UMEÅ UNIVERSITY

A FORMAL AND STRATEGIC APPROACH TO UNDERSTANDING THE ANCIENT VIKING GAME HNEFATAFL

A Game Strategy Approach

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Abstract

The ancient viking board game of Hnefatafl, though partly lost to time, can potentially give us great insight into viking culture, through its strategic play-style. In this paper, we aim to formalize the Fetlar rule set for Hnefatafl and develop multiple strategic approaches to the ancient board game of Hnefatafl. Through consultation with seven players of the game, we, with their help, propose four unique attacking strategies and five defending strategies for Hnefatafl. Through these strategies we explore how the game of Hnefatafl was meant to be played, and what values it may teach us about viking society. While our understanding of Hnefatafl is not complete, our research proposes that the game may be a tale of an epic battle against foreign aggressors unfolding on the game board. Through this analysis, we hope to build a foundation for future Hnefatafl research, hopefully allowing for the creation of greater strategies or even achieving perfect gameplay.

Acknowledgements

I want to give a special thanks to the Hnefatafl community at aagenielsen.dk, who gave me incredible insights into the game and strategies. The people there were incredibly nice and were more than willing to share their knowledge of game strategies everything else. I want to give a special thanks to the users ’Draganov’ and ’Hagbard’ as they were incredibly helpful and showed great interest in my research. They provided incredible insight, and without their help, this paper would not be what it is today. Additionally, I would like to thank my advisor Andreas Brännström for his excellent guidance throughout this process, not only in providing motivation, but also in discussing possible directions to take with my project and ways to go about my research. This process has been really interesting and I could not have done it without all your help.

Contents

1 Introduction

In ancient Scandinavia the game of Hnefatafl was played throughout the land [8]. Only the most skilled of vikings could master the game and prove their skills, not only in physical combat, but also in leadership and strategy[1]. The game has since been lost to time, and though partly restored today thanks to the efforts of Murray [13], we may never know the actual rules or importance of the game. An accurate restoration of Hnefatafl is incredibly difficult to create as the game was played all over Scandinavia during the viking era, and not all games were played with the exact same board, as board lengths would vary between places. This means many small variations of the game exists, such as a thirteen by thirteen variant, weak king games or many others. Additionally, the exact rules for the game was never passed down or documented, so the current available rules were found through analysis of historical remains [13]. This leaves our current understanding of Hnefatafl incredibly vague even though researchers before have attempted to recover the game either through historical artifacts [13] or through artificial intelligence [5]. While these attempts proved relatively successful, there still remain several uncertain aspects of Hnefatafl such as game strategies, exact rules and exact meaning of game narrative. Because of this, we need to further examine our current understanding of not only Hnefatafl as we understand it, but also the rules and strategies to play by. Our approach to this, is to examine Hnefatafl and with input from players of the game, develop various strategies to efficiently win the game. These strategies, we then aim to analyze through both a historical and strategic approach, as well as evaluating how each strategy performs against one another when competing through simulated game play. With this, our contributions to this field of research, are a formal description of the 'Fetlar' rule set for Hnefatafl, nine unique strategic approaches to the game along with algorithms that implement all of them, five distinct classification of strategies, a strategical approach to game narrative analysis and a simple yet effective Hnefatafl simulator that allows easy comparison between developed strategies[2]. This allows for a far greater understanding of Hnefatafl as a game and can hopefully give us an insight into the minds of the greatest vikings of a bygone era.

In the following section of this paper, we cover related work, after which we in Section 3 will provide necessary background information for the game of Hnefatafl and introduce terminology prevalent within the game. In Section 4, we will go through how we developed strategies for Hnefatafl. We will then in Section 5 present our discovered potential Hnefatafl strategies and formalized game rules for the 'Fetlar' rule set of Hnefatfl. In Section 6, we will then evaluate and compare our discovered strategies, where-after we in Section 7 will analyze the inherit strategical advantages of different strategies to better understand how each strategy performs. Lastly, in Section 8, the paper is concluded by highlighting the main contributions and directions for future work.

[1]Source: Jonathan Williamson, "What You Need to Know About Hnefatafl, the 1,000-Year-Old Viking Board Game," *The Viking Herald*, available at `https://thevikingherald.com/article/what-you-need-to-know-about-hnefatafl-the-1-000-year-old-viking-board-game/324` (accessed May 8, 2025).

[2]The developped Hnefatafl simulator can be found at `https://github.com/Hnefa/Hnefatafl`

2 Related Work

The concept of formalizing games in order to solve or get a better understanding of them is not a completely untouched domain. A formal description of games has been used to solve popular board games such as chess and American checkers [6] [11], where formalized higher order logic is used to analyze game states. Here formal language and boolean logic was used to create frameworks that allowed for computers to assist in solving and calculating moves according to rules within the game. This way of using formal language to convey board games however, is not the only method explored for this, as GGP (General Game Play) language developed specifically for board games has been attempted as seen in RBG (Regular Board Game) [7].

Hnefatafl itself has little research done when it comes to the formalization of the game and game strategies themselves remain a relatively unexplored domain. Researches have previously developed players for Hnefatafl using a neural network approach with some success [5]. Though this study has little strategic value, the study brings up an interesting conclusion when discussing their found results. With the neural network approach described in the study by Phillip Hinston [5], it was found that the defending player was able to win the game a majority of the time when playing by Fetlar rules, and they therefore suggested limiting the defensive player, by playing a weak unarmed king version of the Fetlar rules. This however, proved too fatal a handicap for the defending player, as the attacking player now had a significantly higher chance of winning. If we consider other games with more research done on the development of players such as chess, we see great success utilizing alpha-beta algorithms as far back as 2002 [14]. A newer approach to the development of players in chess is that of evolutionary algorithms [3], where agents gradually improve over many generations. This evolutionary algorithm approach is also partly explored in the study by Phillip Hinston [5], where Hinston suggests using this approach to evaluate candidate rule sets for Hnefatafl. The evolutionary algorithm approach to developing players, while efficient, has limited insight into the strategies required of either player in a game of Hnefatafl. This leaves a clear knowledge gap in relation to our understanding of strategies. If we again examine chess, Morales E. highlights the importance of patterns as a part of developing a strategy [12]. Once again Hnefatafl encounters yet another knowledge gap, as there is little documentation on any useful patterns in the game. This leaves the area of Hnefatafl strategies mostly unexplored and with very little documentation. Understanding these key concepts will be a crucial step in being able to recognize patterns and develop strategies for Hnefatafl.

If we instead of examining Hnefatafl specifically, investigate how formal language have been used to understand and explore mechanics of other games such as video games, we see multiple use cases. A relevant example is that of the collectible card game Hearthstone, where formal model to game card descriptions, allowed for improved understandings compared to the natural language used normally [4]. This formalized model allowed all cards to be effectively understood by all players, and exemplifies how formal language can be used to remove doubt from rules, making games universally understood.

Another study focusing on mobile games [9], made use of formal analysis to accurately document, analyze and describe the mobile game "Plants vs Zombies". This accuracy is not only useful for documenting and analyzing, but is also beneficial to deconstruct war games and develop strategies as seen in [2], where the formal language builds the foundation for successful play. From a historical perspective, formal languages have not seen widespread usage when uncovering rule sets or developing strategies for ancient games such as Hnefatafl, which is likely due to the uncertainty of the rules and difference in fields (history and math/computer science), however, documenting rule sets for such games in ways that leave little room for misinterpretation is a common sight in ancient games such as Hnefatafl, Tablut and others as explored in [13]. Formal languages such as boolean logic or RBG should therefore be able to assist in this as this leaves no room for misinterpretation.

3 Background: Hnefatafl

The game of Hnefatafl is an extremely important insight into the forgotten days of ancient Scandinavia, as it not only gives us a glance into viking past times, but can also help give us a significantly better understanding of viking culture, social norms and much morel [8]. The game of Hnefatafl was culturally important enough to be played among the gods as mentioned in both the Friðþjófs-saga and Hervarvar-saga among others [13]. Mastery of the game was considered an important skill for a viking warrior, as ones skill as a leader could be reflected in the game [8]. Nowadays, Hnefatafl is not as relevant to Scandinavian culture, but knowing how to play the game, gives us a further insight into the thought processes for the viking leaders.

3.1 Game Background

The rules for the game of Hnefatafl, though lost to time have been partly restored through analysis of archaeological remains. Rules typically follow a process where an attacker with superior numbers will attempt to capture a defending player's king piece, while the defending player attempts to escape. The most commonly accepted understanding of the rules comes from Murray [13], however, small variations still exist such as Old Hnefatafl[1], Fetlar Hnefatafl [2] and Copenhagen Hnefatafl[3]. In this study we will mainly investigate the Fetlar ruleset, as this is the rule set first adapted to competitive play.

Fetlar Hnefatafl is a game between an attacking player and a defending player, played on an eleven by eleven square board, each taking turns to move a single one of their own pieces. The goal of the game is not the same for both players, as the attacking player aims to capture the defending player's king piece with the use of their twenty-four attacking pieces, while the defending player tries to move their king piece to one of the corners on the board with the help of their twelve defending pieces. If the defending player's king gets captured by the attacking player, the attacking player immediately wins the game. If instead the defending player successfully moves their king piece to one of the four corners on the board, the defending player immediately wins the game. Each piece may move like the rook in chess, meaning as far up, down, left or right as they desire without skipping or jumping over friendly or enemy pieces. Unlike chess however, captures work by effectively 'sandwiching' an enemy piece between two friendly pieces. When a piece is captured, it is removed from play, and the game goes on. Both attacking and defending pieces can be captured in this manner, however, the defending player's king piece may only be captured if they are completely surrounded, and thus four attacking pieces are required for a capture to be successful. Additionally, certain squares on the board serve special purposes and may not be moved to by other pieces than the defending

[1]Source: *Aage Nielsen* Rules : Old Hnefatafl 11x11 Available at: `https://aagenielsen.dk/tafl_rules.php`

[2]Source: *Edoardo De Angelis* Rules of Fetlar Hnefatafl 11x11 Available at: `https://aagenielsen.dk/Rules_of_Fetlar_Hnefatafl_11x11.pdf`

[3]Source:*Edoardo De Angelis* Rules of Copenhagen Hnefatafl 11x11 Available at: `https://aagenielsen.dk/Copenhagen_Hnefatafl_11x11.pdf`

king. One such square is that of the center square that is the king pieces starting position. This square is designated the throne, and only the king may move here. This square however, is not the only one only accessible to the king piece, as all four corner squares mark the escape routes for the king, thus only the king piece may move there. Because of these restrictions put on these squares, an enemy piece may be captured if 'sandwiched' between one of these forbidden squares and a friendly piece. It is worth noting that positions along the edge of the board cannot be 'sandwiched' between outside the board and a friendly piece to be captured, however, an enemy king can be captured as if the position outside the board was an enemy player. In rare cases there might be situations where a player cannot perform a legal move, and in such cases, that player instantly loses the game.

