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ABSTRACT

Assessments of refractivity in a Fabry–Perot (FP) cavity by refractometry often encompass a step in which the penetration depth of the light into the mirrors is estimated to correct for the fraction of the cavity length into which no gas can penetrate. However, as it is currently carried out, this procedure is not always coherently performed. Here, we discuss a common pitfall that can be a reason for this and provide a recipe on how to perform FP-cavity-based refractometry without any influence of mirror penetration depth.

I. INTRODUCTION

Fabry–Perot (FP) based refractometry is a technique that can be used for assessment of gas refractivity, molar density, and pressure. With the latest revision of the SI (International System of Units) system, it also provides an alternative path to realize the pascal.[1,2] By measuring the refractivity and the temperature of a gas, it is possible to calculate its pressure by the use of the Lorentz–Lorenz equation and an equation of state. Such a realization of the pascal does not comprise any mechanical actuator but depends instead on gas parameters, which potentially can decrease uncertainties and shorten calibration chains.[3–12]

Assessments of refractivity in an FP cavity by refractometry often encompass a step in which the penetration depth of the light into the mirrors is estimated to correct for the fraction of the cavity length into which no gas can penetrate. This is of particular importance for systems that are to be used as primary standards. However, there is a general uncertainty about how this should be done. While some authors provide means of compensating for the penetration depth,[4,12,13] others have not found it necessary to do so.[11,14,15] Here, we discuss a common pitfall that can be a reason for this and provide a recipe on how to perform FP-cavity-based refractometry without any influence of mirror penetration depth.

II. CONVENTIONAL MEANS TO CORRECT FOR FINITE PENETRATION DEPTHS OF MIRRORS IN FP-CAVITY-BASED REFRACTOMETRY

When FP-based refractometry is used to assess refractivity, gas molar density, or pressure, it uses a laser to assess the frequency shift of a cavity mode when gas is let in. By locking the frequency of a laser to that of a cavity mode, the shift in the frequency of a cavity mode is transferred to a shift of the frequency of the laser light, which, in turn, is measured.

A. Refractivity assessed by FP cavities with mirrors with phenomenological introduced penetration depth

In the presence of mirrors with a finite penetration depth, as, for example, is the case for mirrors with a coating comprising a distributed Bragg reflector (DBR), it is possible to express the frequency of a laser that addresses an evacuated cavity, ν_0, as

$$\nu_0 = \frac{q_0 c}{2(L_0 + 2L_{pd})} = \frac{q_0 c}{2L_0'}, \tag{1}$$

where q_0 is the number of the mode to which the laser is locked, c is the speed of light in vacuum, L_0 is the length of the cavity (the mirror-to-mirror distance in the center of the cavity), and, as is shown in Fig. 1, L_{pd} is the penetration depth of the light in the mirrors (or in their reflective coating).[16–22] We have in the last step introduced the entity L_0', which is the effective length of the cavity in vacuum, given by $L_0 + 2L_{pd}$.[23] This implies that the free spectral range (*FSR*) of the cavity (likewise in vacuum), defined as the frequency separation of two consecutive cavity modes, ν_{FSR}, is given by

$$\nu_{FSR} = \frac{c}{2L_0'}. \tag{2}$$

When gas with an index of refraction of n is let into such a cavity, the frequency of the laser will change. If we allow for the possibility that the laser simultaneously makes a number of mode jumps, Δq, to closely lying modes, it is customary to assume that the frequency of the laser can be written as

$$\nu_g = \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2L_{pd})}, \tag{3}$$

where δL represents the physical deformation of the cavity due to the presence of the gas (which is positive for elongation and negative for contraction and can include effects both from the altered length of the cavity spacer and the bending, distortion, and potential compression of the mirrors), and where we have acknowledged, as is shown in Fig. 1, that the gas fills the entire volume between the mirror surfaces, but does not penetrate into the mirror coatings,

