APPROXIMATION AND SUBEXTENSION OF NEGATIVE PLURISUBHARMONIC FUNCTIONS

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Licentiate thesis
Research Report in mathematics No. 3, 2008
Department of Mathematics and Mathematical Statistics,
Umeå University, 2008

ISBN 978-91-7264-622-3
ISSN 1653-0810

This thesis was typeset by the author using LaTeX.

Printed by Print & Media, Umeå University 2008.

Abstract

In this thesis we study approximation of negative plurisubharmonic functions by functions defined on strictly larger domains. We show that, under certain conditions, every function u that is defined on a bounded hyperconvex domain Ω in $\mathbb{C}^n$ and has essentially boundary values zero and bounded Monge-Ampère mass, can be approximated by an increasing sequence of functions $\{u_j\}$ that are defined on strictly larger domains, has boundary values zero and bounded Monge-Ampère mass. We also generalize this and show that, under the same conditions, the approximation property is true if the function u has essentially boundary values G, where G is a plurisubharmonic functions with certain properties. To show these approximation theorems we use subextension. We show that if $\Omega \Subset \hat{\Omega}$ are hyperconvex domains in $\mathbb{C}^n$ and if u is a plurisubharmonic function on Ω with given boundary values and with bounded Monge-Ampère mass, then we can find a plurisubharmonic function $\hat{u}$ defined on $\hat{\Omega}$, with given boundary values, such that $\hat{u} \leq u$ on Ω and with control over the Monge-Ampère mass of $\hat{u}$.

2000 *Mathematics Subject Classification*: Primary 32W20; Secondary 32U15.
Key words and phrases. Complex Monge-Ampère operator, approximation, plurisubharmonic function, subextension.

Contents

Papers included in this thesis

I. Cegrell U. & Hed L., Subextension and approximation of negative plurisubharmonic functions, Michigan Math. J. (in press).

II. Czyż R. & Hed L., Subextension of plurisubharmonic functions without increasing the total Monge-Ampère mass, manuscript Krakow and Umeå (2008).

III. Hed L., Approximation of negative plurisubharmonic functions with given boundary values, Research Report in Mathematics No. 2, Umeå University (2008).

1. Introduction

In this thesis we discuss the problem of approximating a negative plurisubharmonic function by functions defined on strictly larger domains. Let $\Omega \Subset \Omega_{j+1} \Subset \Omega_j$ be hyperconvex domains. Here $\Subset$ denotes that Ω_{j+1} is relatively compact in Ω_j. Since we are going to use Monge-Ampère techniques we consider some classes of negative plurisubharmonic functions where the Monge-Ampère operator is well-defined. In this introduction the classes $\mathcal{F}(\Omega)$, $\mathcal{N}(\Omega)$, $\mathcal{E}(\Omega)$ and $\mathcal{F}(\Omega, F)$ will be used and for the definition of these classes see section 2. The problem in paper I is whether, given $u \in \mathcal{F}(\Omega)$ is it possible to find an increasing sequence $\{u_j\}$, $u_j \in \mathcal{F}(\Omega_j)$, such that u_j converges to u a.e. on Ω? Here we don't impose any convergence on the sequence $\{\Omega_j\}$, but in paper I (Theorem 3.1) we prove that the approximation is possible if we can find a function $0 > v \in \mathcal{N}(\Omega)$ and a sequence $\{v_j\}$, $v_j \in \mathcal{N}(\Omega_j)$ such that $v_j \to v$ a.e. on Ω. This condition is of course necessary and is needed to make sure that the sequence $\{\Omega_j\}$ converges to Ω in some sense. For example, this condition imply that $cap(K, \Omega) = \lim_j cap(K, \Omega_j)$ for every compact subset K of Ω. Here $cap(K, \Omega)$ is the relative Monge-Ampère capacity defined by Bedford and Taylor in [5] . We also show (Corollary 3.2 in paper I) that if Ω has C^1-boundary and $\{\Omega_j\}$ is a Stein neighborhood basis, then the approximation is possible. In paper III we consider the same approximation problem with functions in the class $\mathcal{F}(\Omega, H)$, where $H \in \mathcal{E}$ is a maximal function that is continuous up to the boundary. We show that approximating a function $u \in \mathcal{F}(\Omega, H)$, that has bounded Monge-Ampère mass, with functions defined on strictly larger domains that has certain boundary values, is possible if we can approximate one function (not identically zero) that has essentially boundary values zero.