3.2 Terminology

In a competitive Hnefatafl setting, there exists a set of terms describing elements of play. These terms are used to refer to certain situations and elements of strategies. Most terms were coined by early Hnefatafl players[4], and are still relevant in modern play [5].

3.2.1 Hnefi

This term does not see much use in competitive play, however, it is incredibly important for the history of the game, as this is the original name of the $king/Hnefi$ piece, before the concept of a king was introduced. It is also from this term that Hnefatafl has its name as board games were typically referred to as 'tafl', thus giving us 'Hnefi-tafl'.

3.2.2 Hunn

While it is uncertain what exactly the smaller pieces in Hnefatafl are called, the most commonly accepted term is *hunn*. This term was used to refer to the defending player's pieces (excluding the $Hnefi$) in the Hervarvar saga [6], however, it is unclear if this term also applies to the attacking players pieces. For the sake of clarification, *hunn* will in this paper's context refer to both the attacking and defending players' pieces (excluding the $Hnefi$).

3.2.3 Castles

In Hnefatafl, the four corners of the board are often referred to as "castles", as it is here the $Hnefi$ piece will move to win the game. Each of these castles can be seen as a sort of "safe house" for the $Hnefi$ piece, as they escape the battle.

3.2.4 Barricades

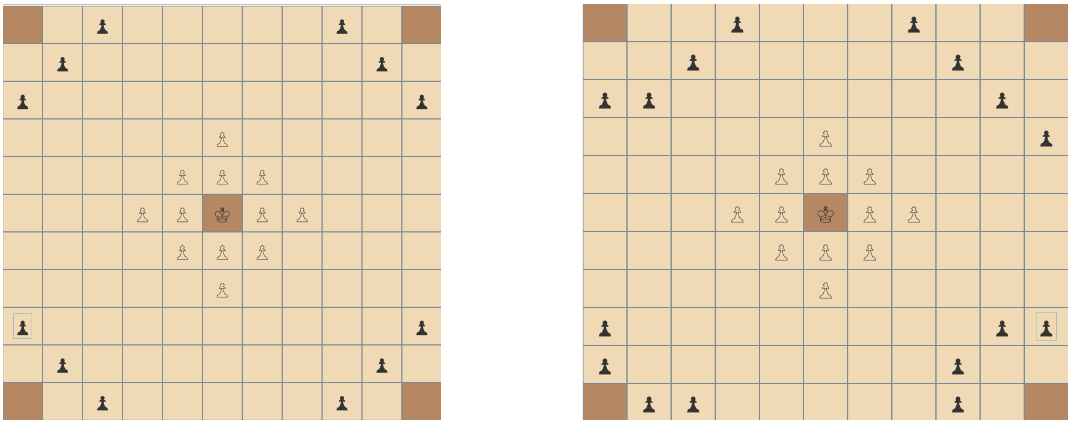
In Hnefatafl, a common strategy revolves around that of controlling the exit castles, so that the player can control which pieces can access them. If a player is successful in taking control of a castle, this is commonly referred to as a "barricade". Typically a barricade is executed by the attacking player and structured as seen in Figure 1a, however, many different

[4]Source: *Tim Miller* on Hnefatafl strategy from March 5, 2018. Available at: https://web.archive.org/web/20180305172919/http://www.tim-millar.co.uk/section509308.html

[5]Source: User *Draganov* on Hnefatafl forum post from August 28, 2023. Available at: https://aagenielsen.dk/hnefataflforum/phpBB3/viewtopic.php?f=3&t=154

[6]Hervarar saga ok Heiðreks, ed. Guðni Jónsson and Bjarni Vilhjálmsson, available at https://heimskringla.no/wiki/Hervarar_saga_ok_Hei%C3%B0reks

versions of barricades exist, such as seen in Figure 1b. It is worth noting that there exist many alternative barricades more complicated than the ones presented here, however, they can all be recognized by their ability to stop the opposing player from reaching the castles.



(a) Example of the primary type of barricades

(b) Example of secondary types of barricades

Figure 1: Example of different types of barricades as executed by the attacking player (black).

3.2.5 Magic Quadrant

The magic quadrant is a term coined by competitive Hnefatafl players, that refers to certain spaces on the board that are incredibly important to control. The magic quadrant is also described as a no-mans-land, where no player initially has control. These board positions are particularly important as they give access to key locations along the edge of the board, that are crucial in forming or preventing barricades. The magic quadrant refers to all locations on the third row from any side as seen on Figure 2.

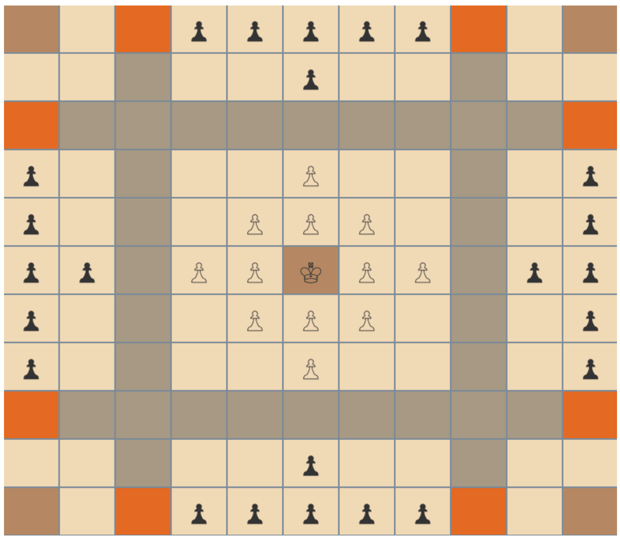


Figure 2: Figure illustrating the magic quadrant with important key edge positions highlighted in orange, and important magic quadrant positions highlighted in gray.

3.2.6 Cordon

The term cordon refers in Hnefatafl to a column or row of eleven *Hunn*, creating an impassable "wall" as seen in Figure 3. A cordon effectively makes certain board positions unavailable for the opposing player, and allows the creator of the cordon to control the board. It is worth noting that a cordon is only effective if there are no enemy pieces on one side of the cordon, as this would allow the opposing player to break the cordon.

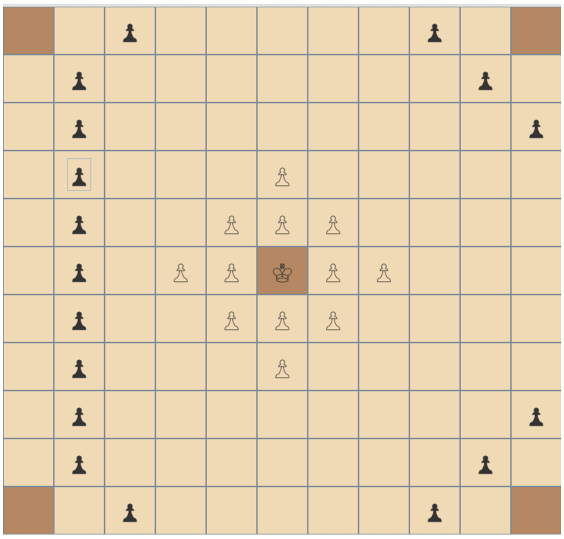


Figure 3: Example of a cordon executed by the attacking player (black), where the attacking player creates a wall that is impossible to pass through for the defending player (white).

3.2.7 Guillotine

In the context of Hnefatafl, a guillotine refers to situations where a player forces their opponent to lose a piece in order to remain in the game. An example of this can be seen in Figure 4, where the defending player can move in and capture the attacking player in the southern left corner. If the attacking player retaliates and captures the defending *Hunn*, an opening is created for the $Hnefi$, allowing the defending player to win the game. If the attacking player does not retaliate, the defending player can now safely move back, leaving an opening for the $Hnefi$, thus forcing the attacking player to place another *Hunn* to block the king, thus creating another victim for the guillotine.

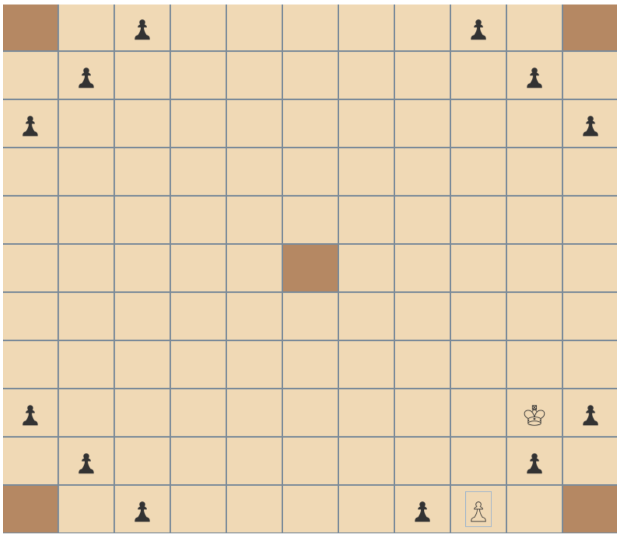


Figure 4: Example of a guillotine executed by the defending player (white), where retaliation against capture of the attacking piece blocking the king will result in additional casualties or loss of the game for the attacking player.

3.2.8 Tower

A tower is a defensive play in Hnefatafl, where a player places 4 pieces next to one another in a square like pattern such that no piece can be captured. A tower such as this can be useful in creating blockades for the opposing player or simply be beneficial when keeping pieces safe. It is worth noting that the integrity of a tower fails when a piece is moved. An example of towers created by both the attacking and defending player can be seen on Figure 5.

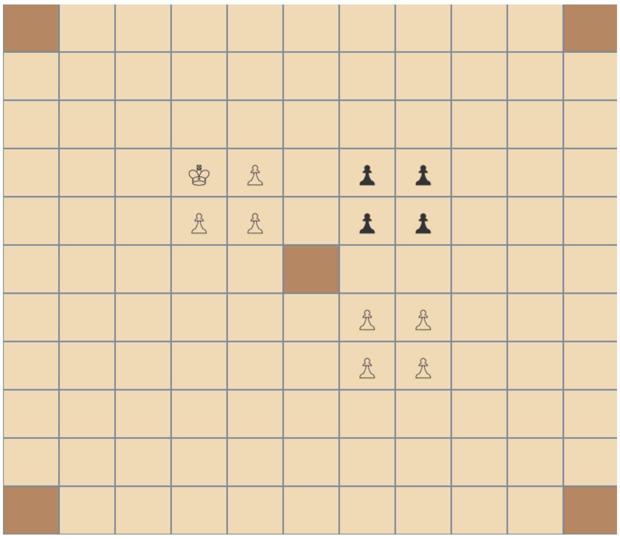


Figure 5: Example of towers executed by both the defending and attacking player, where no piece can be captured unless tower structure is broken by the owner of the tower.