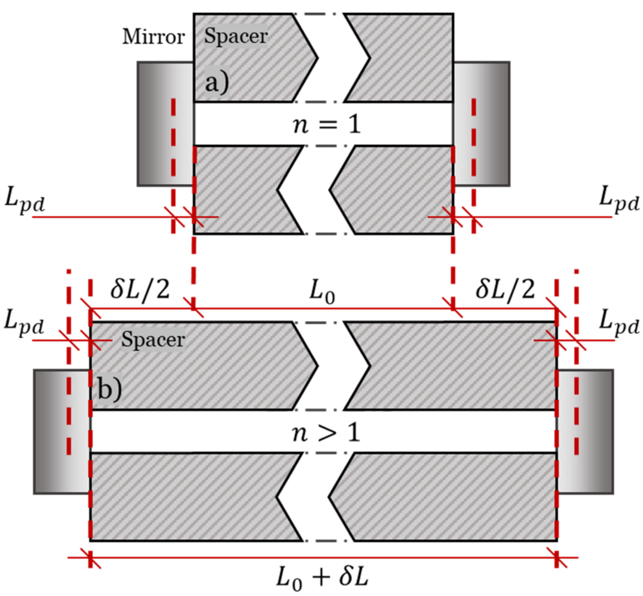

FIG. 1. Schematic illustration of a spacer with (a) an empty cavity and (b) a cavity containing gas. L_0 is the length of the cavity (the mirror-to-mirror distance in the center of the cavity). L_{pd} is the penetration depth of the light in the mirrors. δL represents the physical deformation of the cavity due to the presence of the gas.

and that the penetration depth is an inherent property of the mirrors (whereby it is not affected by the presence of the gas).

Denoting the shift in frequency the laser light makes as a consequence of the alteration of refractivity in the cavity by $\Delta\nu$,[24] it can be shown that, from Eqs. (1) and (3), the refractivity of the gas let into the cavity can be expressed in either of several forms, e.g. (where the first line is exact, derived in Subsection 2 of the Appendix, while the others represent successively simpler, but less accurate forms),[25]

$$\begin{aligned}(n-1) &= \frac{\overline{\Delta\nu} + \overline{\Delta q}}{(1-\overline{\Delta\nu})[1+\varepsilon+\varepsilon(n-1)](1-2\overline{L_{pd}})} \\ &\approx \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1-\overline{\Delta\nu}+\varepsilon}(1+2\overline{L_{pd}}) \\ &\approx \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1-\overline{\Delta\nu}}(1-\varepsilon+2\overline{L_{pd}}) \\ &\approx \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1-\overline{\Delta\nu}}(1+2\overline{L_{pd}}) - \overline{\delta L},\end{aligned} \tag{4}$$

where $\overline{\Delta\nu}$ represents the relative shift of the laser frequency, given by $\Delta\nu/\nu_0$; $\overline{\Delta q}$ is the relative number of modes the laser has jumped, given by $\Delta q/q_0$ (or, alternatively, by $\Delta q \nu_{FSR}/\nu_0$); $\overline{L_{pd}}$ is the relative penetration depth of a mirror, given by L_{pd}/L_0'; $\overline{\delta L}$ is the relative alteration of the length of the cavity due to the presence of the gas, given by $\delta L/L_0'$; and ε is the refractivity normalized relative alteration of the length of the cavity, given by $\overline{\delta L}/(n-1)$. $2\overline{L_{pd}}$ thus represents the fraction of the cavity length into which no gas can penetrate. Expressions of this or similar type appear frequently in the literature, e.g., in the works of Egan and Stone[4] and Zakrisson *et al.*[13]

B. Estimate of the penetration depth

It is common to express the influence of mirrors (with their stack of dielectric layers, henceforth referred to as a Bragg grating) on the propagation of the light in terms of a frequency-dependent optical phase shift at the surface of the mirror, $\phi(\omega)$. It has previously been shown that the mode spacing of longitudinal cavity modes (i.e., the FSR) of an FP-cavity in which there are sources of such phase shifts can be expressed as[26–28]

$$\nu_{FSR}(\omega) = \frac{c}{2\{L_0 + c[\partial\phi(\omega)/\partial\omega]\}}, \tag{5}$$

where ω is the angular frequency of the light.

