
</table>

Figure 17. Optimal asset allocation 1997-2006

5.8 Subperiod V (2007-2016)

Table 9. Descriptive and inferential statistics for calculated monthly excess returns for precious metals and equity for the period 2007-2016

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.010</td>
<td>0.078</td>
<td>0.006</td>
<td>-0.66</td>
<td>3.65</td>
</tr>
<tr>
<td>Gold</td>
<td>0.012</td>
<td>0.100</td>
<td>0.010</td>
<td>0.64</td>
<td>1.32</td>
</tr>
<tr>
<td>Silver</td>
<td>0.013</td>
<td>0.108</td>
<td>0.012</td>
<td>0.37</td>
<td>0.60</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.006</td>
<td>0.093</td>
<td>0.009</td>
<td>0.40</td>
<td>0.89</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>-0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inferential statistics</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta (for Gold)</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation (p-value)</th>
<th>Equity</th>
<th>Gold</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.561</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>0.446</td>
<td>(0.000)</td>
<td>0.788</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.581</td>
<td>(0.000)</td>
<td>0.800</td>
</tr>
</tbody>
</table>
Table 9 shows average return and variability as standard deviation for assets, and beta for gold. Table 9 also shows the correlations for the precious metals and equity. As presented in figure 18, the optimal asset allocation in portfolio included an allocation of 12% gold and 29% silver. This portfolio performed better than a portfolio with 100% equities and a portfolio with only gold and equity. Platinum lacked diversification properties in this portfolio. The hybrid portfolio with gold yielded a return of 0.0107, and the hybrid portfolio with precious metals yielded a return of 0.0109 which gives a slightly higher SR.

![Optimal asset allocation 2007-2016](image)

<table>
<thead>
<tr>
<th></th>
<th>Exposure gold (gold)</th>
<th>Exposure gold &amp; precious metals (gold+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>62%</td>
<td>59%</td>
</tr>
<tr>
<td>Gold</td>
<td>38%</td>
<td>12%</td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td>29%</td>
</tr>
<tr>
<td>Platinum</td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td>Portfolio return</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>Portfolio risk</td>
<td>0.076</td>
<td>0.076</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.140</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Figure 18. Optimal asset allocation 2007-2016

5.9 Comparison (all periods)

Figures 19 & 20 below show comparisons of risks measured by standard deviation and by beta for all the periods. Figure 19 shows the comparison of risk premium per unit of risk for equity and gold during the eight examined periods. The risk is measured by standard deviation. In general equity generated higher risk premiums than gold. Gold almost generated similar risk premiums as equity in subperiods I and V. Figure 20 demonstrates the beta risk for gold, and as the benchmark market always has a beta of one, the Swedish market index that represented equities in this study accordingly had a beta of one (Chua et al., 1990). Gold had a beta lower than one; this means gold is a defensive asset and has the ability to lower the beta risk of the portfolios. However beta values show a slightly upward trend indicating rising systematic risk.
The role of gold in an investment portfolio

Figure 19. Risk premium per unit of risk for all periods

Figure 20. Beta risk for all periods

As presented in the diagram (figure 21) below, correlations between gold and equity for all eight periods exhibited positive values between 0.129 and 0.561. The correlations were significant (p < 0.05) for all subperiods, except for subperiod I (p = 0.187) and subperiod II (p = 0.077). Since the p-value is smaller than the selected α-level for periods F, A, B, III, IV and V the null hypotheses can be rejected in favor of the alternative hypotheses. The results demonstrate that there is sufficient statistical evidence for a significant linear relationship between equity and gold for periods F, A, B, III, IV and V, even though this correlation is small. For the subperiods I and II, there is not enough evidence to reject the null hypothesis. This does not mean that the correlation value is zero which indicates that returns of investments are uncorrelated. Instead it means that there was not sufficient statistical evidence in this study for a linear relationship between assets in subperiods I and II.
The role of gold in an investment portfolio

Figure 21. Correlations between gold and equity for all periods

While not statistically significant, Subperiod I (1968-1976) had the lowest correlation (0.129), and a 38% allocation of gold was suggested. The same allocation was suggested for subperiod V (2007-2016) with the highest correlation (0.561). This was because the allocation of gold was not based only on correlation, but incorporated risk and excess returns. Subperiod V had a small difference in risk premium per unit of risk for equity (0.13) compared to gold (0.12).

