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Does it move?

Euclidean and projective rigidity of hypergraphs.

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Academic dissertation

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Abstract

Rigidity theory is the mathematical study of rigidity and flexibility of discrete structures. Rigidity theory, and the related field of kinematics, have a wide range of applications to fields such as material science, robotics, architecture, and computer aided design.

In rigidity theory, rigidity and flexibility are often studied as properties of an underlying combinatorial object. In this thesis, the aim is to study rigidity theoretic problems where the underlying combinatorial object is an incidence geometry. Firstly, we study rigidity problems for realisations of incidence geometries of rank 2 as points and straight lines in the plane. Finding realisations of incidence geometries as points and straight lines in the plane is an interesting problem in its own right that can be formulates as a problem of realisability of rank 3 matroids over the real numbers.

We study motions of rod configurations, which are realisations of incidence geometries as points and straight line segments in the plane, where each line segment is treated as a rigid rod. Specifically, motions of a rod configuration preserve the distance between any two points on a rod. We introduce and investigate a new notion of minimal rigidity for rod configurations. We also prove that rigidity of a rod configuration is equivalent to rigidity of a graph, under certain geometric conditions on the rod configuration. We also find realisations of v_3-configurations that are flexible as rod configurations for $v \leq 28$. We show that all regular realisations of v_3-configurations for $v \leq 15$, and triangle-free v_3-configurations for $v \leq 20$ are rigid as rod configurations.

We also consider motions of realisations of incidence geometries as points and straight lines in the plane which preserve only incidences between points and lines. We introduce the notion of projective motions, which are motions of realisations of incidence geometries as points and straight lines in the projective plane which preserve incidences. Furthermore, we introduce the basic tools for investigating rigidity with respect to projective motions. We also investigate the relationship between projective rigidity and higher-order projective rigidity.

Finally, we introduce a sparsity condition on graded posets, and introduce an algorithm which can determine whether a given graded poset satisfies the sparsity condition. We also show that sparsity conditions define a greedoid.

Keywords: Rigidity, flexibility, hypergraphs, incidence geometries, the projective plane, algorithms, matroids, greedoids, matroid realization spaces

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