3.2.9 Fort

A fort is a defensive structure mainly used by the defending player when attempting to tie the game. A fort refers to any structure that allows the defending player to repeat a set of moves indefinitely without the attacking player being able to interrupt it. An example can be seen in Figure 6, where the defending player can continuously move to the throne and back with the *Hnefi*, while the attacking player cannot do anything to stop it. Forts come in many different forms, but can never be broken down unless player that built the fort willingly does so.

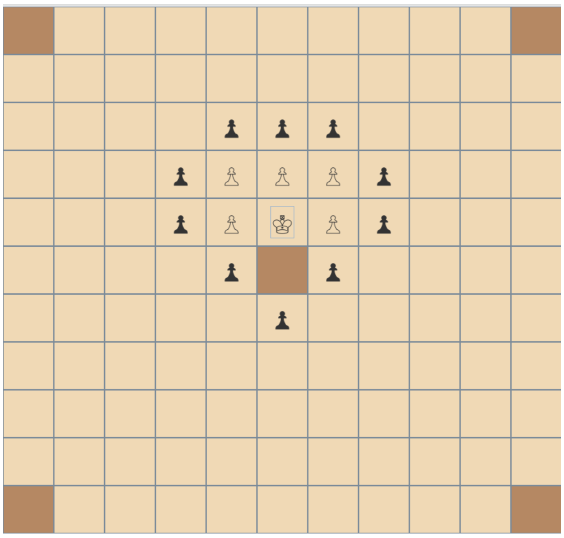


Figure 6: Example of a fort executed by the defending player (white), where the defending player can continuously move to the throne and back with the *Hnefi* piece, resulting in an endless game as the attacking player (black), is unable to do anything to prevent it.

3.3 Game Openings

Within Hnefatafl, formalization of openings is relatively limited, however, after competing in a Old Hnefatafl tournament in 2023, the winner 'Draganov', attempted to formalize a collection of the most used openings at the time[7]. As the Hnefatafl board is symmetrical and different moves are the same if rotated or mirrored, Draganov proposed that all openings should be aligned in the same direction to create formal openings. It was proposed that the rotation of the board should be so that the opening move is made in the most south part of the board, and in case a move could be mirrored, the rightmost move should be selected, as this is thought to be how most right handed players would intuitively approach the game in person. An opening move that had great influence of the modern understanding of the game is that of the Millar Gambit opening[8]. This opening was thought up by Hnefatafl champion Tim Millar in 2010[9]. This opening in particular is incredibly important within Hnefatafl as this is the first known opening that gave the defending player a significant advantage when compared to other plays. The Millar Gambit typically involves sacrificing one of the defending player's *hunn* in order to gain an advantage later in the game, however, several versions of the Millar Gambit exists such as the Millar Gambit (Declined) where the attacking player does not capture the defending player's sacrifice. A full list of recorded openings can be seen

[7]Source: User *Draganov* on Hnefatafl forum post from March 9, 2023. Available at: `https://aagenielsen.dk/lilleforum.php`

[8]Source: User *Hagbard* on Hnefatafl forum post from March 10, 2023. Available at: `https://aagenielsen.dk/lilleforum.php`

[9]Source: User *Hagbard* on Hnefatafl forum post from March 10, 2023. Available at: `https://aagenielsen.dk/lilleforum.php`

on Table 1. Any opening not named here is typically known as an irregular opening.

Table 1 Table displaying known Hnefatafl openings.

Opening Name	First Move	Second Move	Third Move
Millar Gambit (Accepted)	H1 → H3	G5 → G2	H3 → H2
Millar Gambit (Declined)	H1 → H3	G5 → G2	-
Indirect Millar Gambit_1	H1 → H3	G7 → J7	-
Indirect Millar Gambit_2	H1 → H3	E7 → B7	-
Indirect Millar Gambit_3	H1 → H3	E7 → E10	-
Polar Gambit	H1 → H3	H6 → H10	-
Face-to-Face Attack	H1 → H3	D6 → D3	-
Side Attack	H1 → H3	F4 → I4	-
American Opening	F2 → I2	-	-
Bulgarian Opening	F2 → I2	G5 → G4	-
Canadian Opening	H1 → H4	-	-
Dutch Opening	G1 → G2	-	-
English Opening	F2 → I2	F8- I8	-
French Opening	G1 → G3	-	-
German Opening	H1 → H3	D6 → D9	-
Hungarian Opening	H1 → H4	-	-
Italian Opening	H1 → H3	E7 → E9	-
Russian Opening	H1 → H3	H6 → H9	-
Scandinavian Opening	F2 → J2	-	-
Spanish Opening	H1 → H3	F4 → C4	-

4 Methodology

In order to get a better understanding of the strategies within Hnefatafl, a custom Hnefatafl environment was set up following the Fetlar rule set for which a formal language was created to eliminate potential confusion (see Section 5.1). The custom environment was created with the Fetlar rule set in mind [1], to simulate competitive games most accurately. In this environment, we aim to recreate the ancient game of Hnefatafl and explore and discover different strategies, and see how they compare to one another.

Understanding the rules of Hnefatafl may be sufficient for a game play perspective. However, this still leaves a great gap in our understanding of the strategic approach to the game. With a deterministic set of rules, strategic play is a crucial part of the historic value of Hnefatafl as a whole. While game strategies have not been historically preserved, the rules alone might be enough to give us some strategic insight into the game. To get a better understanding of possible strategies, we consulted seven interested participants, to get a sense of possible strategies and strategic approaches to Hnefatafl. If an individual was unfamiliar with the rules, the rules of the game was explained (specifically the Fetlar ruleset as specified in Section 5.1), whereafter the individual played a game as the attacker and defender. Each player was asked questions regarding strategic approaches and game strategies before and after playing the game. The questions asked before playing the game can be seen below.

1. Have you played Hnefatafl before and are you familiar with the rules?
2. How much experience with the game do you have?
3. Knowing the rules, are there any player (defending or attacking) you believe has the advantage from start?
4. Do you have any ideas for strategies that you think would increase your chance of winning?

If a player had already played the game sufficiently (or after playing for both attacker and defender), the player was asked the following questions, in order to gain an insight into their strategic approach to Hnefatafl.

1. Throughout the game, were there any new strategies that emerged?
2. What player do you think have the advantage?
3. Are there any board positions you believe is strategically beneficial or disadvantageous?
4. What is each players strongest feature?
5. If you were to design the ultimate player/strategy for this game. What would their strategy be?

[1] As decribed by Edoardo De Angelis in Rules of Fetlar Hnefatafl 11x11 Available at: `https://aagenielsen.dk/Rules_of_Fetlar_Hnefatafl_11x11.pdf`

6. Consider a stalemate such as this. Would you say it is a tie or the defending player is forced to make a different move to break the stalemate?
7. Consider a situation such as this. Which player (if any) do you believe has the advantage? Do you see any way for white to win?
8. Do you see a critical flaw in the game? If so, how would you fix the game?
9. If you were to guess, do you think people played the game differently back during the viking era?

A table displaying the interviewed participants and a short description of their experience and background can be seen in Table 2.

Table 2 Table displaying background information and game experience of test participants

Identifier	Age (years)	Background	Experience
p1	20-24	Student (Upper Secondary School Teacher)	1-5 hours
p2	20-24	Student (Computer Science)	5-10 hours
p3	25-29	Student (Computer Science)	0 hours
p4	25-29	Student (Computer Science)	0 hours
p5	25-29	Student (Economics)	0 hours
p6	25-29	Student (Biomedical Analysis)	0 hours
p7	35-39	Software Engineer	4 years

Each participant's insights was then used to develop game strategies that were each analyzed and evaluated through simulated games, where each strategy would compete against one another.

5 Results

In the following sections we present some of the main contributions of our research, first formalizing the Fatlar rule set for Hnefatafl, after which we present nine unique strategies developed for the game along with the seven participants.

5.1 Formalized Game Rules

We consider a language L that contains the logical connectives $\neg$, $\vee$ and $\wedge$ as well as the set operator $\cup$, along with a set of board positions B structured as (x, y) where x corresponds to the row of the board position and y represent the column of the board position. We assume that each of these board positions can operate as atoms within boolean logic to describe moves and captures within the game. Using this language we will in the following sections formalize the Fetlar rule set for Hnefatafl. The game of Fetlar Hnefatafl is an asymmetrical hunting game, played by two players; an attacking player, and a defending player. The two players can be denoted by the set P such that $P = \{P_{atk}, P_{def}\}$, where P_{atk} represents the attacking player, and P_{def} represents the defending player. The two players will take turns moving, starting with p_{atk}, until one player has won the game. The game play flow of a game can be described through the protocol seen in Algorithm 1.

Algorithm 1 Simple procedure describing game flow for a game of Hnefatafl

```
INITIALIZE hnfatafl_game
SET board = starting_setup
SET turn = attacker
SET game_state = in_progress

while game_State == in_progress do
    SEND board TO current_player
    WAIT FOR move FROM current_player
    if not isLegalMove(move, board, current_player) then
        SEND "Illegal Move" TO current_player
        CONTINUE
    end if
    APPLY move TO board
    UPDATE board
    if hasPlayerWon(current_player) then
        game_state = getVictoryState(current_player)
        WINNER = current_player
        BREAK
    end if
    SWITCH turn TO opponent
end while
```

5.1.1 Board

Hnefatafl is played on a board of varying size with special squares on the board indicating a throne and four castles. The board used in the Fetlar rule set is of size $l \times l$, where $l = 11$ Additionally, let c indicate the middle row as described by $c = (l+1)/2$, and the set of squares on the board be described such that the board B, is described as $B = \{(1,1),(1,2),(1,3),...,(l,l)\}$. Lastly, let a set of castles be described as C and the throne (also referred to as *konakis* [8]) be described as T where $C = \{(1,1),(1,l),(l,1),(l,l)\}$ and $T = \{(c,c)\}$.

5.1.2 Objective

As Hnefatafl is not a symmetrical game such as chess, players do not have the same objective to win the game. Each player controls a unique set of pieces, they can utilize to fulfill their objective. The pieces available to the attacking player can be described with the set $Q_{atk} = \{atk\}$ and the defending players available pieces can be described using the set $Q_{def} = \{def, king\}$, letting all used pieces be described by $Q = Q_{atk} \cup Q_{def}$. To win the game, the defending player (P_{def} (as defined in Section 5.1), must move their *king* piece (also referred to as *Hnefi* [8]) to any castle in C (as defined by Section 5.1.1). If the defending player succeeds in this, they win the game immediately. The attacking player (P_{atk}) on the other hand, must stop P_{def} from moving to a castle by capturing the defending player's *king*/*Hnefi* piece (see Section 5.1.6). If the defending player's *king* is successfully captured, the attacking player immediately wins the game. Additionally, if a player cannot make a legal move during their turn, they immediately lose the game (see Section 5.1.4).