Figure 22. Portfolio composition for all periods

Subperiod I had the smallest difference in risk return ratio for equity (0.124) and gold (0.123). This was also the subperiod with the lowest portfolio return but also the lowest portfolio risk. The highest allocation of 38% of gold was suggested for subperiod V. However this allocation dropped to 12% when incorporating precious metals. 29% silver (0% platinum) was suggested while decreasing the allocation of equity from 62%
to 59%. The allocation of 36% gold suggested for subperiod IV (1997-2006) also dropped to 0% when incorporating precious metals to the equation, where 35% platinum and 19% silver allocation was suggested along with 46% equity. The trend was the same for all the four subperiods where precious metals were included. Gold allocation decreased, except for subperiod III where an allocation of 0% gold and 100% equity was suggested from the beginning. This did not change when incorporating silver and platinum. The 100% allocation of equity was persistent in subperiod III as displayed in figure 22 and 23. This is also the portfolio with highest risk (0.089). The zero allocation of gold can partly be explained by the equity bull market in the late 1990s (Conover et al., 2009), and partly by the gold bear market in the 1990s (Hillier et al., 2006). This produced the subsequent bottom risk return ratio (0.06) of gold. Interestingly it also had the next highest correlation (0.522) for examined periods. Silver and platinum also had a low risk premium per unit of risk for this subperiod (0.06 and 0.04). The suggested allocation of gold for subperiod III confirms the conclusion of Johnson & Soenen (1997), that adding gold after 1984 was not beneficial since equities dominated gold after this period. The Johnson & Soenen (1997) study was from 1978 to 1995. The period 1987-1996 in subperiod III caught the remaining period after 1984.

Results for subperiod IV and V suggested a higher allocation of gold in the portfolio (36% and 38% respectively) when incorporating the recent bull-run of gold prices from the end of 1999 to the beginning of 2012. However, after incorporating precious metals, platinum outperformed both gold (0.16) and silver (0.18) in subperiod IV, and generated a risk return ratio of 0.22, which was almost as high as that for the equities (0.23). As a result, a high allocation of 35% of platinum along with 19% silver and 0% gold was suggested for the period 1997-2006. However, the risk premium per unit of risk for platinum decreased to 0.06, which was lower than for gold (0.12) and silver (0.12) for subperiod V, and as a consequence platinum was excluded, and gold (12%) and silver (29%) were re-included in portfolio composition.

![Portfolio composition after adding precious metals](image)
For subperiod IV, only a 0% gold allocation was suggested after incorporating precious metals. This does not mean that gold failed to improve risk return characteristics of an equity portfolio, but that silver and platinum improved the risk return characteristics of an equity portfolio better. As displayed in figure 17, a 36% gold allocation improved the Sharpe ratio before adding precious metals to the equity portfolio. The suggested allocation demonstrates that gold was a more beneficial diversifier than silver and platinum for subperiod II, but a less beneficial diversifier for subperiod IV. It was as well a less beneficial diversifier than silver for subperiod V, and a still more beneficial diversifier than platinum in the same period. These results were as well confirmed by Sharpe ratios, which improved when adding platinum for subperiod II, and silver and platinum for subperiod IV. The Sharpe ratio for subperiod V improved when reducing the gold allocation to 12% and adding a silver allocation of 29%.

Figure 24. Sharpe Ratios for all periods

As shown in figure 24, the hybrid portfolio for subperiod II had the highest Sharpe ratio (0.33) with a portfolio composition of 73% equity, 25% gold and 2% platinum. This was also the portfolio that generated the highest returns (0.018). The portfolio for subperiod IV with the next highest Sharpe ratio (0.28) did not include gold. Subperiod V (which included gold and silver) had the portfolio with the lowest Sharpe ratio (0.14).