It is customary to write $\phi(\omega)$ in terms of a Taylor series centered around the design frequency, ω_{des}, as

$$\phi(\omega) = \phi_{des} + T_g(\omega - \omega_{des}) + \frac{D_2}{2}(\omega - \omega_{des})^2, \tag{6}$$

where $\phi_{des} = \phi(\omega_{des})$ and T_g and D_2 are the group delay[29] and the group delay dispersion (GDD)[30] of the mirror, respectively, which often are estimated either from measurements or calculated using Fresnel reflection transfer-matrix methods.

Inserting this expression into that for the free spectral range, i.e., Eq. (5), neglecting the influence from the GDD,[31] and comparing with Eq. (3), show that the penetration depth can be considered to be frequency independent, given by

$$L_{pd} = \frac{c}{2} T_g. \tag{7}$$

This implies that it is customary to assess the refractivity of the gas from a measurement of $\overline{\Delta \nu}$ and $\overline{\Delta q}$ (and a previously assessed value for ε) by any of the expressions in Eq. (4) where $2\overline{L_{pd}}$ is given by $2L_{pd}/L_0$, where, in turn, L_{pd} is given by Eq. (7).

However, regarding calculations of optical phase shifts (and, therefore, penetration depths), which often is done by the use of Fresnel reflection transfer-matrix methods, Rourke recently stated that such an approach "relies on sufficient knowledge of the thickness and refractive index of the thin-film mirror coating, including reference data for absolute refractive index (different in the thin-film form than in the corresponding bulk material), temperature dependence of refractive index (thermo-optic coefficient), and temperature dependence of thickness (thermal expansion coefficient)."[32] This implies that it might not necessarily be trivial to perform such estimates with good accuracy. This has been verified by Ogin, who concluded that reliable reference data on thin-film properties remain lacking.[33] Hence, such estimates can be associated with a significant amount of uncertainty.

Despite this, and based on the fact that the penetration depth of high-reflection coated mirrors suitable for refractometry is a fraction of the Bragg length (which is the sum of a pair of layers of dielectric coatings, which, in turn, typically is half of the wavelength of the light), it is possible to estimate that typical penetration depths can be a few tenths of a μm.[21] This implies that, for cavities with lengths of some tens of centimeter, $2\overline{L_{pd}}$ takes a value of a few (or some) ppm.

However, although the above described procedure seems appropriate, use of it will, in many cases, introduce an error in the assessment of n 1 that is not only given by the uncertainty in the estimate of the penetration depth, but also, in fact, equal to its entire value, i.e., typically of a few ppm. The reason for this is that the aforementioned phenomenological view of the penetration depth neglects two important aspects: (i) the interaction between the electrical field in the cavity and that in the mirror, which makes it dependent on the index of refraction in the cavity, and (ii) the influence this has on the assessment of refractivity.

III. APPROPRIATE MEANS TO TAKE FINITE PENETRATION DEPTHS OF MIRRORS INTO ACCOUNT IN FP-CAVITY-BASED REFRACTOMETRY

A. Interaction between the electrical field in the cavity and that in the mirror—Two types of Bragg grating

Although the penetration depth in a high-reflectivity quarter-wave-stack (QWS) dielectric mirror, comprising alternating layers of dielectric media with high and low indices of refraction, n_H and n_L, respectively, can be estimated from calculations or measurements (e.g. provided by manufacturer)[19] and opposed to what is tacitly assumed when expressing the frequency of the laser probing the cavity in the presence of gas as is done in Eq. (3), it is not generally known among practitioners of refractometry that the penetration depth depends on the index of refraction outside the stack (i.e., in the cavity). In fact, it has repeatedly been shown in the literature that it does so.[16,17,21] This implies that L_{pd} in reality should be seen as $L_{pd}(n)$. It has also been shown that its dependence on n depends on in which order the various dielectric layers are applied.[16,17,21]