5.1.3 Setup

For each game of Hnefatafl, pieces must be placed in their starting positions when beginning the game. If we define a single starting position as $S_{pos} = (plr, pos)$ where $plr \in Q$ (as defined in Section 5.1.2) and $pos \in B$ (as described in 5.1.1). Using this, we can create the set of starting positions for each player as denoted by A (attacking player) and D (defending player) respectively as depicted below. An example of the starting setup can be seen on Figure 7, with the defending player in the center and the attacking player along the edges of the board.

$$\begin{aligned}A = \{&(atk,(c-2,1)),(atk,(c-1,1)),(atk,(c,1)),(atk,(c+1,1)),(atk,(c+2,1)),(atk,(c,2)),\\&(atk,(c-2,l)),(atk,(c-1,l)),(atk,(c,l)),(atk,(c+1,l)),(atk,(c+2,l)),(atk,(c,l-1)),\\&(atk,(1,c-2)),(atk,(1,c-1)),(atk,(1,c)),(atk,(1,c+1)),(atk,(1,c+2)),(atk,(2,c)),\\&(atk,(l,c-2)),(atk,(l,c-1)),(atk,(l,c)),(atk,(l,c+1)),(atk,(l,c+2)),(atk,(l-1,c))\}\end{aligned}$$

$$\begin{aligned}D = \{&(king,(c,c)),(def,(c,c-2)),(def,(c,c-1)),(def,(c,c+1)),(def,(c,c+2)),\\&(def,(c-2,c)),(def,(c-1,c)),(def,(c+1,c)),(def,(c+2,c)),(def,(c-1,c-1)),\\&(def,(c+1,c-1)),(def,(c-1,c+1)),(def,(c+1,c+1))\}\end{aligned}$$

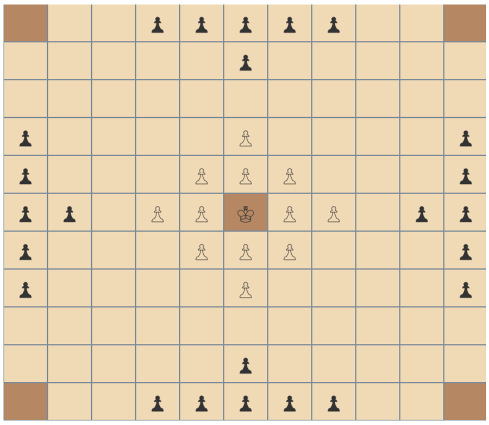


Figure 7: Setup for a game of Fetlar Hnefatafl.

5.1.4 Movement

Any piece in Hnefatafl may move similar to the rook in chess. This means any piece can move as many tiles as it wants either horizontally or vertically but not diagonally. Similarly, a piece may not jump over or move through other pieces. Additionally, only the *king* piece may move through the throne and castle positions on the board. To describe this, let the set of all board positions occupied by P_{atk} be denoted as O_a and all board positions occupied by P_{def} be denoted as O_d (as defined in Section 5.1.2) such that the board positions occupied by either player is denoted as $O = O_a \cup O_d$. The atk and def pieces in Q (as described in Section 5.1.2), may move from their current position $(x_0, y_0) \in B$ to any position on the board as indicated by $M = \{(x, y) \in B \setminus (C \cup T) \mid (x = x_0 \wedge y \neq y_0) \vee (y = y_0 \wedge x \neq x_0)\}$, where no board square between (x_0, y_0) and $(x+1, y+1)$ is present in $(C \cup O \cup T)$. The king however, can also move to thrones and castles such that the movement for $king/Hnefi$ in Q can de described such as $M_k = \{(x, y) \in B \mid (x = x_0 \wedge y \neq y_0) \vee (y = y_0 \wedge x \neq x_0)\}$, where no board square between (x_0, y_0) and $(x + 1, y + 1)$ is present in O.

5.1.5 Captures : Hunn

Upon completing a move, any player may also capture any enemy pieces that can be captured as a result of the completed move. Upon completing a successful capture, the captured piece is taken and removed from the board. An enemy piece is successfully captured if the most recently completed move causes an enemy piece to be trapped between two friendly pieces or a friendly piece and a throne/castle. The $king/Hnefi$ however, cannot be captured in this manner and instead adhere to the rules as described in Section 5.1.6. To determine if an enemy piece is captured ($king/Hnefi$ excluded), let E denote the set of adjacent positions of an enemy piece such that $E = \{((e_x + 1, e_y), (e_x - 1, e_y), (e_x, e_y + 1), (e_x, e_y - 1))\}$ where (e_x, e_y) corresponds to the position on an enemy piece. The attacking player moving from (x_0, y_0) to (x, y) can then capture a defending piece as denoted by boolean logic by C_{atk}. Is the defending player instead moving from (x_0, y_0) to (x, y), the defending player may capture attacking pieces as denoted by C_{def}.

$$
\begin{aligned}
C_{atk} = \{\forall M = (x, y) \in E \mid\ & (x + 1 = e_x \wedge ((x + 2, y) \in (O_a \cup C \cup T)) \vee (x + 2, y) \notin B) \vee \\
& (x - 1 = e_x \wedge ((x - 2, y) \in (O_a \cup C \cup T)) \vee (x - 2, y) \notin B) \vee \\
& (y + 1 = e_y \wedge ((x, y + 2) \in (O_a \cup C \cup T)) \vee (x, y + 2) \notin B) \vee \\
& (y - 1 = e_y \wedge ((x, y - 2) \in (O_a \cup C \cup T)) \vee (x, y - 2) \notin B)\}
\end{aligned}
$$

$$
\begin{aligned}
C_{def} = \{\forall M = (x,y) \in E \mid\ & (x+1 = e_x \wedge ((x+2,y) \in (O_d \cup C \cup T)) \vee (x+2,y) \notin B) \vee \\
& (x-1 = e_x \wedge ((x-2,y) \in (O_d \cup C \cup T)) \vee (x-2,y) \notin B) \vee \\
& (y+1 = e_y \wedge ((x,y+2) \in (O_d \cup C \cup T)) \vee (x,y+2) \notin B) \vee \\
& (y-1 = e_y \wedge ((x,y-2) \in (O_d \cup C \cup T)) \vee (x,y-2) \notin B)\}
\end{aligned}
$$

5.1.6 Captures : King (Hnefi)

The $king/Hnefi$ piece acts differently from any other piece when it comes to being captured. In order to capture the $king/Hnefi$ piece, it must be surrounded by enemies on all sides. We can describe this by letting (k_x, k_y) describe the board position of the $king/Hnefi$ such that the set of board positions directly adjacent to the $king/Hnefi$ can be denoted such that $SK = \{(k_x - 1, k_y), (k_x + 1, k_y), (k_x, k_y - 1), (k_x, k_y + 1)\}$. To capture the $king/Hnefi$, $P_\{atk\}$ must move a piece such that the starting position $(x_0, y_0) \notin SK$ such as described by C_{sk}.

$$
\begin{aligned}
C_{sk} = \{\forall M = (x,y) \in SK \mid & \\
& (x+1 = k_x \wedge ((k_x+1,k_y) \in (O_a \cup C \cup T)) \vee (k_x+1,k_y) \notin B) \wedge \\
& ((k_x,k_y-1) \in (O_a \cup C \cup T)) \vee (k_x,k_y-1) \notin B) \wedge \\
& ((k_x,k_y+1) \in (O_a \cup C \cup T)) \vee (k_x,k_y-1) \notin B) \vee \\
& (x-1 = k_x \wedge ((k_x-1,k_y) \in (O_a \cup C \cup T)) \vee (k_x-1,k_y) \notin B) \wedge \\
& ((k_x,k_y-1) \in (O_a \cup C \cup T)) \vee (k_x,k_y-1) \notin B) \wedge \\
& ((k_x,k_y+1) \in (O_a \cup C \cup T)) \vee (k_x,k_y-1) \notin B) \vee \\
& (y+1 = k_y \wedge ((k_x,k_y+1) \in (O_a \cup C \cup T)) \vee (k_x,k_y+1) \notin B) \wedge \\
& ((k_x-1,k_y) \in (O_a \cup C \cup T)) \vee (k_x-1,k_y) \notin B) \wedge \\
& ((k_x+1,k_y) \in (O_a \cup C \cup T)) \vee (k_x+1,k_y) \notin B) \vee \\
& (y-1 = k_y \wedge ((k_x,k_y-1) \in (O_a \cup C \cup T)) \vee (k_x,k_y-1) \notin B) \wedge \\
& ((k_x-1,k_y) \in (O_a \cup C \cup T)) \vee (k_x-1,k_y) \notin B) \wedge \\
& ((k_x+1,k_y) \in (O_a \cup C \cup T)) \vee (k_x+1,k_y) \notin B) \vee \}
\end{aligned}
$$

5.1.7 Captures : Armed King (Hnefi)

Hnefatafl when playing in accordance with Fetlar rules is played with an armed king. This means that the defending player may utilize their $king/Hnefi$ to capture enemy pieces. An armed king moving from position (x_0, y_0) to (x, y) may capture pieces as denoted by C_{ak}, where E denotes a position $\forall O_a$ where $E = \{((e_x+1, e_y), (e_x-1, e_y), (e_x, e_y+1), (e_x, e_y-1))\}$
.

$$
\begin{aligned}
C_{ak} = \{\forall M_k = (x,y) \in E \mid\ & (x+1 = e_x \wedge ((x+2,y) \in (O_a \cup C \cup T)) \vee (x+2,y) \notin B) \vee \\
& (x-1 = e_x \wedge ((x-2,y) \in (O_a \cup C \cup T)) \vee (x-2,y) \notin B) \vee \\
& (y+1 = e_y \wedge ((x,y+2) \in (O_a \cup C \cup T)) \vee (x,y+2) \notin B) \vee \\
& (y-1 = e_y \wedge ((x,y-2) \in (O_a \cup C \cup T)) \vee (x,y-2) \notin B)\}
\end{aligned}
$$

5.1.8 Game End

A game of Fetlar Hnefatafl can conclude in three distinct ways; a defending win and an attacking loss, an attacking win and a defending loss or a tie. The defending player wins by moving their $king/Hnefi$ piece to one of the corner castles as described by WIN_{def}.

$$WIN_{def} = \{((k_x = 1) \wedge ((k_y = 1) \vee (k_y = l))) \vee (((k_x = l) \wedge ((k_y = 1) \vee (k_y = l))\}$$

The attacking player on the other hand, achieves victory through capture of the defending $king/Hnefi$ such as described by $WIN_{atk} = \{C_{sk}\}$. Alternatively, any player may secure a victory if their opponent cannot make a legal move during their turn, such as WIN_{atks} and WIN_{defs} respectively.

$$WIN_{atks} = \{(\emptyset \in M) \wedge (\emptyset \in M_k)\}$$

$$WIN_{defs} = (\emptyset \in M)$$

Lastly, a game may tie if players repeat the same move endlessly, in a manner where the defending player will never be able to reach an exit, and the attacking player may never be able to capture the king. These games are considered ties, and no victor is declared.