The main tool in the proof of the approximation theorems (Theorem 3.1 in paper I and Theorem 1.1 in paper III) is subextension. Let Ω and $\hat{\Omega}$ be hyperconvex domains such that $\Omega \Subset \hat{\Omega}$. Given a plurisubharmonic function u on Ω, when can we find a plurisubharmonic function v on $\hat{\Omega}$, $v \not\equiv -\infty$, such that $v \le u$ on Ω? The function v is called a subextension of u to $\hat{\Omega}$. The problem of subextending plurisubharmonic functions are discussed in all three papers. The second paper in this thesis is fully dedicated to the problem of subextending functions with given boundary values without increasing the total Monge-Ampère mass. The main result in paper II is the following: Let $\Omega, \hat{\Omega}$ be hyperconvex domains such that $\Omega \subset \hat{\Omega}$. Given two functions $F \in \mathcal{E}(\Omega)$ and a maximal plurisubharmonic function $G \in \mathcal{E}(\hat{\Omega})$, such that $G \le F$ on Ω and given a function $u \in \mathcal{F}(\Omega, F)$, we show that there exists a function $v \in \mathcal{F}(\hat{\Omega}, G)$ such that $v \le u$ on Ω and

$$\int_{\Omega_2} (dd^c v)^n \le \int_{\Omega_1} (dd^c u)^n.$$

Observe that, in this case, we only have control over the total Monge-Ampère mass and not the Monge-Ampère measures. In paper III, we add the conditions that $\Omega \Subset \hat{\Omega}$, $F \in \mathcal{E}(\Omega) \cap C(\bar{\Omega})$ and $\int_\Omega (dd^c u)^n < \infty$, and we get the stronger result that, if v is the maximal subextension of u to $\hat{\Omega}$, then

$$(dd^c v)^n \le \chi_\Omega (dd^c u)^n.$$

Here χ_Ω denotes the characteristic function of Ω. This subextension theorem is used to prove the main result in paper III (Theorem 1.1).

2. Preliminaries

In this section we state some definitions and well known results from pluripotential theory. For further introduction, see the book *Pluripotential theory* by Klimek ([18]). Let $\Omega \subset \mathbb{C}^n$ be a bounded domain (i.e. Ω is an open, bounded and connected set) and let $\mathcal{PSH}(\Omega)$ denote the class of plurisubharmonic functions defined on Ω. Throughout this thesis we have that Ω is a hyperconvex domain.

Definition 2.1. A domain $\Omega \subset \mathbb{C}^n$ is called *hyperconvex* if there exists a plurisubharmonic function $\psi : \Omega \to (-\infty, 0)$ such that the closure of the set $\{z \in \Omega : \psi(z) < c\}$ is compact in Ω, for every $c \in (-\infty, 0)$.

Note that every hyperconvex domain is pseudoconvex and Demailly showed in [14] that a bounded pseudoconvex domain with a Lipschitz boundary is hyperconvex.

In this thesis we only consider non-negative measures and the space of non-negative measures is equipped with the weak*-topology. This means that a sequence $\{\mu_j\}$ of measures defined on Ω, converges to a non-negative measure μ, as $j \to \infty$, in the weak*-topology if, and only if,

$$\int_\Omega \varphi\mu_j \to \int_\Omega \varphi\mu,$$

for every $\varphi \in C_0(\Omega)$. Here $C_0(\Omega)$ denotes the space of continuous functions on Ω with compact support. Observe that if $\{\mu_j\}$ converges to μ in the weak*-topology, doesn't necessarily imply that $\mu_j(\Omega) \to \mu(\Omega)$, even for $\Omega = \mathbb{C}^n$. But we have the following theorem that will be used later. For proof of the theorem and more about measures, see e.g. [19].

Theorem 2.2. *Let $\mu_1, \mu_2, \dots$ be Radon measures on a locally compact metric space X. If $\{\mu_j\}$ converges to μ in the weak*-topology and if $K \subset X$ is compact and $G \subset X$ is open, then*

$$\mu(K) \geq \limsup_{i\to\infty} \mu_j(K)$$

and

$$\mu(G) \leq \liminf_{i\to\infty} \mu_j(G).$$

2.1. The complex Monge-Ampère operator.

In this section we define a generalization of the Laplace operator. Note that the Laplace operator defines a distribution on all locally integrable functions and this distribution is a non-negative measure exactly on the subharmonic functions (and is equal to zero on harmonic functions). The complex Monge-Ampère operator, $(dd^c\ )^n$, defines a non-negative measure on a subset of the plurisubharmonic functions and for those functions we have that the operator is equal to zero on maximal functions (see section 2.4 for the definition of maximal). Let $d = \partial + \bar\partial$ and $d^c = i(\bar\partial - \partial)$. Note that

$$dd^c = 2i\partial\bar\partial,$$

and if $u \in C^2(\Omega)$

$$dd^c u = 2i \sum_{j,k=1}^n \frac{\partial^2 u}{\partial z_j \partial \bar z_k} dz_j \wedge d\bar z_k.$$

