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Mathematical problem solving in textbooks from twelve countries

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ABSTRACT

A selection of secondary school mathematics textbooks from twelve countries on five continents was analysed to better understand the support they might be in teaching and learning mathematical problem solving. Over 5700 tasks were compared to the information provided earlier in each textbook to determine whether each task could be solved by mimicking available templates or whether a solution had to be constructed without guidance from the textbook. There were similarities between the twelve textbooks in the sense that most tasks could be solved using a template as guidance. A significantly lower proportion of the tasks required a solution to be constructed. This was especially striking in the initial sets of tasks. Textbook descriptions indicating problem solving did not guarantee that a task solution had to be constructed without the support of an available template.

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KEYWORDS
Mathematics textbooks; mathematics tasks; mathematical problem solving; secondary school

1. Introduction and purpose

Ambitions to shift towards a richer view of mathematics have appeared in several countries included in this study since the 1990s (Boesen et al., 2014). For example, curricula and national standards in Australia, Canada, India, Ireland, Scotland, Singapore, South Africa, Sweden and the USA now stress the importance of competencies such as problem solving, reasoning and the ability to connect concepts to each other (Boesen et al., 2014; Davis, Smith, Roy, & Bilgic, 2014; Department of Education, Republic of South Africa, 2008; Henderson, 2012; Ministry of Education, Ontario, 2005). It has been shown that a problem-solving approach to teaching and learning mathematics and a certain degree of struggle are important elements in attaining this wider perspective on school mathematics (Hiebert & Grouws, 2007; Schoenfeld, 1985). Although some progress has been made, there is still a large and persistent discrepancy between the richer view of mathematics and what students actually learn (Mullis et al., 2012). In some countries it seems that students still learn mathematics mainly by rote (Boesen et al., 2014; Hiebert, 2003; Lithner, 2008). By 'rote' we mean an imitative approach that focuses on procedures rather than on problem solving and reasoning.

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In classrooms all over the world, textbooks are used to support the teaching and learning of mathematics (Schmidt et al., 2001). In this study, we analyse a selection of secondary school textbooks from twelve countries on five continents and discuss similarities and differences in different parts of the world (Floden, 2002; Pepin & Haggarty, 2001). We focus on the extent to which textbooks may support a problem-solving approach to learning.

2. Background

2.1. Problem solving as a key competence

Mastering mathematics can be seen as possessing competencies such as problem solving, mathematical reasoning, procedural fluency and conceptual understanding (Boesen et al., 2014; National Council of Teachers of Mathematics (NCTM), 2000; Kilpatrick, Swafford, & Findell, 2001; Niss, 2003). Students thus need an education that explicitly focuses on each of these competencies (Hiebert, 2003). Procedural fluency is an essential competence, both in itself and in the sense that it is connected to other competencies (Kilpatrick et al., 2001). Yet a one-sided focus on applying given mathematical procedures does not promote mathematical learning effectively, and there is compelling evidence that it often leads to rote learning, which is a common cause of learning difficulties (Hiebert, 2003). Boaler (1998) argues that mathematics teaching should emphasize a deeper understanding of the subject and pay less attention to computation, rules and procedures.

In this paper, the term *task* is used to refer to a textbook exercise intended to support the student's learning (Halldén, Scheja, & Haglund, 2008). For the purpose of this paper, we distinguish between tasks of routine character and mathematical problems. The latter are tasks in which the student does not have access to a complete solution procedure, but instead has to construct (large parts of) the solution (Schoenfeld, 1985; National Council of Teachers of Mathematics (NCTM), 2000; Niss, 2003). Thus, problem solving means engaging in mathematical problems (National Council of Teachers of Mathematics (NCTM), 2000). There is a distinction between a student practising by constructing solutions and imitating solutions. The latter is not seen as problem solving but is merely completing routine tasks that do little to enhance the development of problem-solving competence (Niss, 2003; Schoenfeld, 1985). Other possible aspects of problem solving, such as challenges (Schoenfeld, 1985) or explorations (National Council of Teachers of Mathematics (NCTM), 2000; Niss, 2003), are not included in our definition since they are not the focus of this study. Whether a task is a mathematical problem or not depends not only on properties of the task but also on the competence of the student (Schoenfeld, 1985).

The problem-solving process may also include the use of competencies such as procedural fluency, which is likely to gain strength from a deeper understanding of mathematics (Hiebert & Grouws, 2007). Fan & Bokhove (2014, p. 481) acknowledge in their literature survey that 'learning of algorithms has suffered from an alleged dichotomy between procedures and understanding' but conclude that 'the majority of more recent research seems to indicate that products and processes, procedures and understanding, go hand in hand.' Problem solving, therefore, is not only a valuable competence in itself, but also a way of approaching mathematics to reach other goals. Problem solving has the potential to elicit mathematical reasoning, which is essential and enhances mathematical understanding (Lithner, 2008).