5.2 Formalizing Game Strategies

In consultation with the participants, several theories and strategies emerged. A common belief among new players is that the attacking and defending player are even at the start of the game, however, once a player has played their first few games, that belief changes. Players with little experience tend to believe that the defending player has the advantage from the start. This is in strong contrast to more advanced players that tend to believe that the attacking player has the advantage. An explanation for this could be that p3, p4 and p5 had difficulties finding a consistent and effective way of capturing the $Hnefi$, whereas more advanced players like p1, p2 and p7 had more thought out strategies for dealing with the capture of the $Hnefi$. A thing in common for both that of novice and experienced players is that of understanding the strengths of each player. In just a few games, all of our participants managed to identify key strengths for each player, such as the value the $Hnefi$ provides the defending player, the starting positions of the attacking player, and the value placed on the corners where barricades can be made. While all participants were able to recognize many of these strengths, there was a larger divide when it came to figuring out how to utilize these strength in a game. Several participants had multiple ideas for approaches, all of which are noted in the following section. A full implementation for each strategy along with a basic simulator can be found on git[1].

5.2.1 Attacking Player : Aggressive

A common theme in both p4 and p5's insights was the value of having more pieces available than your opponent. Taking this to the extreme, this aggressive playstyle focuses on

[1] For full implementation see `https://github.com/Hnefa/Hnefatafl`

capturing pieces whenever possible with the ultimate goal of eventually outnumbering your opponent to such an extent that victory is inevitable. This strategy aims to prevent your opponent from utilizing their pieces effectively, and greatly limits their possible plays.
When attempting to implement this strategy, heavy focus was put on ensuring superiority in numbers, by promoting captures whenever possible, and steps were taken to prevent moves that result in friendly pieces being captured as seen in algorithm 2. The opening move were chosen to be the first part of the Millar Gambit as this move is the most used among the named openings.

Algorithm 2 Algorithm describing a simplified Aggressive strategy, where priority is put on capturing enemy pieces when possible. If no enemy pieces can be captured, this strategy then aims to move closer to the $Hnefi$.

```
SET board = board_state
SET game = game_state
SET player = attacker

if isFirstMove(game) then
   perform first move in the Millar Gambit
end if
if canCaptureKing(player, board) then
   capture king
end if
if canCaptureHunn(player, board) then
   capture hunn
end if

first_closest = getClosestMoveToKing(player, board)
second_closest = getSecondClosestMoveToKing(player, board)
if not canMoveBeCaptured(player, first_closest, board) then
   perform first_closest move
else
   if not canMoveBeCaptured(player, second_closest, board) then
      perform second_closest move
   end if
end if

select random move
```

5.2.2 Attacking Player : Barricading

This strategy operates quite different than the other ones. This strategy proposed by p3, p6 and p7 aims to utilize the attackers superior numbers to create barricades and control each castle (or exit) as seen in Figure 1, thus effectively preventing the defending player from victory. To form optimal barricades, this strategy prefers to create barricades as described by AP_{ec}. It is worth noting that many barricades exist, however due to limited time, only the optimal one will be investigated.

A *Barricading* strategy can be employed either center focused or magic quadrant centered. A magic quadrant centered approach to a *Barricading* strategy will typically focus on controlling key locations indicated by the magic quadrant compared to a centered approach which would focus on gaining control of the center square diagonally in-between the two magic quadrant key locations in any corner. A *Barricading* strategy alone does not guarantee a victory, so the attacking player must take a different approach once all exits are secured.

$$\begin{aligned} AP_{ec} = \{ & ((((1,3) \wedge (2,2), \wedge(3,1)) \in O_a) \vee (((1,2) \wedge (1,3) \wedge (2,1) \wedge (3,1)) \in O_a) \wedge \\ & (((l-2,l) \wedge (l-1,l-1), \wedge(l,l-2)) \in O_a) \vee (((l-1,l) \wedge (l-2,l) \wedge (l,l-1) \wedge \\ & (l,l-2)) \in Oa) \wedge (((l-2,1) \wedge (l-1,2), \wedge(l,3)) \in O_a) \vee (((l-1,1) \wedge (l-2,1) \wedge \\ & (l,2) \wedge (l,3)) \in Oa) \wedge (((3,l) \wedge (2,l-1), \wedge(1,l-2)) \in O_a) \vee (((3,l) \wedge (2,l) \wedge \\ & (1,l-1) \wedge (1,l2)) \in Oa)) \wedge (((1,2) \wedge (2,1) \wedge (l,2) \wedge (2,l) \wedge (l-1,1) \wedge \\ & (1,l-1) \wedge (l-1,l) \wedge (l,l-1)) \notin O_d)\} \end{aligned}$$

5.2.3 Barricading - Aggressive

With all exits walled off, the defending player has no way to escape unless the attacker creates an opening for the defender. Therefore, this strategy attempts to slowly and methodically pick of the defensive players pieces while keeping attacking pieces casualties to a minimum. Once sufficient pieces have been cleared up, the attacking player can then with little obstruction opportunity attempt to capture the defensive players $Hnefi$. This can be seen in Algorithm 3, where the attacking player utilizes a mix of offense and defense to achieve this goal.

Algorithm 3 Algorithm describing a simplified Barricading - Aggressive strategy, where the attacking player prioritizes moving to certain board positions in order to most effectively create barricades around the exit castles, before attempting to capture the $Hnefi$.

```
SET board = board_state
SET game = game_state
SET opponent = defender
SET player = attacker

if isFirstMove(game) then
   perform Dutch opening
end if

if canCaptureKing(player, board) then
   capture king
end if

if canKingReachExit(opponent, board) then
   if canBlockKingReachingExit(player, board) then
      block king from reaching exit
   end if
end if

if doesPlayerOccupyBarricadePositions(opponent, board) then
   defend threatened barricade
end if

if canTakeMiddleBarricadePosition(player, board) then
   take middle barricade position
end if

if canCaptureNonCenterHunn(player, board) then
   capture hunn not in the center of the board
end if

if canTakeBorderBarricadePosition(player, board) then
   take border barricade position
end if
select random move
```

5.2.4 Barricading - Cordon

Another approach to finalize the capture of the defending king is that of board shrinking. This strategy aims to create a cordon as described by AP_{ecv} of the board and slowly and methodically move the wall further up the board, thus effectively shrinking the defensive players possible movement. The eventual goal of this strategy is to utilize the fact that if a player is unable to perform a legal move, they immediately lose the game. A shrinking game board like this, should allow the attacking player to either force a victory by limiting the defending player's movement or force an altercation when most beneficial for the attacking player.

$$AP_{ecv} = \{\forall i \in \{1, 2, ..., l\} | (((2, i) \in O_a) \land ((1, i) \notin O_d)) \lor (((i, 2) \in O_a) \land ((i, 1) \notin O_d)) \lor$$
$$(((i, l-1) \in O_a) \land ((i, l) \notin O_d)) \lor (((l-1, i) \in O_a) \land ((l, i) \notin O_d))\}$$

To best implement this, it was decided to forego the intentional capture of unimportant pieces, and instead focus on mainly creating a cordon. Since it is possible for the opponent to occupy edge positions making a full cordon impossible, our implementation attempts to create one of four possible cordons along the northern, southern, eastern or western edge of the board as seen in Algorithm 4.

Algorithm 4 Algorithm describing a simplified Barricading - Cordon strategy, that performs similarly to the Barricading - Aggressive strategy, only utilizing a Cordon upon completion of the barricades.

```
SET board = board_state, SET game = game_state, SET opponnt = defender, SET player = attacker
SET isCordonComplete = false

if isFirstMove(game) then
    perform Dutch opening
end if
if canCaptureKing(player, board) then
    capture king
end if
if canKingReachExit(opponent, board) then
    if canBlockKingReachingExit(player, board) then
        block king from reaching exit
    end if
end if
if not isCordonComplete then
    if doesPlayerOccupyBarricadePositions(opponent, board) then
        defend threatened barricade
    end if
    if canTakeMiddleBarricadePosition(player, board) then
        take middle barricade position
    end if
    if canTakeBorderBarricadePosition(player, board) then
        take border barricade position
    end if
    if isPlayerOccupyingEdgePositionNorth(opponent, board) then
        build barricade along northern edge
    else
        if isPlayerOccupyingEdgePositionEast(opponent, board) then
            build barricade along eastern edge
        else
            if isPlayerOccupyingEdgePositionSouth(opponent, board) then
                build barricade along southern edge
            else
                if isPlayerOccupyingEdgePositionWest(opponent, board) then
                    build barricade along western edge
                end if
            end if
        end if
    end if
end if
if isCordonComplete then
    if isNorthernCordonBuilt(board) then
        move first hunn in cordon one step south
    end if
    if isEasternCordonBuilt(board) then
        move first hunn in cordon one step west
    end if
    if isSouthernCordonBuilt(board) then
        move first hunn in cordon one step north
    end if
    if isWesternCordonBuilt(board) then
        move first hunn in cordon one step east
    end if
end if
select random move
```

5.2.5 Attacking Player : King Centered

As the attacking player, the main goal is to capture the $Hnefi$. Loss of pieces or positions are thus irrelevant, if the main objective is achieved. This strategy brought up by p3, p4 and p6 thus gets straight to the point and attempts to win the game directly. This is effectively achieved by putting all focus on the defending $Hnefi$, attempting to capture it as fast as possible, while continuously blocking the defending $Hnefi$'s movement as much as possible. To implement this, heavy focus was put on the $Hnefi$'s importance and everything else was disregarded as seen in algorithm 5.

Algorithm 5 Algorithm describing a simplified King Centered strategy, where the attacking player simply moves as close to the $Hnefi$ piece as possible.

```
SET board = board_state
SET game = game_state
SET player = attacker

if canCaptureKing(player, board) then
    capture king
end if

if canKingReachExit(opponent, board) then
    if canBlockKingReachingExit(player, board) then
        block king from reaching exit
    end if
end if

move a piece as close to the Hnefi as possible
```

5.2.6 Defending Player : Defensive

A strategy brought up be p3 and p4, is that of slow methodical play. This strategy focuses on safely capturing enemy pieces to force an opening in the attacking players defense. This strategy proposed by p3 focuses heavily on piece and resource preservation, ensuring all pieces are kept safe. This approach can be seen as similar to chess, where defending your pieces is crucial to victory. This approach can be seen in algorithm 6, where moves are ensured to result in minimal friendly captures.