For the case when the outermost layer has an index of refraction that is higher than the subsequent layer, i.e., n_H, n_L, n_H, n_L, etc., referred to as type H in Fig. 2, the penetration depth, $L^H_{pd}(n)$, is proportional to the index of refraction in the cavity[17,21] and can be written as[17]

$$L^H_{pd}(n) = \frac{n\lambda_B}{4\Delta n_B}, \tag{8}$$

while for the opposite case (i.e., n_L, n_H, n_L, n_H, etc., referred to as type L), the corresponding entity, $L^L_{pd}(n)$, can be expressed as

$$L^L_{pd}(n) = \frac{(\bar{n}_B)^2\lambda_B}{4n\Delta n_B}. \tag{9}$$

Here, n is the index of refraction "outside" the Bragg grating (thus the index of refraction of the gas in the cavity), while λ_B is the Bragg wavelength, defined as $2\bar{n}_B\Lambda$. Moreover, $\bar{n}_B$ is the average refractive index of the high-reflection coating, which for a well-designed mirror (for which $d_H n_H = d_L n_L$, where d_H and d_L are the thicknesses of the layers with high and low index of reflection, respectively) is $2n_H n_L/(n_H + n_L)$. Λ is the grating period (or the physical length of the double layer), given by $d_H + d_L$. Δn_B is the

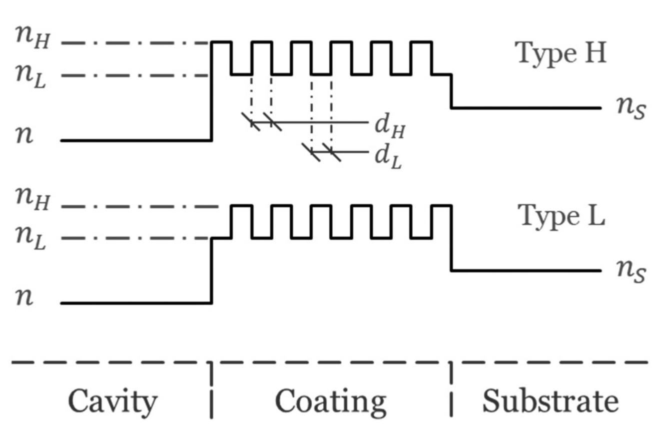

FIG. 2. Schematic illustration of two types of Bragg grating making up high-reflectivity QWS dielectric mirrors. Type H: the case when the outermost layer has an index of refraction that is higher than the subsequent layer, i.e., when they come in order n_H, n_L, n_H, n_L, etc. Type L: the case when the outermost layer has an index of refraction that is lower than the following layer, i.e., for n_L, n_H, n_L, n_H, etc. In the illustration: the leftmost part represents the cavity (with an index of refraction of n), while the rightmost part denotes the mirror substrate.

difference between the indices of refraction of the two layers, i.e., $n_H \quad n_L$.

B. Use of mirrors with Bragg grating of type H for FP-based refractometry—Means to perform Fabry–Perot cavity based refractometry without influence of mirror penetration depth

Equation (8) shows that although there is no gas penetrating into the mirror coating, the penetration depth of a mirror of type H is still proportional to the index of refraction of the gas in the cavity. This implies that for this case, Eq. (3) should in reality be written as

$$v_g = \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2nL_{pd}^{H,0})} = \frac{(q_0 + \Delta q)c}{2nL''}, \qquad (10)$$

where $L_{pd}^{H,0}$ is given by $\lambda_B/(4\Delta n_B)$ and L'' is given by $L_0 + \delta L + 2L_{pd}^{H,0}$. This can be interpreted as if the system does not have any part of the cavity into which gas do not reach; in other words, *the system behaves like an ideal system without penetration depth* (with a length of L'').[34]

This implies, in turn, that when mirrors with coatings of type H are used, various expressions for refractivity given above should be written with no explicit reference to the penetration depth, viz., as

$$\begin{aligned}(n \quad 1) = {} & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{(1 \quad \overline{\Delta\nu})[1 + \varepsilon + \varepsilon(n \quad 1)]} \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu} + \varepsilon} \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu}}(1 \quad \varepsilon) \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu}} \quad \overline{\delta L},\end{aligned} \qquad (11)$$

where, as is shown in Subsection 3 of the Appendix, the first step follows from Eqs. (1) and (10) without approximations, while the following steps represent successively simpler, but less accurate, forms.