Table 10. Increase in returns and reduction in risk after including precious metals in equity portfolio

<table>
<thead>
<tr>
<th>Sample period</th>
<th>Increase in returns</th>
<th>Reduction in risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gold</td>
<td>Precious metals</td>
</tr>
<tr>
<td>F</td>
<td>-8%</td>
<td>-12%</td>
</tr>
<tr>
<td>A</td>
<td>-9%</td>
<td>-17%</td>
</tr>
<tr>
<td>B</td>
<td>-6%</td>
<td>-8%</td>
</tr>
<tr>
<td>I</td>
<td>23%</td>
<td>-7%</td>
</tr>
<tr>
<td>II</td>
<td>-7%</td>
<td>-6%</td>
</tr>
<tr>
<td>III</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>IV</td>
<td>-16%</td>
<td>-6%</td>
</tr>
<tr>
<td>V</td>
<td>8%</td>
<td>11%</td>
</tr>
</tbody>
</table>
Although SR improved with an allocation of gold and precious metals in the equity portfolio, returns decreased with the allocation in all the periods, except for the periods I and V where returns increased. Nonetheless, the risk was reduced more than the reduction in returns with gold and precious metals allocations. Percentage increase/decrease in returns and risk are given in table 10.

6 Discussion

The purpose of this study was to investigate whether gold is an appropriate diversifier for Swedish investors, and whether it can enhance the risk adjusted return in Swedish investors’ financial portfolios, and to find out the optimal weight of gold in a Swedish equity portfolio. The studied period of 47 years was divided into eight different investment horizons to estimate the optimal allocation for different holding periods (as shown in table 1). The study also compared the allocation of gold to other precious metals silver and platinum in subperiods II, III, IV and V (subperiod I was excluded due to lack of price data for silver and platinum). By maximizing the Sharpe ratio, where reward to risk is highest, the optimal allocation of gold in a well-diversified equity-based portfolio was calculated. A number of sub questions were asked to fulfill the overall purpose. The answers that were obtained in the empirical findings section are described and discussed below.

There was a near 0 or weak positive correlation between equity and gold during the examined periods. The correlations were statistically significant (p < 0.05) for all examined periods, except for periods I & II. In contrast to the findings of Pullen et al. (2014) that identified gold as being more of a hedge than a diversifier for US investors, the positive values for correlations indicate that gold is not a hedge against the Swedish equity market and has the ability to act as diversifier for SE investors. This supports the viewpoint that results of studies based on the US market cannot be applied to the Swedish market straightforwardly. Although the correlations between markets are strong, it is not usually 1. Due to the dissimilar macroeconomic factors, equity markets in different countries behave slightly differently. This effect is magnified in an investment portfolio due to the home bias tendency. According to studies (e.g., Johnson & Soenen, 1997; Worthington & Pahlavani, 2007; Wang et al., 2010), gold has an ability to act as an inflation hedge for the US equity market. In contrast to the US, Sweden is a country with strong public finances and a relatively healthy banking system, and the Swedish central bank, Riksbanken, does a good job in keeping the inflation target. Due to the high transparency and the independence of Riksbanken, the public trust is measured to be high in Sweden; this is of course an important factor that helps Riksbanken in their job to maintain price stability. Thus, the need for gold as an inflation hedge in Sweden can be assumed to be negligible. Obtained correlation values less than 1 indicate diversification properties. However, the diagram (figure 21) illustrate that the correlations between gold and equity showed a slight upward trend, suggesting increasing correlations between equity and gold, which may decrease the diversification benefits in the future. This is also reinforced by the upward trend in beta values. Rising correlations and beta values may have a connection to the entrance of the ETFs to the market. This is because ETFs build a bridge between real assets and financial assets. By using derivatives, ETFs give real assets qualities of financial assets. This is desirable only when it comes to accessibility to the asset and not when it comes to correlations. However, the Pearson correlation coefficient is sensitive to extreme values, and outliers in the dataset may have had an effect on the obtained values for
correlations (appendix 2, probability plots). Yet, it is unlikely that the correlations were dramatically affected due to the rather large number of observations. The conclusion that the correlation is less than 1 is very likely valid (appendix 3, scatterplots), and thereby also the conclusion that gold holds properties to diversify Swedish equity portfolios.