The complex Monge-Ampère operator in $\mathbb{C}^n$ is then defined by

$$(u_1, \ldots, u_n) \mapsto (dd^c u_1) \wedge \ldots \wedge (dd^c u_n)$$

where $u_1, \ldots, u_n \in \mathcal{PSH}(\Omega) \cap C^2(\Omega)$. If $u = u_1 = \ldots = u_n$ then

$$(dd^c u)^n = 4^n n! \det \left(\frac{\partial^2 u}{\partial z_j \partial \bar{z}_k} \right) dV,$$

where

$$dV = dx_1 \wedge dy_1 \wedge \ldots \wedge dx_n \wedge dy_n = \left(\frac{i}{2}\right)^n dz_1 \wedge d\bar{z}_1 \ldots \wedge dz_n \wedge d\bar{z}_n$$

is the usual volume form in $\mathbb{C}^n$ or $\mathbb{R}^{2n}$. This operator cannot be extended in a meaningful way to the whole class of plurisubharmonic functions, but Bedford and Taylor showed in [4] that the operator is well-defined on $\mathcal{PSH} \cap L^\infty_{\text{loc}}(\Omega)$. If $u \in \mathcal{PSH} \cap L^\infty_{\text{loc}}(\Omega)$ then $(dd^c u)^n$ is a positive measure.

2.2. **The classes $\mathcal{E}_0$, $\mathcal{F}$, $\mathcal{N}$ and $\mathcal{E}$.** In [11], Cegrell defined a subclass, $\mathcal{E}$, of the negative plurisubharmonic functions that is the natural domain of definition for the complex Monge-Ampère operator, in the sense that it is closed under the operation of taking the maximum of two functions and the Monge-Ampère operator has a continuity property on decreasing limits of functions in this class (see Theorem 4.5 in [11]). In this section we define this class and other related classes. For further reading about these classes, see [3, 10, 11, 12]. Let $\Omega \subset \mathbb{C}^n$, $n \geq 1$, be a hyperconvex domain. Let $\mathcal{E}_0 (= \mathcal{E}_0(\Omega))$ be the convex cone of bounded plurisubharmonic functions φ such that $\lim_{z \to \xi} \varphi(z) = 0$ for every $\xi \in \partial\Omega$ and $\int_\Omega (dd^c \varphi)^n < +\infty$. By Bedford and Taylor, the complex Monge-Ampère operator is well-defined on $\mathcal{E}_0$. Observe that, if $\varphi \in \mathcal{E}_0$, then the maximum principle gives us that either $\varphi < 0$ or $\varphi \equiv 0$ on Ω.

Now let $\mathcal{F}(\Omega)$ be the class of negative plurisubharmonic functions φ on Ω such that there exists a decreasing sequence $\{\varphi_j\}$, $\varphi_j \in \mathcal{E}_0(\Omega)$, that converges to φ on Ω and such that $\sup_j \int_\Omega (dd^c \varphi_j)^n < +\infty$. Let $\mathcal{E}(\Omega)$ be the class of all negative plurisubharmonic functions that are locally in $\mathcal{F}(\Omega)$, i.e. for every $z_0 \in \Omega$ there exists a neighborhood ω of z_0 and a decreasing sequence $\{\varphi_j\}$, $\varphi_j \in \mathcal{E}_0(\Omega)$, that converges to φ on ω and such that $\sup_j \int_\Omega (dd^c \varphi_j)^n < +\infty$.

We now define the class $\mathcal{N}(\Omega)$ that was first defined in [12]. Here a fundamental sequence $\{\Omega^j\}$ of Ω is a sequence of strictly pseudoconvex domains such that $\Omega^j \Subset \Omega^{j+1} \Subset \Omega$ for every j, and $\cup_j \Omega^j = \Omega$.