This shows that *the use of mirrors of type H provides a means to perform FP cavity based refractometry without any influence of mirror penetration depth.*

C. Use of mirrors with Bragg grating of type L for FP-based refractometry—Fabry–Perot cavity based refractometry with excessive influence of mirror penetration depth

Conversely, if mirrors with coatings of type L would be used, the expressions for refractivity would explicitly comprise expressions for the penetration depth. However, since the penetration depth now has an inverse dependence on the index of refraction, it will appear in the expressions for the refractivity with twice the amount assumed by the phenomenological index-of-refraction-independent assumption given by Eq. (4), viz., as[35]

$$\begin{aligned}(n \quad 1) = {} & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{(1 \quad \overline{\Delta\nu})[1 + \varepsilon + \varepsilon(n \quad 1)](1 \quad 4\overline{L_{pd}^{L,0}})} \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu} + \varepsilon}\left(1 + 4\overline{L_{pd}^{L,0}}\right) \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu}}\left(1 \quad \varepsilon + 4\overline{L_{pd}^{L,0}}\right) \\ & \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 \quad \overline{\Delta\nu}}\left(1 + 4\overline{L_{pd}^{L,0}}\right) \quad \overline{\delta L},\end{aligned} \qquad (12)$$

where $L_{pd}^{L,0}$ is given by $(\overline{n}_B)^2\lambda_B/(4\Delta n_B)$. Moreover, it should be noticed that $L_{pd}^{L,0}$ is larger than $L_{pd}^{H,0}$ by an amount $(\overline{n}_B)^2$, which typically is > 3. This implies that a mirror of type L would contribute to the assessment of refractivity six times more than what a similar mirror of type H would do if it would be assessed by the (incorrect) equation (4).[36]

IV. CONCLUSION

This analysis indicates that when FP-based refractometry is used, it is advised to use mirrors with a reflection coating of Type H. For such mirrors, the assessment of refractivity is independent of the penetration depth of the mirrors. The assessment should, therefore, not make use of any expression for refractivity that has an explicit dependence on the penetration depth; the reactivity should instead be assessed by the use of any of the expressions in Eq. (11). The reason for this is that, although no gas is penetrating into the Bragg grating, the penetration depth in this case is proportional to the index of refraction of the gas in the cavity, whereby it simply acts as a part of the cavity into which gas is penetrating.[37] Mirrors of type L should be avoided since they can give rise to substantial penetration depths that need to be corrected for when the measured refractivity should be assessed [by use of Eq. (12)].

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

APPENDIX

1. Derivation of the expressions for the empty cavity mode frequency in the main text, i.e., Eq. (1), from the phase description

According to the phase description, the frequency of a laser that addresses a longitudinal mode in an evacuated cavity can be assessed as[4]

$$\nu_0 = \frac{c[m_0 - \phi^{rt}(\nu)/2\pi]}{2L_0}, \tag{A1}$$

where ϕ^{rt} is the round trip phase. Neglecting the Gouy phase, ϕ^{rt} can be written as 2ϕ, where ϕ is the phase shift of a mirror with its coating. Assuming that the latter can be written as given by Eq. (6) in the main text implies that Eq. (A1) here can be written as

$$\nu_0 = \frac{c[m_0 - 2\phi_{des}/2\pi - 2T_g(\nu_0 - \nu_{des})]}{2L_0}. \tag{A2}$$

Solving Eq. (A2) for ν_0 implies that it can be written as

$$\nu_0 = \frac{q_0 c}{2(L_0 + cT_g)} = \frac{q_0 c}{2(L_0 + 2L_{pd})} = \frac{q_0 c}{2L_0'}, \tag{A3}$$

where we have introduced q_0 as a short hand notation for $m_0 - 2\phi_{des}/2\pi + 2T_g\nu_{des}$ and introduced L_{pd} as a short hand notation for $cT_g/2$. Since Eq. (A3) in this Appendix is identical to Eq. (1), using Eq. (7) for the L_{pd}, both in the main text, this supports those expressions.