Algorithm 6 Algorithm describing a simplified Defensive strategy, where the strategy aims to avoid being captured over all else, only going on the offensive if no pieces are in danger.

```
SET board = board_state
SET opponent = attacker
SET player = defender

if canKingMoveToExitCastle(board) then
   move king to exit castle
end if

if canCaptureKing(opponent, board) then
   if canBlockKingCapture(player, board) then
      block capture of king
   end if
end if

if canCaptureHunn(opponent, board) then
   move capturable hunn to non capturable position
end if

if canCaptureHunn(player, board) then
   capture hunn
end if

if not areAllExitCastlesBarricaded(board) then
   move king as close to an unbarricaded exit castle as possible
end if

select random move that does not allow player's hunn to be captured.
```

5.2.7 Defending Player : Foot-in-the-Door

The foot-in-the-door strategy mentioned by p2 and p7 aims to prevent the attacking player from creating barricades by placing as many pieces as possible in the significant edge positions in accordance with the magic quadrant. This is done with the intent of placing a 'foot-in-the-door', leaving an opening for the $Hnefi$ to reach the exit castle once it is mobilized after sufficient magic quadrants are covered. This is effectively implemented in algorithm 7, where this algorithm attempts to reach magic quadrant corners before attempting to move the $Hnefi$.

Algorithm 7 Algorithm describing a simplified Foot-in-the-Door strategy, in which the strategy aims to utilize *hunn* to occupy certain important board positions before the $Hnefi$ piece can then move to the castles with the help of the occupied board positions.

```
SET board = board_state
SET opponent = attacker
SET player = defender

if canKingMoveToExitCastle(board) then
   move king to exit castle
end if

if canTakeMiddleBarricadePosition(player, board) then
   take middle barricade position
end if

if canTakeBorderBarricadePosition(player, board) then
   take border barricade position
end if

if canCaptureKing(opponent, board) then
   if canBlockKingCapture(player, board) then
      block capture of king
   end if
end if

if not areAllExitCastlesBarricaded(board) then
   move king as close to an unbarricaded exit castle as possible
end if

if canCaptureHunn(player, board) then
   capture hunn
end if

select random move
```

5.2.8 Defending Player : Fortification

While winning the game typically is the goal of any strategy, the defending player can instead play for a tie instead of a victory. This strategy however, is not in the spirit of the game, and is rather uncompetitive. When playing for a tie, the defending player will typically aim to build a fort similar to the one seen in Figure 6.
This was implemented as in algorithm 8, where the defending player attempts to tie the game by building any rotation of the sample fort seen in Figure 6. Do note though, that there exists many forts, so this strategy only applies to the sample fort seen in Figure 6 specifically.

Algorithm 8 Algorithm describing a simplified Fortification strategy, where four different forts are attempted. If a fort is completed, moves are selected to ensure the safe structure of the fort. If required positions to build all forts have been taken by the enemy player, a random move will be selected.

```
SET board = board_state

if isFortBuilt(board) then
    move piece inside fort so that it can't be captured.
else
    if isNorthernFortCompletable(board) then
        move piece to build northern fort
    end if

    if isWesternFortCompletable(board) then
        move piece to build western fort
    end if

    if isSouthernFortCompletable(board) then
        move piece to build southern fort
    end if

    if isEasternFortCompletable(board) then
        move piece to build eastern fort
    end if

    select random move
end if
```

5.2.9 Defending Player : King's Guard

An alternate strategy mentioned by p5 and p6 is that of a King's Guard/Royal Guards. For this strategy, players identified that the defending *Hnefi* has a hard time achieving success alone, and thus a "royal guard" or"King's Guards" consisting of 2-5 smaller pieces following the *Hnefi* and assisting in captures could prove beneficial. Additionally, the "royal guards" serve as excellent defensive tools used to protect the king.
To implement this strategy as accurately as possible, we made the defense of the *Hnefi* as a high priority as seen in algorithm 9. The king's guards were for this implementation represented by the defending *hunn* within close proximity to the *Hnefi* . Please note that algorithm 9 is a simplification of the implemented strategy, and many smaller details had to be omitted for clarity.

Algorithm 9 Algorithm describing a simplified King's Guard strategy, where the king moves as securely as possible by ensuring a royal guard of at least two friendly *hunn* escorts the *Hnefi* to the exit castles.

```
SET board = board_state
SET opponent = attacker
SET player = defender

if not hasKingMovedFromThrone() then
   if king can move at least 3 spaces then
      move king as far as possible
   else
      clear way for king to exit throne
   end if
end if

if canKingMoveToExitCastle(board) then
   move king to exit castle
end if

if canCaptureKing(opponent, board) then
   if canBlockKingCapture(player, board) then
      block capture of king
   end if
end if

if number of defending hunn in a 5 by 5 grid around the king < 2 then
   move hunn not in the 5 by 5 grid as close to king as possible
end if

if not areAllExitCastlesBarricaded(board) then
   move king as close to an unbarricaded exit castle as possible
end if

if canCaptureHunn(player, board) then
   capture hunn
end if

select random move
```

5.2.10 Defending Player : Mobile King

This strategy focuses on escorting the defending *Hnefi* safely to a castle before the attacking player has built a strong defense. This strategy plays heavily into a strong king rule set, and was a relatively common strategy, mentioned by both p1, p2, p5, p6 and p7. This strategy playing into the strength of the *Hnefi* and utilizes this strength to break the the attacking players defenses. This strategy focuses on quickly utilizing the *Hnefi* and is limited by the number of moves required to get the *Hnefi* in play. In order to handle this limitation as best as possible, we attempted to secure an exit for the *Hnefi* as soon as possible, after which we quickly move and fight with it as seen in algortihm 10.

Algorithm 10 Algorithm describing a simplified Mobile King strategy that prioritezes moving and fighting with the $Hnefi$ instead of utilizing *hunn* pieces.

```
SET board = board_state
SET opponent = attacker
SET player = defender

if not hasKingMovedFromThrone() then
  if king can move at least 3 spaces then
    move king as far as possible
  else
    clear way for king to exit throne
  end if
end if

if canKingMoveToExitCastle(board) then
  move king to exit castle
end if

if canCaptureKing(opponent, board) then
  if canBlockKingCapture(player, board) then
    block capture of king
  end if
end if

if not areAllExitCastlesBarricaded(board) then
  move king as close to an unbarricaded exit castle as possible
end if

if canCaptureHunn(player, board) then
  capture hunn (prioritize capture using Hnefi)
end if

select random move
```

6 Evaluation

With a number of potential strategies developed, a way to better investigate each of the strategies is required. For this, we chose to evaluate each of the discovered strategies through simulated game play in accordance with the rules as specified in Section 5.1. For this, we created a simple simulator in Java using Swing, where we implemented each strategy [1]. Each defensive strategy was tested against each attacking strategy. For each game, an artificial time limit was put in place, limiting games to one thousand rounds. If a game was not completed before that, the game will be marked as unfinished. Additionally, a "random" strategy for both the attacking and defending player was implemented as a baseline. This strategy would select a random piece that it can move and select a random valid move that piece could make. As randomness could cause varying results, games will therefore be simulated 10 times each to extract more reliable information. To most accurately record important information, wins, losses, ties as well as number of unfinished games are kept track of along with the average number of rounds for each completed game (wins or losses). If a game does not finish within one thousand rounds, that game is seen as unfinished. Ties will only be noted when a fort is successfully built and maintained for multiple rounds. Ties and unfinished games will not be counted in the average round number.

6.1 Attacking Strategies

With the found strategies in Section 5, each attacking strategy was pitched against each defending strategy for comparison a summary of which we see in Table 3, 4, 5, 6 and 7.

Table 3 Table displaying the attacking *Aggressive* strategy when competing against defensive strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Defensive	0	10	0	0	34
Foot-in-the-Door	5	5	0	0	31
Fortification	1	0	9	0	42
King’s Guard	0	10	0	0	17
Mobile King	9	1	0	0	14
Random	9	1	0	0	47

The *Aggressive* strategy performs well against the $Mobile\ King$ strategy, however, it struggles significantly against all other strategies aside from *Random* $(Defending)$. Additionally, the *Aggressive* and $Foot-in-the-Door$ strategies seem to be evenly matched, as they both retain an equal win rate when competing.

[1]For full implementation see `https://github.com/Hnefa/Hnefatafl`

Table 4 Table displaying the attacking *Barricading – Aggressive* strategy when competing against defensive strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Defensive	2	6	0	2	74
Foot-in-the-Door	0	10	0	0	17
Fortification	0	0	10	0	-
King's Guard	0	10	0	0	17
Mobile King	0	10	0	0	15
Random	10	0	0	0	128.4

Table 5 Table displaying the attacking *Barricading – Cordon* strategy when competing against defensive strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Defensive	2	4	0	4	112
Foot-in-the-Door	0	10	0	0	17
Fortification	0	0	10	0	-
King's Guard	0	10	0	0	17
Mobile King	0	10	0	0	14
Random	10	0	0	0	364.4

If we instead consider the *Barricading – Aggressive* and *Barricading – Cordon* strategies, we see that these strategies perform extremely poor compared to other developed strategies. It is worth noting though, that *Barricading* strategies are much more complex than all other attacking and defending strategies, so without extensive research, these strategies are difficult to implement effectively, which could have caused the *Barricading – Aggressive* and *Barricading – Cordon* strategies to underperform. Additionally, these strategies are finishing games incredibly fast most of the time even compared to other attacking strategies. This however, is due to how quickly they lose against current implementations of defensive strategies. This is particularly seen when competing against the *Random* (*Defensive*) strategy, where they take an astounding 128-364 turns to complete on average which is far higher than all other attacking strategies aside from *Random* (*Attacking*).

Table 6 Table displaying the attacking *King Centered* strategy when competing against defensive strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Defensive	7	3	0	0	48.4
Foot-in-the-Door	2	7	1	0	34.6
Fortification	10	0	0	0	44.6
King's Guard	0	8	2	0	34
Mobile King	1	9	0	0	33.1
Random	10	0	0	0	81.1

The *King Centered* strategy is incredibly unique among attacking strategies as this strategy is able to reliably counter the *Fortification* strategy, which no other attacking strategy is capable of. This however, causes the *King Centered* strategy to seemingly outperform most other attacking strategies as it gains ten victories where all other strategies get one at most. This of course comes at a cost, since the *King Centered* strategy can't handle *Defensive* and *King's Guard* strategies as well as the *Aggressive* strategy.

Table 7 Table displaying the attacking *Random* (*Attacking*) strategy when competing against defensive strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Defensive	0	10	0	0	124.2
Foot-in-the-Door	0	10	0	0	20.8
Fortification	0	0	10	0	-
King's Guard	0	10	0	0	15.2
Mobile King	0	10	0	0	13.6
Random	0	8	0	2	522

A quick glance at the *Random* (*Attacking*) strategy shows just how inefficient it is at playing Hnefatafl. As the attacking player, simply moving pieces around the board is not enough to win, and even in cases where the opponent does not win immediately, the *Random* (*Attacking*) strategy is more likely to get an unfinished game compared to a win or even a tie.

6.2 Defensive Strategies

If we instead examine the found defensive strategies more closely and investigate how they compare to each attacking strategy as seen in Table 8, 9, 10, 11, 12 and 13.