In contrast to mathematical problems are tasks of routine character, where suitable, available solution methods are identified and implemented (Lithner, 2008). Superficial characteristics of the task may guide the student in their strategy choice, and the student may refer to a textbook to find an available algorithm or solution template. An algorithm can be defined as 'a finite sequence of executable instructions which allows one to find a definite result for a given class of problems' (Brousseau, 1997, p. 129). No step in the chain of instructions depends on any unforeseen circumstance earlier in the sequence of instructions, nor 'on the finding of new information, on any new decisions, any interpretation, and therefore on any meaning that one could attribute to them' (Brousseau, 1997, p. 129). This avoidance of meaning may be effective when an algorithm is used only to solve a task, but it is a hindrance when one is supposed to learn something from solving it (Brousseau, 1997). In this study, we use the term *template* to include algorithms, calculations and pre-specified procedures such as rules, solved examples and other task solutions. Including a justification for an action in the template does not seem to alter the learning outcome for the student (Norqvist, 2017).

2.2. The textbook as an artefact used in the teaching and learning of mathematics

How textbooks are used varies a lot, but there is no doubt that textbooks are one of the main influencing factors in the teaching of mathematics (Rezat & Strässer, 2014). Rezat and Strässer (2014) argue that the textbook is part of a didactical tetrahedron, shaping didactical situations together with the teacher, the students and the mathematics. Results from the Trends in International Mathematics and Science Study (TIMSS) 2011 show that more than half of students in secondary school in countries such as Australia, Finland, Singapore, Sweden, South Africa, and Canada are taught in an environment where the mathematics textbook is the basis of instruction. In the USA, the mathematics textbook is the foundation of mathematical education for 48% of students, and in some countries the percentage is higher than 90% (Mullis et al., 2012).

Students and teachers often rely on the textbook to reflect what is to be considered important and part of mathematics education (e.g. Johansson, 2006; Schmidt et al., 2001; Schmidt, Gueudet, Pepin, & Trouche, 2012; Vincent & Stacey, 2008), despite possible discrepancies between textbooks and national curricular goals, standards and research findings. In the framework used in TIMSS, the textbook is mentioned as a factor in the potentially implemented curriculum (Mullis et al., 2009). Schmidt et al. (2001) have also shown there is a clear relationship between textbooks and classroom instruction. The textbook can provide one of many images of students' opportunities to learn (Thompson, Senk, & Johnson, 2012). The opportunities to learn are created by the textbook combined with other factors such as the way it is used by students and teachers (Shield & Dole, 2013; Valverde et al., 2002). Given that textbooks are regarded as an authority by teachers (Pehkonen, 2004), Shield and Dole (2013) argue that textbook analysis is a valuable asset when having discussions about the design of textbooks and teaching. Shield & Dole (2013) suggest textbook analysis as a first level, or content-only analysis, as one way of examining the opportunities to learn that are available to students.

Tasks are a major component in many textbooks. Doyle (1983, p.161) states that 'tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information'. Stein, Remillard, and Smith (2007, p. 346) argue that

'the tasks with which students engage determine what they learn about mathematics'. Stein et al. (2007) conclude that different tasks provide different opportunities to learn, and that the cumulative effect of this explains to students what mathematics is and how one does it. A task may signal whether a student should engage their understanding or just produce an answer (Gresalfi, 2009). Hence, an analysis of tasks can provide information on what is expected from students, and may serve as a complement to content analysis of textbooks (Li, 2000).

Newton and Newton (2007) suggest that textbooks should have a greater focus on conceptual understanding. Previous studies have shown that the proportion of textbook tasks requiring more than using known procedures is low (Brehmer, Ryve, & Van Steenbrugge, 2016; Lithner, 2004; Schmidt et al., 2012). Pointers to solutions, such as key words, leading questions, and step-by-step procedures to memorize, steer students into skill-based task solving without a serious emphasis on understanding (Palm, Boesen, & Lithner, 2011; Schoenfeld, 2012). Textbook examples show the solution to tasks, and sometimes also the reasoning used to come up with the different steps of the solution. Even though this reasoning may be founded on mathematical properties, it can be argued that the whole textbook example is seen mainly as presenting a rule or algorithm (Stacey & Vincent, 2009). Subsequent tasks that can be solved by using the presented algorithm are likely to mediate a focus on rules rather than reasoning.