Obtained risk return ratios for gold and equity in this study show that gold had a lower risk premium per unit of risk than equity for all the examined time periods, except for periods I & V where the difference was negligible. As it appears, the risk that comes with investing in gold is not fully compensated by returns. This contradicts the risk-reward tradeoff hypothesis, where the market usually rewards high risk with high returns and low risk with low returns. The results also contrast with the results of the study by Jaffe (1989) where the average monthly gold returns were similar to small capitalization stocks. The latter were thought of as high risk assets that therefore generated high risk premiums. The results of Chua et al. (1990, p. 78) also support the notion that gold is a risky asset that accordingly generates high returns. However, the studies compared the returns and did not compute the risk premium per unit of risk, where the risk is also compared. One explanation for lower risk premium per unit of risk for gold found in the present study may be an outcome of its other beneficial qualities, like the higher convenience yield and high liquidity value that reward investors. Another explanation is that assets that move independently of the market generate lower returns due to the lower systematic risk. The standard deviation used in the present study measures the total risk that contains both the diversifiable and non-diversifiable risk. As only the non-diversifiable risk is rewarded, beta values are better measures for comparing risk as they measure the systematic risk (Chua et al., 1990, p. 76). In contrast to the standard deviation, beta also incorporates the correlation for determining the risk of an asset. This means that the magnitudes of the correlation between gold and the equities have a considerable effect on risk of portfolios, and it is therefore an important factor when it comes to diversification. Calculated beta values for gold were close to 0 during all examined periods, but with slightly higher betas for subperiod B (0.5) and V (0.7). As the market beta is always 1, the lower values of beta demonstrate that gold is a less risky asset than the equities (market) and provide an explanation for its lower risk premium. This is because the asset-specific risk is diversifiable and investors are only rewarded for systematic risk. Even though the systematic risk is non-diversifiable, it can still be reduced by adding assets with lower betas to portfolios (Jaffe, 1989). With betas lower than one, gold exhibits the characteristic of reducing the systematic risk for equity portfolios. It should also be added that in addition to possibility of extreme losses, the standard deviation as well measures the possibility of extreme high gains as a risk. But for the investors, only the downside risk is a concern (Bodie et al., 2011, p. 157). The standard deviation was used as a risk measure in optimal allocation calculations. Therefore, in addition to lower beta values which signal lower systematic risk, the low risk premium of gold may be an indication of the lower downside risk as well. This is reinforced by the probability plots for assets shown in appendix 2. Most of the outliers for gold lie in the upside returns, as positive values for skewness for gold demonstrates. Gold as well had fatter tails compared to the equities according to values for kurtosis most of the time. Positive skewness, together with values for kurtosis, predicts lower risk than the obtained values for the risk measured with standard deviation. Equities exhibited positive skewness for subperiod II (0.42) and IV (0.06). For all the other periods, equity exhibited negative skewness. Negative skewness indicates the occurrence of extreme negative returns with
higher probability than implied by the normality assumption (Sheikh & Qiao, 2010). Probability plots presented in appendix 2 contain outliers for assets in all eight periods, except for equity in subperiod IV, which also had an almost zero value for skewness. Gold exhibited a skewness of -0.04 for subperiod IV and positive skewness for all the other periods. This indicates occurrences of extreme positive returns with higher probability than implied by the normal distribution. Positive skewness, according to Baur and Lucey (2010), can as well add safe haven properties. According to Emmrich & McGroarty (2013) positive skewness in gold returns can mitigate the negative skewness added by equities to the portfolio returns.

Although risk premium was low, obtained correlations between -1 and +1 indicate that an allocation of gold will improve the risk return characteristics of an investment portfolio only devoted to domestic equity. Results show that on average the Sharpe ratios improve when adding gold to portfolios only devoted to equities, evidencing that gold improves risk return characteristics of equity portfolios (figure 24). However the portfolio returns decreased with the risk in hybrid portfolios. Still, the returns for unit of risk increased in hybrid portfolios compared to equity portfolios. This is consistent with MPT that states that adding assets with low correlations improves portfolio performance even with assets that are not superior to one another at stand-alone basis. This is because in addition to risk and return, MPT also integrates correlation in calculations of asset allocation. Results showed that the reduction in returns are less than the reduction in risk (table 10). By diversification the returns are maximized for the given level of risk. This means that an allocation of gold should improve the risk return characteristics of an investment portfolio only devoted to equity. The results confirm this hypothesis.