Table 8 Table displaying the defending *Defensive* strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	10	0	0	6	34
Barricading - Aggressive	6	2	0	2	74
Barricading - Cordon	4	2	0	2	112
King Centered	3	7	0	0	48.4
Random	10	0	0	0	124.2

When first examining the *Defenisve* strategy, we see that while it does compete well against the *Aggressive* and *Random* strategies, it has trouble when facing all other attacking strategies. This leads to this strategy performing relatively poor especially compared to other defensive strategies.

Table 9 Table displaying the defending $Foot-in-the-Door$ strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	5	5	0	0	31
Barricading - Aggressive	10	0	0	0	17
Barricading - Cordon	10	0	0	0	17
King Centered	7	2	1	0	34.6
Random	10	0	0	0	20.8

The $Foot-in-the-Door$ strategy on the other hand show great promise as this strategy handles most attacking strategies well with the exception of the $Aggressive$ and $King\ Centered$ strategies it can struggle against. While the $Foot-in-the-Door$ strategy shows great promise, it is not the best fairing defensive strategy, but it remains a close second.

Table 10 Table displaying the defending $Fortification$ strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	0	1	9	0	42
Barricading - Aggressive	0	0	10	0	-
Barricading - Cordon	0	0	10	0	-
King Centered	0	10	0	0	44.6
Random	0	0	10	0	-

A rather unique defensive approach is that of the $Fortification$ strategy. As seen in Table 10, this strategy does not win a single game, not even against the $Random(Attacking)$ strategy. What is noteworthy however, is that of its match against the $King\ Centered$ strategy, as the $Fortification$ strategy consistently loses here. This show's that while a fort can reliably be built, there is still counter-play to it. It is worth noting however, that this strategy is severely limited by the implementation, as many forts are possible but only four are attempted.

Table 11 Table displaying the defending $King's\ Guard$ strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	10	0	0	0	17
Barricading - Aggressive	10	0	0	0	17
Barricading - Cordon	10	0	0	0	17
King Centered	8	0	2	0	34
Random	10	0	0	0	15.2

Continuing on to the $King's\ Guard$ strategy, we see that this strategy in particular fairs up exceptionally well against all developed attacking strategies. The $King's\ Guard$ strategy is able to consistently win each game, only encountering mild trouble against the *King Centered* strategy. While this of course is impressive, it is important to note that the implemented strategies are quite simple so complex strategies may underperform.

Table 12 Table displaying the defending *Mobile King* strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	1	9	0	0	14
Barricading - Aggressive	10	0	0	0	15
Barricading - Cordon	10	0	0	0	14
King Centered	9	1	0	0	33.1
Random	10	0	0	0	13.6

Another rather successful defensive strategy is that of the *Mobile King*, as this strategy fairs up well against all attacking strategies but the *Aggressive* one. This loss against the *Aggressive* strategy is hugely detrimental to the *Mobile King* strategy's performance, as an increased performance against the *Aggressive* strategy could prove the *Mobile King* strategy as the best defensive strategy. This is especially relevant as each game was finished in exactly 14 turns, meaning the loss is caused by the current implementation of the strategy. A slight improvement to this implementation could prove highly beneficial for creating a defensive player, especially since this strategy is also highly successful at finishing games faster than any other defensive strategy.

Table 13 Table displaying the defending *Random* (*Defensive*) strategy when competing against attacking strategies

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	1	9	0	0	47
Barricading - Aggressive	0	10	0	0	128.4
Barricading - Cordon	0	10	0	0	364.4
King Centered	0	10	0	0	81.1
Random	8	0	0	2	522

A lesser performing defensive strategy however, is that of the *Random* (*Defensive*) strategy, which highly struggles against all strategies but the *Random* (*Attacking*) one. These wins however, can be attributed to luck due to the nature of this strategy. It is worth noting though that the one victory against the attacking *Aggressive* strategy shows a significant weakness in the *Aggressive* strategy, as it failed capture a randomly moving player.

6.3 Summary and Analysis of Results

Examining the effectiveness of attacking and defending strategies alike, we see that upon competing against one another, the *King's Guard* strategy is standing out among the rest as superior as seen in Table 14. It is worth noting however, that the simple comparison done

Table 14 Table displaying summary of games simulated

	Wins	Losses	Ties	Unfinished	Average Turns (Completed Games)
Aggressive	24	27	9	0	30.8
Barricading - Aggressive	12	36	10	2	50.4
Barricading - Cordon	12	34	10	4	104.9
Defensive	33	11	0	6	78.5
King Centered	30	27	3	0	46
Foot-in-the-Door	42	7	1	0	24.1
Fortification	0	11	39	0	43.3
King's Guard	48	0	2	0	19.6
Mobile King	40	10	0	0	17.9
Random (attacker)	0	48	10	2	139.2
Random (defender)	9	39	0	2	228.6

between the implemented strategies is not sufficient to get a fully accurate understanding of useful strategies. As the game is asymmetrical, the attacking and defending player face far different challenges, so comparing the attacking players performance to the defending players performance can be very misleading. If we take the *Aggressive* strategy for example, its performance seems evenly spread out when examining the wins and losses. While this is correct on the surface, it does not tell the full story, as this does not describe the interactions between this strategy and the different defending strategies. The *Aggressive* strategy faces three main challenges; the *Defensive*, the *King's Guard* and the *Fortification* strategy. While the *Aggressive* strategy can beat other strategies with little problems, the current implementation always lose to the current implementation of the *Defensive* and *King's Guard* strategies. If we consider the last challenging *Fortification* strategy, we see that the *Aggressive* strategy once again fail, as it can't handle the shift of game objectives that the *Fortification* strategy forces upon it. The *Fortification* strategy in particular is problematic for all the attacking strategies (aside from the *King Centered* strategy), as the *Fortification* strategy completely shifts the game's objectives. Each defending strategy on the other hand will never have to face this kind of challenge, which leads to the defending strategies seemingly over performing. It is worth noting though, that the defending strategies seems to benefit from a short game, as indicated by the *King's Guard* and *Mobile King* strategies low average rounds played, where as the better performing attacking strategies such as *Aggressive* and *King Centered* strategies have trouble finishing games quickly. Examining the random strategy matchup, it seems that the attacker requires partial strategy to win, as when moving randomly, the *Random* (*attacker*) strategy failed to win a single game, whereas the *Random* (*defender*) strategy could win when playing against a randomly moving opponent. From this alone we can tell that the defending player can afford to play much less strategically, as it is possible to win without any strategy. The attacker on the other hand must be much more careful with their moves, as winning requires some sort of strategical thinking.

7 Discussion

From our simulated games, we see that attacking strategies that play very aggressively such as the *Aggressive* and *King Centered* strategies perform best when competing against all developed defensive strategies, where as strategies utilizing the *Hnefi* piece such as *Foot − in − the − Door* and *King's Guard* seem to yield the biggest success for the defending player. This indicates that aggressive play may be highly effective against simple defensive strategies and that defensive strategies should put heavy focus on the *Hnefi* piece. It is worth noting though, that the used implementations are relatively simple and easy to break, so strategies that require a complex setup such as *Barricading − Cordon* might not perform as well unless significantly improved. This however, leads us to question the overall effectiveness of each type of strategy as well as what strengths they utilize to achieve victory.

7.1 Classification of strategies

When a strategy is thought up and developed, thought is usually put into a specific strength a player can utilize to achieve their goal. In Hnefatafl, this is typically seen in the main focus of each strategy. If we take the *Mobile King* and *King Centered* strategies for example, these both focus on the fact that the *Hnefi* piece is the strongest, and focus their play style around this. The *Mobile King* strategy for example, utilizes the fact that the *Hnefi* is difficult to capture and can threaten the attacking player in return. The *King Centered* strategy on the other hand takes a different, yet similar approach to this. As this strategy can only be played by the attacking player, the strategy cannot utilize the *Hnefi*, but must instead build their play style around preventing their opponent from using the *Hnefi* to its fullest potential. These two strategies both identify the *Hnefi* as the strongest piece. A similar yet different strategy is that of the *King's Guard*. While this strategy also centers around the *Hnefi*, it is distinctly different as it does not utilize the *Hnefi*'s strength and instead puts the main focus on defending the *Hnefi* with the help of *hunn*. A completely different approach to the game would be that of the *Defensive* and *Agrressive* strategies. Instead of focusing on the *Hnefi*, these strategies are centered around the *hunn*, and aim to achieve victory though strength in numbers and crippling the opponent. These strategies both have heavy focus on the number of pieces controlled by you and your opponent, and give little attribute to any one piece in specific, but instead value numbers instead. Another approach to the game is that of the *Foot-in-the-Door* and *Barricading* strategies, as these strategies does not put any focus on pieces you or your opponent control, but instead attempt to control certain key locations on the board. This strategy does not put any distinct focus on the *Hnefi* or *hunn*, and disregards your opponents movement unless it is directly threatening victory for the opponent. This strategy aims to first control the board and then achieve your main goal once you are in a good position to do so. Lastly, there is the *Fortification* strategy. This strategy heavily focuses on disruption, and instead of aiming to win the game, this strategy simply intents to disrupt the game play. This approach to the game is distinctly different from all other discussed strategies, as this strategy turns the game on its head and changes the entire game. A summary of the different classifications of strategies can be seen on Table 15.

Table 15 Table displaying strategies and their classification

Classification	Strategie(s)
Strong Hnefi	King Centered & Mobile King
Weak Hnefi	King's Guard
Hunn Focused	Aggressive & Defensive
Board Focused	Barricading - Aggressive & Barricading - Cordon & Foot-in-the-Door
Disruption	Fortification

7.2 Effectiveness of Strategies

As each classification of strategy focus on different things, they also provide each player with different levels of usefulness. Evaluating each type of strategy requires a thorough evaluation of each classification and the strength the attempt to utilize.

7.2.1 Competitive Play

When examining the effectiveness of strategies, a good place to start is that of professional competitive play, as players here are familiar with the game and the currently best known strategies. In professional play only four of the found strategies see any use; $Barricading - Aggressive$, $Barricading - Cordon$, *Foot-in-the-Door* and *Mobile King*. This leaves one class of variable strategy for the attacking player and two variable classes of strategies for the defending player. It is worth noting though, that the strategies used in competitive play are not the exact same as the ones we propose. For example, in competitive play, the *Foot-in-the-Door* strategy is used to open up possible exits for the $Hnefi$ before the $Hnefi$ is mobilized. Once the $Hnefi$ is mobilized, you typically see more of a *Mobile King* centered strategy, where the $Hnefi$ working with the *hunn* attempts to conquer the corners of the board. This mobilization can be especially powerful if key locations have been conquered earlier as a result of the *Foot-in-the-Door* strategy's approach to creating an opening using the magic quadrant. This strategy is incredibly powerful for the defending player as the $Hnefi$ (contrary to the king in chess), is the most powerful piece on the board. Since the $Hnefi$ can only be captured when completely surrounded by enemy pieces, this allows the $Hnefi$ to threaten enemy *hunn* without any risk of being captured in return. Additionally, as the $Hnefi$ is the key piece to winning the game as the defending player, this can put immense pressure on the attacking player, as they are forced to prevent the king from reaching an exit castle. This pressure the $Hnefi$ can put on certain board positions, can force your opponent to take focus away from other positions on the board, leaving openings for the $Hnefi$ to walk across the board, suddenly threatening completely different positions, making the attackers past defensive moves null and void. The attacking player on the other hand, only has one class of strategies variable; $Barricading - Aggressive$ and $Barricading - Cordon$. These strategies are particularly powerful as a successful barricade can essentially guarantee that your opponent can never win unless you yourself break the barricade. The reason these strategies see so much play is due to the starting setup of the game, combined with the attackers superior numbers. While the magic quadrant is kind of seen as an area not controlled by any player, the attacking player has the benefit of being able to reach the key edge locations in a single move, where as the defending player must always use two. This gives the attacker an incredibly powerful position, as they are able to control key positions by actively responding to the defending player's move, and block them. This is especially useful as the starting position (see Figure 7), allows the attacking player can move pieces to create any barricade seen in

Figure 1 in either two, three or four moves.