2. Derivation of Eq. (4) for assessment of refractivity by use of FP cavities with mirrors with phenomenological introduced penetration depth

Starting from Eqs. (1) and (3) in the main text, i.e.,

$$\nu_0 = \frac{q_0 c}{2(L_0 + 2L_{pd})} \tag{A4}$$

and

$$\nu_g = \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2L_{pd})}, \tag{A5}$$

implies that the frequency shift, $\Delta\nu$, defined as $\nu_0 - \nu_g$, is given by

$$\Delta\nu = \nu_0 - \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2L_{pd})}, \tag{A6}$$

which alternatively can be rewritten as

$$2(nL_0 + n\delta L + 2L_{pd}) = \frac{(q_0 + \Delta q)c}{\nu_0 - \Delta\nu}. \tag{A7}$$

By expressing δL as $\varepsilon(n-1)L_0$ and n as $1 + (n-1)$, this can, after some algebra, be written as

$$\begin{aligned}(n-1)2L_0[1 + \varepsilon + \varepsilon(n-1)] &= \frac{q_0 c}{\nu_0}\frac{1 + \overline{\Delta q}}{1 - \overline{\Delta\nu}} - 2(L_0 + 2L_{pd})\\ &= \frac{q_0 c}{\nu_0}\frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 - \overline{\Delta\nu}},\end{aligned} \tag{A8}$$

where we in the last step have made use of Eq. (A4).

Again, making use of Eq. (A4) implies that Eq. (A8) can be rewritten as

$$(n-1)[1 + \varepsilon + \varepsilon(n-1)](1 - 2\overline{L}_{pd}) = \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 - \overline{\Delta\nu}}, \tag{A9}$$

which finally yields

$$(n-1) = \frac{\overline{\Delta\nu} + \overline{\Delta q}}{(1 - \overline{\Delta\nu})[1 + \varepsilon + \varepsilon(n-1)](1 - 2\overline{L}_{pd})}, \tag{A10}$$

which, in turn, is the first line of Eq. (4) in the main text.

3. Derivation of Eq. (11) for assessment of refractivity by use of FP cavities with mirrors with Bragg gratings of type H

Again, starting from Eq. (1) in the main text but with the mode frequency given by Eq. (10), which is given by

$$\nu_g = \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2nL_{pd}^{H,0})}, \tag{A11}$$

gives, for the frequency shift, $\Delta\nu$,

$$\Delta\nu = \nu_0 - \frac{(q_0 + \Delta q)c}{2(nL_0 + n\delta L + 2nL_{pd}^{H,0})}. \tag{A12}$$

Performing the same type of algebra as for the case with a phenomenological introduced penetration depth yields, instead of Eq. (A7),

$$2n(L_0 + \delta L + 2L_{pd}^{H,0}) = \frac{(q_0 + \Delta q)c}{\nu_0 - \Delta\nu}. \tag{A13}$$

Rewriting this in a similar fashion as above gives

$$\begin{aligned}(n-1)2(L_0 + 2L_{pd}^{H,0})[1 + \varepsilon + \varepsilon(n-1)]\\ = \frac{q_0 c}{\nu_0}\frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 - \overline{\Delta\nu}}.\end{aligned} \tag{A14}$$

Since, in this case, the $2(L_0 + 2L_{pd}^{H,0})$ term is canceled by the $q_0 c/\nu_0$ term, this can then be succinctly rewritten as

$$(n-1)[1 + \varepsilon + \varepsilon(n-1)] = \frac{\overline{\Delta\nu} + \overline{\Delta q}}{1 - \overline{\Delta\nu}}, \tag{A15}$$

which finally yields

$$(n-1) = \frac{\overline{\Delta\nu} + \overline{\Delta q}}{(1 - \overline{\Delta\nu})[1 + \varepsilon + \varepsilon(n-1)]}, \tag{A16}$$

which, in turn, is the first line of Eq. (11) in the main text.