An allocation of gold improved the risk return characteristics of a portfolio only devoted to equity for all the examined periods, except for subperiod III. The zero allocation of gold in subperiod III can partly be explained by the equity bull market in the late 1990s (Conover et al., 2009), and partly by the gold bear market in the 1990s (Hillier et al., 2006). This produced the subsequent bottom risk return ratio (0.06) of gold, and interestingly also the next highest correlation (0.522) for the examined periods. This is probably due to the observation that demand drives for gold are affected by the wellbeing of the economy. With the exception of subperiod III, Sharpe ratios for equity portfolios improved for the all the other examined periods. Evidencing that gold improve the risk return characteristics of an investment portfolio only devoted to the Swedish domestic equity market.

This study found that an allocation of 26% gold maximized the Sharpe ratio in an equity portfolio with holding periods of more than 40 years. Almost the same average allocation (25%) was suggested for the portfolios with holding periods above 20 years. The average amount was slightly higher (28%) for the portfolios with around 10 year holding periods. The portfolio in sub period II, with 25% allocation of gold, had the highest Sharpe ratio where the reward to volatility is highest. These allocations are consistent with studies by Chua et al. (1990) and Conover et al. (2009), where almost the same amount (25%) of gold allocations were suggested. These studies also examined a long-term investment horizon similar to that of the present study. Even though the suggested allocation was the same for US investors and SE investors, above studies did not examine the optimal allocation that maximize the Sharpe ratio. Therefore it is not clear if the optimal allocation is the same for US and SE investors at long-term investment horizons.
The results showed that beneficial allocation of gold vary depending on the time period examined. This supports the results of the studies by Conover et al. (2009) and Johnson & Soenen (1997). According to the authors, the benefits of gold are not consistent over time. They also showed that the benefits of gold allocation vary depending on the time period examined. The beneficial allocation computed in this study ranged from 0% to 38% depending on the time period examined. For sub periods II and V, the optimal allocation of gold was as high as 38%. This period incorporated the gold bull markets and downturns in the economy. Optimal gold allocation declines during equity bull markets and gold bear markets, and conversely, optimal gold allocation goes up during equity bear markets and gold bull markets. As markets are dynamic, ups and downs are the nature of the markets. According to Conover et al. (2009), benefits of precious metals have a strong connection with the monetary policies. However, due to market efficiency, both strategic and tactical allocation with an ex ante indicator provide similar improvement. Therefore the timing of the market is almost an impossible equation to solve. Since there is no point in using switching indicators, it is essential to find an allocation that can manage both downturns and upturns in asset returns. Therefore, an average allocation is perhaps to prefer. Average allocation also gives more strength to the results by reducing potential influence of outliers. The average optimal allocation was 28% for portfolios with 10 year holding periods. However, this allocation dropped to 9% when incorporating precious metals into the calculations.

Gold was only more beneficial as a diversifier than the other industrial precious metals silver and platinum in subperiod II. Silver was a more beneficial diversifier than platinum in subperiod V, and platinum was a more beneficial diversifier in subperiod IV. Consequently, gold cannot be said to be a more beneficial diversifier than the other industrial precious metals silver and platinum. Therefore the optimal allocation of gold suggested by calculations that maximize the Sharpe ratio is not valid when incorporating other precious metals. The suggested high allocation of gold exposure decreased with silver and platinum exposure. An equity portfolio with silver and platinum performed better than an equity portfolio with gold for subperiod IV. For subperiod II, additional platinum, and for subperiod V, additional silver improved the performance of the equity and gold portfolio. A 0% allocation of gold was suggested for subperiod IV, after incorporating precious metals. Then again, a 36% gold allocation improved the Sharpe ratio before adding precious metals to the equity portfolio, as it is displayed in figure 17 (optimal asset allocation 1997-2006). However, the Sharpe ratio improved further with 19% silver, 35% platinum, and excluding gold. This is understandable since gold performed worst in this period, with a 0.16 risk return ratio. Platinum with a risk return ratio of 0.22, which was almost as high as that for equities (0.23), outperformed both silver (0.18) and gold. Subperiod IV was also the period where Sharpe ratios improved most after including precious metals.