Definition 2.3. Let $u \in \mathcal{PSH}(\Omega)$, $u \leq 0$, and let $\{\Omega^j\}$ be a fundamental sequence of Ω. The function u^j is then defined by

$$u^j = \sup \left\{ \varphi \in \mathcal{PSH}(\Omega) : \varphi \leq u \ \text{ on } \ \mathcal{C}\Omega^j \right\}.$$

Define

$$\tilde{u} = \left(\lim_{j \to \infty} u^j \right)^*,$$

where $(\omega)^*$ denotes the upper semicontinuous regularization of ω.

If $u \in \mathcal{E}(\Omega)$, then by the construction, $u \leq u^j \leq u^{j+1}$, so we have that $\tilde{u} = \left(\lim_{j\to\infty} u^j\right)^* \in \mathcal{E}$ and we define the class $\mathcal{N}$ by

$$\mathcal{N}(\Omega) = \{u \in \mathcal{E} : \tilde{u} = 0\}.$$

Note that, by [8, 9], $\tilde{u}$ is maximal. For more about maximal plurisubharmonic functions see section 2.4.

Remark. Note that $\mathcal{E}_0 \subset \mathcal{F} \subset \mathcal{N} \subset \mathcal{E}$.

By [11], the complex Monge-Ampère operator is well-defined on $\mathcal{E}$ as the limit measure given by the next theorem.

Theorem 2.4. *For k=1,...,n let $u^k \in \mathcal{E}$ and let $\{u_j^k\}, u_j^k \in \mathcal{E}_0$, be a decreasing sequence such that $\{u_j^k\}$ converges pointwise to u^k as $j \to \infty$. Then*

$$(dd^c u_j^1) \wedge (dd^c u_j^2) \wedge \ldots \wedge (dd^c u_j^n)$$

is weak-convergent and the limit measure is independent of the sequence $\{u_j^k\}$.*

Remark. If $u \in \mathcal{F}$, then $\int_\Omega (dd^c u)^n < +\infty$. If $u \in \mathcal{E}$, then $\int_w (dd^c u)^n < +\infty$ for any open set w such that $\bar{w} \subset \Omega$.

We now remind us of some well known results about these classes that is extensively used throughout this thesis.

Theorem 2.5. *Let $\mathcal{K} \in \{\mathcal{E}_0, \mathcal{F}, \mathcal{E}\}$, $u \in \mathcal{K}$ and $v \in \mathcal{PSH}^-(\Omega)$. Then*

$$\max\{u, v\} \in \mathcal{K}.$$

Proof. See [10, 11]. □

Theorem 2.6. *Let $u \in \mathcal{F}$, then*

$$\limsup_{z\to\xi} u(z) = 0$$

for every $\xi \in \partial\Omega$.

Proof. See Corollary 1.5 in [2]. □

Although functions in $\mathcal{F}$ have essentially boundary values zero, note that, in Example 1.6 in [2], Åhag constructed a function $u \in \mathcal{F}$ that has the property that $\liminf_{z\to\xi} u(z) = -\infty$, for every $\xi \in \partial\Omega$. Since functions in $\mathcal{F}$ have essentially boundary values zero and finite total Monge-Ampère mass we have a formula for integration by parts with no boundary terms. This makes the class $\mathcal{F}$ very useful.

Theorem 2.7. *If v, u_1,...,$u_n \in \mathcal{F}$ then*

$$\int_\Omega v(dd^c u_1) \wedge \ldots \wedge (dd^c u_n) = \int_\Omega u_1(dd^c v) \wedge \ldots \wedge (dd^c u_n) .$$

Proof. See [11]. □

This theorem gives us the following comparison theorem.

Corollary 2.8. *Let $u, v \in \mathcal{F}$ be such that $u \leq v$ on Ω, then*

$$\int_\Omega \varphi(dd^c u)^n \leq \int_\Omega \varphi(dd^c v)^n,$$

where $\varphi \in \mathcal{PSH}^-(\Omega)$.

Proof. See Corollary 2.11 in [1]. □

2.3. **The classes $\mathcal{E}_0(H)$, $\mathcal{F}(H)$ and $\mathcal{N}(H)$.** We now define some classes of negative plurisubharmonic functions that can be seen as generalizations of the classes above. These functions have more general boundary values and orginates from [10]. Let $\mathcal{K} \in \{\mathcal{E}_0, \mathcal{F}, \mathcal{N}\}$. We say that a plurisubharmonic function u defined on Ω belongs to the class $\mathcal{K}(\Omega, H)$, $H \in \mathcal{E}$, if there exists a function $\varphi \in \mathcal{K}$ such that

$$H \geq u \geq \varphi + H\,.$$

Note that $\mathcal{K}(\Omega, 0) = \mathcal{K}$ and that functions belonging to $\mathcal{K}(\Omega, H)$ not necessarily have finite total Monge-Ampère mass. For more details about these classes see [3, 12]. The following theorem was proved in [3] (Lemma 3.3) and is extensively used in this thesis.