Studies of the frequency of reasoning-and-proving tasks in textbooks (Bieda, Ji, Drwencke, & Picard, 2014; Stylianides, 2009) have shown that textbooks offer limited opportunities to practise anything other than empirical argumentation, that is, examining specific cases rather than verifying answers through a more general approach. This is especially so in the algebra sections of textbooks (Stylianides, 2009). Furthermore, as Bieda et al. (2014) point out, the reasoning-and-proving tasks are more frequently found in the extension sections of textbooks, which are less likely to be used in the classroom. Instead, procedures, earlier task sets and the easier tasks, seem to be prioritized (Kaur, 2010; Sidenvall, Lithner, & Jäder, 2015; Stein et al., 2007).

A textbook may be regarded as something that describes or even defines the mathematics being taught (Rezat & Strässer, 2014). Some mathematics textbooks are explicit about the performance expectations for students (Schmidt et al., 2001) and so label the tasks to indicate specific levels of difficulty (Brehmer et al., 2016), or approaches such as 'Investigate' or 'Apply' or specific competencies such as 'Fluency' or 'Reasoning'. Regarding the textbook as an artefact influencing the teaching and learning of mathematics, all attributes of a task may be taken into consideration, including the structure of tasks in terms of their sequencing and the general descriptors used.

2.3. Textbook task analysis

Students spend a large proportion of their learning time with textbook tasks (Kaur, 2010; Mullis et al., 2012; Stein et al., 2007), which can be solved either by imitating a template or by constructing a solution. It has been shown that students tend to avoid constructing a solution when explicit guidance is available (Sidenvall et al., 2015). Even when faced with a mathematical problem (i.e. that requires constructing a solution method), it has been shown that students will strive to use familiar algorithms, even though they are not applicable (Jäder, Sidenvall, & Sumpter, 2017). They may use the guidance from resources such

as the textbook (Boesen, Lithner, & Palm, 2010; Brousseau, 1997; Hiebert, 2003; Lithner, 2003) or, if more easily accessible, from peers or teachers (Sidenvall et al., 2015; Stein et al., 2007). When working on textbook tasks, it may be possible for students to connect a task to an example or other information in the textbook, perhaps by using superficial properties of the task. This provides the student with a template for how to solve the task, which may be done in a routine way without considerations of mathematical properties (Brousseau, 1997).

One way for students to develop problem-solving competency is by working with tasks where the option of imitation of an available solution template is not as easily accessible. It is thus interesting to investigate the relation between a specific task and other information the textbook provides the student. Although templates for solving tasks may be found anywhere in the textbook, it is likely that students will search for solution templates in the text preceding a task rather than later in the book (Lithner, 2003; Sidenvall et al., 2015). Therefore, although it is difficult to know how the textbook is used in practice, in this study only text that precedes a task is considered when determining relatedness. The template may be found in a solved example, or in a theory paragraph, or in a previous task. Students may proceed by linking a task to previously used procedures (Silver, 1986).

By comparing textbook tasks, and specifically the required solution, to the preceding material in the textbook, it is possible to distinguish three kinds of tasks. *High Relatedness tasks (HR)* can be solved using a template presented in the textbook, *Global Low Relatedness tasks (GLR)* are tasks for which there is no template to support solution, and *Local Low Relatedness (LLR)* are tasks that can be completed using an available template, but with minor modifications.

3. Research questions

Problem solving is a central competence in mathematics, but rote learning and a focus on recalling algorithms often still dominate the teaching and learning of mathematics (Mullis et al., 2012). Given that textbooks and the tasks in them constitute an influential artefact used in the teaching and learning of mathematics, it is important to investigate them. The proportion of different kinds of tasks and what they require and enable students to do may indicate the extent to which a textbook lines up with contemporary competence frameworks (National Council of Teachers of Mathematics (NCTM), 2000; Kilpatrick et al., 2001; Niss, 2003). Identification of differences and similarities between the textbooks used in different countries also allows for discussion of opportunities to learn (Floden, 2002; Pepin & Haggarty, 2001).

The sequencing of different sets of tasks, and the descriptive labels or headings sometimes attached to a task or set of tasks, may also influence the students' view of different mathematical competencies and their opportunities to learn them. It is therefore of interest to investigate the extent to which textbooks provide students with tasks with or without solution templates, and also how each task is presented within the structure of the textbook in terms of sequencing and possible descriptors.

We thus set out to investigate the following questions:

- What is the percentage of HR, LLR and GLR tasks in chapters on algebra and geometry in common mathematics textbooks used in secondary school in twelve countries?