A gold allocation of 0% to as high as 38% was suggested, before including precious metals silver and platinum into the portfolio. After including silver and platinum the highest gold allocation dropped to 25%. In a strategic portfolio the optimal allocation should be somewhat constant, and therefore average weightings are to be considered. Correlations between gold, silver and platinum were high (around 0.7-0.8) indicating that common factors affect their returns. These findings are reinforced by the values for correlations between precious metals and equity that formed an accompanying pattern. Subperiod II had correlations around 0.1-0.2 for gold, silver, and platinum with equities. Correlations for subperiod III lay around 0.5-0.6, and for subperiod IV and V.
correlations lay around 0.3–0.4 and 0.4–0.6, respectively. These correlation values indicate that a moderate allocation of all three precious metals is to prefer to allocation of just one. However, the suggested allocations of precious metals in the portfolios fluctuate, but a strategic portfolio strategy needs steadfast allocations. Since there are no valid switching indicators, mean weightings are implied. Mean weightings for subperiods (holding period >8) with precious metals suggested a 70% dominant weighting for equity, and a 9% moderate weight for gold, as well as weights of 12% for silver and 9% for platinum. This allocation of gold is in line with the suggested allocations from other studies (e.g., Jaffe, 1989; Lucey et al., 2006; Hillier et al., 2006; Dempster & Artigas 2010; Emmrich & McGroarty, 2013). An allocation of silver, gold and platinum, in contrast to only gold, confirm the results of earlier studies (e.g., Conover et al., 2009; Daskalaki & Skiadopoulos, 2011; Hillier et al., 2006). Although the allocations are equal to those of earlier research, beta values are not as low as in earlier studies. This indicates that gold is not a defensive asset to the same degree for SE investors as it is for US investors (e.g., Jaffe, 1989 (beta 0.09); Chua et al., 1990 (beta 0.11)).

Risk management is not only maximization of returns; it is also critical thinking and optimization of returns. Gold mining is associated with damaging footprints on the planet. Although the suggested allocations by the Sharpe ratio maximize the portfolio performance, it may not optimize the portfolio performance. This is because the Sharpe ratio only calculates the risk given by the volatility of the returns at a short-term monetary basis. Although this makes calculations somewhat manageable, risk can arise from many other sources. Environmental damage threatens the stability of long-term returns and is therefore a risk to consider. Therefore sustainability should be included in optimal asset allocation calculations.

7 Conclusions

The prolonged and ongoing turbulence in economies and uncertainty in financial markets make risk management strategies like portfolio diversification more crucial in the investment world today. At the same time, studies show that the correlations among traditional assets, like equities and fixed incomes, are rising and resulting in shrinking diversification benefits. This, together with the low fixed income yields and poor returns in equity markets, has forced irresolute investors to search for alternative investments to diversify their financial portfolios (Pullen et al., 2014, p. 77, Ratner & Klein, 2008; Bernhart et al., 2011). At the same time, the latest Bull Run in gold, together with its monetary history, has caught the attention of the investors. Since the demand drivers differ for real assets compared to financial asset, at least in theory, real assets are supposed to be less correlated with financial assets. Therefore, the commodity gold should be less correlated with equities and consequently be a proper diversifier in a financial portfolio.

However, in the investment world there are different views on whether to invest in gold or not. Some gold bearish investors more or less banish the metal from their investment portfolios, arguing that gold is an asset with no intrinsic value. At the same time, gold bullish investors tend to over-weight their investment portfolios with gold. While the need for academic literature that gives clarity to the matter is vast, there is not much academic literature that investigates the role of gold as an investment. In addition, the existing financial literature in the subject focuses mostly on US investors. Since factors
like home bias and exchange rate differentiate SE investors from US investors, the results from those academic studies cannot be applied to SE investors straightforwardly.

Therefore, the purpose of this study is to fill the gap in the existing literature and shed some light on the investment role of gold for Swedish investors. Accordingly, this study takes the perspective of Swedish investors to examine whether gold is an appropriate diversifier in a well-diversified domestic equity portfolio. Further, the study also aims to find the optimal allocation of gold in an investment portfolio that is only devoted to domestic equity. There are several ways of obtaining exposure to gold, and this study examined the exposure produced by gold bullion. In addition, benefits of gold allocation were compared with benefits of direct exposure to other precious metals silver and platinum. Since this study took the perspective of Swedish investors, a domestic market proxy was used to represent the well diversified domestic equity portfolio. Precious metals, including gold prices, were converted to Swedish krona using up to date exchange rates. Optimal allocation of gold, silver and platinum were calculated under the Markowitz MPT theory framework and maximization of the Sharpe ratio.