REFERENCES

[1]M. Stock, R. Davis, E. de Mirandés, and M. J. T. Milton, Metrologia **56**, 022001 (2019).
[2]M. Stock, R. Davis, E. de Mirandés, and M. J. T. Milton, Metrologia **56**, 049502 (2019).
[3]M. Andersson, L. Eliasson, and L. R. Pendrill, Appl. Opt. **26**, 4835 (1987).
[4]P. F. Egan and J. A. Stone, Appl. Opt. **50**, 3076 (2011).
[5]P. F. Egan, J. A. Stone, J. E. Ricker, and J. H. Hendricks, Rev. Sci. Instrum. **87**, 053113 (2016).
[6]K. Jousten *et al.*, Metrologia **54**, S146 (2017).
[7]D. Mari, M. Pisani, and M. Zucco, Measurement **132**, 402 (2019).
[8]Y. Takei, K. Arai, H. Yoshida, Y. Bitou, S. Telada, and T. Kobata, Measurement **151**, 107090 (2020).
[9]Z. Silvestri, D. Bentouati, P. Otal, and J. P. Wallerand, Acta IMEKO **9**, 303 (2020).
[10]V. N. Thakur, S. Yadav, and A. Kumar, MAPAN **35**, 595 (2020).
[11]I. Silander, C. Forssén, J. Zakrisson, M. Zelan, and O. Axner, J. Vac. Sci. Technol. B **39**, 044201 (2021).
[12]Y. Yang, T. Rubin, and J. Sun, Vacuum **194**, 110598 (2021).
[13]J. Zakrisson, I. Silander, C. Forssén, M. Zelan, and O. Axner, J. Vac. Sci. Technol. B **38**, 054202 (2020).
[14]O. Axner, I. Silander, T. Hausmaninger, and M. Zelan, e-print arXiv:1704.01187v2 (2017).
[15]I. Silander, T. Hausmaninger, M. Zelan, and O. Axner, J. Vac. Sci. Technol. A **36**, 03E105 (2018).
[16]D. Babic and S. Corzine, IEEE J. Quantum Electron. **28**, 514 (1992).
[17]L. R. Brovelli and U. Keller, Opt. Commun. **116**, 343 (1995).
[18]C. J. Hood, H. J. Kimble, and J. Ye, Phys. Rev. A **64**, 033804 (2001).
[19]E. Garmire, Appl. Opt. **42**, 5442 (2003).
[20]M. Malak, A.-F. Obaton, F. Marty, N. Pavy, S. Didelon, P. Basset, and T. Bourouina, AIP Adv. **2**, 022143 (2012).
[21]C. Koks and M. P. van Exter, Opt. Express **29**, 6879 (2021).
[22]It is worth noting that, as is well described by Koks and van Exter,[21] there are several different definitions of penetration depth in a DBR equipped mirror; there is a frequency penetration depth, a phase penetration depth, and a modal penetration depth. Of these, it is the former that is of importance for Fabry–Perot based refractometry since it is this penetration depth that determines the frequency spacing between consecutive longitudinal modes in an FP cavity. Therefore, it is the frequency penetration depth, here denoted L_{pd}, that is considered in this work. It quantifies the group delay, T_g, and, therefore, also the prolongation in length, $cT_g/2$, that light experiences upon reflection from a DBR. Its definition is such that a hard mirror positioned at a distance L_{pd} in the coating will produce the same group delay (and, therefore, the same length prolongation) as the DBR and hence mimic its time/frequency response. The frequency penetration depth is also the entity that attracts most attention when the effect of DBR on the properties of Fabry–Perot cavities is considered.[16–21]
[23]It is worth noting that although the wavelength description of the condition for the cavity mode frequencies given by Eq. (1), in which q_0 represents the number of half wavelengths that make up the standing wave that can exist in an FP cavity with an effective length of L_0', differs slightly from the phase description that is given by Eq. (A1) in the Appendix; as is shown there, the two are equivalent.