Theorem 2.9. *Let $u, v \in \mathcal{N}(H)$, $H \in \mathcal{E}(\Omega)$, be such that $u \leq v$ and $\int_\Omega \varphi(dd^c u)^n < +\infty$, where $\varphi \in \mathcal{PSH}(\Omega)$, $\varphi \leq 0$. Then*

$$\int_\Omega (-\varphi)(dd^c u)^n \geq \int_\Omega (-\varphi)(dd^c v)^n.$$

Some other well known results are formulated in the proposition below.

Proposition 2.10. *Let $\Omega \subset \mathbb{C}^n$ be a hyperconvex domain.*

a) Let $H \in \mathcal{E}$. If $\{u_j\}$, $u_j \in \mathcal{N}(H)$ is a decreasing sequence that converges pointwise to a function $u \in \mathcal{N}(H)$ as $j \to +\infty$, then

$$\lim_{j \to +\infty} \int_\Omega (dd^c u_j)^n = \int_\Omega (dd^c u)^n.$$

b) If $u \in \mathcal{E}(\Omega)$ and $\int_\Omega (dd^c u)^n < +\infty$, then $u \in \mathcal{F}(\tilde{u})$.

Proof. See Corollary 3.4 in [3] and Theorem 2.1 in [12]. □

2.4. **Maximal plurisubharmonic functions.** In the same way as we can describe harmonic functions as maximal subharmonic functions we want to generalize this to higher dimensions. In the definition below, we use the same terminology as Sadullaev in [21].

Definition 2.11. A plurisubharmonic function $u : \Omega \to \mathbb{R}$ is said to be *maximal* if for each relatively compact open subset ω of Ω and for each upper semicontinuous function v defined on $\bar{\omega}$ such that $v \in \mathcal{PSH}(\omega)$ and $v \leq u$ on $\partial\omega$, we have that $v \leq u$ on ω. The maximal plurisubharmonic functions on a domain Ω is denoted by $\mathcal{MPSH}(\Omega)$.

Remark. In one dimension, the maximal subharmonic functions and the harmonic functions are the same.

Maximal plurisubharmonic functions play an important role in pluripotential theory since, by [8, 9], if $u \in \mathcal{E}(\Omega)$ then

$$u \text{ is maximal} \Leftrightarrow (dd^c u)^n = 0 \text{ on } \Omega.$$

3. Subextension

In this section we discuss the problem of subextending plurisubharmonic functions. The main problem is, given two domains Ω and $\hat{\Omega}$ in $\mathbb{C}^n$, with $\Omega \subset \hat{\Omega}$. Then given $u \in \mathcal{PSH}(\Omega)$, when can we find $\hat{u} \in \mathcal{PSH}(\hat{\Omega})$, $\hat{u} \not\equiv -\infty$, such that $\hat{u} \leq u$ on

Ω? The function $\hat{u}$ is called a subextension of u to $\hat{\Omega}$.

In [15], El Mir constructed an example of a plurisubharmonic function defined on the unit bidisc in $\mathbb{C}^2$ for which the restriction to any smaller bidisc admits no subextension to a larger domain. Bedford and Taylor improved, in [6], an example by Fornæss and Sibony ([16]) by constructing, for any C^2-smooth domain in $\mathbb{C}^n$, a smooth negative plurisubharmonic function that does not subextend. Here we are interested in subextension without increasing the total Monge-Ampère mass. In [13], Cegrell and Zeriahi showed that if a function is in the class $\mathcal{F}$ (i.e. has bounded total Monge-Ampère mass and essentially boundary values zero), then one can always find a subextension, without increasing the total Monge-Ampère mass. It was shown by Wiklund (Theorem 3.2 in [23]) that, for every hyperconvex domain Ω, there exists a function in the class $\mathcal{E}$ which cannot be subextended. He also points out that having control over the Monge-Ampère mass does not imply that the function has a subextension. The boundary values of the function seem to play an important role. But note that, in Example 5.2 in paper III, we show that the function constructed by Wiklund is in $\mathcal{N}\backslash\mathcal{F}$. So although a function has boundary values zero it is possible that the function cannot be subextended.