- How does the organization of the material in a textbook, in terms of sequencing and descriptors of tasks or task sets, reflect the percentages of HR, LLR and GLR tasks?

4. Method

4.1. Selection of countries and textbooks

The selection of countries was based on a desire to include different parts of the world, but was also affected by pragmatic constraints. Firstly, in order to avoid the methodological difficulty of making translations, the selection was limited to countries where English or Swedish is a common language in secondary school. Secondly, as there have been few studies of textbooks from more than two or three countries that could guide the selection procedure, the selection in this study is partially explorative. This led to the decision to adopt a geographical distribution including samples from five continents, since an underlying question is whether textbooks are different in different parts of the world. The countries selected also show variation in their achievement in international tests such as TIMSS (Mullis et al., 2012) and PISA (OECD, 2014).

A questionnaire inquiring what textbooks or textbook series (by the same authors and/or from the same publisher, and with a common name) were commonly used was sent to informants, such as mathematics educators, teachers, school administrators and researchers in thirteen countries. All countries were included in the study, except for England, from where no reply to the questionnaire was received. We decided to focus our analysis on the textbooks or textbook series most frequently mentioned in the replies to our questionnaire. In cases where two or more textbook series were said to be in common use, textbooks from all series were initially examined to see whether there were any major differences. No such differences were found, so it was decided to analyse only one textbook series per country. A list of the countries and the textbooks analysed is provided in Appendix 1.

The decision to include textbook series in the selection along with individual textbooks was based on the need to meet the selection requirements regarding mathematical content. This content was labelled in accordance with the TIMSS content framework at its most detailed level, which covers 44 mathematics topics (Mullis et al., 2008). Two topics that are central to mathematical education internationally were selected: geometry and algebra. The rationale for this choice was that these two topics were found and could be compared in all textbooks or textbook series. Geometry seems to stand out as a topic where students are given greater opportunities to practise their reasoning-and-proving competence (Stylianides, 2009). Algebra, on the other hand, is as a topic where previous studies have shown a greater algorithmic focus (Stylianides, 2009; Vincent & Stacey, 2008), but at the same time more complex performance demands (Schmidt et al., 2001). Within these two topics, we chose to focus on *equations and formulas* and *perimeter, area and volume* (Mullis et al., 2008). These specified topic areas were components of all textbooks or textbook series, although intended for somewhat different grade levels. Nevertheless, all the chapters analysed were part of secondary school education covering similar material, and were deemed comparable.

4.2. Data analysis and classification procedure

Tasks were analysed in relation to the information in the textbook and categorized as having high (HR), local low (LLR), or global low relatedness (GLR) to presumptive templates.

A task is seen as including *one* question or prompt. In most cases this also lined up with one delimited exercise. More comprehensive exercises, however, could be broken down into two or more tasks, requiring different and unrelated solutions. The analysis considered each of these tasks separately. Thus, in the analysis no task was significantly different from the others regarding the extent of the required solution, which makes the proportions in terms of the number of tasks more valid.

The analysis was based on a method proposed by Lithner (2004) for analysing tasks in a university mathematics textbook. It was later refined by Boesen et al. (2010) and Palm et al. (2011) for analysing mathematics test tasks in secondary school. The overall approach was to determine for each task whether a template that could be used to solve the task was easily available in the textbook.

Preliminary analysis identified three situations in which finding the template was possible, establishing a relatedness between template and task. The first situation was when the template was close to the task in that it appeared earlier in the same textbook section. A section of a book was defined as a well-delimited space within a chapter, with its own (sub)title in the contents section. Sections often spanned 1–10 pages. A second situation was when a template could be found through the solutions to previous tasks within the same section. The relatedness in these two situations was thus supported by a common section heading. The third situation was when the template was not found within the same section, as described in the first two situations, but instead appeared in other sections, earlier in the textbook. The more the parameters in a task resemble the ones in a template at hand, the less cognitively demanding the identification of a suitable template will be (Palm et al., 2011). In this study, we will use a set of task characteristics suggested by Palm et al. (2011) as important for helping students detect the relatedness between a task and the corresponding template.

The preliminary analysis also showed that in most textbooks the tasks or task sets had descriptors such as labels or headings. For task sets in some textbooks, these descriptors used recurring sequences within each section (e.g. Level 1-Level 2-Level 3-review or Practice-Apply-Challenge). A label or heading sometimes described an approach or competency (e.g. Investigate, Apply, Fluency or Reasoning). This general information was recorded for each task.

For each task, the analysis procedure comprised the following steps:

Step 1: Recording general information about the task.