[24]For the case when the shift of the frequency of the laser is assessed by measuring the frequency of the beat with respect to a fixed laser frequency, f, this entity is often denoted Δf in the literature.
[25]As is shown in Subsection 2 of the Appendix, the first step in Eq. (4) follows directly from Eqs. (1) and (3) without approximations. In the second step, terms in the order of $\varepsilon(n \quad 1)$ (and less) have been neglected. In the third and fourth step, terms with sizes in the order of ε^2 have additionally been neglected. This implies that for a system using 1.5 μm light probing a cavity with a free spectral range of 1 GHz (whereby $\overline{\Delta\nu} \quad 5 \quad 10^{\;6}$) and a relative distortion coefficient (ε) of $2 \quad 10^{\;3}$ assessing nitrogen at atmospheric pressure (for which $n \quad 1 = 3 \quad 10^{\;4}$), as recently has been developed and characterized by Silander *et al.*,[11] the second, third, and fourth expressions in Eq. (4) are accurate to levels of $6 \quad 10^{\;7}$, $4 \quad 10^{\;6}$, and $4 \quad 10^{\;6}$, respectively. The various expressions are valid to other levels of accuracy for other pressures and other types of systems.
[26]R. G. DeVoe, C. Fabre, K. Jungmann, J. Hoffnagle, and R. G. Brewer, Phys. Rev. A **374**, 1802 (1988).
[27]M. J. Thorpe, R. J. Jones, K. D. Moll, J. Ye, and R. Lalezari, Opt. Express **13**, 882 (2005).
[28]A. Schliesser, C. Gohle, T. Udem, and T. W. Hansch, Opt. Express **14**, 5975 (2006).
[29]R. Paschotta, "Group Delay," RP Photonics Encyclopedia; see: https://www.rp-photonics.com/group_delay.html; accessed 22 September 2021.
[30]R. Paschotta, "Group Delay Dispersion," RP Photonics Encyclopedia; see: https://www.rp-photonics.com/group_delay_dispersion.html; accessed 22 September 2021.
[31]For the case when the refractometer is making use of mode jumps, the maximum detuning is the free spectral range (FSR). Since, for a cavity with a length of 0.15 m, this is in the order of 1 GHz, the GDD term in the denominator of Eq. (5) is negligible with respect to L_0 [for the case with a GDD (D_2) of 10 (fs)^2, it is 18 orders of magnitude smaller].
[32]P. M. C. Rourke, J. Phys. Chem. Ref. Data **50**, 033104 (2021).
[33]G. H. Ogin, "Measurement of thermo-optic properties of thin film dielectric coatings," Ph.D. thesis (California Institute of Technology, Pasadena, CA, 2013).
[34]Since $\delta L = 0$ in vacuum, the ν_{FSR} is given by, and represents, $c/2(L_0 + 2L_{pd}^{H,0})$.
[35]While n can be expressed as $1 + (n \quad 1)$, $1/n$ can be written as $1/1 + (n \quad 1)$, which, for small values of $n \quad 1$, approximately can be written as $1 \quad (n \quad 1)$. Hence, the latter expression differs from the first one by an amount of $2(n \quad 1)$.
[36]For the case with a Bragg grating of type L made by the same materials and cavity length as were considered for the type H Bragg grating in the estimate above, it would thus provide a contribution to the estimate of refractivity by $25 \quad 10^{\;6}$.
[37]Equation (4) should never be used for high-reflectivity QWS dielectric mirrors. If refection coatings of type L are used, the appropriate expressions for the refractivity are those that are given by Eq. (12).