Non-negative, low correlations of gold, silver and platinum are consistent with the expectation of beneficial diversification properties. Gold had the lowest correlation with the Swedish equity market all through the studied periods, and data revealed that the correlations of gold were less than plus one for all examined periods. However, the results also show that the correlations are not consistent over time. Overall, an allocation of gold improved the risk return characteristics of an investment portfolio only devoted to equity. Portfolio performance improved with exposure to gold during much of the studied periods. The only exception was subperiod III, where all the precious metals failed to improve the Sharpe ratio. Subperiod III incorporated the latest Bull Run in equities. This period also contained the portfolio with the highest portfolio risk (0.089). At stand-alone basis, silver and platinum outperformed gold only in subperiod IV, supporting the findings of earlier studies (e.g., Hillier et al., 2006; Conover et al., 2009) that gold is superior to other precious metals as an investment at stand-alone basis. This is probably because the gold demand drivers are slightly different from that of other precious metals. In addition to industrial use, gold demand is also coloured by its symbolic value and its long monetary history. Though allocations as high as 36-38% was suggested for subperiods IV and V, which incorporated the recent bull run in gold, the suggested allocation dropped drastically (0%-12%) after adding precious metals. At stand-alone basis, equities outperformed precious metals, including gold, throughout the study period. This indicates that the gains of precious metals lie in their diversification benefits rather than in extraordinary returns.

The results showed that an average allocation of 25% maximized the Sharpe ratio in a strategic well-diversified domestic equity portfolios. This is consistent with earlier studies by Chua et al. (1990) and Conover et al. (2009). However, this allocation dropped to 9% when incorporating precious metals into portfolios. Then again, in a risk management perspective, a moderate allocation of less than 10% can be considered more desirable in a well-diversified portfolio than devoting a high allocation of 25% to a single asset. In summary, the answer to the research question “Is gold an appropriate diversifier in a financial portfolio for a Swedish investor, and can gold enhance the risk adjusted return in Swedish investors’ financial portfolios?” is yes. The results showed that the correlation between Swedish equity and gold is low and that on average an allocation of gold enhances the risk adjusted return in Swedish investors’ financial
portfolios, and therefore gold is an appropriate diversifier in a financial portfolio for a Swedish investor.

7.1 Theoretical contributions
The academic literature that investigates the role of gold as a diversifier is scarce, and the literature that takes the perspective of Swedish investors is non-existent in my knowledge. Sweden does not have tradition of investing in gold, but gold is entering the investment world as a much needed alternative investment. This study fills the gap in academic literature needed in this area by taking the perspective of Swedish investors. In addition, by gathering and combining historical data from different sources, the study extends the earlier studies by examining 47 years of data and presenting benefits of adding gold to a Swedish equity portfolio. Further, the strategic allocation of gold was examined by dividing the examined period into long- and short-term holding periods to observe the constancy of the allocation. The study also examined the direct exposure produced by the investment channel gold bullion and compared the diversification benefit of industrial precious metals silver and platinum with diversification benefits of gold. Optimal allocation of gold is found by using mathematics in modern portfolio theory and maximizing the Sharpe ratios.

7.2 Practical implications and recommendations
Gold is an appropriate diversifier in a financial portfolio for a Swedish investor, and gold enhances the risk adjusted return in Swedish investors’ financial portfolios. However, optimal allocation vary depending on the examined time period and diversification benefits increased with addition of the precious metals silver and platinum. Therefore, an average weighting of 9% gold, 12% silver, and 9% platinum is to prefer to enhance the performance of a strategic Swedish equity portfolio, rather than just including gold.

Although suggested allocations maximize the risk return characteristics of the long-term investment portfolio, the returns and risks are only calculated on a short-term monetary basis. Since precious metals, especially gold, leave long-term damaging footprints on the environment, the suggested allocation may not be sustainable in the long term. Since investors have the means to influence social and environmental sustainability positively, this should as well be considered when making investment decisions. Therefore, optimal asset allocation to optimize rather than to maximize portfolio performance is to be preferred.