Step 2: Formulating reasonable solutions to the task.

Step 3: Searching the same textbook section for templates that could be used to solve the task. The template could, for instance, be found in solved examples, highlighted facts, or the solution to previous tasks.

Step 4: If no template was found in the same section as the task, an extended search was conducted in previous sections. A template that could be used to solve the task was considered reasonably likely to be found by students if it had similar characteristics to the task (see Palm et al. (Gresalfi, 2009) for details), such as assignment, contextual features, representations, linguistic features, explicit hints and response format.

Step 5: Concluding and arguing for categorization of a task as a high, local low, or global low relatedness task (HR, LLR or GLR).

The main author analysed all the tasks. Initially the analysis and classification of tasks was discussed with two fellow researchers conversant with the framework and the method of analysis. This strengthened the reliability of the method as inter-rater agreement was reached in relation to more than 98% of the tasks analysed.

Some textbooks included extended learning opportunities in the form of (home-)work books or practice books. These were surveyed but not included in the final analysis as they were judged not to significantly change the outcome of the analysis and were only available for some textbooks.

4.2.1. Classification categories

4.2.1.1. High relatedness tasks. For a task to be classified as an HR task, two conditions needed to be fulfilled. First, there had to be a solution template that could be applied to solve the task. Second, this solution algorithm had to be easily identified, either by being positioned earlier within the same section as the task or in terms of the task characteristics presented above.

4.2.1.2. Local low relatedness tasks. If the above requirements for an HR task were essentially fulfilled but the template needed minor modifications, the task was classified as an LLR task.

4.2.1.3. Global low relatedness tasks. If no template that could be used to solve the task had appeared earlier in the textbook, or if such a template existed but the student could not reasonably be expected to connect it to the task, the task was classified as a GLR task.

4.3. Examples of task categorization

4.3.1. HR categorization (Secondary Basic Mathematics, Book 1, Tanzania, task 7.5-2)

HR tasks set in a purely mathematical context were common in all the analysed textbooks. An example of such a task is $x - 8 = 8$. Within the same section, a solved example, $x - 8 = 15$, was provided, using the same algorithm.

4.3.2. HR categorization (Geometry, USA, task 1.7-40)

The task read: 'The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answer to the nearest tenth.'

Step 1: General information: This textbook was structured with sets of tasks, and this particular task was found in a set with the common heading 'Problem Solving'. Throughout the book, 'Practice' task sets were followed in a sequence by these 'Problem Solving' task sets.

Step 2: Possible solution: Calculate the radius by dividing the diameter in half. Use the formula $C = 2\pi r$ to calculate the circumference and the formula $A = \pi r^2$ to calculate the area of the circular pad. Round the answer.

Step 3: Template: In the same textbook section as the task, the area and circumference of a circular cloth patch were calculated, with a figure of the patch with the diameter shown. In that example, the diameter was divided by two to get the radius, and the formulas $C = 2\pi r$ and $A = \pi r^2$ are used. The answers were also rounded to the nearest tenth.

Step 4: Not applicable since a template was already identified.

Step 5: Conclusion: Since a template that could be used to solve the task was found in the same section as the task, the task was categorized as an HR task.

4.3.3. LLR categorization (Speedy Maths, Book 10, Nepal, task 5.1-4d)

The task was 'If the perimeter of an equilateral triangle is 12 cm, find its area.'

Step 1: General information: The task was placed in a set of tasks with the heading 'General Section'. Throughout the book, the 'General Section' preceded the 'Creative Section'.

Step 2: Possible solution: Use the fact that the triangle is equilateral to find the length of the sides by dividing the perimeter by three. Calculate the height of the triangle by using Pythagoras' theorem, and finally use the formula $A = \frac{base \cdot height}{2}$ to calculate the area of the triangle.

Step 3: Template: A previous task (5.1-4a) asked for the area of an equilateral triangle for which the sides were known. Thus, the solution included all necessary aspects, except finding the length of each side.

Step 4: No additional template was found in other, previous sections.

Step 5: Conclusion: To calculate the area one needed to find the height and base of the triangle. The height was found by using Pythagoras' theorem in both the analysed task and the previous task. But the analysed task started at an earlier point, because the base (i.e. the length of the sides) had to be calculated. There was no available solution template indicating the need to find the side length and presenting such calculations, which makes the categorization LLR.

