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# Non-parametric methods for functional data

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**Abstract**

In this thesis we develop and study non-parametric methods within three major areas of functional data analysis: testing, clustering and prediction. The thesis consists of an introduction to the field, a presentation and discussion of the three areas, and six papers.

In Paper I, we develop a procedure for testing for group differences in functional data. In case of significant group differences, the test procedure identifies which of the groups that significantly differ, and also the parts of the domain they do so, while controlling the type I error of falsely rejecting the null hypothesis. In Paper II, the methodology introduced in Paper I is applied to knee kinematic curves from a one-leg hop for distance to test for differences within and between three groups of individuals (with and without knee deficits). It was found that two of the groups differed in their knee kinematics. We also found that the individual kinematic patterns differed between the two legs in one of the groups. In Paper III, we test for group differences in three groups with respect to joint kinematics from a vertical one-leg hop using a novel method that allows accounting for multiple joints at the same time. The aim of Paper III, as one of few within the field of biomechanics, is to illustrate how different choices prior to the analysis can result in different contrasting conclusions. Specifically, we show how the conclusions depend on the choice of type of movement curve, the choice of leg for between-group comparisons and the included joints.

In Paper IV, we present a new non-parametric clustering method for dependent functional data, the double clustering bagging Voronoi method. The objective of the method is to identify latent group structures that slowly vary over domain and give rise to different frequency patterns of functional data object types. The method uses a bagging strategy based on random Voronoi tessellations in which local representatives are formed and clustered. Combined with the clustering method, we also propose a multiresolution approach which allows identification of latent structures at different scales. A simulated dataset is used to illustrate the method's potential in finding stable clusters at different scales. The method is also applied to varved lake sediment data with the aim of reconstructing the climate over the past 6000 years, at different resolutions. In Paper V, we expand and modify the bagging strategy used in Paper IV, by considering different methods of generating the tessellations and clustering the local representatives of the tessellations. We propose new methods for clustering dependent categorical data (e.g., labelled functional data) along a one-dimensional domain, which we also compare in a simulation study.

In Paper VI, two kriging approaches to predict spatial functional processes are compared, namely functional kriging and spatio-temporal kriging. A simulation study is conducted to compare their prediction performance and computational times. The overall results show that prediction performance is about the same for stationary spatio-temporal processes while functional kriging works better for non-stationary spatio-temporal processes. Furthermore, the computational time for (ordinary) kriging for functional data, was considerably lower than spatio-temporal kriging. Conditions are also formulated under which it is proved that the two functional kriging methods: ordinary kriging for functional data and pointwise functional kriging coincide.

**Keywords**

Functional data analysis, testing, clustering, prediction, inference, bagging Voronoi strategy, kriging, dependency

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