The next theorem is useful when we want to subextend functions in the classes $\mathcal{E}_0$ and $\mathcal{F}$. The proof uses an idea of Phạm in [20].

Proposition 3.1. *Let Ω and $\hat{\Omega}$ be hyperconvex domains such that $\Omega \Subset \hat{\Omega}$. If $\mathcal{K}(\Omega) \in \{\mathcal{E}_0(\Omega) \cap C(\bar{\Omega}), \mathcal{E}_0(\Omega), \mathcal{F}(\Omega)\}$ and if $u \in \mathcal{K}(\Omega)$ then*

$$\hat{u}(z) = \sup\{\varphi(z) \in \mathcal{PSH}^-(\hat{\Omega}) : \varphi \leq u \text{ on } \Omega\}$$

belongs the class $\mathcal{K}(\hat{\Omega})$ and $(dd^c\hat{u})^n \leq \chi_\Omega (dd^c u)^n$.

Proof. See Proposition 3.1 in paper III. □

In paper III, we generalize this to the classes $\mathcal{E}_0(\Omega, F) \cap C(\bar{\Omega})$ and $\mathcal{F}(\Omega, H)$, where $F \in \mathcal{E}(\Omega)$ and $H \in \mathcal{E}(\Omega) \cap C(\bar{\Omega})$.

4. Approximation

In this thesis we study approximation of plurisubharmonic functions by functions defined on strictly larger domains. This question has been studied by, for example, Sibony ([22]) where he showed that if Ω is a pseudoconvex domain with smooth boundary then every function $f \in \mathcal{PSH}(\Omega) \cap C(\bar{\Omega})$ can be approximated uniformly on $\bar{\Omega}$ by functions in $\mathcal{PSH}(\bar{\Omega}) \cap C^\infty(\bar{\Omega})$. Fornæss and Wiegerinck then showed in [17] that this is true even for arbitrary bounded domains with C^1-boundary. In paper I we show that if $\Omega \Subset \Omega_{j+1} \Subset \Omega_j$ are hyperconvex domains and if we can find a function $0 > v \in \mathcal{N}(\Omega)$ and a sequence $\{v_j\}$, $v_j \in \mathcal{N}(\Omega_j)$, such that $v_j \to v$ a.e. on Ω, then every function $u \in \mathcal{F}(\Omega)$ can be approximated from below on Ω by functions $u_j \in \mathcal{F}(\Omega_j)$. This generalizes a theorem by Benelkourchi ([7]) where he considers functions in $\mathcal{F}^a$, i.e. functions in $\mathcal{F}$ whose Monge-Ampère measure vanishes on pluripolar subsets of Ω. In paper III we generalize this further and show that if $G \in \mathcal{MPSH}(\Omega_1) \cap C(\bar{\Omega})$, $G \leq 0$, and if we can approximate one function with essentially boundary values zero (not identically zero), then to every function $u \in \mathcal{F}(\Omega, G|_\Omega)$, such that

$$\int_\Omega (dd^c u)^n < +\infty,$$

there exists an increasing sequence of functions $u_j \in \mathcal{F}(\Omega_j, G|_{\Omega_j})$ such that $\lim_j u_j = u$ a.e. on Ω. To show both of these theorems we define the functions u_j as the maximal subextension of u to Ω_j (subextension were discussed in section 3).

Acknowledgements - Tack

Först och främst vill jag tacka min handledare Urban Cegrell för all hans hjälp och för hans otroligt stora kunskap inom ämnet. Jag vill även tacka min biträdande handledare Anders Fällström för all inspiration och peppning. Ett stort tack till Per Åhag för den hjälp och uppmuntran jag har fått samt för att han har korrekturläst avhandlingen. Även Phạm Hoàng Hiệp förtjänar ett stort tack för hans bra förslag och idéer. Tack till forskningsgrupperna i komplex analys både i Umeå och i Sundsvall och speciellt till Berit Kemppe som har varit ett stort stöd när det ibland har känts tungt och till Linus Carlsson för värdefulla kommentarer på kappan. Institutionen för matematik och matematisk statistik förtjänar ett stort tack för bl.a. trevliga fikaraster och sist men inte minst vill jag tacka min kära familj och min man Andreas för att de alltid finns där för mig.

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