4.3.4. GLR categorization (Matematik 5000 1b, Sweden, task 3324, a-task)

The task was to 'use words to describe the relation between a and b'.

a	b
1	6
2	7
4	9

Step 1: General information: In this textbook, tasks were sequenced according to their difficulty. In each section, a task set labelled 'a' (indicating the lowest level of difficulty) was followed by increasingly difficult task sets 'b' and 'c'. This particular task was in an 'a' set.

Step 2: Possible solution: Compare the values of a and b in each row. A reasonable approach is to search for elementary arithmetic relations, and to find the relation that b is 5 more than a. In this case, this can be done by performing the subtractions 6–1, 7–2 and 9–4 and seeing that the result is always 5.

Step 3: Template: No template was found within the same section as the task.

Step 4: No template was found in other, previous sections.

Step 5: Conclusion: Since there is no template to apply, the task is categorized as a GLR task.

4.4. Categorization of task labels

An inductive process of comparing and contrasting the organization of tasks was initiated, using the general information from step 1 (see section 4.2) in the analysis procedure. The aim was to find general descriptions that would align with several of the textbooks.

5. Results and analysis

5.1. The percentage of HR, LLR and GLR tasks

In total 5738 tasks were analysed, divided into 4505 algebra tasks and 1233 geometry tasks.

The two most salient results were, firstly, that the percentages of HR, LLR and GLR tasks were relatively similar in textbooks from all twelve countries and, secondly, that HR tasks dominated all textbooks (Table 1). The percentage of HR tasks was slightly higher in algebra chapters than in geometry chapters in textbooks from all countries except Singapore and Canada, where the two percentages were similar. On average, 81% of tasks in an algebra chapter were HR tasks, while in geometry chapters this percentage was 71% (with a standard deviation of 8% in both cases). On average, the percentage of GLR tasks was 8% and 12% respectively for the algebra and geometry sections, with standard deviations of 4% and 6%.

5.2. The percentage of HR, LLR and GLR tasks in relation to the sequencing and descriptors of the tasks and task sets

The descriptors used in the textbooks provided information about such things as the level of difficulty of a task, the focus on a specific competency, or the way to approach the task. Most of the time, a descriptor applied to a task set, although on a few occasions descriptors referred to individual tasks scattered throughout a chapter. In several of the textbooks, some of the task sets and associated descriptors were sequenced in the same way throughout the textbook, making the sets recur in the same order in each section. The results recognize the recurring sequencing of tasks (Table 2), as well as the descriptors in relation to competencies or approaches (Table 3). The descriptors and sequencing varied between textbooks and countries, and therefore the tasks were regrouped to present these results. Sometimes task descriptors were used both for a recurring sequencing of tasks and as an indicator of a focus on a specific competency or approach, or as an expressed level of difficulty. Consequently, some tasks appear in both Tables 2 and 3. On the other hand, not

Table 1. The percentage of tasks belonging to each of the categories in the textbooks' algebra and geometry sections respectively. Number of tasks analysed shown within parenthesis. A: Algebra, G: Geometry. The average is calculated using the percentages for each country.

		HR	LLR	GLR			HR	LLR	GLR
AUS	A (361)	94	4	3	SCO	A (851)	96	3	1
	G (78)	74	14	12		G (114)	85	12	3
CAN	A (235)	75	12	13	SGP	A (327)	74	10	16
	G (53)	75	11	13		G (243)	74	16	9
FIN	A (221)	74	16	10	ZAF	A (260)	82	10	8
	G (87)	62	17	21		G (107)	69	22	8
IND	A (111)	68	25	6	SWE	A (649)	74	15	11
	G (75)	59	25	16		G (105)	65	15	20
IRL	A (419)	86	10	5	TZA	A (398)	84	11	5
	G (129)	82	11	7		G (88)	63	24	14
NPL	A (193)	82	14	4	USA	A (480)	80	10	10
	G (68)	74	19	7		G (86)	65	16	19
					Avg.	Algebra	81	12	8
						Geometry	71	17	12

Note: Table contains rounding errors.

Table 2. The percentage of tasks belonging to each of the categories in different re-occurring, sequenced task sets of the textbooks. The number of analysed tasks shown within parenthesis.

	HR	LLR	GLR
Initial activities (133)*	74	8	18
Introductory tasks and first set of tasks (1249)	88	8	4
Second set of tasks (538)	68	18	14
Third and fourth sets of tasks (202)	51	19	30
Review (684)	84	9	7

* Task sets that precede any mathematical information in the section.

Table 3. The percentage of tasks belonging to each of the categories with different task descriptors. The number of tasks analysed shown within parenthesis.