7.3 Truth criteria
Validity and reliability are fundamental aspects of empirical research. In general, validity concerns whether the research methods and subsequent observations (e.g., measurements) provide an adequate reflection of the truth (Roe & Just, 2009, p. 1266). Reliability refers to repeatability or consistency of measurements (Trochim, 2006). When conducting measurement it is always going to be errors. It is important to minimize these errors in order to secure the reliability and in extension the validity of a study. While measurements like the behaviour of organisms and psychological constructs has some degree of subjectivism, numerical data that are based on quantities like the price of gold can be considered as an objective measurement. Objectiveness gives a consistency of measurement and makes data repeatable and more reliable. Clear conceptualization of research (i.e. what is to be measured) and standardization is a way of making data less subjective and more consistent (i.e., reliable). This as well improves
the validity of the study, since this makes sure that used implements measure the desired subject and not something else (Bryman & Bell, 2011, p. 42). As reliability and validity are quality assessments of a study and provide more strength to the result, the source and a brief description of all the variables used in this study are given in the methodology chapter to increase the transparency. The research process is also described thoroughly. Data was collected from reliable sources and the names of the sources and the search codes are listed. Well-established software programs (Minitab and Excel) were used to process the data. Data was collected from homogenous data sources and the most reliable primary data source was used when available. This makes repetition of the study easier. In addition, well-known theories and calculation methods are applied to ensure the validity of the conclusions.

7.4 Limitations and future research
Despite old wisdom, many individual investors still do not fully take advantage of the power of diversification. The major reason for this may be the complicated algorithm behind MPT. The Single-index model derived from MPT simplifies the MPT mathematics. There are pros and cons with these models. When computing a portfolio with a large set of assets, the estimations needed to apply MPT escalate. This can magnify simple errors. As a result, with the increasing number of assets, MPT calculations can both be a formidable task and increase the probability of errors. The Single-index-model addresses these shortcomings by using a proxy for the common macroeconomic factor, which reduces the number of needed estimations. However, this simplistic decomposition of risk can result in an inefficient diversification. One of the reasons for increased interest in investing in gold is the perceived risk in the economy. Investors are demanding a high risk premium for bearing the additional risk in equity. The risk that an asset holds over the risk-free rate is compensated by a risk premium. The Risk-free rate used for this study was close to zero (-0.40% to 0.37%) during the entire examined period. To make comparisons easier with other studies, the risk-free rate was not adjusted for inflation and the results can therefore be deceptive. As the nominal rates were used in the calculations for both the risk-free and the risky assets, the real rates of returns are likely somewhat lower than the values given for optimal portfolio returns (Bodie et al., 2011, p. 146). However, this does not affect the optimal allocation of assets in the portfolio since nominal prices were used for all the assets in the study.

Since expected returns or risks are not directly observable, optimal allocation of assets were calculated by analysing realized returns and risk. The use of historical data can be rightfully questioned. Past data can give a fairly good indication of future prices. However, if the markets are efficient, they incorporate all the information in asset prices. That means that it is impossible to predict and outperform the market. At the same time, the past is the only way to predict the future market. However, the returns of most of the examined assets contained outliers and diverted from a normal distribution. Since this can lead calculations of optimum allocations astray, there is a need to investigate methods that are not dependent on normality assumptions to determine the risk and return of an asset. Although a large set of data often compensate for the deviations from the normality assumption, log returns can be used to satisfy the normality assumption. Then again, this makes diversification calculations difficult. The standard deviation measures both downside and upside risk, but extreme gains are not a risk for investors. Therefore, other risk measures may be preferred. Value at risk (VaR) calculations can be applied to exclude the upside risk and thereby only measure the
downside risk of a portfolio. Finding statistical measures that compensate for above shortcomings and confirming the results of the optimal allocations of precious metals in a Swedish equity portfolio may be a suitable topic for future studies. Since negative environmental and negative social aspects add long-term risks to investments portfolios, the ethical risks and sustainability as well should be included in calculations. Another area for future research is extending the allocation to include precious stones, such as diamonds, since their demand drivers are hybrids of precious metals and equities.
The role of gold in an investment portfolio

References


The role of gold in an investment portfolio

John Wiley & Sons.


### Appendix 1 (Covariances for assets)

#### Covariances: Equity (68-16); Gold (68-16)

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**Covariances: Equity (07-16); Gold (07-16); Silver (07-16); Platinum (07-16)**

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Platinum (07-16) 0,00873593
Appendix 2 (probability plots for assets)
Appendix 3 (scatterplots for gold vs equity)
Appendix 4 (gold bull-run)

Gold bull-run 2012