7.2.2 Strong Hnefi Strategies

Strong Hnefi strategies have incredible potential, yet can also prove disastrous for advanced players. While these strategies put heavy focus on the strongest piece on the board, they are not always useful. This is due to the inherit asymmetrical game design of Hnefatafl. *Strong Hnefi* strategies aim to either utilize the *Hnefi* or prevent the *Hnefi* from being utilized. The issue here lies with the main goal for each player, as the defending player utilizing their *Hnefi* to the fullest potential can give massive advantages, the same can not be said for the attacking player. While an attacking player that utilizes this class of strategy has correctly recognized their main threat, this class of strategy plays into your opponents strength instead of your own. In a theoretical perfect situation where the attacker manages to block the *Hnefi* at every turn, the defending player may still move around and avoid capture. This means that the attacker rarely gets closer to completing their main objective, especially when four moves are required to capture the *Hnefi*, compared to the one move the *Hnefi* has to break completely free. This essentially means that the attacking player has to limit movement for the *Hnefi* enough, to the point where the *Hnefi* is limited in their movement for four or more turns, which is very unlikely in advanced games.

7.2.3 Weak Hnefi Strategies

The *Weak Hnefi* class of strategies can be seen as a misunderstanding of the *Hnefi*'s role in the game. This class of strategies recognize the importance of the *Hnefi*, but fail by not recognizing the strength it has. These strategies much like in chess focus on protecting the king/*Hnefi* from harm. The thought behind this strategy falls in line with that of battles or chess, where the soldiers or pawns has to protect the leader/king from harm. Hnefatafl, however, operates of a different approach, as the *Hnefi* behaves more like the queen in chess instead of the king, thus the focus on protecting the *Hnefi* from harm instead of utilizing its combat potential, leaves this class of strategies with limited effectiveness when played against advanced players. This strategy however, is relatively common among beginners.

7.2.4 Hunn Focused Strategies

The effectiveness of *Hunn Focused* strategies are often quite limited, as these strategies are not playing into their respective side's biggest strength, and instead focus on the *hunn* which has little to do with the main objective of the game. These strategies are commonly seen utilized by beginner players who approaches Hnefatafl with a chess-like mindset. This way of thinking is useful in chess, however, Hnefatafl's main objectives stand in strong contrast to these strategies. Since the *Hnefi* is what determines a win or loss for either player, putting your main focus on the other pieces, is a common mistake seen in beginner players.

7.2.5 Board Focused Strategies

Board Focused strategies are incredibly interesting, as these strategies play into factors outside either players control. This means that their effectiveness is purely a reflection of the game's design, with limited influence from your opponent. It is noteworthy that all found *Board Focused* strategies see some use in competitive play, not only by attacking player, but also the defending. This indicates that this approach to the game is likely to be strategically useful when playing the game on an advanced level. It is worth noting though, that the attacking player has the main advantage in *Board Focused* class strategies as mentioned in Section

7.2.1, however, the defending players ability to threaten two edge positions at once using the magic quadrant allows them to utilize key locations on the board. The defending player's approach to *Board Focused* strategies, should therefore be to poke holes in the attacking players barricades, leaving openings for the *Hnefi*, meaning the defending player can only partly use this strategy, committing to this strategy plays into the strengths of the attacking player, and could therefore prove disadvantageous in the long run.

7.2.6 Disruption Strategies

Lastly, *Disruption* strategies have no practical usefulness when it comes to winning the game, however, can be extremely effective when attempting to tie the game. This is because this strategy is particularly difficult to prevent, due to the sheer number of forts possible. Additionally, this class of strategies are especially difficult to counter, as you have limited time to prevent it, since once a fort is setup, the game is decided. A player will typically expect their opponent to play to win, so this class of strategy will catch many players by surprise. Since this type of strategy shifts the goal of the game, there is limited counter-play. If a player recognizes a *Disruption* type strategy, they are presented with the choice to either play the best game for themselves (which is almost guaranteed to end in a tie), or to shift their goal to allocating all their resources to preventing their opponent from creating a fort. While preventing a fort may seem like a variable option, this often comes at the cost of your defenses, potentially leaving gaps for your opponent to exploit. Because of this, *Disruption* type strategies could theoretically be highly effective if your opponent attempts to respond, however, more often than not, the most strategically sound play for your opponent is to avoid engaging, thus almost guaranteeing a tie.

7.3 Strategic Analysis of Game Narrative

What exactly Hnefatafl represents may never be known. Several educated guesses have been made as to what the game represents, all from an epic tale of a smart and heroic leader defeating a stronger enemy [10] to overthrowing a cowardly leader [8]. If we study Hnefatafl though its Swedish cousin Tablut [1], we see the two battling forces in this case being described as the swedes (the defending player) and the moscovites (the attacking player). With the many similarities Hnefatafl shares with the game of Tablut, this could indicate that Hnefatafl might also be a game representing an epic battle between a viking tribe and foreign aggressor. This falls in line with the commonly used terms such as the attacking and defending player. If we analyze these possible representations through the eyes of strategic analysis, we can see how our strategies fit into the different understandings of the game. *Strong Hnefi* strategies for example play well into the epic tale and invading tribe narrative, as these strategies put great importance on the *Hnefi*. *Hunn Focused* strategies on the other hand are more in line with Hnefatafl being a representation of two equally strong viking tribes in battle, as these strategies put little to no emphasis on on individual pieces but hold all pieces of equal value. This is in strong contradiction to *BoardFocused* strategies, as these strategies fall in line with Hnefatafl as a representation of a cowardly leader, as these strategies has heavy focus on controlling where the *Hnefi* can go on the battlefield. In this understanding of the game, the attacking player would typically be seen as the hero for capturing the cowardly leader, so it would make sense for this type of strategies to play to the attacking players strengths. Where the lines get a bit more blurry is for that of *Disruption* and *Weak Hnefi* strategies, as these do not fall neatly within any common interpretations of the game. This makes sense for the *Disruption* class of strategies, as these intentionally break the game, however, the

$Weak\ Hnefi$ is a bit of an outlier. While it can be argued that $Weak\ Hnefi$ strategies could represent the invading tribe narrative, the $Weak\ Hnefi$ play style does not put focus on defeating the enemy forces or moving to a secure location, but instead focus on keeping the $Hnefi$ safe. This extreme defense would be more in line with protecting a leader or outlasting a siege, however, this can not secure a victory according to the game rules. A summary of game narratives along with matching strategy types can be seen in Table 16.

If we work under the assumption that the current interpretation of the rules is mostly correct, we can through our current understanding of game strategies conclude that the narrative that fits the game best strategically is that of an epic battle, possibly against foreign aggressors. It is worth noting that this interpretation of the game heavily play into the defending player being the main character of the narrative, as this narrative puts the defending player in a significantly better light. If we instead work under the assumption that the equal battle narrative is correct, this does not line up with our strategic findings as these strategies are not particularly effective. If we instead focus on another incredibly effective class of strategies such as the *Board Focused* class, we see that this narrative that matches here, heavily favors the attacking player. We can therefore say that from a purely strategical perspective with the current understanding of the rule set, the currently known effective strategies could imply that Hnefatafl represents an epic battle against foreign aggressors or the capture of a cowardly and unfit leader, depending on which player is the main focus of the game.

Table 16 Table displaying theorized narratives of Hnefatafl and strategy classes that matches.

Narrative Interpretation	Strategy Classification(s)
Epic Battle	Strong Hnefi
Equal Battle	Hunn Focused
Foreign Agressors	Strong Hnefi
Cowardly Hnefi	Board Focused

8 Future Work

The ancient Scandinavian pastime of Hnefatafl though lost to time has still things to teach us today. Though our current understanding of Hnefatafl strategies is quite limited, understanding *Strong Hnefi*, *Weak Hnefi*, *Board Focused*, *Hunn Focused* and *Disruption* strategies and the elements that create them are a crucial step in figuring out what elements of the game are important for a Hnefatafl player. Our research identified the *Hnefi* as a key piece for the defending player, and thus any strategy the defending player employs should heavily center around utilizing the *Hnefi* to the maximum possible extent. This could indicate that the leader figure which the *Hnefi* depicts, is a representation of a powerful and strong warrior, that leads a friendly/antagonistic force of men. The attacking player on the other player on the other hand has their greatest strength in their board control, and should therefore employ strategies that allow them to control the board as effectively as possible, such as building barricades and utilizing cordons. This strategic yet aggressive play style could show the value the vikings put on combat strategy and the use of wits and numbers to outmaneuver and defeat an enemy much stronger than one self. This understanding of successful game strategies not only helps up develop better players for Hnefatafl, but may also help shed light on the narrative of the game itself. Through strategical analysis of the game, our research suggests that with the current understanding of Hnefatafl through the Fetlar rule set, effective game strategies indicate that Hnefatafl could tell the story of an epic battle against foreign aggressors or that of a cowardly and unfit leader being dethroned. These two narratives greatly helps our understanding of the game, and what it meant to the vikings of a bygone era. With this new examination of potential historical significance through the lens of strategy, we hope that along with the formalization of game rules (see Section 5.1) as well as nine distinct strategic approaches, that we inspire future researches to further develop our understanding of this long forgotten game. In future research we propose further investigating *Board Focused* and *Strong Hnefi* strategies, as these strategies show incredible potential that has yet to fully realized, and it may even be possible to fully solve Hnefatafl with the current rule set. Another area of research that remain unexplored is that of *Disruption* strategies, as these strategies may prove more problematic than they seem at first sight. One potential way of exploring these strategies could be a formal mapping of the formalisms into logic programming, such as answer set programming. This would allow to generate alternative solutions (sequences of moves) for a given strategy and game state, to further explore the dynamics of the game. Fully understanding the limitations and benefits of each strategy would prove incredibly interesting, and help us further understand this wonderful ancient game.

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