	HR	LLR	GLR
Fluency, practice (1123)[1]	87	9	4
Apply (216)[2]	66	17	17
Investigate, explore (250)[3]	68	12	20
Reason, discuss, think, understand (134)[4]	62	9	29
Problem solving (32)[5]	69	6	25

[1] Includes task sets from AUS (Fluency, 371 pcs), CAN (Practice the concept, 120), NPL (General, 33), SGP (Level 1-2/Basic and Further practice, Try it 286), USA-Algebra (Practice, Do you know how 176), USA-Geometry (Guided and Skill practice, 46), ZAF (Check and Extend your skills, 91)

[2] Includes task sets from CAN (Apply the concept, 57), SGP (Maths@work, 24), SWE (Theme, 27), USA-Algebra (Apply, 108)

[3] Includes task sets from CAN (Investigate, 20), IRL (Activity, 45), SGP (Class activity, 85), SWE (Activity, Activity-explore, 73), USA-Algebra (Activity exercises, 22), USA-Geometry (Activity, 5)

[4] Includes task sets from AUS (Understanding, Reasoning, Reflection 55), CAN (Discuss the concept, 15), FIN (To think about, 11), SGP (Brainworks, Discuss 16), SWE (Activity-discuss, 14), USA-Algebra (Do you understand, 23)

[5] Includes task sets from AUS (Problem solving, 11), USA-Geometry (Problem solving, Problem solving workshop, 21).

all countries used sequenced sets or descriptors, and so some analysed tasks (see Table 1) do not appear at all in Tables 2 and 3. As the analysed textbooks had different designs, the number of analysed tasks also varied between textbooks and countries. However, the following results do not hold information about the individual textbooks or countries.

The percentages of GLR and LLR tasks were lower in the first task set, where the vast majority of the tasks were positioned. This was the case in textbooks where specific properties were not attributed to the task sets and their descriptors, as well as in textbooks where the task sets were described in terms of increasing levels of difficulty, such as those from Sweden and Singapore.

Regardless of the descriptor, even if it was 'problem solving', HR tasks were dominant.

6. Discussion

6.1. Similarities among textbooks internationally

The range of TIMSS (Mullis et al., 2012) and PISA (OECD, 2014) results is an indication that there are major differences between countries when it comes to learning opportunities.

Despite this and the wide geographical diversity among the twelve countries, the results show that the percentages of different types of tasks are relatively similar in the sample of textbooks from all twelve countries. There may, of course, be other textbook series and textbooks from other countries that do not fit in with this relative uniformity.

6.2. Opportunities to practise problem solving

The importance of problem solving and procedural fluency are both recognized in relation to a rich view of mathematical competence (Boesen et al., 2014; National Council of Teachers of Mathematics (NCTM), 2000; Kilpatrick et al., 2001; Niss, 2003). Problem solving is often supported by procedures (Hiebert & Grouws, 2007), which fits with our observation that most tasks classified as GLR and LLR include the application of algorithmic sub procedures (see, for example, the tasks in section 4.3.3 [the use of Pythagoras' theorem and the calculation of the area of a triangle] and section 4.3.4 [subtraction]). Hence, another way to express the result of this study is to say that almost all of the tasks provide an opportunity to practise using algorithms, while in a significantly lower proportion of the tasks it was necessary to construct a new solution method, and thus practise this specific aspect of problem solving. There is also a possibility that even if a template is not provided by the textbook, it may be provided to a student by a teacher or a peer (Sidenvall et al., 2015; Stein et al., 2007), further diminishing the opportunities to learn problem solving. Other aspects of problem solving, such as explorations (National Council of Teachers of Mathematics (NCTM), 2000; Niss, 2003) and that problem solving should be challenging (Schoenfeld, 1985) were not investigated in this study. As students tend to rely on templates when these are available (Boesen et al., 2014; Hiebert, 2003; Sidenvall et al., 2015), the GLR and LLR tasks are important to include.

Problem-solving competency may be developed if students spend a substantial part of their time with the textbook working on GLR and LLR tasks. However, earlier research (Kaur, 2010; Sidenvall et al., 2015) indicates that students work mainly on the first task sets in their textbooks, which include a much lower proportion of GLR and LLR tasks than other sets of tasks. Brehmer et al. (2016) and Stein et al. (2007) also show that conventional textbooks sequence tasks so that those requiring competencies such as problem solving and reasoning are preceded by tasks solvable by standard algorithms. It seems that at least some textbooks include more tasks than an average student can reasonably be expected to work through. This implies that a selection has to be made, either by the individual student or by the teacher. Bieda et al. (2014) argue that the opportunities to learn diminish if specific tasks are placed in extension sections of the textbook, making it easier for a teacher to deselect them from the classroom practice. When the first sets of tasks in textbook sections, and basic skills are prioritized (Kaur, 2010; Sidenvall et al., 2015; Stein et al., 2007) an even greater emphasis on procedural fluency over problem solving is implied.

The approach to differentiating between tasks varies among the textbooks from different countries. While some textbooks explain that later tasks are more difficult but do not necessarily require different types of solution skills, others distinguish tasks according to the kind of mathematics needed to solve the tasks within a particular group. The percentages of LLR and GLR tasks are noticeably higher among tasks said to be more difficult or challenging, which students and teachers may interpret as meaning that such tasks are not for all students. Similar to other textbook analyses carried out, using other methods (Brehmer

et al., 2016; Vincent & Stacey, 2008), our results indicate a skewed balance towards procedural tasks rather than problem solving in textbooks, as well as a lack of emphasis on the importance of problem solving for all students.

The results show that the percentages of GLR and LLR tasks are higher for geometry than for algebra in textbooks from all twelve countries. The difference can partly be attributed to the fact that algebra and geometry are different in nature, and that the approaches to work with problems are different. These results can be compared with earlier studies indicating that opportunities to practise reasoning-and-proving skills are more limited in school algebra than in geometry (Stylianides, 2009). The reason for this might be the historical tradition of Euclidean geometry in secondary mathematics. Proofs may be included in the curriculum through the Euclidian axiomatic system (Stylianides, 2009). Work with algebra on the other hand rarely seems to be done by reference to axioms of real numbers. This means that algebraic proofs are more difficult to do, as one does not know where to start (i.e. what facts to take for granted, and what needs to be proven).

6.3. Textbook task descriptors in relation to problem solving

There were a number of task descriptors used in the textbooks where we assumed HR tasks to be few. Descriptors such as 'Problem solving', 'Reasoning' or 'Investigate' supposedly indicate a greater focus on competencies other than procedural fluency. Such descriptors may be a good idea as they might suggest different aspects of mathematical knowledge and so support the creation of a beneficial learning situation (Rezat & Strässer, 2014). This could include an understanding of what different, necessary mathematical competencies entail. However, in the textbooks analysed, the descriptors did not seem to be in line with common definitions of problem solving concerning the construction of a solution method new to the student (e.g. National Council of Teachers of Mathematics (NCTM), 2000; Kilpatrick et al., 2001; Niss, 2003). The percentages of HR tasks with descriptors such as 'investigate, explore' and 'reason, discuss, think, understand' and 'problem solving' were all above 60%. An example of such a task is given in section 4.3.2, where a task was described by the textbook as 'problem solving' even though it could be solved by applying the template provided. Thus, it is an HR task and not a problem-solving task (National Council of Teachers of Mathematics (NCTM), 2000; Kilpatrick et al., 2001; Niss, 2003). This confusion may lead to an ambiguous view of what problem solving actually entails.

In line with results from Schmidt (Schmidt et al., 2012), the review task sets seem to reflect what is presented by the exercise sets in each chapter, implying that an analysis of review sections may indicate the general focus of a textbook.

6.4. Implications

Problem solving has been proven to establish a conceptual sense of mathematics and to promote competencies such as reasoning and procedural fluency (Hiebert, 2003). Considering the large percentage of HR tasks in the textbooks, we suggest, as do Newton & Newton (2007), that textbooks can be improved by providing a greater proportion of LLR and GLR tasks. We would like to highlight this especially for the initial task sets and among tasks described as easier. However, it is difficult to determine exactly what students' learning opportunities are without considering not only what tasks are presented in the textbook,

but also how the textbook is actually used (Valverde et al., 2002). Nevertheless, the textbook is one of the influential factors in the outcome of mathematics education (Rezat & Strässer, 2014). Schoenfeld (1985) claims that to develop a problem-solving competency, one needs to work with problems. However, as the great majority of tasks in all textbooks analysed were HR tasks where solution templates were available to students, there is a need for careful selection (by teachers and students) to create opportunities to develop problem-solving competency in this sense. The results of the study also emphasize that teachers and students need to make conscious decisions when using textbook tasks and should carefully investigate the learning opportunities of tasks rather than relying on the textbook's own descriptors. The framework and method in this study provide teachers, researchers, and textbook authors and designers with an additional tool with which they can examine mathematics textbooks more closely.

Given that all the textbooks do include some GLR tasks, there is potential for the teaching and learning of problem solving supported by textbooks. It is possible to increase the amount of problem-solving practice a student gets by consciously selecting tasks from the whole textbook.

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No potential conflict of interest was reported by the authors.

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